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Thermal Instability in a Horizontal Layer of Walter's (Model B') Visco-Elastic Nanofluid- A More Realistic Approach

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Abstract

Thermal instability in a horizontal layer of Walter's (Model B') visco-elastic nanofluid is investigated for more realistic boundary conditions. The flux of volume fraction of nanoparticles is taken to be zero on the isothermal boundaries. The model used for nanofluid incorporates the effect of Brownian diffusion and thermophoresis. Perturbation method, normal mode technique and Galerkin method are used in the solution of the eigenvalue problem. Oscillatory convection has been ruled out for the problem under consideration. The influences of the Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number on the stationary convection are shown both analytically and graphically.

Keywords: Nanofluid; Walter's (B') visco-elastic fluid; normal mode technique; oscillatory convection; Galerkin method; Rayleigh number; Lewis number

MSC 2010 No.: 76D05; 76E06; 76M55; 76N20

1. Introduction

Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes and hybrid powered engines, domestic refrigerator, chiller, heat exchanger and nuclear reactor, in grinding, in machining, in space, defense and ships and in boiler flue gas temperature reduction. Nanofluid is a fluid colloidal mixture of nano sized particles in base fluid. Nanoparticle materials may be taken as oxide ceramics (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. and base fluids are water, ethylene or tri-ethylene- glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common fluids. The term 'nanofluid' was coined by Choi (1995). Since Choi proposed his theory on nanofluids a continuous effort has ensued to look for the causes of the so-called anomalous increase in thermal conductivity of nanofluids. The presence of nanoparticles in the fluid significantly increases the effective thermal conductivity of the mixture. Xuan and Li (2003) investigated convective heat transfer and flow features of Cu-water nanofluid. They observed that the suspended nanoparticles remarkably enhance heat transfer process and the nanofluid has larger heat transfer coefficient than that of the original base liquid under the same Reynolds number. The characteristic feature of nanofluid is the thermal conductivity enhancement, a phenomena observed by Masuda et al. (1993). Keblinski et al. (2009), Wong and Leon (2010), Yu and Xie (2012), Philip and Shima (2012), Taylor et al. (2013) reported the developments and applications in the study of heat transfer using nanofluids.

Buongiorno (2006) studied almost all aspects of the convective transport in nanofluids. Tzou (2008a, b) studied the thermal instability problems of nanofluid using the method of eigen-function expansion and observed that nanofluids are less stable than regular fluid. Thermal convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov (2010) and found that the critical Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of nanoparticles. Thermal instability in a horizontal nanofluid layer in porous medium was investigated by Nield and Kuznetsov (2009) and then by Darcy model, while Kuznetsov and Nield (2010) studied the same by Brinkman model. Chand and Rana (2012a) investigated the oscillating convection in a Darcy porous medium and found that "Principle of Exchange of Stabilities" is not valid. Alloui et al. (2010) studied the natural convection of nanofluids in a shallow cavity heated from below. They observed that the presence of nanoparticles in a fluid is found to reduce the strength of flow field, this behavior being more pronounced at low Rayleigh number. Thermal instability of rotating nanofluid layer was studied by Yadav et al. (2011), Chand and Rana (2012b), Chand (2013a) and they observed that rotation has a significant effect on the onset of thermal instability. Magneto- convection in a horizontal layer of nanofluid was investigated by Chand (2013b) and Chand and Rana (2015a), while the effect of Hall current in a horizontal layer of nanofluid was investigated by Chand and Rana (2014a, b) and found that Hall effect destabilizes the fluid layer.

Recently Nield and Kuznetsov (2014), Chand et al. (2014), Chand and Rana (2014c, 2015a), studied the thermal instability of nanofluid by taking normal component of the nanoparticle flux zero at boundary which is more physically realistic. Zero-flux for nanoparticles means one could control the value of the nanoparticles fraction at the boundary in the same way as the temperature there could be controlled. Under the circumstances, it is desirable to investigate convective instability problems by utilizing these boundary conditions to get meaningful insight into the problems.

