Dispersion of a Solute in Hartmann Two-Fluid Flow between Two Parallel Plates

J. Prathap Kumar and J.C. Umavathi
Department of Mathematics
Gulbarga University, Gulbarga-585106
Karnataka, India
p_rathap@yahoo.com;jc_uma11@yahoo.com

Abstract

The paper presents an analytical solution for the dispersion of a solute in a conducting immiscible fluid flowing between two parallel plates in the presence of a transverse magnetic field. The fluids in both the regions are incompressible, electrically conducting and the transport properties are assumed to be constant. The channel walls are assumed to be electrically insulating. Separate solutions for each fluid are obtained and these solutions are matched at the interface using suitable matching conditions. The results are tabulated for various values of viscosity ratio, pressure gradient and Hartman number on the effective Taylor dispersion coefficient and volumetric flow rate in the absence and in the presence of chemical reactions. It is found that the solute is dispersed relative to a plane moving with the mean speed of flow with an effective Taylor diffusion coefficient which decreases with an increase in magnetic field with or without chemical reactions. The validity of the results obtained for conducting two fluid model is verified by comparison with the available one-fluid model and the values tally very well.

Keywords: MHD, two-fluid model, dispersion, chemical reaction

AMS-MSC 2010 No.: 35B30, 76W05, 80A32

Nomenclature

B - constant

B_0 - applied magnetic field

C_i - concentration of the solute

D_i - molecular diffusion coefficient

D - ratio of molecular diffusion coefficient, (D_2 / D_1)

E - electric load parameter

E_0 - applied electric field

h - distance between the plates

K_i - first-order reaction rate constant

L - typical length along the flow direction

Q_i - volumetric flow rate

U_i - velocity

$\bar{u}_i$ - non-dimensional average velocity

u_i - non-dimensional velocity

$\dfrac{dP}{dX}$ - pressure gradient

M - Hartman number

m - viscosity ratio, $\left(\mu_2 / \mu_1 \right)$

n - density ratio, $\left(\rho_1 / \rho_2 \right)$

p - non-dimensional pressure gradient

Greek symbols

η - dimensionless length

α_i - dimensionless reaction rate parameters

β_i - wall catalytic parameter

μ_i - dynamic viscosity

σ_{ei} - electrical conductivities

σ_r - ratio of electrical conductivity

ρ_i - density of the fluid

Subscripts

1, 2   - quantities for region-1 and region-2, respectively

1.   Introduction

The dispersion of a soluble matter in a non-conducting viscous fluid flowing in a circular pipe under laminar conditions has been discussed by Taylor (1953, 1954a, b). He has shown that

relative to a plane moving with the mean speed of the flow, the solute is dispersed with an apparent diffusion coefficient $R^2 \bar{V}_x^2 / 48D$, where R, $\bar{V}_x$ and D are the radius of the pipe, the average velocity and the molecular diffusion coefficient respectively. Taylor has further shown that this condition is valid, when $4L/R >> \bar{V}_x R/D >> 6.9$. Aris (1956), while extending Taylor's analysis has shown that the rate of growth of the variance of the solute distribution is proportional to the sum of the molecular diffusion coefficient and the Taylor diffusion coefficient. His analysis incidentally removes the above restrictions of Taylor. Fan and Hwang (1965) have extended Taylor's analysis to a non-Newtonian fluid obeying the power law model due to Ostwald-de Waele. Further Fan and Wang (1966) have analyzed the dispersion of a solute in the laminar flow of the Bingham plastic and the Ellis model in a pipe.

The study of hydromagnetic convection with heat transfer being important in the design of MHD generators, cross-field accelerators, shock tubes, pumps and cooling system of reactors have been investigated by several authors. A comprehensive review of these works was given by Romig (1964). Gershuni and Zhukhovitskii (1958) have investigated convective MHD flow in a vertical channel when the wall temperatures are constant while Yu (1965) has investigated the same problem when the plate temperatures vary linearly with vertical distance. Recently there are some experimental and theoretical studies on hydrodynamic and hydromagnetic aspects of two phase flows available in the literature. The interest in these types of problems stems from their abundance in technological applications such as MHD power generators, thermo-nuclear power generations and nuclear engineering. Lohrasbi and Sahai (1988) dealt with two-phase MHD flow and heat transfer in a parallel-plate channel. Malashetty and Leela (1991, 1992) analyzed the Hartmann flow characteristics of two-fluids in a horizontal channel. A two-phase MHD flow and heat transfer in an inclined channel was investigated by Malashetty and Umavathi (1997).

Malashetty et al. (2000, 2001) analyzed the problem of fully developed two-fluid MHD flow with or without applied electric field in an inclined channel. Chamkha (2000) considered the steady, laminar flow of two viscous, incompressible electrically conducting and heat generating or absorbing immiscible fluids in an infinitely long, impermeable parallel-plate channel filled with a uniform porous medium. Recently Prathap Kumar et al. (2011) analyzed the mixed convective flow and heat transfer in a vertical channel with one region filled with conducting fluid and another region with non-conducting fluid. For measuring flow rate, velocity etc. tracer elements are introduced into the concerned flows and the study of dispersion of these solutes introduced is therefore very important. Such studies were initiated by Taylor (1953, 1954a). Aris (1956) further elaborated on such studies. Mazumbar (1981) has studied Taylor dispersion for a natural convective flow through a vertical channel when the plate temperatures vary linearly with vertical distance.

All the investigations mentioned above deal with flows where the solute does not chemically react with the liquid through which it is dispersed. But in practice we have to deal with a wide variety of problems where the diffusion of the solute takes place with simultaneous chemical reactions, for example hydrolysis of ester, gas absorption in an agitated tank with chemical reaction and so on (Bird et al., 1960). Cleland and Wilhelm (1956) discussed the problem of a finite first-order homogeneous reaction in a pipe under laminar flow conditions by a finite difference method. He supported his results with experimental data. Katz (1959) investigated the

effect of homogeneous reaction at the wall on the concentration profiles, while combined first order heterogeneous and homogeneous reactions were studied by Walker (1962) and Solomon and Hudson (1961). Mandal et al. (1983) analyzed the effect of dispersion coefficient on the mean concentration distribution using the generalized dispersion model of Gill and Sankarsubramanian (1972). Shivakumar et al. (1987) studied the effect of time dependent dispersion coefficient on the concentration distribution. Dutta et al. (2006) reviewed the effect of commonly micro fabricated channel cross sections on the Taylor-Aris dispersion of solute slugs in simple pressure-driven flow systems. Suvadip Paul and Mazumder (2008) presented the longitudinal dispersion of passive contaminant released in an incompressible viscous fluid flowing between two infinite parallel flat walls, in which the flow is driven by the application of both the periodic pressure gradient and the oscillation of upper plate in its own plane with a constant velocity.

The above investigations were confined to chemical reactions taking place under steady state conditions. Gupta and Gupta (1972) discussed the unsteady dispersion of a solute with simultaneous chemical reaction in a laminar flow of liquid flowing between two parallel plates using Taylor's approach.

One of the important factors is to study how the external magnetic field influences the dispersion. The paper presents a study of dispersion of a solute with or without chemical reaction of immiscible electrically conducting fluids between two parallel plates in the presence of an external magnetic field. In the absence of the Hartman number, and considering the same viscous fluid in both the regions we obtain the case discussed by Gupta and Gupta (1972).

2. Mathematical Formulation

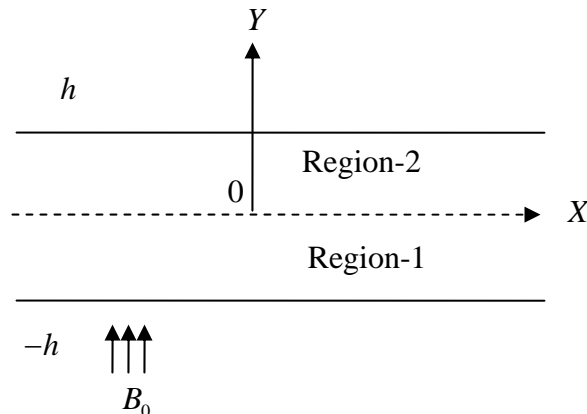The physical configuration considered in this study is shown in Figure 1.

Figure 1. Physical configuration

Consider the laminar flow of two conducting immiscible fluids between two parallel plates distant $2h$ apart, taking X-axis along the mid-section of the channel and Y-axis perpendicular

to the walls. Region-1 $(-h \leq Y \leq 0)$ is filled with the conducting fluid of conductivity σ_{e1}, density ρ_1, and viscosity μ_1, whereas region-2 $(0 \leq Y \leq h)$ is filled with another conducting fluid of conductivity σ_{e2}, density ρ_2, viscosity μ_2. A uniform magnetic field B_0 is applied perpendicular to the flow field. The fluids in both the regions are Newtonian fluids.

It is assumed that the flow is steady, laminar, fully developed, and that fluid properties are constant. Further it is also assumed that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The flow in both regions is assumed to be driven by a common constant pressure gradient. Under these assumptions, the governing equations of motion for incompressible fluids are

Region-1

$$\mu_1 \frac{d^2 U_1}{dY^2} - \frac{dP}{dX} - \sigma_{e1} B_0^2 U_1 = 0. \tag{1}$$

Region-2

$$\mu_2 \frac{d^2 U_2}{dY^2} - \frac{dP}{dX} - \sigma_{e2} B_0^2 U_2 = 0, \tag{2}$$

where B_0 is the applied magnetic field, σ_{ei} are the conductivities of the fluids, U_i are the X-component of fluid velocities and P is the pressure. The subscript i (=1, 2) denote the values for region-1 and region-2 respectively.

