



## A Theoretical Model for Blood Flow in Small Vessels

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### Abstract

A two-fluid model consisting of a core region of suspension of all the erythrocytes (particles) in plasma (fluid) assumed to be a particle-fluid mixture and a peripheral layer of cell-free plasma (Newtonian fluid), has been proposed to represent blood flow in small diameter tubes. The analytical results obtained in the proposed model for effective viscosity, velocity profiles and flow rate have been evaluated numerically for various values of the parameters available from published works. Quantitative comparison has shown that present model suitability represents blood flow at hematocrit ( $\leq 40\%$ ) and in vessels up to  $70\mu\text{m}$  in diameter. Using experimental values of the parameters, the flow rate for normal and diseased blood has been computed and compared with corresponding values obtained from a well known experimentally tested model in the literature.

**Keywords:** Erythrocytes, plasma, hematocrit, peripheral layer, velocity profile, flow rate, effective viscosity

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### 1. Introduction

The study of blood flow through mammalian circulatory system has been the subject of scientific research for about a couple of centuries. Like most of the problems of nature and life sciences, it is complex one due to the complicated structure of blood, the circulatory system and their constituent materials. The experimental studies and the theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes and practical applications.

A large number of theoretical and experimental efforts have been made in the literature to explain the blood flow behavior when it flows through the vessels of circulatory system of living beings. To account for the new evidences uncovered through improved experimental theories of blood flow from the numerous relevant and important contributions of Bayliss (1952), Womersley (1954,1955,1957,1958), Muller(1951,1959), McDonald (1960),

Whitmore (1963), Copley and Stainsby (1960), Attinger (1964), Fung (1964), Lew and Fung (1970) and many others, mathematical modeling of blood flow has been subject to constant changes and modifications. Above listed investigators have used single-phase homogeneous Newtonian viscous fluid, a classical approach that does not account for the presence of red cells (one of the main constituents of blood, which is responsible for many of the blood properties and diseases (Srivastava, 1995)) in blood while flowing through the circulatory system. Although, this approach provides satisfactory tools to describe certain aspects of blood flow in aorta and large arteries, it fails to give an adequate representation of flow field, especially in the vessels of small diameter (2400 – 8 $\mu$ m, Srivastava and Srivastava (1983)).

Several researchers (Casson, 1959; Haynes, 1959; Charm and Kurland, 1964; Eringen, 1964; Gupta et al., 1982; Chaturani and Upadhyaya, 1981) have proposed various representative models for blood in small vessels and narrow capillaries. A number of investigators including Haynes and Burton (1959), Merrill et al. (1963), Charm and Kurland (1965), Hershey et al. (1964), Cokelet (1972) and Lih (1975) have pointed out that blood being a suspension of corpuscles, behaves like a non-Newtonian fluid at low shear rates. In particular, Hershey et al. (1966) and Huckaba et al. (1968) have shown that blood flowing through a tube of diameter less than 0.2 mm and at low shear rate less than 20/s, behaves as a power law- fluid while Casson (1959), Reiner and Blair (1959), Charm and Kurland (1965,1974) and Merrill et al. (1964) have suggested that blood inhibits yield stress and behaves as a Casson model fluid at a shear rate equal to 0.1/s.

Experimental investigation of Cokelet (1972) and theoretical observations of Haynes (1960) indicate that blood can no longer be treated as a single-phase homogenous viscous fluid in small size vessels (of diameter  $\leq 1000\mu$ m). It is surprising to note that the individuality of the red cells (of diameter 8 $\mu$ m) is important even in such large vessels (with diameter up to 100 cells diameter). Skalak (1972) concluded that in capillary vessels whose diameter (4-10 $\mu$ m) are equal or smaller than that of a red blood cell, an accurate description of flow requires consideration of red cells as discrete particles. Also, certain observed phenomena in blood flow including Fahraeus-Lindqvist effect (the decrease of apparent viscosity with decreasing diameter of blood vessels), non-Newtonian behavior, etc. can not be explained fully by considering blood as a single-phase homogenous fluid. Thus, in dealing with the problem of microcirculation also, the individuality of red blood cells cannot be ignored. It seems to be therefore important and necessary to consider the whole blood as a particle-fluid system while flowing through small vessels. Including some recent studies, a number of investigations have been conducted in the literature using particulate suspension theory to describe the flow of blood in small vessels. Srivastava and Srivastava (1983) proposed a two-phase theoretical model to address pulsatile blood flow in the entrance region of an artery. Srivastava et al. (1994) applied the theory to study the effects of an external body acceleration on blood flow through small diameter tubes while Srivastava (1995,2002) dealt with the problem of blood flow through stenotic vessels representing blood by an erythrocytes-plasma suspension. And most recently, Jung et al. (2006a, 2006b) discussed steady and pulsatile flow of particulate buildup on the inside curvature of coronary artery using multiphase of dense suspension hemodynamics. In addition, Bugliarello and Sevilla (1970), Cokelet (1972) and Thurston (1989) have shown experimentally that for blood flowing through small vessels, there is a cell-free plasma (Newtonian fluid) layer and a core region of suspension of all the erythrocytes. Haynes (1960) presented a two-fluid theoretical model for blood flow consisting of a core region of suspension of all the erythrocytes as a homogeneous Newtonian viscous fluid and a cell-free plasma layer as a Newtonian fluid of constant viscosity (equal to the viscosity of water). Bugliarello and Sevilla (1970) presented