The above study deals with nanofluid as Newtonian nanofluid. There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology, petroleum, biological and material industries. The study of non-Newtonian nanofluid is desirable. There are many visco-elastic fluids which cannot be characterized by Maxwell's constitutive relations. One such class of visco-elastic fluid is Walters' (Model B') fluids. Walters (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5g of polymer per liter with density 0.98g per liter behaves very nearly as the Walters' (Model B') fluid. Walters' (Model B') visco- elastic fluid forms the basis for the manufacture of many important polymers and useful products. Sharma and Kumar (1997), Sunil et al. (2000) studied the thermosolutal instability of Walter's (Model B') liquids whereas Chand and Rana (2012c) studied the thermosolutal Rivlin-Ericksen elastico-viscous fluid in porous medium.

Non-Newtonian nanofluid Bénard- convection problems were studied by Nield (2010), Sheu (2011), Chand and Rana (2012d, 2015b) and Rana et al. (2014), Rana and Chand (2015a, b) by taking different non-Newtonian fluids. In this paper paper an attempt has been made to revise the Chand and Rana (2015b) findings by considering physically realistic boundary conditions. The interest for investigations of visco-elastic nanofluids is also motivated by a wide range of engineering applications.

2. Mathematical Formulations of the Problem

Consider an infinite horizontal layer of Walter's (Model B') elastico-viscous nanofluid of thickness 'd' bounded by planes $z = 0$ and $z = d$ and heated from below. Fluid layer is acted upon by gravity force $\boldsymbol{g}(0, 0, -g)$ as shown in Figure 1. The normal component of the nanoparticles flux has to vanish at an impermeable boundary and the temperature T is taken to be T_0 at $z = 0$ and T_1 at $z = d$ ($T_0 > T_1$). The reference scale for temperature and nanoparticles fraction is taken to be T_1 and φ_0 respectively.

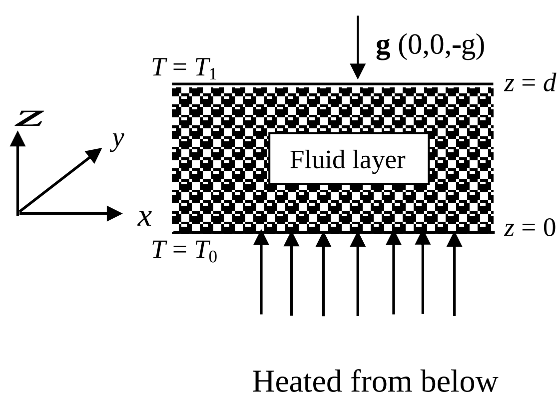

Figure 1: Physical configuration of the problem

2.1 Assumptions

The mathematical equations describing the physical model are based upon the following assumptions:

i) thermophysical properties of fluid, except for density in the buoyancy force (Boussinesq Hypothesis), are constant,
ii) the fluid phase and nanoparticles are in thermal equilibrium state,
iii) dilute mixture,
iv) nanoparticles are spherical,
v) no chemical reactions take place in fluid layer, and
vi) negligible viscous dissipation.

2.2 Governing Equations

The governing equations for Walter's (Model B') elastico-viscous nanofluid [Chandrasekhar (1981), Sharma and Kumar (1997), Chand and Rana (2015b)] are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\rho \frac{d\mathbf{q}}{dt} = -\nabla p + \left(\varphi\rho_p + (1-\varphi)\{\rho(1-\alpha(T-T_0))\}\right)\mathbf{g} + \left(\mu + \mu' \frac{\partial}{\partial t}\right)\nabla^2 \mathbf{q}\ , \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{q} \cdot \nabla)$ is stands for convection derivative, $\mathbf{q}(u, v, w)$ is the velocity vector, p is the hydrostatic pressure, μ is viscosity, μ' kinematic visco-elasticity, α is the coefficient of thermal expansion, φ is the volume fraction of the nanoparticles, ρ_p density of nanoparticles and ρ_f density of base fluid and $\mathbf{g}(0, 0, -g)$ is acceleration due to gravity.

