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Investigation of Nonlinear Problems of Heat Conduction in Tapered Cooling Fins Via Symbolic Programming

Hooman Fatoorehchi and Hossein Abolghasemi
Center for Separation Processes Modeling and Nano-Computations
Oil and Gas Center of Excellence
School of Chemical Engineering
University of Tehran
Tehran, Iran
abolghasemi.ha@gmail.com

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Abstract

In this paper, symbolic programming is employed to handle a mathematical model representing conduction in heat dissipating fins with triangular profiles. As the first part of the analysis, the Modified Adomian Decomposition Method (MADM) is converted into a piece of computer code in MATLAB to seek solution for the mentioned problem with constant thermal conductivity (a linear problem). The results show that the proposed solution converges to the analytical solution rapidly. Afterwards, the code is extended to calculate Adomian polynomials and implemented to the similar, but more generalized, problem involving a power law dependence of thermal conductivity on temperature. The latter generalization imposes three different nonlinearities and extremely intensifies the complexity of the problem. The code successfully manages to provide parametric solution for this case. Finally, for the sake of exemplification, a relevant practical and real-world case study, about a silicon fin, for the complex nonlinear problem is given. It is shown that the numerical results are very close to those calculated by the classical Finite Difference Method (FDM).

Keywords: Tapered fin, Nonlinear differential equation, Modified Adomian decomposition, MATLAB.

MSC 2010 No.: 34L30; 46Txx

1. Introduction

Fins are echo-friendly and economic means of convective heat transfer enhancement. They are encountered quite often in practice: from industrial compact heat exchangers to CPU heat sink modules of personal computers. Finned structures, better known as heat sinks, have well served thermal management of electronic systems for many years [Anandan and Ramalungam (2008), Dewan et al. (2009)]. The literature is rich in publications on heat transfer in fins of various profile shapes, viz. rectangular, circular, convex/concave parabolic, trapezoidal, triangular, etc. [Rong-Hua (1995), Bejan and Kraus (2003), Kraus et al. (2001)]. Fins with variable thermal conductivity are more realistic and have been paid attention so far. Linearly temperature dependent thermal conductivity for a straight longitudinal fin has been studied by Arslanturk (2005). A very similar problem has been solved by Joneidi et al. (2009) through the Differential Transform Method (DTM).

Tapered fins particularly are of interest in airborne and space applications, where weight is a decisive factor, as for dissemination of a given heat load, they result normally in lighter structures than rectangular fins and are easier to fabricate compared to convex/concave fin profiles [Khani and Aziz (2010), Krikkis and Razelos (2002)]. In a numerical effort, Abrate and Newnham (1995) utilized Finite Element Method (FEM) to analyze the performance of installed triangular fins. Naphon and Sookkasem (2007) chose a finite volume method with an unstructured non-uniform girding to evaluate tapered cylindrical pin fins. Campo and Morrone (2004) combined Finite Difference Method and a mesh-free approach to carry out thermal analysis of annular fins having tapered cross section. Recently, Fatoorehchi and Abolghasemi (2011) came up with accurate approximations for steady-state temperature distributions in triangular fins of various fin parameters via Integral Approximation Method (IAM). Bert (2002) investigated steady state performance of triangular fins having constant physical properties. Khani and Aziz (2010) studied trapezoidal fins analytically with linear dependence of thermal conductivity employing Homotopy Analysis Method (HAM).

In this paper, we incorporate symbolic programming into a modified Adomian decomposition scheme to obtain solutions to a triangular fin problem involving both constant and power-law dependent thermal conductivity (highly nonlinear). A similar power-law dependence problem has been followed recently by Moitsheki et al. (2010), however, their methodology (Lie Symmetry Method) and fin type (constant cross-section) differs from our work greatly. The provided comparisons and error analysis ascertain the magnificent performance of our proposed scheme. The approach has several merits such as, fast convergence to analytical solutions, if any, high accuracy, simplicity, algorithmic nature, and not requiring any linearization, discretization, or perturbation. Moreover, a realistic case study about silicon tapered fins, which obey the aforesaid power-law dependence, is given in the final section.

2. Basics of ADM

Since MADM is a fine modification on ADM, we essentially begin with fundamentals of ADM before describing MADM. Having been initially developed and introduced by the seasoned mathematician George Adomian in 1984, ADM provides convenient solutions to a wide span of linear, as well as nonlinear, differential/integral equations ingeniously [Adomian (1984), (1988), (1994), (1998)]. In this section we only present a concise introduction to ADM.

Consider a very general differential equation as follows:

$$Lu + Nu + Ru = g \ , \tag{1}$$

where L is an easily invertible linear operator, N is a nonlinear part and R stands for the remaining part. By defining the inverse operator of L as L^{-1}, it is directly concluded that:

$$L^{-1}Lu + L^{-1}Nu + L^{-1}Ru = L^{-1}g \, . \tag{2}$$

Taking L as an n-th order derivative operator into account, L^{-1} becomes an n-fold integration operator. Thus, it is followed that $L^{-1}Lu=u+a$, where a is emerged from the integrations. ADM proposes the final solution in form of $u = \sum_{n=0}^{\infty} u_n$ (that is why it is called *decomposition*). Identifying u_0 as $L^{-1} - ga$, equation (2) yields:

$$u = u_0 - L^{-1}Nu - L^{-1}Ru \, . \tag{3}$$

Furthermore, Nu shall be decomposed into an infinite series of Adomian polynomials as follows:

$$Nu = \sum_{n=0}^{\infty} A_n \ , \tag{4}$$

where A_n is classically suggested to be computed from [Jiao et al. (2008)]:

$$A_n = A_n\left(u_0, u_1, \ldots, u_n\right) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0} . \tag{5}$$

