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Effects of an Inserted Endoscope on Chyme Movement in Small Intestine – A Theoretical Model

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Abstract

The effects of an inserted endoscope on chyme movement in small intestine (gastrointestinal tract) have been investigated. The flow of chyme is induced by a progressive wave of area contraction along the length of intestinal wall under long wavelength approximation. It is found that the chyme movement is significantly influenced due to the presence of the endoscope. The pressure drop assumes lower values for higher values of the endoscope radius for small flow rates but the property reverses with increasing flow rates. The friction forces at intestinal wall and endoscope possess character similar to the pressure drop for any given set of parameters. The friction force at the intestinal wall is found much higher than at the endoscope.

Keywords: endoscope, chyme, peristaltic wave, flow rate, pressure rise, friction force.

1. Introduction

Theoretical study of biological systems has been the subject of scientific research for over a couple of centuries. Like most of the problems of nature sciences, mathematical modelling of bio-systems is complex one due to the complicated structure of organs and their constituent materials. The walls of many body passages contain a special type of muscle called smooth muscle. The muscle contracts in sequence, sending waves of contraction along the walls of the passage. These waves cause the Contents in the passage to move forward in the direction of the waves. Physiologists term the phenomenon of such transport as peristalsis. It is a form of fluid transport, which occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible duct containing liquid or mixture. This property is put to use by the body to transport bio-fluids in many biological organs including small intestine (Srivastava and Srivastava, 1984). Latham (1966) was probably the first to investigate the mechanism of peristaltic transport in his M.S. thesis and since then several theoretical and experimental attempts have been made to understand peristaltic action in various situations. A review of much of the early literature is presented in an excellent article by Jaffrin and Shapiro (1971). Most of the investigations reported up to the year 1983, arranged according to the geometry, the fluid, the Reynolds number, the wavelength parameter, the wave amplitude parameter, and the wave shape, as well as an account of the experimental attempts on the subject have been given in Srivastava and Srivastava (1984). The literature beyond this is well referenced in Srivastava and coworkers (1995, 2002).

Human gastrointestinal tract (small intestine) whose functions are digestion and absorption is a convoluted tube which lies in the central and lower parts of abdomen and extends from pylorus to ileocaecal valve where it joins with the large intestine. Its length is about 6-7 m and average radius about 1.25 cm, which are correlated with the height of an individual but not with the weight or age (Piersol, 1930; Fulton, 1946; Vander et al., 1975). Peristalsis which is nearly of sinusoidal in nature (Lew et al., 1871, Vander et al., 1975, Srivastava and Srivastava, 1985)), occurs most obviously in the

digestive tract with a wave speed of about 2-2.5 cm/min and the chyme takes about
4.5 hours to pass through intestine (Fulton, 1946). As each group of muscle fibers in the intestinal wall contracts, it narrows that part of the passage, squeezing the food (chyme) bolus into adjoining section where muscle fibers are relaxed. When the walls of intestine are stretched, a circular peristaltic wave is formed behind the point of stimulation, which passes (along the rectum) towards the intestine. The response to this stretch is known as myenteric reflux. Each wave lasts for about a seconds and is then followed by a quiescent period of few seconds to few minutes.

Due to the increasing rate of environmental pollution, particularly the water pollution, intestinal infections, which may result into distention, constipation, over formation of gas, etc., have become a common disease in living systems. Under infectious conditions a powerful wave called peristaltic rush occurs which travels long distances in small intestine in few minutes. They sweep the contents of intestine into the colon, thereby relieving the small intestine of irritant or excessive distention, as it occurs in the case of diarrhea. It is well known that endoscopy has been very much helpful and one of the most powerful means in diagnosis and management of various intestinal diseases for past decades. It appears, however that no rigorous attention, latest to the author’s knowledge, has been paid in the literature to study the influence of an inserted endoscope on chyme flow in small intestine (gastrointestinal tract).

With the above discussion in mind, the present investigation is therefore devoted to observe the effects of an inserted endoscope on chyme movement in small intestine. The mathematical model considers the flow of a Newtonian viscous fluid between the annular space (gap) of two concentric tubes; the outer tube (circular cylindrical) corresponds to the intestine and inner one (solid circular cylinder) to the endoscope. The flow is induced by sinusoidal peristaltic waves along the length of the outer tube wall (intestinal wall).

