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Mathematical Model of Blood Flow through a Composite Stenosis in Catheterized Artery with Permeable Wall

Rupesh K. Srivastav
Department of Mathematics
Ambalika Institute of Management & Technology
Integral University
Lucknow-226001, India
rupeshk.srivastav@gmail.com

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Abstract

The present mathematical analysis, the study of blood flow through the model of a composite stenosed catheterized artery with permeable wall, has been performed to investigate the blood flow characteristics. The expressions for the blood flow characteristics-the impedance (resistance to flow), the wall shear stress distribution in stenosis region, the shear stress at the throat of the stenosis have been derived. The results obtained are displayed graphically and discussed briefly.

Keywords: Stenosis; catheter; permeable wall; Darcy number; slip parameter

MSC2010: 76Z05

1. Introduction

The cause and development of many cardiovascular diseases, most common types are ischemia, angina pectoris, myocardial infraction and cerebral strokes, are related to the nature of blood flow and mechanical behavior of the blood vessel wall, that is why the study of the blood flow through a stenosed artery is very important. The generic medical term stenosis or arteriosclerosis is the narrowing of any body passage, tube or orifice; comes from the Greek words arthero (meaning gruel or paste) and sclerosis (hardness). A common form of arterial narrowing or stenosis is that caused by atheroma, a deposition of fats and fibrous tissue in the arterial lumen. Such constriction of the arterial lumen grows inward and restricts the normal flow of blood where the transport of blood to the region beyond the narrowing is reduced considerably. Moreover, under normal physiological conditions, the transport of blood in the human circulatory system depends entirely on the pumping action of the heart which produces a pressure gradient throughout the arterial system.

Since the first investigation of Mann et al. (1938), a large number of studies including the important contributions of Young (1968,1979), Young and Tsai (1973), Caro et al. (1978), Shukla et al.(1980), Ahmed and Giddens (1983), Sarkar and Jayaraman (1998), Pralhad and Schultz (2004), Jung et al. (2004), Liu et al. (2004) Srivastava and coworkers (1996, 2009, 2010a,b), Mishra et al. (2006), Ponalagusamy (2007), Layek et al. (2005, 2009), Joshi et al. (2009), Mekheimer and El-Kot (2008), Tzirtzilakis (2008), Mandal and coworkers (2005, 2007a, b), Politis et al. (2007, 2008), Siddiqui et al. (2009), Singh et al. (2010), Medhavi (2011) and many others, have been conducted in the literature in various context.

It has been established that the development of stenosis in its early stage of the disease, is strongly related to the characteristics of the blood flow by [Gidden et al. (1993)]. It is reported that at high shear rates and in larger vessel, blood behaves like a Newtonian fluid, [Taylor (1959)], [Young (1968)] has analyzed the effects of stenosis on flow characteristics of blood treating blood as a Newtonian fluid. He reported that an increase in the stenosis size increases both the impedance to flow and wall shear stress.

The insertion of a catheter into an artery forms the annular region between the catheter wall and the arterial wall. A catheter is composed of polyster based thermoplastic polyurethane, medical grade polyvinyl choloride, etc. The insertion of the catheter will change the flow field, modify the pressure distribution and increase the resistance. Even though the catheter tool devices are used for the measurement of arterial blood pressure or pressure gradient and flow velocity or flow rate, X-ray angiography and intravascular ultrasound diagnosis and coronary ballon angioplasty treatment of various arterial diseases, a little attention has been given in literature to the flow in catheterized arteries.

Kanai et al. (1970), has reported that when a catheter is inserted in a stenosed artery, it further increases the impedance to flow and changes the pressure distribution. Jayaraman and Tewari (1995) have studied blood flow in a catheterized curved artery, by assuming the artery as a curved pipe and the catheter to be co-axial to it. Young and Tsai (1973), Lee (1974), Mcdonald (1979), Ahmad and Giddens (1983), Ponalagusami (1986), Back (1994) and Back et al. (1996) studied the mean flow resistance increase during coronary artery catheterization in normal as well as stenosed arteries. Srivastava and Srivastava (2009) have presented a brief review of the literature on artery catheterization with and without stenosis. Layek et al. (2009) investigated the effect of an overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial direction of the arterial segment under consideration. Srivastava et al. (2010) investigated the effect on flow characteristics of blood due to the presence of overlapping stenosis in an artery assuming that the flowing blood is represented by Newtonian fluid.

The plasma membrane is a thin elastic membrane around the cell which usually allows the movement of small irons and molecules of various substances through it. This nature of plasma membrane is termed as ‘permeability’. The flow in the permeable boundary is described by

Darcy law which states that the rate at which a fluid flows through a permeable substance per unit area is equal to the permeability times the pressure drop per unit length of flow, divided by the viscosity of fluid. Most recently, Srivastava et al. (2009) studied impedance and wall shear stress in blood flow through a bell shaped stenosis in an artery with permeable wall.

In view of the above discussion, the research reported here is devoted to the study of blood flow through a composite stenosis in catheterized artery with permeable wall; the flow in the permeable boundary is described by Darcy law; assuming that blood is represented by a Newtonian fluid.