blood in small diameter tubes by a two-layered model assuming peripheral and core fluids as Newtonian fluids of different viscosities. Following the theoretical study of Haynes (1960) and experimentally tested model of Bugliarello and Sevilla (1970), two-fluid modeling of blood flow has been discussed and used by a good number of researchers. Shukla et al. (1980) applied a two-fluid model to discuss the flow of blood through a stenosis. Chaturani and Upadhyaya (1979, 1981) addressed the flow of blood in small diameter tubes using the two-layered model of micropolar and couple stress fluids, respectively. Pralhad and Scultz (1988) used a two-fluid model of polar fluid to analyze the flow of blood through stenotic arteries. Two-fluid model analyses have been carried out by Srivastava (2000, 2002) to observe the effects of a non-symmetrical stenosis on blood flow characteristics. Sharan and Popel suggested a modification on the models of Haynes(1960) and Bugliarello and Sevilla (1970) assuming the viscosity in the peripheral layer to be higher than that of plasma due to additional dissipation of energy caused by the red cells motion near the cell-free layer. Wang and Bassingthwaight (2003) applied the two-layered models of Haynes (1960) and Sharan and Popel (2001) to discuss the flow of blood in narrow curved tubes, etc.

The studies mentioned just above on two-fluid modeling have represented blood either by a single-phase Newtonian or non-Newtonian fluid in the core region. With increasing interest in two-phase flows and its applications to blood flow problems, it is however regretted that no rigorous effort, at least to the author's knowledge, has been made in the literature to represent blood as a two-phase system (i.e., erythrocytes and plasma mixture) in the core region. The purpose of this paper is therefore to investigate the flow of blood in small vessels involving a two-fluid model. The mathematical model considers a two-layered model of blood, consisting of a core region of suspension of all the erythrocytes (small spherical non-flexible particles), assumed to be a particle-fluid suspension (i.e., a suspension of red cells in plasma) and a peripheral layer of plasma (Newtonian fluid). The study thus presents a theoretical model for blood, seems to be the only one of its kind which enables one to observe the simultaneous effects of hematocrit and the peripheral layer on the flow characteristics while flowing through small vessels.

## 2. Formulation of the Problem and Analysis

Consider the axisymmetric flow of blood in an uniform circular tube of radius  $R$ . Blood is represented by a two-fluid model consisting of a core region (central layer) of suspension of all the erythrocytes assumed to be a particle-fluid mixture (i.e., a suspension of red cells in plasma) of radius  $R_1$  and a peripheral layer of plasma (Newtonian fluid) of thickness  $(R-R_1)$  as shown in Fig. 1. Under the simplified assumptions stated in Srivastava and Srivastava (1983), the equations governing the flow are therefore written (Srivastava, 1995; Drew, 1974) as

$$\frac{dp}{dz} = \frac{\mu_o}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) u_o, \quad R_1 < r < R, \quad (1)$$

$$(1-C) \frac{dp}{dz} = (1-C) \frac{\mu_s(C)}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) u_f + C S (u_p - u_f), \quad 0 \leq r \leq R_1, \quad (2)$$

$$C \frac{dp}{dz} = C S (u_f - u_p), \quad 0 \leq r \leq R_1, \quad (3)$$

where  $(r, z)$  are (radial, axial) coordinates,  $(u_f, u_p)$  are the axial velocity of (fluid, particle) in the core region ( $0 \leq r \leq R_1$ ),  $(\mu_o, u_o)$  are the (viscosity, fluid velocity) in the peripheral region ( $R_1 \leq r \leq R$ ),  $\mu_s \cong \mu_s(C)$  is the suspension viscosity in the core region,  $C$  denotes the constant volume fraction density of the particles (called hematocrit) and  $S$  is the drag coefficient of interaction between the two phases (fluid and particle). The expression for the drag coefficient of interaction  $S$  and empirical relation for the viscosity of suspension  $\mu_s$  may be selected (Charm and Kurland, 1974; Srivastava, 1995) as

$$S = \frac{9}{2} \frac{\mu_o}{a_o^2} \frac{[4 + 3(8C - 3C^2)^{1/2} + 3C]}{(2 - 3C)^2}, \quad (4)$$

$$\mu_s = \frac{\mu_o}{1 - qC}; \quad q = 0.07 \exp[2.49 C + \frac{1107^0 K}{T} \exp(-1.69C)], \quad (5)$$

where  $a_o$  is the radius of a particle and  $T$  is measured in absolute temperature.

