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Controlling and Synchronizing Combined Effect of Chaos Generated in Generalized Lotka-Volterra Three Species Biological Model using Active Control Design

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Abstract

In this work, we study hybrid projective combination synchronization scheme among identical chaotic generalized Lotka-Volterra three species biological systems using active control design. We consider here generalized Lotka-Volterra system containing two predators and one prey population existing in nature. An active control design is investigated which is essentially based on Lyapunov stability theory. The considered technique derives the global asymptotic stability using hybrid projective combination synchronization technique. In addition, the presented simulation outcomes and graphical results illustrate the validation of our proposed scheme. Prominently, both the analytical and computational results agree excellently. Comparisons versus others strategies exhibiting our proposed technique in generalized Lotka-Volterra system achieved asymptotic stability in a lesser time.

Keywords: Chaotic system; Hybrid projective synchronization; Combination synchronization; Generalized Lotka-Volterra model; Active control design; Lyapunov stability theory; MATLAB

MSC 2010 No.: 34K23, 34K35, 37B25, 37N35

1. Introduction

The interactions among several systems found in nature such as biological species, political parties, businesses, countries and others systems include competition or cooperation (Goel et al. (1971)). Examining the aforementioned interactions by employing mathematical modelling gives us a mechanism to obtain profound understanding of such systems. Lotka (1926) and Vito Volterra (Scudo (1971)) in 1920s introduced the vastly known biological models involving quadratic differential equations to investigate numerous key aspects of population dynamics, for instance, parasitism or predation among two species. Lotka-Volterra (LV) model was described as a biological notion, yet it has been applied to several diverse fields of research, which are, studying transaction counts and interactions between cryptocurrencies (Gatabazi et al. (2019), Gatabazi et al. (2019)), controlling congestion in wireless sensor networks (Antoniou et al. (2010)), and others. The LV model has comprehensively contributed to the biological research describing inter species interactions by giving a broad spectrum of varied parameters which modulate population dynamics (Gavin et al. (2006), Tonnang et al. (2009), Tsai et al. (2016), Perhar et al. (2016), Reichenbach et al. (2006), Silva-Dias and Löpez-Castillo (2018), Hening and Nguyen (2018), Xiong et al. (2019), Nag (2020)) and others. Nevertheless, L-V model acquires few limitations, for example, interactions in various species of similar ecosystem, interplaying with the natural habitat etc.

Specifically, generalized Lotka-Volterra (GLV) model including three species has become the most significant in all existential population's oscillatory interactions. In 1980, Arneodo et al. (1980) have established that it may acquires chaotic, i.e., utterly complex pattern for a very specific choice of parameters. Additionally, Samardzija and Greller (1988) in 1988 performed a comprehensive analysis in GLV model exhibiting its chaotic behaviour. Chaos synchronization (CS) of chaotic systems is prescribed as a procedure of adapting identical or non-identical chaotic systems in a typical way that both depict the similar conduct owing to pairing to gain stability.

After pioneered work of Pecora and Carroll (1990) introduced in 1990, a wide spectrum of researches have been conducted describing different kinds of chaos synchronization and control (CSC) techniques such as complete (Singh et al. (2017)), anti (Li and Zhou (2007)), hybrid (Sudheer and Sabir (2009)), hybrid projective (Khan and Chaudhary (2020b), Khan and Chaudhary (2020a)), function projective (Zhou and Zhu (2011)), lag (Li and Liao (2004)), phase (Ma et al. (2017)), projective (Ding and Shen (2016)), combination synchronization (Khan and Chaudhary (2020a)), combination-combination (Khan and Chaudhary (2020b)), modified projective (Li (2007)), combination difference (Khan and Chaudhary (2020c)), triple compound (Yadav et al. (2019)), active (Delavari and Mohadeszadeh (2018)), adaptive (Khan and Chaudhary (2020a), Khan and Bhat (2017a)), backstepping design (Rasappan and Vaidyanathan (2012)), feedback (Chen and Han (2003)), sliding mode (Vaidyanathan and Sampath (2012); Jahanzaib et al. (2020)), impulsive (Li and Zhang (2016)) to achieve stability in chaotic systems. CSC among chaotic dynamical systems using active control design was first described by Bai and Lonngren (1997) in 1997. More importantly, Mainieri and Rehacek (1999) in 1999 developed the idea of projective synchronization in chaotic systems. In addition, combination synchronization was introduced firstly in 2011 by Runzi et al. (2011). Further, some significant researches (Wu (2013);

Runzi and Yinglan (2012)) are established in this direction. Also, Dongmo et al. (2018) examined in detail a novel strategy, mentioned as difference synchronization, in the year 2018. Moreover, in 2019 Yadav et al. (2019) studied difference synchronization of chaotic systems with exponential terms. Also, a detailed analysis of chaos synchronization in chaotic systems has been described in Liao and Tsai (2000), Yassen (2003), Li and Xu (2004), Li et al. (2012), and Wu et al. (2012). In addition, an optimal control strategy for Lotka-Volterra model has been studied rigorously in (El-Gohary and Yassen (2001)). Further, in Khan and Bhat (2017b), Khan and Tyagi (2017b), and Khan and Tyagi (2017a), numerous control schemes are analyzed in detail in newly constructed chaotic systems. Moreover, in Vaidyanathan (2016), and Vaidyanathan (2015), adaptive control technique is discussed for synchronizing GLV biological system.

Considering the aforementioned discussions and literature review, our immediate goal in this article is to propose and study a hybrid projective combination synchronization (HPCS) among three identical GLV systems via active control design (ACD). Basically, combination synchronization scheme involve three chaotic systems (identical or non-identical) out of which two are taken as master systems and one is selected as a slave system. We here consider GLV model (master as well as slave system) since this system acquires numerous oscillatory characteristics relating to populations, although the considered GLV model is non-realistic.

