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Exact Solutions of the Generalized Benjamin Equation and (3 + 1)-Dimensional Gkp Equation by the Extended Tanh Method

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Abstract

In this paper, the extended tanh method is used to construct exact solutions of the generalized Benjamin and (3 + 1)-dimensional gKP equation. This method is shown to be an efficient method for obtaining exact solutions of nonlinear partial differential equations. It can be applied to nonintegrable equations as well as to integrable ones.

Keywords: Extended tanh method; Generalized Benjamin equation; (3+1)-dimensional gKP equation

MSC 2010: 47F05, 35G20

1. Introduction

It is well known that the nonlinear partial differential equations (NLPDEs) are widely used to describe complex phenomena in various fields of sciences, such as physics, biology, chemistry, etc. Therefore, seeking exact solutions of NLPDEs is very important and significant in the nonlinear sciences. In the past decades, great effort has been made towards this task and many powerful methods have been presented, such as the homogeneous balance method [Khalafallah (2009) and Wang (1995, 1996)], the modified simplest equation method [Kudryashov and Loguinova (2008)], the tanh method [Malfliet and Hereman (1992, 1996)], the extended tanh method [Taghizadeh et al. (2011) and Wazwaz (2007, 2008)], the Jacobian elliptic function expansion method [Liu et al. (2001)], the first integral method [Feng et al. (2002, 2005)], and so on.

In recent years, many authors have used the extended tanh method to obtain the exact solutions of partial differential equations as the method is deemed an efficient method for obtaining exact solutions of NLPDEs. The aim of this paper is to find exact soliton solutions of the generalized Benjamin equation and (3 + 1)-dimensional gKP equation, using the extended tanh method.

2. The Extended Tanh Method and Tanh Method

A PDE

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{xxx}, \ldots) = 0, \tag{1}$$

can be converted to an ODE

$$G(u, u', u'', u''', \ldots) = 0, \tag{2}$$

upon using a wave variable $\xi = x - \lambda t$. Eq. (2) is then integrated as long as all terms contain derivatives where the integration constants are considered zeros.

The standard tanh method is developed by Malfliet [9 − 11] where the tanh is used as a new variable, since all derivatives of a tanh are represented by a tanh itself. Introducing a new independent variable

$$Y = \tanh(\mu\xi), \quad \xi = x - \lambda t, \tag{3}$$

leads to the change of derivatives:

$$\frac{d}{d\xi} = \mu(1-Y^2)\frac{d}{dY},$$

$$\frac{d^2}{d\xi^2} = -2\mu^2 Y(1-Y^2)\frac{d}{dY} + \mu^2(1-Y^2)^2\frac{d^2}{dY^2}. \tag{4}$$

The extended tanh method admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k}, \tag{5}$$

where M is a positive integer, in most cases, that will be determined. Expansion (5) reduces to the standard tanh method for $b_k = 0,\ (k = 1,\dots,M)$. Substituting (5) into the ODE (2) results in an algebraic equation in powers of Y.

To determine the parameter M, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $a_k\,(k = 0,\dots,M),\ b_k\,(k = 1,\dots,M)$, μ and λ . Having determined these parameters we obtain an analytic solution $u(x,t)$ in a closed form.

3. Exact Solutions of the Generalized Benjamin Equation

Let us consider the generalized Benjamin equation:

$$u_{tt} + \alpha(u^n u_x)_x + \beta u_{xxxx} = 0, \tag{6}$$

where α and β are constants. This kind of equation is one of the most important NLPDEs, used in the analysis of long wave in shallow water [Hereman et al. (1986)]. By using the wave variable $u(x,t) = U(\mu\xi)$, $\xi = k(x - \lambda t)$, (6) becomes the ODE

$$k^2\lambda^2 U'' + \alpha(kU^n U')' + \beta k^4 U'''' = 0. \tag{7}$$

Integrating Equation (7), twice and setting the constant of integrating to zero, we have

