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An Exponential Matrix Method for Numerical Solutions of Hantavirus Infection Model

Şuayip Yüzbaşi
Department of Mathematics
Akdeniz University
Antalya, Turkey
syuzbasi@akdeniz.edu.tr

Mehmet Sezer
Department of Mathematics
Celal Bayar University
Manisa, Turkey
mehmet.sezer@cbu.edu.tr

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Abstract

In this paper, a new matrix method based on exponential polynomials and collocation points is proposed to obtain approximate solutions of Hantavirus infection model corresponding to a class of systems of nonlinear ordinary differential equations. The method converts the model problem into a system of nonlinear algebraic equations by means of the matrix operations and the collocation points. The reliability and efficiency of the proposed scheme is demonstrated by the numerical applications and all numerical computations have been made by using a computer program written in Maple.

Keywords: Hantavirus infection model; Exponential approximation; Numerical solution; System of nonlinear differential equations; Matrix method; Collocation points

MSC 2010 No.: 93A30; 35A24; 34K28; 33B10; 65L05; 65L60; 65L80

1. Introduction

In this study, we develop an exponential approximation to obtain approximate solutions of the Hantavirus infection model considered in [Abramson and Kenkre (2002), Abramson et al.

(2003), Goh et al. (2009) and Gökdoğan et al. (2012)]. The Hantavirus infection model is given by a system of the nonlinear differential equations

$$\begin{cases} \dfrac{dS}{dt} = b\left(S+I\right) - cS - \dfrac{S\left(S+I\right)}{k} - aSI \\ \dfrac{dI}{dt} = -cI - \dfrac{I\left(S+I\right)}{k} + aSI \end{cases}, \qquad S(0)=\lambda_1,\ I(0)=\lambda_2, \quad 0 \le t \le R < \infty, \tag{1}$$

where $S(t)$ and $I(t)$ are the populations of susceptible and infected mice, respectively, and $M(t) = S(t) + I(t)$ is the total population of mice.

In this model problem, the meanings of other terms are as follows:

Births: $b\left(S+I\right)$ denotes the birth of susceptible newborn mice, all with a rate proportional to the total population, since all mice make contribution to breeding regardless of whether it is susceptible or infected.

Deaths: c represents the rate of depletion by death for natural reasons, proportional to the corresponding density.

Competition: $-SI\left(S+I\right)/k$ shows the process of limiting the increase of population due to conflict of resource sharing. k shows that the carrying capacity of all means to protect the population. Higher values of k represents good environmental conditions which these are water, food, housing availability, favorable climatic conditions.

Infection: aSI represents the number of susceptible mice that get infected, due to an encounter with an infected (and consequently infectious) mouse, at a rate a that we assume constant.

On the other hand, exponential polynomials or exponential functions have interesting applications in many optical and quantum electronics [Alharbi (2010)], some nonlinear phenomena modeled by partial differential equations [Alipour et al. (2011)], many statistical discussions (especially in data analysis) [Shanmugam (1988)], the safety analysis of control synthesis [Xu et al. (2010)], the problem of expressing mean-periodic functions [Ouerdiane and Ounaies (2012)], and the study of spectral synthesis [Debrecen (2000) and Ross (1963)]. These polynomials are based on the exponential base set $\left\{1, e^{-t}, e^{-2t}, \ldots\right\}$.

Lately, Yüzbaşı et al. [Yüzbaşı (2012a), (2012b) and Yüzbaşı et al. (2012)] have studied the Bessel polynomial approximation, based on the collocation points, for the continuous population models for single and interacting species, the HIV infection model of CD4^{+} T cells and the pollution model of lakes.

Our purpose in this study, is to develop a new matrix method, which is based on the exponential basis set $\{1, e^{-t}, e^{-2t}, \ldots\}$ and the collocation points, to obtain the approximate solutions of the model (1) in the exponential forms

$$S(t) \cong S_N(t) = \sum_{n=0}^{N} a_{1,n} e^{-nt} \quad \text{and} \quad I(t) \cong I_N(t) = \sum_{n=0}^{N} a_{2,n} e^{-nt}, \tag{2}$$

where the exponential basis set is defined by $\{1, e^{-t}, e^{-2t}, \ldots, e^{-Nt}\}$ and $a_{1,n}$, $a_{2,n}$ $(n = 0,1,2,...,N)$ are unknown coefficients.

Note that $S(t)$ and $I(t)$ are the exact solutions of the problem (1)-(2) and $S_N(t)$ and $I_N(t)$ the approximate solution of the problem (1)-(2).

2. Exponential-Matrix Method

Firstly, let us show model (1) in the form

$$\begin{cases} \dfrac{dS}{dt} = (b-c)S + bI - \dfrac{1}{k}S^2 - \left(\dfrac{1}{k} + a\right)SI \\ \dfrac{dI}{dt} = -cI - \dfrac{1}{k}I^2 - \left(\dfrac{1}{k} - a\right)SI. \end{cases} \tag{3}$$

Now, let us consider the approximate solutions $S(t)$ and $I(t)$ of system (3) defined by the exponential basis set (2). The approximate solutions $S(t)$ and $I(t)$ can be written in the matrix forms

$$S(t) = \mathbf{E}(t)\mathbf{A}_1 \text{ and } I(t) = \mathbf{E}(t)\mathbf{A}_2, \tag{4}$$

where

$$\mathbf{E}(t) = \begin{bmatrix} 1 & e^{-t} & e^{-2t} & \ldots & e^{-Nt} \end{bmatrix}, \ \mathbf{A}_1 = \begin{bmatrix} a_{1,0} & a_{1,1} & \cdots & a_{1,N} \end{bmatrix}^T, \ \mathbf{A}_2 = \begin{bmatrix} a_{2,0} & a_{2,1} & \cdots & a_{2,N} \end{bmatrix}^T.$$

Secondly, the relation between $\mathbf{E}(t)$ and its first derivative $\mathbf{E'}(t)$ is given by

$$\mathbf{E'}(t) = \mathbf{E}(t)\mathbf{M}, \tag{5}$$

so that

$$\mathbf{M}=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & -2 & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \cdots & -N \end{bmatrix}.$$

By placing Equation (5) into the first derivative of the solution functions, we have the matrix forms

$$S^{(1)}(t)=\mathbf{E}(t)\mathbf{M}\mathbf{A}_1 \text{ and } I^{(1)}(t)=\mathbf{E}(t)\mathbf{M}\mathbf{A}_2 . \tag{6}$$

By using the relations (4) and (5), we construct the matrices $\mathbf{y}(t)$ and $\mathbf{y}^{(1)}(t)$ as follows:

$$\mathbf{y}(t)=\overline{\mathbf{E}}(t)\mathbf{A} \quad \text{and} \quad \mathbf{y}^{(1)}(t)=\overline{\mathbf{E}}(t)\overline{\mathbf{M}}\mathbf{A}, \tag{7}$$

where

$$\mathbf{y}(t)=\begin{bmatrix} S(t) \\ I(t) \end{bmatrix},\ \mathbf{y}^{(1)}(t)=\begin{bmatrix} S^{(1)}(t) \\ I^{(1)}(t) \end{bmatrix},\ \overline{\mathbf{E}}(t)=\begin{bmatrix} \mathbf{E}(t) & 0 \\ 0 & \mathbf{E}(t) \end{bmatrix},\ \overline{\mathbf{M}}=\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{M} \end{bmatrix}, \mathbf{A}=\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}.$$

