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MHD Mixed Convective Flow of Viscoelastic and Viscous Fluids in a Vertical Porous Channel

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Abstract

In this paper, we analyze the problem of steady, mixed convective, laminar flow of two incompressible, electrically conducting and heat absorbing immiscible fluids in a vertical porous channel filled with viscoelastic fluid in one region and viscous fluid in the other region. A uniform magnetic field is applied in the transverse direction, the fluids rise in the channel driven by thermal buoyancy forces associated with thermal radiation. The equations are modeled using the fully developed flow conditions. An exact solution is obtained for the velocity, temperature, skin friction and Nusselt number distributions. The physical interpretation to these expressions is examined through graphs and table for the shear stress and rate of heat transfer coefficients at the channel walls.

Keywords: MHD, mixed convection, viscoelastic fluid, thermal radiation, porous channel

MSC 2010: 74H10, 65L10, 76A10, 76S05

Nomenclature

X, Y - space coordinates
U_1, U_2 - dimensional velocity distributions
g - gravitational force
P - dimensional pressure
E_0 - dimensional viscoelastic parameter
B_0 - strength of transverse magnetic field
K_0 - dimensional permeability of the porous medium
$k_1,\ k_2$ - coefficient of thermal conductivities
$T_1,\ T_2$ - dimensional temperature distributions
T_0 - static temperature
c_p - specific heat at constant pressure
Q - dimensional heat sink parameter
$K_{\lambda w}$ - radiation absorption coefficient at the wall
$e_{b\lambda}$ - Plank's function
a - ratio of electric conductivity, $a = \sigma_2 / \sigma_1$
b - density ratio, $b = \rho_2 / \rho_1$
c - viscosity ratio, $c = \mu_1 / \mu_2$
k - thermal conductivity ratio, $k = k_1 / k_2$
h - channel width ratio, $h = h_2 / h_1$
E - viscoelasic parameter
R - suction parameter
M^2 - Hartmann number
K - permeability of the porous medium
Gr - thermal Grashof number
Re - Reynolds number
Pr - Prandtl number
F - thermal radiation parameter
u - velocity distribution
$Nu_1,\ Nu_2$ - Nusselt number at the walls

Greek letters

$\rho_1,\ \rho_2$ - densities
$\mu_1,\ \mu_2$ - dynamic viscosities
$\nu_1,\ \nu_2$ - kinematic viscosities
$\sigma_1,\ \sigma_2$ - coefficient of electric conductivities
$\beta_1,\ \beta_2$ - thermal expansion coefficients

β - thermal expansion coefficient ratio, $\beta = \beta_2 / \beta_1$
α - heat source parameter
θ - temperature distribution
τ_1, τ_2 - skin friction at the walls

Subscript
1, 2 - reference quantities for Region-I and Region-II, respectively.

1. Introduction

The interaction between the conducting fluid and the magnetic field radically modifies the flow, with attendant effects on such important flow properties as pressure drop and heat transfer, the detailed nature of which is strongly dependent on the orientation of the magnetic field. The advent of technology that involves the MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting fluids. Thome (1964) initiated the first investigation associated with two phase liquid metal magneto-fluid-mechanics generator. Postlethwaite and Sluyter (1978) presented an overview of the heat transfer problems related to MHD generators. Lohrasbi and Sahai (1988) considered MHD two-phase flow and heat transfer in a horizontal channel and reported analytical solutions for the velocity and temperature profiles for the case where only one of the fluids is electrically conducting. Malashetty and Leela (1992) reported closed-form solutions for the two-phase flow and heat transfer situation in a horizontal channel for which both phases are electrically conducting. Malashetty and Umavathi (1997) studied two-phase MHD flow and heat transfer in an inclined channel in the presence of buoyancy effects for the situation where only one of the phases is electrically conducting. Malashetty et al. (2006) examined the magneto convection of two-immiscible fluids in vertical enclosure.

The flow and heat transfer aspects of immiscible fluids is of special importance in petroleum extraction and transport. For example, the reservoir rock of an oil field always contains several immiscible fluids in its pores. Part of the pore volume is occupied by water and the rest may be occupied either by oil or gas or both. These examples show the importance of knowledge of the laws governing immiscible multi-phase flows for proper understanding of the processes involved. The subject of two-fluid flow and heat transfer has been extensively studied due to its importance in chemical and nuclear industries. Identification of the two-fluid flow region and determination of the pressure drop, void fraction, quality reaction and two-fluid heat transfer coefficient are of great importance for the design of two-fluid systems. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. There have been some studies on various aspects of two-phase flow reported in the literature [Packham and Shail (1971), Malashetty et al. (2000, 2001), Umavathi et al. (2005), Muthuraj and Srinivas (2010) and Prathap Kumar et al. (2010)].

The theory of non-Newtonian fluids has become a field of very active research for the last few decades as this class of fluids represents many industrially important fluids such as plastic films and artificial fibers in industry. The second grade fluids are one of the most popular models for non-Newtonian fluids. In general the equation of motion of incompressible second-grade fluid is of higher order than the Navier-Stokes equations. A marked difference between the case of the Navier-Stokes theory and that for fluids of second grade is that of ignoring the nonlinearity in the Navier-Stokes equation. Ignoring the nonlinearity in the Navier-Stokes equation does not reduce the order of the equation but ignoring the higher-order nonlinearities in the case of the second-grade fluid reduces the order of the equation. The extra stress tensor of the second-grade fluids is of the form $\sigma^* = \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2$, where μ is the dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients that are related to the material modulus and the kinematic first two Rivlin-Ericksen tensors A_1 and A_2 are given by $A_1 = gradV + (gradV)^T$ and $A_2 = A_1 gradV + (gradV)^T A_1$. The Cauchy stress tensor for a general incompressible and homogeneous Rivlin-Ericksen fluid of second-grade is given by $T^* = -pI + \sigma^*$ … (i), here p is the pressure and $-pI$ is the spherical part of the stress due to the constraint of incompressibility and V is velocity. If the fluid modeled by (i) is to be compatible with the thermodynamics in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid is a minimum in equilibrium, then we can get $\mu \geq 0,\ \alpha_1 \geq 0,\ \alpha_1 + \alpha_2 = 0$. The fluid exhibited anomalous behavior not to be expected of any fluid of rheological interest if $\alpha_1 < 0$ and $\alpha_1 + \alpha_2 \neq 0$. It is very important that solutions to steady flow problems can be found when $\alpha_1 < 0$. We can find brief details of the above discussed results from the works of [Dunn and Fosdick (1974), Fosdick and Rajagopal (1978, 1979), Rajagopal (1992), Dunn and Rajagopal (1995), Sadeghy and Sharifi (2004), Hayat et al. (2008), Sajid et al. (2010), Kumar and Gursharn (2010) and Kumar and Sivaraj (2011)].