The boundary conditions are the standard no-slip conditions of velocities and shear stresses at the tube wall and the interface, and are given as

$$u_o = 0 \quad \text{at} \quad r = R, \quad (6)$$

$$u_o = u_f \quad \text{and} \quad \tau_o = \tau_f \quad \text{at} \quad r = R_1, \quad (7)$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_p}{\partial r} = 0 \quad \text{at} \quad r = 0, \quad (8)$$

with  $\tau_o = \mu_o \partial u_o / \partial r$  and  $\tau_f = (1 - C) \mu_s \partial u_f / \partial r$  as the shear stresses of the peripheral and central layers, respectively.

The expressions for the velocities  $u_o$ ,  $u_f$  and  $u_p$  obtained as the solutions of equations (1) - (3), subject to the boundary conditions (6) - (8), are given as

$$u_o = - \frac{R^2}{4\mu_o} \frac{dp}{dz} \left\{ 1 - (r/R)^2 \right\}, \quad R_1 \leq r \leq R, \quad (9)$$

$$u_f = - \frac{R^2}{4(1-C)\mu_o} \frac{dp}{dz} \left\{ \mu' \left[ \left( \frac{R_1}{R} \right)^2 - \left( \frac{r}{R} \right)^2 \right] + (1-C) \left[ 1 - \left( \frac{R_1}{R} \right)^2 \right] \right\}, \quad 0 \leq r \leq R_1, \quad (10)$$

$$u_p = - \frac{R^2}{4(1-C)\mu_o} \frac{dp}{dz} \left\{ \mu' \left[ \left( \frac{R_1}{R} \right)^2 - \left( \frac{r}{R} \right)^2 \right] + (1-C) \left[ 1 - \left( \frac{R_1}{R} \right)^2 \right] + \frac{4(1-C)\mu_o}{SR^2} \right\}, \quad 0 \leq r \leq R_1, \quad (11)$$

with  $\mu' = \mu_o / \mu_s$ .

The flow flux (volumetric flow rate) is now calculated as

$$Q = Q_o + Q_f + Q_p, \quad (12)$$

Where,  $Q_o = 2\pi \int_{R_1}^R ru_o dr$ ,  $Q_f = 2\pi (1-C) \int_0^{R_1} ru_f dr$ , and

$$Q_p = 2\pi C \int_0^{R_1} ru_p dr.$$

Using equations (9) – (11) into equation (12), one obtains the expression for flow flux as

$$Q = - \frac{\pi R^4}{8(1-C)\mu_0} \frac{dp}{dz} \left\{ (1-C) \left[ 1 - (R_1/R)^4 \right] + \mu' (R_1/R)^4 + \eta^2 (R_1/R)^2, \right\} \quad (13)$$

with  $\eta^2 = 8 C (1-C) \mu_0 / SR^2$ , a non-dimensional suspension parameter. The use of the fact that total flux is equal to the sum of the fluxes across the two regions (peripheral and core) determines the relation (Haynes, 1960; Bugliarello and Sevilla, 1970)

$$R_1 = \alpha R. \quad (14)$$

An application of relation (14) into equation (13), yields the following expression for the effective (apparent) viscosity as

$$\mu_e = \frac{(1-C)\mu_0}{(1-C)(1-\alpha^4) + \mu' \alpha^4 + \eta^2 \alpha^2}. \quad (15)$$

When  $R_1 = R$  (i.e., in the absence of the peripheral layer), above results reduces to the case of a single layered model of a particle-fluid suspension as

$$\mu_{es} = \frac{(1-C)\mu_0}{\mu' + \eta^2}. \quad (16)$$

It is worth mentioning that in the absence of the particles (i.e.,  $C = 0$ ), the core mixture changes to the same fluid as in the peripheral region and thus the role of the peripheral layer automatically disappears. In addition, when core mixture behaves as a single-phase fluid of constant viscosity (i.e.,  $\mu_s = \mu_1 \neq \mu_0$ ), one obtains the same expression for effective viscosity derived from steady Newtonian fluid model of Bugliarello and Sevilla (1970) as

$$\mu_{eb} = \frac{\mu_0}{1 - \alpha^4 + \mu' \alpha^4} \quad (17)$$

Equation (17) recovers the result obtained in Haynes (1960) when  $\mu_0 = 1\text{cp}$ .