Therefore, a recurrence can be established to calculate the remnant solution terms as:

$$u_{i+1} = -L^{-1}A_i - L^{-1}Ru_i \quad ; \quad i \geq 0 \, . \tag{6}$$

3. Problem No. I: *Constant Thermal Conductivity*

The differential equation governing temperature distribution inside a triangular fin with invariant thermal conductivity can be expressed as:

$$\frac{d}{dx}\left(x\frac{d\theta}{dx}\right) - m^2\theta = 0 \, , \tag{7}$$

or equally:

$$x\frac{d^2\theta}{dx^2}+\frac{d\theta}{dx}-m^2\theta=0$$

with boundary conditions:

$$\left.\frac{d\theta}{dx}\right|_{x=0}=0\quad, \tag{9}$$

and

$$\theta(L)=\theta_b\,, \tag{10}$$

where θ is the temperature measured above the ambient temperature, subscript b stands for fin base and L is the fin length. In addition, m^2 is the fin parameter defined as:

$$m^2=\frac{(h_1+h_2)L}{kb}\quad, \tag{11}$$

where h_1 and h_2 are convective heat transfer coefficients of fin's either sides, k is the thermal conductivity and b is fin's vertical dimension at its base. Please note that the origin of coordinates is placed at the tapered end of the fin for this formulation.

3.1. Analysis by MADM

Although the classical ADM is very powerful, it fails in treating of some singular boundary value problems. MADM has been proposed to alleviate this deficiency. In fact, MADM is a slight refinement to the original ADM and it only modifies the involved differential operator. Generally, MADM proposes the following differential and inverse operators [Hasan and Zhu (2008), (2009)]:

$$L(\cdot)=x^{-1}\frac{d^{n-1}}{dx^{n-1}}x^{n-m}\frac{d}{dx}x^{m-n+1}\frac{d}{dx}(\cdot) \tag{12}$$

$$L^{-1}(\cdot)=\int_0^x x^{n-m-1}\int_b^x x^{m-n}\underbrace{\int_0^x\cdots\int_0^x}_{n-1} x(\cdot)dx\cdots dx \tag{13}$$

for treatment of singular boundary value problem of:

$$y^{(n+1)}+\frac{m}{x}y^{(n)}+Ny=g\left(x\right). \tag{14}$$

Accordingly, we find the appropriate operators for the equation (8) as:

$$L_{xx}=x^{-1}\frac{d}{dx}x\frac{d}{dx}\left(\cdot\right) \quad , \tag{15}$$

$$L_{xx}^{-1}(\cdot)=\int_L^x x^{-1}\int_0^x x(\cdot)dxdx. \tag{16}$$

Therefore, the equation (8) can be expressed in its operator form:

$$L_{xx}\theta=m^2\frac{\theta}{x}. \tag{17}$$

Applying the inverse operator, we have:

$$\theta\left(x\right)=\theta\left(L\right)+L_{xx}^{-1}\left(m^2\frac{\theta}{x}\right) \tag{18}$$

Recalling the boundary condition at the fin base, we obtain the first decomposition term as:

$$\theta_0=\theta_b. \tag{19}$$

And the recursive relation is yielded:

$$\theta_{k+1}=\int_L^x x^{-1}\int_0^x m^2\theta_k dxdx \quad ; \quad k\geq 0 \tag{20}$$

Therefore, the decomposition terms of the solution can easily be calculated by the following simple code in MATLB.

```
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
clear all
clc
syms L m x xx s theta_b
nth=input('How many decomposition terms do you want to include in your solution? ');
f=1; s=0;
for n=1:nth
   s=s+f;
   disp(sprintf('%s%d', 'Theta_', n-1,'='))
disp(f*theta_b)
   f=int((1/xx)*int(m*m*f,x,0,xx),xx,L,x);
end
solution=s*theta_b
% The End
```

The first seven components of the solution computed by the presented code are given for the sake of demonstration.

$$\theta_0 = \theta_b$$
$$\theta_1 = m^2\theta_b x - m^2\theta_b L$$
$$\theta_2 = \frac{m^4\theta_b}{4}x^2 - m^4\theta_b L x + \frac{3m^4\theta_b L^2}{4}$$
$$\theta_3 = \frac{m^6\theta_b}{36}x^3 - \frac{m^6\theta_b L}{4}x^2 + \frac{3m^6\theta_b L^2}{4}x - \frac{19m^6\theta_b L^3}{36}$$
$$\theta_4 = \frac{m^8\theta_b}{576}x^4 - \frac{m^8\theta_b L}{36}x^3 + \frac{3m^8\theta_b L^2}{16}x^2 - \frac{19m^8\theta_b L^3}{36}x + \frac{211m^8\theta_b L^4}{576}$$
$$\theta_5 = \frac{m^{10}\theta_b}{14400}x^5 - \frac{m^{10}\theta_b L}{576}x^4 + \frac{m^{10}\theta_b L^2}{48}x^3 - \frac{19m^{10}\theta_b L^3}{144}x^2 + \frac{211m^{10}\theta_b L^4}{576}x - \frac{1217m^{10}\theta_b L^5}{4800}$$
$$\theta_6 = \frac{m^{12}\theta_b}{518400}x^6 - \frac{m^{12}\theta_b L}{14400}x^5 + \frac{m^{12}\theta_b L^2}{768}x^4 - \frac{19m^{12}\theta_b L^3}{1296}x^3 + \frac{211m^{12}\theta_b L^4}{2304}x^2 - \frac{1217m^{12}\theta_b L^5}{4800}x + \frac{30307m^{12}\theta_b L^6}{172800}$$

3.2. Error Analysis

Arpaci (1966) has presented the relevant analytical solution to the mathematical modeling as:

$$\frac{\theta}{\theta_b} = \frac{I_0\left(2mx^{0.5}\right)}{I_0\left(2mL^{0.5}\right)} \quad , \tag{21}$$

where I_0 denotes Bessel's function of second kind.