Convective heat transfer and fluid flow in a system simultaneously containing a fluid reservoir and a porous medium saturated with fluid is of great mathematical and physical interest. More specifically the existence of a fluid layer adjacent to another layer of fluid saturated porous medium is a common occurrence in both geophysical and engineering environments. The systematic study of flow through porous medium constitutes a comparatively recent development in fluid mechanics, with applications in science, engineering and technology. Important technological applications include thermally enhanced oil recovery, subsurface contamination and remediation, drying process, chemical and nuclear reactor safety analysis and geothermal energy exploitation [Whitaker (1977), Dhir (1994), Chamkha (2000), Malashetty et al. (2004) and Nield and Bejan (2006)]. The hot walls and the working fluid are usually emitting the thermal radiation within the systems. The role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles. [Grosan and Pop (2007), Joshi and Kumar (2010) and Srinivas and Muthuraj (2010)] have made investigations of fluid flow with thermal radiation.

To the best of the author's knowledge the problem of MHD flow of viscoelastic and viscous fluid in a vertical porous channel has not been studied before. Therefore the purpose of the present paper is to report the analytical solutions for fully developed MHD mixed convective flow of viscoelastic and viscous fluid through a vertical channel filled with porous medium in the presence of thermal buoyancy, heat sink and thermal radiation. The solutions for the flow, heat transfer, wall shear stress and rate of heat transfer were obtained using the exact solution method. The effects of various significant parameters of this problem are illustrated through graphs and a table. The rest of the paper is structured as follows: The problem is formulated in Section 2. Section 3 comprises the solutions for flow and heat transfer analysis. The graphical and tabular results are presented and discussed in Section 4 and we present the conclusions in Section 5.

2. Formulation of the Problem

The fully developed, mixed convective, laminar flow of two immiscible fluids in a vertical channel filled with a porous medium is shown in the Figure 1. A coordinate system is chosen such that the X-axis is taken vertically upwards and the Y-axis is perpendicular to it. The walls Y_1 and Y_2 are maintained at constant temperatures T_{w1} and T_{w2} $\left(T_{w1} > T_{w2} \;\&\; T_{w2} = T_0\right)$ respectively. The region $-h_1 \le Y \le 0$ is occupied by viscoelastic fluid and the region $0 \le Y \le h_2$ is occupied by viscous fluid. An external uniform magnetic field of strength B_0 is applied normal to the vertical walls, the effect of thermal buoyancy; radiation and a temperature dependent heat sink are taken into account. The transport properties of both fluids are assumed to be constant. It is of worth to mention here that the viscoelastic and viscous fluids are immiscible and the constitutive equations for the fluids are different. Also, the viscosities, conductivities, densities $\rho_1 = \rho_0\left[1-\beta_1\left(T_1 - T_0\right)\right]$, $\rho_2 = \rho_0\left[1-\beta_2\left(T_2 - T_0\right)\right]$ and thermal expansions of both fluids are different. Since our model is general, one can choose any two different fluids which are immiscible. Under the assumptions stated above, we employ the Boussinesq approximation for the governing equations.

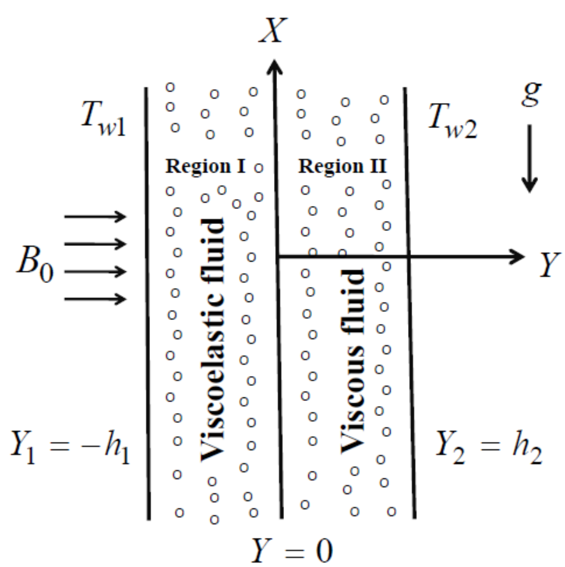

Figure 1. Flow geometry of the problem

It is assumed that the only non-zero components of the velocity q is the X-component $U_i\left(i = 1, 2\right)$. Thus, as a consequence of the mass balance equation, one can obtain

$$\frac{\partial U_i}{\partial X} = 0 \tag{1}$$

So that U_i depends only on Y. The momentum and energy balance equations are as follows
Region-I

$$E_0 \frac{d^3 U_1}{dY^3} + \mu_1 \frac{d^2 U_1}{dY^2} - \rho_1 \frac{dU_1}{dY} - \sigma_1 B_0^2 U_1 - \frac{\mu_1}{K_0} U_1 + \rho_1 g \beta_1 \left(T_1 - T_0\right) = \frac{dP}{dX} \tag{2}$$

$$\frac{k_1}{\rho_1 c_p} \frac{d^2 T_1}{dY^2} - \frac{dT_1}{dY} - \frac{Q\left(T_1 - T_0\right)}{\rho_1 c_p} - \frac{1}{\rho_1 c_p} \frac{dq_r}{dY} = 0. \tag{3}$$

