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New Results for Compatible Mappings of Type A And Subsequential Continuous Mappings

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Abstract

In this paper, we corroborated some common fixed point theorems for two pairs of self mappings by using the impression of compatibility of type A and subsequential continuity (alternatively subcompatiblity and reciprocal continuity) in multiplicative metric spaces (MMS). The proven results are the improved version in a manner that the completeness, closedness and continuity of the mappings are relaxed.

Keywords: Fixed point; Multiplicative metric space (MMS); Compatible mappings of type (A); Subsequential continuous maps; Reciprocal continuity; Sub-compatible maps

MSC 2010 No.: 47H10, 54H25

1. Introduction

In the year 2008, Bashirov et al. brought up multiplicative calculus into the attention of researchers involved in the field of analysis and introduced the concept of multiplicative metric space (MMS).

In 2011, Bashirov et al. in their work on modeling with multiplicative differential equations claimed that multiplicative differential equations are more appropriate tool than ODE's while investigating and analyzing some of the problems in various fields of engineering applications.

Ozavsar and Cevikel (2012) gave some fixed point theorems by using the concept of multiplicative contractive mappings in multiplicative metric spaces. Working on the same line, He et al. (2014) came out with some interesting fixed point results for four self-maps in multiplicative metric spaces.

Kang et al. (2015) gave some common fixed point theorems in multiplicative metric spaces by using the concepts of compatible mappings and compatible mappings of types (A) and (B).

Bouhadjera and Thobie (2009) proved some common fixed point theorems for the pairs of mapping using the notion of sub-compatible and sub sequential maps which are weaker than that of occasionally weakly compatible maps introduced by Thagafi and Shahzad (2008) and reciprocal continuity by Pant (1999) respectively.

Afrah (2016) took under consideration compatible mappings of types (A) and (B) along with continuous and closedness of mappings to establish fixed point theorems in a multiplicative metric space. For more details, we refer to Sessa (1982), Jungck and Rhoades (1998), Kang et al. (2014), Kumar et al. (2014), Kumar et al. (2015), Khan et al. (2018), Sharma and Thakur (2019).

2. Preliminaries

In 2008, Bashirov et al. initiated the new type of space called multiplicative metric space (MMS) by replacing usual triangle inequality with multiplicative triangle inequality.

Bouhadjera (2016) introduced the concept of sub-sequential mappings in metric spaces. We redefine it in the framework of a multiplicative metric space and used it in the claimed result.

Jungck (1986) introduced the concept of compatible and compatible of Type A mappings in metric spaces. We'll use these in the framework of a multiplicative metric space (MMS).

Pant (1999) gave a new continuity condition called reciprocal continuity in metric spaces whereas Bouhadjera and Thobie (2009) introduced the concept of sub-compatible maps in metric spaces. We'll use these in the setting of a multiplicative metric spaces (MMS).

3. Main Results

Theorem 3.1.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are sub sequentially continuous and compatible of type (A). Then,
(i) C and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[\phi[max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Fy)}{1 + d(Ax, Dy)}, \frac{d(Ax, Fy)d(Dy, Ex)}{1 + d(Ax, Dy)}\}]\right]^{\lambda}, \quad (1)$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A, D, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} AEx_n = Ap, \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility of type (A) implies that

$$\lim_{n\to\infty} d(EAx_n, AAx_n) = d(Ep, Ap) = 1,$$
$$\lim_{n\to\infty} d(AEx_n, EEx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$, whereas in respect of the pair (D, F) being subsequentially continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X$$
$$\lim_{n\to\infty} DFy_n = Ds, \quad \lim_{n\to\infty} FDy_n = Fs.$$

Also, the pair (D, F) is compatible of type (A), therefore,

$$\lim_{n\to\infty} d(FDy_n, DDy_n) = d(Fs, Ds) = 1,$$
$$\lim_{n\to\infty} d(DFy_n, FFy_n) = d(Ds, Fs) = 1.$$

Therefore, $Ds = Fs$. Hence, p is a coincidence point of the pair (A, E) whereas s is a coincidence point of the pair (D, F).