### 3. Numerical Results, Discussion and Biological Relevance

In order to discuss the results of the theoretical model proposed in the study quantitatively and to point out its biological relevance, computer codes are developed to evaluate the analytical results for effective viscosity, velocity profiles and flow rate obtained in equations (9) - (15) for various values of the parameters involved. For the purpose of comparison, the corresponding results obtained in the theoretical model of Haynes (1960) considering a two-phase fluid in the core region and experimentally tested steady flow model of Bugliarello and Sevilla (1970) using a single-phase fluid (blood) of constant viscosity for a given hematocrit have been evaluated for the experimental values of the parameters available from the published literature of (Bugliarello and Sevilla, 1970; Sud and Sekhon, 1985; Srivastava et al., 1994) at the temperature of 25.5<sup>o</sup> C. The value of  $\alpha$  is calculated from the relation:  $\alpha = 1 - \varepsilon/R$ , in which  $\varepsilon \cong \varepsilon$  (C) denotes the peripheral layer thickness for a given hematocrit (Haynes, 1960).

Owing to the significance of viscosity, the effective viscosity at 20% and 40% hematocrit (red cell concentration) has been computed for different size blood vessels. The results obtained are arranged in Table 1 and compared with the corresponding theoretical values of Haynes (1960), Chaturani and Upadhyya (1979) and experimental values of Bugliarello and Sevilla (1970). For numerical evaluation of the result for effective viscosity given in equation (15), the mixture viscosity  $\mu_s$  has been computed using empirical relation (5) for two values of the pressure gradient,  $-dp/dz = 67.5 \text{ dyne/mm}^3$  and  $76.0 \text{ dyne /mm}^3$ . An inspection of the Table 1 reveals that effective viscosity computed from the proposed model is in agreement within the acceptable range to the corresponding values of the effective viscosity obtained in the theoretical models of Haynes (1960), Bugliarello and Sevilla (1970) and Chaturani and Upadhyya (1979), particularly at low concentration of red cells (20% hematocrit) and in Vessels of diameter  $\leq 40\mu\text{m}$ . The effective viscosity deviates from experimental value with increasing hematocrit and also with the vessel size. However, one notices that present model exhibits Fahraeus – Lindqvist effect (i.e., apparent viscosity of blood decreases with decreasing diameter of blood vessel).

The axial velocity profiles ( $u_f$ ,  $u_p$  and  $u_o$ ), computed from the present theory (equations (9) - (11)), the corresponding model derived (i.e., using erythrocytes-plasma suspension to represent blood in the core region similar to the present proposed model) from Haynes (1960) and the steady flow model of Bugliarello and Sevilla (1970) at 20% and 40% hematocrit are displayed graphically in Figs. 2 and 3 respectively. To evaluate the results obtained for velocity profiles in Haynes (1960) and Bugliarello and Sevilla (1970), the mixture viscosity (or blood viscosity) has been taken to be 2.18 cp and 3.10 cp for 20% and 40% hematocrit, respectively from published literature (Bugliarello and Sevilla, 1970; Sud and Sekhon, 1985; Srivastava et al., 1994). One observes that erythrocyte velocity at the tube axis assumes

higher magnitude than the plasma velocity but the difference in their magnitudes decreases with increasing radial coordinate  $r$  towards the interface and at the interface the plasma velocity (in present and Haynes, 1960 models) coincides with the blood velocity obtained in Bugliarello and Sevilla (1970).

**Table 1.**

Vessel Diameter ( $\mu\text{m}$ )	$\alpha$	Effective Viscosity (CP)			
		Present	Haynes(1960)	Bugliarello & Sevilla(1970)	Chaturani& Upadhy(1979)
20% Hematocrit, $\varepsilon = 4.67\mu\text{m}$					
20	0.382	1.211	1.247	1.245	1.261
30	0.588	1.240	1.340	1.334	1.389
40	0.766	1.265	1.429	1.420	1.511
70	0.866	1.315	1.753	1.607	1.679
100	0.906	1.340	1.753	1.723	1.869
40% Hematocrit, $\varepsilon = 3.12\mu\text{m}$					
20	0.688	1.243	1.423	1.391	1.445
30	0.792	1.282	1.656	1.581	1.709
40	0.844	1.390	1.979	1.741	2.080
70	0.910	1.355	2.307	2.075	2.440
100	0.937	1.377	2.624	2.272	2.667

The volumetric flow rate  $Q$  vs pressure gradient  $-dp/dz$  computed from the proposed model (equation (13)) and the experimentally tested model of Bugliarello and Sevilla (1970), at 20% and 40% hematocrits have been plotted in Fig 4. It may be noted that the magnitudes of the flow rate  $Q$  obtained in the proposed theory are in reasonable agreement with the corresponding value obtained in Bugliarello and Sevilla (1970), particularly for low pressure gradients. To emphasize further on the study presented above, flow rate  $Q$  vs pressure gradient  $-dp/dz$  for normal and diseased blood (Hb SS, plasma cell dycrasias, hypertension(controlled), hypertension (uncontrolled) and polycythemia) in a  $70\mu\text{m}$  diameter vessel using the present theoretical approach and the experimental data available from published literature of shu (1982) and Bugliarello and Sevilla (1970), has been shown in Fig. 5 and compared with the corresponding values obtained in Bugliarello and Sevilla (1970). The various values of the parameters used for the purpose are arranged in Table 2.