2. Formulation of the Problem

Consider the axisymmetric flow of blood through a composite stenosis, specified at the position shown in Figure 1 in an inserted catheterized artery with permeable wall. The geometry of the stenosis which is assumed to be manifested in the arterial wall segment is described as:

$$\frac{R(z)}{R_0} = 1 - \frac{2\delta}{R_0 L_0}(z-d); \quad d \le z \le d + L_0/2, \tag{1}$$

$$= 1 - \frac{\delta}{2R_0}\left\{1 + \cos\frac{2\pi}{L_0}(z - d - L_0/2)\right\}; \; d + L_0/2 \le z \le d + L_0, \tag{2}$$

$$= 1; \qquad \text{otherwise}, \tag{3}$$

where $(R(z),\ R_0)$ are the radii of the tube (with, without) stenosis, L_0 is the stenosis length and d indicates its location, δ is the depth of the stenosis at throat.

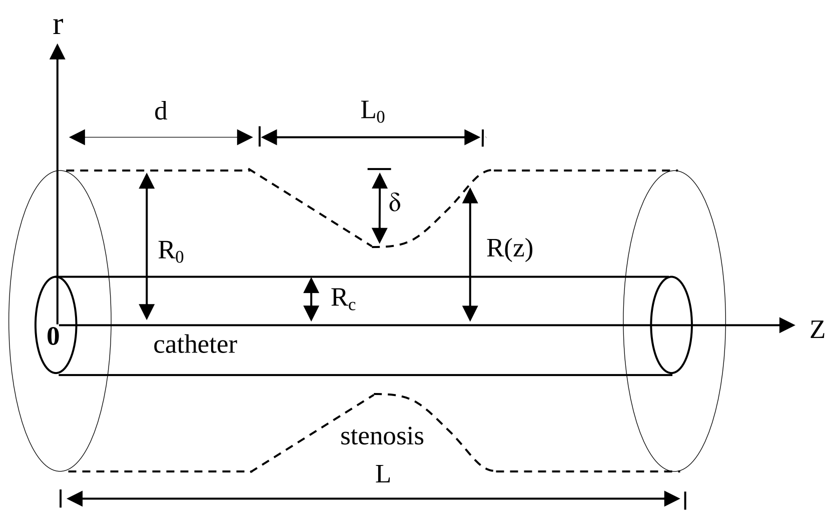

Figure 1. The geometry of a composite stenosis in inserted catheterized artery with permeable wall

The flowing blood is assumed to be represented by a Newtonian fluid; using thus a continuum approach, the equations governing the linear momentum and the conservation of mass for a Newtonian fluid are given by

$$\rho\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\mu\nabla^2 u, \tag{4}$$

$$\rho\left\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\mu(\nabla^2-\frac{1}{r^2})v, \tag{5}$$

$$\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial z}=0, \tag{6}$$

where $\nabla^2=\partial^2/\partial r^2+(1/r)(\partial/\partial r)+\partial^2/\partial z^2$ is a two-dimensional Laplacian operator, r is the radial coordinate measured in the direction normal to the tube axis, (u,v) denotes the (axial, radial) components of velocity of the fluid, p is the pressure, ρ and μ are respectively the fluid density and the viscosity.

Due to the non-linearity of convective acceleration terms, to obtain the solution of equations (4)–(6) is a formidable task. Depending, therefore, on the size of the stenosis, however, certain terms in these equations are of less importance than other (Young, 1968). Considering thus the case of a mild stenosis $(\delta/R_0<<1)$, the general constitutive equations (4)-(6) in the case of an axisymmetric, laminar, steady one-dimensional flow of blood in an artery reduce (Young, 1968; Srivastava and Rastogi, 2009) to

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) \tag{7}$$

where u is the axial velocity, μ is the fluid viscosity, r is the radial co-ordinate and p is the pressure.

The condition that are specified at the artery wall and the interface for present study may now be stated (Beavers and Joseph, 1967; Srivastava et al., 2012) as

$$u=0,\quad \text{at } r=R_c, \tag{8}$$

$$u=u_B \text{ and } \frac{\partial u}{\partial \mathrm{r}}=\frac{\alpha}{\sqrt{D_a}}(u_B-u_{porous}),\quad \text{at } r=R(z), \tag{9}$$

where $u_{porous}=\frac{-D_a}{\mu}\frac{dp}{dz}$, u_{porous} is the velocity in the permeable boundary, u_B is the slip velocity, D_a is the Darcy number and α, called slip parameter, is a dimensionless quantity depending on the material parameters which characterize the structure of the permeable material within the boundary region.