Region-II

$$\mu_2 \frac{d^2 U_2}{dY^2} - \rho_2 \frac{dU_2}{dY} - \sigma_2 B_0^2 U_2 - \frac{\mu_2}{K_0} U_2 + \rho_2 g \beta_2 \left(T_2 - T_0\right) = \frac{dP}{dX} \tag{4}$$

$$\frac{k_2}{\rho_2 c_p} \frac{d^2 T_2}{dY^2} - \frac{dT_2}{dY} - \frac{Q\left(T_2 - T_0\right)}{\rho_2 c_p} - \frac{1}{\rho_2 c_p} \frac{dq_r}{dY} = 0\,. \tag{5}$$

The appropriate boundary and interface conditions on the velocity and the temperature are

$$U_1 = 0, \qquad T_1 = T_{w1} \qquad at \qquad Y_1 = -h_1 \tag{6}$$

$$U_2 = 0, \qquad T_2 = T_{w2} \qquad at \qquad Y_2 = h_2 \tag{7}$$

$$U_1 = U_2, \quad T_1 = T_2 \qquad at \qquad Y = 0 \tag{8a}$$

$$\mu_1 \frac{dU_1}{dY} = \mu_2 \frac{dU_2}{dY}, \qquad k_1 \frac{dT_1}{dY} = k_2 \frac{dT_2}{dY} \qquad at \qquad Y = 0\,. \tag{8b}$$

The pressure gradient dP/dX in Equations (2) and (4) is unknown and must be evaluated via the overall mass conservation equation

$$\int_{Y=-h}^{Y=0} U dY = Q^*\,. \tag{9}$$

The radiative heat flux [Cogley et al. (1968)] is given by

$$\frac{\partial q_r}{\partial Y} = 4\left(T_i - T_0\right) I' . \tag{10}$$

We introduce the following non-dimensional variables

$$x = \frac{X_i}{h_i},\ y = \frac{Y_i}{h_i},\ u_i = \frac{U_i}{U_0},\ \theta_i = \frac{T_i - T_0}{T_{w1} - T_{w2}},\ p = \frac{h_i P}{\mu_i U_0},\ E = \frac{E_0}{\mu_i h_i},\ R = \frac{U_0 d}{\nu_i},\ M^2 = \frac{\sigma_i B_0^2 h_i^2}{\mu_i},$$

$$\frac{1}{K} = \frac{h_i^2}{K_0},\ Gr = \frac{g\beta_i\left(T_{w1} - T_{w2}\right)h_i^2}{U_0 \nu_i},\ \mathrm{Re} = \frac{U_0 h_i}{\nu_i},\ Pr = \frac{\mu_i c_p}{k_i},\ \alpha = \frac{Q h_i^2}{k_i},\ F = \frac{4I' h_i^2}{k_i},\ i = 1,2. \tag{11}$$

In view of Equation (11), the dimensionless form of the momentum and energy equations become

Region-I

$$ER\frac{d^3u_1}{dy^3} + \frac{d^2u_1}{dy^2} - R\frac{du_1}{dy} - \left(M^2 + \frac{1}{K}\right)u_1 + \frac{Gr}{\mathrm{Re}}\theta_1 = B \tag{12}$$

$$\frac{d^2\theta_1}{dy^2} - PrR\frac{d\theta_1}{dy} - \left(\alpha + F\right)\theta_1 = 0 . \tag{13}$$

Region-II

$$\frac{d^2u_2}{dy^2} - R\frac{du_2}{dy} - \left(M^2 + \frac{1}{K}\right)u_2 + \frac{Gr}{\mathrm{Re}}\theta_2 = B \tag{14}$$

$$\frac{d^2\theta_2}{dy^2} - PrR\frac{d\theta_2}{dy} - \left(\alpha + F\right)\theta_2 = 0. \tag{15}$$

The dimensionless form of the boundary and interface conditions become

$$u_1 = 0, \quad \theta_1 = 1 \qquad at \quad y_1 = -1 \tag{16}$$

$$u_2 = 0, \quad \theta_2 = 0 \qquad at \quad y_2 = 1 \tag{17}$$

$$u_1 = u_2, \quad \theta_1 = \theta_2 \qquad at \quad y = 0 \tag{18a}$$

$$\frac{du_1}{dy} = \frac{1}{ch}\frac{du_2}{dy}, \quad \frac{d\theta_1}{dy} = \frac{1}{kh}\frac{d\theta_2}{dy} \qquad at \quad y = 0 , \tag{18b}$$

along with the overall mass conservation equation

$$\int_{y=-1}^{y=0} u\,dy = 1 . \tag{19}$$

3. Solution of the problem

When $E = 0$ (Newtonian fluid) the solution of the momentum equation (12) subject to the boundary and interface conditions (16) to (18b) is given by

$$u_1(y) = B_5 e^{l_1 Y} + B_6 e^{l_2 Y} + B_7 + B_{11} e^{l_5 Y} + B_{12} e^{l_6 Y}, \tag{20}$$

when $E \neq 0$ the solution for the velocity of the of the second-grade fluid obtained from the Equation (12) is

$$u_1(y) = B_{15} e^{l_1 Y} + B_{16} e^{l_2 Y} + B_{17} + C_1 e^{l_9 Y} + C_2 e^{l_{10} Y} + C_3 e^{l_{11} Y}, \tag{21}$$

where C_1, C_2 & C_3 are constants of integration. We have given one boundary and interface condition for u_1. Therefore we need to manipulate the solution before applying the boundary and interface conditions. The behavior of the $l_9,\ l_{10}$ & l_{11} is of crucial importance for small E.

