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Dust-Acoustic Solitary Waves in Magnetized Dusty Plasma with Dust Opposite Polarity

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Abstract

The nonlinear propagation of small but finite amplitude dust-acoustic solitary waves (DAWs) in magnetized collision less dusty plasma has been investigated. The fluid model is a four component magnetized dusty plasma, consisting of positive and negative dust species, isothermal electrons and ions in the presence of an external magnetic field. A reductive perturbation method was employed to obtain the Zakharov Kuznetsov (ZK) equation for the first-order potential. The effects of the presence of positively charged dust fluid, the external magnetic field, and the obliqueness are obtained. The results of the present investigation may be applicable to some plasma environments, such as cometary tails, upper mesosphere and Jupiter's magnetosphere.

Keywords: magnetized dusty plasma; opposite polarity; dust-acoustic waves; ZK equation; Solitary solution

MSC 2010 No.: 82D10, 34A34, 34G20, 47J35

1. Introduction

The physics of charged dust, which are ubiquitous in space plasmas, has received a great deal of interest in understanding the electrostatic density perturbations and solitary potential structures that are observed in different regions of space, viz. mesosphere, cometary tails, planetary rings, planetary magnetospheres, interplanetary space, interstellar media, etc. [Verheest (2000), Shukla et al. (2002), Carlile et al. (1991), Horanyi et al. (1986)]. The charging of dust grains occurs due to a variety of processes [Merlino et al. (1998), Barkan et al. (1994), Whipple (1981)]. Actually dust grains of different sizes can acquire different polarities; large grains become negatively charged and ounces become small positively charged [Chow et al. (1993), Mendis and Rosenberg (1994), Mendis and Rosenberg (1995)]. In fact, positively charged dust particles have been observed in different regions of space, viz. cometary tails [Chow et al. (1993), Mendis and Rosenberg (1994), Mendis and Rosenberg (1995)], Jupiter's magnetosphere [Horanyi et al. (1993), etc.]

There are three principal mechanisms by which a dust grain becomes positively charged [Fortov et al. (1998)]. These are photoemission in the presence of a flux of ultraviolet (UV) photons, thermionic emission induced by radiative heating, and secondary emission of electrons from the surface of the dust grains. In other words, there is also direct evidence of the existence of both positively and negatively charged dust particles in the earth's mesosphere [Havens et al. (2001), Klumov et al. (2000), Smiley (2003)], as well as in cometary tails and comet [Horanyi et al. (1986), Mendis and Horanyi (1991)].

Mamun and Shukla (2002) have considered dusty plasma model, which consists of positive and negative dust only, and have theoretically investigated the properties of linear and nonlinear electrostatic waves in such dusty plasma. Their model of Mamun and Shukla (2002) is valid only if a complete depletion of the background electrons and ions is possible, and both positive and negative dust fluids are cold.

Recently, El-Wakil et al. (2006b) investigated theoretically the higher-order contributions to nonlinear dust-acoustic waves that propagate in a mesospheric dusty plasma with a completely depleted of background (or electrons and ions). However, in most space dusty plasma systems a complete depletion of the background electrons and ions is not possible [Sayed and Mamun (2007), Abdelwahed et al. (2008), Mamun (2008), El Wakil et al. (2006a), Mowafy et al. (2008)] and the positive dust component is of finite temperature.

Later, Attia et al. (2010) investigated the higher order effects of positive and negative dust charge fluctuation on the propagation of dust ion acoustic waves (DIAWs) in a weakly inhomogeneous, weakly coupled, collision less and unmagnetized mesospheric dusty Plasma consisting of four components dusty plasma. The present work is therefore attempted to see how DAWs characteristics are adapted in magnetized plasma with two charged dust cyclotron frequencies. This paper is organized as follows: In Section 2 we present the basic set of fluid equations governing our plasma model. In Section 3 we derive the ZK equation with lowest-order nonlinearity and dispersion. The stationary solitary wave solution of the ZK equation is analyzed in Section 4. Finally, discussions are given in Section 5.