3. Analysis

Using boundary conditions (8) and (9), the expression for the velocity obtained as solution of equation (7), is given as

$$u = \frac{-1}{4\mu}\frac{dp}{dz}\left[R^2 - r^2 + \frac{(R^2 - R_c^2)}{\log(R/R_c)}\log\left(\frac{r}{R}\right)\right] + \left[\frac{\log(r/R)}{\log(R/R_c)} + 1\right] u_B. \tag{10}$$

Also the slip velocity, u_B is determined as

$$u_B = \frac{-1}{4\mu}\frac{dp}{dz}\left[-2R + \frac{(R^2 - R_c^2)}{R\log(R/R_c)} + 4\alpha\sqrt{D_a}\right]\frac{1}{\{\alpha/\sqrt{D_a} - 1/R\log(R/R_c)\}}. \tag{11}$$

An application of equation (10) into (11), yields

$$u = \frac{-1}{4\mu}\frac{dp}{dz}\left[\begin{array}{c}\left\{R^2 - r^2 + \dfrac{(R^2 - R_c^2)}{\log(R/R_c)}\log\left(\dfrac{r}{R}\right)\right\} + \left\{\dfrac{\log(r/R)}{\log(R/R_c)} + 1\right\} \\ \left\{\left(-2R + \dfrac{(R^2 - R_c^2)}{R\log(R/R_c)} + 4\alpha\sqrt{D_a}\right)\Bigg/\left(\dfrac{\alpha}{\sqrt{D_a}} - \dfrac{1}{R\log(R/R_c)}\right)\right\}\end{array}\right]. \tag{12}$$

The volumetric flow rate, Q is now calculated as

$$Q = 2\pi\int_{R_c}^{R} urdr$$

$$Q = \frac{-\pi R_0^4}{8\mu}\frac{dp}{dz}\left[\begin{array}{r}\left(\dfrac{R}{R_0}\right)^4\left(1 - \dfrac{1}{\log(R/R_c)}\right) + \left(\dfrac{R}{R_0}\right)^2\left(\dfrac{2\varepsilon^2}{\log(R/R_c)} - \dfrac{\beta}{R_0^2\log(R/R_c)}\right. \\ \left. - \dfrac{2\beta}{R_0^2}\right) - \dfrac{\varepsilon^2}{\log(R/R_c)}\left(\varepsilon^2 - \dfrac{\beta}{R_0^2}\right) - \varepsilon^4 + \left(\dfrac{4\beta\varepsilon^2}{R_0^2}\right)\end{array}\right], \tag{13}$$

where $\varepsilon = \dfrac{R_c}{R_0}$, and $\beta = \left(-2R + \dfrac{(R^2 - R_c^2)}{R\log(R/R_c)} + 4\alpha\sqrt{D_a}\right)\Bigg/\left(\dfrac{\alpha}{\sqrt{D_a}} - \dfrac{1}{R\log(R/R_c)}\right)$.

From equation (13), one now obtains

$$\frac{dp}{dz} = -\frac{8\mu Q}{\pi R_0^4}\phi(z), \tag{14}$$

where $\phi(z) = \dfrac{1}{\Delta(z)}$ and with

$$\Delta(z) = \left[\begin{array}{r} \left(\dfrac{R}{R_0}\right)^4 \left(1 - \dfrac{1}{\log(R/R_c)}\right) + \left(\dfrac{R}{R_0}\right)^2 \left(\dfrac{2\varepsilon^2}{\log(R/R_c)} - \dfrac{\beta}{R_0^2 \log(R/R_c)} - \dfrac{2\beta}{R_0^2}\right) \\ - \dfrac{\varepsilon^2}{\log(R/R_c)}\left(\varepsilon^2 - \dfrac{\beta}{R_0^2}\right) - \varepsilon^4 + \left(\dfrac{4\beta\varepsilon^2}{R_0^2}\right) \end{array} \right].$$

The pressure drop, $\Delta p (= p \text{ at } z = 0, \text{-}p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz} \right) dz = \frac{8\mu Q}{\pi R_0^4} \psi \,, \tag{15}$$

where

$$\psi = \int_0^d [\varphi(z)]_{R/R_0=1} dz + \int_d^{d+L_0/2} [\varphi(z)]_{R/R_0 \text{ from } (1)} dz + \int_{d+L_0/2}^{d+L_0} [\varphi(z)]_{R/R_0 \text{ from } (2)} dz + \int_{d+L_0}^{L} [\varphi(z)]_{R/R_0=1} dz \,.$$

The first and the fourth integrals in the expression obtained above are straight forward whereas the analytical evaluation of second and third integrals are almost a formidable task and therefore shall be evaluated numerically.

The flow resistance (resistive impedance), $\bar{\lambda}$, the wall shear stress in stenotic region, $\overline{\tau_w}$, and shearing stress at stenotic throat, $\overline{\tau_s}$ are now calculated as

$$\bar{\lambda} = \frac{\Delta p}{Q} = \frac{8\mu}{\pi R_0^4} \left[\frac{L - L_0}{\Omega} + \int_d^{d+L_0/2} [\phi(z)]_{R/R_0 \text{ from } (1)} dz + \int_{d+L_0/2}^{d+L_0} [\phi(z)]_{R/R_0 \text{ from } (2)} dz \right], \tag{16}$$

$$\bar{\tau}_w = -\frac{R}{2}\frac{dp}{dz} = \frac{4\mu Q}{\pi R_0^3} \left(\frac{R}{R_0}\right) \phi(z) \,, \tag{17}$$

$$\tau_s = [\tau_w]_{R/R_0 = 1 - \delta/R_0} \,. \tag{18}$$

Following now the reports of Srivastava et al. (2009), one derives the expressions for the impedance, λ, the wall shear stress in the stenotic region, τ_w, and shearing stress at stenotic throat, $\overline{\tau_s}$ in their non-dimensional form as

