Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 05, Issue 2 (December 2010), pp. 331 – 344
(Previously, Vol. 05, Issue 10, pp. 1428 – 1441)

Applications and Applied Mathematics:
An International Journal
(AAM)

Bound for the Complex Growth Rate in Thermosolutal Convection Coupled with Cross-diffusions

Hari Mohan Sharma
Department of Mathematics
ICDEOL Himachal Pradesh University
Summer Hill SHIMLA-5, India
hm_math_hpu@rediffmail.com

Received: May 29, 2010; Accepted: August 11, 2010

Abstract

Thermosolutal convection problem of the Veronis' type coupled with cross–diffusion is considered in the present paper. A semi -circle theorem that prescribes upper limit for the complex growth rate of oscillatory motions of neutral or growing amplitude in such a manner that it naturally culminates in sufficient conditions precluding the non- existence of such motions is derived. Further, results for thermosolutal convection problems with or without the individual consideration of Dufour and Soret effects follow as a consequence.

Keywords: Thermosolutal convection; Rayleigh numbers; Dufour number; Soret number; Lewis number; Prandtl number

MSC 2000 No.: 76E 99, 80A20

1. Introduction

The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour [Fitts (1962), Groot (1962)] effects. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, arises when the mass flux contains a term that depends upon the temperature gradient. The analogous effect that arises from a concentration gradient dependent term in the heat flux is called the Dufour effect. Although it is clear that the thermosolutal and Soret-Dufour problems are quite closely related, their relationship has never been carefully elucidated. They are in fact, formally identical and identification is done by means of a linear transformation that takes the equations and boundary conditions for the latter problem into those for the former. The analysis of double diffusive convection becomes complicated in case when the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously, they interfere with each other and produce cross diffusion effect. The Soret and Dufour coefficients describe the flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient respectively.

The coupling of the fluxes of the stratifying agents is a prevalent feature in multicomponent fluid systems. In general, the stability of such systems is also affected by the cross-diffusion terms. Generally, it is assumed that the effect of cross diffusions on the stability criteria is negligible. However, there are liquid mixtures for which cross diffusions are of the same order of magnitude as the diffusivities.

There are only few studies available on the effect of cross diffusion on double diffusion convection largely because of the complexity in determining these coefficients. Hurle and Jakeman (1971) have studied the effect of Soret coefficient on the double–diffusive convection. They have reported that the magnitude and sign of the Soret coefficient were changed by varying the composition of the mixture. McDougall (1983) has made an in depth study of double diffusive convection where in both Soret and Dufour effects are important.

Mohan (1998, 1996) mollified the nastily behaving governing equations of Dufour-driven thermosolutal convection and Soret–driven thermosolutal convection problems of the Veronis (1965) type by the construction of a linear transformation and derived the desired results concerning the linear growth rate and the behavior of oscillatory motions on the lines suggested by Banerjee et al. (1981, 1993).

The present paper purports to deal with the more general thermosolutal convection problems of Veronis type coupled with cross-diffusion and derives semi- circle theorem that prescribe upper limits for the complex growth rate of oscillatory motions of neutral or growing amplitude in such a manner that it naturally culminates in sufficient conditions precluding the non- existence of such motions. Further, results for thermosolutal convection problems with or without the individual consideration of Dufour and Soret effects follow as a consequence.

2. Mathematical Formulation and Analysis

Following the usual steps of linear stability theory the non- dimensional linearized perturbation equations governing the thermosolutal convection problem coupled with cross-diffusion with slight change in notations are easily seen to given by [Neild (1967), Krusin (1979)]

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = R_T\, a^2\theta - R_s a^2\phi \quad , \tag{2.1}$$

$$\left(D^2 - a^2 - p\right)\theta + D_T\,(D^2 - a^2)\,\phi = -w \quad , \tag{2.2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi + S_T\,(D^2 - a^2)\,\theta = -\frac{w}{\tau}\,, \tag{2.3}$$

together with the boundary condition

$$w = 0 = \theta = \phi = Dw \quad \text{at z =0 and z =1} \qquad \text{(both boundaries rigid)} \tag{2.4}$$

or

$$w = 0 = \theta = \phi = D^2 w \quad \text{at z =0 and z =1} \qquad \text{(both boundaries dynamically free)} \tag{2.5}$$

or

$$\begin{aligned} &w = 0 = \theta = \phi = Dw \quad \text{at z =0} \\ &w = 0 = \theta = \phi = D^2 w \quad \text{at z =1,} \end{aligned} \tag{2.6}$$

(lower boundary rigid and upper boundary dynamically free).