2. Basic Equations

Let us consider a homogeneous system of a magnetized collision less plasma consisting of four-component dusty plasma with massive, micron-sized, positively, negatively dust grains and isothermal electrons and ions. This study is based on the condition that, the negative dust particles are much more massive than positive ones [Horanyi et al. (1993)], Fortov et al. (1998)]. The dynamics of the nonlinear DA waves in the presence of an external magnetic field

$\boldsymbol{B}_0 = \overleftrightarrow{e_x} B_0$ is governed by:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\, \mathbf{u}) = 0\,, \tag{1a}$$

$$\mu\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u}\right) + \nabla\phi + \frac{\sigma_d}{n}\nabla \boldsymbol{p} - (\boldsymbol{u} \times \boldsymbol{\Omega}_1 \overleftrightarrow{e_x}) = 0\,, \tag{1b}$$

$$\frac{\partial \boldsymbol{p}}{\partial t} + \boldsymbol{u}\nabla \boldsymbol{p} + \gamma \boldsymbol{p}\nabla \boldsymbol{u} = 0\,, \tag{1c}$$

for Positive dust plasma and

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\ \boldsymbol{v}) = 0\,, \tag{2a}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \mathbf{v}.\nabla \mathbf{v} - \nabla\phi + (\boldsymbol{v} \times \boldsymbol{\Omega}_2 \overleftrightarrow{e_x}) = 0\,, \tag{2b}$$

for negative dust plasma.

Equations (1-2) are supplemented by Poisson's equation

$$\nabla^2\phi = N - \mu_1 n + \mu_2 n_e - \mu_3 n_i\,, \tag{3a}$$

$$n_e = e^{\sigma_e \phi}\,, \tag{3b}$$

$$n_i = e^{-\phi}\,. \tag{3c}$$

In the above equations, n and u are the density and velocity of positively charged dusty grains while N and v are the density and velocity of negatively charged dusty grains, n_e and n_i are the density of electrons and ions, ϕ and p are the electric potential of dust fluid and the thermal pressure of the positively charged dust fluid, respectively. Here, n and N are normalized by their equilibrium values n_0 and N_0, u and v are normalized by $C_s = \sqrt{\rho} V_T$,

$\rho = Z_n m_1 / (Z_p m_2)$, and $V_T = (Z_p k_B T_i \ / m_1)^{\frac{1}{2}}$, $Z_p(Z_n)$ represents the number of the positive (negative) charges on the dust grain surface, $m_1(m_2)$ represents the mass of the positive (negative) dust particle, k_B is the Boltzmann constant, T_i is the temperature of the ions, and p is normalized by $n_{10} k_B T_p$. Since T_p is the temperature of the positively charged dust fluid and ϕ is normalized by $k_B T_i / e$, x is the space variable normalized by $\lambda_{D1} = (k_B T_i / 4\pi N_0 Z_n e^2)^{\frac{1}{2}}$, t is the time variable normalized by $\omega_{p2}^{-1} = (m_2 / 4\pi N_0 Z_n^2 e^2)^{\frac{1}{2}}$, where $\sigma_d = (T_p / T_i Z_p)$, $\sigma_e = (T_e \ /T_i)$, $\mu_1 = n_0 Z_p / (N_0 Z_n)$, $\mu_2 = n_{e0} / (N_0 Z_n)$ and $\mu_3 = n_{i0} / (N_0 Z_n)$. Ω_1 and Ω_2 are the positive and negative charged dust cyclotron frequencies normalized to plasma frequency.

3. Zakharov-Kuznetsov Equation

To derive the ZK equation describing the behavior of the system for longer times and small but finite amplitude DA waves, we introduce the slow stretched co-ordinates:

$$\tau = \varepsilon^{\frac{3}{2}} t \ , \ X = \varepsilon^{\frac{1}{2}} (x - \lambda t) \, , Y = \varepsilon^{\frac{1}{2}} y \ , \ Z = \varepsilon^{\frac{1}{2}} z \ , \tag{4}$$

where ε is a small dimensionless expansion parameter and λ is the speed of DA waves. All physical quantities appearing in (1-3) are expanded as power series in ε about their equilibrium values as:

