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Variants of Meir-Keeler Fixed Point Theorem And Applications of Soft Set-Valued Maps

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Abstract

In this paper, we prove a Meir-Keeler type common fixed point theorem for two mappings for which the range set of the first one is a family of soft sets, called soft set-valued map and the second is a point-to-point mapping. In addition, it is also shown that under some suitable conditions, a soft set-valued map admits a selection having a unique fixed point. In support of the obtained result, nontrivial examples are provided. The novelty of the presented idea herein is that it extends the Meir-Keeler fixed point theorem and the theory of selections for multivalued mappings from the case of crisp mappings to the frame of soft set-valued maps. Finally, an application of soft set-valued maps in decision making problems is considered.

Keywords: E-compatible; E-sequence; E-soft fixed point; Fixed point; Soft set-valued map; Selection

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1. Introduction

Fixed point theory is a highly significant tool in the study of nonlinear analysis as it is considered a pivotal link between pure and applied mathematics with enormous applications in social, physical, and life sciences such as economics, physics, biology, and in several branches of engineering (for example, see Caballero et al. (2010), Chern and Huang (2010), Huang et al. (2011), Jitpeera and Kumam (2011), Mohammed and Azam (2019c)). In the field of engineering, there are many problems in complex adaptive systems where convergence and stability analysis are prime issues. In this direction, different case studies having engineering applications can be described by contraction mappings including their fixed point iterations, such as linear and nonlinear filters, image restoration, and in several areas where this theory helps to comprehend the phenomena.

Banach contraction principle (or Banach fixed point theorem) (Banach (1922)) is the first most applicable result in metric fixed point theory. Following Banach (1922), more than a handful generalizations of this result are available in the literature. For example, see Agarwal et al. (2015), Kadelburg et al. (2016), Kadelburg and Radenović (2011), Ljubomir (2003), Mohammed et al. (2019). The interested reader is also referred to Rhoades (1977) or Taskovic (1978) for varying definitions of contractive type mappings.

Meir and Keeler (1969) (M-K) introduced the notion of weakly uniformly strict contraction and established a fixed point theorem which is one of the notable improvements of Banach fixed point theorem. The publication of M-K fixed point theorem motivated many researchers, consequently, a number of extensions followed. See, for example, Mirjana and Stojan (2019), Mitrovic and Radenovic (2019) and the references therein.

It is well-known that every weakly uniformly strict contraction is continuous. Rhoades et al. (1990) removed this restriction, and by using the idea of compatibility of two maps, they established a fixed point theorem which includes the M-K fixed point theorem as a special result. Lim (2001) defined the notion of an L-function and characterized the weakly uniformly strict contraction. Thereafter, Suzuki (2006) introduced the idea of asymptotic contraction of Meir-Keeler type by utilizing the characterization of weakly uniformly strict contraction proved in Lim (2001) and hence presented a fixed point theorem which generalized the M-K fixed point theorem. Samet (2010) proved some coupled fixed point theorems under a generalized weakly uniformly strict contraction conditions in partially ordered metric spaces.

Later on, similar significant extensions of M-K fixed point theorem in ordered metric spaces have been emerging in the literature (e.g., Abdeljawad et al. (2012), Harjani et al. (2011, Karapínar et al. (2013a), Popa and Patriciu (2017))). Recently, Kanwal and Azam (2018) proved some common fixed point theorems for intuitionistic fuzzy maps in the setting of (α, β)-cut sets of intuitionistic fuzzy sets on a complete metric space in connection with the Hausdorff metric. They (Kanwal and Azam (2018)) further applied the technique of weakly uniformly strict contraction to establish common fixed point of intutionistic fuzzy compatible maps.

Along the line, the arena of applied mathematics witnessed tremendous developments as a result of the introduction of soft set theory by Molodtsov (1999). The method of handling problems in classical mathematics is in the opposite of the technique of soft set theory. In conventional mathematics, to describe any system or object, we first construct its mathematical model and then attempt to obtain the exact solution. If the exact solution is too complicated, then we define the notion of approximate solution. On the other hand, in soft set theory, the initial description of an object takes an approximate nature with no restriction, and the notion of exact solution is not essential. In other words, to describe an object in soft set theory, any convenient parametrization tools which may be words, sentences, numbers, mappings, functions, to mention a few, may be used; thereby, making the theory more easier and flexible in terms of applications in every day life. Molodtsov (1999), highlighted several directions for the applications of soft sets, such as smoothness of functions, game theory, Riemann-integration, operation research, probability and so on. At present, a lot of work is going on rapidly in the area of soft set theory (see, for instance Mohammed (2020d), Mohammed and Azam (2020) and the references therein).

Recently, Mohammed and Azam (2019a), (2019b)) introduced the concept of soft set-valued map, that is, a map whose range set lies in a family of soft sets, and thus established fixed point theorems of Nadler's and Edelstein's type. On the other hand, Repovs and Semenov (2013) provided a detailed study of the theory of continuous selections for multivalued mappings. They noted that this subdivision of modern topology was initiated by Michael (1956) and, thereafter, the notion has attracted keen interests with diverse applications outside the mainstream of topology, for example, control theory, differential inclusions, economics, fixed point theory, and so on. Most fundamental is to establish existence conditions for selections, under different regularity assumptions, such as measurability and Lipschitz-continuity.