To draw a comparison between the results by MADM and the analytical solution, we define an overall relative error which covers the whole fin length:

$$overall\ ARD = \frac{1}{5}\left(\left|\frac{\theta_{exc}-\theta_{MADM}}{\theta_{exc}}\right|_{@x=0} + \left|\frac{\theta_{exc}-\theta_{MADM}}{\theta_{exc}}\right|_{@x=\frac{L}{4}} + \left|\frac{\theta_{exc}-\theta_{MADM}}{\theta_{exc}}\right|_{@x=\frac{2L}{4}} + \left|\frac{\theta_{exc}-\theta_{MADM}}{\theta_{exc}}\right|_{@x=\frac{3L}{4}} + \left|\frac{\theta_{exc}-\theta_{MADM}}{\theta_{exc}}\right|_{@x=L}\right) \quad , \tag{22}$$

where θ_{exc} and θ_{MADM} stand for the exact analytical solution and the solution obtained by MADM, respectively. Such a definition for ARD gives a global and average sense of deviation from exact solution and is more valid than focusing on error at a single fixed point. Figures 1-3 show the values of the overall ARD plotted against the number up to which MADM solution series is expanded, for various fin parameter (m). It can be interpreted from these figures, for every fin parameters, that the aforesaid deviation vanishes quickly as more terms of the MDAM series solution are included.

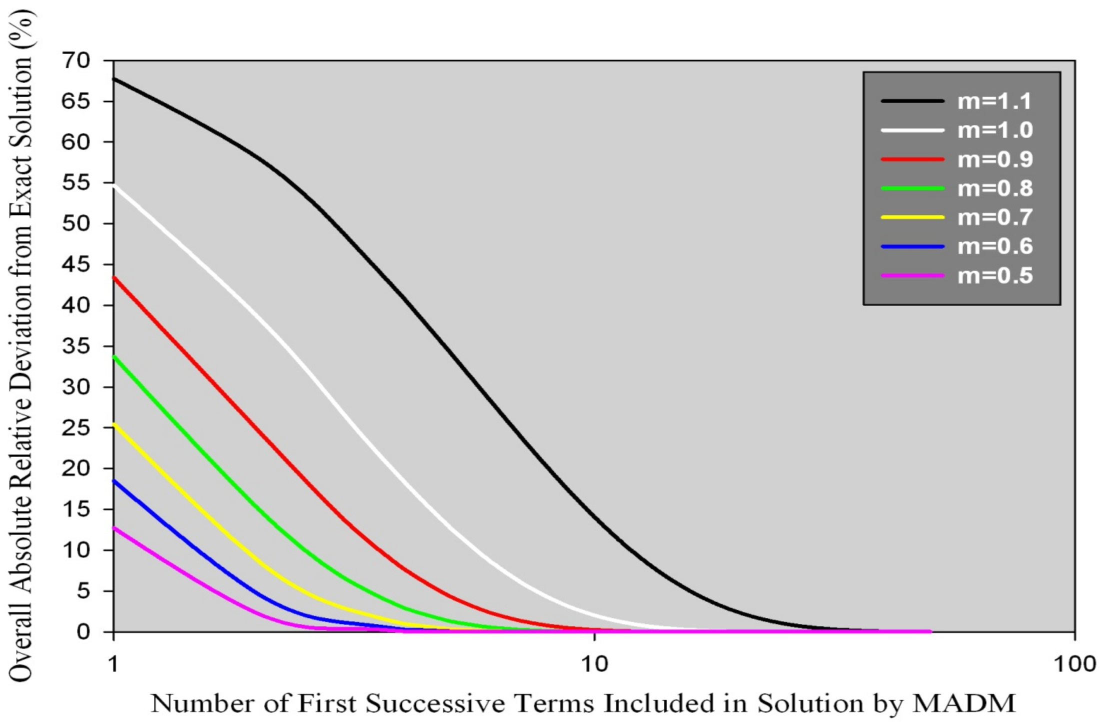

Figure 1. Values of the defined ARD vs. number of terms in MADM solution series expansion for *m* ranging from 0.5 to 1.1. Please note that the x-axis is displayed on a logarithmic scale for better visualization.