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + + \varepsilon^3 n_3 + \dots, \tag{5a}$$

$$u_x = \varepsilon u_{x1} + \varepsilon^2 u_{x2} + \varepsilon^3 u_{x3} + \dots, \tag{5b}$$

$$u_y = \varepsilon^{\frac{3}{2}} u_{y1} + \varepsilon^2 u_{y2} + \varepsilon^{\frac{5}{2}} u_{y3} + \dots, \tag{5c}$$

$$u_z = \varepsilon^{\frac{3}{2}} u_{z1} + \varepsilon^2 u_{z2} + \varepsilon^{\frac{5}{2}} u_{z3} + \dots, \tag{5d}$$

$$N = 1 + \varepsilon N_1 + \varepsilon^2 N_2 + + \varepsilon^3 N_3 + \dots, \tag{5e}$$

$$v_x = \varepsilon v_{x1} + \varepsilon^2 v_{x2} + \varepsilon^3 v_{x3} + \dots, \tag{5f}$$

$$v_y = \varepsilon^{\frac{3}{2}} v_{y1} + \varepsilon^2 v y_2 + \varepsilon^{\frac{5}{2}} v_{y3} + \dots, \tag{5g}$$

$$v_z = \varepsilon^{\frac{3}{2}} v_{z1} + \varepsilon^2 v_{z2} + \varepsilon^{\frac{5}{2}} v_{z3} + \dots, \tag{5h}$$

$$p = 1 + \varepsilon p_1 + \varepsilon^2 p_2 + \varepsilon^3 p_3 + \dots, \tag{5i}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \dots . \tag{5j}$$

The charge-neutrality condition in the dusty plasma is always maintained through the relation:

$$1 - \mu_1 + \mu_2 - \mu_3 = 0 . \tag{6a}$$

We impose the boundary conditions that as:

$$|x, y, z| \to \infty,\ n = N = 1,\ p = 1,\ u = v = 0,\ \phi = 0 . \tag{6b}$$

Substituting (4) and (5) into (1-3) and equating coefficients of like powers in the lowest-order equations in ε, the following results are obtained:

$$x_1 = \frac{\rho_1\,\phi_1}{\lambda\mu_1}, \rho_1 = \lambda^2\left(\mu_3 + \mu_2\sigma_e\right) - 1, \tag{7a}$$

$$n_1 = \frac{\rho_1\,\phi_1}{\lambda^2\mu_1}, \qquad uu_{y_1} = -\frac{\left(\mu_1\lambda^2 + \gamma\rho_1\sigma_d\right)}{\lambda^2\mu_1\Omega_1}\frac{\partial\phi_1}{\partial z}, \tag{7b}$$

$$u_{z_1} = \frac{\left(\mu_1\lambda^2 + \gamma\rho_1\sigma_d\right)}{\lambda^2\mu_1\Omega_1}\frac{\partial\phi_1}{\partial y}, \tag{7c}$$

$$p_1 = \frac{\gamma\rho_1\phi_1}{\lambda^2\mu_1}, \tag{7d}$$

$$N_1 = \frac{-\phi_1}{\lambda^2}, v_{x1} = -\frac{\phi_1}{\lambda}, \qquad v_{y_1} = -\frac{1}{\Omega_2}\frac{\partial\phi_1}{\partial z}, \qquad v_{z_1} = -\frac{1}{\Omega_2}\frac{\partial\phi_1}{\partial y}. \tag{7e}$$

Poisson's equation gives the linear dispersion relation

$$\frac{\lambda^2\mu_1 - \rho_1\left(\lambda^2\rho - \gamma\sigma_d\right)}{\lambda^2\mu_1} = 0 . \tag{8}$$

The next-order of the perturbation gives:

$$\frac{\partial n_1}{\partial\tau} - \lambda\frac{\partial n_2}{\partial x} + n_1\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{x2}}{\partial x} + \frac{\partial u_{y2}}{\partial y} + \frac{\partial u_{z2}}{\partial z} = 0, \tag{9a}$$

$$-\lambda\frac{\partial u_{x2}}{\partial x} + \rho\frac{\partial u_{x1}}{\partial\tau} + \rho u_{x1}\frac{\partial u_{x1}}{\partial x} - \sigma_d n_1\frac{\partial P_1}{\partial x} + \sigma_d\frac{\partial P_2}{\partial x} + \frac{\partial\phi_2}{\partial x}, \tag{9b}$$

$$-\Omega_1 u_{z\,3} + \sigma_d \frac{\partial P_2}{\partial y} - \sigma_d \; n_1 \frac{\partial P_1}{\partial y} + \frac{\partial \phi_2}{\partial y} - \lambda\rho \frac{\partial u_{y\,2}}{\partial x} = 0\,, \tag{9c}$$

$$\Omega_1 u_{y\,3} - \sigma_d \; n_1 \frac{\partial P_1}{\partial z} + \sigma_d \frac{\partial P_2}{\partial z} - \lambda\rho \frac{\partial u_{z\,2}}{\partial x} + \frac{\partial \phi_2}{\partial z} = 0\,, \tag{9d}$$