Now, we can show the system (3) with the matrix form

$$\mathbf{y}^{(1)}(t)=\mathbf{B}\mathbf{y}(t)+\mathbf{K}\overline{\mathbf{y}}(t)\mathbf{y}(t)+\mathbf{L}\mathbf{y}_{1,2}(t), \tag{8}$$

so that

$$\mathbf{B}=\begin{bmatrix} b-c & b \\ 0 & -c \end{bmatrix},\ \mathbf{K}=\begin{bmatrix} -1/k & 0 \\ 0 & -1/k \end{bmatrix},\ \mathbf{L}=\begin{bmatrix} -(1/k)-a \\ -(1/k)+a \end{bmatrix},\ \overline{\mathbf{y}}(t)=\begin{bmatrix} S(t) & 0 \\ 0 & I(t) \end{bmatrix} \text{ and } \mathbf{y}_{1,2}(t)=\left[S(t)I(t)\right].$$

By placing the collocation points, defined by

$$t_i=\frac{R}{N}i,\ i=0,1,\ldots,N, \tag{9}$$

in Equation (8) , we have the matrix equation system

$$\mathbf{y}^{(1)}(t_i)=\mathbf{B}\mathbf{y}(t_i)+\mathbf{K}\overline{\mathbf{y}}(t_i)\mathbf{y}(t_i)+\mathbf{L}\mathbf{y}_{1,2}(t_i), \quad i=0,1,\ldots,N .$$

Briefly, this system can be expressed in the matrix form

$$\mathbf{Y}^{(1)}-\overline{\mathbf{B}}\mathbf{Y}-\overline{\mathbf{K}}\overline{\mathbf{Y}}\mathbf{Y}-\overline{\mathbf{L}}\overline{\overline{\mathbf{Y}}}=\mathbf{Z}, \tag{10}$$

where

$$\mathbf{Y}^{(1)}=\begin{bmatrix}\mathbf{y}^{(1)}(t_0)\\ \mathbf{y}^{(1)}(t_1)\\ \vdots\\ \mathbf{y}^{(1)}(t_N)\end{bmatrix},\ \bar{\mathbf{B}}=\begin{bmatrix}\mathbf{B}&0&0&0\\0&\mathbf{B}&0&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{B}\end{bmatrix}_{(N+1)\times(N+1)},\ \mathbf{Y}=\begin{bmatrix}\mathbf{y}(t_0)\\ \mathbf{y}(t_1)\\ \vdots\\ \mathbf{y}(t_N)\end{bmatrix},\ \bar{\bar{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,2}(t_0)\\ \mathbf{y}_{1,2}(t_1)\\ \vdots\\ \mathbf{y}_{1,2}(t_N)\end{bmatrix}$$

$$\bar{\mathbf{K}}=\begin{bmatrix}\mathbf{K}&0&0&0\\0&\mathbf{K}&0&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{K}\end{bmatrix}_{(N+1)\times(N+1)},\ \bar{\mathbf{Y}}=\begin{bmatrix}\bar{\mathbf{y}}(t_0)&0&0&0\\0&\bar{\mathbf{y}}(t_1)&0&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\bar{\mathbf{y}}(t_N)\end{bmatrix}_{(N+1)\times(N+1)},\ \mathbf{Z}=\begin{bmatrix}0\\0\\ \vdots\\0\end{bmatrix}_{2(N+1)\times 1},$$

$$\bar{\mathbf{L}}=\begin{bmatrix}\mathbf{L}&0&0&0\\0&\mathbf{L}&0&0\\ \vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\mathbf{L}\end{bmatrix}_{(N+1)\times(N+1)}.$$

By putting the collocation points (9) into the relations given in Equation (7), we have the systems

$$\mathbf{y}(t_i)=\bar{\mathbf{E}}(t_i)\mathbf{A}\quad\text{and}\quad\mathbf{y}^{(1)}(t_i)=\bar{\mathbf{E}}(t_i)\bar{\mathbf{M}}\mathbf{A},\ i=0,1,\ldots,N.$$

Briefly, we can write these systems in the following matrix forms, respectively,

$$\mathbf{Y}=\mathbf{E}\mathbf{A}\ \text{ and }\ \mathbf{Y}^{(1)}=\mathbf{E}\bar{\mathbf{M}}\mathbf{A},\tag{11}$$

where

$$\mathbf{E}=\begin{bmatrix}\bar{\mathbf{E}}(t_0)\\ \bar{\mathbf{E}}(t_1)\\ \vdots\\ \bar{\mathbf{E}}(t_N)\end{bmatrix},\ \bar{\mathbf{E}}(t_i)=\begin{bmatrix}\mathbf{E}(t_i)&0\\0&\mathbf{E}(t_i)\end{bmatrix},\ i=0,1,\ldots,N.$$

By aid of Equation (4), the matrix $\bar{\mathbf{y}}(t)$ given in Equation (8) can be expressed with the matrix form

$$\bar{\mathbf{y}}(t)=\begin{bmatrix}S(t)&0\\0&I(t)\end{bmatrix}=\bar{\mathbf{E}}(t)\bar{\mathbf{A}},\tag{12}$$

where

$$\overline{\mathbf{E}}(t)=\begin{bmatrix}\mathbf{E}(t) & 0\\ 0 & \mathbf{E}(t)\end{bmatrix},\ \overline{\mathbf{A}}=\begin{bmatrix}\mathbf{A}_1 & 0\\ 0 & \mathbf{A}_2\end{bmatrix}.$$

By using the collocation points (9) and the relation (12), we gain the relation

$$\overline{\mathbf{Y}}=\begin{bmatrix}\overline{\mathbf{y}}(t_0) & 0 & 0 & 0\\ 0 & \overline{\mathbf{y}}(t_1) & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \overline{\mathbf{y}}(t_N)\end{bmatrix}=\begin{bmatrix}\overline{\mathbf{E}}(t_0)\overline{\mathbf{A}} & 0 & 0 & 0\\ 0 & \overline{\mathbf{E}}(t_1)\overline{\mathbf{A}} & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \overline{\mathbf{E}}(t_N)\overline{\mathbf{A}}\end{bmatrix}=\overline{\mathbf{E}}\overline{\overline{\mathbf{A}}}, \tag{13}$$

so that

$$\overline{\mathbf{E}}=\begin{bmatrix}\overline{\mathbf{E}}(t_0) & 0 & 0 & 0\\ 0 & \overline{\mathbf{E}}(t_1) & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \overline{\mathbf{E}}(t_N)\end{bmatrix},\ \overline{\mathbf{E}}(t_i)=\begin{bmatrix}\mathbf{E}(t_i) & 0\\ 0 & \mathbf{E}(t_i)\end{bmatrix},\ \overline{\overline{\mathbf{A}}}=\begin{bmatrix}\overline{\mathbf{A}} & 0 & 0 & 0\\ 0 & \overline{\mathbf{A}} & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \overline{\mathbf{A}}\end{bmatrix},$$

$$\overline{\mathbf{A}}=\begin{bmatrix}\mathbf{A}_1 & 0\\ 0 & \mathbf{A}_2\end{bmatrix}.$$