The boundary conditions on velocity are no-slip conditions requiring that the velocity must vanish at the walls. In addition, continuity of velocity and shear stress at the interface is assumed. With these assumptions, the boundary and interface conditions on velocity become

$$U_1 = 0 \quad at \quad Y = -h,$$
$$U_2 = 0 \quad at \quad Y = h,$$
$$U_1 = U_2 \quad at \quad Y = 0,$$
$$\mu_1 \frac{dU_1}{dY} = \mu_2 \frac{dU_2}{dY} \quad at \quad Y = 0. \tag{3}$$

Using the non-dimensional parameters,

$$\eta = \frac{Y}{h}, \ u_1 = \frac{\rho_1 h}{\mu_1} U_1, \ u_2 = \frac{\rho_2 h}{\mu_2} U_2, \ x = \frac{X}{h}, \ p^* = \frac{P}{\rho_1 (v_1/h)^2}, \ M = B_0 h \sqrt{\frac{\sigma_{e1}}{\mu_1}}, \tag{4}$$

the equations in (1) to (3) become

Region-1

$$\frac{d^2 u_1}{d\eta^2} - \frac{dp^*}{dx} - M^2 u_1 = 0, \tag{5}$$

Region-2

$$\frac{d^2 u_2}{d\eta^2} - \frac{dp^*}{dx} - \frac{\sigma_r M^2}{m} u_2 = 0. \tag{6}$$

$$u_1 = 0 \quad \text{at} \quad \eta = -1,$$

$$u_2 = 0 \quad \text{at} \quad \eta = 1,$$

$$u_1 = mn\, u_2, \quad \frac{du_1}{d\eta} = m^2\, n \frac{du_2}{d\eta} \quad \text{at} \quad \eta = 0, \tag{7}$$

where

$$m = \frac{\mu_2}{\mu_1}, \quad n = \frac{\rho_1}{\rho_2}, \quad \sigma_r = \frac{\sigma_{e2}}{\sigma_{e1}}, \quad \text{and} \quad p = \frac{dp^*}{dx}.$$

Solutions of (5) and (6) using boundary and interface conditions (7) are

$$u_1 = a_1 \cosh(M\eta) + a_2 \sinh(M\eta) - \frac{p}{M^2}, \tag{8}$$

$$u_2 = a_3 \cosh(B\eta) + a_4 \sinh(B\eta) - \frac{p}{B^2}. \tag{9}$$

From (8) and (9) the average velocities become

$$\bar{u}_1 = \frac{1}{2} \int_{-1}^{0} u_1 \, d\eta, \tag{10}$$

and

$$\bar{u}_2 = \frac{1}{2} \int_{0}^{1} u_2 \, d\eta. \tag{11}$$

Case 1a: Diffusion of a Tracer in the Absence of First-Order Chemical Reaction

The equation for the concentration C_1 of the solute for the region-1 satisfies:

$$\frac{\partial C_1}{\partial t} + u_1 \frac{\partial C_1}{\partial X} = D_1 \left(\frac{\partial^2 C_1}{\partial X^2} + \frac{\partial^2 C_1}{\partial Y^2} \right). \tag{12}$$

Similarly, the equation for the concentration C_2 of the solute for the region-2 satisfies:

$$\frac{\partial C_2}{\partial t} + u_2 \frac{\partial C_2}{\partial X} = D_2 \left(\frac{\partial^2 C_2}{\partial X^2} + \frac{\partial^2 C_2}{\partial Y^2} \right),$$
(13)

in which D_1 and D_2 are the molecular diffusion coefficients (assumed constants) for the region-1 and region-2, respectively.

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane the fluid velocities are given by:

Region-1

$$u_{1x} = u_1 - \bar{u} = a_1 \cosh(M\eta) + a_2 \sinh(M\eta) + l_1.$$
(14)

Region-2

$$u_{2x} = u_2 - \bar{u} = a_3 \cosh(B\eta) + a_4 \sinh(B\eta) + l_2,$$
(15)

where $\bar{u}$ is the sum of average velocities of region-1 and region-2.

Introducing the dimensionless quantities

$$\theta_1 = \frac{t_1}{\bar{t}_1}, \ \bar{t}_1 = \frac{L_1}{\bar{u}_1}, \ \xi_1 = \frac{x_1 - \bar{u}_1 t}{L}, \ \theta_2 = \frac{t_2}{\bar{t}_2}, \ \bar{t}_2 = \frac{L_2}{\bar{u}_2}, \ \xi_2 = \frac{x_2 - \bar{u}_2 t}{L},$$
(16)

and using (14) and (15), the equations (12) and (13) (assuming that

$$\frac{\partial^2 C_1}{\partial X^2} << \frac{\partial^2 C_1}{\partial Y^2} \ \text{ and } \ \frac{\partial^2 C_2}{\partial X^2} << \frac{\partial^2 C_2}{\partial Y^2})$$

become:

Region-1

$$\frac{1}{t} \frac{\partial C_1}{\partial \theta_1} + \frac{u_{1x}}{L} \frac{\partial C_1}{\partial \xi_1} = \frac{D_1}{h^2} \frac{\partial^2 C_1}{\partial \eta^2}.$$
(17)

Region-2

$$\frac{1}{t}\frac{\partial C_2}{\partial \theta_2} + \frac{u_{2x}}{L}\frac{\partial C_2}{\partial \xi_2} = \frac{D_2}{h^2}\frac{\partial^2 C_2}{\partial \eta^2}, \tag{18}$$

where L is the typical length along the flow direction. Following Taylor (1953), we now assume that partial equilibrium is established in any cross-section of the channel so that the variations of C_1 and C_2 with η are calculated from (17) and (18) as:

Region-1

$$\frac{\partial^2 C_1}{\partial \eta^2} = \frac{h^2}{D_1 L}u_{1x}\frac{\partial C_1}{\partial \xi_1}. \tag{19}$$

Region-2

$$\frac{\partial^2 C_2}{\partial \eta^2} = \frac{h^2}{D_2 L}u_{2x}\frac{\partial C_2}{\partial \xi_2}. \tag{20}$$

To solve these equations we use the following boundary conditions,

$$\frac{\partial C_1}{\partial \eta} = 0 \text{ at } \eta = -1 \text{ and } \frac{\partial C_2}{\partial \eta} = 0 \text{ at } \eta = 1. \tag{21}$$

Equations (19) and (20) are solved exactly for C_1 and C_2 which are given by

Region-1

$$C_1 = Z_1\left(\frac{a_1}{M^2}\cosh(M\eta) + \frac{a_2}{M^2}\sinh(M\eta) + \frac{l_1}{2}\eta^2\right) + b_1\eta + b_2. \tag{22}$$

Region-2

$$C_2 = Z_2\left(\frac{a_3}{B^2}\cosh(B\eta) + \frac{a_4}{B^2}\sinh(B\eta) + \frac{l_2}{2}\eta^2\right) + b_3\eta + b_4, \tag{23}$$

where b_2 and b_4 being constants to be determined using the entry conditions (following Rudraiah and Ng, 2007) and the values of b_2 and b_4 are taken as zero for the present evaluation.

The volumetric flow rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) using (16), (17) and (22), (23), respectively, are given by

$$Q_1 = h \int_{-1}^{0} C_1 u_{1x} \, d\eta = -Z_1 h \int_{-1}^{0} C_{11} u_{1x} \, d\eta, \tag{24}$$

and

$$Q_2 = h \int_{0}^{1} C_2 u_{2x} \, d\eta = -Z_2 h \int_{0}^{1} C_{22} u_{2x} \, d\eta, \tag{25}$$

where

$$C_{11} = -\frac{a_1 \cosh(M\eta)}{M^2} - \frac{a_2 \sinh(M\eta)}{M^2} - \frac{l_1 \eta^2}{2} - b_1 \eta,$$

and

$$C_{22} = -\frac{a_3 \cosh(B\eta)}{B^2} - \frac{a_4 \sinh(B\eta)}{B^2} - \frac{l_2 \eta^2}{2} - b_3 \eta.$$

Following Taylor (1953), we assume that the variations of C_1 and C_2 with η are small compared with those in the longitudinal direction, and if C_{m1} and C_{m2} are the mean concentration over a section, $\partial C_1 / \partial \xi_1$ and $\partial C_2 / \partial \xi_2$ are indistinguishable from $\partial C_{m1} / \partial \xi_1$ and $\partial C_{m2} / \partial \xi_2$ respectively so that equations (24) and (25) may be written as:

Region-1

$$Q_1 = -D_1^* \frac{\partial C_{m1}}{\partial \xi_1}. \tag{26}$$

Region-2

$$Q_2 = -D_2^* \frac{\partial C_{m2}}{\partial \xi_2}. \tag{27}$$

The fact that no material is lost in the process is expressed by the continuity equation for C_{m1} and C_{m2}, namely

Region-1

$$\frac{\partial Q_1}{\partial \xi_1} = -2 \frac{\partial C_{m1}}{\partial t}. \tag{28}$$

Region-2

$$\frac{\partial Q_2}{\partial \xi_2} = -2\frac{\partial C_{m2}}{\partial t}.$$ (29)

Equations (28) and (29) using equations (24) and (25) become

Region-1

$$\frac{\partial C_{m1}}{\partial t} = \frac{D_1^*}{2}\frac{\partial^2 C_{m1}}{\partial \xi_1^2}.$$ (30)

Region-2

$$\frac{\partial C_{m2}}{\partial t} = \frac{D_2^*}{2}\frac{\partial^2 C_{m2}}{\partial \xi_2^2},$$ (31)

which are the equations governing the longitudinal dispersion, where

$$D_1^* = \frac{h^2}{2D_1}\int_{-1}^{0} C_{11}\, u_{1x}\, d\eta = \frac{h^2}{2D_1} F_1\left(M, \sigma_r, p, m, n\right),$$

and

$$D_2^* = \frac{h^2}{2D_2}\int_{-1}^{0} C_{12}\, u_{1x}\, d\eta = \frac{h^2}{2D_2} F_2\left(M, \sigma_r, p, m, n\right).$$

Equations (30) and (31) are the well-known heat conduction equations which can be solved easily for a given initial conditions.