$$\frac{\partial P_1}{\partial \tau} - \lambda \frac{\partial P_2}{\partial x} + u_{x\,1} \frac{\partial P_1}{\partial x} + \gamma P_1 \frac{\partial u_{x\,1}}{\partial x} + \gamma (\frac{\partial u_{x\,2}}{\partial x} + \frac{\partial u_{y\,2}}{\partial y} + \frac{\partial u_{z\,2}}{\partial z}) = 0\,, \tag{9e}$$

$$u_{y\,2} = \frac{\rho\left(\mu_1\lambda^2 + \gamma\rho_1\sigma_d\right)}{\lambda\mu_1\Omega_1^2} \frac{\partial^2 \phi_1}{\partial xy}\,, \tag{9f}$$

$$u_{z\,2} = \frac{\rho\left(\mu_1\lambda^2 + \gamma\rho_1\sigma_d\right)}{\lambda\mu_1\Omega_1^2} \frac{\partial^2 \phi_1}{\partial xz}\,, \tag{9g}$$

for Positive dust plasma and

$$\frac{\partial N_1}{\partial \tau} - \lambda \frac{\partial N_2}{\partial x} + \frac{\partial}{\partial x}(v_{x\,1} N_1 + v_{x\,2}) + \frac{\partial v_{y\,2}}{\partial y} + \frac{\partial v_{z\,2}}{\partial z} = 0\,, \tag{10a}$$

$$\frac{\partial v_{x\,1}}{\partial \tau} + v_{x\,1} \frac{\partial v_{x\,1}}{\partial x} - \lambda \frac{\partial v_{x\,2}}{\partial x} - \frac{\partial \phi_2}{\partial x} = 0\,, \tag{10b}$$

$$-\Omega_2 v_{z\,3} - \frac{\partial \phi_2}{\partial y} - \lambda \frac{\partial v_{y\,2}}{\partial x} = 0\,, \tag{10c}$$

$$-\Omega_2 v_{y\,3} - \frac{\partial \phi_2}{\partial y} - \lambda \frac{\partial v_{z\,2}}{\partial x} = 0\,, \tag{10d}$$

$$v_{y\,2} = \frac{\lambda}{\Omega_2^2} \frac{\partial^2 \phi_1}{\partial xy}\,, \tag{10e}$$

$$v_{z\,2} = \frac{\lambda}{\Omega_2^2} \frac{\partial^2 \phi_1}{\partial xz}\,, \tag{10f}$$

for negative dust plasma and Poisson's equation is

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\phi_1 - \frac{1}{2}\mu_2\sigma_e^2 \; \phi_1^2 + \frac{1}{2}\mu_3\phi_1^2 + \mu_1 n_2 - N_2 - \mu_3 \; \phi_2 - \mu_2\sigma_e \; \phi_2 = 0\,. \tag{11}$$

Eliminating the second order perturbed quantities n_2, u_2, N_2, ν_2 and ϕ_2 in (9-11), we derive the desired ZK equation

$$\frac{\partial\phi_1}{\partial\tau}+\mathbf{A}\phi_1\frac{\partial\phi_1}{\partial x}+C\,(\frac{\partial^3\phi_1}{\partial x\,\partial y^2}+\frac{\partial^3\phi_1}{\partial x\,\partial z^2})+\mathbf{B}\frac{\partial^3\phi_1}{\partial x^3}=0, \tag{12}$$

where

$$\mathbf{A}=\frac{\left(3\rho\lambda^2+(\gamma-2)\gamma\sigma_d\right)\left(\lambda^2\left(\mu_3+\mu_2\sigma_e\right)-1\right)^2+\mu_1\left(\lambda^2\rho-\gamma\sigma_d\right)\left(\lambda^4\left(\mu_3-\mu_2\sigma_e^2\right)-3\right)}{2\lambda\mu_1\left(\lambda^4\rho\left(\mu_3+\mu_2\sigma_e\right)-\gamma\sigma_d\right)},$$

$$\mathbf{B}=\frac{\lambda^5\rho-\gamma\lambda^3\sigma_d}{2\left(\lambda^4\rho\left(\mu_3+\mu_2\sigma_e\right)-\gamma\sigma_d\right)},$$

$$\mathbf{C}=\frac{\lambda^3\left(\left(\lambda^2\rho-\gamma\sigma_d\right)\left(\Omega_2-1\right)\left(\Omega_2+1\right)\Omega_1^2+\rho^2\left(\mu_1\lambda^2+\gamma\sigma_d\left(\lambda^2\left(\mu_3+\mu_2\sigma_e\right)-1\right)\right)\Omega_2^2\right)}{2\left(\lambda^4\rho\left(\mu_3+\mu_2\sigma_e\right)-\gamma\sigma_d\right)\Omega_1^2\Omega_2^2}.$$