The paper is arranged as follows. Section 2 consists of some preliminaries containing few notations and essential terminology which has been used in upcoming sections. Section 3 describes some elementary structured features of GLV model for which HPCS is investigated. Section 4 outlines comprehensively the synchronization theory using ACD approach. The active control functions are appropriately designed for investigating HPCS strategy. Section 5 deals with the discussions regarding numerical simulations performed in MATLAB environment and illustrations of the experimental results. In addition, a comparison study keeping previously published work in view has been done. Conclusions and discussions are finally presented in Section 6.

2. Mathematical Preliminaries

In this section, to achieve combination synchronization (Runzi et al. (2011)), a methodology based on master-slave framework has been presented which is required in coming up sections.

Two master systems and one slave system taken into consideration are written as:

$$\dot{z}_{m1} = f_1(z_{m1})\gamma_1 + F_1(z_{m1}), \tag{1}$$
$$\dot{z}_{m2} = f_2(z_{m2})\gamma_2 + F_2(z_{m2}), \tag{2}$$
$$\dot{z}_{s1} = f_3(z_{s1})\gamma_3 + F_3(z_{s1}) + V_1(z_{m1}, z_{m2}, z_{m3}), \tag{3}$$

where $z_{m1} = (z_{m11}, z_{m12}, \dots, z_{m1n})^T \in R^n$, $z_{m2} = (z_{m21}, z_{m22}, \dots, z_{m2n})^T \in R^n$, $z_{s1} = (z_{s11}, z_{s12}, \dots, z_{s1n})^T \in R^n$ are the state vectors of master and slave systems (1), (2) and (3) respectively, $F_1, F_2, F_3 : R^n \to R^n$ are three nonlinear continuous functions, $\gamma_1 = (\gamma_{11}, \gamma_{12}, \dots, \gamma_{1p_1})^T$ is a $p_1 \times 1$ unknown parameter vector of the first master system (1), $\gamma_2 = (\gamma_{21}, \gamma_{22}, \dots, \gamma_{2p_2})^T$ is a $p_2 \times 1$ unknown parameter vector of the second master system (2), $\gamma_3 = (\gamma_{31}, \gamma_{32}, \dots, \gamma_{3p_3})^T$ is a $p_3 \times 1$ unknown parameter vector of the slave system (3), $f_1 : R^n \to R^{n \times p_1}$, $f_2 : R^n \to R^{n \times p_2}$, $f_3 : R^n \to R^{n \times p_3}$, $V_1 : R^n \times R^n \times R^n \to R^n$ are the controllers to be properly determined.

Definition 2.1.

If there exist three constant matrices $P_1, P_2, P_3 \in R^n \times R^n$ and $P_3 \neq 0$ such that

$$\lim_{t\to\infty} \|E(t)\| = \lim_{t\to\infty} \|(P_1 z_{m1}(t) + P_2 z_{m2}(t) - P_3 z_{s1}(t))\| = 0,$$

then combination of two chaotic master systems (1) and (2) is said to perform combination synchronization with one chaotic slave system (3) and $\|.\|$ denotes vector norm.

Remark 2.1.

The constant matrices P_1, P_2 and P_3 are called the scaling matrices. Moreover, P_1, P_2 and P_3 can be extended as matrices of functions of state variables z_{m1}, z_{m2} and z_{s1}.

Remark 2.2.

The problem of combination synchronization would be converted into traditional chaos control issue for $P_1 = P_2 = 0$.

Remark 2.3.

If $P_3 = -I$ and $P_1 = P_2 = -\delta I$, then for $\delta = 1$ it will be reduced to combination complete synchronization and for $\delta = -1$ it turns into combination anti-synchronization. Therefore, the combination of anti-synchronization and complete synchronization makes hybrid projective synchronization. Hence, the hybrid projective combination synchronization (HPCS) error takes the form:

$$E = z_{s1} - \delta(z_{m2} + z_{m1}), \tag{4}$$

where $\delta = diag(\delta_1, \delta_2, \dots, \delta_n)$.

Remark 2.4.

Definition 2.1 exhibits that combination of master and corresponding slave systems may be expanded to more such chaotic systems. In addition, the chosen master systems as well as slave system of combination synchronization scheme may be identical or non-identical.

3. Generalized Lotka-Volterra System: Existence of Chaos and a Brief Elucidation

In this section, we describe in brief the chaotic system, widely known as Generalized Lotka-Volterra (GLV) three species biological system, to be selected for HPCS technique using active control design (ACD).

We now represent the GLV system as:

$$\begin{cases} \dot{z}_1 = z_1 - z_1 z_2 + b_3 z_1^2 - b_1 z_1^2 z_3, \\ \dot{z}_2 = -z_2 + z_1 z_2, \\ \dot{z}_3 = -b_2 z_3 + b_1 z_1^2 z_3, \end{cases} \tag{5}$$

where $(z_1, z_2, z_3)^T \in R^3$ is the state vector and b_1, b_2 and b_3 are positive parameters. Also, in (5), z_1 represents the prey population and z_2, z_3 denotes the predator populations. The parameter data associated with GLV system (5) which display chaotic behavior is listed as $b_1 = 2.9851$, $b_2 = 3$ and $b_3 = 2$. Also, Figure 1(a-d) display the phase plots of (5). Moreover, the detailed theoretical study and numerical simulation results for (5) can be found in Samardzija and Greller (1988).