2. Formulation of the Problem and Analysis

Consider the axisynmetric flow of an incompressible Newtonian Viscous fluid between the annular space (gap) of two concentric tubes (the inner tube as the endoscope) with a sinusoidal peristaltic wave travelling down the wall of the outer tube (the intestine). The geometry of the outer wall surface may therefore be described as (Fig.1)

$$H(x,t) = a+b \sin \frac{2\pi}{\lambda} (x - ct), \tag{1}$$

where a is the radius of the outer tube, b is the amplitude of the wave, λ is the wavelength, c is the wave propagation speed, t is the time and x is the axial coordinate.

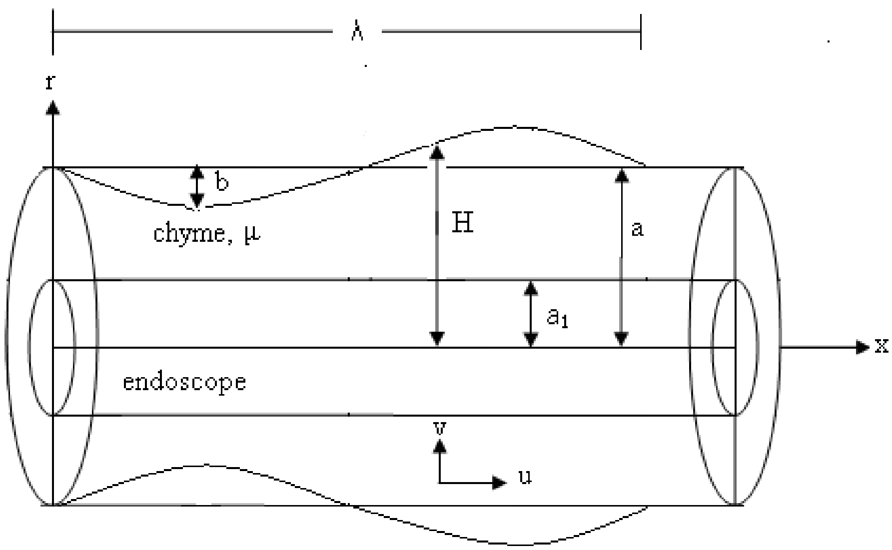

Fig.1 Flow geometry of an inserted endoscope in small intestine.

The appropriate equations of momentum and continuity in the wave frame of reference (moving with speed c) may therefore be written as

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2}\right), \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2}\right), \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0, \tag{4}$$

where ρ is the density of the fluid, μ is the fluid viscosity, p is the pressure, (u, v) are velocity components in (axial, radial) directions and r is the radial coordinate.

The boundary conditions in wave frame of reference are

$$u = -c \quad \text{at} \quad r = H = a + b \sin\frac{2\pi x}{\lambda}, \tag{5}$$

$$u = v = -c \quad \text{at} \quad r = a_1, \tag{6}$$

with a_1 as the radius of the inner tube (endoscope).

An introduction of the following dimensionless variables

$$R' = \frac{r}{a}, \quad x' = \frac{x}{\lambda}, \quad u' = \frac{u}{c}, \quad v' = \frac{\lambda v}{ac},$$

$$t' = \frac{c\,t}{\lambda}, \quad p' = \frac{a^2}{\lambda c\mu}p, \tag{7}$$

Into equations (2) – (4), after dropping primes, yields

$$Re \in \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \in^2\frac{\partial^2 u}{\partial x^2}, \tag{8}$$

$$Re \in^3 \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \in^2\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial (rv)}{\partial r}\right\} + \in^2\frac{\partial^2 v}{\partial x^2}\right], \tag{9}$$

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0, \tag{10}$$

with Re (= ρ c a/ μ) and $\in$ = (a/ λ) are Reynolds and wave numbers, respectively.

Now using the long wavelength approximation (i.e. $\in <<1$) and vanishing Reynolds number theory of Shapiro et al. (1969), equations (8) – (10) reduces to

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{11}$$

$$\frac{\partial p}{\partial r} = 0. \tag{12}$$

The Non-dimensional boundary conditions are thus obtained as

$$u = -1 \quad \text{at} \quad r = \delta = \frac{a_1}{a}, \tag{13}$$

$$u = -1 \quad \text{at} \quad r = h = 1 + \phi \,\text{Sin}\, 2\pi x, \tag{14}$$

with ϕ (= b/a) as amplitude ratio.