$$k^2\lambda^2 U + \frac{\alpha k}{n+1} U^{n+1} + \beta k^4 U'' = 0. \tag{8}$$

Balancing U'' with U^{n+1} in Equation (8) gives

$$M + 2 = (n+1)M,$$

then

$$M = \frac{2}{n}.$$

To get a closed form solution, M should be an integer. To achieve our goal, we use the Transformation

$$U(\mu\xi) = V^{\frac{1}{n}}(\mu\xi). \tag{9}$$

This transformation (9) changes Equation (8) into the ODE

$$k\lambda^2 n^2 (n+1)V^2 + \alpha n^2 V^3 - \beta k^3[(n^2-1)V'^2 - n(n+1)VV''] = 0. \tag{10}$$

Balancing VV'' with V^3 in Equation (10) gives

$$2M + 2 = 3M,$$

so

$$M = 2.$$

In this case, the extended tanh method in the form (5) admits the use of the finite expansion

$$U(\mu\xi) = S(Y) = a_0 + a_1 Y + a_2 Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}. \tag{11}$$

Substituting the form (11) into Equation (10) and using (4), while collecting the coefficients of Y we obtain:

Coefficients of Y^6:

$$\alpha n^2 a_2^3 + 2\beta k^3 \mu^2 (n+1)(n+2)a_2^2.$$

Coefficients of Y^5:

$$3\alpha n^2 a_1 a_2^2 + 4\beta k^3 \mu^2 (n+1)^2 a_1 a_2.$$

Coefficients of Y^4:

$$k\lambda^2 n^2 (n+1)a_2^2 + 3\alpha n^2[a_0 a_2^2 + a_1^2 a_2] + \beta k^3 \mu^2 (n+1)[(n+1)a_1^2 - 8a_2^2 + 6na_0 a_2].$$

Coefficients of Y^3:

$$2k\lambda^2 n^2 (n+1)a_1 a_2 + 3\alpha n^2[a_2^2 b_1 + 2a_0 a_1 a_2] + \alpha n^2 a_1^3 \\ + 2\beta k^3 \mu^2 (n+1)[na_0 a_1 - (n+4)a_1 a_2 + (5n-2)a_2 b_1].$$

Coefficients of Y^2:

$$k\lambda^2 n^2 (n+1)[a_1^2 + 2a_0 a_2] + 3\alpha n^2 [a_0 a_1^2 + a_0^2 a_2 + a_2^2 b_2 + 2a_1 a_2 b_1]$$
$$- 2\beta k^3 \mu^2 (n+1)[a_1^2 + (n-2)a_2^2 + 4na_0 a_2 - (2n-1)a_1 b_1 - 4(2n-1)a_2 b_2].$$

Coefficients of Y^{1}:

$$2k\lambda^2 n^2 (n+1)[a_0 a_1 + a_2 b_1] + 3\alpha n^2 [a_0^2 a_1 + a_1^2 b_1 + 2a_0 a_2 b_1 + 2a_1 a_2 b_1]$$
$$- 2\beta k^3 \mu^2 (n+1)[na_0 a_1 + (n-2)a_1 a_2 + (9n-4)a_2 b_1 - 2(2n-1)a_1 b_2].$$

Coefficients of Y^{0}:

$$k\lambda^2 n^2 (n+1)[a_0^2 + 2a_1 b_1 + 2a_2 b_2] + 3\alpha n^2 [a_1^2 b_2 + a_2 b_1^2 + 2a_0 a_1 b_1 + 2a_0 a_2 b_2]$$
$$+ \alpha n^2 a_0^3 - 2\beta k^3 \mu^2 (n+1)[-na_0 a_2 - na_0 b_2 + 2(2n-1)a_1 b_1 + 8(2n-1)a_2 b_2]$$
$$- \beta k^3 \mu^2 (n+1)(n-1)[a_1^2 + b_1^2].$$