Case 1b: Diffusion of a Tracer in the Absence of First Order Chemical Reaction and for Purely Viscous Fluid (Two-Fluid Model)

To validate the results of the present model, the problem is solved in the absence of magnetic field and compared with the results of Gupta and Gupta (1972).

The non-dimensional equations of motion for incompressible, viscous fluids are:

Region-1

$$\frac{d^2 u_1}{dy^2} - \frac{dp^*}{dx} = 0.$$ (32)

Region-2

$$\frac{d^2 u_2}{dy^2} - \frac{dp^*}{dx} = 0.$$

(33)

The boundary and interface conditions are defined as in (7). Using (7) in (32) and (33), the solutions become

$$u_1 = \frac{p\eta^2}{2} + a_1\eta + a_2,$$

(34)

and

$$u_2 = \frac{p\eta^2}{2} + a_3\eta + a_4.$$

(35)

The average velocities become:

$$\bar{u}_1 = \frac{1}{2}\left(\frac{p}{6} - \frac{a_1}{2} + a_2\right),$$

(36)

and

$$\bar{u}_2 = \frac{1}{2}\left(\frac{p}{6} + \frac{a_3}{2} + a_4\right).$$

(37)

The solutions of (19) and (20) in the absence of magnetic field B_0 yields:

$$C_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}\left(\frac{p\eta^4}{24} + \frac{a_1\eta^3}{6} + \frac{lc_1\eta^2}{2} + b_1\eta\right) + C_{01},$$

(38)

and

$$C_2 = \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}\left(\frac{p\eta^4}{24} + \frac{a_3\eta^3}{6} + \frac{lc_2\eta^2}{2} + b_3\eta\right) + C_{02},$$

(39)

where C_{01} and C_{02} being constants to be determined using entry conditions.

The volumetric rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) and the evaluation of effective dispersion coefficients F_{ii} are evaluated as explained in the case 1a. The values of $F_i(p,m,n)$ are computed for different values of the dimensionless parameters p_i and m and are shown in Table 4.

Case 1c: Diffusion of a Tracer in the Absence of First Order Chemical Reaction and for Purely Viscous Fluid (One-Fluid Model).

The non-dimensional equation of motion is

$$\frac{d^2 u}{d\eta^2} = \frac{dp}{dx},$$ (40)

along with boundary conditions

$$u = 0 \quad at \quad \eta = \pm 1.$$ (41)

The solution of (40) is

$$u = -p(1 - \eta^2)/2.$$

The average velocity is given by

$$\bar{u} = -p/3.$$

The concentration equation for one-fluid model using Taylor (1953) becomes:

$$\frac{\partial^2 C}{\partial \eta^2} = \frac{h^2}{DL} \frac{\partial C}{\partial \xi} u_x,$$ (42)

where

$$u_x = \frac{p\eta^2}{2} - \frac{p}{6}.$$

The solution of (42) using boundary conditions $\frac{\partial C}{\partial \eta} = 0$ at $\eta = \pm 1$ is

$$C = \frac{h^2}{DL} \frac{\partial C}{\partial \xi} \left(\frac{p}{24} \eta^4 - \frac{p}{12} \eta^2 \right) + C_0,$$ (43)

where C_0 being constant to be determined using entry conditions.

The volumetric flow rate in which the solute is transported across a section of the channel of unit breadth is

$$Q = h \int_{-1}^{1} C u_x d\eta = -\frac{h^2 p^2}{D} \frac{\partial C}{\partial \xi} \left(\frac{2}{945} \right).$$ (44)

Hence, the value for D^* can be written as $\frac{h^2 p^2}{D} \frac{2}{945}$ by comparing with Fick's law of diffusion which agrees with the results of Wooding (1960) where p is the non-dimensional pressure

gradient. D^* is also the effective dispersion co-efficient obtained by Gupta and Gupta (1972) in the absence of chemical reactions.

Case 2a: Diffusion of a Tracer in the Presence of Homogeneous First-Order Chemical Reaction.

The physical model and the assumptions made in case 1a are true here, except that we have the chemical reaction. In this case we assume that the chemical reaction is first order and it occurs under such conditions that the gas film resistance is negligible. This means that the reaction term is $-KC_1$ (region-1) and $-KC_2$ (region-2) mol cm^{-3} s^{-1}, which represents the volume rate of disappearance of the solute due to chemical reaction. Here K represents the first-order reaction rate constant.

The velocity and average velocity are exactly the same as in (8) to (11). The equations for concentration, instead of (12) and (13) are:

Region-1

$$\frac{\partial C_1}{\partial t} + u_1 \frac{\partial C_1}{\partial X} = D_1 \left(\frac{\partial^2 C_1}{\partial X^2} + \frac{\partial^2 C_1}{\partial Y^2} \right) - K_1 C_1. \tag{45}$$

Region-2

$$\frac{\partial C_2}{\partial t} + u_2 \frac{\partial C_2}{\partial X} = D_2 \left(\frac{\partial^2 C_2}{\partial X^2} + \frac{\partial^2 C_2}{\partial Y^2} \right) - K_2 C_2. \tag{46}$$

Along with the boundary condition (21), the continuity of concentration and continuity of mass flux at the interface is considered to evaluate the integrating constants. That is,

$$C_1 = C_2 \text{ and } \frac{\partial C_1}{\partial \eta} = \frac{D_2}{D_1} \frac{\partial C_2}{\partial \eta} \text{ at } \eta = 0. \tag{47}$$

Following the analysis of case 1, the non-dimensional form of (45) and (46) are:

Region-1

$$\frac{\partial^2 C_1}{\partial \eta^2} - \alpha_1^2 C_1 = \frac{h^2}{D_1 L} u_{1x} \frac{\partial C_1}{\partial \xi_1}. \tag{48}$$

Region-2

$$\frac{\partial^2 C_2}{\partial \eta^2} - \alpha_2^2 C_2 = \frac{h^2}{D_2 L} u_{2x} \frac{\partial C_2}{\partial \xi_2}, \tag{49}$$

where

$$\alpha_1 = h\sqrt{K_1/D_1} \text{ and } \alpha_2 = h\sqrt{K_2/D_2} .$$

The solutions of (48) and (49) become:

Region-1

$$C_1 = b_1 \cosh(\alpha_1 \eta) + b_2 \sinh(\alpha_1 \eta) + Z_1 \left(\frac{a_1 \cosh(M\eta) + a_2 \sinh(M\eta)}{M^2 - \alpha_1^2} + \frac{l_1}{\alpha_1^2} \eta^2 \right). \tag{50}$$

Region-2

$$C_2 = b_3 \cosh(\alpha_2 \eta) + b_4 \sinh(\alpha_2 \eta) + Z_2 \left(\frac{a_3 \cosh(B\eta) + a_4 \sinh(B\eta)}{B^2 - \alpha_2^2} + \frac{l_2 \eta^2}{\alpha_2^2} \right). \tag{51}$$

The expressions for C_1 and C_2 can also be written as

$$C_1 = \frac{h^2}{D_1 L} \frac{\partial C_1}{\partial \xi_1} C_{11} + \frac{h^2}{D_2 L} \frac{\partial C_2}{\partial \xi_2} C_{12} \text{ and } C_2 = \frac{h^2}{D_1 L} \frac{\partial C_1}{\partial \xi_1} C_{21} + \frac{h^2}{D_2 L} \frac{\partial C_2}{\partial \xi_2} C_{22} .$$

The volumetric flow rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) using (14), (15) and (50), (51), respectively, are given by

Region-1

$$Q_1 = h \int_{-1}^{0} C_1 u_{1x} \, d\eta = -(Q_{11} + Q_{12}). \tag{52}$$

Region-2

$$Q_2 = h \int_{0}^{1} C_2 u_{2x} \, d\eta = -(Q_{21} + Q_{22}), \tag{53}$$

where

$$Q_{11} = -Z_1 h \int_{-1}^{0} C_{11} \, u_{1x} \, d\eta, \; Q_{12} = -Z_2 h \int_{-1}^{0} C_{12} \, u_{1x} \, d\eta,$$

$$Q_{21} = -Z_1 h \int_0^1 C_{21} u_{2x} \, d\eta \,, \quad Q_{22} = -Z_2 h \int_0^1 C_{22} u_{2x} \, d\eta \,.$$

Following the procedure explained in case 1 and using the fact that no material is lost in the process expressed by the continuity equation for C_1 and C_2, given by (50) and (51), we obtain an effective dispersion coefficient D^* in the form:

$$D_{11}^* = \frac{h^2}{2D_1} \int_{-1}^0 C_{11} u_{1x} \, d\eta = \frac{h^2}{2D_1} F_{11}\left(M, \sigma_r, p, \alpha_1, \alpha_2, m, n\right),$$

$$D_{12}^* = \frac{h^2}{2D_2} \int_{-1}^0 C_{12} u_{1x} \, d\eta = \frac{h^2}{2D_2} F_{12}\left(M, \sigma_r, p, \alpha_1, \alpha_2, m, n\right),$$

$$D_{21}^* = \frac{h^2}{2D_1} \int_0^1 C_{21} u_{2x} \, d\eta = \frac{h^2}{2D_1} F_{21}\left(M, \sigma_r, p, \alpha_1, \alpha_2, m, n\right),$$

$$D_{22}^* = \frac{h^2}{2D_2} \int_0^1 C_{22} u_{2x} \, d\eta = \frac{h^2}{2D_2} F_{22}\left(M, \sigma_r, p, \alpha_1, \alpha_2, m, n\right). \tag{54}$$

Values of F_{ii} are computed for different values of dimensionless parameters such as Hartman number M, viscosity ratio m, pressure gradient p and conductivity ratio σ_r for variations of α_1 and α_2. Volumetric flow rate is also computed for variations of Hartman number, viscosity ratio, pressure gradient and height of the channel.