The meanings of symbols from physical point of view are as follows:

z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $\tau = \frac{\eta_1}{\kappa}$ is the Lewis number, $R_T = \frac{g\alpha\beta_1 d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $R_S = \frac{g\alpha\beta_2 d^4}{\kappa\upsilon}$ is the concentration Rayleigh number, $D_T = \frac{\beta_2 D_f}{\beta_1\kappa}$ is the Dufour number, $S_T = \frac{\beta_1 S_f}{\beta_2\eta_1}$ is the Soret number, ϕ is the concentration, θ is the temperature, p is the complex growth rate and w is the vertical velocity.

In equations (2.1)–(2.6), z is real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , a^2 is a constant, $\sigma > 0$ is a constant, $\tau > 0$ is a constant, R_T and R_S are positive constants for the Veronis' configuration, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$ and $\phi(z) = \phi_r(z) + i\phi_i(z)$ are complex valued functions(and their real and imaginary parts are real valued).

We now prove the following theorem:

Theorem 1. If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equations (2.1)– (2.3) together with one of the boundary conditions (2.4)-(2.6) with, $R_T' > 0$, $R'_S > 0$, then

$$|p| < \frac{R_T' \sigma \sqrt{M^2 - 1}}{4\pi^2 (\tau\, k_2 + \sigma)},$$

where

$$M = \frac{4R_T' \sigma}{27\pi^4 (\tau\, k_2 + \sigma)\, k_1}.$$

Proof:

We introduce the transformations

$$\tilde{w} = (S_T + B)\, w,$$

$$\tilde{\theta} = E\theta + F\phi,$$

$$\tilde{\phi} = S_T \theta + B\phi \quad, \tag{2.7}$$

where

$$\mathrm{B} = -\frac{1}{\tau} A,\ \mathrm{E} = \frac{S_T + B}{D_T + A} A,\ \mathrm{F} = \frac{S_T + B}{D_T + A} D_T$$

and A is a positive root of the equation

$$A^2 + (\tau - 1)A - \tau\, S_T D_T = 0.$$

The system of equations (2.1)-(2.6), upon using the transformation (2.7), assumes the following form:

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \frac{p}{\sigma}\right) w = R_T{}' \, a^2 \theta - R_S{}' \, a^2 \phi \quad, \tag{2.8}$$

$$\left(k_1\left(D^2 - a^2\right) - p\right)\theta = -w, \tag{2.9}$$

$$\left(k_2(D^2-a^2)-\frac{p}{\tau}\right)\phi=-\frac{w}{\tau} \tag{2.10}$$

with

$$w=0=\theta=\phi=Dw \quad \text{at z =0 and z =1} \tag{2.11}$$

or

$$w=0=\theta=\phi=D^2w \quad \text{at z =0 and z =1} \tag{2.12}$$

or

$$w=0=\theta=\phi=Dw \quad \text{at z =0}$$

$$w=0=\theta=\phi=D^2w \quad \text{at z =1 ,} \tag{2.13}$$

where

$$k_1=1+\frac{\tau\, D_T S_T}{A},\ k_2=1-\frac{S_T D_T}{A}\ \textit{are positive constants}$$

$$\textit{and } R_T'=\frac{(D_T+A)(R_T B+R_S S_T)}{BA-S_T D_T},\ R_S'=\frac{(S_T+B)(R_S A+R_T D_T)}{BA-S_T D_T}$$

are respectively the modified thermal Rayleigh number and the modified concentration Rayleigh number.

The tilde has been omitted for simplicity.