In the similar way, substituting the collocation points (9) into the $\mathbf{y}_{1,2}(t)$ given in Equation (8), we have the matrix form

$$\overline{\overline{\mathbf{Y}}}=\begin{bmatrix}\mathbf{y}_{1,2}(t_0) & \mathbf{y}_{1,2}(t_1) & \dots & \mathbf{y}_{1,2}(t_N)\end{bmatrix}^T=\begin{bmatrix}S(t_0)I(t_0) & S(t_1)I(t_1) & \dots & S(t_N)I(t_N)\end{bmatrix}^T=\overline{\mathbf{S}}\overline{\mathbf{I}}, \tag{14}$$

where

$$\overline{\mathbf{S}}=\tilde{\mathbf{E}}\overline{\mathbf{A}}_1 \ \text{ and } \ \overline{\mathbf{I}}=\tilde{\tilde{\mathbf{E}}}\mathbf{C}\mathbf{A}, \tag{15}$$

$$\tilde{\mathbf{E}}=\begin{bmatrix}\mathbf{E}(t_0) & 0 & 0 & 0\\ 0 & \mathbf{E}(t_1) & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{E}(t_N)\end{bmatrix},\ \overline{\mathbf{A}}_1=\begin{bmatrix}\mathbf{A}_1 & 0 & 0 & 0\\ 0 & \mathbf{A}_1 & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{A}_1\end{bmatrix}_{(N+1)\times(N+1)},\ \tilde{\tilde{\mathbf{E}}}=\begin{bmatrix}\mathbf{E}(t_0)\\ \mathbf{E}(t_1)\\ \vdots\\ \mathbf{E}(t_N)\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}\tilde{\mathbf{Z}} & \tilde{\mathbf{I}}\end{bmatrix},$$

$\tilde{\mathbf{Z}}=[0]_{(N+1)\times(N+1)}$, $\tilde{\mathbf{I}}$ is the unit matrix in dimension $(N+1)\times(N+1)$ and $\mathbf{A}=\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix}$.

Now, let us substitute relations (11), (13)-(15) into Equation (10) and thus we obtain the fundamental matrix equation

$$\left\{\mathbf{E}\bar{\mathbf{M}}-\bar{\mathbf{B}}\mathbf{E}-\bar{\mathbf{K}}\bar{\mathbf{E}}\bar{\bar{\mathbf{A}}}\mathbf{E}-\bar{\mathbf{L}}\tilde{\mathbf{E}}\bar{\mathbf{A}}_1\tilde{\bar{\mathbf{E}}}\right\}\mathbf{A}=\mathbf{Z}. \tag{16}$$

Briefly, we can express the matrix equation (16) in the form

$$\mathbf{W}\mathbf{A}=\mathbf{Z} \text{ or } [\mathbf{W};\mathbf{Z}]; \quad \mathbf{W}=\mathbf{E}\bar{\mathbf{M}}-\bar{\mathbf{B}}\mathbf{E}-\bar{\mathbf{K}}\bar{\mathbf{E}}\bar{\bar{\mathbf{A}}}\mathbf{E}-\bar{\mathbf{L}}\tilde{\mathbf{E}}\bar{\mathbf{A}}_1\tilde{\bar{\mathbf{E}}}\mathbf{C}, \tag{17}$$

which corresponds to a system of the $2(N+1)$ nonlinear algebraic equations with the unknown coefficients $a_{1,n}$ and $a_{2,n}$, ($n=0,1,2,...,N$).

From the relation (7), the matrix form for conditions in the model (1) becomes

$$\mathbf{U}\mathbf{A}=[\lambda] \quad \text{or} \quad [\mathbf{U};\lambda], \tag{18}$$

where

$$\mathbf{U}=\bar{\mathbf{E}}(0)\mathbf{A} \text{ and } \lambda=\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix}.$$

As a result, by replacing the rows of the matrix $[\mathbf{U};\lambda]$ by two rows of the augmented matrix $[\mathbf{W};\mathbf{Z}]$, we have the new augmented matrix

$$[\tilde{\mathbf{W}};\tilde{\mathbf{Z}}] \text{ or } \tilde{\mathbf{W}}\mathbf{A}=\tilde{\mathbf{Z}}, \tag{19}$$

which is a nonlinear algebraic system. The unknown coefficients are computed by solving this system. The unknown coefficients $a_{i,0},a_{i,1},\ldots,a_{i,N}$, $(i=1,2)$ are substituted in Equation (4). Hence, we gain the exponential polynomial solutions

$$S_N(t)=\sum_{n=0}^{N}a_{1,n}e^{-nt} \quad \text{and} \quad I_N(t)=\sum_{n=0}^{N}a_{2,n}e^{-nt}.$$

We can easily check the accuracy of these solutions as follows:

Since the truncated series (2) are approximate solutions of system (1), when the function $S_N(t)$, $I_N(t)$ and theirs derivatives are substituted in system (1), the resulting equation must be satisfied approximately; that is, for $t=t_q\in[0,R]$ $q=0,1,2,\ldots$,

$$\begin{cases} E_{1,N}(t_q) = \left| S_N^{(1)}(t_q) - b\left(S_N(t_q) + I_N(t_q)\right) + cS_N(t_q) + \dfrac{S_N(t_q)\left(S_N(t_q) + I_N(t_q)\right)}{k} + aS_N(t_q)I_N(t_q) \right| \cong 0, \\ E_{2,N}(t_q) = \left| I_N^{(1)}(t_q) + cI_N(t_q) + \dfrac{I_N(t_q)\left(S_N(t_q) + I_N(t_q)\right)}{k} - aS_N(t_q)I_N(t_q) \right| \cong 0, \end{cases} \tag{20}$$

and

$$E_{i,N}(t_q) \le 10^{-k_q}, \ i = 1,2 \ \ (k_q \text{ positive integer}).$$

If max $10^{-k_q} = 10^{-k}$ (k positive integer) is prescribed, then the truncation limit N is increased until the difference $E_{i,N}(t_q)$ at each of the points becomes smaller than the prescribed 10^{-k}, see [Yüzbaşı (2012a), (2012b)].

3. Numerical Applications

In this section, we apply the presented method to obtain the approximate exponential solutions in interval $0 \le t \le 3$ for solving Hantavirus infection model [Abramson and Kenkre (2002), Abramson et al. (2003), Goh et al. (2009) and Gökdoğan et al. (2012)]. Firstly, we consider the model (1) for $a = 0.1$, $b = 1$, $c = 0.5$, $k = 20$ with the initial conditions $S(0) = 10$ and $I(0) = 10$.