Case 2b: Diffusion of a Tracer with Combined Homogeneous and Heterogeneous First-Order Chemical Reaction.

We now discuss the problem of diffusion in a channel with a first-order chemical reaction taking place both in the bulk of the fluid as well as at the walls which are assumed to be catalytic. In this case the diffusion equations remain the same as defined in (48) and (49) subject to the dimensionless boundary and interface conditions as

$$\frac{\partial C_1}{\partial \eta} - \beta_1 C_1 = 0 \quad at \quad \eta = -1,$$

$$\frac{\partial C_2}{\partial \eta} + \beta_2 C_2 = 0 \quad at \quad \eta = 1, \tag{55}$$

$$C_1 = C_2 \qquad\qquad at \quad \eta = 0,$$

$$D_1 \frac{\partial C_1}{\partial \eta} = D_2 \frac{\partial C_2}{\partial \eta} \quad at \; \eta = 0,$$

where $\beta_1 = f_1 h$ and $\beta_2 = f_2 h$ are the heterogeneous reaction rate parameters (or wall catalytic parameter) corresponding to catalytic reaction at the walls.

The solutions of (19) and (20) are same as in (50) and (51). The integrating constants b_1, b_2, b_3 and b_4 are obtained using boundary and interface conditions as defined in (55) and given as follows:

$$b_1 = Z_1 b_{11} + Z_2 b_{12}, \; b_2 = Z_1 b_{21} + Z_2 b_{22}, \; b_3 = Z_1 b_{31} + Z_2 b_{32}, \; b_4 = Z_1 b_{41} + Z_2 b_{42}.$$

The procedure of evaluating the volumetric flow rate and effective dispersion coefficient is same as in (52) to (54).

Case 2c: Diffusion of a Tracer in the Presence of Homogeneous First-Order Chemical Reaction and in the Absence of Magnetic Field for Purely Viscous Fluid (Two-Fluid Model).

We justify our results by comparing with the results obtained by Gupta and Gupta (1972) (one fluid model) with first order chemical reaction for a purely viscous fluids.

The solutions of velocities and average velocities are same as in (34) to (37).

The solutions of (19) and (20) for purely viscous fluid yields:

$$C_1 = b_1 \cosh(\alpha_1 \eta) + b_2 \sinh(\alpha_1 \eta) + Z_1 \left(l_1 \eta^2 + l_2 \eta + l_3 \right), \tag{56}$$

and

$$C_2 = b_3 \cosh(\alpha_2 \eta) + b_4 \sinh(\alpha_2 \eta) + Z_2 \left(l_4 \eta^2 + l_5 \eta + l_6 \right). \tag{57}$$

The volumetric rates at which the solute is transported across a section of the channel of unit breadth Q_1 (region-1) and Q_2 (region-2) and the evaluation of effective dispersion coefficients F_{ii} are evaluated as explained in the case1a. The values of $F_{ii} (\alpha_1, \alpha_2, p, m, n)$ are computed for different values of the dimensionless reaction rate parameters α_i, p_i and m and are shown in Table 4.

Case 2d: The Channel Filled with Only Viscous Fluid (One-Fluid Model) for Homogeneous Chemical Reaction.

The solutions of velocities and average velocities are given as in case 1c.

The concentration equation for one-fluid model using Taylor (1953) becomes:

$$\frac{\partial^2 C}{\partial \eta^2} - \alpha^2 C = \frac{h^2}{DL}\frac{\partial C}{\partial \xi}u_x,$$ (58)

where

$$u_x = \frac{p\eta^2}{2} - \frac{p}{6}.$$

The solution of (58) using boundary conditions $\frac{\partial C}{\partial \eta}=0$ at $\eta=\pm 1$ is:

$$C = A\cosh(\alpha\eta) - \frac{h^2}{\alpha^2 DL}\frac{\partial C}{\partial \xi}\left(\frac{p}{2}\eta^2 - \frac{p}{6} + \frac{p}{\alpha^2}\right).$$ (59)

The volumetric flow rate in which the solute is transported across a section of the channel of unit breadth is

$$Q = h\int_{-1}^{1} C u_x d\eta = \frac{h^2 p^2}{\alpha^2 D}\frac{\partial C}{\partial \xi}\left(\frac{1}{\alpha^4} + \frac{1}{3\alpha^2} - \frac{\coth(\alpha)}{\alpha} - \frac{1}{45}\right).$$ (60)

Comparing (60) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = \frac{h^2 p^2}{D}F(\alpha),$$ (61)

where

$$F(\alpha) = \frac{1}{\alpha^2}\left(\frac{\coth(\alpha)}{\alpha} - \frac{1}{\alpha^4} - \frac{1}{3\alpha^2} + \frac{1}{45}\right).$$

Values of $F(\alpha)$ are computed for different values of the dimensionless reaction rate parameter α and are shown in Table 4. When $\alpha \to 0$, (61) gives

$$\lim_{\alpha \to 0} F(\alpha) = \frac{2}{945},$$ (62)

so that the value for D^* can be written as $\frac{h^2 p^2}{D}\frac{2}{945}$, which agrees with the results of Wooding (1960), where p is the non-dimensional pressure gradient.

The solution for heterogeneous chemical reaction is also found for two-fluid and one-fluid model and the results are shown in Table 4. To further justify the mean velocity for two-fluid and one-fluid model the following analysis is made.

For the present problem, $\bar{u}_1 = -0.0647483$ (equation. 10) and $\bar{u}_2 = -0.0647483$ (equation (11)) for $m = 1$, $p = 1$, and $M = 2$.

For one-fluid model the governing equation is

$$\frac{d^2u}{dy^2} - M^2u = p,$$ (63)

where

$$p = \frac{dp/dx}{\rho(v/h)^2},$$

whose solution is

$$u = \frac{p}{M^2}\left(\frac{\cosh(M\eta)}{\cosh(M)} - 1\right).$$ (64)

The average velocity is

$$\bar{u} = \int_{-1}^{1} u \, d\eta.$$ (65)

The $\bar{u} = -0.129497\left(= \bar{u}_1 + \bar{u}_2\right)$ for $M = 2$ and $p = 1$. Hence, the mean velocity for two-fluid and one-fluid model is the same.

All the constants appeared in the above equations are shown in the Appendix

3. Results and Discussion

The dispersion of a solute in a two-fluid flow between two parallel plates in the presence of transverse magnetic field is analyzed. The dispersion of a solute is analyzed with or without first order chemical reactions following the Taylor diffusion model.

The average velocities in both the regions are evaluated using no-slip conditions at the boundaries and continuity of velocity and shear stress at the interface. The volumetric flow rate

and effective Taylor dispersion coefficient in each region is evaluated for various governing parameters.

Case 1: Diffusion of a Tracer in the Absence of First-Order Chemical Reaction.

The effect of the Hartman number M on the velocity is shown in Figure 2. We observe that an increase in the value of the Hartman number M decreases the velocity. This is the classical Hartman effect. Values of effective Taylor dispersion coefficient (ETDC) in each region for various values of Hartman number M, viscosity ratio m, pressure gradient p and ratio of electrical conductivity σ_r is shown in Table-1. It can be seen that the ETDC decreases with increase in M. This is to be expected on physical grounds, since the velocity profile becomes flatter (Fig. 2) with the increase of Hartmann number M.