The conservation of the nanoparticle mass [Nield and Kuznetsov (2010)] requires that

$$\frac{\partial \varphi}{\partial t} + \mathbf{q} \cdot \nabla \varphi = D_B \nabla^2 \varphi + \frac{D_T}{T_1} \nabla^2 T, \tag{3}$$

where D_B is the Brownian diffusion coefficient and D_T is the thermoporetic diffusion coefficient of the nanoparticles.

The energy equation in nanofluid [Nield and Kuznetsov (2010)] is given by

$$\rho c \left(\frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T \right) = k \nabla^2 T + (\rho c)_p \left(D_B \nabla \varphi \cdot \nabla T + \frac{D_T}{T_1} \nabla T \cdot \nabla T \right), \tag{4}$$

where ρc is heat capacity of fluid, $(\rho c)_p$ is heat capacity of nanoparticles and k is thermal conductivity.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. Thus, boundary conditions [Chandrasekhar (1981), Nield and Kuznetsov (2014)] are

$$w=0, \quad T=T_{0,} \quad D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z} \quad =0 \text{ at } \quad z=0 \text{ and}$$

$$w=0, \quad T=T_1 \quad D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \quad at \quad z=d. \tag{5}$$

Introducing non-dimensional variables as

$$(x',y',z')=\left(\frac{x,y,z}{d}\right), \mathbf{q}'(u',v',w')=\mathbf{q}\left(\frac{u,v,w}{\kappa}\right)d, t'=\frac{\kappa}{d^2}t,$$
$$p'=\frac{p}{\rho\kappa^2}d^2, \varphi'=\frac{(\varphi-\varphi_0)}{\varphi_0}, T'=\frac{(T-T_1)}{(T_0-T_1)},$$

where $\kappa=\dfrac{k}{\rho c}$ is thermal diffusivity of the fluid.

Equations (1)- (5) in non-dimensional form can be written as

$$\nabla\cdot\mathbf{q}=0, \tag{6}$$

$$\frac{1}{Pr}\frac{\partial\mathbf{q}}{\partial t}=-\nabla p+(1-nF)\nabla^2\mathbf{q}\text{-}Rm\hat{\mathbf{e}}_{\mathbf{z}}+RaT\hat{\mathbf{e}}_{\mathbf{z}}-Rn\varphi\hat{\mathbf{e}}_{\mathbf{z}}, \tag{7}$$

$$\frac{\partial\varphi}{\partial t}+\mathbf{q}\cdot\nabla\varphi=\frac{1}{Le}\nabla^2\varphi+\frac{N_A}{Le}\nabla^2T, \tag{8}$$

$$\frac{\partial T}{\partial t}+\mathbf{q}\cdot\nabla T=\nabla^2T+\frac{N_B}{Le}\nabla\varphi\cdot\nabla T+\frac{N_AN_B}{Le}\nabla T\cdot\nabla T \quad . \tag{9}$$

[The dashes (') have been dropped for simplicity]. Here, non-dimensional parameters are given as

$$Pr=\frac{\mu}{\rho\kappa}$$

is the Prandtl number,

$$Le=\frac{\kappa}{D_B}$$

is the Lewis number,

$$Ra=\frac{\rho g\alpha d^3(T_0-T_1)}{\mu\kappa}$$

is the thermal Rayleigh number,

$$Rm=\frac{\left(\rho_p\varphi_0+\rho\left(1-\varphi_0\right)\right)gd^3}{\mu\kappa}$$

is the basic-density Rayleigh number,

$$Rn = \frac{(\rho_p - \rho)(\varphi_1\text{-}\varphi_0)gd^3}{\mu\kappa}$$

is nanoparticle Rayleigh number,

$$N_A = \frac{D_T(T_0\text{-}T_1)}{D_B T_1 \varphi_0}$$

is the modified diffusivity ratio,

$$N_B = \frac{(\rho c)_p \varphi_0}{(\rho c)_f}$$

is the modified particle-density increment,

$$F = \frac{\mu'}{\rho d^2}$$

is the kinematic visco-elasticity parameter.

