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A New Algorithm for Solving Shortest Path Problem on a Network with Imprecise Edge Weight

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Abstract

Nayeem and Pal (Shortest path problem on a network with imprecise edge weight, Fuzzy Optimization and Decision Making 4, 293-312, 2005) proposed a new algorithm for solving shortest path problem on a network with imprecise edge weight. In this paper the shortcomings of the existing algorithm, (Nayeem and Pal, 2005) are pointed out and to overcome these shortcomings a new algorithm is proposed. To show the advantages of the proposed algorithm over existing algorithm the numerical examples presented in (Nayeem and Pal, 2005) are solved using the proposed algorithm and obtained results are discussed.

Keywords: Fuzzy shortest path problem, Ranking function, Interval numbers, Triangular fuzzy numbers

MSC 2010 No.: 03E72, 74P10, 52B05

1. Introduction

The shortest path problem concentrates on finding the path with minimum distance. To find the shortest path from a source node to the other nodes is a fundamental matter in graph theory. In conventional shortest path problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in the real life situations there always exists uncertainty about the parameters of shortest path problems. To deal with such type of problems, the parameters of shortest path problems are represented by fuzzy numbers (Zadeh, 1965).

Klein (1991) presented new models based on fuzzy shortest paths and also given a general algorithm based on dynamic programming to solve the new models. Lin and Chern (1993) considered the case that the arc lengths are fuzzy numbers and proposed an algorithm for finding the single most vital arc in a network. Okada and Gen (1994) discussed the problem of finding the shortest paths from a fixed origin to a specified node in a network with arcs represented as intervals on real line. Li et al. (1996) introduced the neural networks for solving fuzzy shortest path problems. Gent et al. (1997) investigated the possibility of using genetic algorithms to solve shortest path problems. Shih and Lee (1999) investigated multiple objective and multiple hierarchies minimum cost flow problems with fuzzy costs and fuzzy capacities in the arcs. Okada and Soper (2000) concentrated on a shortest path problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length. Okada (2001) concentrated on a shortest path problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length. Liu and Kao (2004) investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Seda (2005) dealed with the steiner tree problem on a graph in which a fuzzy number, instead of a real number, is assigned to each edge.

Takahashi (2005) discussed the shortest path problem with fuzzy parameters. He proposed a modification in Okada's (2001) algorithm, using some properties observed by other authors. He also proposed a genetic algorithm to seek an approximated solution for large scale problems. Chuang and Kung (2005) represented each arc length as a triangular fuzzy number and proposed a new algorithm to deal with the fuzzy shortest path problems. Nayeem and Pal (2005) considered a network with its arc lengths as imprecise number, instead of a real number, namely, interval number and triangular fuzzy number. Ma and Chen (2005) proposed an algorithm for the on-line fuzzy shortest path problems, based on the traditional shortest path problem in the domain of the operations research and the theory of the on-line algorithms. Kung and Chuang (2005) proposed a new algorithm composed of fuzzy shortest path length procedure and similarity measure to deal with the fuzzy shortest path problem. Gupta and Pal (2006) presented an algorithm for the shortest path problem when the connected arcs in a transportation network are represented as interval numbers.

Moazeni (2006) discussed the shortest path problem from a specified node to every other node on a network in which a positive fuzzy quantity with finite support is assigned to each arc as its arc length. Chuang and Kung (2006) pointed out that there are several methods reported to solve this kind of problem in the open literature. In these methods, they can obtain either the fuzzy shortest length or the shortest path. In their paper, a new algorithm was proposed that can obtain both of them. The discrete fuzzy shortest length method is proposed to find the fuzzy shortest length, and the fuzzy similarity measure is utilized to get the shortest path. Ji et al. (2007) considered the shortest path problem with fuzzy arc lengths. According to different decision criteria, the concepts of expected shortest path, a-shortest path and the shortest path in fuzzy environment are originally proposed, and three types of models are formulated. In order to solve these models, a hybrid intelligent algorithm integrating simulation and genetic algorithm is provided and some numerous examples are given to illustrate its effectiveness.

Hernandes et al. (2007) proposed an iterative algorithm that assumes a generic ranking index for comparing the fuzzy numbers involved in the problem, in such a way that each time in which the decision-maker wants to solve a concrete problem (s)he can choose (or propose) the ranking index that best suits that problem. Yu and Wei (2007) proposed a simple linear multiple objective programming to deal with the fuzzy shortest path problem. Mahdavi et al. (2009) proposed a dynamic programming approach to solve the fuzzy shortest chain problem using a suitable ranking method.

In this paper the shortcomings of the existing algorithm, (Nayeem and Pal, 2005) are pointed out and to overcome these shortcomings a new algorithm is proposed. To show the advantages of the proposed algorithm over existing algorithm the numerical examples presented in (Nayeem and Pal, 2005) are solved using the proposed algorithm and obtained results are discussed.

This paper is organized as follows: In Section 2, some basic definitions, addition of interval and triangular fuzzy numbers and comparison methods between such numbers are reviewed and also the notations used throughout the paper are presented. In Section 3, an algorithm is proposed for finding the fuzzy shortest path and fuzzy shortest distance of each node from source node. In Section 4, to illustrate the proposed algorithm and to point out the shortcomings of the existing algorithm (Nayeem and Pal, 2005) numerical examples presented in Nayeem and Pal (2005) are solved by using the proposed algorithm. In Section 5, shortcomings of the existing comparison method (Nayeem and Pal, 2005) are pointed out. In Section 6, the comparison methods proposed by Liou and Wang (1992) are reviewed. In Section 7, the comparison methods, reviewed in Section 6, are used to solve the numerical examples presented in Section 4. The obtained results are discussed in Section 8. The conclusions are discussed in Section 9.