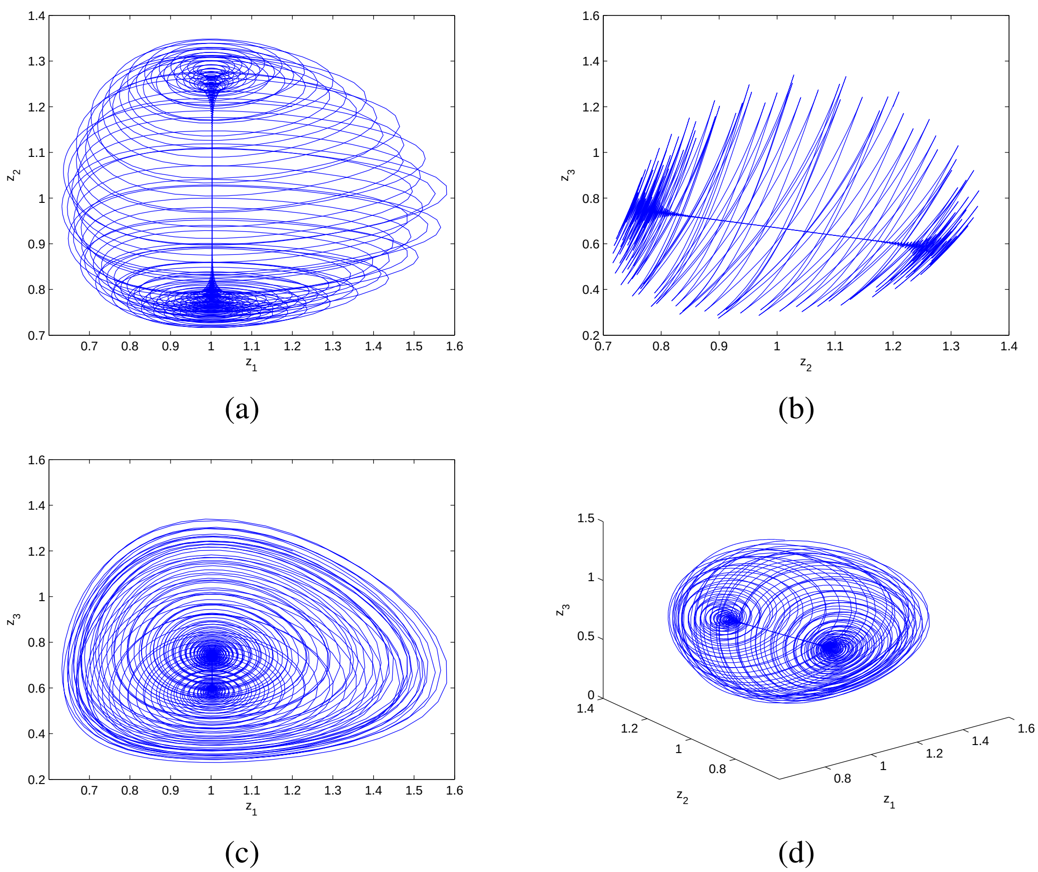

Figure 1. Phase plots for chaotic GLV system in (a) $z_1 - z_2$ plane, (b) $z_2 - z_3$ plane, (c) $z_1 - z_3$ plane, (d) $z_1 - z_2 - z_3$ space

The following section presents the HPCS scheme to control chaos generated by (5) using ACD approach.

4. Synchronization Theory via Active Control Design

In this section, for illustration purpose, we consider three identical GLV systems to investigate the proposed HPCS scheme via ACD. Lyapunov stability theory (LST) based active control design (ACD) is employed and required stability criterion is derived.

Therefore, we consider two master systems (GLV) and one slave system (GLV) with required controllers as:

$$\begin{cases} \dot{z}_{m11} = z_{m11} - z_{m11}z_{m12} + b_3 z_{m11}^2 - b_1 z_{m11}^2 z_{m13}, \\ \dot{z}_{m12} = -z_{m12} + z_{m11}z_{m12}, \\ \dot{z}_{m13} = -b_2 z_{m13} + b_1 z_{m11}^2 z_{m13}, \end{cases} \tag{6}$$

$$\begin{cases} \dot{z}_{m21} = z_{m21} - z_{m21}z_{m22} + b_3 z_{m21}^2 - b_1 z_{m21}^2 z_{m23}, \\ \dot{z}_{m22} = -z_{m22} + z_{m21}z_{m22}, \\ \dot{z}_{m23} = -b_2 z_{m23} + b_1 z_{m21}^2 z_{m23}, \end{cases} \tag{7}$$

$$\begin{cases} \dot{z}_{s31} = z_{s31} - z_{s31}z_{s32} + b_3 z_{s31}^2 - b_1 z_{s31}^2 z_{s33} + V_{11}, \\ \dot{z}_{s32} = -z_{s32} + z_{s31}z_{s32} + V_{12}. \\ \dot{z}_{s33} = -b_2 z_{s33} + b_1 z_{s31}^2 z_{s33} + V_{13}, \end{cases} \tag{8}$$

where V_{11},V_{12} and V_{13} are active controllers to be determined in such a manner that HPCS between three identical GLV chaotic systems will be attained.

Define now the error functions as

$$\begin{cases} E_{11} = z_{s31} - \delta_1(z_{m21} + z_{m11}), \\ E_{12} = z_{s32} - \delta_2(z_{m22} + z_{m12}), \\ E_{13} = z_{s33} - \delta_3(z_{m23} + z_{m13}). \end{cases} \tag{9}$$

The immediate goal in this work is to design controllers V_{1i}, $(i = 1, 2, 3)$ ensuring that error functions defined in (9) satisfy

$$\lim_{t\to\infty} E_{1i}(t) = 0 \quad \text{for } (i = 1, 2, 3).$$

The resulting error dynamics turns into:

$$\begin{cases} \dot{E}_{11} = E_{11} - z_{s31}z_{s32} + b_3 z_{s31}^2 - b_1 z_{s31}^2 z_{s33} - \delta_1(-z_{m21}z_{m22} + b_3 z_{m21}^2 \\ \qquad -b_1 z_{m21}^2 z_{m23} + z_{m11}z_{m12} - b_3 z_{m11}^2 + b_1 z_{m11}^2 z_{m13}) + V_{11}, \\ \dot{E}_{12} = -E_{12} + z_{s31}z_{s32} - \delta_2(-z_{m11}z_{m12} + z_{m21}z_{m22}) + V_{12}, \\ \dot{E}_{13} = -b_2 E_{13} + b_1 z_{s31}^2 z_{s33} - \delta_3(b_1 z_{m21}^2 z_{m23} - b_1 z_{m11}^2 z_{m13}) + V_{13}. \end{cases} \tag{10}$$