Now, we prove that $p = s$. By putting $x = x_n$ and $y = y_n$ in inequality (1), we have

$$d(Ex_n, Fy_n) \\ \leq \left[\phi[max\{d(Ax_n, Dy_n), \frac{d(Ax_n, Ex_n)d(Dy_n, Fy_n)}{1 + d(Ax_n, Dy_n)}, \frac{d(Ax_n, Fy_n)d(Dy_n, Ex_n)}{1 + d(Ax_n, Dy_n)}\}]\right]^{\lambda}.$$

Taking the limit as $n \to \infty$, we get

$$d(p, s) \leq \left[\phi[max\{d(p, s), \frac{d(p, p)d(s, s)}{1 + d(p, s)}, \frac{d(p, s)d(s, p)}{1 + d(p, s)}\}]\right]^{\lambda} \\ = [\phi[d(p, s)]]^{\lambda} \\ \leq [d(p, s)]^{\lambda}.$$

We get $d(p, s) = 1$ and so $p = s$.

Next we will show that $Ep = p$. By putting $x = p$ and $y = y_n$ in the inequality (1), we get

$$d(Ep, Fy_n) \\ \leq \left[\phi[max\{d(Ap, Dy_n), \frac{d(Ap, Ep)d(Dy_n, Fy_n)}{1 + d(Ap, Dy_n)}. \frac{d(Ap, Fy_n)d(Dy_n, Ep)}{1 + d(Ap, Dy_n)}\}]\right]^{\lambda}.$$

Taking the limit as $n \to \infty$, we get

$$d(Ep, p) \\ \leq \left[\phi[max\{d(Ep, p), \frac{d(Ep, Ep)d(p, p)}{1 + d(Ep, p)}, \frac{(Ep, p)d(p, Ep)}{1 + d(Ep, p)}\}]\right]^{\lambda} \\ = [\phi[d(Ep, p)]]^{\lambda} \leq [d(Ep, p)]^{\lambda}.$$

This gives, $d(Ep, p) = 1$ and so $Ep = p$. Therefore, $Ap = Ep = p$. Hence, p is a common fixed point of (A, E).

Now, we show that p is a common fixed point of (D, F). By putting $x = \{x_n\}$ and $y = p$ in the inequality (1), we get

$$d(Ex_n, Fp) \\ \leq \left[\phi[max\{d(Ax_n, Dp), \frac{d(Ax_n, Ex_n)d(Dp, Fp)}{1 + d(Ax_n, Dp)}, \frac{d(Ax_n, Fp)d(Dp, Ex_n)}{1 + d(Ax_n, Dp)}\}]\right]^{\lambda}.$$

Taking the limit as $n \to \infty$, we get

$$d(p, Fp) \\ \leq \left[\phi[max\{d(p, Fp), \frac{d(p, p)d(Fp, Fp)}{1 + d(p, Fp)}, \frac{d(p, Fp)d(Fp, p)}{1 + d(p, Fp)}\}]\right]^{\lambda} \\ = [\phi[d(p, Fp)]]^{\lambda} \leq [d(p, Fp)]^{\lambda},$$

which implies $d(p, Fp) = 1$ and so $Fp = p$. This gives $Dp = Fp = p$ and p is a common fixed point of (D, F). Hence, $p = Ap = Dp = Ep = Fp$, i.e., p is a common fixed point of A, D, E and F.

For uniqueness, let p and s be two common fixed points of A, D, E and F. Then, by using (1) we have

$$\begin{aligned} d(Ep, Fs) &= d(p, s)) \\ &\leq \left[\phi[max\{d(Ap, Ds), \frac{d(Ap, Ep)d(Ds, Fs)}{1 + d(Ap, Ds)}, \frac{d(Ap, Fs)d(Ds, Ep)}{1 + d(Ap, Ds)}\}]\right]^{\lambda} \\ &= [\phi(d(p, s))]^{\lambda} \leq [d(p, s)]^{\lambda}. \end{aligned}$$

This implies that $p = s$. Therefore, p is a unique common fixed point of A, D, E and F. This completes the proof. ∎

If we put $A = D$ in Theorem 3.1, we have the following corollary for three mappings.