2. Preliminaries

In this section some basic definitions, addition of interval numbers and triangular fuzzy numbers and comparison methods of such numbers are reviewed. Also the notations used throughout the paper are presented.

2.1 Basic Definitions

In this section, some basic definitions are reviewed.

Definition 2.1. (Nayeem and Pal, 2005) An interval number is defined as $A=[a_L,a_R]=\{a: a_L \le a \le a_R\}$, where, a_L and a_R are the real numbers called the left end point and the right end point of the interval A.

Another way to represent an interval number in terms of midpoint and width is $A=\langle m(A), w(A)\rangle$, where $m(A)=$ midpoint of $A=\dfrac{a_R+a_L}{2}$ and $w(A)=$ half width of $A=\dfrac{a_R-a_L}{2}$.

Definition 2.2. (Nayeem and Pal, 2005) Two interval numbers $A = \langle m_A, w_A \rangle$ and $B = \langle m_B, w_B \rangle$ are said to be non-dominating if

i. $m_A = m_B$ and
ii. $w_A = w_B$.

Definition 2.3. (Nayeem and Pal, 2005) A triangular fuzzy number is represented by a triplet $\widetilde{A} = \langle m, \alpha, \beta \rangle$ with the membership function

$$\mu_{\widetilde{A}}(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha} & \text{for } m - \alpha < x \le m \\ 1 - \dfrac{x-m}{\beta} & \text{for } m < x < m + \beta \\ 0 & \text{otherwise} \end{cases}$$

where $m \in R$ and $\alpha, \beta > 0$.

Definition 2.4. (Nayeem and Pal, 2005) Two triangular fuzzy numbers $\widetilde{A} = \langle a, \alpha, \beta \rangle$ and $\widetilde{B} = \langle b, \gamma, \delta \rangle$ are said to be non-dominating if

i. $a = b$ and
ii. $\alpha = \gamma$ or $\beta = \delta$ but, not both simultaneously.

2.2. Addition of Interval Numbers and Triangular Fuzzy Numbers (Kaufmann and Gupta, 1985)

The addition of two interval numbers $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is given by

$$A \oplus B = [a_L + b_L, a_R + b_R].$$

Alternately, in mean-width notations, if $A = \langle m_1, w_1 \rangle$ and $B = \langle m_2, w_2 \rangle$ then,

$$A \oplus B = \langle m_1 + m_2, w_1 + w_2 \rangle.$$

Let $\widetilde{A} = \langle m_1, \alpha_1, \beta_1 \rangle$ and $\widetilde{B} = \langle m_2, \alpha_2, \beta_2 \rangle$ be two triangular fuzzy numbers then,

$$\widetilde{A} \oplus \widetilde{B} = \langle m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2 \rangle.$$

2.3. Comparison of Interval Numbers (Nayeem and Pal, 2005)

Nayeem and Pal (2005) used acceptability index ($\mathcal{A}$-index) to the proposition '*A* is inferior to *B*' as $\mathcal{A}(A \prec B) = \dfrac{m_2 - m_1}{w_1 + w_2}$.

In connection with this acceptability index, Nayeem and Pal (2005) defined the total dominance and partial dominance of two interval numbers $A = \langle m_1, w_1 \rangle$ and $B = \langle m_2, w_2 \rangle$ one over another as follow:

i. If $\mathcal{A}(A \prec B) \geq 1$ then, A is said to be totally dominating over B in the sense of minimization and *B* is said to be totally dominating over *A* in the sense of maximization. We denote this by $A \prec B$, i.e., minimum $\{A, B\} = A$.
ii. If $0 < \mathcal{A}(A \prec B) < 1$ then, A is said to be partially dominating over B in the sense of minimization and *B* is said to be partially dominating over *A* in the sense of maximization. We denote this by $A \prec_P B$, i.e., minimum $\{A, B\} = A$.
iii. But when, $\mathcal{A}(A \prec B) = 0$ i.e., $m_1 = m_2$ then we may not get an order relation from the above cases. Then we may emphasize on the widths of the interval numbers *A* and *B*.

If $w_1 < w_2$ then the left end point of A is less than that of B and on finding a minimum distance, there is a chance that the distance may lie on A. But at the same time, since the right end point of *A* is greater than that of *B*, if one prefers *A* to *B* in minimization then in worst case, he may be looser than one who prefers *B* to *A*. Thus in such a situation an optimistic decision-maker would prefer *A* to *B* whereas a pessimistic decision-maker would do the converse.

2.4. Comparison of Triangular Fuzzy Numbers (Nayeem and Pal 2005)

The acceptability index ($\mathcal{A}$-index) to the proposition '$\widetilde{A} = \langle a, \alpha, \beta \rangle$ is preferred to $\widetilde{B} = \langle b, \delta, \gamma \rangle$' is given by $\mathcal{A}(\widetilde{A} \prec \widetilde{B}) = \dfrac{b - a}{\beta + \gamma}$.