Let us now define the active controllers as:

$$\begin{cases} V_{11} = -E_{11} + z_{s31}z_{s33} - b_3 z_{s31}^2 + b_1 z_{s31}^2 z_{s33} + \delta_1(-z_{m21}z_{m22} + b_3 z_{m21}^2 \\ \qquad -b_1 z_{m21}^2 z_{m23} + z_{m11}z_{m12} - b_3 z_{m11}^2 + b_1 z_{m11}^2 z_{m13} - K_1 E_{11}, \\ V_{12} = E_{12} - z_{s31}z_{s32} + \delta_2(-z_{m11}z_{m12} + z_{m21}z_{m22}) - K_2 E_{12}, \\ V_{13} = b_2 E_{13} - b_1 z_{s31}^2 z_{s33} + \delta_3(b_1 z_{m21}^2 z_{m23} - b_1 z_{m11}^2 z_{m13}) - K_3 E_{13}, \end{cases} \tag{11}$$

where K_1, K_2 and K_3 are positive gain constants.

On substituting the active controllers (11) into error dynamics (10), we get

$$\begin{cases} \dot{E}_{11} = -K_1 E_{11}, \\ \dot{E}_{12} = -K_2 E_{12}, \\ \dot{E}_{13} = -K_3 E_{13}. \end{cases} \tag{12}$$

The Lyapunov function is described as

$$V(E(t)) = \frac{1}{2}[E_{11}^2 + E_{12}^2 + E_{13}^2]. \tag{13}$$

We find that Lyapunov function $V(E(t))$ is positive definite.

Then, the derivative of Lyapunov function $V(E(t))$ may be expressed as:

$$\dot{V}(E(t)) = E_{11}\dot{E}_{11} + E_{12}\dot{E}_{12} + E_{13}\dot{E}_{13}. \tag{14}$$

Theorem 4.1.

The chaotic systems (6)-(8) are globally and asymptotically hybrid projective combination synchronized in each initial states by the active controllers (11).

Proof:

Certainly, Lyapunov function $V(E(t))$ that is described in (13) is positive definite function in R^3.

Using (12) in (14), we obtain

$$\begin{aligned} \dot{V}(E(t)) &= -K_1 E_{11}^2 - K_2 E_{12}^2 - K_3 E_{13}^2 \\ &< 0, \end{aligned}$$

which displays that $\dot{V}(E(t))$ is negative definite.

Thus, by using LST, we find that discussed HPCS synchronization error $E(t) \to 0$ asymptotically with $t \to \infty$ for each initial values $E(0) \in R^3$ which completes the proof. ■

5. Numerical Simulations and Discussions

In this section, we conduct few simulation experiments to illustrate the effectivity and feasibility of proposed HPCS scheme using ACD. For achieving this, we utilize the typical 4th-order Runge-Kutta methodology for solving systems of ordinary differential equations. For the parameters $b_1 = 2.9851$, $b_2 = 3$ and $b_3 = 2$, the system (5) displayed chaotic behaviour without the presence of controllers. Initial values of master systems (6)-(7) and corresponding slave system (8) are $(z_{m11}(0) = 27.5, z_{m12}(0) = 23.1, z_{m13}(0) = 11.4)$, $(z_{m21}(0) = 1.2, z_{m22}(0) = 1.2, z_{m23}(0) = 1.2)$ and $(z_{s31}(0) = 2.9, z_{s32}(0) = 12.8, z_{s33}(0) = 20.3)$, respectively.

We achieve hybrid projective combination synchronization between two master (6)-(7) and corresponding one slave systems (8) by taking a scaling matrix δ with $\delta_1 = 8, \delta_2 = -4, \delta_3 = 3$.