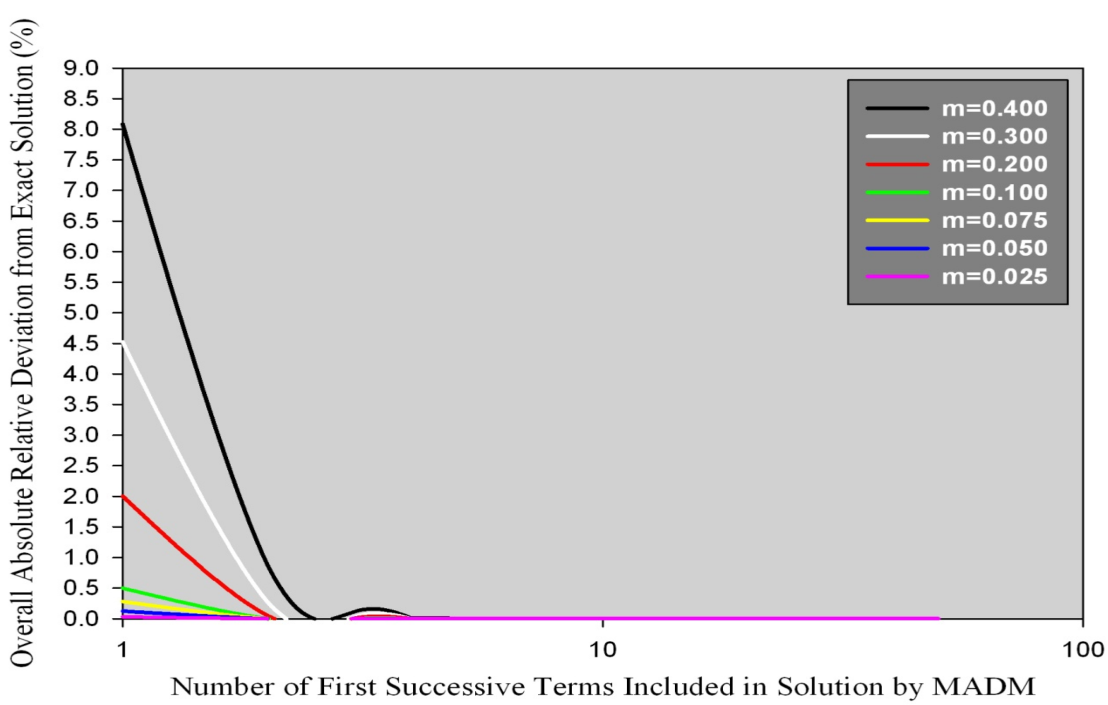

Figure 2. Values of the defined ARD vs. number of terms in MADM solution series expansion for *m* ranging from 0.025 to 0.400. Please note that the x-axis is displayed on a logarithmic scale for better visualization.

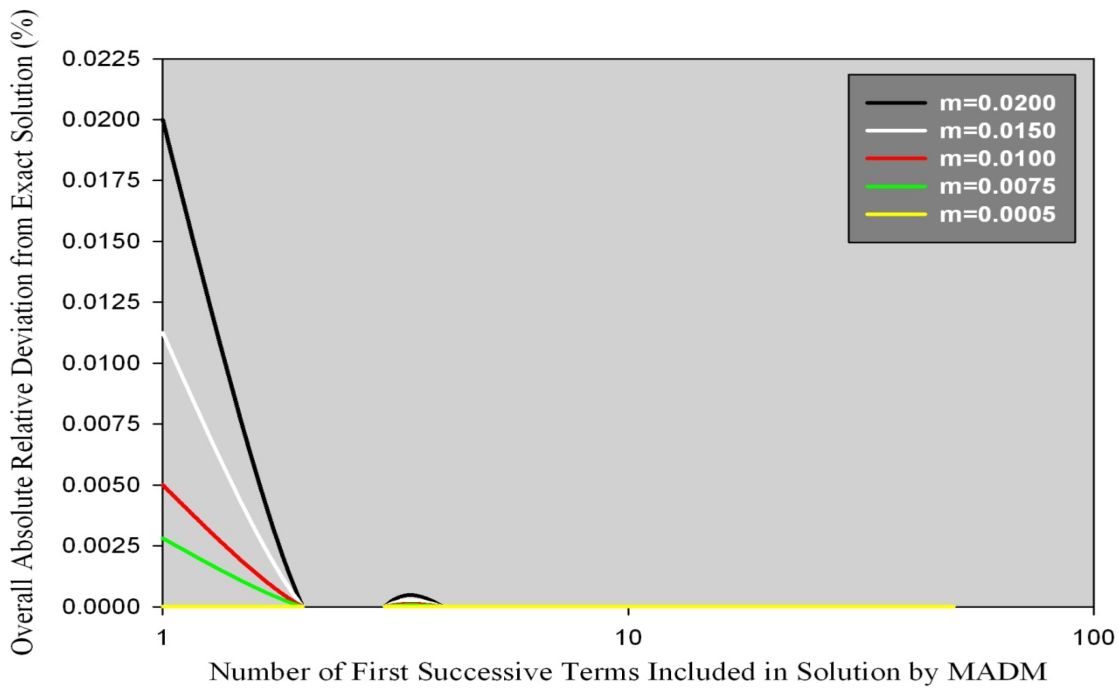

Figure 3. Values of the defined ARD vs. number of terms in MADM solution series expansion for *m* ranging from 0.0005 to 0.0200. Please note that the x-axis is displayed on a logarithmic scale for better visualization.

4. Problem No. II: *Power Law Dependence of Thermal Conductivity*

Assuming general power-law dependence for thermal conductivity of the fin material in form of:

$$k = k_0 T^{\beta} \tag{23}$$

one can easily derive the succeeding governing equation by setting a heat balance over a differential control volume on the fin.

$$\frac{d}{dx}\left(xT^{\beta} \frac{dT}{dx} \right) - \frac{2hL}{k_0 b}\left(T - T_{\infty} \right) = 0\,. \tag{24}$$

Or also as:

$$T^{\beta} \frac{dT}{dx} + xT^{\beta} \frac{d^2T}{dx^2} + \beta x T^{\beta-1} \left(\frac{dT}{dx} \right)^2 - \frac{2hL}{k_0 b}\left(T - T_{\infty} \right) = 0\,, \tag{25}$$

$$\frac{d^2T}{dx^2} + \frac{1}{x}\frac{dT}{dx} + \beta \frac{1}{T}\left(\frac{dT}{dx} \right)^2 - \frac{2hL}{k_0 bx}\left(T^{1-\beta} - T_{\infty} T^{-\beta} \right) = 0\,. \tag{26}$$

with $h = \dfrac{h_1 + h_2}{2}$.

4.1. Analysis by MADM

Similar to the previous part, MADM candidates the following operator for dealing with the equation (26):

$$L_{xx} = x^{-1} \frac{d}{dx} x \frac{d}{dx}(\cdot), \tag{27}$$

$$L_{xx}^{-1}(\cdot) = \int_L^x x^{-1} \int_0^x x(\cdot) dx dx. \tag{28}$$

Consequently, one can easily convert the equation (26) to its operator form equivalent as:

$$L(T) + \beta \frac{1}{T}\left(\frac{dT}{dx}\right)^2 - \frac{2hL}{k_0 bx}\left(T^{1-\beta} - T_\infty T^{-\beta}\right) = 0. \tag{29}$$

Taking the inverse transform on both sides of equation (29), we achieve:

$$T(x) = T(L) - \beta L_{xx}^{-1}\left(\frac{1}{T}\left(\frac{dT}{dx}\right)^2\right) + \frac{2hL}{k_0 b} L_{xx}^{-1}\left(\frac{T^{1-\beta}}{x}\right) - \frac{2hLT_\infty}{k_0 b} L_{xx}^{-1}\left(\frac{T^{-\beta}}{x}\right). \tag{30}$$

As observed, three different nonlinearities exist in this problem [equation (26)].