$$\lambda = \eta\left[\frac{1-L_0/L}{\Omega} + \frac{1}{L}\left(\int_d^{d+L_0/2}[\phi(z)]_{R/R_0 \text{ from (1)}}\,dz + \int_{d+L_0/2}^{d+L_0}[\phi(z)]_{R/R_0 \text{ from (2)}}\,dz\right)\right], \tag{19}$$

$$\tau_w = \eta\frac{(R/R_0)}{\left[\begin{array}{l}\left(\frac{R}{R_0}\right)^4\left(1-\frac{1}{\log(R/R_0)}\right)+\left(\frac{R}{R_0}\right)\left(\frac{2\varepsilon^2}{\log(R/R_0\varepsilon)}-\frac{\beta}{R_0^2\log(R/R_0\varepsilon)}\right.\\ \left.\qquad -\frac{2\beta}{R_0^2}\right)-\frac{\varepsilon^2}{\log(R/R_0\varepsilon)}\left(\varepsilon^2-\frac{\beta}{R_0^2}\right)-\varepsilon^4+4\varepsilon^2\frac{\beta}{R_0^2}\end{array}\right]}, \tag{20}$$

$$\tau_s = [\tau_w]_{R/R_0=1-\delta/R_0}, \tag{21}$$

where,

$$\lambda = \bar{\lambda}/\lambda_0,\ (\tau_w,\tau_s) = (\bar{\tau}_w,\bar{\tau}_s)/\tau_0,\ \eta = 1-\frac{4\sqrt{D_a}}{\alpha R_0^2}(R_0-2\alpha\sqrt{D_a}),$$

$$\frac{\beta}{R_0^2} = \frac{\left[-2\left(\frac{R}{R_0}\right)+\frac{(R/R_0)^2-(R_c/R_0)^2}{(R/R_0)\log(R/R_0\varepsilon)}+\frac{4\alpha\sqrt{D_a}}{R_0}\right]}{\left[\frac{R_0\alpha}{\sqrt{D_a}}-\frac{1}{(R/R_0)\log(R/R_0\varepsilon)}\right]},$$

$$\Omega = (1-\varepsilon^2)\left(1-\varepsilon^2+\frac{\beta}{R_0^2}\right)\frac{1}{\log\varepsilon}-(1-2\varepsilon^2)\frac{2\beta}{R_0^2}+1-\varepsilon^4,$$

and $\lambda_0 = \frac{8\mu L}{\pi\eta R_0^4}$ and $\tau_0 = \frac{4\mu Q}{\pi\eta R_0^3}$ are, respectively, the resistive impedance and wall shear stress for an uncatheterized normal artery (no stenosis).

In the absence of the catheter (i.e., under the limit $\varepsilon \to 0$), one derives the expressions for λ, τ_w, respectively, through a stenosed artery with permeable wall as

$$\lambda = \eta\left[\frac{1-L_0/L}{\Omega}+\frac{1}{L}\int_d^{d+L_0}\frac{dz}{(R/R_0)^2\left\{(R/R_0)^2+\frac{4\sqrt{D_a}}{\alpha R_0^2}(R-2\alpha\sqrt{D_a})\right\}}\right], \tag{22}$$

$$\tau_w = \frac{\eta}{(R/R_0)\left\{(R/R_0)^2 + \dfrac{4\sqrt{D_a}}{\alpha R_0^2}(R - 2\alpha\sqrt{D_a})\right\}}, \tag{23}$$

$$\tau_s = [\tau_w]_{R/R_0 = 1-\delta/R_0}. \tag{24}$$

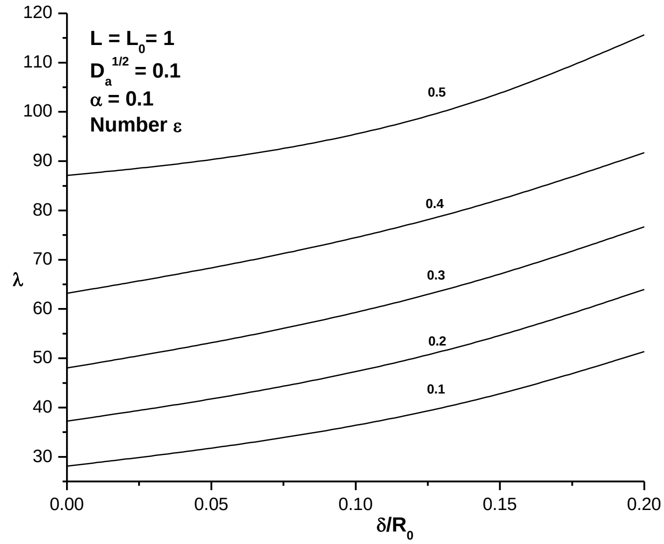

Figure:2. Impedance, λ versus stenosis height, δ / R_0 for different catheter size, ε.