Multiplying equation (2.8) by w^* (the complex conjugate of w) and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*(D^2-a^2)\,(D^2-a^2-\frac{p}{\sigma})\,w\,dz=R_T'a^2\int_0^1\theta\, w^*\,dz-R_S'a^2\int_0^1\phi\, w^*\,dz. \tag{2.14}$$

Taking the complex conjugate of equations (2.9) and (2.10) and using the resulting equations in equation (2.14), we get

$$\int_0^1 w^*(D^2-a^2)(D^2-a^2-\frac{p}{\sigma})w\,dz=-R_T'a^2\int_0^1\theta\left[k_1(D^2-a^2)-p^*\right]\theta^*dz+R_S'a^2\tau\int_0^1\phi\left[k_2(D^2-a^2)-\frac{p^*}{\tau}\right]\phi^*dz. \tag{2.15}$$

Integrating equations (2.15) by parts a suitable number of times, using either of the boundary conditions (2.10)-(2.13) and one of the following inequalities

$$\int_0^1 \psi^* D^{2n}\psi \, dz = (-1)^n \int_0^1 \left|D^n\psi\right|^2 dz \,, \tag{2.16}$$

where

$\psi = \theta = \phi,$ for $n = 0$, 1 and $\psi = w,$ for $n = 0$, 1, 2,

we have

$$\begin{aligned}
&\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2\left|Dw\right|^2 + a^4\left|w\right|^2\right)dz + \frac{p}{\sigma}\int_0^1\left(\left|Dw\right|^2 + a^2\left|w\right|^2\right)dz \\
&\qquad = R_T' a^2 \int_0^1 \left[k_1\left(\left|D\theta\right|^2 + a^2\left|\theta\right|^2\right) + p^*\left|\theta\right|^2\right]dz \\
&\qquad\quad - R_S' a^2\,\tau \int_0^1 \left[k_2\left(\left|D\varphi\right|^2 + a^2\left|\varphi\right|^2\right) + \frac{p^*}{\tau}\left|\varphi\right|^2\right]dz.
\end{aligned} \tag{2.17}$$

Equating the real and imaginary parts of equation (2.17) equal to zero and using $p_i \neq 0$, we get

$$\begin{aligned}
&\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2\left|Dw\right|^2 + a^4\left|w\right|^2\right)dz + \frac{p_r}{\sigma}\int_0^1\left(\left|Dw\right|^2 + a^2\left|w\right|^2\right)dz \\
&\qquad - R_T' a^2 \int_0^1 \left[k_1\left(\left|D\theta\right|^2 + a^2\left|\theta\right|^2\right) + p_r\left|\theta\right|^2\right]dz \\
&\qquad - R_S' a^2\,\tau \int_0^1 \left[k_2\left(\left|D\varphi\right|^2 + a^2\left|\varphi\right|^2\right) + \frac{p_r}{\tau}\left|\varphi\right|^2\right]dz = 0
\end{aligned} \tag{2.18}$$

and

$$\frac{1}{\sigma}\int_0^1 (\left|Dw\right|^2 + a^2\left|w\right|^2)dz + R_T' a^2\int_0^1 \left|\theta\right|^2 dz - R_S' a^2 \int_0^1 \left|\phi\right|^2 dz = 0\,. \tag{2.19}$$

Multiplying equation (2.19) by p_r and adding the resulting equation to (2.18), we have

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2\left|Dw\right|^2 + a^4\left|w\right|^2\right)dz - R'_T a^2 \int_0^1 \left[k_1\left(\left|D\theta\right|^2 + a^2\left|\theta\right|^2\right)\ \right]dz + R'_S a^2\,\tau \int_0^1 \left[k_2\left(\ \left|D\varphi\right|^2 + a^2\left|\varphi\right|^2\right)\ \right]dz + \frac{2p_r}{\sigma}\int_0^1 \left|Dw\right|^2 + a^2\left|w\right|^2)dz = 0. \tag{2.20}$$

Equation (2.19) implies that

$$\frac{1}{\sigma}\int_0^1 (\left|Dw\right|^2 + a^2\left|w\right|^2)dz \ < R'_S a^2 \int_0^1 \left|\phi\right|^2 dz \tag{2.21}$$

Since w, θ, ϕ vanish at z = 0 and z = 1, therefore Rayleigh-Ritz inequality [Shultz (1973)] yields

$$\int_0^1 \left|Dw\right|^2 dz \geq \pi^2 \int_0^1 \left|w\right|^2 dz\,, \tag{2.22}$$

$$\int_0^1 \left|D\theta\right|^2 dz \geq \pi^2 \int_0^1 \left|\theta\right|^2 dz\,, \tag{2.23}$$

$$\int_0^1 \left|D\varphi\right|^2 dz \geq \pi^2 \int_0^1 \left|\varphi\right|^2 dz\,. \tag{2.24}$$