Coefficients of Y^{-1}:

$$2k\lambda^2 n^2 (n+1)[a_0 b_1 + a_1 b_2] + 3\alpha n^2 [a_0^2 b_1 + a_1 b_1^2 + 2a_0 a_1 b_2 + 2a_2 b_1 b_2]$$
$$- 2\beta k^3 \mu^2 (n+1)[na_0 b_1 + (n-2)b_1 b_2 + (9n-4)a_1 b_2 - 2(2n-1)a_2 b_1].$$

Coefficients of Y^{-2}:

$$k\lambda^2 n^2 (n+1)[b_1^2 + 2a_0 b_2] + 3\alpha n^2 [a_0 b_1^2 + a_0^2 b_2 + a_2 b_2^2 + 2a_1 b_1 b_2]$$
$$- 2\beta k^3 \mu^2 (n+1)[b_1^2 + (n-2)b_2^2 + 4na_0 b_2 - (2n-1)a_1 b_1 - 4(2n-1)a_2 b_2].$$

Coefficients of Y^{-3} :

$$2k\lambda^2 n^2 (n+1)b_1 b_2 + 3\alpha n^2 [a_1 b_2^2 + 2a_0 b_1 b_2] + \alpha n^2 b_1^3$$
$$+ 2\beta k^3 \mu^2 (n+1)[na_0 b_1 - (n+4)b_1 b_2 + (5n-2)a_1 b_2].$$

Coefficients of Y^{-4}:

$$k\lambda^2 n^2 (n+1)b_2^2 + 3\alpha n^2 [a_0 b_2^2 + b_1^2 b_2] + \beta k^3 \mu^2 (n+1)[(n+1)b_1^2 - 8b_2^2 + 6na_0 b_2].$$

Coefficients of Y^{-5}:

$$3\alpha n^2 b_1 b_2^2 + 4\beta k^3 \mu^2 (n+1)^2 b_1 b_2.$$

Coefficients of Y^{-6}:

$$\alpha n^2 b_2^3 + 2\beta k^3 \mu^2 (n+1)(n+2) b_2^2.$$

Setting these coefficients equal to zero, and solving the resulting system, by using Maple, we find the following sets of solutions:

$$a_0 = \frac{-1}{2}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}, \quad a_1 = 0, \quad a_2 = \frac{1}{2}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha},$$

$$b_1 = 0, \quad b_2 = 0, \quad \mu = \pm\frac{1}{2}\frac{n\lambda}{k\sqrt{-\beta}}. \tag{12}$$

$$a_0 = \frac{-1}{2}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}, \quad a_1 = 0, \quad a_2 = 0,$$

$$b_1 = 0, \quad b_2 = \frac{1}{2}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}, \quad \mu = \pm\frac{1}{2}\frac{n\lambda}{k\sqrt{-\beta}}. \tag{13}$$

$$a_0 = \frac{-1}{4}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}, \quad a_1 = 0, \quad a_2 = \frac{1}{8}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha},$$

$$b_1 = 0, \quad b_2 = \frac{1}{8}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}, \quad \mu = \pm\frac{1}{4}\frac{n\lambda}{k\sqrt{-\beta}}. \tag{14}$$

Recall that

$$U = V^{\frac{1}{n}}.$$

For $\beta < 0$, the sets (12)-(14) give the solitons solutions

$$u_1(x,t) = \{\frac{-1}{2}\frac{\lambda^2 k\,(n+1)(n+2)}{\alpha}\operatorname{sech}^2[\frac{1}{2}\frac{n\lambda}{\sqrt{-\beta}}(x-\lambda t)]\}^{\frac{1}{n}}, \tag{15}$$

$$u_2(x,t)=\{\frac{-1}{2}\frac{\lambda^2 k(n+1)(n+2)}{\alpha}\operatorname{csc}h^2[\frac{1}{2}\frac{n\lambda}{\sqrt{-\beta}}(x-\lambda t)]\}^{\frac{1}{n}}, \tag{16}$$

$$u_3(x,t)=\{\frac{-1}{8}\frac{\lambda^2 k(n+1)(n+2)}{\alpha}(2-\tanh^2[\frac{1}{4}\frac{n\lambda}{\sqrt{-\beta}}(x-\lambda t)] -\coth^2[\frac{1}{4}\frac{n\lambda}{\sqrt{-\beta}}(x-\lambda t)])\}^{\frac{1}{n}}. \tag{17}$$