The dimensionless boundary conditions are

$$w=0,\quad T=1,\quad \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \;\; at \quad z=0 \text{ and } w=0,\quad T=0,\quad \frac{\partial\phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \;\; at \quad z=1. \tag{10}$$

2.3. Basic Solutions

The basic state of the nanofluid is assumed to be time independent and is described by

$$\mathbf{q}'(\mathbf{u},\mathbf{v},\mathbf{w})=0,\ p'=p'(z),\ T'=T_b(z),\quad \varphi'=\varphi_b(z).$$

Equations (6) – (9) using boundary conditions (10) give solution as

$$T_b = 1 - z\,,\ \varphi_b = \varphi_0 + N_A z\,, \tag{11}$$

where φ_0 is reference value for nanoparticles volume fraction. These basic solutions are identical with solutions obtained by Nield and Kuznetsov (2014).

2.4. Perturbation Solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are written in following forms

$$\mathbf{q}' = 0+\mathbf{q}'', T'=T_b+T'', \varphi'=\varphi_b + \varphi'', p'=p_b+p'' \text{ with } T_b=1\text{-z},\ \varphi_b = z+N_A z. \tag{12}$$

[Hereafter, the dashes (") are suppressed for convenience].

Using Equation (12) in Equations (6) – (9) and linearizing the resulting equations by neglecting the product of the prime quantities, we obtain the following equations

$$\nabla \cdot \mathbf{q} = 0, \tag{13}$$

$$\frac{1}{Pr}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + (1 - nF)\nabla^2 \mathbf{q} + RaT\hat{\mathbf{e}}_{\mathbf{z}} - Rn\varphi\hat{\mathbf{e}}_{\mathbf{z}}, \tag{14}$$

$$\frac{\partial \varphi}{\partial t} + wN_A = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T, \tag{15}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Le}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2N_A N_B}{Le}\frac{\partial T}{\partial z}. \tag{16}$$

The boundary conditions are given by

$$w = 0, \ \ \frac{\partial^2 w}{\partial z^2} = 0, \ \ T = 0, \ \ \frac{\partial \varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \ \ at \quad z = 0,1. \tag{17}$$

It will be noted that the parameter *Rm* is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient and not appear in the subsequent equations [Nield and Kuznetsov(2010)].

The six unknown's u, v, w, p, T and φ can be reduced to three unknowns by operating Equation (14) with $\mathrm{e}_z.\mathrm{curl\,curl},$ we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 w - (1 - nF)\nabla^4 w = Ra\nabla_H^2 T - Rn\nabla_H^2\varphi, \tag{18}$$

where

$$\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the two-dimensional Laplacian operator on the horizontal plane.

3. Normal Modes Analysis

We analyze the disturbances into the normal modes and assume that the perturbed quantities are of the form

$$[w, \theta, \varphi] = [W(z), \Theta(z), \Phi(z)]\,exp(ik_x x + ik_y y + nt), \tag{19}$$

where k_x and k_y are wave numbers in x and y directions respectively, while n (complex constant) is the growth rate of disturbances.

Using Equation (19), Equations (18), (15) and (16) become

$$\left(D^2 - a^2\right)\left(\left(1 - nF\right)\left(D^2 - a^2\right) - \frac{n}{Pr}\right)W - a^2 Ra\Theta + a^2 Rn\Phi = 0, \tag{20}$$

$$WN_A - \frac{N_A}{Le}\left(D^2 - a^2\right)\Theta - \left(\frac{1}{Le}\left(D^2 - a^2\right) - n\right)\Phi = 0, \tag{21}$$

$$W + \left(D^2 - a^2 - n + \frac{N_A}{Le}D - \frac{2N_A N_B}{Le}D\right)\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{22}$$

where

$$D \equiv \frac{d}{dz}$$

and

$$a = \sqrt{k_x^2 + k_y^2}$$

is the dimensionless resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are written as

$$W = 0,\ D^2W = 0,\ \Theta = 0,\ D\Phi + N_A D\Theta = 0 \quad \text{at } z = 0, 1. \tag{23}$$