Corollary 3.2.

Let A, E and F be three self mappings of a multiplicative metric space (X). If the pairs (A, S) and (A, T) are sub sequentially continuous and compatible of type (A), then
(i) A and E have a coincidence point
(ii) A and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$\begin{aligned} & d(Ex, Fy) \\ & \leq \left[\phi[max\{d(Ax, Ay), \frac{d(Ax, Ex)d(Ay, Fy)}{1 + d(Ax, Ay)}, \frac{d(Ax, Fy)d(Ay, Ex)}{1 + d(Ax, Ay)}\}]\right]^{\lambda}, \end{aligned} \tag{2}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} AEx_n = Ap, \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility of type (A) implies that

$$\lim_{n\to\infty} d(EAx_n, AAx_n) = d(Ep, Ap) = 1,$$
$$\lim_{n\to\infty} d(AEx_n, EEx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$, whereas in respect of the pair (A, F) being subsequentially continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X,$$
$$\lim_{n\to\infty} AFy_n = As, \quad \lim_{n\to\infty} FAy_n = Fs.$$

Also, the pair (A, F) is compatible of type (A), therefore

$$\lim_{n\to\infty} d(FAy_n, AAy_n) = d(Fs, As) = 1,$$
$$\lim_{n\to\infty} d(AFy_n, FFy_n) = d(As, Fs) = 1.$$

Therefore, $As = Fs$.

Hence, p is a coincidence point of the pair (A, E), whereas s is a coincidence point of the pair (A, F). The rest of the proof can be completed on the lines of Theorem 3.1. ■

Alternatively, if we set $E = F$ in Theorem 3.1, we have the following corollary for other three self mappings.

Corollary 3.3.

Let A, D and E be three self mappings of a multiplicative metric space (X). If the pairs (A, S) and (B, S) are sub sequentially continuous and compatible of type (A), then
(i) A and E have a coincidence point
(ii) D and E have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Ey) \leq \left[\phi[max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Ey)}{1 + d(Ax, Dy)}, \frac{d(Ax, Ey)d(Dy, Ex)}{1 + d(Ax, Dy)}\}]\right]^{\lambda}, \tag{3}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A, D and E have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$

this gives

$$\lim_{n\to\infty} AEx_n = Ap, \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility of type (A) implies that

$$\lim_{n\to\infty} d(EAx_n, AAx_n) = d(Ep, Ap) = 1,$$
$$\lim_{n\to\infty} d(AEx_n, EEx_n) = d(Ap, Ep) = 1.$$

This shows that $Ap = Ep$, whereas in respect of the pair (D, E) being subsequentially continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Ey_n = s \text{ for some } s \in X,$$
$$\lim_{n\to\infty} DEy_n = Ds \quad \text{and} \quad \lim_{n\to\infty} EDy_n = Es.$$

Also, the pair (D, E) is compatible of type (A), therefore

$$\lim_{n\to\infty} d(EDy_n, DDy_n) = d(Es, Ds) = 1$$
$$\lim_{n\to\infty} d(DEy_n, EEy_n) = d(Ds, Es) = 1.$$

This gives $Ds = Es$.

Hence, p is a coincidence point of the pair (A, E) whereas s is a coincidence point of the pair (D, E). The rest of the proof can be completed on the lines of Theorem 3.1. ∎

If we put $E = F$ in corollary 3.2, we have the following result for two self mappings.

Corollary 3.4.