The expression for the fluid velocity profile obtained as the solution of equations (11) and (12) subject to the boundary conditions (13) and (14), is given as

$$u = -1 - \frac{1}{4}\frac{dp}{dx}\left\{\delta^2 - r^2 + \frac{\delta^2 - h^2}{\ln(\delta / h)} \ln\,(r/\delta)\right\}. \tag{15}$$

The dimensionless volume flow rate, q (= q' / π a^2c ; q' being the flux in moving system which is same as in stationary system) is thus calculated as

$$q = 2\int_{\delta}^{h} rudr$$

$$= (\delta^2 - h^2)\quad\left[1 + \frac{1}{8}\frac{dp}{dx}\left\{\delta^2 + h^2 - \frac{\delta^2 - h^2}{\ln(\delta / h)}\right\}\right]. \tag{16}$$

It is worthmentioning that the above expressions for the velocity and flow rate reduce to Saxena and Srivastava (1997) under the limit $\delta \rightarrow$o for single phase Newtonian fluid.

From equation (16), one now obtains

$$\frac{dp}{dx} = 8\quad\frac{q^2 + h^2 - \delta^2}{\{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln\,(\delta / h)\}}. \tag{17}$$

Since the pressure drop, Δp = p (o) - p (λ), across one wavelength is same whether measured in stationary or moving coordinate system, it can therefore be calculated from equation (17) as follows

$$\Delta p = -\int_{o}^{1}\frac{dp}{dx}\,dx$$

$$= -8\int_{o}^{1}\frac{q + h^2 - \delta^2}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\,dx\,. \tag{18}$$

The non-dimensional friction forces, $F_{(i,o)}$ (= $F'_{(i,o)}/\pi\,\lambda\,c\,\mu$; F'_i, F'_o, are respectively, the friction forces at the inner and the outer tubes wall, which are also same in moving or stationary coordinate systems) are obtained using equation (17) as

$$F_i = -\int_{o}^{1}\delta^2\frac{dp}{dx}\,dx$$

$$= -8\int_{o}^{1}\frac{(q + h^2 - \delta^2)\,\delta^2}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\quad dx, \tag{19}$$

$$F_o = -\int_o^1 h^2 \frac{dp}{dx}\, dx$$

$$= -8 \int_o^1 \frac{(q \;\; + h^2 - \delta^2)\; h^2}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\; dx. \qquad (20)$$

Following Shapiro et al. (1969), the mean volume flow, Q over a period is given by

$$Q = q + 1 + \frac{\phi^2}{2}. \qquad (21)$$

An application of the relation (21) into equations (18-20), yields the following final form expressions for the dimensionless pressure drop, Δp, the friction forces at the inner and outer tubes wall, F_i and F_o respectively, as

$$\Delta p = -8 \int_o^1 \frac{Q + h^2 - \delta^2 - \phi^2/2 - 1}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\; dx, \qquad (22)$$

$$F_i = -8 \int_o^1 \frac{(Q + h^2 - \delta^2 - \phi^2\; 2 - 1)\;\; \delta^2}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\; dx, \qquad (23)$$

$$F_o = -8 \int_o^1 \frac{(Q + h^2 - \delta^2 - \phi^2 / 2 - 1)\; h^2}{\delta^4 - h^4 - (\delta^2 - h^2)^2 / \ln(\delta / h)}\; dx. \qquad (24)$$

In the limit $\delta \rightarrow$o (i.e., in the absence of the endoscope), the results obtained in equations (22) – (24), reduce to

$$\Delta p = 8 \int_o^1 \frac{Q + h^2 - 1 - \phi^2 / 2}{h^4}\; dx, \qquad (25)$$

$$F_o = 8 \int_o^1 \frac{Q + h^2 - 1 - \phi^2 / 2}{h^2}\; dx, \qquad (26)$$

which are the same results as derived from Srivastava and Saxena (1995) for single – layered Newtonian fluid. Evaluating integrals in the equations (25) and (26) in closed form, one arrives to the results

$$\Delta p = \frac{2 + 3\,\phi^2}{4\,(1 - \phi^2)^{7/2}}\; (Q - 1 - \phi^2 / 2) + \frac{8}{(1 - \phi^2)^{3/2}}\;, \qquad (27)$$

$$F_o = 8\left\{1 + \frac{Q - 1 - \phi^2 / 2}{(1 - \phi^2)^{\;3/2}}\right\}. \qquad (28)$$