Comparison of equations (20) and (21) clearly indicates that for small value of E, we need to have $C_1 = 0$ in equation (21), so that the solution matches the corresponding solution in the Newtonian case. Therefore the velocity of second-grade fluid is given by

$$u_1(y) = B_{15} e^{l_1 Y} + B_{16} e^{l_2 Y} + B_{17} + C_2 e^{l_{10} Y} + C_3 e^{l_{11} Y} . \tag{22}$$

Solutions of the velocity and temperature distributions are obtained by solving the Equations (12) – (15) using the boundary and interface conditions (16) – (18b) are

$$u_1(y) = B_{15} e^{l_1 Y} + B_{16} e^{l_2 Y} + B_{17} + B_{18} e^{l_{10} Y} + B_{19} e^{l_{11} Y} \tag{23}$$

$$u_2(y) = B_8 e^{l_3 Y} + B_9 e^{l_4 Y} + B_{10} + B_{13} e^{l_7 Y} + B_{14} e^{l_8 Y} \tag{24}$$

$$\theta_1(y) = B_1 e^{l_1 Y} + B_2 e^{l_2 Y} \tag{25}$$

$$\theta_2(y) = B_3 e^{l_3 Y} + B_4 e^{l_4 Y}, \tag{26}$$

where

$$I' = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda \,;\; Q^* = U_0 h_i \text{ (say)};\; B = dp/dx\,;\; N = \left(M^2 + \frac{1}{K} \right);$$

$$l_1 = l_3 = \frac{PrR + \sqrt{Pr^2R^2 + 4(\alpha + F)}}{2};\; l_2 = l_4 = \frac{PrR - \sqrt{Pr^2R^2 + 4(\alpha + F)}}{2};$$

$$l_5 = l_7 = \frac{R + \sqrt{R^2 + 4N^2}}{2};\; l_6 = l_8 = \frac{R - \sqrt{R^2 + 4N^2}}{2};$$

$$l_9 = \left(\frac{-1}{3ER} \right) - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left[n_1 + \sqrt{n_1^{\,2} - n_2} \right]} - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left[n_1 - \sqrt{n_1^{\,2} - n_2} \right]};$$

$$l_{10} = \left(\frac{-1}{3ER} \right) + \left(\frac{1 + i\sqrt{3}}{6} \right)\sqrt[3]{\frac{1}{2}\left[n_1 + \sqrt{n_1^{\,2} - n_2} \right]} + \left(\frac{1 - i\sqrt{3}}{6} \right)\sqrt[3]{\frac{1}{2}\left[n_1 - \sqrt{n_1^{\,2} - n_2} \right]};$$

$$l_{11} = \left(\frac{-1}{3ER} \right) + \left(\frac{1 - i\sqrt{3}}{6} \right)\sqrt[3]{\frac{1}{2}\left[n_1 + \sqrt{n_1^{\,2} - n_2} \right]} + \left(\frac{1 + i\sqrt{3}}{6} \right)\sqrt[3]{\frac{1}{2}\left[n_1 - \sqrt{n_1^{\,2} - n_2} \right]};$$

$$n_1 = \frac{2}{E^3R^3} - \frac{9}{E^2R} - \frac{27N^2}{ER};\; n_2 = 4\left(\frac{1}{E^2R^2} - \frac{3}{E} \right)^3;\; B_1 = \frac{b_2}{b_2 e^{-\beta_1} - b_1 e^{-\beta_2}};\; B_2 = \frac{1 - B_1 e^{-l_1}}{e^{-l_2}};$$

$$B_3 = \frac{(B_1 + B_2)e^{l_4}}{e^{l_4} - e^{l_3}};\; B_4 = \frac{-B_3 e^{l_3}}{e^{l_4}};\; B_5 = \frac{-B_1 Gr}{\mathrm{Re}\left(l_1^2 - Rl_1 - N \right)};\; B_6 = \frac{-B_2 Gr}{\mathrm{Re}\left(l_2^2 - Rl_2 - N \right)};$$

$$B_7 = B_{10} = B_{17} = \frac{-B}{N};\; B_8 = \frac{-B_3 Gr}{\mathrm{Re}\left(l_3^2 - Rl_4 - N \right)};\; B_9 = \frac{-B_4 Gr}{\mathrm{Re}\left(l_4^2 - Rl_4 - N \right)};$$

$$B_{11} = \frac{b_3 b_8 + b_9 e^{-l_6}}{b_7 e^{-l_6} - b_8 e^{-l_5}};$$

$$B_{12} = \frac{b_3 + B_{11} e^{-l_5}}{-e^{-l_6}};\, B_{13} = \frac{b_4 + (B_{11} + B_{12})e^{l_8}}{e^{l_8} - e^{l_7}};\; B_{14} = \frac{b_4 + B_{13} e^{l_7}}{-e^{l_8}};$$

$$B_{15} = \frac{-B_1 Gr}{\mathrm{Re}\left(ERl_1^3 + l_1^2 - Rl_1 - N \right)};$$

$$B_{16} = \frac{-B_2 Gr}{\mathrm{Re}\left(ERl_2^3 + l_2^2 - Rl_2 - N\right)};\ B_{18} = \frac{b_{12}b_{17} + b_{18}e^{-l_{11}}}{b_{16}e^{-l_{11}} - b_{17}e^{-l_{10}}};\ B_{19} = \frac{b_{12} + B_{18}e^{-l_{10}}}{-e^{-l_{11}}};$$

$$b_1 = khl_1e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4e^{l_3} - l_3e^{l_4});\ b_2 = khl_2e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4e^{l_3} - l_3e^{l_4});$$