4. Stationary Solution

In order to obtain a stationary solution for equation (12), let us introduce the following traveling variable

$$\eta=(Lx+My+\Gamma z-\nu\tau), \tag{13}$$

where η is the transformed coordinate relative to a frame which moves with the velocity ν, L, M and Γ are the directional cosines of the wave vector k along the x, y and z, respectively, in the way that $L^2+M^2+\Gamma^2=1$. By integrating equation (12) with respect to the variable η and using the vanishing boundary condition for $\phi_1(\eta)$ and its derivatives up to the second-order for $|\eta|\to\infty$, we have the one-soliton solution

$$\phi_1=\phi_0\ \mathrm{sech}^2\left(\eta/\Delta\right), \tag{14}$$

where the soliton amplitude ϕ_0 and the soliton width Δ are given by

$$\phi_0=\frac{3\nu}{G_1},$$

$$\Delta = 2\frac{\sqrt{G_2}}{\sqrt{\nu}},$$

$$G_1 = LA ,$$

$$G_2 = BL^3 + (1 - L^2)C .$$

5. Results and Discussion

Three dimensional nonlinear dust-acoustic solitary waves (DAWs) in a magnetized collision less plasma consisting of four-component dusty plasma with massive, micron-sized, positively, negatively dust grains and isothermal electron and ion have been investigated. The application of the reductive perturbation theory to the basic set of fluid equations leads to a ZK equation (12) which describes the nonlinear evolution of the DAWs.

To make our result physically relevant, numerical calculations were performed referring to typical dusty plasma parameters as given in [Havens et al. (2001), Smiley (2003)]. Since one of our motivations was to study the effect of some plasma parameters such as σ_e , L , Ω_1 and Ω_2 on the existence of solitary waves. For example, the basic properties amplitude and width of the small amplitude electrostatic solitary structures are displayed in Figures 1-4. It is obvious from Figures 1 and 2 that the magnitude of the soliton amplitude decrease (increase) and the width increase (decrease) with the increase of $cos(\theta)$ (σ_e). The variations of the bell type solitons with respect to ν and η are displayed in Fig 3. On the other hand, the effect of Ω_1 and Ω_2 on the soliton width are shown in Figure 4.

The results in this paper show that the parameters σ_e , $cos(\theta)$, Ω_1 and Ω_2 modify significantly the properties of dust-acoustic solitary waves. Therefore the present investigation can help us to identify the origin of charge separation as well as dust coagulation in plasma containing positive and negative dust. We would like to point out that the lowest order amplitude of the DAWs soliton ϕ_0 is independent of the magnitude of the external magnetic field and the positive and negative charged dust cyclotron frequencies Ω_1 and Ω_2. So, we stress the need to include the higher-order amplitude of the DAWs effect in the analysis to take the dependence of higher-order corrections on the external magnetic field and the positive and negative charged dust cyclotron frequencies. This is outside the scope of the current investigation and may be addressed in subsequent work.

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Figure captions:

Figure (1): The variation of the soliton amplitude and width with respect to η for different values of $Cos\theta$ (L=0.8 solid line, L=0.9 dotted line) for $\gamma = \frac{5}{3}, \sigma_d = 0.1, \rho = 0.4$, $\sigma_e = 0.3$, $\upsilon = 0.4$, $\mu_2 = 0.3$, $\Omega_1 = 0.62$, $\Omega_2 = 0.6$ and $\mu_3 = 0.5$.

Figure (2): The variation of the soliton amplitude and width with respect to η for different values of σ_e ($\sigma_e = 0.01$ solid line, $\sigma_e = 0.5$ dotted line) for $\gamma = \frac{5}{3}, \sigma_d = 0.1, \rho = 0.4$, $L = 0.8$, $\upsilon = 0.4$, $\mu_2 = 0.3$, $\Omega_1 = 0.62$, $\Omega_2 = 0.6$ and $\mu_3 = 0.5$.