The results obtained in equations (27) and (28) are the same as given in Shapiro et al. (1969). These results are also derived from Shukla et. Al. (1980) in the absence of the peripheral layer.

3. Numerical Results and Discussion

In order to have an estimate of quantitative effects of various parameters involved on the results of the analysis, particularly the radius ratio parameter, δ which is our main contribution to the study, computer codes were developed for numerical evaluation of analytical results obtained in equations (22)-(24). Some of the critical results are displayed graphically in Figs.2-7.The parameter values for male small intestine used in the numerical evaluations are chosen from Barton and Raynor (1968) as

a = 1.25 cm, c = 2 cm/min, λ = 8.01 cm.

Further as reported in Cotton and Williams (1990), the most routine upper gastrointestinal endoscopes are between 8 and 11 mm. This calculates the values of radius ratio parameter, δ to be between 0.32 and 0.44. It is important to mention that the theory of long wavelength and zero Reynolds number of Shapiro et al. (1969) remains applicable in the present investigation as the radius of the small intestine a = 1.25 cm, is small as compared to the wavelength, λ = 8.01 cm. It has also been observed by Lew et al. (1971) that the Reynolds number in the small intestine was very small.

One notices that there exists a linear relationship between the pressure drop, Δp and flow rate Q. pressure drop, Δp assumes lower magnitudes for higher values of radius ratio parameter, δ but this property reverses with increasing flow rate Q (Fig. 2). It is clearly observed that for any given flow rate, Q, the magnitude of the pressure drop, Δp depends on the radius ratio parameter, δ (Fig.2). Pressure rise (negative of pressure drop, $-\Delta p$) increases with decreasing values of the flow rate, Q. Thus the

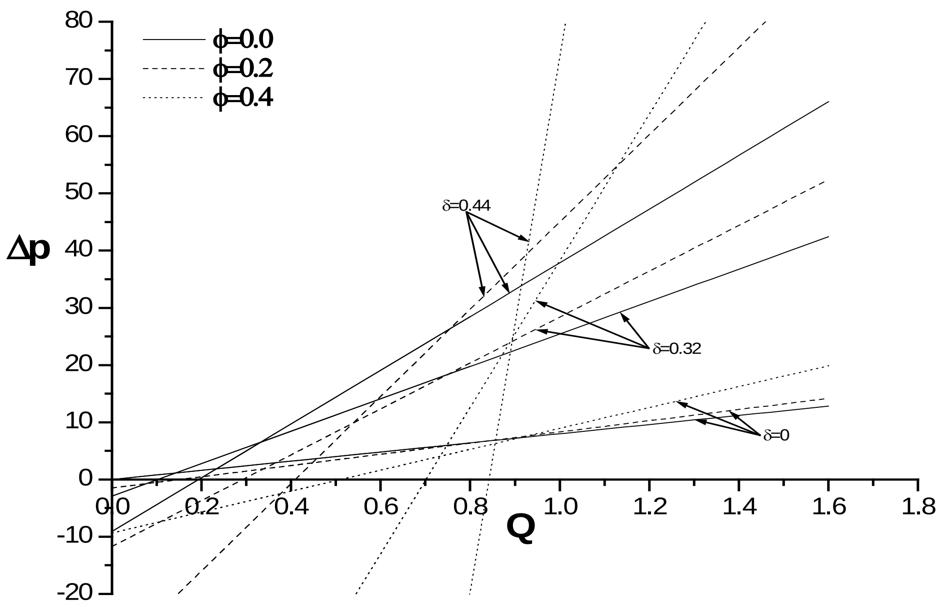

Fig.2 Variation of Δp with Q for different ϕ and δ.