However for $\beta>0$, we obtain the travelling wave solutions

$$u_4(x,t)=\{\frac{-1}{2}\frac{\lambda^2 k(n+1)(n+2)}{\alpha}\sec^2[\frac{1}{2}\frac{n\lambda}{\sqrt{\beta}}(x-\lambda t)]\}^{\frac{1}{n}}, \tag{18}$$

$$u_5(x,t)=\{\frac{-1}{2}\frac{\lambda^2 k(n+1)(n+2)}{\alpha}\csc^2[\frac{1}{2}\frac{n\lambda}{\sqrt{\beta}}(x-\lambda t)]\}^{\frac{1}{n}}, \tag{19}$$

$$u_6(x,t)=\{\frac{-1}{8}\frac{\lambda^2 k(n+1)(n+2)}{\alpha}(2+\tan^2[\frac{1}{4}\frac{n\lambda}{\sqrt{\beta}}(x-\lambda t)] +\cot^2[\frac{1}{4}\frac{n\lambda}{\sqrt{\beta}}(x-\lambda t)])\}^{\frac{1}{n}}. \tag{20}$$

4. The (3 + 1)-dimensional gKP Equation

The (3 + 1)-dimensional gKP equation, given by

$$(u_t+6u^n u_x+u_{xxx})_x+3u_{yy}+3u_{zz}=0, \tag{21}$$

describes the dynamics of solitons and nonlinear waves in plasmas physics and fluid dynamics [Alagesan et al.(1997)].

By using the wave transformation $u(x,y,z,t)=U(\mu\xi),\quad \xi=k(x+ly+mz-\lambda t)$, carries Equation (21) into the ODE

$$(3l^2+3m^2-\lambda)U''+6U^nU''+6nU^{n-1}(U')^2+k^2U''''=0. \tag{22}$$

Twice integrating of Equation (22), setting the constant of integrating to zero, we will have

$$(3l^2+3m^2-\lambda)U+\frac{6}{n+1}U^{n+1}+k^2U''=0. \tag{23}$$

Balancing U'' with U^{n+1} in Equation (23) gives

$$M+2=(n+1)M,$$

then

$$M=\frac{2}{n}.$$

To get a closed form solution, M should be an integer. To achieve our goal, we use the Transformation

$$U(\mu\xi)=V^{\frac{1}{n}}(\mu\xi). \tag{24}$$

This transformation (24) will change Equation (23) into the ODE

$$n^2(n+1)(3l^2+3m^2-\lambda)V^2+6n^2V^3-k^2[(n^2-1)V'^2-n(n+1)VV'']=0. \tag{25}$$

Balancing VV'' with V^3 in Equation (25) gives

$$2M+2=3M,$$

then

$$M=2.$$

In this case, the extended tanh method the form (5) admits the use of the finite expansion

$$U(\mu\xi)=S(Y)=a_0+a_1Y+a_2Y^2+\frac{b_1}{Y}+\frac{b_2}{Y^2}. \tag{26}$$

Substituting the form (26) into Equation (25) and using (4), collecting the coefficients of Y we obtain:

Coefficients of Y^6:

$$6n^2a_2^3+2k^2\mu^2(n+1)(n+2)a_2^2.$$

Coefficients of Y^5:

$$18n^2a_1a_2^2 + 4k^2\mu^2(n+1)^2a_1a_2.$$

Coefficients of Y^4:

$$n^2(n+1)(3l^2+3m^2-\lambda)a_2^2 + 18n^2[a_0a_2^2 + a_1^2a_2] + k^2\mu^2(n+1)[(n+1)a_1^2 - 8a_2^2 + 6na_0a_2].$$