As the viscosity ratio m increases ETDC decreases for values of $m < 1$ and increases in magnitude for values of $m > 1$. This is due to the fact that viscosity ratio m has significant effect on velocity. As m becomes small, the velocity profiles become flat in region-1 and parabolic in region-2 which causes for reduction in F for values of $m < 1$ and increases F for values of $m > 1$. The values of ETDC, (F) is symmetric for pressure gradient $p > 0$ and for $p < 0$. Further as p increases, F increases for values of $p > 0$ and decreases as p increases for values of $p < 0$. This is due to the fact that as p increases for values of $p > 0$, velocity increases which causes an increase in F. (It should be noted here that p is defined as $-\dfrac{dp^*}{dx}$). The ratio of electrical conductivity $\left(\sigma_r = \dfrac{\sigma_{e2}}{\sigma_{e1}} \right)$ reduces the ETDC in both the regions for values of $\sigma_r < 1$ and increases for values of $\sigma_r > 1$. It is also observed from Table 1 that the values of F_1 and F_2 for different values of M and p are equal, because when $\sigma_r = 1$ and $m = 1$ imply for considering the two-fluids having the same conductivity and viscosity.
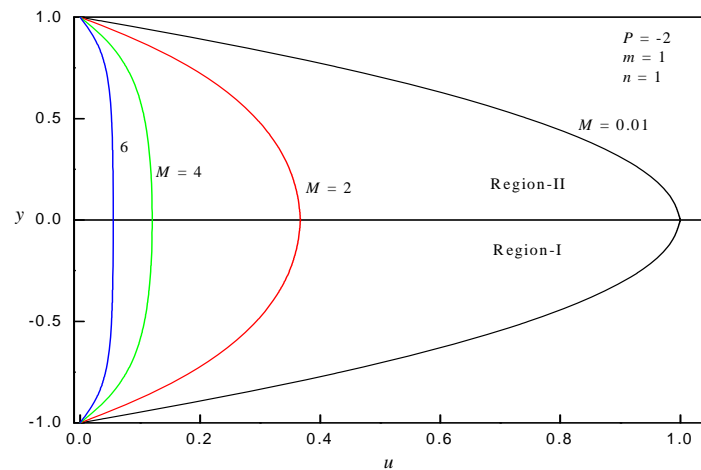

Figure 2. Velocity profiles for different values of Hartman number M

The effects of the Hartmann number M, the viscosity ratio m, the pressure gradient p and the height of the channel h on the volumetric flow rate Q is shown in Figure 3. As the Hartmann number increases the volumetric flow rate decreases for $M = 3$ and remains constant for $M \geq 3$. The volumetric flow rate remains almost constant for values of viscosity ratio m up to 2 and increases for values of $m > 2$ as m increases. The volumetric flow rate is symmetric for negative and positive values of the pressure gradient p and the optimal flow rate is attained in the absence of a pressure gradient. As the height ratio h increases the flow rate decreases in magnitude.

The results obtained (two-fluid model) in the absence of first order chemical reactions agree with the results obtained by Gupta and Chatterjee (1968) for the effect of Hartmann number on effective Taylor dispersion co-efficient. That is, as M increases F decreases. Letting $M \to 0$ and fixing $m = 1, p = 1$ and $h = 1$ (i.e., considering same fluid in both the regions) we obtain the results of Gupta and Gupta (1972) in the absence of first order chemical reactions which are also the results of Wooding (1960) as shown in Table 4.

Table 1: Values of ETDC for variations of Hartman number, viscosity ratio, pressure gradient and conductivity ratio in the absence of first order chemical reaction.

| M	$F_1(M, \sigma_r, m, p)$	$F_2(M, \sigma_r, m, p)$	$F(M, \sigma_r, m, p)$
0.1	0.0010491	0.0010491	0.0020982
2	1.2587E-4	1.2587E-4	2.5174E-4
4	1.0046E-5	1.0046E-5	2.0092E-5
6	1.4882E-6	1.4882E-6	2.9764E-6
8	3.3776E-7	3.3776E-7	6.7552E-7
m			
0.1	1.8799E-4	3.9672E-4	5.8471E-4
0.5	-1.81E-5	6.0698E-5	4.2598E-5
1	1.2587E-4	1.2587E-4	2.5174E-4
2	-2.4809E	-0.001616	-0.001865
3	-0.003563	-0.012556	-0.016119
4	-0.012059	-0.044209	-0.056268
p			
-15	0.0283211	0.0283211	0.0566422
-10	0.0125871	0.0125871	0.0251743
-5	0.0031468	0.0031468	0.0062936
0.1	1.2587E-6	1.2587E-6	2.5174E-6
5	0.0031468	0.0031468	0.0062936
10	0.0125871	0.0125871	0.0251743
15	0.0283211	0.0283211	0.0566422
σ_r			
0.1	6.6905E-4	0.067592	0.068262
0.5	2.4622E-4	8.5864E-4	0.001105
1	1.2587E-4	1.2587E-4	2.5174E-4
2	1.9672E-4	1.53E-4	3.4972E-4
3	3.2215E-4	1.8199E-4	5.0414E-4
4	4.3116E-4	1.9476E-4	6.2592E-4
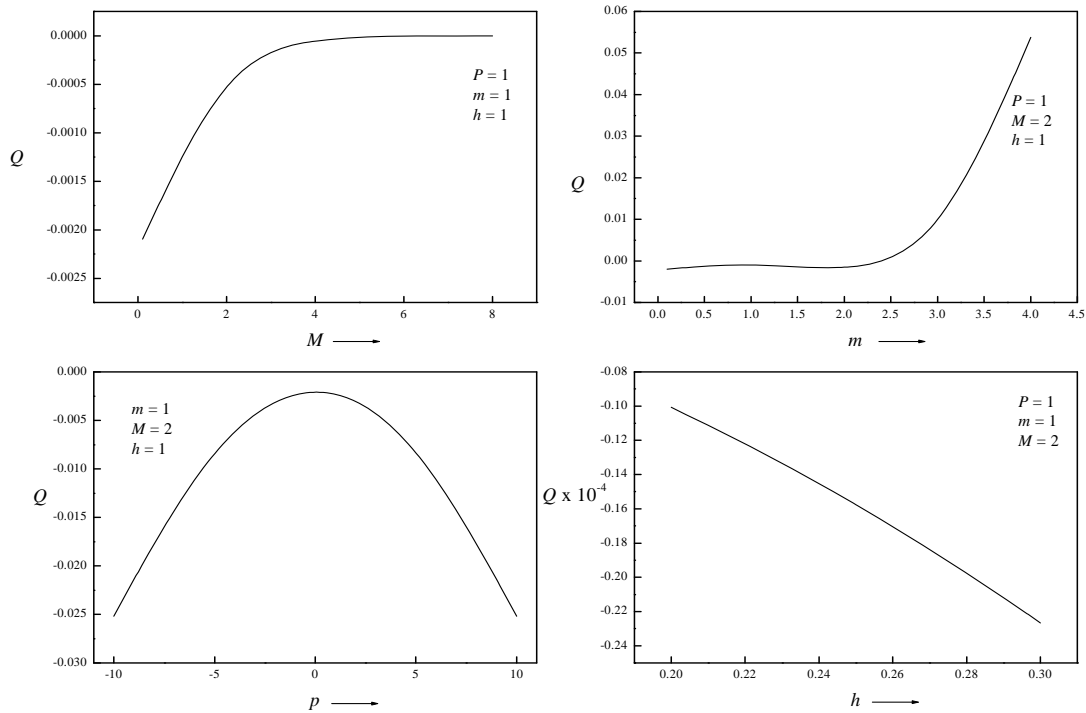
Figure 3. Volumetric flow rate Q versus Hartman number M, viscosity ratio m, pressure gradient p and height of the channel h in the absence of first order chemical reaction

Case 2: Diffusion of a Tracer in the Presence of Combined Homogeneous and Heterogeneous First-Order Chemical Reaction.

The effects of the viscosity ratio m, the pressure gradient p, the Hartman number M, the conductivity ratio σ_r and the reaction rate parameter α on ETDC for homogeneous first order chemical reaction is displayed in Table 2. As the reaction rate parameter $\alpha\ (=\alpha_1=\alpha_2)$ increases, ETDC increases in both the regions for all values of m, p, M and σ_r. This is due to the fact that, increase in α signifies that increasing number of moles of solute undergoing chemical reaction results in a drop in dispersion coefficient. As the viscosity ratio m increases, the total effective dispersion coefficient $F\ (=F_1=F_2)$ decreases for values of $m \le 1$ and increases for $m \ge 1$. F decreases as p increases for $p < 1$ and increases as p increases for values of $p > 1$. As the Hartmann number M increases ETDC decreases in both the regions. As the conductivity ratio σ_r increases, F decreases for values of $\sigma_r \le 1$ and increases for $\sigma_r \ge 1$.

Table 2: Values of ETDC for variations of reaction rate parameter, Hartman number, viscosity ratio, pressure gradient and conductivity ratio in the presence of first order chemical reaction