maximum flow rate is achieved at zero pressure rise and maximum pressure occurs at zero flow rate. The friction forces F_o and F_i at outer and inner tube wall possess character similar to the pressure drop, Δp (an opposite character to the pressure rise, $-\Delta p$) for any set of other given parameters (Figs. 2, 3 and 4). Also, it is observed that the friction force, F_o at outer tube wall always assumes higher magnitudes than the friction force, F_i at the inner tube wall for any given set of parameters (Figs. 3 and 4). Pressure drop, Δp and friction forces, F_o and F_i decrease with increasing values of

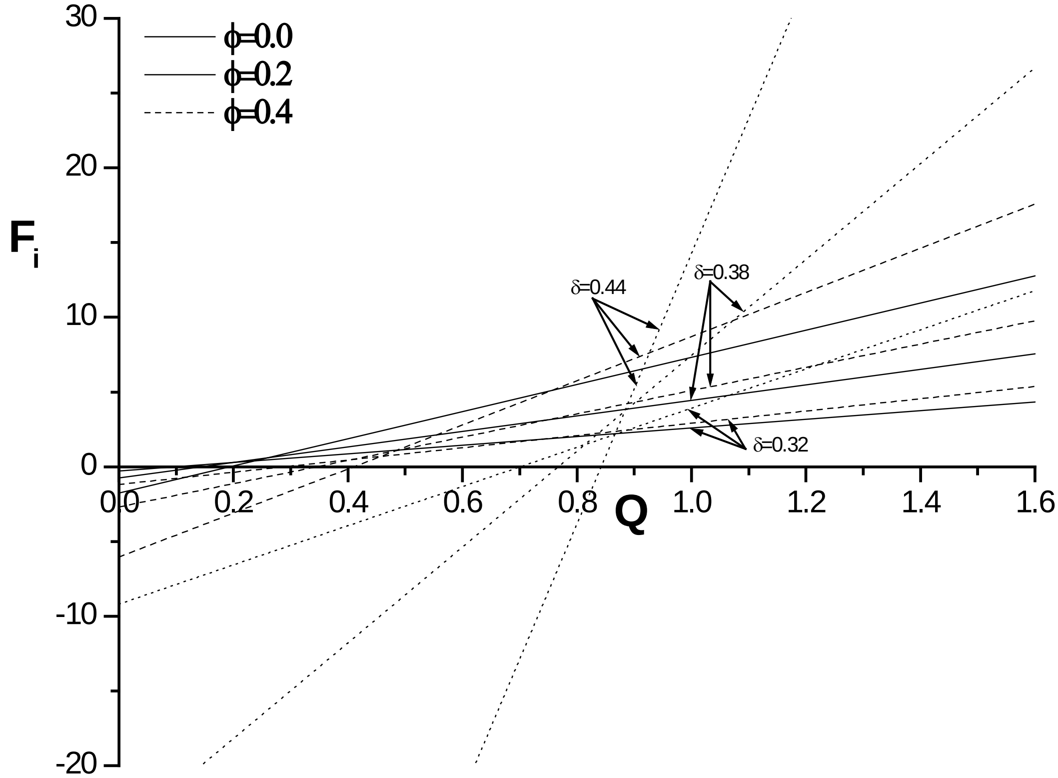

Fig.3 Variation of F_i with Q for different ϕ and δ.

the amplitude ratio, ϕ (Figs. 5, 6 and 7). We observe that the flow characteristics, Δp, F_o and F_i assume higher magnitudes for small values of ϕ with increasing radius ratio parameter, δ but the property reverses for large values of ϕ. It is worth mentioning here that friction force F_i always attains lower magnitude than F_o with increasing amplitude ratio ϕ for any given set of other parameters(Figs. 6 and 7).