Coefficients of Y^3:

$$2n^2(n+1)(3l^2+3m^2-\lambda)a_1a_2 + 18n^2[a_2^2b_1 + 2a_0a_1a_2] + 6n^2a_1^3 + 2k^2\mu^2(n+1)[na_0a_1 - (n+4)a_1a_2 + (5n-2)a_2b_1].$$

Coefficients of Y^2:

$$n^2(n+1)(3l^2+3m^2-\lambda)[a_1^2 + 2a_0a_2] + 18n^2[a_0a_1^2 + a_0^2a_2 + a_2^2b_2 + 2a_1a_2b_1] - 2k^2\mu^2(n+1)[a_1^2 + (n-2)a_2^2 + 4na_0a_2 - (2n-1)a_1b_1 - 4(2n-1)a_2b_2].$$

Coefficients of Y^1:

$$2n^2(n+1)(3l^2+3m^2-\lambda)[a_0a_1 + a_2b_1] + 18n^2[a_0^2a_1 + a_1^2b_1 + 2a_0a_2b_1 + 2a_1a_2b_1] - 2k^2\mu^2(n+1)[na_0a_1 + (n-2)a_1a_2 + (9n-4)a_2b_1 - 2(2n-1)a_1b_2].$$

Coefficients of Y^0:

$$n^2(n+1)(3l^2+3m^2-\lambda)[a_0^2 + 2a_1b_1 + 2a_2b_2] + 18n^2[a_1^2b_2 + a_2b_1^2 + 2a_0a_1b_1 + 2a_0a_2b_2] + 6n^2a_0^3 - 2k^2\mu^2(n+1)[-na_0a_2 - na_0b_2 + 2(2n-1)a_1b_1 + 8(2n-1)a_2b_2] - k^2\mu^2(n+1)(n-1)[a_1^2 + b_1^2].$$

Coefficients of Y^{-1}:

$$2n^2(n+1)(3l^2+3m^2-\lambda)[a_0b_1 + a_1b_2] + 18n^2[a_0^2b_1 + a_1b_1^2 + 2a_0a_1b_2 + 2a_2b_1b_2] - 2k^2\mu^2(n+1)[na_0b_1 + (n-2)b_1b_2 + (9n-4)a_1b_2 - 2(2n-1)a_2b_1].$$

Coefficients of Y^{-2}:

$$n^2(n+1)(3l^2+3m^2-\lambda)[b_1^2+2a_0b_2]+18n^2[a_0b_1^2+a_0^2b_2+a_2b_2^2+2a_1b_1b_2]$$
$$-2k^2\mu^2(n+1)[b_1^2+(n-2)b_2^2+4na_0b_2-(2n-1)a_1b_1-4(2n-1)a_2b_2].$$

Coefficients of Y^{-3}:

$$2n^2(n+1)(3l^2+3m^2-\lambda)b_1b_2+18n^2[a_1b_2^2+2a_0b_1b_2]+6n^2b_1^3$$
$$+2k^2\mu^2(n+1)[na_0b_1-(n+4)b_1b_2+(5n-2)a_1b_2].$$

Coefficients of Y^{-4}:

$$n^2(n+1)(3l^2+3m^2-\lambda)b_2^2+18n^2[a_0b_2^2+b_1^2b_2]$$
$$+k^2\mu^2(n+1)[(n+1)b_1^2-8b_2^2+6na_0b_2].$$