**Table 2.** Experimental data for diseased and normal blood in a  $70\mu\text{m}$  diameter vessel (Bugliarello and Sevilla, 1970; Shu, 1982).

Disease	Hematocrit (%)	$\mu_0$ (cp)	$\mu_s$ (cp)	$\alpha$
Hb SS (Sickle cell)	24.80	1.30	5.10	0.795
Plasma cell dycrasias	28.00	3.09	5.43	0.816
Normal blood	42.60	1.24	4.03	0.920
Hypertension (uncontrolled)	43.25	1.52	5.15	0.925
Hypertension (uncontrolled)	43.31	1.28	4.86	0.928
Polycythemia	63.20	1.50	7.69	0.990

It is clearly visible that the flow rates obtained in the present analysis are in good agreement with those obtained in Bugliarello and Sevilla (1970) for relatively low pressure gradients,

particularly for Hb SS (24.8% hematocrit) and plasma cell dyscrasias (28% hematocrit). The flow rates obtained from the study deviate from those obtained in Bugliarello and Sevilla (1970), with increasing values of the pressure gradient and also with increasing hematocrit.

#### 4. Conclusions

A two-layered model consisting of a core region of suspension of all the erythrocytes in plasma (i.e., particle- fluid mixture) and a peripheral layer of plasma (Newtonian fluid) has been proposed to describe blood flow in small diameter vessels. As evident from the numerical results presented above, it appears that the present theoretical model suitably describes blood flow in small vessels (of diameter  $\leq 70\mu\text{m}$ ) and at low concentration of red cells ( $\leq 40\%$ ). The results of the analysis deviate from the experimental works with increasing diameter of blood vessel and also with increasing hematocrit. The reason behind this is the empirical formula used for the mixture viscosity  $\mu_s$  in the proposed theoretical model which is based on the Einstein's theory of particulate suspension, and is therefore applicable only for low particle concentration (Drew, 1974; Srivastava et al., 1994). A modification in the empirical formula for mixture viscosity  $\mu_s$  or development an equivalent close mathematical model thus seems to be necessary in order to increase the range of the usefulness of the present theoretical model. Author is already in the course of constructing a close mathematical model for the viscosity of suspension and would present in his subsequent communication. The assumption that the red cells are small spherical non-flexible particles remains another approximation to the study. The proposed model certainly enabled one to observe the simultaneous effects of hematocrit and the peripheral layer on flow characteristics of blood, seems to be the only one of its kind in the published literature. It is however felt that a considerable amount of further research is essential to make the model useful for higher parameter values (hematocrit and vessel size) and also to overcome some other approximations used in the formulation.

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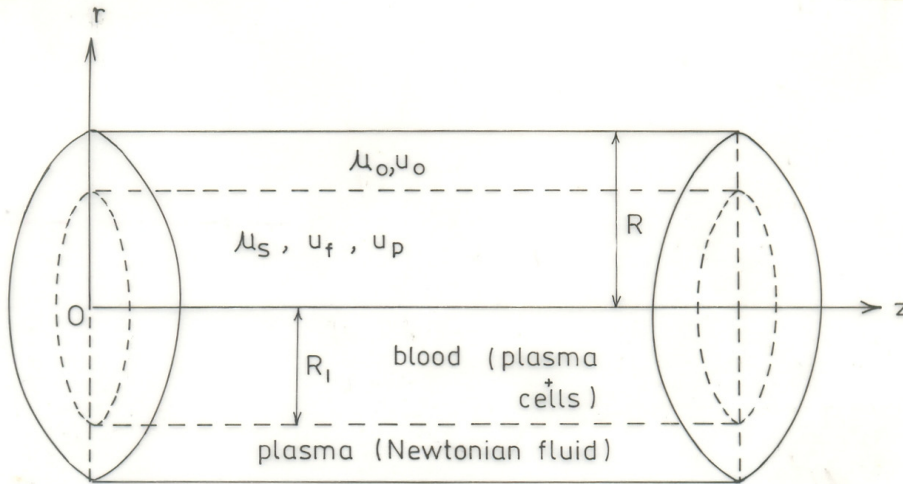


Fig.1. Flow geometry of blood in small vessels .