By applying the procedure in Section 2, we obtain the approximate solutions for $N = 3,5,8$, respectively,

$$\begin{aligned} S_3(t) = {} & 7.97961086872372246666 - 1.3577048868675920297\mathrm{e}^{-t} + 3.7765771675640166596\mathrm{e}^{-2t} \\ & - 0.39848314942014709651\mathrm{e}^{-3t}, \end{aligned}$$

$$\begin{aligned} I_3(t) = {} & 2.8100391985861038614 + 13.011369515975257778\mathrm{e}^{-t} - 9.4528566277088271393\mathrm{e}^{-2t} \\ & + 3.6314479131474655003\mathrm{e}^{-3t}, \end{aligned}$$

$$\begin{aligned} S_5(t) = {} & 8.1062106573203768838 - 4.1539364503803987741\mathrm{e}^{-t} + 19.983801525528864957\mathrm{e}^{-2t} \\ & - 40.417559636037345816\mathrm{e}^{-3t} + 41.968407210407807439\mathrm{e}^{-4t} - 15.486923306839304690\mathrm{e}^{-5t}, \end{aligned}$$

$$\begin{aligned} I_5(t) = {} & 2.4654747495280979769 + 20.279637386816532149\mathrm{e}^{-t} - 51.412352423470310580\mathrm{e}^{-2t} \\ & + 106.22137100041973618\mathrm{e}^{-3t} - 106.65160802533515909\mathrm{e}^{-4t} + 39.097477312041103369\mathrm{e}^{-5t}, \end{aligned}$$

$$S_8(t)=8.3986256360790617831-3.7914786186071451877e^{-t}+53.787946143568880295e^{-2t}$$
$$-300.90798479851296821e^{-3t}+1098.5575432950269574e^{-4t}-2475.3960662942672625e^{-5t}$$
$$+3287.0301032241887022e^{-6t}-2325.9385883729143652e^{-7t}+668.25989978543813943e^{-8t},$$

$$I_8(t)=1.9700160282326584478+21.266752484031151884e^{-t}-137.16044310971876387e^{-2t}$$
$$+738.51164961488074596e^{-3t}-2622.3802236466945357e^{-4t}+5815.8217970419928128e^{-5t}$$
$$-7639.9688738358019135e^{-6t}+5362.7702661022330538e^{-7t}-1530.8309406791552099e^{-8t}.$$

For $N=5$, $a=0.1$, $b=1$, $c=0.5$, $k=20$, $S(0)=10$ and $I(0)=10$, the approximate solutions are compared with the solutions of the fourth-order Runge-Kutta (RK4) method in Figure 1. From this comparison, it is observed that our method and RK4 method are consistent. We note that the classical fourth-order Runge-Kutta method was used for Lorenz system which is a system of nonlinear ordinary differential equations in [Ababneh et al. (2009)]. Figure 2(a) denotes a plot of the approximate solutions $S_N(t)$ obtained for $S(0)=10$, $I(0)=10$ and $N=3,5,8$. The approximate solutions $I_N(t)$ obtained for $S(0)=10$, $I(0)=10$ and $N=3,5,8$ are given in Figure 2(b). We compute the error functions for $S(0)=10$, $I(0)=10$ and $N=3,5,8$ by using Equation (20). Figure 3(a) displays the error functions for $S_N(t)$ for $N=3,5,8$. The error functions for $I_N(t)$ for $N=3,5,8$ are shown in Figure 3(b). Figure 2(a) shows that the population of suspected mice $S(t)$ slowly approaches to 8 for $N=3,5,8$. It is seen from Figure 2(b) that the population of infected mice $I(t)$ decreases to 3 for $N=3,5,8$.

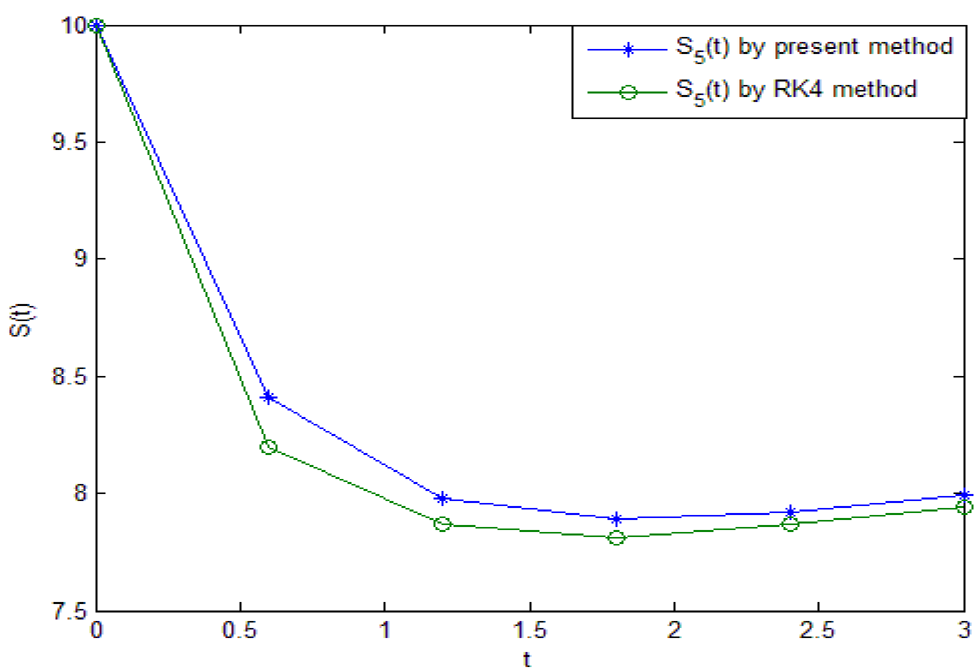

Figure 1(a). Comparison of the solutions $S_5(t)$ of the present method and the RK4 method for $a=0.1$, $b=1$, $c=0.5$, $k=20$, $S(0)=10$ and $I(0)=10$

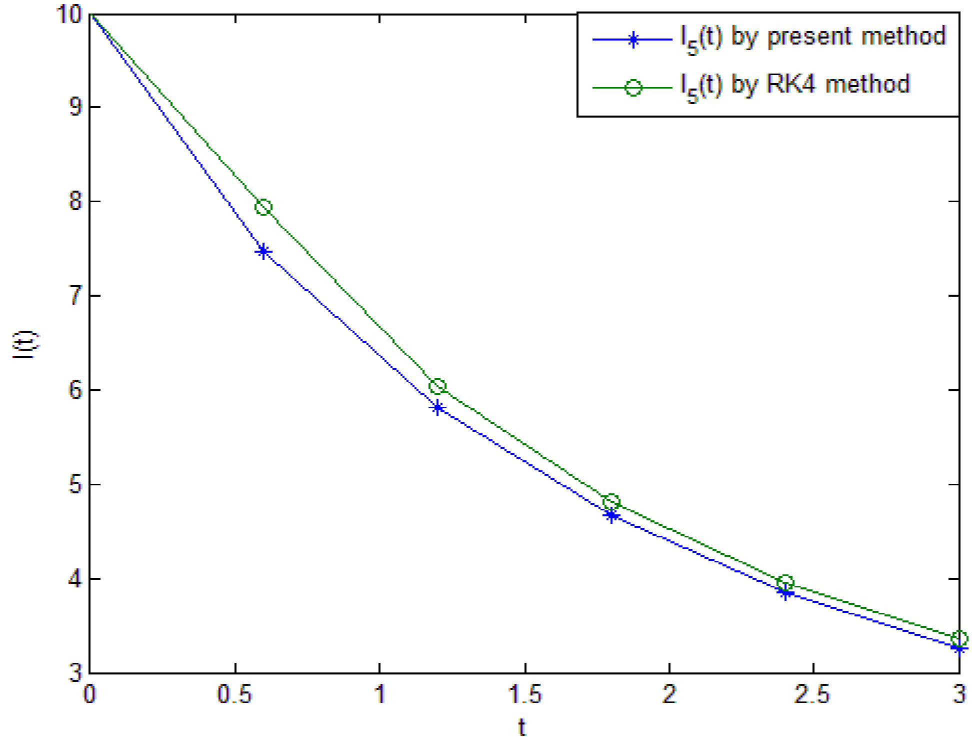

Figure 1(b). Comparison of the solutions $I_5(t)$ of the present method and the RK4 method for $a = 0.1$, $b = 1$, $c = 0.5$, $k = 20$, $S(0) = 10$ and $I(0) = 10$