Using this $\mathcal{A}$-index Nayeem and Pal (2005) defined the following ranking orders.

i. If $\mathcal{A}(\widetilde{A} \prec \widetilde{B}) \geq 1$ then, $\widetilde{A}$ is said to be totally dominating over $\widetilde{B}$ in case of minimization and the case is converse in case of maximization and this is denoted by $\widetilde{A} \prec \widetilde{B}$, i.e., minimum $\{\widetilde{A}, \widetilde{B}\} = \widetilde{A}$.
ii. If $0 < \mathcal{A}(\widetilde{A} \prec \widetilde{B}) < 1$ then, $\widetilde{A}$ is said to be partially dominating over $\widetilde{B}$ in the sense of minimization and $\widetilde{B}$ is said to be partially dominating over $\widetilde{A}$ in the sense of maximization. This is denoted by $\widetilde{A} \prec_P \widetilde{B}$, i.e., minimum $\{\widetilde{A}, \widetilde{B}\} = \widetilde{A}$.

2.5. Notation

In this section the notation that will be used throughout the paper are presented.

$N = \{1,2,\ldots,n\}$	: The set of all nodes in a network.
$Np(j)$	: The set of all predecessor nodes of node j.
e_i	: The distance between node i and first (source) node.
e_{ij}	: The distance between node i and j.
$\tilde{e}_i$	: The fuzzy distance between node i and first (source) node.
$\tilde{e}_{ij}$	: The fuzzy distance between node i and j.

Remark 1. A node i is said to be predecessor node of node j if

(i) Node i is directly connected to node j.
(ii) The direction of path, connecting node i and j, is from i to j.

3. Proposed Algorithm

In this section a new algorithm is proposed for finding the fuzzy shortest path and fuzzy shortest distance of each node from source node.

The steps of the algorithm are summarized as follows:

Step1
Assume $\tilde{e}_1 = \langle 0,0,0 \rangle$ (or $\langle 0,0 \rangle$ interval number) and label the source node (say node 1) as $[\langle 0,0,0 \rangle, -]$ (or $[\langle 0,0 \rangle, -]$).

Step 2
Find $\tilde{e}_j = \text{minimum}\{\tilde{e}_i \oplus \tilde{e}_{ij} \,/\, i \in Np(j)\};\ j \neq 1,\ j = 2,3,\ldots,n.$

Step 3
If minimum occurs corresponding to unique value of i i.e., $i = r$ then label node j as $[\tilde{e}_j, r]$. If minimum occurs corresponding to more than one values of i then it represents that there are more than one fuzzy path between source node and node j but fuzzy distance along all paths is $\tilde{e}_j$, so choose any value of i.

Step 4
Let the destination node (node n) be labeled as $[\tilde{e}_n, l]$, then the fuzzy shortest distance between source node and destination node is $\tilde{e}_n$.

Step 5
Since destination node is labeled as $[\widetilde{e}_n, l]$. So, to find the fuzzy shortest path between source node and destination node, check the label of node l. Let it be $[\widetilde{e}_l, p]$, now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

Step 6
Now the fuzzy shortest path can be obtained by combining all the nodes obtained by the Step 5.

Remark 2. If there is no uncertainty about any parameter then the proposed algorithm is also a new algorithm for finding the optimal solution for conventional shortest path problems.

4. Illustrative Examples

To show the advantages of the proposed algorithm over the existing algorithm (Nayeem and Pal, 2005), the numerical examples presented in Nayeem and Pal (2005) are solved by using the proposed algorithm and the results of existing and the proposed algorithms are compared.

Example 1. (Nayeem and Pal, 2005) The problem is to find the shortest path between source node (say node 1) and destination node (say node 6) on the network consists of 6 vertices {1,2,3,4,5,6} and 11 edges $\{e_{12}, e_{13}, e_{14}, e_{23}, e_{24}, e_{34}, e_{35}, e_{36}, e_{45}, e_{46}, e_{56}\}$ the arc lengths of the network, shown in Figure 1 are all interval numbers and given by

$e_{12} = [10,12], e_{13} = [25,28]$, $e_{14} = [19,20]$, $e_{23} = [20,21]$, $e_{24} = [30,35]$, $e_{34} = [6.5,7.5]$, $e_{35} = [38,40]$, $e_{36} = [43,44]$, $e_{45} = [35,40]$, $e_{46} = [49,51]$, $e_{56} = [12,13]$.

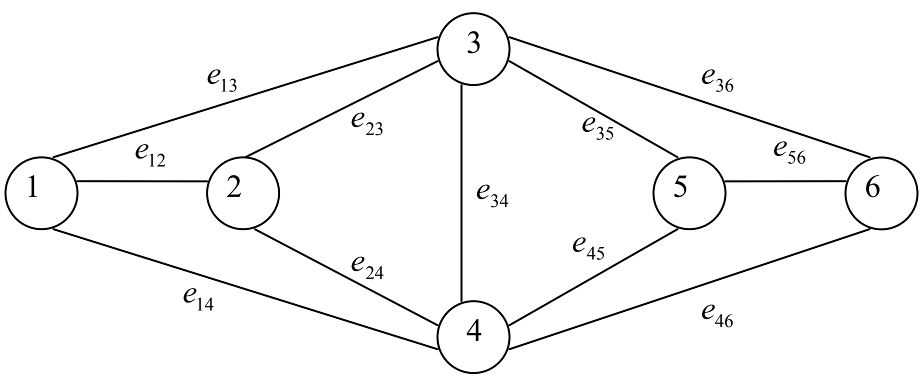

Figure 1. A network

Solution The mean-width notations of interval numbers as follow:

$e_{12}=\langle 11,1\rangle$, $e_{13}=\langle 26.5,1.5\rangle$, $e_{14}=\langle 19.5,0.5\rangle$, $e_{23}=\langle 20.5,0.5\rangle$, $e_{24}=\langle 32.5,2.5\rangle$, $e_{34}=\langle 7,0.5\rangle$, $e_{35}=\langle 39,1\rangle$, $e_{36}=\langle 43.5,0.5\rangle$, $e_{45}=\langle 37.5,2.5\rangle$, $e_{46}=\langle 50,1\rangle$, $e_{56}=\langle 12.5,0.5\rangle$.

Since node 6 is the destination node, so $n=6$.

Assume $e_1=\langle 0,0\rangle$ and label the source node (say node 1) as $[\langle 0,0\rangle,-]$, the values of e_j ; $j=2,3,4,5,6$ can be obtained as follows:

Iteration 1
Since only node 1 is the predecessor node of node 2, so putting $i=1$ and $j=2$ in Step 2 of the proposed algorithm, the value of e_2 is

$$e_2=\text{minimum}\,\{e_1\oplus e_{12}\}=\text{minimum}\,\{\langle 0,0\rangle\oplus\langle 11,1\rangle\}=\langle 11,1\rangle.$$

Since minimum occurs corresponding to $i=1$, so label node 2 as $[\langle 11,1\rangle,1]$.

Iteration 2
The predecessor nodes of the node 3 are node 1 and 2, so putting $i=1,2$ and $j=3$ in step 2 of the proposed algorithm, the value of e_3 is

$$e_3=\text{minimum}\,\{e_1\oplus e_{13},e_2\oplus e_{23}\}$$

$$=\text{minimum}\,\{\langle 0,0\rangle\oplus\langle 26.5,1.5\rangle,\langle 11,1\rangle\oplus\langle 20.5,0.5\rangle\}$$

$$=\text{minimum}\,\{\langle 26.5,1.5\rangle,\langle 31.5,1.5\rangle\}.$$

$$A(\langle 26.5,1.5\rangle\prec\langle 31.5,1.5\rangle)=\frac{31.5-26.5}{1.5+1.5}=1.66>1.$$

Using Section 2.3, $\text{minimum}\,\{\langle 26.5,1.5\rangle,\langle 31.5,1.5\rangle\}=\langle 26.5,1.5\rangle$.
i.e., $e_3=\langle 26.5,1.5\rangle$.

Since minimum occurs corresponding to $i=1$, so label node 3 as $[\langle 26.5,1.5\rangle,1]$.

Iteration 3
The predecessor node of the node 4 is node 1, 2 and 3, so putting $i=1$, 2, 3 and $j=4$ in step 2 of the proposed algorithm, the value of e_4 is

$$e_4=\text{minimum}\,\{e_1\oplus e_{14},e_2\oplus e_{24},e_3\oplus e_{34}\}$$

$$= \text{minimum}\left\{\langle 0,0\rangle \oplus \langle 19.5,0.5\rangle, \langle 11,1\rangle \oplus \langle 32.5,2.5\rangle, \langle 26.5,1.5\rangle \oplus \langle 7,0.5\rangle\right\} = \langle 19.5,0.5\rangle .$$

Since minimum occurs corresponding to $i = 1$, so label node 4 as $[\langle 19.5,0.5\rangle,1]$.

Iteration 4

The predecessor nodes of the node 5 are node 3 and 4, so putting $i = 3$, 4 and $j = 5$ in Step 2 of the proposed algorithm, the value of e_5 is

$$e_5 = \text{minimum}\{e_3 \oplus e_{35}, e_4 \oplus e_{45}\}$$

$$= \text{minimum}\left\{\langle 26.5,1.5\rangle \oplus \langle 39,1\rangle, \langle 19.5,0.5\rangle \oplus \langle 37.5,2.5\rangle\right\}$$

$$= \text{minimum}\left\{\langle 65.5,2.5\rangle, \langle 57,3\rangle\right\} = \langle 57,3\rangle .$$

Since minimum occurs corresponding to $i = 4$, so label node 5 as $[\langle 57,3\rangle,4]$.

Iteration 5

The predecessor nodes of the node 6 are node 3, 4 and 5, so putting $i = 3, 4, 5$ and $j = 6$ in step 2 of the proposed algorithm, the value of e_6 is

$$e_6 = \text{minimum}\{e_3 \oplus e_{36}, e_4 \oplus e_{46}, e_5 \oplus e_{56}\}$$

$$= \text{minimum}\left\{\langle 26.5,1.5\rangle \oplus \langle 43.5,0.5\rangle, \langle 19.5,0.5\rangle \oplus \langle 50,1\rangle, \langle 57,3\rangle \oplus \langle 12.5,0.5\rangle\right\}$$

$$= \text{minimum}\left\{\langle 70,2\rangle, \langle 69.5,1.5\rangle, \langle 69.5,3.5\rangle\right\}.$$

$$e_6 = \langle 69.5,1.5\rangle \text{ or } \langle 69.5,3.5\rangle .$$

Since minimum occurs corresponding to $i =$ 4, 5 so we can label node 6 as $[\langle 69.5,1.5\rangle,4]$ or $[\langle 69.5,3.5\rangle,5]$, if we label node 6 as $[\langle 69.5,1.5\rangle,4]$ then the corresponding shortest distance is 69.5 . Now the fuzzy shortest path between node 1 and node 6 can be obtained by using the following procedure:

Since node 6 is labeled by $[\langle 69.5,1.5\rangle,4]$, which represents that we are coming from node 4. Node 4 is labeled by $[\langle 19.5,0.5\rangle,1]$, which represents that we are coming from node 1. Now the fuzzy shortest path between node 1 and node 6 is obtained by joining all the obtained nodes. Hence the fuzzy shortest path is $1 \to 4 \to 6$ and in the second case if we label node 6 as $[\langle 69.5,3.5\rangle,5]$ then the corresponding shortest distance is same i.e., 69.5 but the shortest path is $1 \to 4 \to 5 \to 6$.

Example 2. (Nayeem and Pal, 2005) Let us consider the same network, shown in Fig. 1, with its arc lengths as triangular fuzzy numbers given by

$\tilde{e}_{12} = (10,11,12)$, $\tilde{e}_{13} = (25,27,28)$, $\tilde{e}_{14} = (19,20,22)$, $\tilde{e}_{23} = (20,21,21)$, $\tilde{e}_{24} = (30,34,35)$, $\tilde{e}_{34} = (6.5,7,8)$, $\tilde{e}_{35} = (30,30,32)$, $\tilde{e}_{36} = (43,44,45)$, $\tilde{e}_{45} = (39,40,40)$, $\tilde{e}_{46} = (49,50,52)$, $\tilde{e}_{56} = (9,9,10)$ and we are interested to find the fuzzy shortest path and fuzzy shortest path between the nodes 1 and 6.

Solution:

The triangular fuzzy numbers in the form of $\langle m,\alpha,\beta\rangle$, i.e., in terms of mean and the left-spreads and right-spreads are as follow:

$\tilde{e}_{12} = \langle 11,1,1\rangle$, $\tilde{e}_{13} = \langle 27,2,1\rangle$, $\tilde{e}_{14} = \langle 20,1,2\rangle$, $\tilde{e}_{23} = \langle 21,1,0\rangle$, $\tilde{e}_{24} = \langle 34,4,1\rangle$, $\tilde{e}_{34} = \langle 7,0.5,1\rangle$, $\tilde{e}_{35} = \langle 30,0,2\rangle$, $\tilde{e}_{36} = \langle 44,1,1\rangle$, $\tilde{e}_{45} = \langle 40,1,0\rangle$, $\tilde{e}_{46} = \langle 50,1,2\rangle$, $\tilde{e}_{56}\langle 9,0,1\rangle$.
Since node 6 is the destination node, so $n = 6$.

Assume $\tilde{e}_1 = \langle 0,0,0\rangle$ and label the source node (say node 1) as $[\langle 0,0,0\rangle,-]$, the values of $\tilde{e}_j$; $j = 2,3,4,5,6$ can be obtained as follows:

Iteration 1
Since only node 1 is the predecessor node of node 2, so putting $i = 1$ and $j = 2$ in Step 2 of the proposed algorithm, the value of $\tilde{e}_2$ is

$$\tilde{e}_2 = \text{minimum}\{\tilde{e}_1 \oplus \tilde{e}_{12}\} = \text{minimum}\{\langle 0,0,0\rangle \oplus \langle 11,1,1\rangle\} = \langle 11,1,1\rangle.$$

Since minimum occurs corresponding to $i = 1$, so label node 2 as $[\langle 11,1,1\rangle,1]$.

Iteration 2
The predecessor nodes of the node 3 are node 1 and 2, so putting $i = 1, 2$ and $j = 3$ in Step 2 of the proposed algorithm, the value of $\tilde{e}_3$ is

$$\tilde{e}_3 = \text{minimum}\{\tilde{e}_1 \oplus \tilde{e}_{13}, \tilde{e}_2 \oplus \tilde{e}_{23}\}$$

$$= \text{minimum}\{\langle 0,0,0\rangle \oplus \langle 27,2,1\rangle, \langle 11,1,1\rangle \oplus \langle 21,1,0\rangle\}$$

$$= \text{minimum}\{\langle 27,2,1\rangle, \langle 32,2,1\rangle\}.$$

Since $\tilde{A} = \langle a,\alpha,\beta\rangle = \langle 27,2,1\rangle$ and $\tilde{B} = \langle b,\gamma,\delta\rangle = \langle 32,2,1\rangle$.

$$\mathcal{A}\ (\widetilde{A} \prec \widetilde{B}) = \frac{32-27}{1+2} = 1.66 > 1 .$$

So using Section 2.3, minimum $\{\langle 27,2,1\rangle, \langle 32,2,1\rangle\} = \langle 27,2,1\rangle$, i.e., $\widetilde{e}_3 = \langle 27,2,1\rangle$.

Since minimum occurs corresponding to $i = 1$, so label node 3 as $[\langle 27,2,1\rangle,1]$.

Similarly, $\widetilde{e}_4 = \langle 20,1,2\rangle$, label node 4 as $[\langle 20,1,2\rangle,1]$,
$\widetilde{e}_5 = \langle 57,2,3\rangle$, label node 5 as $[\langle 57,2,3\rangle,3]$,
$\widetilde{e}_6 = \langle 66,2,4\rangle$, label node 6 as $[\langle 66,2,4\rangle,5]$.