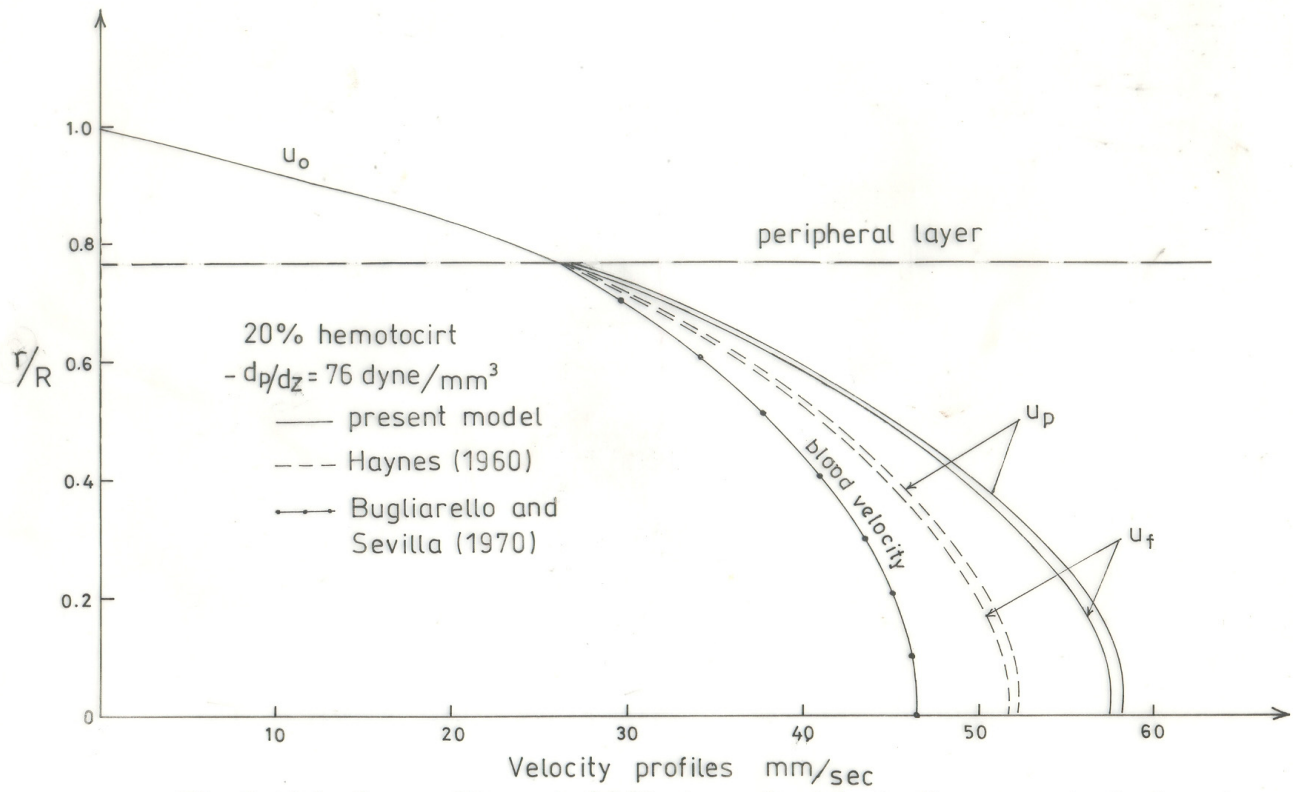


Fig.2. Velocity profiles at 20% hematocrit in the vessel of diameter 40  $\mu\text{m}$ .