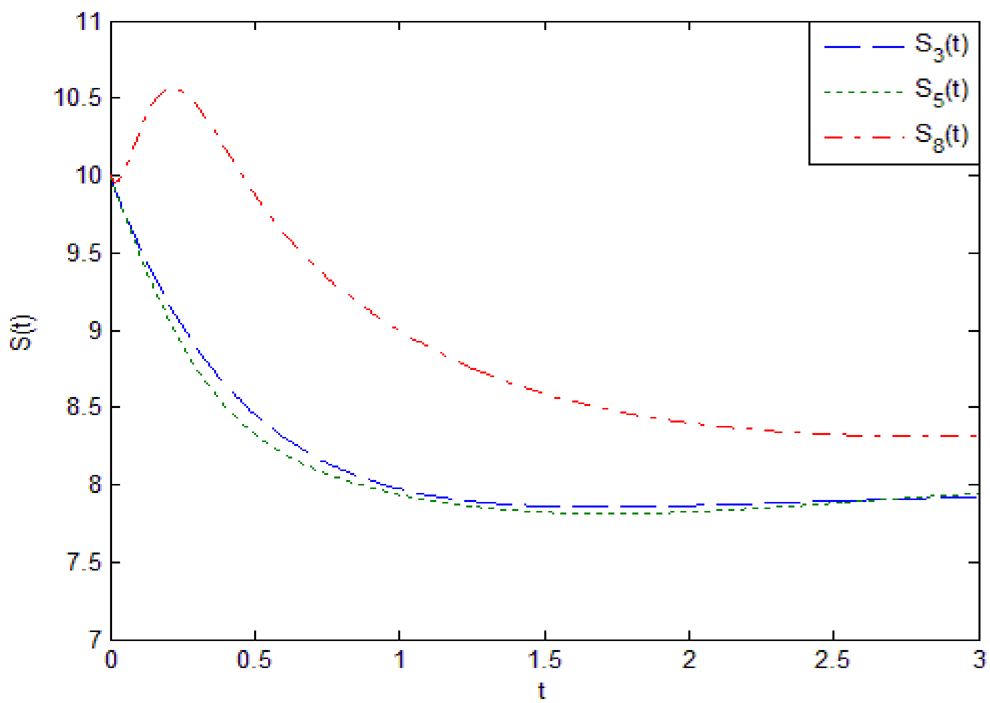

Figure 2(a). Graph of the solutions $S_N(t)$ for $S(0)=10$, $I(0)=10$ and $N=3,5,8$

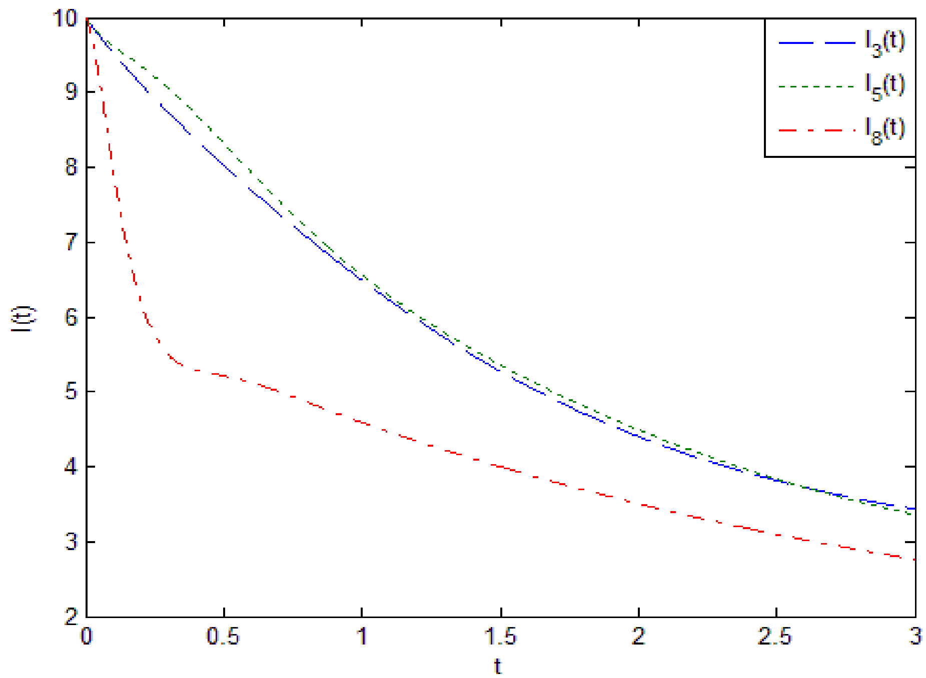

Figure 2(b). Graph of the solutions $I_N(t)$ for $S(0)=10$, $I(0)=10$ and $N=3,5,8$

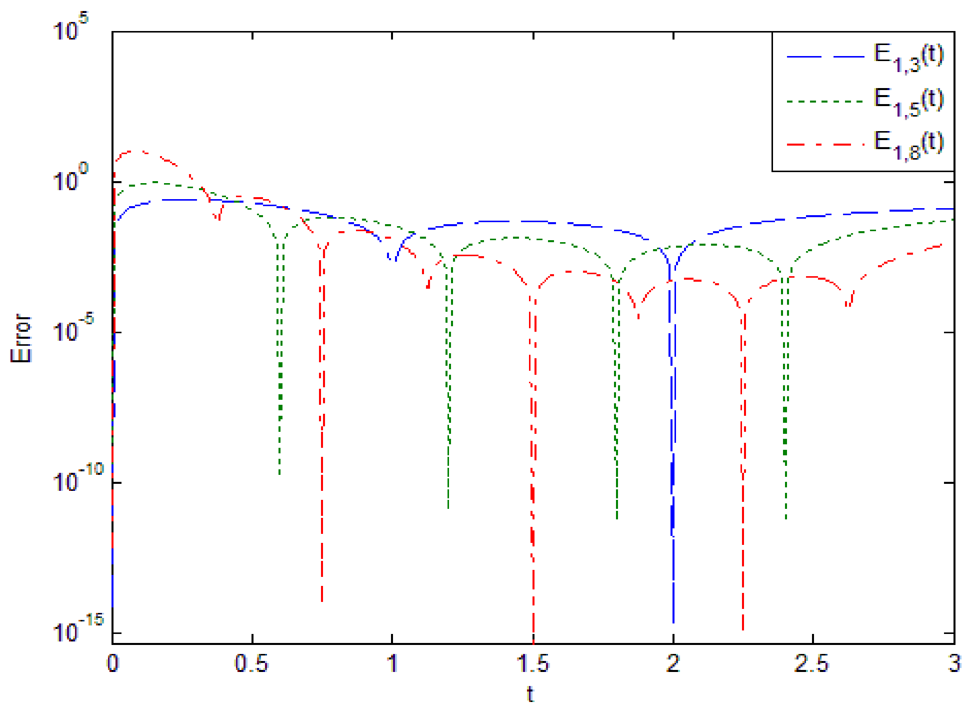

Figure 3(a). Graph of the error functions obtained with accuracy of the solutions $S_N(t)$ for $S(0)=10$, $I(0)=10$ and $N=3,5,8$