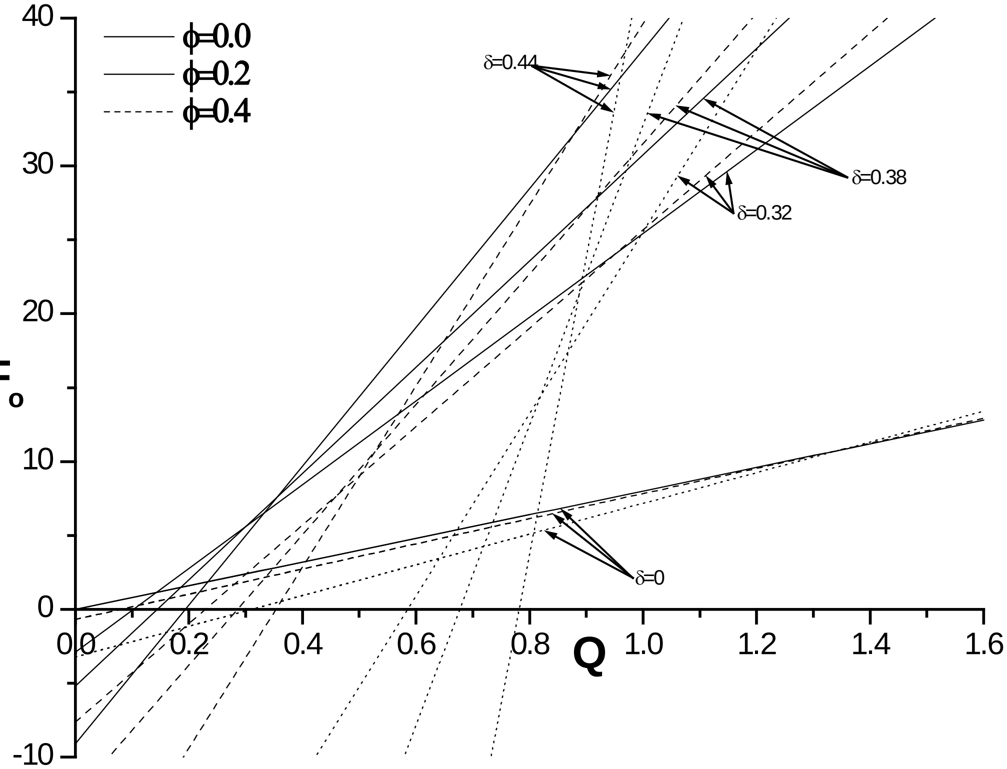

Fig.4 Variation of F_o with Q for different ϕ and δ.

The study presented above is subject to certain assumptions and approximations, comments therefore seem to be essential for these. In view of the investigations of Lew et al. (1971) and Srivastava and Srivastava (1985), it appears that a non-Newtonian (particularly, a power-law) fluid would be more adequate to represent chyme in small intestine, the mathematical tool for the same is yet not available and needs to be developed before discussing the intestinal flow with an inserted endoscope using a power-law fluid. The use of a Newtonian fluid to represent chyme in small intestine thus remains as an approximation to the study. From the published literature of Vander et al.

(1975), Lew et al. (1971) and Srivastava and Srivastava (1985), it is established that peristaltic waves in intestinal wall are of sinusoidal in nature. However, the use of waves of arbitrary shape would be a further improvement of the study and closer to the realistic situation. Finally, the low Reynolds number Re and long wavelength approximation used to discuss intestinal flow definitely require certain explanation as well. The condition that Re<<1 (i.e., low Reynolds number) usually applies to the low velocity and highly viscous flows. Chyme is known to be a highly viscous fluid and as pointed out earlier flow velocity (2.54 cm/min) in small intestine is low (Barton and Raynor, 1968; Srivastava and Srivastava, 1985). Thus the conditions for the Stokes approximation for slow creeping flows are well met in the case of intestinal flow. As a result, the inertia terms become negligible in the flow equation (White, 2006). On the other