Let A and E be two self mappings of a multiplicative metric space (X). If the pair (A, E) is sub sequentially continuous and compatible of type (A), then A and E have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Ey) \leq \left[\phi[max\{d(Ax, Ay), \frac{d(Ax, Ex)d(Ay, Ey)}{1 + d(Ax, Ay)}, \frac{d(Ax, Ey)d(Ay, Ex)}{1 + d(Ax, Ay)}\}]\right]^{\lambda}, \tag{4}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A and E have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} AEx_n = Ap \quad \text{and} \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility of type (A) implies that

$$\lim_{n\to\infty} d(EAx_n, AAx_n) = d(Ep, Ap) = 1,$$
$$\lim_{n\to\infty} d(AEx_n, EEx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$. Hence, p is a coincidence point of the pair (A, E). The rest of the proof can be completed on the lines of Theorem 3.1. ■

Theorem 3.5.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are sub compatible and reciprocally continuous, then
(i) A and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[\phi[max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Fy)}{1 + d(Ax, Dy)}, \frac{d(Ax, Fy)d(By, Ex)}{1 + d(Ax, Dy)}\}]\right]^{\lambda}, \quad (5)$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A, D, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subcompatible and reciprocally continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} d(AEx_n, EAx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$, whereas in respect of the pair (D, F) being subcompatible and reciprocally continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X,$$
$$\lim_{n\to\infty} d(DFy_n, FDy_n) = d(Ds, Fs) = 1.$$

This shows that $Ds = Fs$. Hence, p is a coincidence point of the pair (A, E) whereas s is a coincidence point of the pair (D, F). The rest of the proof can be completed on the lines of Theorem 3.1. ■

Remark 3.6.

The results similar to Corollaries 3.2, 3.3 and 3.4 can also be obtained in the respect of Theorem 3.5.

Theorem 3.7.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are compatible and subsequentially continuous, then
(i) A and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[\phi[max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Fy)}{1 + d(Ax, Dy)}, \frac{d(Ax, Fy)d(Dy, Ex)}{1 + d(Ax, Dy)}\}]\right]^{\lambda}, \quad (6)$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and monotonic increasing function such that $\phi(0) < t$ for all $t > 0$. Then, A, D, E and F have a unique common fixed point in X.

Proof:

The proof can be completed on the lines of Theorem 3.1. ■

Remark 3.8.

The results similar to Corollaries 3.2, 3.3 and 3.4 can also be obtained in the respect of Theorem 3.7.

The following result can be derived from Theorem 3.1. on setting $\phi(t) = kt$

Corollary 3.9.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are sub sequentially continuous and compatible of type (A), then
(i) A and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[k\, max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Ty)}{1 + d(Ax, Dy)}, \frac{(Ax, Fy)d(Dy, Ex)}{1 + d(Ax, Dy)}\}\right]^{\lambda}, \quad (7)$$

then A, D, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} AEx_n = Ap \quad \text{and} \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility of type (A) implies that

$$\lim_{n\to\infty} d(EAx_n, AAx_n) = d(Ep, Ap) = 1$$
$$\lim_{n\to\infty} d(AEx_n, EEx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$, whereas in respect of the pair (D, F) being subsequentially continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X,$$
$$\lim_{n\to\infty} DFy_n = Ds, \quad \lim_{n\to\infty} FDy_n = Fs.$$

Also, the pair (D, F) is compatible of type (A), therefore

$$\lim_{n\to\infty} d(FDy_n, DDy_n) = d(Fs, Ds) = 1,$$
$$\lim_{n\to\infty} d(DFy_n, FFy_n) = d(Ds, Fs) = 1.$$

This shows that $Ds = Fs$.

Hence, p is a coincidence point of the pair (A, E) whereas s is a coincidence point of the pair (D, F). The rest of the proof can be completed on the lines of Theorem 3.1. ■

We can derive the following result if we take $\phi(t) = k(t)$ in Theorem 3.5.