α	$m = 0.1$			$m = 1$		
	$F_1(\alpha_1,\alpha_2)$	$F_2(\alpha_1,\alpha_2)$	$F(\alpha_1,\alpha_2)$	$F_1(\alpha_1,\alpha_2)$	$F_2(\alpha_1,\alpha_2)$	$F(\alpha_1,\alpha_2)$
0.4	8.00589E-6	1.49616E-4	1.57622E-4	9.8934E-6	9.8934E-6	1.97868E-5
0.8	8.35876E-6	1.36619E-4	1.44978E-4	9.46219E-6	9.46219E-6	1.89244E-5
1.2	8.50548E-6	1.20835E-4	1.29341E-4	8.82393E-6	8.82393E-6	1.76479E-5
1.6	8.30537E-6	1.05557E-4	1.13862E-4	8.06677E-6	8.06677E-6	1.61335E-5
2.0	7.82051E-6	9.20293E-5	9.98498E-4	7.27065E-6	7.27065E-6	1.45413E-5
α	$m = 2$			$p = -5$		
0.4	6.297E-5	2.11697E-5	8.41398E-5	2.47335E-4	2.47335E-4	4.9467E-4
0.8	5.85902E-5	2.08786E-5	7.94688E-5	2.36555E-4	2.36555E-4	4.73109E-4
1.2	5.29001E-5	2.01194E-5	7.30195E-5	2.20598E-4	2.20598E-4	4.41196E-4
1.6	4.69458E-5	1.88913E-5	6.58371E-5	2.01669E-4	2.01669E-4	4.03338E-4
2.0	4.12782E-5	1.73595E-5	5.86377E-5	1.81766E-4	1.81766E-4	3.63532E-4
α	$p = 0.1$			$p = 5$		
0.4	9.8934E-8	9.8934E-8	1.97868E-7	2.47335E-4	2.47335E-4	4.9467E-4
0.8	9.46219E-8	9.46219E-8	1.89244E-7	2.36555E-4	2.36555E-4	4.73109E-4
1.2	8.82393E-8	8.82393E-8	1.76479E-7	2.20598E-4	2.20598E-4	4.41196E-4
1.6	8.06677E-8	8.06677E-8	1.61335E-7	2.01669E-4	2.01669E-4	4.03338E-4
2.0	7.27065E-8	7.27065E-8	1.45413E-7	1.81766E-4	1.81766E-4	3.63532E-4
α	$M = 0.1$			$M = 5$		
0.4	0.00103000	0.00103000	0.00206514	3.56801E-6	3.56801 E-6	7.13602 E-6
0.8	9.86023E-4	9.86023E-4	0.00197205	3.41511 E-6	3.41511 E-6	6.83022 E-6
1.2	9.17199E-4	9.17199E-4	0.00183440	3.18869 E-6	3.18869 E-6	6.37737 E-6
1.6	8.35679E-4	8.35679E-4	0.00167136	2.91988 E-6	2.91988 E-6	5.83976 E-6
2.0	7.50142E-4	7.50142E-4	0.00150028	2.63695 E-6	2.63695 E-6	5.27390 E-6
α	$M = 10$			$\sigma_r = 0.1$		
0.4	1.00268 E-7	1.00268 E-7	2.00536 E-7	2.68126E-4	1.74734E-4	4.42859E-4
0.8	9.63199 E-8	9.63199 E-8	1.92640 E-7	2.3091E-4	1.57484E-4	3.88394E-4
1.2	9.04596 E-8	9.04596 E-8	1.80919 E-7	1.88526E-4	1.37013E-4	3.25539E-4
1.6	8.34778 E-8	8.34778 E-8	1.66956 E-7	1.50963E-4	1.17758E-4	2.68721E-4
2.0	7.60945 E-8	7.60945 E-8	1.52189 E-7	1.20901E-4	1.01186E-4	2.22087E-4
α	$\sigma_r = 1$			$\sigma_r = 2$		
0.4	9.8934E-6	9.8934E-6	1.97868E-5	1.04941E-5	1.63515E-5	2.68456E-5
0.8	9.46219E-6	9.46219E-6	1.89244E-5	9.62562E-5	1.41923E-5	2.38179E-5
1.2	8.82393E-6	8.82393E-6	1.76479E-5	8.56359E-5	1.17153E-5	2.02789E-5
1.6	8.06677E-6	8.06677E-6	1.61335E-5	7.52492E-5	9.49710E-5	1.7022E-5
2.0	7.27065E-6	7.27065E-6	1.45413E-5	6.59314E-5	7.69917E-5	1.42923E-5

Table 3: Values of ETDC for variations of wall catalytic parameter, Hartman number, viscosity ratio, pressure gradient and conductivity ratio in the presence of first order chemical reaction

β	$m = 0.1$			$m = 1$		
	$F_1(\alpha_i,\beta_i)$	$F_2(\alpha_i,\beta_i)$	$F(\alpha_i,\beta_i)$	$F_1(\alpha_i,\beta_i)$	$F_2(\alpha_i,\beta_i)$	$F(\alpha_i,\beta_i)$
2	8.80015E-6	1.25461E-4	1.34261E-4	8.97079 E-6	8.97079 E-6	1.79416E-5
4	1.06137E-5	1.09507E-4	1.20121E-4	8.22100 E-6	8.22100 E-6	1.6442E-5
6	1.15313E-5	1.0242E-4	1.13951E-4	7.96109 E-6	7.96109E-6	1.59222E-5
8	1.20341E-5	9.8707E-5	1.10741E-4	7.83936 E-6	7.83936E-6	1.56787E-5
10	1.23509E-5	9.64181E-5	1.08769E-4	7.76877 E-6	7.76877E-6	1.55375E-5
β	$m = 2$			$p = -5$		
2	5.42686E-5	2.04981E-5	7.47668E-5	2.2427E-5	2.2427E-5	4.48539E-5
4	4.7363E-5	2.06885E-5	6.80515E-5	2.05525E-5	2.05525E-5	4.1105E-5
6	4.4527E-5	2.09914E-5	6.55185E-5	1.99027E-5	1.99027E-5	3.98054E-5
8	4.3087E-5	2.11933E-5	6.42803E-5	1.95984E-5	1.95984E-5	3.91968E-5
10	4.22133E-5	2.1331E-5	6.35443E-5	1.94219E-5	1.94219E-5	3.88438E-5
β	$p = 0.1$			$p = 5$		
2	8.97079E-8	8.97079 E-8	1.79416E-7	2.2427E-5	2.2427E-5	4.48539E-5
4	8.22100 E-8	8.22100 E-8	1.64420 E-7	2.05525E-5	2.05525E-5	4.1105E-5
6	7.96109 E-8	7.96109 E-8	1.59222 E-7	1.99027E-5	1.99027E-5	3.98054E-5
8	7.83936 E-8	7.83936 E-8	1.56787 E-7	1.95984E-5	1.95984E-5	3.91968E-5
10	7.76877 E-8	7.76877 E-8	1.55375 E-7	1.94219E-5	1.94219E-5	3.88438E-5
β	$M = 0.1$			$M = 5$		
2	9.35748E-4	9.35748E-4	0.00187150	3.23731E-6	3.23731 E-6	6.47463 E-6
4	8.65498E-4	8.65498E-4	0.00173100	2.95774 E-6	2.95774 E-6	5.91549 E-6
6	8.41145E-4	8.41145E-4	0.00168229	2.86083 E-6	2.86083 E-6	5.72166 E-6
8	8.2974E-4	8.2974E-4	0.00165948	2.81544 E-6	2.81544 E-6	5.63089 E-6
10	8.23126E-4	8.23126E-4	0.00164625	2.78912 E-6	2.78912 E-6	5.57824 E-6
β	$M = 10$			$\sigma_r = 0.1$		
2	9.13367 E-8	9.13367 E-8	1.82673 E-7	1.9335E-4	1.51182E-4	3.44532E-4
4	8.26084 E-8	8.26084 E-8	1.65217 E-7	1.19893E-4	1.64294E-4	2.84187E-4
6	7.95827 E-8	7.95827 E-8	1.59165 E-7	8.88153E-5	1.67752E-4	2.56567E-4
8	7.81657 E-8	7.81657 E-8	1.56331 E-7	7.28403E-5	1.69096E-4	2.41936E-4
10	7.73439 E-8	7.73439 E-8	1.54688 E-7	6.30865E-5	1.6978E-4	2.32867E-4
β	$\sigma_r = 1$			$\sigma_r = 2$		
2	8.97079 E-6	8.97079 E-6	1.79416E-5	9.42998 E-6	1.18866E-5	2.13166E-5
4	8.22100 E-6	8.22100 E-6	1.6442E-5	1.06612E-5	7.14330 E-6	1.78044E-5
6	7.96109 E-6	7.96109E-6	1.59222E-5	1.10628E-5	5.16619 E-6	1.6229E-5
8	7.83936 E-6	7.83936E-6	1.56787E-5	1.12445E-5	4.15603 E-6	1.54005E-5
10	7.76877 E-6	7.76877E-6	1.55375E-5	1.13477E-5	3.54119 E-6	1.48889E-5

From Table 3, we find the variations of ETDC and the wall catalytic parameter β, on viscosity ratio m, pressure gradient p, Hartmann number M and conductivity ratio σ_r for fixed value of homogeneous reaction rate parameter $\alpha (= \alpha_1 = \alpha_2)$. As the wall catalytic parameter $\beta (= \beta_1 = \beta_2)$ increases, ETDC decreases for all values of m, p and M (similar results observed for homogeneous chemical reaction). Further the effects of m, p and M on ETDC are the similar results observed for homogeneous chemical reaction. The effect of viscosity ratio m, pressure gradient p Hartmann number M and height of the channel on the volumetric flow rate

for both homogeneous and heterogeneous chemical reaction shows the similar result as observed for the diffusion without chemical reaction (case 1) and hence not shown graphically. The effects of Hartmann number on the ETDC, reaction rate parameter and wall catalytic parameter for homogeneous and heterogeneous chemical reactions of the present model (two-fluid) agree with Sundhanshu et al. (1976) (one-fluid model). That is, as the Hartman number M increases, the reaction rate parameter α, wall catalytic parameter β and ETDC decreases. Letting $M \rightarrow 0$, $m = 1$ and $p = 1$ agree with the results of Gupta and Gupta (1972) as shown in Table 4.