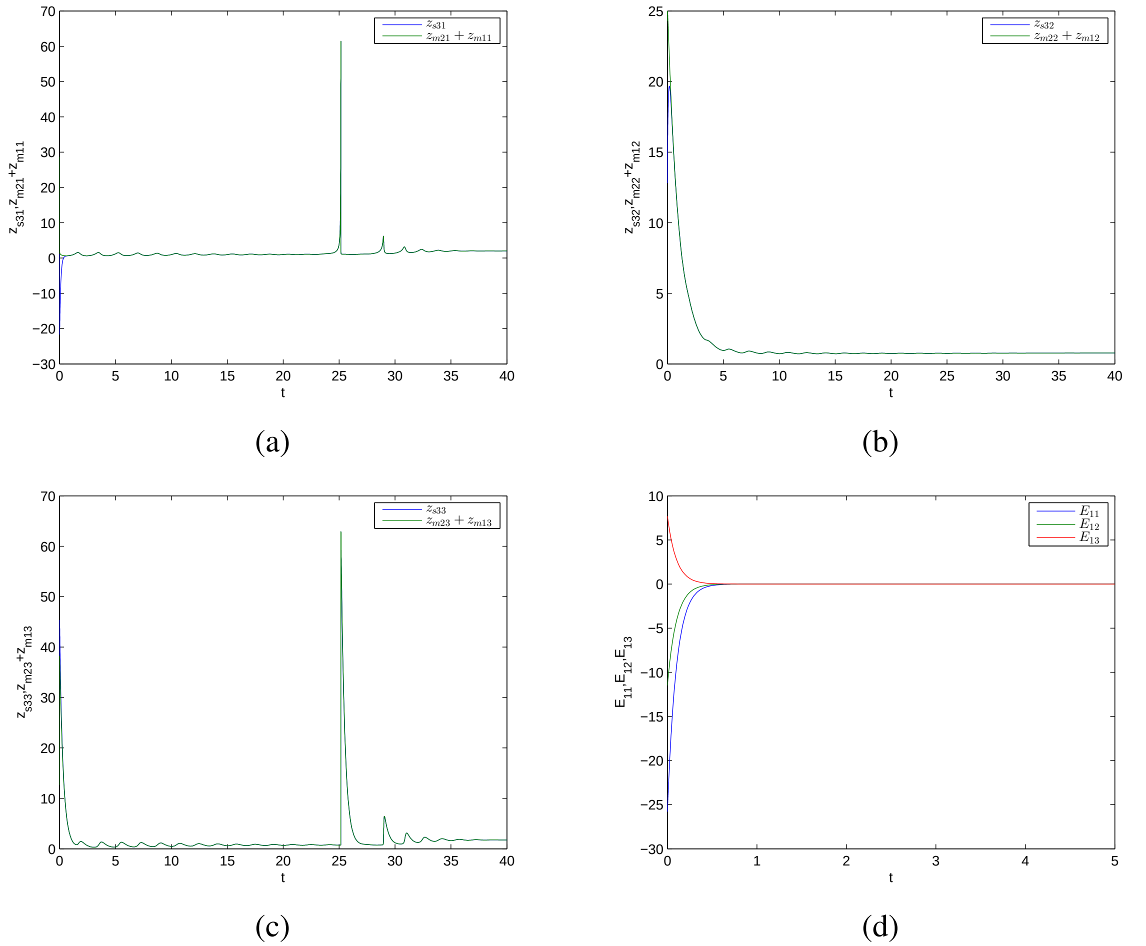

Figure 2. Complete synchronized trajectories for GLV system (a) between $z_{s31}(t)$ and $z_{m21}(t) - z_{m11}(t)$, (b) between $z_{s32}(t)$ and $z_{m22}(t) - z_{m12}(t)$, (c) between $z_{s33}(t)$ and $z_{m23}(t) - z_{m13}(t)$, (d) synchronization errors

Here, the control gains have been taken as $K_i = 10$ for $i = 1, 2, 3$. Further, simulation results are depicted in Figure 4(a-c) which exhibit the HPCS synchronized state trajectories of master (6)-(7) and slave system (8). In addition, synchronization errors $(E_{11}, E_{12}, E_{13}) = (213.3, -74.8, 50.9)$ approaching zero as t tending to infinity have been displayed in Figure 4(d). Hence, the proposed HPCS scheme among master and slave systems is justified computationally. Furthermore, Figure 2 and Figure 3 exhibit the particular cases, namely, combination complete synchronization and combination anti-synchronization respectively of HPCS scheme in GLV systems.

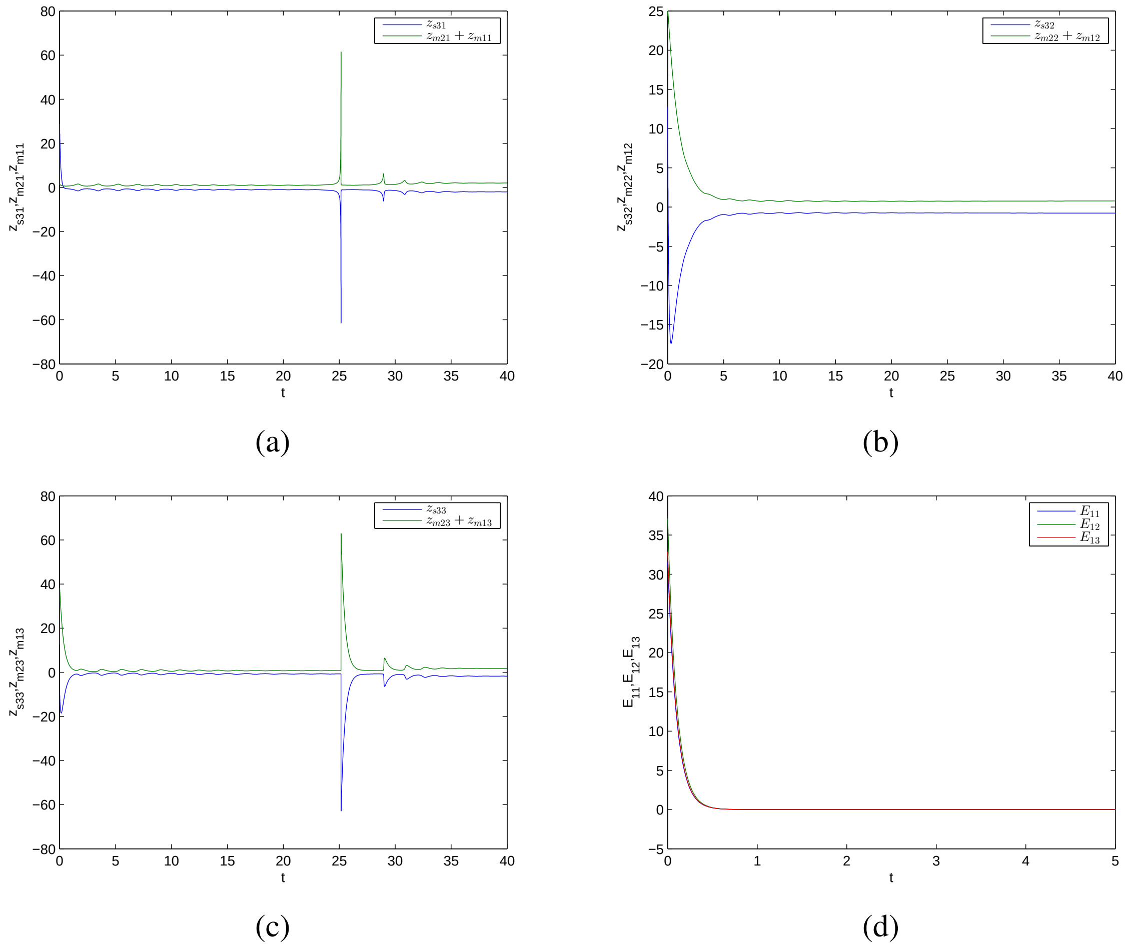

Figure 3. Anti-synchronized trajectories for GLV system (a) between $z_{s31}(t)$ and $z_{m21}(t) - z_{m11}(t)$, (b) between $z_{s32}(t)$ and $z_{m22}(t) - z_{m12}(t)$, (c) between $z_{s33}(t)$ and $z_{m23}(t) - z_{m13}(t)$, (d) synchronization errors