Figure (3): The variation of the bell type soliton with respect to ν and η for different values of for $\gamma = \frac{5}{3}, \sigma_d = 0.1, \rho = 0.4$, $L = 0.8$, $\upsilon = 0.4$, $\sigma_e = 0.1$, $\mu_2 = 0.3$, $\Omega_1 = 0.5$, $\Omega_2 = 0.55$ and $\mu_3 = 0.5$.

Figure (4): The variation of the soliton width with respect to Ω_1 *and* Ω_2 for $\gamma = \frac{5}{3}, \sigma_d = 0.1$, $\rho = 0.4$, $\sigma_e = 0.3, \upsilon = 0.4, \sigma_e = 0.1, \mu_2 = 0.3$ and $\mu_3 = 0.5$.

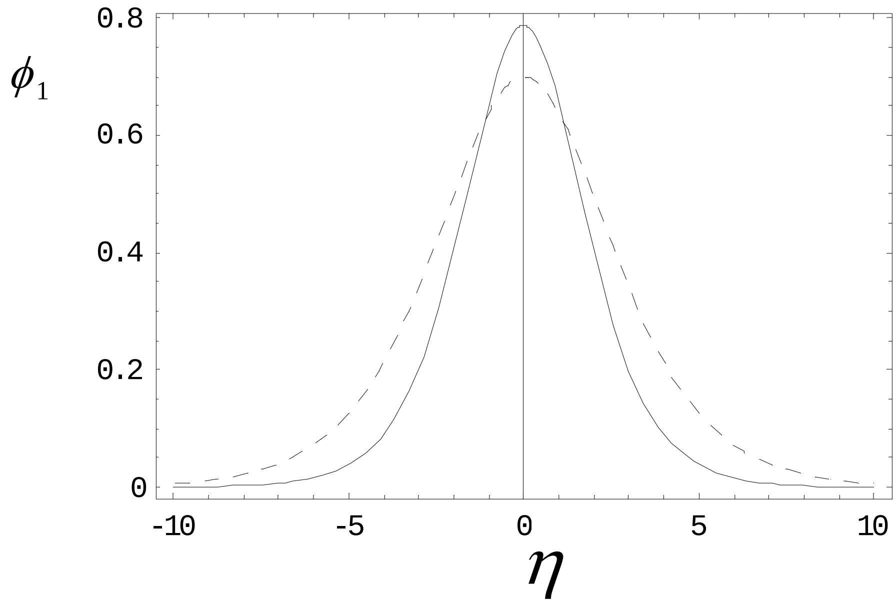


Fig. (1)