4. Method of Solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the system of Equations (20) – (22) with the corresponding boundary conditions (23). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{N} A_p W_p,\ \Theta = \sum_{p=1}^{N} B_p \Theta_p, \Phi = \sum_{p=1}^{N} C_p \Phi_p\ , \tag{24}$$

where A_p, B_p and C_p are unknown coefficients, p = 1, 2, 3, ..., N and the base functions W_p, Θ_p and Φ_p are assumed in the following form

$$W_p = \Theta_p = z^p - z^{p+1}, \Phi_1 = N_A\left(z^2 - z\right) \text{ and } \Phi_p = \frac{1}{2} N_A z^2,\ p = 2, 3, 4, ..., N. \tag{25}$$

such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and Φ in Equations (20) – (22) and multiplying first equation by W_p second equation by Θ_p and third by Φ_p and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p = 1, 2, 3, …, N. For the existence of a non-trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number *Ra*.

5. Linear Stability Analysis

We confine ourselves to the one- term Galerkin approximation. The eigenvalue equation of the problem is given as

$$Ra=\frac{\left(a^2+10\right)\left(1+nF-\frac{n}{Pr}\right)\left(a^2+10+n\right)}{a^2}-\frac{\left(a^2+10\right)\left(\frac{N_A}{Le}+1\right)+n}{\frac{\left(a^2+10\right)}{Le}+n}Rn. \tag{26}$$

For neutral stability, the real part of n is zero. Hence, on putting $n = i\omega$, (where ω is real and dimensionless frequency of oscillation) in Equation (26), we get

$$Ra=\Delta_1+i\omega\Delta_2\,, \tag{27}$$

where

$$\Delta_1=\frac{\left(a^2+10\right)}{a^2}\left(\left(a^2+10\right)^2-\left(\frac{1}{Pr}-\left(a^2+10\right)F\right)\omega^2\right)-\frac{\left(a^2+10\right)^2\left(\left(\frac{N_A}{Le}+1\right)+\omega^2\right)}{\left(\frac{a^2+10}{Le}\right)^2+\omega^2}Rn \tag{28}$$

and

$$\Delta_2=\frac{\left(a^2+10\right)}{a^2}\left(\left(\frac{1}{Pr}-\left(a^2+10\right)F\right)\left(a^2+10\right)+\left(a^2+10\right)\right)-\frac{\left(\frac{a^2+10}{Le}\right)-\left(a^2+10\right)\left(\frac{N_A}{Le}+1\right)}{\left(\frac{a^2+10}{Le}\right)^2+\omega^2}Rn. \tag{29}$$

Since *Ra* is a physical quantity, it must be real. Hence, it follows from Equation (26) that either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$ over-stability or oscillatory onset).

5.1. Stationary Convection

When the stability sets as stationary convection, the marginal state will be characterized by $n = 0$ ($\omega = 0$), and then Equation (26) gives the stationary Rayleigh number as

$$\left(Ra\right)_s=\frac{\left(a^2+10\right)^2}{a^2}-Rn\left(Le+N_A\right). \tag{30}$$

We find that for the stationary convection, the kinematic visco-elasticity parameter F vanishes with n and hence, elastico-viscous fluid behaves like an ordinary Newtonian fluid.

It is also clear from Equation (30) that stationary Rayleigh number, Ra, depends upon dimensionless wave number 'a', Lewis number, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn, but independent of modified particle-density increment $N_{B,}$ Prandtl number Pr and density Rayleigh number Rm.

The interweaving behaviors of Brownian motion and thermophoresis of nanoparticles evidently do not change the critical size of the Bénard cell at the onset of instability. As such, the critical size is not a function of any thermophysical properties of nanofluid.