$$b_3 = B_5e^{-l_1} + B_6e^{-l_2} + B_7;\ b_4 = B_8e^{l_3} + B_9e^{l_4} + B_{10};\ b_5 = B_7l_3 + B_8l_4 - B_5l_1 - B_6l_2;$$

$$b_6 = b_5e^{l_8} - b_4e^{l_8};\ b_7 = chl_5e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8e^{l_7} - l_7e^{l_8});$$

$$b_8 = chl_6e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8e^{l_7} - l_7e^{l_8});\ b_9 = b_4(l_8e^{l_7} - l_7e^{l_8}) + b_6(e^{l_7} - e^{l_8}),$$

$$b_{10} = khl_1e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4e^{l_3} - l_3e^{l_4});\ b_{11} = khl_2e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4e^{l_3} - l_3e^{l_4});$$

$$b_{12} = B_{15}e^{-l_1} + B_{16}e^{-l_2} + B_{17};\ b_{13} = B_8e^{l_3} + B_9e^{l_4} + B_{10};\ b_{14} = B_8l_3 + B_9l_4 - B_{15}l_1 - B_{16}l_2;$$

$$b_{15} = b_{14}e^{l_8} - b_{13}e^{l_8};\ b_{16} = chl_{10}e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8e^{l_7} - l_7e^{l_8});$$

$$b_{17} = chl_{11}e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8e^{l_7} - l_7e^{l_8});\ b_{18} = b_{13}(l_8e^{l_7} - l_7e^{l_8}) + b_{15}(e^{l_7} - e^{l_8}).$$

The shear stress and coefficient of the rate of heat transfer at any point in the fluid may be characterized by

$$\tau_i^* = \mu_i u_i'; \qquad Nu_i^* = -k_i T_i'. \tag{27}$$

In the dimensionless form

$$\tau_i = \frac{\tau_i^* h_i}{\mu_i U_0} = u_i'; \qquad Nu_i = \frac{Nu_i^*}{k_i\left(T_i - T_0\right)} = -\theta_i'. \tag{28}$$

The skin friction (τ_i) and the Nusselt number (Nu_i) at the walls $y = -1$ and $y = 1$ are given by

$$\tau_1 = u_1'\big|_{y=-1} \qquad \tau_2 = u_2'\big|_{y=1} \tag{29}$$

$$Nu_1 = -\theta'\big|_{y=-1} \qquad Nu_2 = -\theta'\big|_{y=1}, \tag{30}$$

where $i = 1, 2$ and the primes are the differential derivative with respect to y.

4. Result and Discussion

A numerical evaluation for the analytical solutions of this problem is performed and the results are illustrated graphically in Figures. 2 – 21 to show the interesting features of the significant parameters on the velocity, temperature, skin friction and Nusselt number distributions for both viscoelastic and viscous fluids in the porous channel. Throughout the computations we employ $E=1$, $M=2$, $K=0.2$, $Gr=5$, $Re=2$, $B=0.1$, $c=0.5$, $R=2$, $Pr=1$, $\alpha=1$, $F=0.5$, $k=1$ and $h=1$ unless otherwise stated. Figures 2-12 present the effect of M, K, E, c, h, R, Gr, Re, Pr, α and F on the velocity distribution respectively. Figure 2 shows that the application of a magnetic field normal to the flow direction has the tendency to slow down the movement of the fluids in the channel because it gives rise to a resistive force called the Lorentz force which acts opposite to the flow direction. It is clear from Fig. 3 that the increase in the porosity parameter leads to an enhanced velocity because it reduces the drag force. Figure 4 illustrates the increment of the viscoelastic parameter decreases the velocity of the fluid throughout the boundary layer except for near the plate $y=-1$. Physically speaking, the higher values of the viscoelastic parameter are having greater stability than the smaller values. So, the effect of the viscoelastic parameter is to destabilize the fluid flow system for higher values. Figures 5 and 6 shows that an increase in the density ratio and height ratio has the tendency to decrease the viscoelastic fluid velocity but increase the viscous fluid velocity in the porous channel. It is apparent from Figure 7 that an increase in the values of the suction parameter leads to an increase in the velocity. Figure 8 illustrates the influence of inclusion of the buoyancy effects. The presence of the buoyancy effect complicates the problem, by coupling of the flow problem with a thermal problem for both fluids. The presence of the thermal buoyancy currents enhances the velocities of both fluids which is taking place through the application of a pressure gradient. Increases in the Reynolds number strictly diminish the fluid velocity throughout the boundary layer and the curves could be seen in Figure 9. It is observed from Figure 10 that increasing the Prandtl number results in an increase of the fluid velocity. Figures 11 and 12 elucidate that the fluid velocity of both fluids notably decreases for the higher values of heat absorption and thermal radiation parameters.