4. Numerical Results and Discussion

The development of a stenosis in an artery can obviously create many serious problems and in general disrupt the normal function of the circulatory system. In order to observe the quantitative effects of the various parameters involved in the present analysis, computer codes are developed to evaluate the analytical results obtained in equations (19)-(21) for dimensionless resistance to flow, λ, the wall shear stress, τ_w, in the stenotic region, the shear stress, τ_s, at the throat of the stenosis in a tube of radius $R_0 = 0.01cm$ for various parameter values: $d = 0$; $L_0(cm) = 1$; $L\,(cm) = 1, 2, 5$; ε (non-dimensional catheter radius)= 0.1, 0.2, 0.3, 0.4, 0.5; α = 0.1, 0.2, 0.3, 0.4, 0.5; $\sqrt{D_a}$ (square root of Darcy number, D_a and hereafter referred as Darcy number) = 0.1, 0.2, 0.3, 0.4, 0.5; δ/R_0 = 0, 0.05, 0.10, 0.15, 0.20.

The impedance, λ, increases with increasing value of the catheter size, ε, also increases with the stenosis height, δ/R_0, for any given set of parameters which turns difficult for blood to flow in the blood vessel (Figure 2).

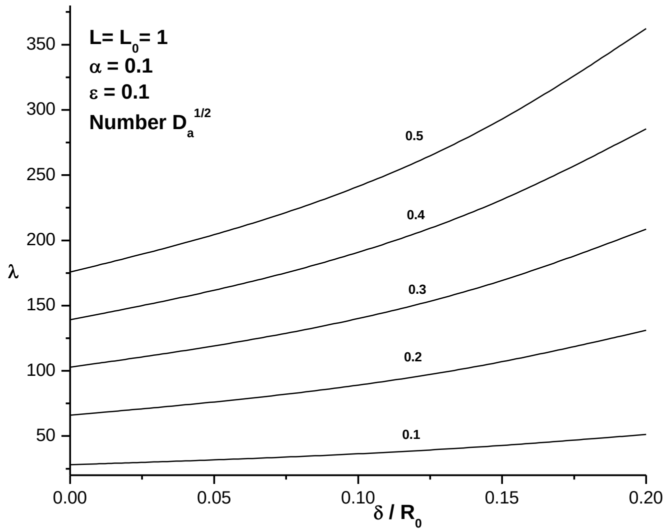

Figure:3. Impedance, λ versus stenosis height, δ / R_0 for different Darcy number, $D_a^{1/2}$.

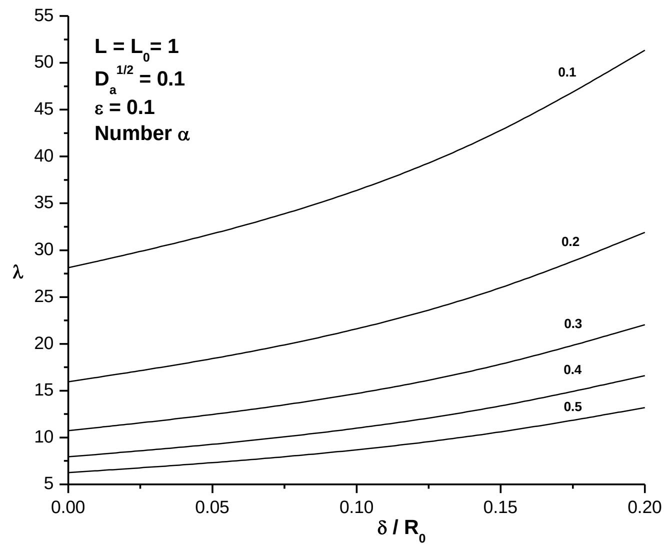

Figure:4. Impedance, λ versus stenosis height, δ / R_0 for different slip parameter, α.

It is observed that the presence of the catheter causes increase in the magnitude of impedance, D_a, in addition to that due to the presence of the stenosis (Figure 3) and possesses similar characteristics with catheter size, ε, (Figure 5).

It is also clear that the blood flow characteristic, impedance, λ, decreases with increasing value of the slip parameter, α, increases with the stenosis height, δ/R_0, for given value of Darcy number, $\sqrt{D_a}$, (Figures 4).

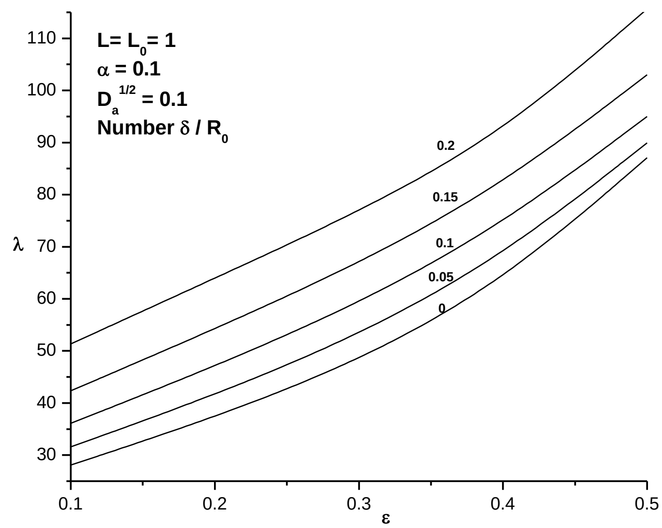

Figure:5. Impedance, λ versus catheter size, ε for different stenosis height, δ / R_0 .