Coefficients of Y^{-5}:

$$18n^2b_1b_2^2+4k^2\mu^2(n+1)^2b_1b_2.$$

Coefficients of Y^{-6}:

$$6n^2b_2^3+2k^2\mu^2(n+1)(n+2)b_2^2.$$

Setting these coefficients equal to zero, and solving the resulting system, by using Maple, we find the following sets of solutions:

$$a_0=\frac{-1}{12}(3l^2+3m^2-\lambda)(n+1)(n+2),\quad a_1=0,$$

$$a_2=\frac{1}{12}(3l^2+3m^2-\lambda)(n+1)(n+2),\quad b_1=0,\quad b_2=0, \tag{27}$$

$$\mu=\pm\frac{1}{2}\frac{n\sqrt{\lambda-3m^2-3l^2}}{k}.$$

$$a_0=\frac{-1}{12}(3l^2+3m^2-\lambda)(n+1)(n+2),\quad a_1=0,\quad a_2=0,\quad b_1=0, \tag{28}$$

$$b_2 = \frac{1}{12}(3l^2 + 3m^2 - \lambda)(n+1)(n+2), \quad \mu = \pm \frac{1}{2}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}.$$

$$a_0 = \frac{-1}{24}(3l^2 + 3m^2 - \lambda)(n+1)(n+2), \quad a_1 = 0,$$

$$a_2 = \frac{1}{48}(3l^2 + 3m^2 - \lambda)(n+1)(n+2), \quad b_1 = 0, \tag{29}$$

$$b_2 = \frac{1}{48}(3l^2 + 3m^2 - \lambda)(n+1)(n+2), \quad \mu = \pm \frac{1}{4}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}.$$

Recall that

$$U = V^{\frac{1}{n}}.$$

For $\lambda - 3m^2 - 3l^2 < 0$, the sets (27)-(29) give the solitons solutions

$$u_1(x,y,z,t) = \{\frac{-1}{12}(3l^2 + 3m^2 - \lambda)(n+1)(n+2) \times \sec h^2[\frac{1}{2}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}\xi]\}^{\frac{1}{n}}, \tag{30}$$

$$u_2(x,y,z,t) = \{\frac{-1}{12}(3l^2 + 3m^2 - \lambda)(n+1)(n+2) \times \csc h^2[\frac{1}{2}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}\xi]\}^{\frac{1}{n}}, \tag{31}$$

$$u_3(x,y,z,t) = \{\frac{-1}{48}(3l^2 + 3m^2 - \lambda)(n+1)(n+2) \times (2 - \tanh^2[\frac{1}{4}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}\xi] - \coth^2[\frac{1}{4}\frac{n\sqrt{\lambda - 3m^2 - 3l^2}}{k}\xi])\}^{\frac{1}{n}}, \tag{32}$$

where $\xi = k(x + ly + mz - \lambda t)$.

However, for $\lambda - 3m^2 - 3l^2 > 0$, we obtain the travelling wave solutions

$$u_4(x,y,z,t) = \{\frac{-1}{12}(3l^2 + 3m^2 - \lambda)(n+1)(n+2)\sec^2[\frac{1}{2}\frac{n\sqrt{3l^2 + 3m^2 - \lambda}}{k}\xi]\}^{\frac{1}{n}}, \tag{33}$$

$$u_5(x,y,z,t) = \{\frac{-1}{12}(3l^2 + 3m^2 - \lambda)(n+1)(n+2)csc^2[\frac{1}{2}\frac{n\sqrt{3l^2 + 3m^2 - \lambda}}{k}\xi]\}^{\frac{1}{n}}, \tag{34}$$

$$u_6(x,y,z,t) = \{\frac{-1}{48}(3l^2 + 3m^2 - \lambda)(n+1)(n+2) \times (2 + \tan^2[\frac{1}{4}\frac{n\sqrt{3l^2 + 3m^2 - \lambda}}{k}\xi] + \cot^2[\frac{1}{4}\frac{n\sqrt{3l^2 + 3m^2 - \lambda}}{k}\xi])\}^{\frac{1}{n}}, \tag{35}$$

where $\xi = k(x + ly + mz - \lambda t)$.

5. Conclusion

The extended tanh method has been successfully applied here to find the exact solutions for generalized Benjamin equation and the (3+1)-dimensional gKP equation. It is also evident that the proposed method can be extended to solve the problems of nonlinear partial differential equations arising in the theory of solitons and other areas.

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