Combining inequalities (2.21) and (2.22), we get

$$\frac{\pi^2 + a^2}{\sigma}\int_0^1 \left|w\right|^2 dz \leq R'_S a^2 \int_0^1 \left|\varphi\right|^2 dz. \tag{2.25}$$

Also upon using inequality (2.24), we have

$$R'_S a^2 \int_0^1 (\left|D\varphi\right|^2 + a^2\left|\varphi\right|^2)\,dz \geq (\pi^2 + a^2)\,R'_S a^2 \int_0^1 \left|\varphi\right|^2 dz. \tag{2.26}$$

Combining inequalities (2.25) and (2.26), we have

$$R'_S\ a^2 \int_0^1 \left(\left|D\varphi\right|^2 + a^2\left|\varphi\right|^2\right)dz \geq \frac{\pi^2 + a^2}{\sigma}\int_0^1 \left|w\right|^2 dz. \tag{2.27}$$

Further, utilizing Schwartz inequality, we have

$$\int_0^1 \left(\left|w^2\right|\right)^{\frac{1}{2}} dz \int_0^1 \left(\left|Dw\right|^2\right)^{\frac{1}{2}} dz \geq -\int_0^1 \left|w^* D^2 w\right| dz = \int_0^1 \left|Dw\right|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz,$$ (Using (2.21))

which on simplification yields

$$\int_0^1 \left(\left|D^2 w\right|^2\right) \geq \pi^4 \int_0^1 |w|^2. \tag{2.28}$$

Inequality (2.22) together with inequality (2.28) yields

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 \left|Dw\right|^2 + a^4 |w|^2\right) dz \geq \left(\pi^2 + a^2\right)^2 \int_0^1 |w|^2 \, dz. \tag{2.29}$$

Multiplying equation (2.9) by the complex conjugate of equation (2.9) and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 \left[\left(k_1\left(D^2 - a^2\right) - p\right)\theta\left(k_1\left(D^2 - a^2\right) - p^*\right)\theta^*\right] dz = \int_0^1 ww^* \, dz.$$

Integrating the above equation by parts an appropriate number of times and using either of the given boundary conditions, we get

$$\int_0^1 \left|k^2{}_1\left(D^2 - a^2\right)\theta\right|^2 + 2p_r k\int_0^1 \left(\left|(D\theta)\right|^2 + a^2|\theta|^2\right) dz + |p|^2 \int_0^1 |\theta|^2 \, dz = \int_0^1 |w|^2 \, dz \tag{2.30}$$

Since $p_r \geq 0$, therefore from equation (2.30) , we have

$$k^2{}_1 \int_0^1 \left|\left(D^2 - a^2\right)\theta\right|^2 dz + |p|^2 \int_0^1 |\theta|^2 \, dz \leq \int_0^1 |w|^2 \, dz. \tag{2.31}$$

Also emulating the derivation of inequalities (2.28) and (2.29) we derive the following inequality

$$\int_0^1 \left|\left(D^2 - a^2\right)\theta\right|^2 dz = \int_0^1 \left|D^2\theta\right|^2 + 2a^2 \left|D\theta\right|^2 + a^4 |\theta|^2 dz \geq \left(\pi^2 + a^2\right)^2 \int_0^1 |\theta|^2 \, dz. \tag{2.32}$$

Using inequality (2.32) in equality (2.31), we get

$$k_1^2\left(\pi^2+a^2\right)^2\left[1+\frac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right]\int_0^1|\theta|^2dz \le \int_0^1|w|^2dz\,. \tag{2.33}$$

Now,

$$\int_0^1\left(|D\theta|^2+a^2|\theta|^2\right)dz=\left|-\int_0^1\theta^*\left(D^2-a^2\right)\theta\right|$$

$$\le \int_0^1|\theta|\left|\left(D^2-a^2\right)\theta\right|dz$$

$$\le\left\{\int_0^1\left|\theta^2\right|\right\}^{\frac{1}{2}}\left\{\int_0^1\left|\left(D^2-a^2\right)\theta\right|^2\right\}^{\frac{1}{2}} \qquad \text{(using Schwartz inequality)}$$

$$\le \frac{1}{k_1^2\left(\pi^2+a^2\right)^2}\left\{1+\frac{|p|^2}{k_1^2\left(\pi^2+a^2\right)}\right\}^{-1/2}\int_0^1|w|^2dz\,. \tag{2.34}$$