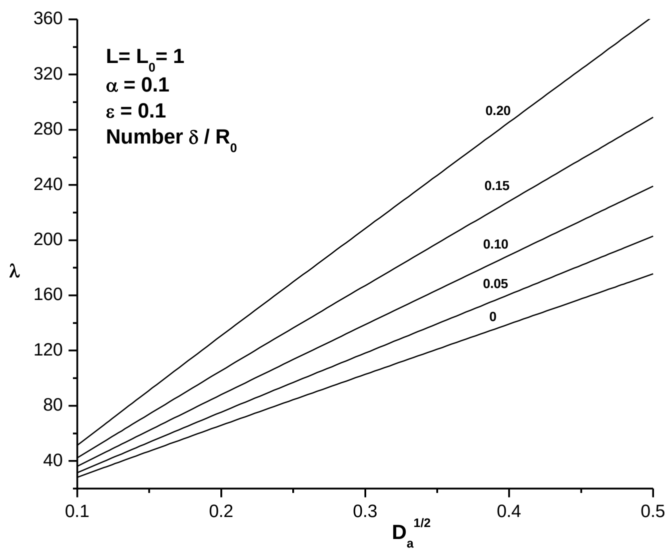

Figure:6. Impedance, λ versus Darcy number, $D_a^{1/2}$ for different stenosis height, δ / R_0 .

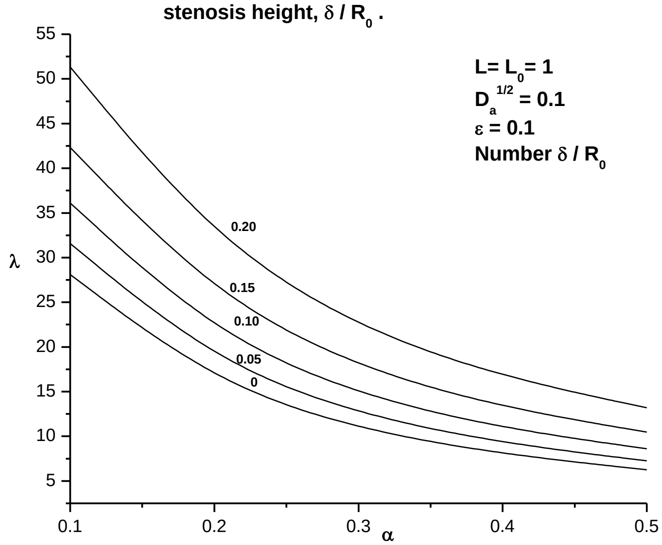

Figure: 7. Impedance, λ versus slip parameter, α for different stenosis height, δ / R_0 .

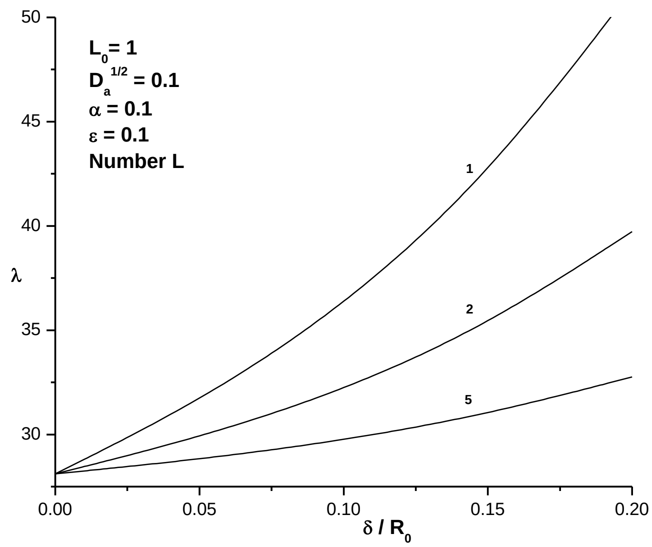

Figure:8. Impedance, λ versus stenosis height, δ / R_0 for different tube length L .

For other given set of parameters, the blood flow characteristic, λ, increases with increasing Darcy number, $\sqrt{D_a}$, (Figure:6). One observes that the flow resistance, λ, decreases rapidly with increasing value of the slip parameter from its maximal amplitude at $\alpha = 0.1$ (Figure:7). For a given Darcy number, $\sqrt{D_a}$, the catheter size, ε , the impedance, λ , decreases with increasing tube length, L , increases with stenosis size (Figure: 8). The wall shear stress in the stenotic region, τ_w, increases rapidly in the upstream of the stenosis throat from its approached value at $Z/L_0 = 0$ and achieves its maximal magnitude at the stenosis throat (i.e., at $Z/L_0 = 0.5$), it then decreases rapidly in the downstream of the throat to its approached value at the end point of the constriction profile (at $Z/L_0 = 1$)(Figure: 9, 10, 11 and 12). The curves representing the shear stress distribution have features almost analogous to the geometry of a composite stenosis under