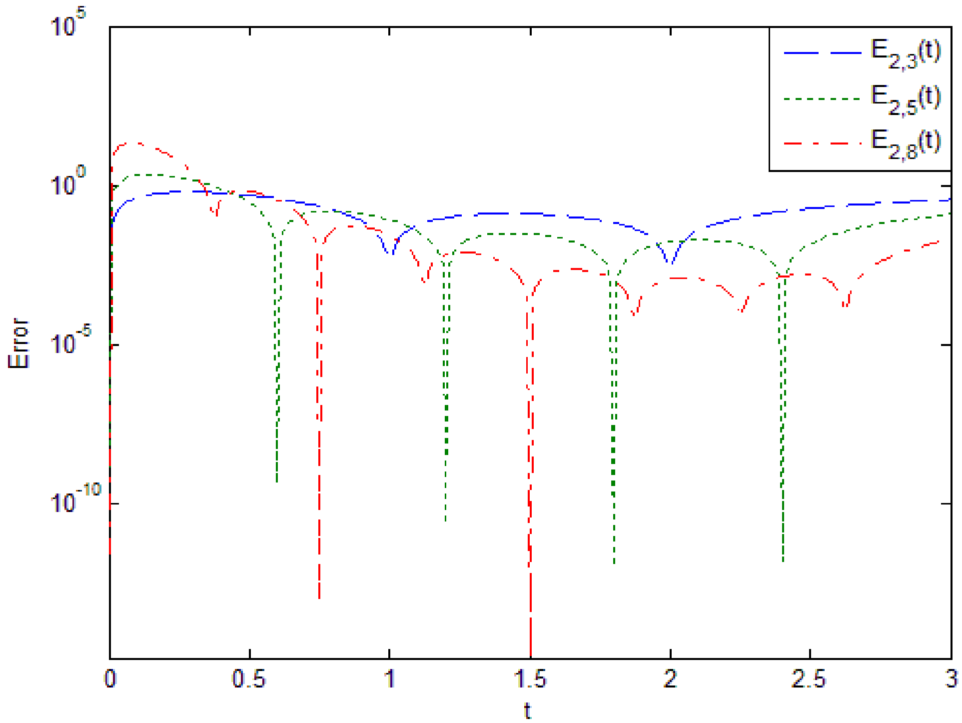

Figure 3(b). Graph of the error functions obtained with accuracy of the solutions $I_N(t)$ for $S(0)=10$, $I(0)=10$ and $N=3,5,8$

Secondly, let us consider the model (1) by selecting $a=0.1$, $b=1$, $c=0.5$, $k=20$, $S(0)=20$ and $I(0)=8$. By following the method in Section 2, the approximate solutions for $N=3,5,8$ are computed as follows, respectively,

$$S_3(t)=7.9785791910339515167+2.0977724901029807716\mathrm{e}^{-t}+5.8687174466921839069\mathrm{e}^{-2t}$$
$$+4.0549308721708838049\mathrm{e}^{-3t},$$

$$I_3(t)=3.0008588899987160837+14.193577267970781365\mathrm{e}^{-t}-12.589731205937710981\mathrm{e}^{-2t}$$
$$+3.3952950479682135323\mathrm{e}^{-3t},$$

$$S_5(t)=8.0964426892753099268-0.85251493218771395312\mathrm{e}^{-t}+22.355041398527925686\mathrm{e}^{-2t}$$
$$-35.960235247891340219\mathrm{e}^{-3t}+41.783192582573209558\mathrm{e}^{-4t}-15.421926490297390999\mathrm{e}^{-5t},$$

$$I_5(t)=2.6287826727296234772+22.433717400030604592\mathrm{e}^{-t}-59.245440301504902762\mathrm{e}^{-2t}$$
$$+118.31107746833087355\mathrm{e}^{-3t}-120.96461699591757457\mathrm{e}^{-4t}+44.836479756331375714\mathrm{e}^{-5t},$$

$$
\begin{aligned}
S_8(t) = {} & 8.4700074822947369071 + 4.1940925874831515074e^{-t} + 34.949026159768877167e^{-2t} \\
& - 186.39763397675789407e^{-3t} + 708.47714649755769076e^{-4t} - 1643.0056622272026225e^{-5t} \\
& + 2239.2445356886538861e^{-6t} - 1624.2053657314944162e^{-7t} + 478.27385351969659032e^{-8t},
\end{aligned}
$$

$$
\begin{aligned}
I_8(t) = {} & 2.0344897099603089328 + 20.843875482002528078e^{-t} - 145.70232728100196255e^{-2t} \\
& + 778.21306485826602621e^{-3t} - 2765.0083922309125786e^{-4t} + 6144.0074841549497186e^{-5t} \\
& - 8090.5986550166078558e^{-6t} + 5694.9740198329983464e^{-7t} - 1630.7635595096545313e^{-8t}.
\end{aligned}
$$

For $N = 5$, $a = 0.1$, $b = 1$, $c = 0.5$, $k = 20$, $S(0) = 20$ and $I(0) = 8$, the comparisons between the approximation solutions of our method and the fourth-order Runge-Kutta (RK4) method are given in Figure 4. It is seen from Figure 4 that our method is to be in harmony with the fourth-order Runge-Kutta (RK4). The approximate solutions $S_N(t)$ and $I_N(t)$ obtained for $S(0) = 20$, $I(0) = 8$ and $N = 3,5,8$ are shown in Figure 5(a) and Figure 5-(b), respectively. Equation (20). From Equation (20), we calculate the error functions for these approximate solutions and we display them in Figure 6(a) and Figure 6(b). We see from Figure 5(a) that population of suspected mice $S(t)$ slowly approaches to 8 for $N = 3,5,8$. It is seen from that the population of infected mice $I(t)$ decreases to 3 for $N = 3,5,8$.

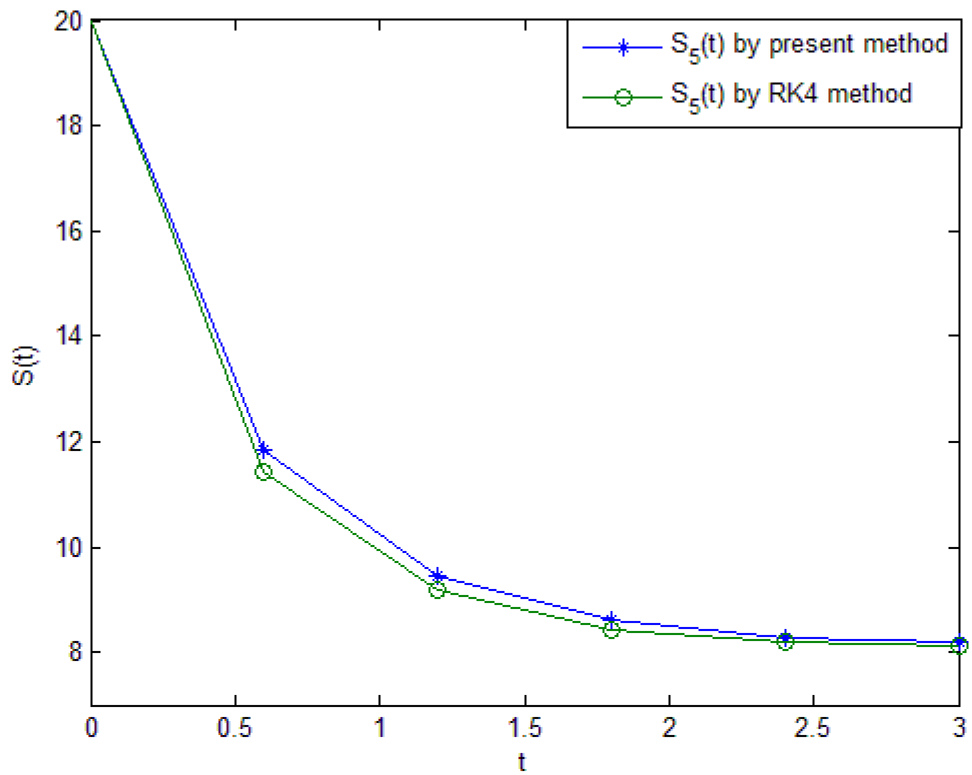

Figure 4(a). Comparison of the solutions $S_5(t)$ of the present method and the RK4 method for $a = 0.1$, $b = 1$, $c = 0.5$, $k = 20$, $S(0) = 20$ and $I(0) = 8$