We represent them as series decomposition of three different Adomian polynomials given below:

$$NA = \frac{1}{T}\left(\frac{dT}{dx}\right)^2 = \sum_{n=0}^{\infty} A_n, \tag{31}$$

$$NB = T^{1-\beta} = \sum_{n=0}^{\infty} B_n, \tag{32}$$

and

$$NC = T^{-\beta} = \sum_{n=0}^{\infty} C_n. \tag{33}$$

Following the decomposition technique, we find:

$$\sum_{n=0}^{\infty} T_n = T(L) - \beta L_{xx}^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + \frac{2hL}{k_0 b} L_{xx}^{-1}\left(\frac{\sum_{n=0}^{\infty} B_n}{x}\right) - \frac{2hLT_\infty}{k_0 b} L_{xx}^{-1}\left(\frac{\sum_{n=0}^{\infty} C_n}{x}\right). \tag{34}$$

Thus,

$$\begin{cases} T_0 = T(L) \\ T_{k+1} = -\beta L_{xx}^{-1}(A_k) + \dfrac{2hL}{k_0 b} L_{xx}^{-1}\left(\dfrac{B_k}{x}\right) - \dfrac{2hLT_\infty}{k_0 b} L_{xx}^{-1}\left(\dfrac{C_k}{x}\right); k \geq 0 \end{cases}. \tag{35}$$

4.2. Computational Work

To reach the very final solution, we need to compute each decomposed component of the series $T = \sum_{k=0}^{\infty} T_k$ recursively. For this purpose, it is necessary to obtain Adomian polynomials components of A_k, B_k, and C_k at each iteration. To handle this task neatly, we have built three functions returning symbolic representations for A_k, B_k, and C_k, a function to take inverse transform and a core code to calculate the ultimate solution with the help of these functions. All these MATLAB codes are given in appendix A. Also, for the sake of demonstration and interest of the reader the first five components of Adomian polynomials pertaining to the discussed nonlinearities, computed by the established MATLAB functions, are presented in Appendix B.

Using the mentioned computational code, the MADM solution series can be expanded up to any desired component.

$$T_0 = T_b,$$

$$T_1 = \frac{2hL}{k_0 b} T_b^{1-\beta}(x-L) - \frac{2hLT_\infty}{k_0 b} T_b^{-\beta}(x-L),$$

$$T_2 = -\frac{h^2 L^2 T_b^{-1-2\beta}}{k_0^2 b^2}\begin{pmatrix} -T_b^2 x^2 + 4T_b^2 xL + T_b x^2 T_\infty - 4T_b x T_\infty L + T_b^2 x^2 \beta - 4T_b^2 xL\beta - 2T_b x^2 \beta T_\infty + 8T_b xL\beta T_\infty \\ -3T_b^2 L^2 + 3T_b T_\infty L^2 + 3T_b^2 L^2 \beta - 6\beta T_b T_\infty L^2 + \beta T_\infty^2 x^2 - 4\beta T_\infty^2 xL + 3\beta T_\infty^2 L^2 \end{pmatrix}$$

$$\vdots$$

5. Case Study

Silicon, being an efficient thermal conductor, has been of extensive interest in fabrication of cooling fins and packed heat sink modules especially for thermal management in microelectronics [Tullius et al. (2011), Sadri-Lonbani et al. (2003)].

For temperatures ranging within 300-1400K, a power law correlation for thermal conductivity of silicon is proposed [Sze (1981), Shanks and Maycock (1963)]:

$$k = k_{300}\left(\frac{T}{300}\right)^{\alpha}, \tag{36}$$

where

$$k_{300} = 148\frac{W}{Km}, \alpha = -1.3.$$

Regarding equation (23), $k_0 = 148 \times 300^{1.3} \dfrac{WK^{0.3}}{m}$.

Now we take advantage of the described computational work based on MADM to investigate the temperature distribution in a triangular silicon fin with dimensions *L=0.05 m* and *b=0.005 m*, subject to constant a base temperature of T_b=423K (150^{o}C), and ambient temperature of T_∞=298K (25^{o}C). To perform a comparative study and cross check the accuracy of the results, the problem is resolved by the classical numerical approach of Finite Difference Method (FDM). Accordingly, equation (26) is discretized into an arbitrary number of nodes (n) as follows:

For the first node which is located at the fin's tip, let us denote it with index 0, the insulation conditions gives:

$$\frac{T_1 - T_0}{\Delta x} = 0 \rightarrow T_0 = T_1 . \tag{37}$$

For the i^{th} node, where $0 < i \le n-2$, we write derivatives in terms of forward finite differences:

$$\begin{aligned} & i\Delta x T_i^{\beta} \frac{1}{(\Delta x)^2}\left(T_{i+2} - 2T_{i+1} + T_i\right) + T_i^{\beta}\frac{1}{\Delta x}\left(T_{i+1} - T_i\right) + i\Delta x \beta T_i^{\beta-1}\frac{1}{(\Delta x)^2}\left(T_{i+1} - T_i\right)^2 \\ & -\frac{2hL}{k_0 b}(T_i - T_\infty) = 0 \end{aligned} . \tag{38}$$