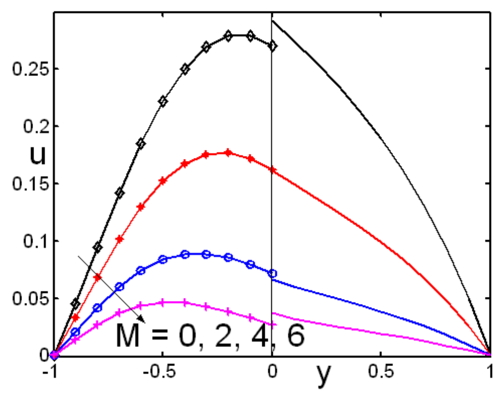

Figure 2. Effect of magneic field in velocity distribution

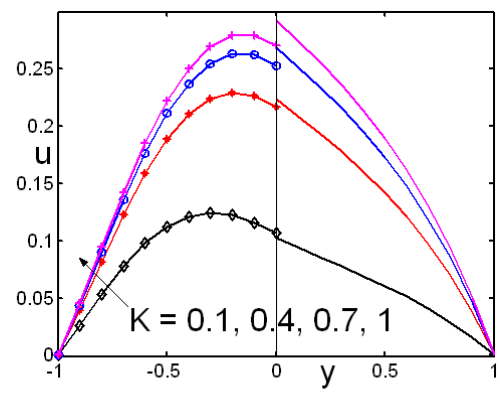

Figure 3. Effect of permiability parameter in velocity distribution

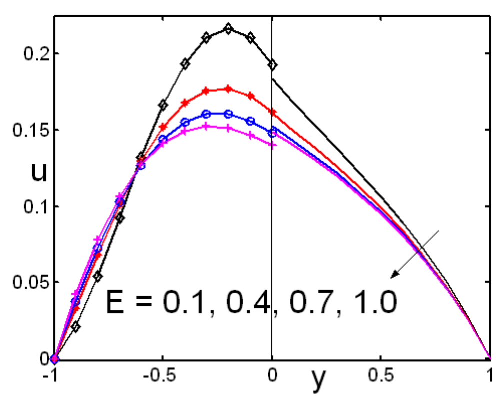

Figure 4. Effect of viscoelastic parameter in velocity distribution

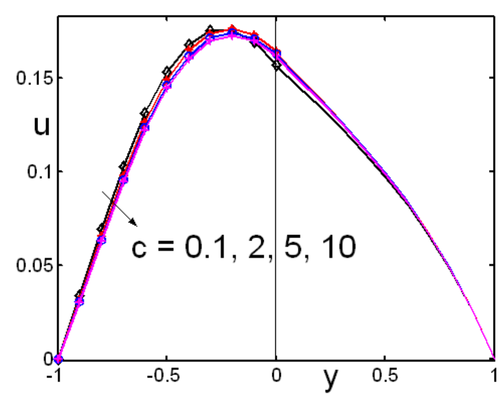

Figure 5. Effect of viscosity ratio in velocity distribution

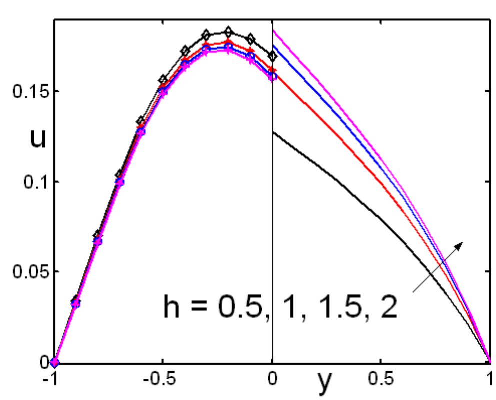

Figure 6. Effect of height ratio in velocity distribution

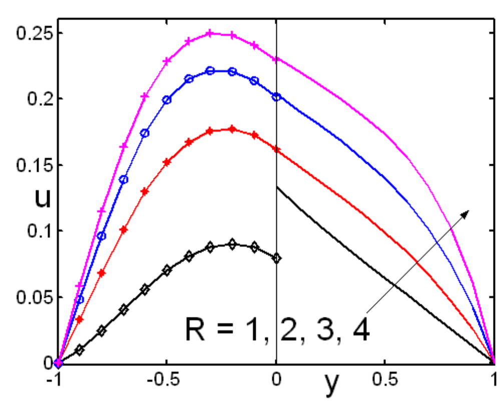

Figure 7. Effect of suction parameter in velocity distribution

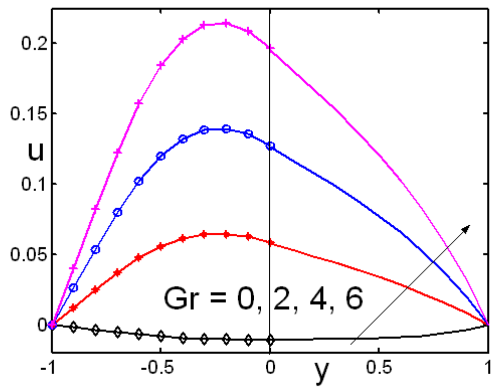

Figure 8. Effect of Grashof number in velocity distribution

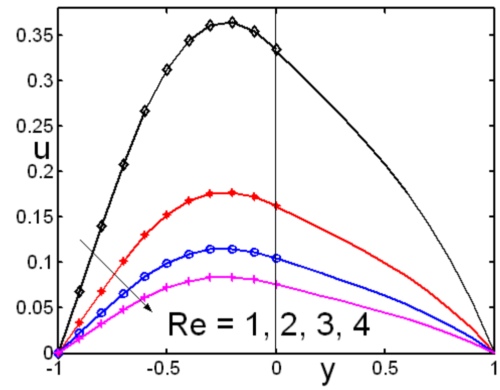

Figure 9. Effect of Reynolds number in velocity distribution

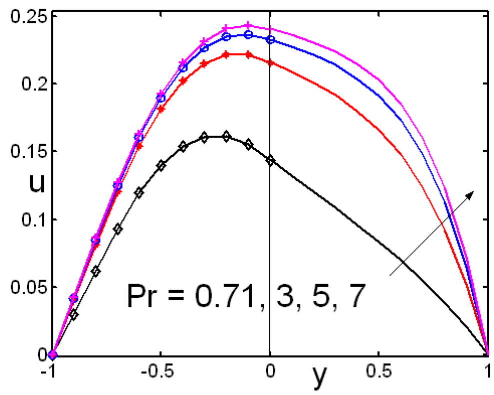

Figure 10. Effect of Prandtl number in velocity distribution

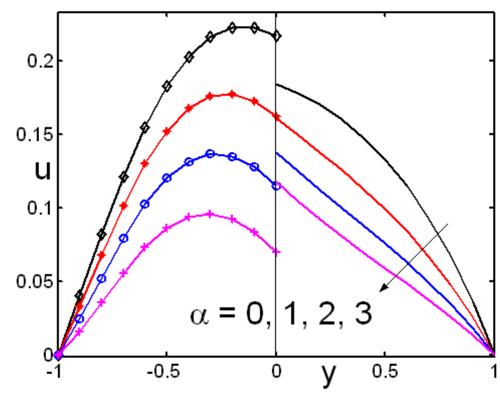

Figure 11. Effect of heat absorption parameter in velocity distribution

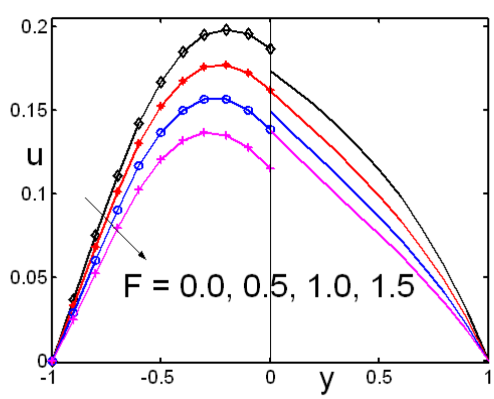

Figure 12. Effect of radiation parameter in velocity distribution

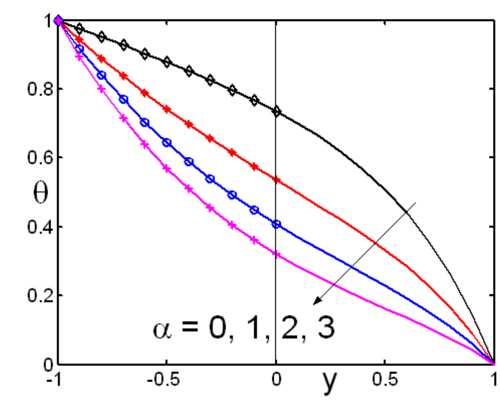

Figure 13. Effect of heat absorption parameter in temperature distribution

Figures 13-17 show the influence of α , F , k , Pr and R on the temperature distribution respectively. Figure 13 illustrates that the fluid temperature monotonically decreases for increasing the heat absorption parameter. Figure 14 show that an increase in the radiation parameter decreases the temperature distribution because large values of the radiation parameter enhance the conduction over radiation, thereby decreasing the thickness of the thermal boundary layer. Figure 15 shows that the thermal conductivity ratio seems to increase the temperature of