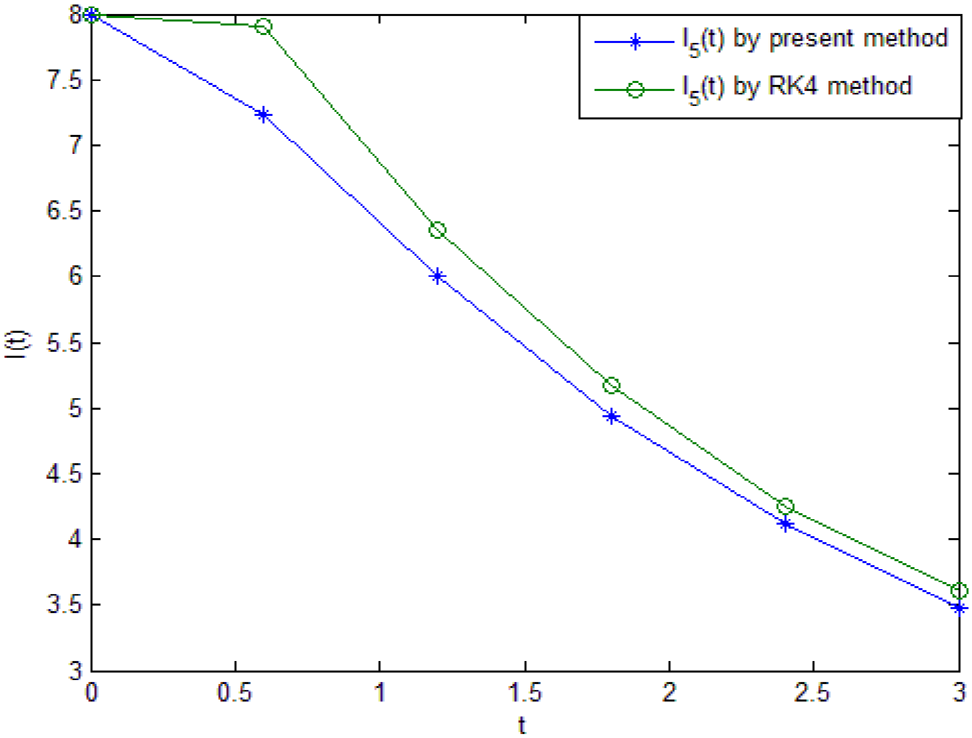

Figure 4(b). Comparison of the solutions $I_5(t)$ of the present method and the RK4 method for $a = 0.1$, $b = 1$, $c = 0.5$, $k = 20$, $S(0) = 20$ and $I(0) = 8$

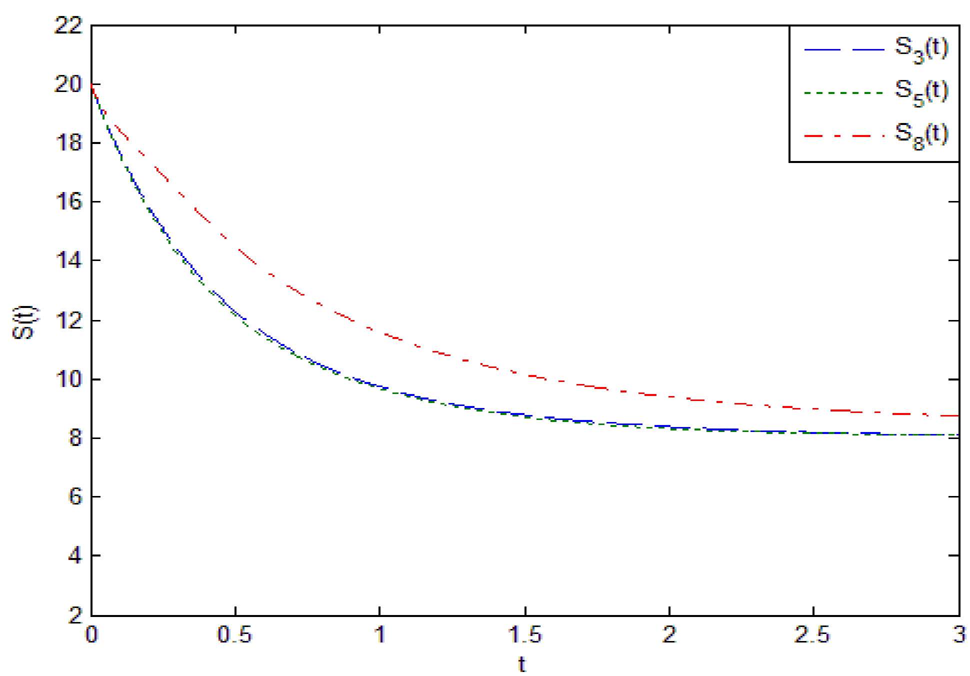

Figure 5. (a). Graph of the solutions $S_N(t)$ for $S(0) = 20$, $I(0) = 8$ and $N = 3, 5, 8$