The minimum value of the first term of RHS of Equation (30) is attained at wave number $a \approx \sqrt{10}$, so that the critical Rayleigh number for stationary number is given by

$$(Ra)_c = 40 - (N_A + Le)Rn. \tag{31}$$

This is the same result which was obtained by Nield and Kuznetsov (2014).

In the absence of nanoparticles ($Rn = Le = N_A = 0$) i.e. for ordinary fluid, the Rayleigh number Ra for steady onset is given by

$$(Ra)_c = 40.$$

This is the same result which was obtained by Nield and Kuznetsov (2014) and is approximately equal to well-known result of Chandrasekhar (1981) for regular fluid that the critical Rayleigh number is

$$Ra_c = 4\pi^2.$$

In order to investigate the effects of Lewis number Le, modified diffusivity ratio N_A and nanoparticles Rayleigh number Rn on the stationary convection, we examine the behavior of

$$\frac{\partial Ra}{\partial Le}, \frac{\partial \mathrm{R}a}{\partial N_A} \text{ and } \frac{\partial \mathrm{R}a}{\partial Rn}$$

analytically. From Equation (30), we have

$$\frac{\partial Ra}{\partial Le} < 0, \quad \frac{\partial Ra}{\partial N_A} < 0, \text{ and } \frac{\partial Ra}{\partial Rn} < 0.$$

It implies that for stationary convection Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effect on the fluid layer. These results are equivalent to the results obtained by Nield and Kuznetsov (2014), Chand and Rana (2014b, 2015a).

5.2. Oscillatory Convection

Here, we considered the possibility of oscillatory convection. For oscillatory convection ($\omega \neq 0$), we must have $\Delta_2 = 0$, which gives

$$\omega^2 = \left(\frac{a^2}{a^2+10}\right)\left(\frac{\frac{1}{Le} - \left(\frac{N_A}{Le}+1\right)}{1-\left(a^2+10\right)F + \frac{1}{Pr}}\right)Rn - \frac{\left(a^2+10\right)^2}{Le^2}. \tag{32}$$

Equation (32) gives the frequency of oscillatory mode. For the value of parameters considered in the range of $10^2 \leq Ra \leq 10^5$ (thermal Rayleigh number), $Rn > 0$ (nanoparticles Rayleigh number), $10^2 \leq Le \leq 10^6$ (Lewis number) [Nield and Kuznetsov (2014)], we get the negative value of ω^2. Thus, oscillatory convection is not possible. Oscillatory convection is ruled out because of the absence of the two opposing buoyancy forces.

6. Results and Discussion

Thermal instability in a horizontal layer of Walters (Model B') visco-elastic nanofluid for more physically realistic boundary conditions is investigated. The numerical computations are carried out for different values of Lewis number *Le*, modified diffusivity ratio N_A and nanoparticles Rayleigh number *Rn*. The parameters considered are in the range of $10^2 \leq Ra \leq 10^5$ (thermal Rayleigh number), $Rn > 0$ (nanoparticles Rayleigh number), $10^2 \leq Le \leq 10^5$ (Lewis number).The convection curves for Lewis number *Le*, modified diffusivity ratio N_A and concentration nanoparticles *Rn* in the (*Ra*, *a*) plane are shown in Figures 2-4.

Figure 2 represents the variation of stationary Rayleigh number with wave number for different value of Lewis number *Le* and it is found that stationary Rayleigh number decreases with an increase in the value of Lewis number; thus, Lewis number has destabilizing effect on the stationary convection. This is in agreement with the result obtained by Chand and Rana (2014b, 2015a).

Figure 3 represents the variation of stationary Rayleigh number with wave number for different value of concentration Rayleigh number *Rn* and it is found that stationary Rayleigh number decreases with an increase in the value of concentration Rayleigh number *Rn*, which imply that concentration Rayleigh number destabilizes the stationary convection. This is in agreement with the result obtained by Chand and Rana (2014b, 2015a).