For the node number n-1, backward finite differences are used:

$$\begin{aligned} & (n-1)\Delta x T_{n-1}^{\beta}\frac{1}{(\Delta x)^2}\left(T_{n-1} - 2T_{n-2} + T_{n-3}\right) + T_{n-1}^{\beta}\frac{1}{\Delta x}\left(T_{n-1} - T_{n-2}\right) + \\ & (n-1)\Delta x \beta T_{n-1}^{\beta-1}\frac{1}{(\Delta x)^2}\left(T_n - T_{n-1}\right)^2 - \frac{2hL}{k_0 b}(T_{n-1} - T_\infty) = 0 \end{aligned} . \tag{39}$$

The prescribed boundary condition prevails at the last node:

$$T_n = T_b . \tag{40}$$

In this way, FDM converts the governing differential equation (26) into a set of nonlinear algebraic equations which can be solved simultaneously by appropriate algorithms like Gauss-Newton or Levenberg-Marquardt. Herein, we have employed the *fsolve* command in MATLAB, which is based on nonlinear least square algorithm, to solve this case study.

The results to this case study by MADM and FDM are given in table I and as shown, their absolute differences are very small despite that the MADM was continued only up to the summation of its first 8 components. This much accuracy is due to the efficiency of MADM.

Table I- Comparison between the solutions by MADM and FDM for the silicon fin

x	T(x) by MADM*	T(x) by FDM	Absolute Difference
0	144.92076590748	145.08489521307	0.16412930558734
L/10	145.42039325042	145.08489522111	0.33549802931392
2*L/10	145.92184015669	145.98805088945	0.66210732759028e-1
3*L/10	146.42511552698	146.47817830047	0.53062773494269e-1
4*L/10	146.93022831971	146.97231454755	0.42086227837910e-1
5*L/10	147.43718755148	147.46987748987	0.32689938385951e-1
6*L/10	147.94600229744	147.97052349930	0.24521201864250e-1
7*L/10	148.45668169176	148.47400203941	0.17320347654017e-1
8*L/10	148.96923492799	148.98015411903	0.10919191044296e-1
9*L/10	149.48367125952	149.48886637971	0.51951201871899e-2
10*L/10	150	150.00001999462	0.19994623755792e-4

*The first 8 components are included.

6. Conclusion

Convective triangular fins with invariant and power-law temperature-dependent thermal conductivities were studied by Modified Adomian Decomposition Method (MADM). Through symbolic programming in MATLAB, we managed to computerize MADM and offer reliable parametric solutions for both problems. The MADM solutions were compared with an exact analytical solution and a numerical solution by Finite Difference Method for the constant *k* and power-law temperature-dependent *k* cases, respectively. A practical and realistic case study regarding a silicon fin of specific dimensions was carried out as a numerical illustrative example. The all obtained results ascertained the magnificent efficiency and rapid convergence of MADM. Other researchers may benefit from the pieces of MATLAB codes provided herein for their complex analyses of nonlinear differential equations.