Since node 6 is the destination node of the given network, so the fuzzy shortest distance between node 1 and 6 is $\langle 66,2,4\rangle$ and the fuzzy shortest path is $1 \to 3 \to 5 \to 6$.

5. Shortcomings of Existing Comparison Methods (Nayeem and Pal, 2005)

To show the advantages of the proposed algorithm over existing algorithm (Nayeem and Pal, 2005) the numerical examples presented in Nayeem and Pal (2005) are solved by the proposed algorithm and it is found that the results of existing and proposed algorithm are same, while the existing algorithm is very confusing to understand and to apply for finding the optimal solution compare to the proposed algorithm.

For solving the numerical examples the comparison methods presented in Nayeem and Pal (2005) are used but there are the following shortcomings in these comparison methods:

(i) To show that the existing comparison method (Nayeem and Pal, 2005) can't be used for finding the fuzzy shortest path of real life problems the fuzzy shortest path and fuzzy shortest distance between node 1 and 4 of the network, shown in Fig. 2, is obtained by the existing comparison method (Nayeem and Pal, 2005) and the obtained results are as follows:

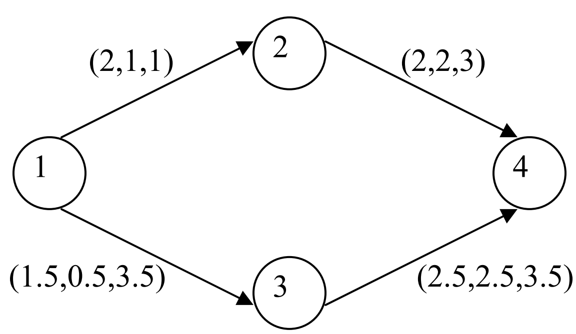

Figure 2. A network

In the network shown in Figure 2, there are two possible paths $1 \to 2 \to 4$ and $1 \to 3 \to 4$ between node 1 and 4. Using the existing comparison method (Nayeem and Pal, 2005) the distance between node 1 and node 4 along the first path i.e. $1 \to 2 \to 4$ is (4,3,4) while along the second path i.e. $1 \to 3 \to 4$ the distance between the node 1 and node 4 is (4,3,7). It is obvious from definition 2.4 that the distances (4,3,4) and (4,3,7) are non-dominating and it is not possible to find the minimum between these distances so according to existing comparison method (Nayeem and Pal, 2005) the decision maker can choose either $1 \to 2 \to 4$ or $1 \to 3 \to 4$ i.e. using the existing comparison method it is not possible to choose the best from $1 \to 2 \to 4$ and $1 \to 3 \to 4$. But it is obvious from the values of the distances of paths that a decision maker will choose the path $1 \to 2 \to 4$. Since along this path the traveled distance will be between 1 unit and 8 unit and the maximum possibility is that it will be 4 unit while along the second path the traveled distance will be between 1 and 11 unit and the maximum possibility is that it will be 4 unit. Hence it can be concluded that existing comparison method Nayeem and Pal (2005) should not be used to compare the fuzzy numbers for solving real life problems.

(ii) Nayeem and Pal (2005) have pointed out that their method for comparison of different numbers is particular case of Okada and Soper (2000) method but from the network, shown in Fig. 3, it is clear that the results are different using both existing methods.

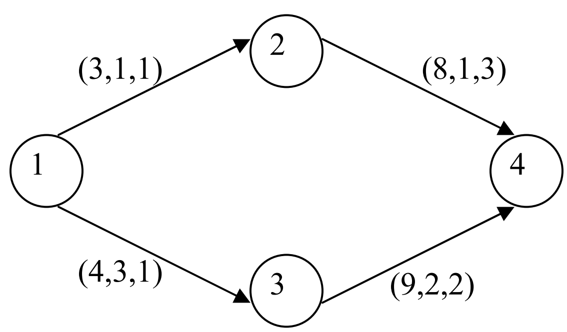

Figure 3. A network

According to comparison method presented in Okada and Soper (2000) the fuzzy shortest path and fuzzy shortest distance are $1 \to 3 \to 4$ and (13,5,3) respectively, while using the comparison method presented in Nayeem and Pal (2005) the fuzzy shortest path and fuzzy shortest distance are $1 \to 2 \to 4$ and (11,2,4) respectively, i.e., according to Okada and Soper (2000) the fuzzy shortest path is $1 \to 3 \to 4$ while according to Nayeem and Pal (2005) the fuzzy shortest path is $1 \to 2 \to 4$.

(iii) Both the existing algorithms (Okada and Soper, 2000; Nayeem and Pal, 2005) are very difficult and confusing to understand and to apply for a new decision maker, for finding the fuzzy optimal solution of shortest path problems occurring in real life problems.

(iv) In the real life problems, it is required to compare more than two fuzzy numbers (or interval numbers) simultaneously. But it is very difficult to compare a large number of fuzzy numbers simultaneously using the existing comparison method (Nayeem and Pal, 2005). For example in 3rd and 5th iteration of Example 1 and 2, it is required to calculate the acceptability index of each pair i.e., it is not easy to find the minimum of three numbers.

To overcome the above shortcomings the existing comparison method (Liou and Wang, 1992) is used for solving the Examples 1 and 2.