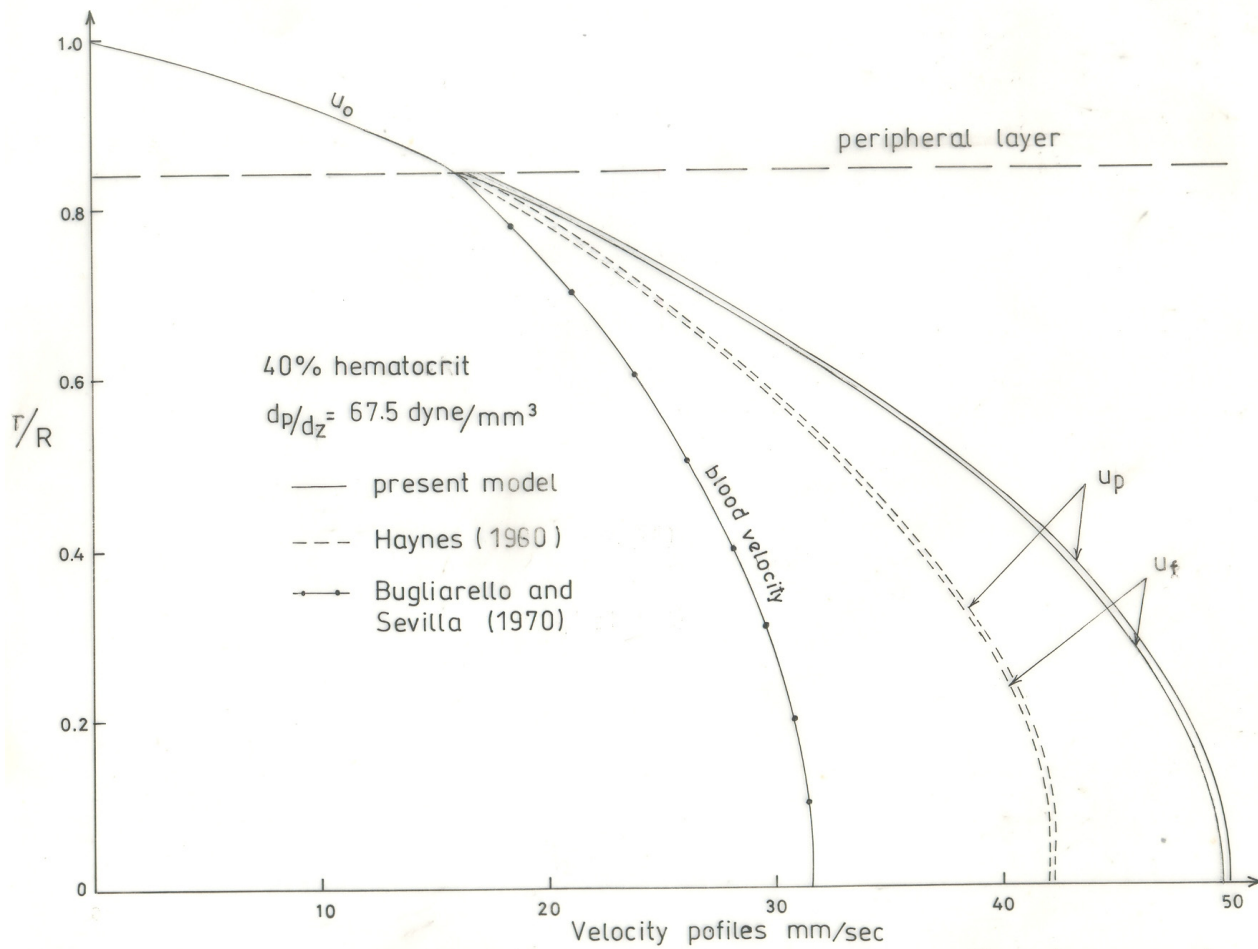


Fig.3. Velocity profiles at 40% hematocrit in a 40  $\mu\text{m}$  diameter vessel.