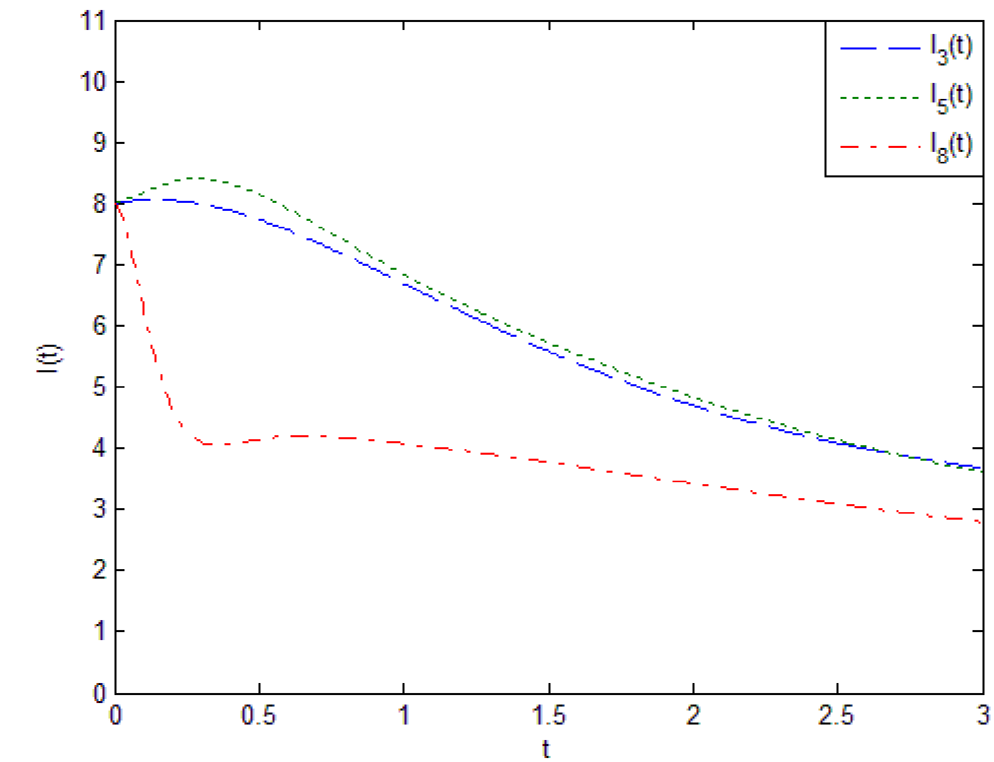

Figure 5. (b). Graph of the solutions $I_N(t)$ for $S(0) = 20$, $I(0) = 8$ and $N = 3, 5, 8$

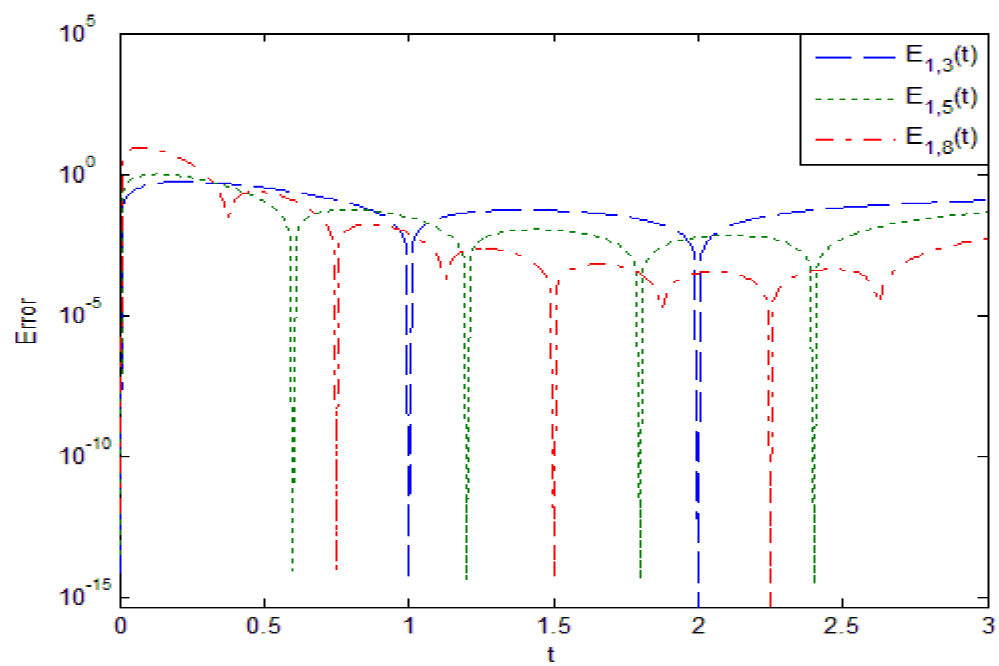

Figure 6. (a). Graph of the error functions obtained with accuracy of the solutions $S_N(t)$ for $S(0) = 20$, $I(0) = 8$ and $N = 3, 5, 8$

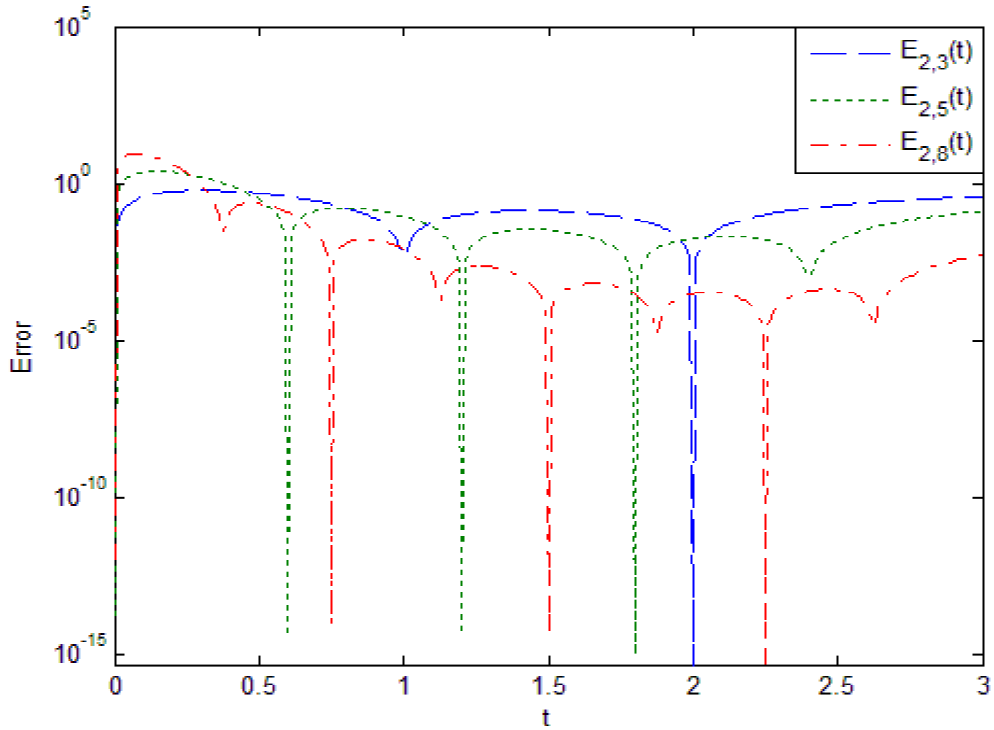

Figure 6. (b). Graph of the error functions obtained with accuracy of the solutions $I_N(t)$ for $S(0)=20$, $I(0)=8$ and $N=3,5,8$

4. Conclusions

The solutions of some problems encountered in Science and Engineering are in form of exponential functions. Therefore, it is important to find the approximate solutions in terms of the exponential functions of ordinary differential equations. In this paper, a new exponential approximation is presented for solving the Hantavirus infection model by means of exponential polynomials bases. This model corresponds to a class of systems of nonlinear ordinary differential equations. Application of the technique is simple and practical. A applications of the suggested method have been given to demonstrate the accuracy and efficiency of this method for the model problem. Comparisons between the solutions of our method and the fourth-order Runge-Kutta (RK4) method are given. From these comparisons, it is observed that our method and RK4 method are consistent. We assured the correctness of the obtained solutions by putting them back into the original equation with the aid of Maple; it provides a measure for confidence of the results. It is seen from Figure 3(a), (b) and Figure 6(a), (b) that the accuracies of the solutions increase when value of N is increased. However, the calculation errors may be too big for large values of N. Therefore, it is recommended that value of N is to select large enough. The computations associated with the application have been performed using a computer code written in Maple. The basic idea described in this study can be used to be further employed to solve other similar nonlinear problems.

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