(using inequalities (2.32) and (2.33))

Making use of inequalities (2.27), (2.29) and (2.34), equation (2.20) yields

$$\left(\pi^2+a^2\right)^2\int_0^1|w|^2dz-\frac{R_T'a^2}{k_1\left(\pi^2+a^2\right)\left[1+\frac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right]^{\frac{1}{2}}}\int_0^1|w|^2dz+\frac{\tau\,k_2\left(\pi^2+a^2\right)^2}{\sigma}\int_0^1|w|^2dz$$

$$+\frac{2p_r}{\sigma}(\pi^2+a^2)\int_0^1|w|^2dz<0. \tag{2.35}$$

Since, $p_r \ge 0$, it follows from inequality (2.35) that

$$\left(\pi^2+a^2\right)^2\left(1+\frac{\tau k_2}{\sigma}\right)\int_0^1|w|^2dz-\frac{R_T'a^2}{k_1\left(\pi^2+a^2\right)\left[1+\frac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right]^{\frac{1}{2}}}\int_0^1|w|^2dz<0$$

or

$$k_1\left(\frac{\tau k_2+\sigma}{\sigma}\right)\frac{(\pi^2+a^2)^3}{a^2}\left[1+\frac{|p|^2}{k_1^2(\pi^2+a^2)^2}\right]^{\frac{1}{2}}<R'_T\,. \tag{2.36}$$

Since, minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect a^2 is $\frac{27\pi^4}{4}$, it follows from inequality (2.36) that

$$k_1\left(\frac{\tau k_2+\sigma}{\sigma}\right)\frac{27\pi^4}{4}\left[1+\frac{|p|^2}{k_1^2(\pi^2+a^2)^2}\right]^{\frac{1}{2}}<R'_T$$

or

$$\left[1+\frac{|p|^2}{k_1^2(\pi^2+a^2)^2}\right]^{\frac{1}{2}}<\frac{4R'_T\sigma}{27\pi^4(\tau k_2+\sigma)k_1}(=M)\,. \tag{2.37}$$

Therefore, we have

$$|p|<k_1(\pi^2+a^2)\sqrt{M^2-1}\,. \tag{2.38}$$

Further, since $\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}>1$, therefore, it follows from inequality (2.36) that

$$(\pi^2+a^2)<\frac{R'_T a^2}{(\tau k_2+\sigma)(\pi^2+a^2)^2}\,. \tag{2.39}$$

Now, the maximum value of $\frac{a^2}{(\pi^2+a^2)^2}$ with respect to a^2 is $\frac{1}{4\pi^2}$, therefore inequality (2.39) yields

$$(\pi^2+a^2)<\frac{R'_T\sigma}{4\pi^2(\tau k_2+\sigma)}\,. \tag{2.40}$$

Using inequality (2.40) in inequality (2.38), we get

$$|p| < \frac{R_T' \sigma \sqrt{M^2 - 1}}{4\pi^2 (\tau k_2 + \sigma)}.$$

This completes the proof of the theorem.

Theorem 1 from the point of view of hydrodynamic stability theory may be stated as:

The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude ($p_r \geq 0$) in thermosolutal convection problem of Veronis' type coupled with cross-diffusion lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius

$$\frac{R_T' \sigma \sqrt{M^2 - 1}}{4\pi^2 (\tau k_2 + \sigma)}.$$

Corollary 1. If (p, w, θ, ϕ), $p = p_r + ip_i$, $p_r \geq 0$ $p_i \neq 0$ is a non -trivial solution of equation (2.1)– (2.3) together with one of the boundary conditions (2.4)-(2.6) with, $R_T' > 0$, $R'_S > 0$ and M ≤ 1 , then $p_r < 0$.

Proof:

Follows from Theorem 1.

Corollary 1 implies that oscillatory motions of growing amplitude are not allowed in thermosolutal convection problem of Veronis type coupled with cross- diffusion if $M \left(= \frac{4 R_T' \sigma}{27\pi^4 (\tau k_2 + \sigma) k_1} \right) \leq 1$.