Appendix A

All MATLAB codes used in this paper.

```
% Function Ak, returning the kth component of the Adomian polynomials corresponding % to the nonlinearity A.
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
function Ak =f(k)
syms x s h
sym('u0(x)');sym('u1(x)');sym('u2(x)');sym('u3(x)');sym('u4(x)');sym('u5(x)');sym('u6(x)');sym('u7(x)');sym('u8(x)');sym('u9(x)');sym('u10(x)');sym('u11(x)');sym('u12(x)');sym('u13)');sym('u14');sym('u15)');sym('u16)'); sym('u17)'); sym('u18)'); sym('u19)'); sym('u20)');
s='u0(x)'+h*'u1(x)'+h^2*'u2(x)'+h^3*'u3(x)'+h^4*'u4(x)'+h^5*'u5(x)'+h^6*'u6(x)'+h^7*'u7(x)'+h^8*'u8(x)'+h^9*'u9(x)'+h^10*'u10(x)'+h^11*'u11(x)'+h^12*u12+h^13*u13+h^14*u14+h^15*u15+h^16*u16+h^17*u17+h^18*u18+h^19*u19+h^20*u20;
Ak=(1/factorial(k))*subs(diff(1/s*(diff(s,x))^2,h,k),h,0);
% The End
```

```
% Function Bk, returning the kth component of the Adomian polynomials corresponding % to the nonlinearity B.
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
function Bk =f(k)
syms x s h Z
sym('u0(x)');sym('u1(x)');sym('u2(x)');sym('u3(x)');sym('u4(x)');sym('u5(x)');sym('u6(x)');sym('u7(x)');sym('u8(x)');s
ym('u9(x)');sym('u10(x)');sym('u11(x)');sym('u12(x)');sym('u13)');sym('u14');sym('u15)');sym('u16)'); sym('u17)');
sym('u18)'); sym('u19)'); sym('u20)');
s='u0(x)'+h*'u1(x)'+h^2*'u2(x)'+h^3*'u3(x)'+h^4*'u4(x)'+h^5*'u5(x)'+h^6*'u6(x)'+h^7*'u7(x)'+h^8*'u8(x)'+h^9*'u9
(x)'+h^10*'u10(x)'+h^11*'u11(x)'+h^12*u12+h^13*u13+h^14*u14+h^15*u15+h^16*u16+h^17*u17+h^18*u18+h^
19*u19+h^20*u20;
Bk=(1/factorial(k))*subs(diff(s^(1-Z),h,k),h,0);
% The End

% Function Ck, returning the kth component of the Adomian polynomials corresponding % to the nonlinearity C.
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
function Ck =f(k)
syms x s h Z
sym('u0(x)');sym('u1(x)');sym('u2(x)');sym('u3(x)');sym('u4(x)');sym('u5(x)');sym('u6(x)');sym('u7(x)');sym('u8(x)');s
ym('u9(x)');sym('u10(x)');sym('u11(x)');sym('u12(x)');sym('u13)');sym('u14');sym('u15)');sym('u16)'); sym('u17)');
sym('u18)'); sym('u19)'); sym('u20)');
s='u0(x)'+h*'u1(x)'+h^2*'u2(x)'+h^3*'u3(x)'+h^4*'u4(x)'+h^5*'u5(x)'+h^6*'u6(x)'+h^7*'u7(x)'+h^8*'u8(x)'+h^9*'u9
(x)'+h^10*'u10(x)'+h^11*'u11(x)'+h^12*u12+h^13*u13+h^14*u14+h^15*u15+h^16*u16+h^17*u17+h^18*u18+h^
19*u19+h^20*u20;
Ck=(1/factorial(k))*subs(diff(s^(-Z),h,k),h,0);
% The End

% Function Linverse, returning inverse transformed function of its input.
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
function Linverse =f(y)
syms L x xx;
Linverse = int(1/xx*int(y*x,x,0,xx),xx,L,x);
% The End

% The Core Code
% coded by H.F. and H.A. ___ Apr. 2011
% Beginning
clc
clear all
syms U x u0 T0 T1 T2 BETA h L k0 b Tinf Tb Z sumsol
T0=Tb;
```

```
T1=-BETA*Linverse(subs(Ak(0),sym('u0(x)'),T0))+2*h*L/k0/b*Linverse(subs(Bk(0)/x,sym('u0(x)'),T0))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(0)/x,sym('u0(x)'),T0));
T2=-
BETA*Linverse(subs(Ak(1),{sym('u0(x)'),sym('u1(x)')},{T0,T1}))+2*h*L/k0/b*Linverse(subs(Bk(1)/x,{sym('u0(x
)'),sym('u1(x)')},{T0,T1}))-2*h*L*Tinf/k0/b*Linverse(subs(Ck(1)/x,{sym('u0(x)'),sym('u1(x)')},{T0,T1}));
T3=-
BETA*Linverse(subs(Ak(2),{sym('u0(x)'),sym('u1(x)'),sym('u2(x)')},{T0,T1,T2}))+2*h*L/k0/b*Linverse(subs(Bk(
2)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)')},{T0,T1,T2}))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(2)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)')},{T0,T1,T2}));
T4=-
BETA*Linverse(subs(Ak(3),{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)')},{T0,T1,T2,T3}))+2*h*L/k0/b*Li
nverse(subs(Bk(3)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)')},{T0,T1,T2,T3}))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(3)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)')},{T0,T1,T2,T3}));
T5=-
BETA*Linverse(subs(Ak(4),{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)')},{T0,T1,T2,T3,T4}))
+2*h*L/k0/b*Linverse(subs(Bk(4)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)')},{T0,T1,T2,T
3,T4}))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(4)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)')},{T0,T1,
T2,T3,T4}));
T6=-
BETA*Linverse(subs(Ak(5),{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)'),sym('u5(x)')},{T0,T1,
T2,T3,T4,T5}))+2*h*L/k0/b*Linverse(subs(Bk(5)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)
'),sym('u5(x)')},{T0,T1,T2,T3,T4,T5}))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(5)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)'),sym('u5(x
)')},{T0,T1,T2,T3,T4,T5}));
T7=-
BETA*Linverse(subs(Ak(6),{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)'),sym('u5(x)'),sym('u6(
x)')},{T0,T1,T2,T3,T4,T5,T6}))+2*h*L/k0/b*Linverse(subs(Bk(6)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u
3(x)'),sym('u4(x)'),sym('u5(x)'),sym('u6(x)')},{T0,T1,T2,T3,T4,T5,T6}))-
2*h*L*Tinf/k0/b*Linverse(subs(Ck(6)/x,{sym('u0(x)'),sym('u1(x)'),sym('u2(x)'),sym('u3(x)'),sym('u4(x)'),sym('u5(x
)'),sym('u6(x)')},{T0,T1,T2,T3,T4,T5,T6}));
L=0.05;Tinf=273+25;Tb=273+150;h=4;BETA=-1.3;Z=-1.3;k0=148*300^-BETA;b=0.005;
Disp(‘The solution by MADM is: ‘);
sumsol=T0+T1+T2+T3+T4+T5+T6+T7
% The End
```

Appendix B

First five components of Adomian polynomials series for the nonlinearity: $NA = \frac{1}{T}\left(\frac{dT}{dx}\right)^2$
$A_0 = \dfrac{\left(\frac{dT_0}{dx}\right)^2}{T_0}$
$A_1 = \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1}{T_0^2} + 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)}{T_0}$
$A_2 = \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1^2}{T_0^3} - 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_1}{T_0^2} - \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_2}{T_0^2} + \dfrac{\left(\frac{dT_1}{dx}\right)^2}{T_0} + 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_2}{dx}\right)}{T_0}$
$A_3 = -\dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1^3}{T_0^4} + 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_1^2}{T_0^3} + 2\dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1 T_2}{T_0^3} - \dfrac{\left(\frac{dT_1}{dx}\right)^2 T_1}{T_0^2} - 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_2}{T_0^2}$ $-2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_2}{dx}\right)T_1}{T_0^2} - \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_3}{T_0^2} + \dfrac{\left(\frac{dT_1}{dx}\right)\left(\frac{dT_2}{dx}\right)}{T_0} + \dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_3}{dx}\right)}{T_0}$
$A_4 = 2\dfrac{\left(\frac{dT_1}{dx}\right)\left(\frac{dT_3}{dx}\right)}{T_0} - \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_4}{T_0^2} - \dfrac{\left(\frac{dT_1}{dx}\right)^2 T_2}{T_0^2} + \dfrac{\left(\frac{dT_2}{dx}\right)^2}{T_0} + 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_4}{dx}\right)}{T_0} + \dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1^4}{T_0^5}$ $-2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_1^3}{T_0^4} - 3\dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1^2 T_2}{T_0^4} + \dfrac{\left(\frac{dT_1}{dx}\right)^2 T_1^2}{T_0^3} + 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_2}{dx}\right)T_1^2}{T_0^3} + \dfrac{\left(\frac{dT_0}{dx}\right)^2\left(\frac{dT_1}{dx}\right)T_2^2}{T_0^3}$ $+4\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_1 T_2}{T_0^3} + 2\dfrac{\left(\frac{dT_0}{dx}\right)^2 T_1 T_3}{T_0^3} - 2\dfrac{\left(\frac{dT_1}{dx}\right)\left(\frac{dT_2}{dx}\right)T_1}{T_0^2} - 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_1}{dx}\right)T_3}{T_0^2}$ $-2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_2}{dx}\right)T_2}{T_0^2} - 2\dfrac{\left(\frac{dT_0}{dx}\right)\left(\frac{dT_3}{dx}\right)T_1}{T_0^2}$