Corollary 3.10.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are subcompatible and reciprocally continuous, then
(i) A and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[k\, max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Fy)}{1 + d(Ax, Dy)}, \frac{d(Ax, Fy)d(Dy, Ex)}{1 + d(Ax, Dy)}\}\right]^{\lambda}, \quad (8)$$

then A, D, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subcompatible and reciprocally continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} d(AEx_n, EAx_n) = d(Ap, Ep) = 1.$$

Therefore, $Ap = Ep$, whereas in respect of the pair (D, F) being subcompatible and reciprocally continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X$$
$$\lim_{n\to\infty} d(DFy_n, FDy_n) = d(Ds, Fs) = 1.$$

This shows that $Ds = Fs$. Hence, p is a coincidence point of the pair (A, E) whereas s is a coincidence point of the pair (D, F). The rest of the proof can be completed on the lines of Theorem 3.5. ∎

We'll get the following result if we take $\phi(t) = k(t)$ in Theorem 3.7.

Corollary 3.11.

Let A, D, E and F be four self mappings of a multiplicative metric space (X). If the pairs (A, E) and (D, F) are compatible and subsequentially continuous, then
(i) A and E have a coincidence point
(ii) D and F have a coincidence point.

Further, if there exists a real constant $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Ex, Fy) \leq \left[k\, max\{d(Ax, Dy), \frac{d(Ax, Ex)d(Dy, Fy)}{1 + d(Ax, Dy)}, \frac{d(Ax, Fy)d(Dy, Ex)}{1 + d(Ax, Dy)}\}\right]^{\lambda}, \quad (9)$$

Then, A, D, E and F have a unique common fixed point in X.

Proof:

Since the pair (A, E) is subsequentially continuous, therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ex_n = p \text{ for some } p \in X,$$
$$\lim_{n\to\infty} AEx_n = Ap, \quad \lim_{n\to\infty} EAx_n = Ep.$$

The compatibility implies that

$$\lim_{n\to\infty} d(AEx_n, EAx_n) = d(Ap, Ep) = 1.$$

This gives $Ap = Ep$, whereas in respect of the pair (D, F) being subsequentially continuous, there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Dy_n = \lim_{n\to\infty} Fy_n = s \text{ for some } s \in X,$$
$$\lim_{n\to\infty} DFy_n = Ds, \quad \lim_{n\to\infty} FDy_n = Fs.$$

Also, the pair (D, F) is compatible, implies

$$\lim_{n\to\infty} d(FDy_n, DFy_n) = d(Fs, Ds) = 1.$$

Therefore, $Ds = Fs$. Hence, p is a coincidence point of the pair (A, E), whereas s is a coincidence point of the pair (D, F). The rest of the proof can be completed on the lines of Theorem 3.7. ∎

If we set $k = 1, E = F$, and $A = D = I$ (the identity mapping on X) in the corollaries of Theorems (3.1, 3.5, 3.7) with completeness as an additional condition, then we'll have the Banach fixed point theorem in a complete multiplicative metric space (X, d) as follows:

Corollary 3.12.

Let E be a mapping of a complete multiplicative metric space (X, d) into itself satisfying the following condition:

$$d(Ex, Ey) \leq [d(x, y)]^{\lambda} \text{ for all } x, y \in X, \text{ where } \lambda \in [0, 1).$$

Then, E has a unique common fixed point in X.

Proof:

The proof is given in Ozavsar and Cevikel (2012). ∎

4. Conclusion

In this article, we presented some common fixed point theorems for two pairs of self mappings with compatibility of type A and sub sequential continuity (alternatively sub compatibility and reciprocal continuity) in multiplicative metric spaces. We have also derived some corollaries from the established results and deduced Banach fixed point theorem in a complete multiplicative metric space. Based on the results in this paper, interesting future research may be prospective. In the future study, one can establish the integral version of fixed point theorem in the multiplicative sense and can also think of establishing some new fixed point results in multiplicative metric spaces using $C-$ class functions. The work presented here is likely to provide a ground to the researchers to do work in different structures by using these conditions. Moreover, the technique used in this paper is suggestive to discuss the related problem in the general case.

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