Table 4: Values of effective dispersion coefficient for purely viscous fluid

	Two-fluid model (present model)			One-fluid model Gupta and Gupta (1972)
	In the absence of first-order chemical reaction			
$m = p$	$F_1(m, p)$	$F_2(m, p)$	$F(m, p)$	$F(m, p)$
1	0.0010582	0.0010582	0.0021164	0.0021164
	In the presence of first-order homogeneous chemical reaction			
α	$F_1(\alpha_1, \alpha_2)$	$F_2(\alpha_1, \alpha_2)$	$F(\alpha_1, \alpha_2)$	$F(\alpha)$
0.4	0.0010099	0.0010099	0.0020199	0.00201987
0.8	9.1846E-4	9.1846E-4	0.0018369	0.00183692
1.2	8.2952E-4	8.2952E-4	0.0016590	0.00165904
1.6	7.474E-4	7.474E-4	0.0014948	0.00149480
2	6.70579E-4	6.70579E-4	0.0013412	0.00134116
	In the presence of first-order combined homogeneous and heterogeneous chemical reaction			
β	$F_1(\alpha_i, \beta_i)$	$F_2(\alpha_i, \beta_i)$	$F(\alpha_i, \beta_i)$	$F(\alpha, \beta)$
2	8.48463E-4	8.48463E-4	0.00169693	0.0016969
4	8.30289E-4	8.30289E-4	0.00166058	0.0016606
6	8.22866E-4	8.22866E-4	0.00164573	0.0016457
8	8.18832E-4	8.18832E-4	0.00163766	0.0016377
10	8.16298E-4	8.16298E-4	0.00163260	0.0016326

5. Conclusions

1. The ETDC decreases with an increase in the Hartman number with or without chemical reactions.
2. The ETDC decreases for $m \leq 1$, $\sigma_r \leq 1$ and $p < 1$ whereas it increases for $m > 1$, $\sigma_r > 1$ and $p > 1$ as m, p and σ_r increases in the presence or in the absence of chemical reactions.
3. As the homogeneous reaction rate increases, the ETDC decreases for all values of the viscosity ratio, the pressure gradient, the Hartman number and the conductivity ratio. A Similar effect was observed on wall catalytic parameter.
4. The results for the two-fluid model agree with the results for the one-fluid models of Gupta and Chatterjee (1968), Sudhanshu et al. (1976) and Gupta and Gupta (1972).

Acknowledgements

Appendix

Case 1: Diffusion of a Tracer in the Absence of First-Order Chemical Reaction.

$$p = \frac{dp_1^*}{dx}, \ B = \frac{\sigma_r M^2}{m}, \ a_3 = \frac{-Mp\left(\left(\frac{1}{M^2} - \frac{mn}{B^2}\right)\cosh(M)\sinh(B) - \frac{1}{M^2}\sinh(B)\right)}{M\, mn\cosh(M)\sinh(B) + m^2 n\, B\cosh(B)\sinh(M)}$$

$$a_4 = -\frac{\left(M^2 a_3 \cosh(B) - p\right)}{M^2 \sinh(B)}, \ a_2 = \frac{\left(M^2 a_1 \cosh(M) - p\right)}{M^2 \sinh(M)}, \ a_1 = \frac{p}{M^2} + mn\left(a_3 - \frac{p}{B^2}\right),$$

$$l_1 = -\frac{p}{M^2} + lc_1, \ l_2 = -\frac{p}{B^2} + lc_1,$$

$$lc_1 = -\frac{a_2}{2M} - \frac{a_1 \sinh(M)}{2M} + \frac{a_2 \cosh(M)}{2M} + \frac{p}{2M^2} - \frac{a_3 \sinh(B)}{2B} - \frac{a_4 \cosh(B)}{2B} + \frac{p}{2B^2} + \frac{a_4}{2B},$$

$$Z_1 = \frac{h^2}{D_1 L}\frac{\partial C_1}{\partial \xi_1}, \ Z_2 = \frac{h^2}{D_2 L}\frac{\partial C_2}{\partial \xi_2}, \ b_1 = Z_1\left(\frac{a_1}{M}\sinh(M) - \frac{a_2}{M}\cosh(M) + l_1\right),$$

$$b_3 = -Z_2\left(\frac{a_3}{B}\sinh(B) + \frac{a_4}{B}\cosh(B) + l_2\right).$$

Case 1b: Diffusion of a Tracer in the Absence of First Order Chemical Reaction and for Purely Viscous Fluid (Two-Fluid Model)

$$p = \frac{dp^*}{dx}, \ a_2 = mna_4, \ a_1 = m^2 na_3, \ a_4 = -\frac{p}{2} - a_3, \ a_3 = \frac{p - pmn}{2(m+1)mn}.$$

Case 2a: Diffusion of a Tracer in the Presence of Homogeneous First-Order Chemical Reaction.

$$D = \frac{D_2}{D_1}, \ g_1 = \frac{a_2 M \cosh(M)}{M^2 - \alpha_1^2} - \frac{a_1 M \sinh(M)}{M^2 - \alpha_1^2}, \ g_2 = \frac{a_3 B \sinh(B)}{B^2 - \alpha_2^2} + \frac{a_4 B \cosh(B)}{B^2 - \alpha_2^2},$$

$$g_3 = \frac{a_1}{M^2 - \alpha_1^2} - \frac{l_1}{\alpha_1^2}, \ g_4 = \frac{l_2}{\alpha_2^2} - \frac{a_3}{B^2 - \alpha_2^2}, \ g_5 = \frac{a_2 M}{M^2 - \alpha_1^2}, \ g_6 = -\frac{a_4 BD}{B^2 - \alpha_2^2},$$

$$Dr = D\alpha_2^2 \cosh(\alpha_1)\sinh(\alpha_2) + \alpha_1 \alpha_2 \sinh(\alpha_1)\cosh(\alpha_2),$$

$$b_{41} = \frac{-1}{Dr}\left(g_3 \alpha_1 \alpha_2 \sinh(\alpha_1)\sinh(\alpha_2) - g_5 \alpha_2 \cosh(\alpha_1)\sinh(\alpha_2) + g_1 \alpha_2 \sinh(\alpha_2)\right),$$

$$b_{42} = \frac{-1}{Dr}\left(g_4 \alpha_1 \alpha_2 \sinh(\alpha_1)\sinh(\alpha_2) - g_6 \alpha_2 \cosh(\alpha_1)\sinh(\alpha_2) + g_2 \alpha_1 \sinh(\alpha_1)\right),$$

$$b_{31} = -\frac{b_{41} \cosh(\alpha_2)}{\sinh(\alpha_2)}, \ b_{32} = \frac{-b_{42}\alpha_2 \cosh(\alpha_2) - g_2}{\alpha_2 \sinh(\alpha_2)}, \ b_{11} = b_{31} - g_3, \ b_{12} = b_{32} - g_4,$$

$$b_{21} = \frac{b_{11}\alpha_1 \sinh(\alpha_1) - g_1}{\alpha_1 \cosh(\alpha_1)}, \ b_{22} = \frac{b_{12} \sinh(\alpha_1)}{\cosh(\alpha_1)},$$

$$C_{11} = -b_{11}\cosh(\alpha_1\eta) - b_{21}\sinh(\alpha_1\eta) - \frac{a_1}{M^2 - \alpha_1^2}\cosh(M\eta) - \frac{a_2}{M^2 - \alpha_1^2}\sinh(M\eta) - \frac{l_1}{\alpha_1^2}\eta^2,$$

$$C_{12} = -b_{12}\cosh(\alpha_1\eta) - b_{22}\sinh(\alpha_1\eta),$$

$$C_{21} = -b_{31}\cosh(\alpha_2\eta) - b_{41}\sinh(\alpha_2\eta),$$

$$C_{22} = b_{32}\cosh(\alpha_2\eta) + b_{42}\sinh(\alpha_2\eta) + \left(\frac{a_3}{B^2 - \alpha_2^2}\cosh(B\eta) + \frac{a_4}{B^2 - \alpha_2^2}\sinh(B\eta) + \frac{l_2}{\alpha_2^2}\eta^2\right).$$

Case 2b: Diffusion of a Tracer with Combined Homogeneous and Heterogeneous First-Order Chemical Reaction

$$g_7 = -\alpha_1 \sinh(\alpha_1) - \beta_1 \cosh(\alpha_1), \ g_8 = \alpha_1 \cosh(\alpha_1) + \beta_1 \sinh(\alpha_1),$$

$$g_9 = -\frac{a_1 M \sinh(M)}{M^2 - \alpha_1^2} + \frac{a_2 M \cosh(M)}{M^2 - \alpha_1^2} - \beta_1\left(\frac{a_1 \cosh(M)}{M^2 - \alpha_1^2} - \frac{a_2 \sinh(M)}{M^2 - \alpha_1^2} - \frac{l_1}{\alpha_1^2}\right),$$

$$g_{10} = \alpha_2 \sinh(\alpha_2) + \beta_2 \cosh(\alpha_2), \ g_{11} = \alpha_2 \cosh(\alpha_2) + \beta_2 \sinh(\alpha_2),$$

$$g_{12} = \frac{a_3 B \sinh(B)}{B^2 - \alpha_2^2} + \frac{a_4 B \cosh(B)}{B^2 - \alpha_2^2} + \beta_2 \left(\frac{a_3 \cosh(B)}{B^2 - \alpha_2^2} + \frac{a_4 \sinh(B)}{B^2 - \alpha_2^2} - \frac{l_2}{\alpha_2^2} \right),$$

$$b_{41} = \frac{\alpha_1}{g_8 g_{10} D \alpha_2 - g_{11} g_5 \alpha_7} \left(g_7 g_{10} \left(\frac{a_1}{M^2 - \alpha_1^2} - \frac{l_1}{\alpha_1^2} \right) - \frac{g_8 g_{10} M a_2}{\alpha_1 (M^2 - \alpha_1^2)} - g_9 g_{10} \right),$$

$$b_{42} = \frac{\alpha_1}{g_8 g_{10} D \alpha_2 - g_7 g_8 \alpha_1} \left(g_7 g_{12} - \frac{g_8 g_{10} D a_4}{\alpha_1 (B^2 - \alpha_2^2)} - g_7 g_{10} \left(\frac{a_3}{B^2 - \alpha_2^2} - \frac{l_2}{\alpha_2^2} \right) \right), \quad b_{31} = -\frac{g_{11} b_{41}}{g_{10}},$$

$$b_{32} = \frac{-g_{11} b_{42} - g_{12}}{g_{10}}, \quad b_{11} = b_{31} - \frac{a_1}{M^2 - \alpha_1^2} + \frac{l_1}{\alpha_1^2}, \quad b_{12} = b_{32} + l_5, \quad b_{21} = \frac{-g_7 b_{11} - g_9}{g_8},$$

$$b_{22} = -\frac{g_7 b_{12}}{g_8}.$$

Case 2c: Diffusion of a Tracer in the Presence of Homogeneous First-Order Chemical Reaction in the Absence of Magnetic Field for Purely Viscous Fluid (Two-Fluid Model).