Figure 4 represents the variation of stationary Rayleigh number with wave number for the different values of modified diffusivity ratio N_A and it is noted that stationary Rayleigh number decreases with an increase in the value modified diffusivity ratio N_A. Thus, the modified diffusivity ratio N_A has a destabilizing effect on the stationary convection. This is in agreement with the result obtained by Chand and Rana (2014b, 2015a).

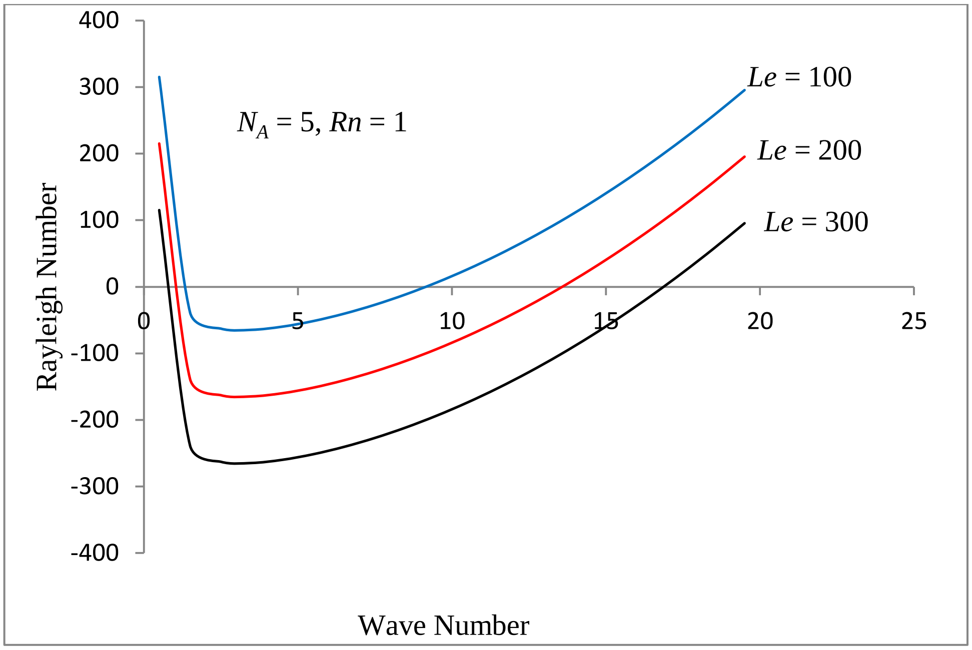

Figure 2. Variation of stationary Rayleigh number *Ra* with wave number *a* for different values of Lewis number *Le*