6. Comparison of Interval and Triangular Fuzzy Numbers (Liou And Wang, 1992)

Due to the shortcomings of the existing comparison methods (Nayeem and Pal, 2005) it is better to use the following comparison method (Liou and Wang, 1992).

6.1 Comparison of Triangular Fuzzy Numbers

Let $\widetilde{A} = (m_1, \alpha_1, \beta_1)$ and $\widetilde{B} = (m_2, \alpha_2, \beta_2)$ be two triangular fuzzy number then

(i) $\widetilde{A} \succ \widetilde{B}$ if $\Re(\widetilde{A}) \succ \Re(\widetilde{B})$.

(ii) $\widetilde{A} \prec \widetilde{B}$ if $\Re(\widetilde{A}) \prec \Re(\widetilde{B})$.

(iii) $\widetilde{A} \approx \widetilde{B}$ if $\Re(\widetilde{A}) \approx \Re(\widetilde{B})$,

where $\Re(\widetilde{A}) = m_1 - \frac{1}{4}(\alpha_1 - \beta_1)$ and $\Re(\widetilde{B}) = m_2 - \frac{1}{4}(\alpha_2 - \beta_2)$.

6.2 Comparison of Interval Numbers

Let $A = [a_L, a_R]$ and $B = [b_L, b_R]$ be two interval numbers then the symmetric triangular fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ corresponding to A and B are given by $\widetilde{A} = \left(\frac{a_L + a_R}{2}, \frac{a_R - a_L}{2}, \frac{a_R - a_L}{2}\right)$,

$\widetilde{B} = \left(\frac{b_L + b_R}{2}, \frac{b_R - b_L}{2}, \frac{b_R - b_L}{2}\right)$ and $\Re(\widetilde{A}) = \frac{a_L + a_R}{2}$, $\Re(\widetilde{B}) = \frac{b_L + b_R}{2}$.

(i) $A \succ B$ if $\frac{a_L + a_R}{2} \succ \frac{b_L + b_R}{2}$.

(ii) $A \prec B$ if $\frac{a_L + a_R}{2} \prec \frac{b_L + b_R}{2}$.

(iii) $A \approx B$ if $\frac{a_L + a_R}{2} \approx \frac{b_L + b_R}{2}$.

7. Illustrative Examples Using Existing Comparison Method (Liou and Wang, 1992)

In Section 4, to solve the numerical examples the proposed algorithm is used with existing comparison method (Nayeem and Pal, 2005) but due to shortcomings in the existing comparison method in this section the same numerical examples are solved using the proposed algorithm with existing comparison method (Liou and Wang, 1992).

Example 3. Let the arc lengths of the network shown in Fig. 1 be all interval numbers and be given by

$e_{12} = [10,12]$, $e_{13} = [25,28]$, $e_{14} = [19,20]$, $e_{23} = [20,21]$, $e_{24} = [30,35]$, $e_{34} = [6.5,7.5]$, $e_{35} = [38,40]$, $e_{36} = [43,44]$, $e_{45} = [35,40]$, $e_{46} = [49,51]$, $e_{56} = [12,13]$ then we have to find out the shortest path between the vertices 1 and 6.

Solution:

Since node 6 is the destination node, so $n = 6$.

Assume $e_1 = [0,0]$ and label the source node (say node 1) as $\left[[0,0],-\right]$, the values of e_j ; $j = 2,3,4,5,6$ can be obtained as follows:

Iteration 1
Since only node 1 is the predecessor node of node 2, so putting $i = 1$ and $j = 2$ in step 2 of the proposed algorithm, the value of e_2 is

$$e_2 = \text{minimum}\,\{e_1 \oplus e_{12}\} = \text{minimum}\,\{[0,0] \oplus [11,12]\} = [11,12].$$

Since minimum occurs corresponding to $i = 1$, so label node 2 as $\left[[11,12],1\right]$.

Iteration 2
The predecessor nodes of the node 3 are node 1 and 2, so putting $i = 1, 2$ and $j = 3$ in Step 2 of the proposed algorithm, the value of e_3 is

$$e_3 = \text{minimum}\,\{e_1 \oplus e_{13}, e_2 \oplus e_{23}\}$$

$$= \text{minimum}\,\{[0,0] \oplus [25,28], [10,12] \oplus [20,21]\}$$

$$= \text{minimum}\,\{[25,28], [30,33]\}.$$

Since $\frac{25+28}{2} \prec \frac{30+33}{2}$.

So using Section 6.2, minimum $\{[25,28],[30,33]\} = [25,28]$., i.e., $e_3 = [25,28]$.

Since minimum occurs corresponding to $i = 1$, so label node 3 as $[[25,28],1]$.

Similarly, $\widetilde{e}_4 = [19,20]$, label node 4 as $[[19,20],1]$,

$\widetilde{e}_5 = [54,60]$, label node 5 as $[[54,60],4]$.

$\widetilde{e}_6 = [68,71]$ or $[66,73]$, Since minimum occurs corresponding to $i = 4$, 5 so we can label node 6 as $[[68,71],4]$ or $[[66,73],5]$, if we label node 6 as $[[68,71],4]$ then the corresponding shortest distance is 69.5 and path is $1 \to 4 \to 6$ and in the second case if we label node 6 as $[[66,73],5]$ then the corresponding shortest distance is same i.e., 69.5 but the shortest path is $1 \to 4 \to 5 \to 6$.