Corollary 2. For thermohaline convection ($D_T = S_T = 0$) the complex growth rate **p** = $p_r + ip_i$, of an arbitrary oscillatory perturbation of growing amplitude lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T \sigma \sqrt{M'^2 - 1}}{4\pi^2 (\tau + \sigma)},$$

where

$$M' = \frac{4R_T\sigma}{27\pi^4(\tau+\sigma)}.$$

Corollary 3. For Soret –driven thermosolutal convection ($D_T = 0$) the complex growth rate $p = p_r + ip_i$, of an arbitrary oscillatory perturbation of growing amplitude lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{\left(R_T - \frac{\tau R_T S_T}{(1-\tau)} \right)\sigma\sqrt{M''^2 - 1}}{4\pi^2(\tau+\sigma)},$$

where

$$M'' = \frac{4\left(R_T - \frac{R_T D_T \tau}{(1-\tau)} \right)\sigma}{27\pi^4(\tau+\sigma)}.$$

Corollary 4. For Dufour –driven thermosolutal convection ($S_T = 0$) the complex growth rate $p = p_r + ip_i$, of an arbitrary oscillatory perturbation of growing amplitude lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{\left(R_T + \frac{R_T D_T}{(1-\tau)} \right)\sigma\sqrt{M'''^2 - 1}}{4\pi^2(\tau+\sigma)},$$

where

$$M''' = \frac{4R_T\left(1 + \frac{D_T}{(1-\tau)} \right)\sigma}{27\pi^4(\tau+\sigma)}.$$

3. Conclusions

In the present paper, thermosolutal convection problem of Veronis' type configuration coupled with cross- diffusion is considered. The investigation of cross –diffusion effect is motivated by its interesting complexities as a thremosolutal or double diffusive phenomenon which has its importance in various field such as high quality crystal production, oceanography, production of pure medication, solidification of molten alloys, exothermally heated lakes and magmas. The analysis made brings out the following main conclusions:

(i) The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory perturbation of growing amplitude ($p_r \geq 0$) in thermosolutal convection problem of Veronis' type coupled with cross- diffusion lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R'_T \sigma \sqrt{M^2 - 1}}{4\pi^2(\tau k_2 + \sigma)}.$$

(ii) The oscillatory motions of growing amplitude are not allowed in thermosolutal convection problem of Veronis type coupled with cross- diffusion if M $\left(= \frac{4R'_T \sigma}{27\pi^4(\tau k_2 + \sigma)k_1} \right) \leq 1$.

Acknowledgements

The author expresses his sincere thanks to the referees for their perspicacious comments which results in appreciable enhancement of this paper.

REFERENCES

Banerjee, M. B., Katoch D. C., Dube G.S., Banerjee K. (1981). Bounds for Growth Rate of a perturbation in Thermohaline Convection. Proc. Roy. Soc. London, Ser. A vol. **378**, 301-304.

Banerjee, M.B., Gupta J.R., Parkash J. (1993). On Thermohaline Convection of the Veronis Type. J. Math. Anal. Appl., vol. **179**, 327-.

de Groot, S.R., Mazur P. (1962). *Nonequilibrium Thermodynamics*, North Holland, [8] Amsterdam.

Fitts, D.D. (1962). *Nonequlibrium Thermodynamics*, McGraw Hill, New York.

Hurle, D.T.J., Jakeman E. (1971). Soret-Driven Thermosolutal Convection, *J. Fluid Mech*., vol. **47**, pp. 668-687.

Krusin-Gutkowicz, D., Collins, M.A., Ross (1979). Rayleigh-Benard Instability in Non-Reactive Binary Fluids. J., Phy. Fluids, vol.**22**, 1443.

Mcdougall, T.J. (1983). Double-diffusive Convection Caused by Coupled Molecular Diffusion, *J. Fluid Mech*., vol. **126**, pp. 379.

Mohan, H. (1996). The Soret Effect on the Rotatory Thermosolutal Convection of the Veronis Type, Indian J.Pure Appl.Math., vol.**27,** pp. 609-619.

Mohan, H. (1998). Dufour-Driven Thermosolutal Convection of the Veronis Type, J. Math.Anal.Appl,vol. **218**, pp.569-580.

Neild, D. (1967). The Thermohaline Rayleigh-Jaffreys Problems J. Fluid Mach., vol.**29**, pp. 545-558.

Schultz, M.H. (1973). *Spline Analysis*, Prentice-Hall, Englewood Cliffs, N.J.

Veronis, G. (1965). On Finite Amplitude Instability in Thermohaline Convection, J. Mar. Res., vol. **23**, pp.1-17.