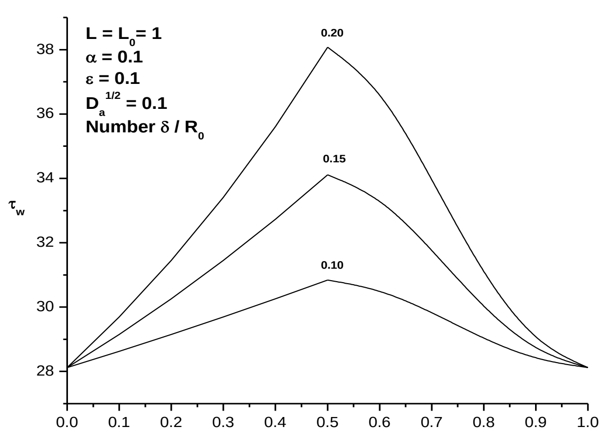

Figure:9. Wall shear stress distribution, τ_w in stenotic region for different stenosis height, δ / R_0.

consideration. The wall shear stress are found to be compressive in nature and they are all downwardly concave.

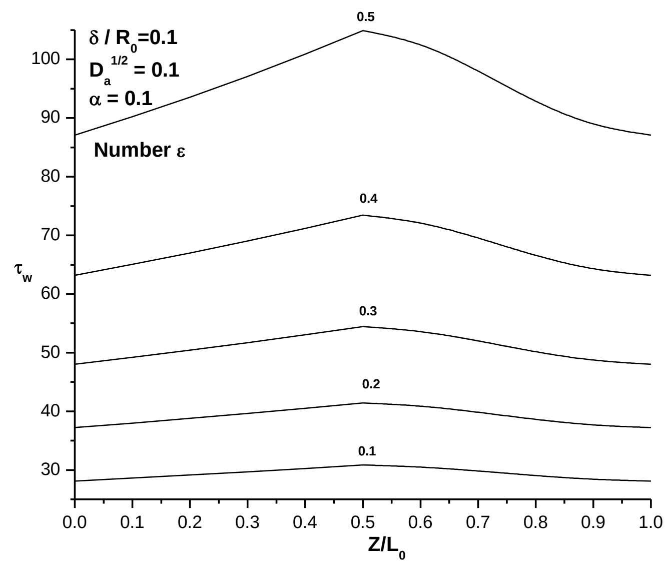

Figure:10. Wall shear stress distribution, τ_w in stenotic region for different catheter size, ε .

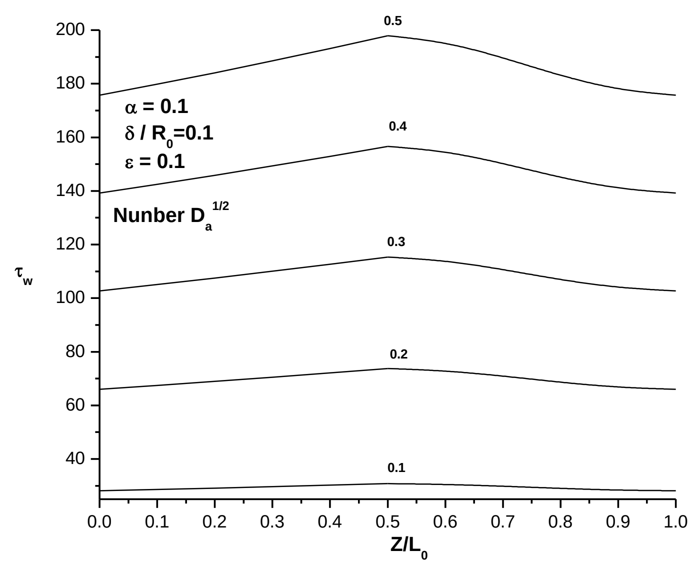

Figure:11. Wall shear stress distribution, τ_w in stenotic region for different Darcy number $D_a^{1/2}$.

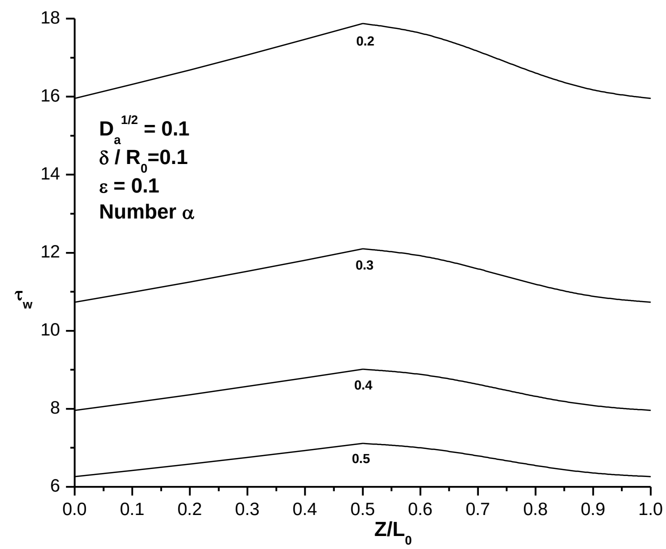

Figure:12. Wall shear stress distribution, τ_w in stenotic region for different slip parameter, α .