both fields. Figure 16 demonstrates that increasing the Prandtl number increases the heat transfer because the larger Prandtl number corresponds to the stronger thermal diffusivity and thicker boundary layer. Physically, these behaviors are all valid qualitatively for air $(Pr = 0.71)$ and water $(Pr = 7)$. Furthermore increases in the boundary layer suction also increase the fluid temperature which is shown in Figure 17.

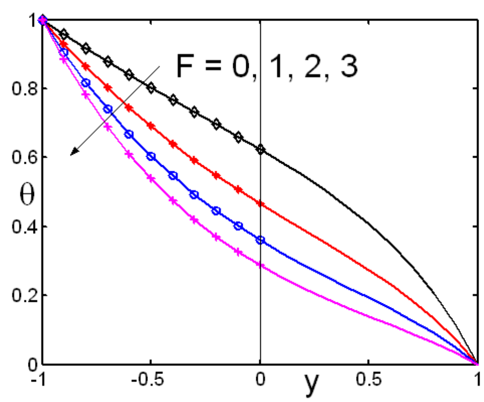

Figure 14. Effect of radiation parameter in temperature distribution

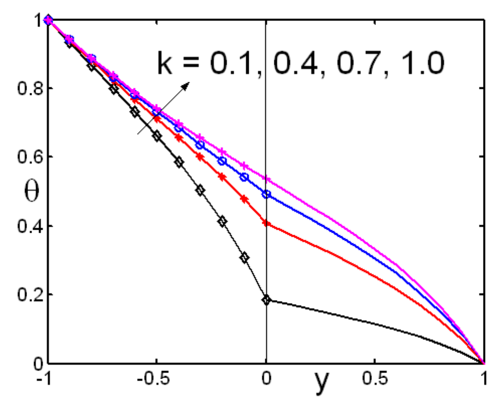

Figure 15. Effect of thermal conductivity ratio in temperature distribution

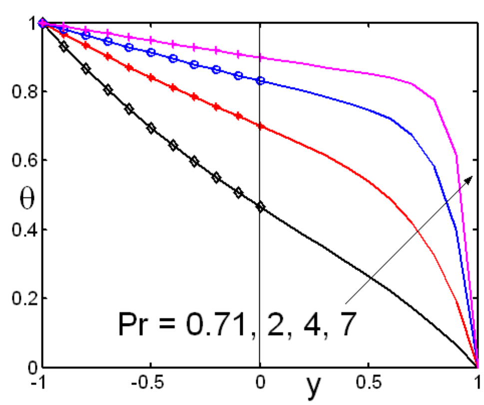

Figure 16. Effect of Prandtl number in temperature distribution

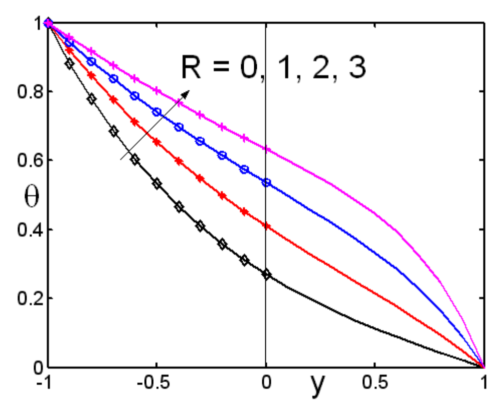

Figure 17. Effect of suction parameter in temperature distribution

We present the effect of skin friction co-efficient against the thermal Grashof number for various values of M and K through the Figures. 18 and 19. It is noticed from Fig. 18 that value of shear stress falls rapidly for increasing the magnetic field parameter at the wall $y = -1$ but the trend is reversed at the other wall $y = 1$. The increments of porous permeability parameter increase the skin friction at the wall $y=-1$ whereas the opposite effect is observed at the other wall $y=1$ displayed in Figure 19. The Nusselt number with respect to the heat absorption parameter for various values of F and Pr are graphically displayed in Figures 20 and 21. It is observed from Figure 20 that an increase in the thermal radiation parameter enhances the rate of heat transfer at $y=-1$ while it reverses the effect at $y=1$. An increase in the Prandtl number decreases the rate of heat transfer at $y=-1$ but the opposite effect is observed at $y=1$ shown in Figure 21.