Example 4. (Nayeem and Pal, 2005) Let us consider the same network with its arc lengths as triangular fuzzy numbers as shown in Example 2.

Solution:

Since node 6 is the destination node, so $n = 6$.

Assume $\widetilde{e}_1 = \langle 0,0,0 \rangle$ and label the source node (say node 1) as $[\langle 0,0,0 \rangle, -]$, the values of $\widetilde{e}_j$; $j = 2,3,4,5,6$ can be obtained as follows:

Iteration1

Since only node 1 is the predecessor node of node 2, so putting $i = 1$ and $j = 2$ in Step 2 of the proposed algorithm, the value of $\widetilde{e}_2$ is

$$\widetilde{e}_2 = \text{minimum}\ \{\widetilde{e}_1 \oplus \widetilde{e}_{12}\} = \text{minimum}\ \{\langle 0,0,0 \rangle \oplus \langle 11,1,1 \rangle\} = \langle 11,1,1 \rangle .$$

Since minimum occurs corresponding to $i = 1$, so label node 2 as $[\langle 11,1,1 \rangle, 1]$.

Iteration 2

The predecessor nodes of the node 3 are node 1 and 2, so putting $i = 1, 2$ and $j = 3$ in Step 2 of the proposed algorithm, the value of $\widetilde{e}_3$ is

$$\widetilde{e}_3 = \text{minimum}\ \{\widetilde{e}_1 \oplus \widetilde{e}_{13}, \widetilde{e}_2 \oplus \widetilde{e}_{23}\}$$

$$= \text{minimum}\left\{\langle 0,0,0\rangle \oplus \langle 27,2,1\rangle, \langle 11,1,1\rangle \oplus \langle 21,1,0\rangle\right\}$$

$$= \text{minimum}\left\{\langle 27,2,1\rangle, \langle 32,2,1\rangle\right\}.$$

Since $\widetilde{A} = \langle 27,2,1\rangle$ and $\widetilde{B} = \langle 32,2,1\rangle$, using Section 6.1, $\Re(\widetilde{A}) = 26.75$ and $\Re(\widetilde{B}) = 31.75$.
Since $\Re(\widetilde{A}) \prec \Re(\widetilde{B})$, so minimum $\left\{\langle 27,2,1\rangle, \langle 32,2,1\rangle\right\} = \langle 27,2,1\rangle$, i.e., $\widetilde{e}_3 = \langle 27,2,1\rangle$.

Since minimum occurs corresponding to $i = 1$, so label node 3 as $[\langle 27,2,1\rangle, 1]$.

Similarly, $\widetilde{e}_4 = \langle 20,1,2\rangle$, label node 4 as $[\langle 20,1,2\rangle, 1]$,
$\widetilde{e}_5 = \langle 57,2,3\rangle$, label node 5 as $[\langle 57,2,3\rangle, 3]$,
$\widetilde{e}_6 = \langle 66,2,4\rangle$, label node 6 as $[\langle 66,2,4\rangle, 5]$.

Since node 6 is the destination node of the given network, so the fuzzy shortest distance between node 1 and 6 is $\langle 66,2,4\rangle$ and the fuzzy shortest path is $1 \to 3 \to 5 \to 6$.

7.1. Advantages of Existing Comparison Method (Liou and Wang, 1992)

In this section, it is shown that if we apply the proposed algorithm with existing comparison method (Liou and Wang, 1992) to solve the fuzzy shortest path problems then it overcomes all the shortcomings described in Section 5.

(i) Using the proposed algorithm with existing comparison method (Liou and Wang, 1992) the fuzzy shortest path and fuzzy shortest distance between node 1 and 4, of the network shown in Fig. 2, are $1 \to 2 \to 4$ and (4,3,4) respectively.
(ii) The proposed algorithm is very easy to understand and to apply for a new decision maker, for finding the fuzzy shortest path problems.
(iii) It is very easy to compare more then two fuzzy numbers (interval numbers) simultaneously.

8. Results and Discussion

To compare the proposed algorithm with existing algorithm (Nayeem and Pal, 2005) the numerical examples presented in Nayeem and Pal (2005) are solved using the proposed algorithm and the following results are obtained.

(i) If the proposed algorithm is applied with existing comparison method (Nayeem and Pal, 2005) then the obtained shortest path and shortest distance are same as obtained by the existing algorithm (Nayeem and Pal, 2005) but the existing algorithm is very confusing to understand and to apply for finding the optimal solution of shortest path problems for a new decision maker while the proposed algorithm is very easy to understand and to apply for the same.

(ii) If the proposed algorithm is applied with the existing comparison method (Liou and Wang, 1992) then it overcomes all the shortcomings, described in Section 5 and the shortest path and shortest distance are same as obtained by the existing algorithm.

On the basis of above results it can be suggested that it is better to use the proposed algorithm with existing comparison method (Liou and Wang, 1992) compare to existing method (Nayeem and Pal, 2005) for finding the fuzzy shortest path and fuzzy shortest distance of fuzzy shortest path problems occurring in real life situations.

9. Conclusions

The shortcomings of the existing algorithm for finding the fuzzy shortest path and fuzzy shortest distance of any node from source node are pointed out and to overcome these shortcomings a new algorithm is proposed for the same. To show the advantage of the proposed algorithm over existing algorithm the results of some fuzzy shortest path problems, obtained by using the existing algorithm and proposed algorithm are compared.

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