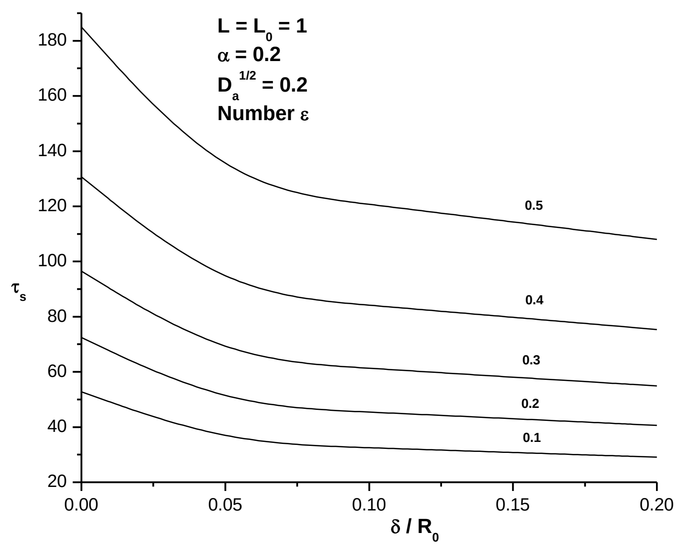

Figure:13. Wall shear stress at stenosis throat, τ_S versus stenosis height, δ / $R_0$for different catheter size, ε .

The wall shear stress at the stenosis throat, τ_S increases with increasing value of the stenosis height, δ/R_0 and attains its maximal magnitude and after that assumes an asymptotic value with increasing value of the stenosis height, δ/R_0 , (Figure 14) and also wall shear stress, τ_S corresponding to catheter size, ε , possesses similar characteristics (Figure 13).

The wall shear stress at the stenosis throat, τ_S, decreases with increasing value of the slip parameter, α, decreases and assumes an asymptotic value with increasing value of the stenosis height, δ/R_0, (Figure 15).

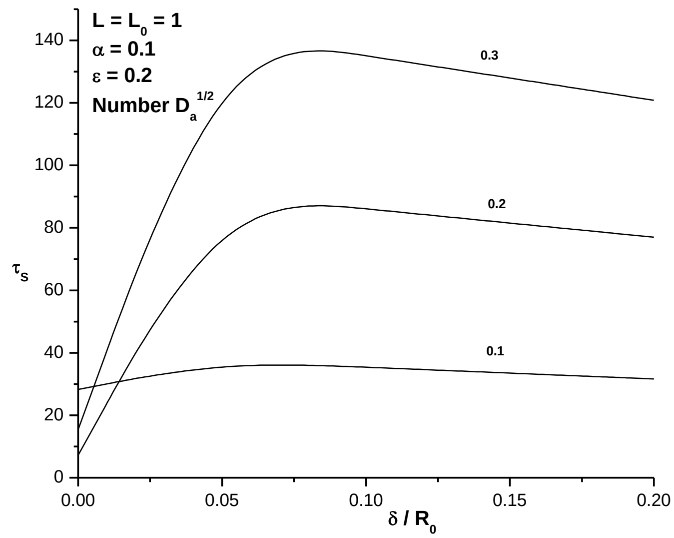

Figure:14. Wall shear stress at stenosis throat, τ_S versus stenosis height, δ / R_0 for different Darcy number, D_a.

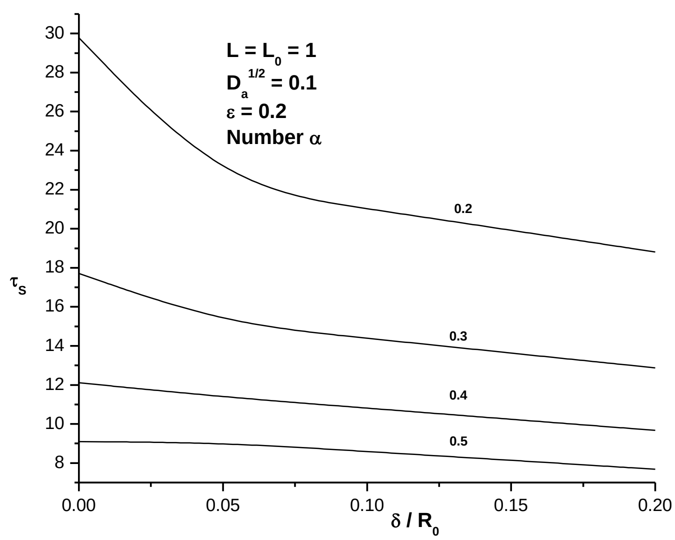

Figure:15. Wall shear stress at stenosis throat, τ_S versus stenosis height, δ / R_0 for different slip parameter, α .

5. Conclusions

In the present paper, we have discussed the effect on flow characteristics of a Newtonian fluid in an inserted catheterized stenosed artery with permeable wall. A significant change in the magnitude of the blood flow characteristics occurs with respect to the flow parameters and the catheter size. Thus, the size of the catheter must be chosen, keeping in view, the stenosis height during the medical treatment.

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