First five components of Adomian polynomials series for the nonlinearity: $NB = T^{1-\beta}$
$B_0 = T_0^{1-\beta}$
$B_1 = \frac{(1-\beta) T_0^{1-\beta} T_1}{T_0}$
$B_2 = \frac{1}{2}\frac{(1-\beta)^2 T_0^{1-\beta} T_1^2}{T_0^2} + \frac{(1-\beta) T_0^{1-\beta} T_2}{T_0} - \frac{1}{2}\frac{(1-\beta) T_0^{1-\beta} T_1^2}{T_0^2}$
$B_3 = \frac{1}{6}\frac{(1-\beta)^3 T_0^{1-\beta} T_1^3}{T_0^3} + \frac{(1-\beta)^2 T_0^{1-\beta} T_1 T_2}{T_0^2} - \frac{1}{2}\frac{(1-\beta)^2 T_0^{1-\beta} T_1^3}{T_0^3} + \frac{(1-\beta) T_0^{1-\beta} T_3}{T_0} - \frac{(1-\beta) T_0^{1-\beta} T_1 T_2}{T_0^2} + \frac{1}{3}\frac{(1-\beta) T_0^{1-\beta} T_1^3}{T_0^3}$
$B_4 = \frac{1}{24}\frac{(1-\beta)^4 T_0^{1-\beta} T_1^4}{T_0^4} + \frac{1}{2}\frac{(1-\beta)^3 T_0^{1-\beta} T_1^2 T_2}{T_0^3} - \frac{1}{4}\frac{(1-\beta)^3 T_0^{1-\beta} T_1^4}{T_0^4} + \frac{1}{2}\frac{(1-\beta)^2 T_0^{1-\beta} T_2^2}{T_0^2} - \frac{3}{2}\frac{(1-\beta)^2 T_0^{1-\beta} T_1^2 T_2}{T_0^3} + \frac{(1-\beta) T_0^{1-\beta} T_4}{T_0^2} + \frac{11}{24}\frac{(1-\beta)^2 T_0^{1-\beta} T_1^4}{T_0^4} + \frac{(1-\beta) T_0^{1-\beta} T_4}{T_0} - \frac{(1-\beta) T_0^{1-\beta} T_1 T_3}{T_0^2} + \frac{(1-\beta) T_0^{1-\beta} T_1^2 T_2}{T_0^3} - \frac{1}{2}\frac{(1-\beta) T_0^{1-\beta} T_2^2}{T_0^2} - \frac{1}{4}\frac{(1-\beta) T_0^{1-\beta} T_1^4}{T_0^4}$

First five components of Adomian polynomials series for the nonlinearity: $NC = T^{-\beta}$
$C_0 = T_0^{-\beta}$
$C_1 = -\frac{\beta T_0^{-\beta} T_1}{T_0}$
$C_2 = \frac{1}{2}\frac{\beta^2 T_0^{-\beta} T_1^2}{T_0^2} - \frac{\beta T_0^{-\beta} T_2}{T_0} + \frac{1}{2}\frac{\beta T_0^{-\beta} T_1^2}{T_0^2}$
$C_3 = -\frac{1}{6}\frac{\beta^3 T_0^{-\beta} T_1^3}{T_0^3} + \frac{\beta^2 T_0^{-\beta} T_1 T_2}{T_0^2} - \frac{1}{2}\frac{\beta^2 T_0^{-\beta} T_1^3}{T_0^3} - \frac{\beta T_0^{-\beta} T_3}{T_0} + \frac{\beta T_0^{-\beta} T_1 T_2}{T_0^2} - \frac{1}{3}\frac{\beta T_0^{-\beta} T_1^3}{T_0^3}$
$C_4 = \frac{1}{24}\frac{\beta^4 T_0^{-\beta} T_1^4}{T_0^4} - \frac{1}{2}\frac{\beta^3 T_0^{-\beta} T_1^2 T_2}{T_0^3} + \frac{1}{4}\frac{\beta^3 T_0^{-\beta} T_1^4}{T_0^4} + \frac{1}{2}\frac{\beta^2 T_0^{-\beta} T_2^2}{T_0^2} - \frac{3}{2}\frac{\beta^2 T_0^{-\beta} T_1^2 T_2}{T_0^3} + \frac{\beta^2 T_0^{-\beta} T_1 T_3}{T_0^2} + \frac{11}{24}\frac{\beta^2 T_0^{-\beta} T_1^4}{T_0^4} - \frac{\beta T_0^{-\beta} T_4}{T_0} + \frac{\beta T_0^{-\beta} T_3 T_1}{T_0^2} - \frac{\beta T_0^{-\beta} T_2 T_1^2}{T_0^3} + \frac{1}{2}\frac{\beta T_0^{-\beta} T_2^2}{T_0^2} + \frac{1}{4}\frac{\beta T_0^{-\beta} T_1^4}{T_0^4}$

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