$$lc_1 = -\frac{1}{2} \left(\frac{p_1}{6} - \frac{a_1}{2} - a_2 + \frac{p_2}{6} + \frac{a_3}{2} + a_4 \right), \quad lc_2 = -\frac{1}{2} \left(\frac{p_1}{6} - \frac{a_1}{2} + a_2 + \frac{p_2}{6} + \frac{a_3}{2} - a_4 \right),$$

$$l_1 = \frac{-p_1}{2\alpha_1^2}, \quad l_2 = \frac{-a_1}{\alpha_1^2}, \quad l_3 = -\frac{lc_1}{\alpha_1^2} - \frac{p_1}{\alpha_1^4}, \quad l_4 = \frac{-p_2}{2\alpha_2^2}, \quad l_5 = \frac{-a_3}{\alpha_2^2}, \quad l_6 = -\frac{lc_2}{\alpha_2^2} - \frac{p_2}{\alpha_2^4},$$

$$Dr = \alpha_1 \alpha_2 \sinh(\alpha_1) \cosh(\alpha_2) + \alpha_2^2 D \sinh(\alpha_2) \cosh(\alpha_1),$$

$$b_{41} = \frac{-1}{Dr} \left(l_3 \alpha_1 \alpha_2 \sinh(\alpha_1) \sinh(\alpha_2) - l_2 \alpha_2 \cosh(\alpha_1) \sinh(\alpha_2) + \alpha_2 \sinh(\alpha_2)(l_2 - 2l_1) \right),$$

$$b_{42} = \frac{-1}{Dr} \left(\alpha_1 \sinh(\alpha_2)(l_5 + 2l_4) - l_6 \alpha_1 \alpha_2 \sinh(\alpha_1) \sinh(\alpha_2) + l_5 D \alpha_2 \cosh(\alpha_1) \sinh(\alpha_2) \right),$$

$$b_{31} = \frac{-b_{41} \cosh(\alpha_2)}{\sinh(\alpha_2)}, \quad b_{32} = \frac{-b_{42} \alpha_2 \cosh(\alpha_2) - 2l_4 - l_5}{\alpha_2 \sinh(\alpha_2)}, \quad b_{11} = b_{31} - l_3, \quad b_{12} = b_{32} + l_6,$$

$$b_{21} = \frac{b_{11} \alpha_1 \sinh(\alpha_1) + 2l_1 - l_2}{\alpha_1 \cosh(\alpha_1)}, \quad b_{22} = \frac{b_{12} \sinh(\alpha_1)}{\cosh(\alpha_1)}.$$

REFERENCES

Aris, R. (1956). On the dispersion of solute in a fluid flowing through a tube, Proc. Roy. Soc. Lond. A, vol. 235, pp. 67-77.

Bird, R. B., Stewart, W.E. and Lightfoot, E. N. (1960). Transport phenomena, John Wiley & Sons, pp. 542–546.

Chamkha, A. J. (2000). Flow of two-immiscible fluids in porous and nonporous channels, J. Fluids Eng., vol. 122, pp. 117–124.

Cleland, F. A. and Wilhelm, R. H. (1956). Diffusion and reaction in viscous-flow tubular reactor, AIChE. J., vol. 2, pp. 489.

Dutta, D., Arun Ramachandran, David, T. and Leighton, J. R. (2006). Effect of channel geometry on solute dispersion in pressure-driven microfluidic systems, Microfluid Nanofluid, vol. 2, pp. 275-290.

Fan, L. T. and Hwang, W. S. (1965). Dispersion of Ostwald-de Waele fluid in a laminar flow through a cylindrical tube, Proc. Roy. Soc. Lond. A, vol. 283, pp. 576–582.

Fan, L. T. and Wang, C. B. (1966). Dispersion of matter in non-Newtonian laminar flow through a circular tube, Proc. Roy. Soc. Lond. A, vol. 292, pp. 203.

Gershuni, G. Z. and Zhukhovitskii, E. M. (1958). Stationary convection flow of an electrically conducting liquid between parallel plates in a magnetic field, Sov. Phys. JETP, vol. 34, pp. 461.

Gill, W. N. and Sankarasubramanian, R. (1972). Dispersion of non-uniformly distributed time variable continuous source in time-dependent flow, Proc. Roy. Soc. Lond. A, vol. 327, pp. 191-208.

Gupta, A. S. and Chatterjee, P. S. (1968). Dispersion of a soluble matter in the hydromagnetic laminar flow between two parallel plates, Proc. Camb. Phi. Soc., vol. 64, pp. 1209-1214.

Gupta, P. S. and Gupta, A. S. (1972). Effect of homogeneous and heterogeneous reactions on the dispersion of solute in a laminar flow between two plates, Proc. Roy. Soc. Lond. A, vol. 330, pp. 59-63.

Katz, S. (1959). Chemical reactions catalysed on a tube wall, Chem. Eng. Sci., vol. 10(4), pp. 202.

Lohrasbi, J. and Sahai, V. (1988). Magnetohydrodynamic heat transfer in two-phase flow between parallel plates, Appl. Sci. Res., vol. 45, pp. 53–66.

Malashetty, M. S. and Leela, V. (1991). Magnetohydrodynamic heat transfer in two-fluid flow, Proceedings of the ASME/AIChE 27th National Heat Transfer Conference and Exposition, pp. 28–31.

Malashetty, M. S. and Leela, V. (1992). Magnetohydrodynamic heat transfer in two phase flow, Int. J. Eng. Sci., vol. 30, pp. 371–377.

Malashetty, M. S. and Umavathi, J. C. (1997). Two-phase magnetohydrodynamic flow and heat transfer in an inclined channel, Int. J. Multiphase Flow, vol. 22, pp. 545–560.

Malashetty, M. S., Umavathi, J. C. and Prathap Kumar, J. (2000). Two-fluid magnetoconvection flow in an inclined channel, Int. J. Trans. Pheno., vol. 3, pp. 1–12.

Malashetty, M. S., Umavathi, J. C. and Prathap Kumar, J. (2001). Convection magnetohydrodynamic two-fluid flow and heat transfer in an inclined channel, Heat Mass Transfer, vol. 37, pp. 259–264.

Mandal, K. K., Mandal, G. and Mandal, M. (1983). Analysis between dispersion in porous media and in MHD, ASME. J. Appl Mech., vol. 50, pp. 674-675.

Mazumdar, B. S. (1981). Taylor diffusion for a natural convective flow through a vertical channel, Int. J. Eng.. Sci., vol. 19, pp. 771-777.

Prathap Kumar, J., Umavathi, J. C. and Basavaraj M. Biradar, (2011). Mixed convection of magneto hydrodynamic and viscous fluid in a vertical channel, Int. J. Non-Linear Mech., vol. 46, pp. 278–285.

Roming, M. F. (1964). The influence of electric and magnetic fields on heat transfer to electrically conducting fluids, Advances in Heat Transfer (Ed. By Trvine T.F., Jr. and Hartnett, J.P) vol.1 Academic press, New York.

Rudraiah, N. and Chiu-On-Ng. (2007). Dispersion in porous media with and without reaction: A review, J. Porous Media, vol. 10(3), pp. 219-248.

Shivakumar, P. N., Rudraiah, N., Dulal pal and Siddheshwar, P. G. (1987). Closed form solution for unsteady convective diffusion in a fluid-saturated sparsely packed porous medium. Int. Comm. Heat Mass Transfer, vol. 14, pp. 137-145.

Solomon, R. L. and Hudson, J. L. (1967). Heterogeneous and homogeneous reactions in a tubular reactor, AIChE J., vol. 13, pp. 545-550, 1967.

Sudhanshu, K. Ghoshal, Subhash Ch. Sikdar and Ajit K. Choudhury (1976). Dispersion of solutes in laminar hydromagnetic flows with homogeneous and heterogeneous chemical reactions, Proc.Ind. Nati. Sci. Acad. Part A, Physical Sciences, vol. 43, pp. 370-379.

Suvadip Paul and Mazumder, B. S. (2008). Dispersion in unsteady Couette-Poiseuille flows, Int. J. Eng. Sci., vol. 46, pp. 1203-1217.

Taylor, G. I. (1953). Dispersion of soluble matter in solvent flowing slowly through a tube, Proc. Roy. Soc. Lond. A, vol. 219, pp. 186-203, 1953.

Taylor, G. I. (1954a). The dispersion of matter in turbulent flow through a pipe, Proc. Roy. Soc. Lond. A, vol. 223, pp. 446-468.

Taylor, G. I. (1954b). Conditions under which dispersion of a solute in a stream of solvent can be used to measure molecular diffusion, Proc. Roy. Soc. Lond. A, vol. 225, pp. 473-477.

Walker, R. (1961). First order homogeneous and heterogeneous reactions, Phy. Fluids, vol. 4, pp. 1211–1216.

Wooding, R. A. (1960). Instability of a viscous liquid of variable density in a vertical Hele-Shaw cell, J. Fluid Mech., vol. 7, pp. 501–515.

Yu, C. P. (1965). Combined free and forced convection channel flows in magnetohydrodynamics, AIAA. J., vol. 3(6), pp. 1184-1186.