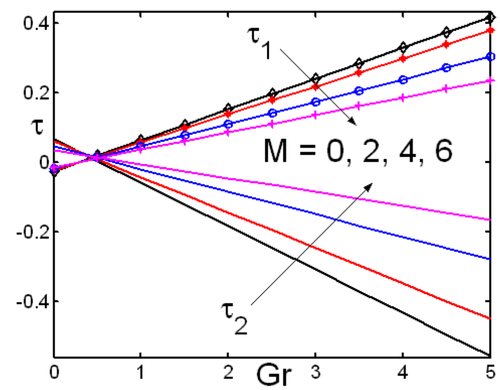

Figure 18. Effect of magnetic field in skin friction distribution

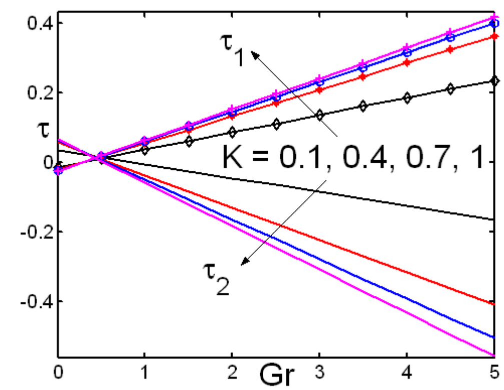

Figure 19. Effect of permiability parameter in skin friction distribution

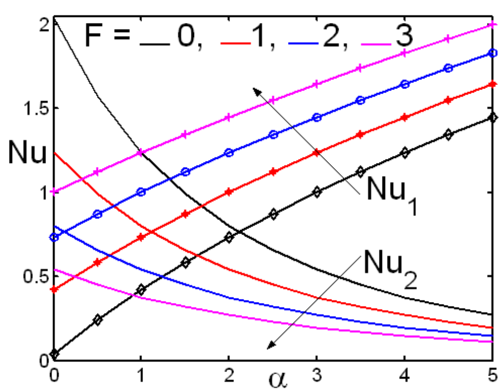

Figure 20. Effect of radiation parameter in Nusselt number distribution

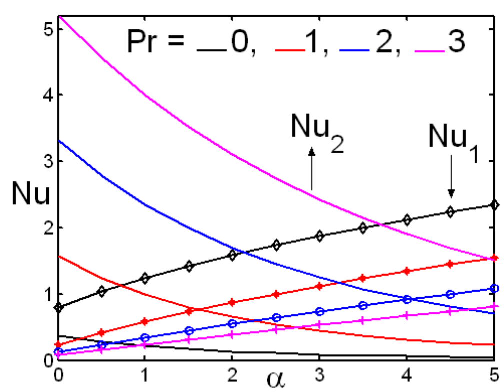

Figure 21. Effect of Prandtl number in Nusselt number distribution

Table 1 illustrates the variations of the skin friction and Nusselt number distributions at the walls $y=-1$ and $y=1$ for different values of α, Pr, R and F. It is observed that τ_1 increases for increasing Pr and R but decreases for increasing α and F while a reverse trend is noticed for τ_2. It is elucidated that Nu_1 increases for increasing α and F but decreases for increasing Pr and R whereas the reverse trend is observed for Nu_2.

Table 1: The effect of α , Pr , R and F on τ_1 , τ_2 , Nu_1 and Nu_2

Physical Parameters	Values	τ_1	τ_2	Nu_1	Nu_2
α	0.0 1.0 2.0	0.3841 0.3029 0.2116	-0.4015 -0.2790 -0.1999	0.2431 0.5868 0.8729	1.5744 0.9909 0.6560
Pr	0.0 1.0 2.0	0.0724 0.3029 0.3537	-0.1066 -0.2790 -0.4526	1.2431 0.5868 0.3456	0.2131 0.9909 2.3518
R	0.0 1.0 2.0	-0.2653 0.0737 0.3029	-0.0177 -0.1354 -0.2790	1.2431 0.8363 0.5868	0.2131 0.5129 0.9909
F	0.0 0.5 1.0	0.3443 0.3029 0.2584	-0.3334 -0.2790 -0.2355	0.4241 0.5868 0.7354	1.2396 0.9909 0.8020

5. Conclusions

Analytical solutions are obtained for MHD mixed convective flow of viscoelastic and viscous fluids in a vertical porous channel and matched at the interface using suitable matching conditions. The results are evaluated numerically and displayed graphically. It is found that an increase in the Grashof number, permeability parameter, suction parameter and Prandtl number enhances the velocity of both the fluids whereas increasing the viscoelastic parameter, Reynolds number, magnetic field, heat absorption parameter and radiation parameters reverses the effect. The higher values of the viscosity ratio and the channel width ratio decrease the viscoelastic fluid velocity whereas they increase the viscous fluid velocity. The heat transfer increases for increasing conductivity ratio, suction parameter and Prandtl number but the trend is reversed for increasing heat absorption and radiation parameters. Furthermore, the table provides the effect of various significant parameters on skin friction and Nusselt number distributions. This study is expected to be useful in understanding the influence of thermal buoyancy and a magnetic field on enhanced oil recovery and filtration systems

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