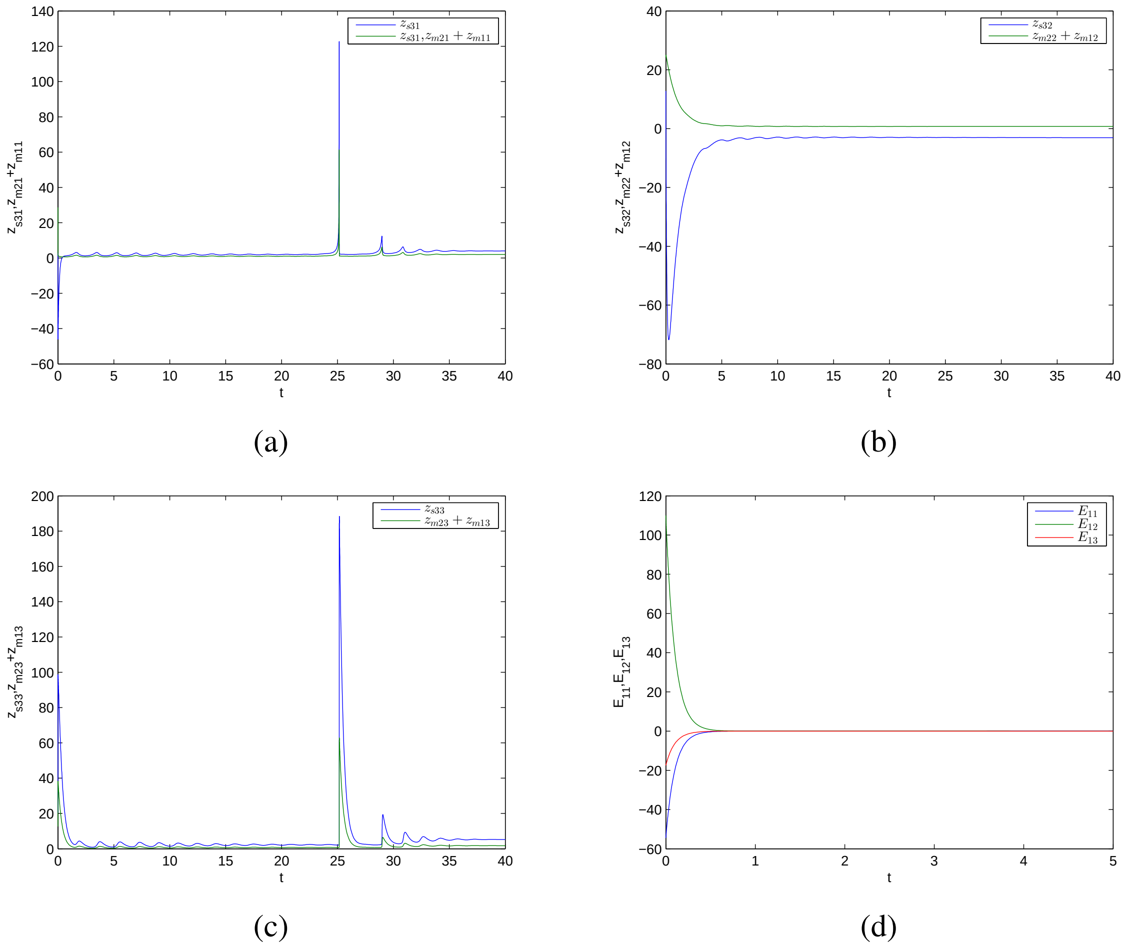

Figure 4. HPCS trajectories for GLV system (a) between $z_{s31}(t)$ and $z_{m21}(t) - z_{m11}(t)$, (b) between $z_{s32}(t)$ and $z_{m22}(t) - z_{m12}(t)$, (c) between $z_{s33}(t)$ and $z_{m23}(t) - z_{m13}(t)$, (d) synchronization errors

5.1. Comparative study for the proposed HPCS technique and the related published work

Hybrid synchronization of two chaotic systems is achieved using adaptive control design in Vaidyanathan (2016) when performed on same GLV systems. It is recorded that synchronization errors converge to zero at $t = 0.8$ (approx), whereas in this study, HPCS scheme is achieved using ACD, in which it is noted that the synchronization errors converge to zero at $t = 0.5$ (approx) as displayed in Figure 5. This clearly depicts that our proposed HPCS scheme via ACD has more preference over earlier published literature.

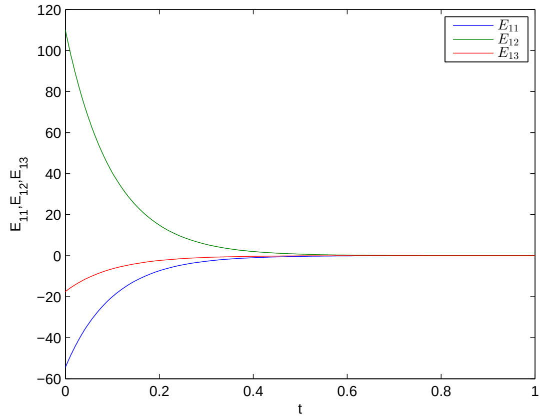

Figure 5. HPCS synchronization error plot

6. Conclusion and Future work

In this paper, the proposed HPCS technique among identical chaotic 3D GLV biological systems using active control design is investigated. By describing proper active controllers based on LST, the discussed HPCS scheme is achieved. The specific cases of anti-synchronization, complete synchronization, and chaos control problem are further discussed. Simulation outcomes through MATLAB environment indicate that the proposed active controllers are effective in controlling the chaotic behaviour of the GLV system to desired set points which shows the effectiveness of the proposed HPCS technique. Significantly, the analytical approach and the numerical outcomes both agree excellently. Even though our discussed technique is simple yet it has numerous applications in encryption, control theory and secure communication. While investigating it is noted that the time taken by synchronized errors in converging to zero as time tending to infinity is less in comparing with other related published work. The proposed active controller designing can be explored to the GLV model interrupted by system uncertainties and external disturbances as a future work.