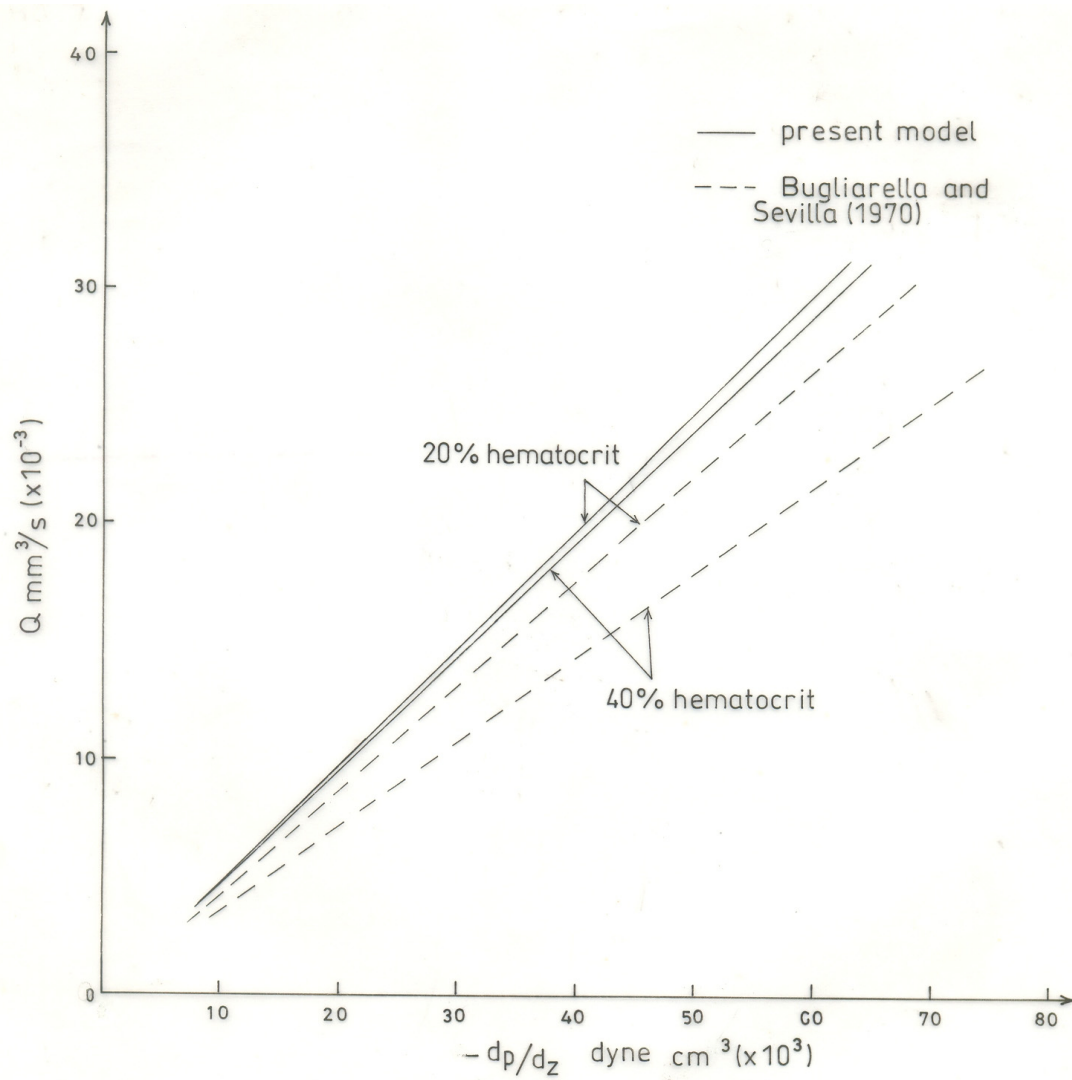


Fig.4. Pressure-flow rate relationship in a  $40\mu\text{m}$  diameter vessel for different hematocrit.

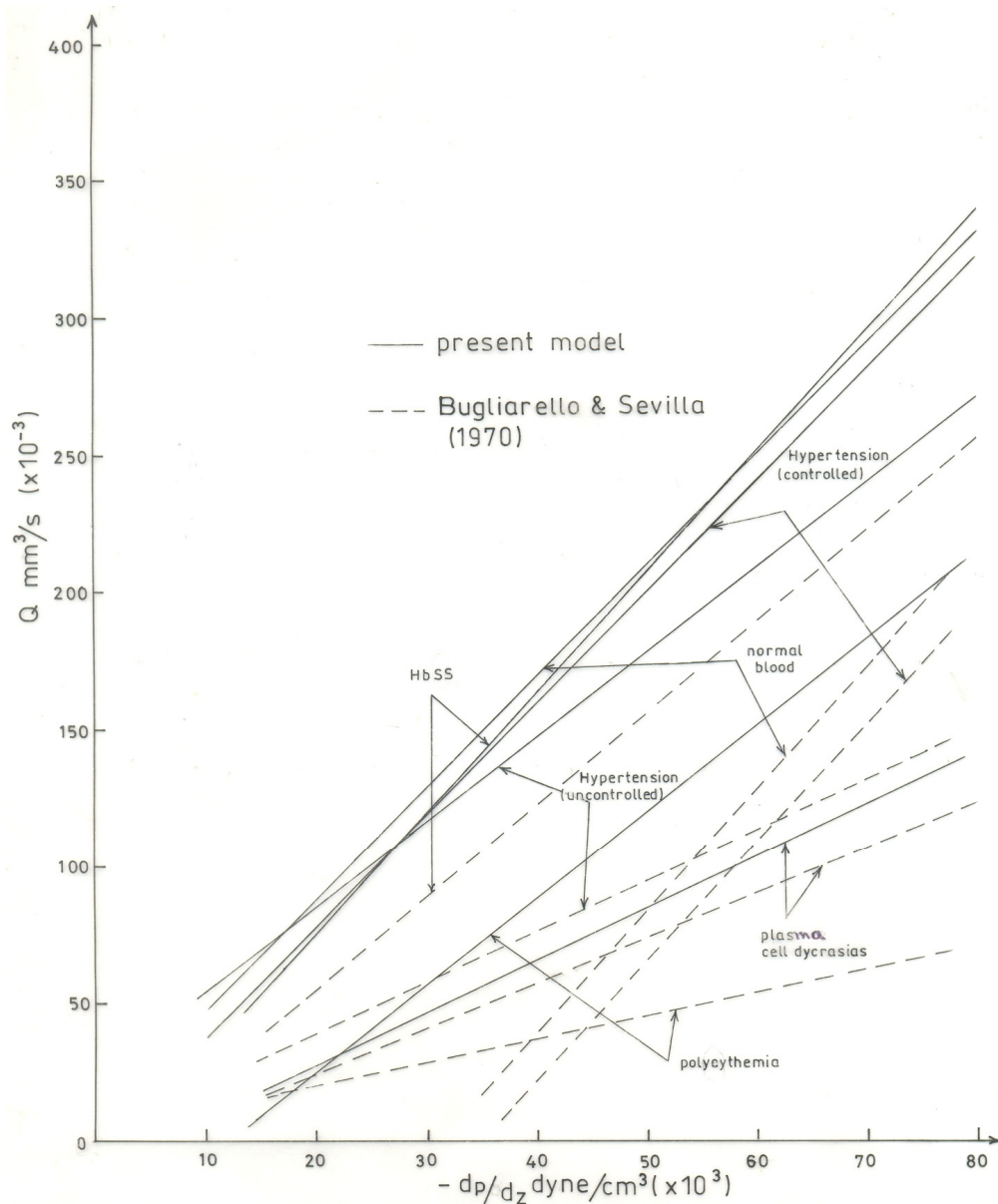


Fig.5. Flow rate for normal and diseased blood in a 70  $\mu\text{m}$  diameter vessel.