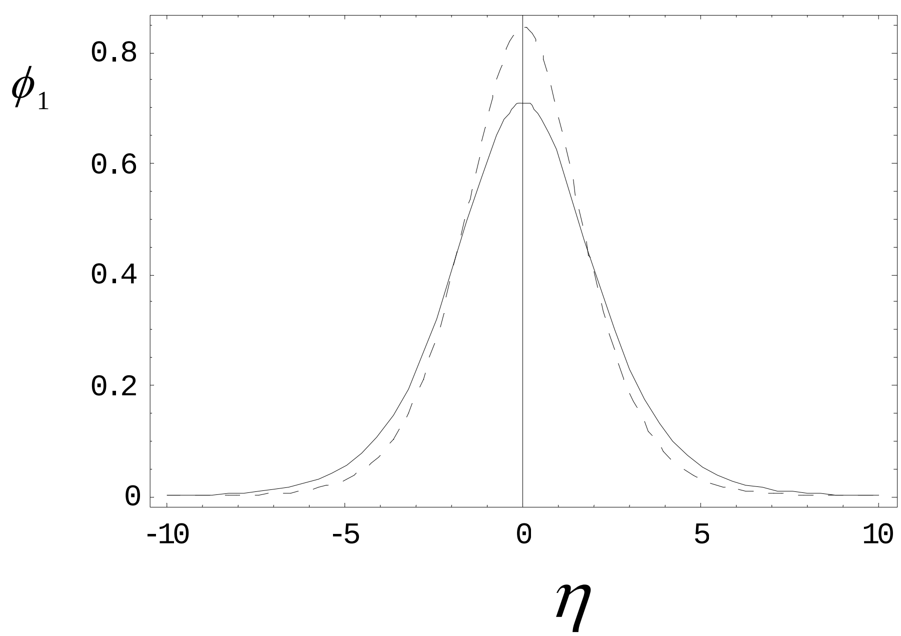
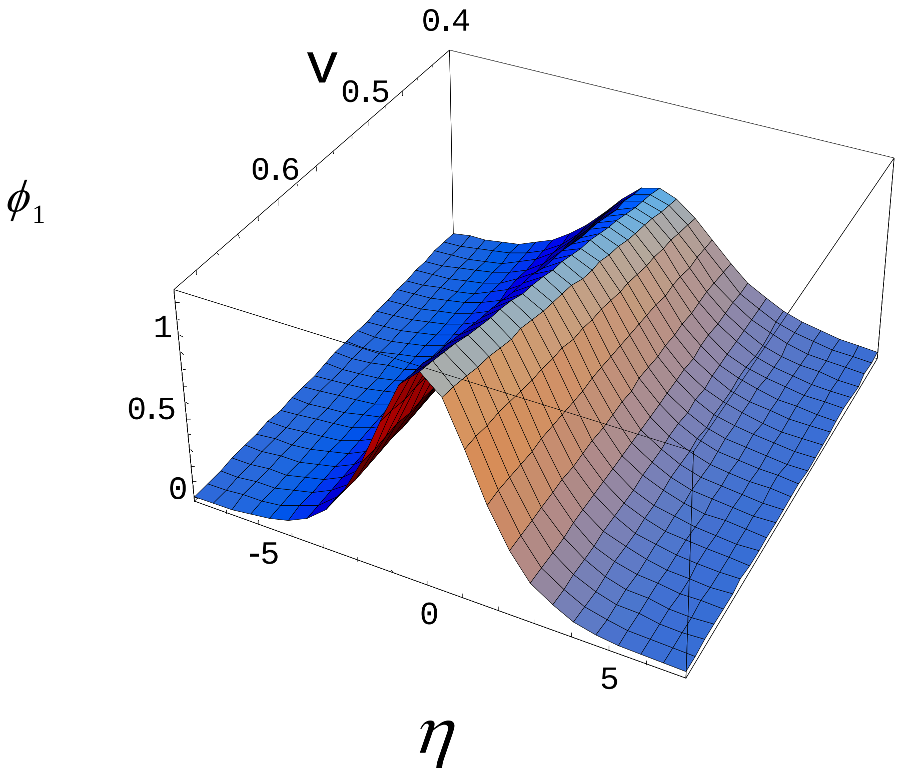


Fig. (2)



0.4
V
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-5
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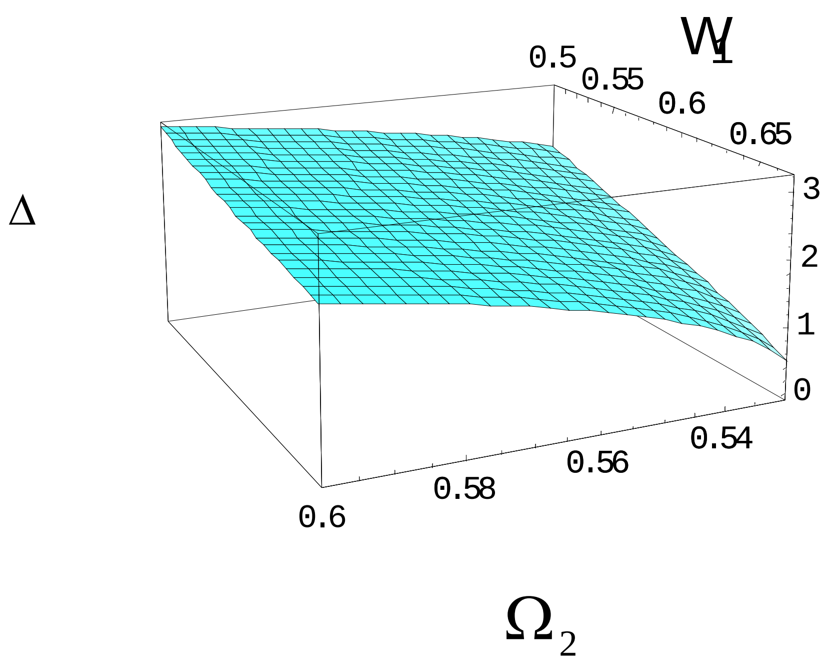
Fig. (3)



W1
0.5
0.55
0.6
0.65
Δ
3
2
1
0
0.54
0.56
0.58
0.6
Ω_2

Fig. (4)