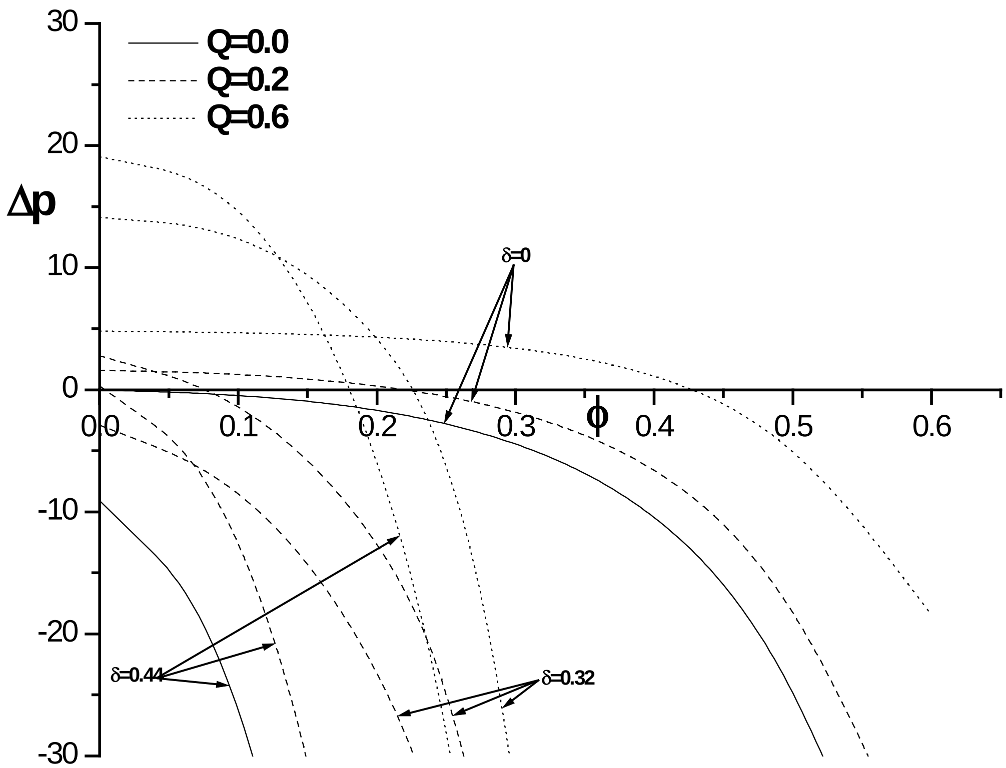

Fig.5 Variation of Δp with ϕ for Q and δ.

hand, under a long wavelength approximation (i.e., $\in$<<1), the inertial terms may be neglected (Shapiro et al., 1969). However, in the case of intestinal flow the conditions of low Reynolds number and long wavelength approximation both exist (Lew et al., 1971, Srivastava and Srivastava, 1985). Consequently, Re $\in$ and Re $\in^3$ are negligible quantities which justifies the use of a fully developed flow in the case of intestine. It is worth mentioning here that for an arbitrary value of the wave number $\in$ and for $0 \neq$ Re<< 1, the solution may be obtained using the perturbation technique. However, being of less significant to the problem discussed above, this case has not been considered here.