REFERENCES

Antoniou, P. and Pitsillides, A. (2010). A bio-inspired approach for streaming applications in wireless sensor networks based on the lotka–volterra competition model, Computer Communications, Vol. 33, No. 17, pp. 2039–2047.

Arneodo, A., Coullet, P. and Tresser, C. (1980). Occurence of strange attractors in three-dimensional volterra equations, Physics Letters A, Vol. 79, No. 4, pp. 259–263.

Bai, E.-W. and Lonngren, K.-E. (1997). Synchronization of two lorenz systems using active control, Chaos, Solitons & Fractals, Vol. 8, No. 1, pp. 51–58.

Chen, M. and Han, Z. (2003). Controlling and synchronizing chaotic genesio system via nonlinear feedback control, Chaos, Solitons & Fractals, Vol. 17, No. 4, pp. 709–716.

Delavari, H. and Mohadeszadeh, M. (2018). Hybrid complex projective synchronization of complex chaotic systems using active control technique with nonlinearity in the control input, Journal of Control Engineering and Applied Informatics, Vol. 20, No. 1, pp. 67–74.

Ding, Z. and Shen, Y. (2016). Projective synchronization of nonidentical fractional-order neural networks based on sliding mode controller, Neural Networks, Vol. 76, pp. 97–105.

Dongmo, E. D., Ojo, K. S., Woafo, P. and Njah, A. N. (2018). Difference synchronization of identical and nonidentical chaotic and hyperchaotic systems of different orders using active backstepping design, Journal of Computational and Nonlinear Dynamics, Vol. 13, No. 5.

El-Gohary, A. and Yassen, M. T. (2001). Optimal control and synchronization of lotka–volterra model, Chaos, Solitons & Fractals, Vol. 12, No. 11, pp. 2087–2093.

Gatabazi, P., Mba, J. C. and Pindza, E. (2019). Modeling cryptocurrencies transaction counts using variable-order fractional grey Lotka-Volterra dynamical system, Chaos, Solitons & Fractals, Vol. 127, pp. 283–290.

Gatabazi, P., Mba, J. C., Pindza, E. and Labuschagne, C. (2019). Grey Lotka–Volterra models with application to cryptocurrencies adoption, Chaos, Solitons & Fractals, Vol. 122, pp. 47–57.

Gavin, C., Pokrovskii, A., Prentice, M. and Sobolev, V. (2006). Dynamics of a Lotka-Volterra type model with applications to marine phage population dynamics, Journal of Physics: Conference Series, Vol. 55, p. 80.

Goel, N. S., Maitra, S. C. and Montroll, E. W. (1971). On the Volterra and other nonlinear models of interacting populations, Reviews of Modern Physics, Vol. 43, No. 2, p. 231.

Hening, A. and Nguyen, D. H. (2018). Stochastic Lotka–Volterra food chains, Journal of Mathematical Biology, Vol. 77, No. 1, pp. 135–163.

Jahanzaib, L. S., Trikha, P., Chaudhary, H., Haider, S. M., et al. (2020). Compound synchronization using disturbance observer based adaptive sliding mode control technique, J. Math. Comput. Sci., Vol. 10, No. 5, pp. 1463–1480.

Khan, A. and Bhat, M. A. (2017a). Hyper-chaotic analysis and adaptive multiswitching synchronization of a novel asymmetric non-linear dynamical system, International Journal of Dynamics and Control, Vol. 5, No. 4, pp. 1211–1221.

Khan, A. and Bhat, M. A. (2017b). Hyperchaotic analysis and adaptive projective synchronization of nonlinear dynamical system, Computational Mathematics and Modeling, Vol. 28, No. 4, 517–530.

Khan, A. and Chaudhary, H. (2020a). Adaptive control and hybrid projective combination synchronization of chaos generated by generalized lotka-volterra biological systems, Bloomsbury India, p. 174.

Khan, A. and Chaudhary, H. (2020b). Hybrid projective combination–combination synchronization in non-identical hyperchaotic systems using adaptive control, Arabian Journal of Mathematics, p. 1–15.

Khan, A. and Tyagi, A. (2017a). Analysis and hyper-chaos control of a new 4-d hyperchaotic system by using optimal and adaptive control design, International Journal of Dynamics and Control, Vol. 5, No. 4, 1147–1155.

Khan, A. and Tyagi, A. (2017b). Hybrid projective synchronization between two identical new

4-d hyper-chaotic systems via active control method, Int. J. Nonlinear. Sci., Vol. 23, No. 3, 142–150.

Khan, T. and Chaudhary, H. (2020c). Estimation and identifiability of parameters for generalized lotka-volterra biological systems using adaptive controlled combination difference anti-synchronization.

Li, C. and Liao, X. (2004). Complete and lag synchronization of hyperchaotic systems using small impulses, Chaos, Solitons & Fractals, Vol. 22, No. 4, pp. 857–867.

Li, D. and Zhang, X. (2016). Impulsive synchronization of fractional order chaotic systems with time-delay, Neurocomputing, Vol. 216, pp. 39–44.

Li, G. H. (2007). Modified projective synchronization of chaotic system, Chaos, Solitons & Fractals, Vol. 32, No. 5, pp. 1786–1790.

Li, G.-H. and Zhou, S.-P. (2007). Anti-synchronization in different chaotic systems, Chaos, Solitons & Fractals, Vol. 32, No. 2, pp. 516–520.

Li, S.-Y., Yang, C.-H., Lin, C.-T., Ko, L.-W. and Chiu, T.-T. (2012). Adaptive synchronization of chaotic systems with unknown parameters via new backstepping strategy, Nonlinear Dynamics, Vol. 70, No. 3, pp. 2129–2143.