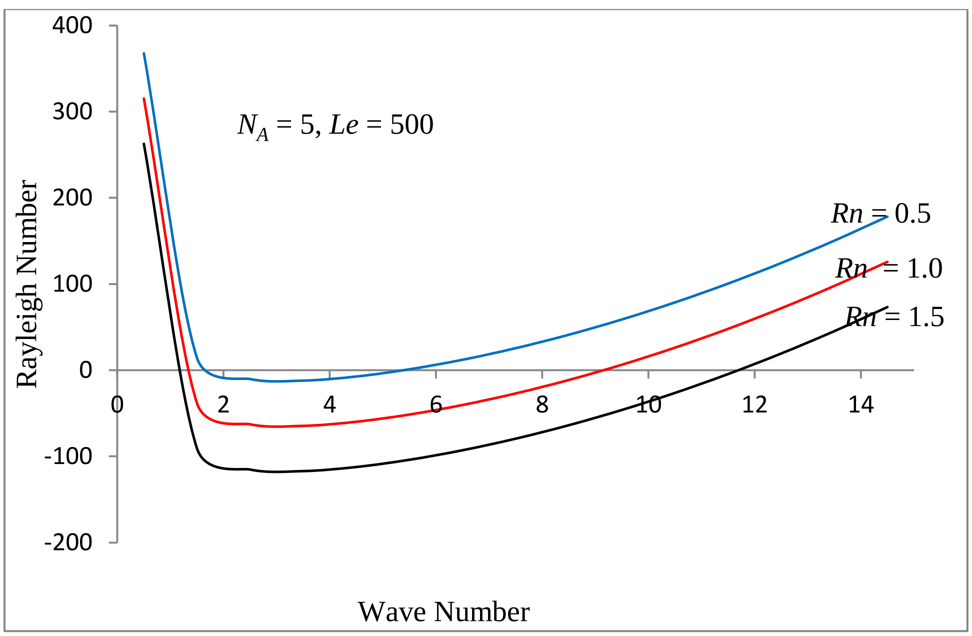

Figure 3. Variation of stationary Rayleigh number *Ra* with wave number *a* for different values concentration Rayleigh number *Rn*

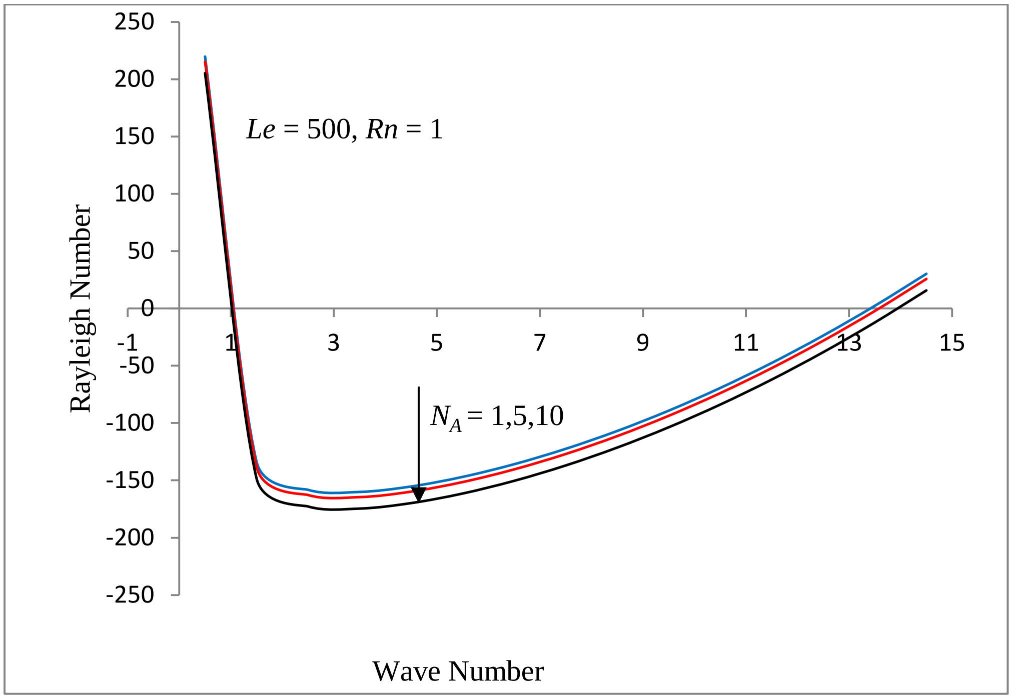

Figure 4. Variation of stationary Rayleigh number Ra with wave number a for different values of modified diffusivity ratio N_A

7. Conclusions

Thermal instability in a horizontal layer of Walter's (Model B') elastico-viscous nanofluid layer is investigated for more realistic boundary conditions. The effect of various parameters such as Lewis number, modified diffusivity ratio and concentration Rayleigh number has been investigated analytically and graphically. Main conclusions from the analysis of this paper are as follows:

(i) The critical cell size is not a function of any thermophysical properties of nanofluid.

(ii) Instability is purely phenomenon due to buoyancy coupled with the conservation of nanoparticles.

(iii) For the stationary convection, the visco-elastic fluid behaves like an ordinary Newtonian fluid.

(iv) Oscillatory convection is ruled out because of the absence of the two opposing buoyancy forces.

(v) Lewis number, modified diffusivity ratio and concentration Rayleigh number have destabilizing effect on the stationary convection.

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