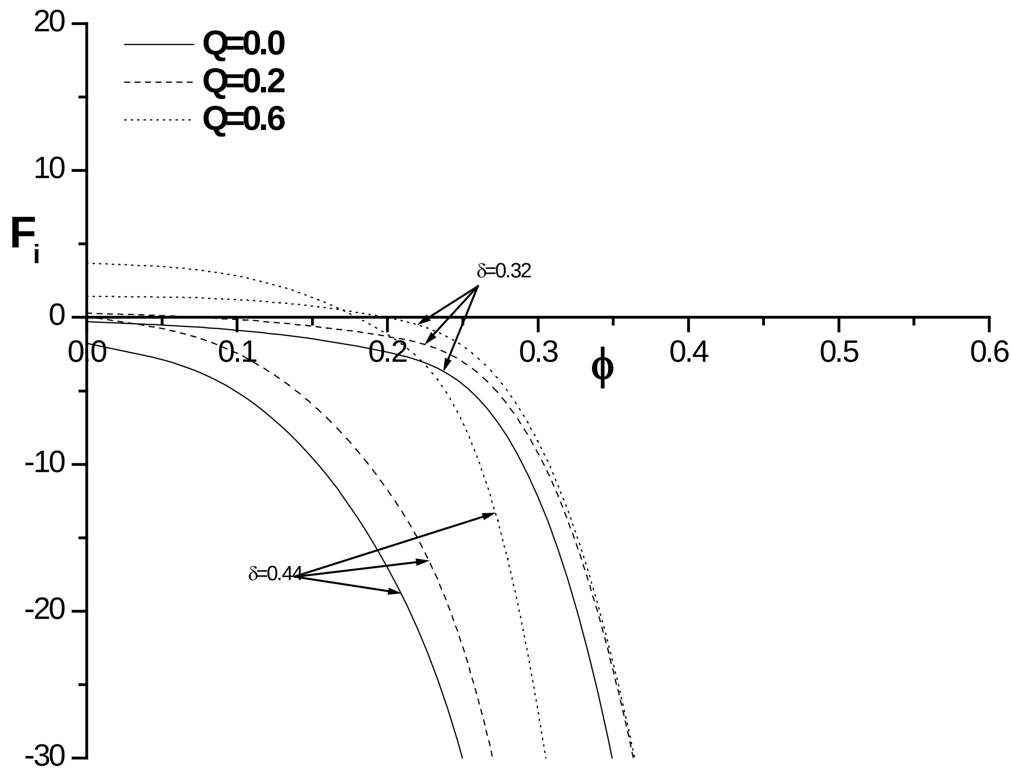

Fig.6 Variation of F_i with ϕ for different Q and δ.

4. Conclusions

An attempt has been made in the study to observe the influence of an inserted endoscope on chyme movement in small intestine. The impacts on the results due to the presence of the endoscope, as discussed above seems of be of significant clinical application. In view of the discussion presented above, it appears that a power-law fluid would adequately represent the chyme in small intestine. Author is already in the course of developing the required mathematical tool to discuss the intestinal flow with an inserted endoscope using a power-law fluid and would address the problem in his subsequent communication. The use of the Newtonian fluid therefore remains a major approximation to the study. Although, the study has been carried out under several simplifications and approximations, it still enables one to have a qualitative and quantitative view of the influence of the inserted endoscope on transport of chyme in small intestine. It is however felt that considerable amount of investigations are necessary to discuss the problem adequately and closer to realistic situations.

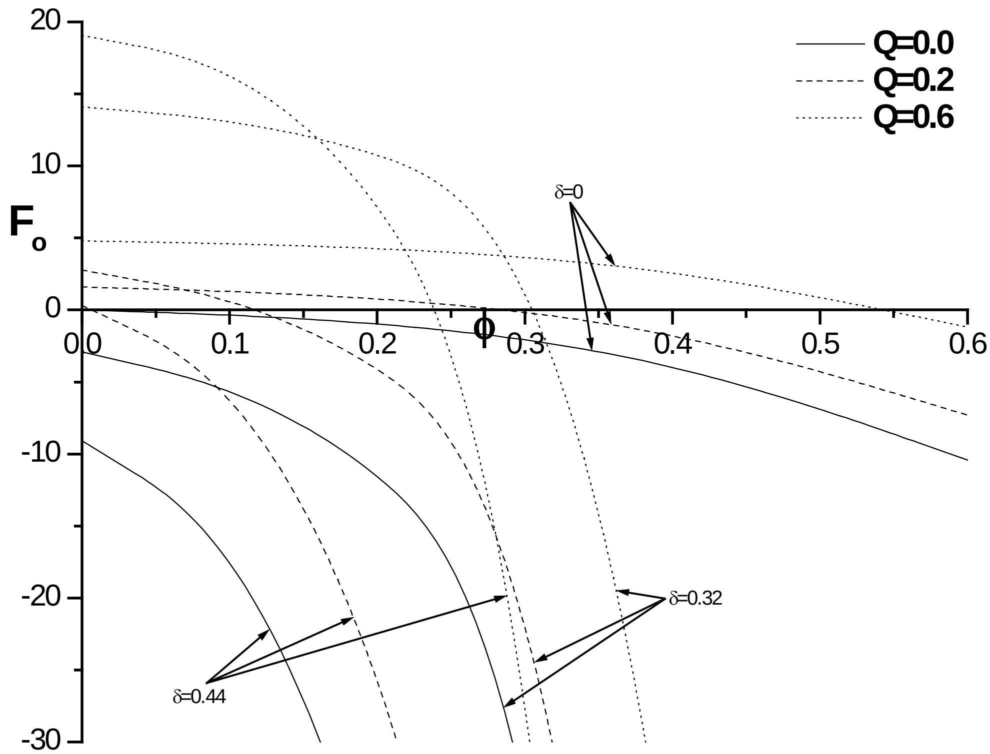

Fig.7 Variation of F_o with ϕ for different Q and δ.

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