Li, Z. and Xu, D. (2004). A secure communication scheme using projective chaos synchronization, Chaos, Solitons & Fractals, Vol. 22, No. 2, pp. 477–481.

Liao, T.-L. and Tsai, S.-H. (2000). Adaptive synchronization of chaotic systems and its application to secure communications, Chaos, Solitons & Fractals, Vol. 11, No. 9, pp. 1387–1396.

Lotka, A. J. (1926). Elements of physical biology, Science Progress in the Twentieth Century (1919-1933), Vol. 21, No. 82, pp. 341–343.

Ma, J., Mi, L., Zhou, P., Xu, Y. and Hayat, T. (2017). Phase synchronization between two neurons induced by coupling of electromagnetic field, Applied Mathematics and Computation, Vol. 307, pp. 321–328.

Mainieri, R. and Rehacek, J. (1999). Projective synchronization in three-dimensional chaotic systems, Physical Review Letters, Vol. 82, No. 15, pp. 3042.

Nag, S. (2020). A mathematical model in the time of COVID-19.

Pecora, G. M. and Carroll, L. M. (1990). Synchronization in chaotic systems, Physical Review Letters, Vol. 64, No. 8, p. 821.

Perhar, G., Kelly, N.E., Ni, F. J., Simpson, M. J., Simpson, A. J. and Arhonditsis, G. B. (2016). Using daphnia physiology to drive food web dynamics: A theoretical revisit of Lotka-Volterra models, Ecological Informatics, Vol. 35, pp. 29–42.

Rasappan, S. and Vaidyanathan, S. (2012). Synchronization of hyperchaotic liu system via backstepping control with recursive feedback, International Conference on Eco-friendly Computing and Communication Systems, pp. 212–221.

Reichenbach, T., Mobilia, M. and Frey, E. (2006). Coexistence versus extinction in the stochastic cyclic Lotka-Volterra model, Physical Review E, Vol. 74, No. 5, p. 051907.

Runzi, L., Yinglan, W. and Shucheng, D. (2011). Combination synchronization of three classic chaotic systems using active backstepping design, Chaos: An Interdisciplinary Journal of Nonlinear Science, Vol. 21, No. 4, 043114.

Runzi, L. and Yinglan, W. (2012). Finite-time stochastic combination synchronization of three different chaotic systems and its application in secure communication, Chaos: An Interdisciplinary Journal of Nonlinear Science, Vol. 22, No. 2, 023109.

Samardzija, N. and Greller, L. D. (1988). Explosive route to chaos through a fractal torus in a generalized Lotka-Volterra model, Bulletin of Mathematical Biology, Vol. 50, No. 5, pp. 465–491.

Scudo, F. M. (1971). Vito Volterra and theoretical ecology, Theoretical Population Biology, Vol. **2**, No. 1, pp. 1–23.

Silva-Dias, L. and Löpez-Castillo, A. (2018). Spontaneous symmetry breaking of population: Stochastic Lotka–Volterra model for competition among two similar preys and predators, Mathematical Biosciences, Vol. 300, pp. 36–46.

Singh, A. K., Vijay, Yadav, K. and Das, S. (2017). Synchronization between fractional order complex chaotic systems, International Journal of Dynamics and Control, Vol. 5, No. 3, pp. 756–770.

Sudheer, K. S. and Sabir, M. (2009). Hybrid synchronization of hyperchaotic lu system, Pramana, Vol. 73, No. 4, p. 781.

Tonnang, H. E. Z., Nedorezov, L. V., Ochanda, H., Owino, J. and Löhr, B. (2009). Assessing the impact of biological control of plutella xylostella through the application of Lotka–Volterra model, Ecological Modelling, Vol. 220, No. 1, pp. 60–70.

Tsai, B.-H., Chang, C.-J., and Chang, C.-H. (2016). Elucidating the consumption and CO2 emissions of fossil fuels and low-carbon energy in the United States using Lotka–Volterra models, Energy, Vol. 100, pp. 416–424.

Vaidyanathan, S. (2015). Adaptive biological control of generalized Lotka-Volterra three-species biological system, International Journal of PharmTech Research, Vol. 8, No. 4, pp. 622–631.

Vaidyanathan, S. (2016). Hybrid synchronization of the generalized Lotka-Volterra three-species biological systems via adaptive control, Int. J. PharmTech. Res., Vol. 9, No. 1, pp. 179–192.

Vaidyanathan, S. and Sampath, S. (2012). Anti-synchronization of four-wing chaotic systems via sliding mode control, Int. J. Autom. Comput., Vol. 9, No. 3, pp. 274–279.

Wu, Z. and Fu, X. (2013). Combination synchronization of three different order nonlinear systems using active backstepping design, Nonlinear Dynamics, Vol. 73, No. 3, pp. 1863–1872.

Wu, Z., Duan, J. and Fu, X. (2012). Complex projective synchronization in coupled chaotic complex dynamical systems, Nonlinear Dynamics, Vol. 69, No. 3, pp. 771–779.

Xiong, J., Li, X., and Wang, H. (2019). The survival analysis of a stochastic Lotka-Volterra competition model with a coexistence equilibrium, Mathematical Biosciences, Vol. 1, (k1k2):1–k1k2.

Yadav, V. K., Prasad, G., Srivastava, M. and Das, S. (2019). Triple compound synchronization among eight chaotic systems with external disturbances via nonlinear approach, Differ Equ Dyn Syst, pp. 1–24.

Yadav, V. K., Shukla, V. K. and Das, S. (2019). Difference synchronization among three chaotic systems with exponential term and its chaos control, Chaos, Solitons & Fractals, Vol. 124, pp. 36–51.

Yassen, M. T. (2003). Adaptive control and synchronization of a modified chua's circuit system, Appl. Math. Comput., Vol. 135, No. 1, pp. 113–128.

Zhou, P. and Zhu, W. (2011). Function projective synchronization for fractional-order chaotic systems, Nonlinear Analysis: Real World Applications, Vol. 12, No. 2, pp. 811–816.