In this paper, our aim is twofold. First, utilizing the notion of soft set-valued maps presented in Mohammed and Azam (2019a), a common E-soft fixed point theorem for two compatible maps is established by employing the technique of weakly uniformly strict contraction of Meir and Keeler (1969). Secondly, we establish a theorem in which under suitable conditions, a soft set-valued map admits a selection having a unique fixed point. Moreover, examples are furnished to support the hypotheses of our results. Finally, we also consider an application of soft set-valued maps in decision making problems to show possible usage of the concepts of soft set-valued maps. In a nutshell, the ideas discussed herein are extensions of fixed point theorem due to Meir and Keeler (1969) and the concepts of selections for multi-valued mappings.

2. Preliminaries

In this section, we recall specific notations, definitions and results which are needed in the sequel. Most of these preliminaries are recorded from Azam and Beg (2009), Heilpern (1981), Mohammed and Azam (2019a), and Molodtsov (1999). Throughout this paper, $\mathbb{N}$ and $\mathbb{R}$ represent the sets of natural and real numbers, respectively. Let X be a reference set and E be the universe of discourse of all parameters related to the elements in X. In this case, each parameter is a word or sentence. The power set of X is denoted by $P(X)$. Molodtsov (1999) defined the notion of soft set in the following manner.

Definition 2.1.

A pair (F, A) is called a soft set over X, where $A \subseteq E$ and F is a set- valued mapping $F : A \longrightarrow P(X)$. In this way, a soft set over X is a parameterized family of subsets of X.

Example 2.2.

Suppose the soft (F, E) describes the structures of certain number of men. Let the reference set of all men be

$$X = \{x_1, x_2, x_3, x_4, x_5\}$$

and the universe of all parameters be represented by

$$E = \{e_1, e_2, e_3, e_4\} = \{\text{fat}, \text{tall}, \text{muscular}, \text{lanky}\}.$$

In this case, to define a soft set means to point out fat men, tall men, muscular men, and lanky men. Thus, we may define $F : E \longrightarrow P(X)$ by $F(e_1) = \{x_1, x_2, x_5\}$, $F(e_2) = \{x_2, x_4, x_5\}$, $F(e_3) = \{x_5\}$, $F(e_4) =$ empty. So, the soft set (F, E) is a family $\{F(e_i) : i = 1, 2, 3, 4\}$ of $P(X)$. In order to store a soft set in a computer, a table with binary entries is usually used. Table 1 represents the soft set (F, E) of Example 2.2. Notice that if $x_i \in F(e_j)$, then $x_{ij} = 1$, otherwise, $x_{ij} = 0$, where x_{ij} are the entries for ith row and jth column of the table.

Suppose that Mr. P who is a tug of war coach is interested in selecting two men to participate in the upcoming national tug of war competition on the basis of his choice parameters "fat," " muscular," and so on. According to the choice value shown in Table 1, Mr. P will choose x_5 and x_2.

Table 1. Tabular representation of the soft set in Example 2.2

X	e_1	e_2	e_3	e_4	Choice Value
x_1	1	0	0	0	1
x_2	1	1	0	0	2
x_3	0	0	0	0	0
x_4	0	1	0	0	1
x_5	1	1	1	0	3

Let (X, σ) be a metric space and $\mathcal{X}^*$ be the set of all nonempty closed and bounded subsets of X. Denote by $[P(X)]^E$, the family of soft sets over X. Then consider two soft sets (F, A) and (G, B), $(a, b) \in A \times B$. Assume that $F(a), G(b) \in \mathcal{X}^*$. For $\epsilon > 0$, define $N^\sigma(\epsilon, F(a))$, $S_{EX}^{(a,b)}(F, G)$ and $E^\sigma_{(F_a,G_b)}$, respectively, as follows:

$$N^\sigma(\epsilon, F(a)) = \{x \in X : \sigma(x, y) < \epsilon, \text{ for some } y \in F(a)\},$$

$$E^\sigma_{(F_a,G_b)} = \{\epsilon > 0 : F(a) \subseteq N^\sigma(\epsilon, G(b)), \quad G(b) \subseteq N^\sigma(\epsilon, F(a))\},$$

and

$$S_{EX}^{(a,b)}(F, G) = \inf E^\sigma_{(F_a,G_b)},$$

Define a distance function $S^\infty_{EX} : [P(X)]^E \times [P(X)]^E \longrightarrow \mathbb{R}$ by

$$S_{EX}^{\infty}(F,G) = \sup_{(a,b)\in\overline{A}\times\overline{B}} S_{EX}^{(a,b)}(F,G),$$

where

$$\overline{A}\times\overline{B} = \{(a,b)\in A\times B : F(a), G(b)\in CB(X)\}.$$

The completeness of (X,σ) implies that $(\mathcal{X}^*, S_{EX}^{\infty})$ is complete (see Aubin (1977)).

Definition 2.3.

A mapping $T : X \longrightarrow [P(X)]^E$ is called a soft set-valued map. A point $u \in X$ is called an e-soft fixed point of T if $u \in (Tu)(e)$, for some $e \in E$. This is also written as $u \in Tu$, for short. If $Dom\ Tx = E$ and $u \in (Tu)(e)$ for all $e \in E$, then u is said to be an E-soft fixed point of T. We shall denote the set of all E-soft fixed points of a soft set-valued map T by $E_{Fix(T)}$.

Notice that if $T : X \longrightarrow [P(X)]^E$ is a soft set-valued map, then (Tx, E) is a soft set over X, for all $x \in X$. Hereafter, if $T : X \longrightarrow [P(X)]^E$ is a soft set-valued map, then the set $(Tx)(e)$ shall be written as $(T_e x)$.

Example 2.4.

Let $X = [-0.6, 0.6] = E$. Define

$$T : X \longrightarrow [P(X)\setminus\emptyset]^{[-0.6,0.6]}$$

by

$$(T_e x) = \left[-\sqrt{1-x^2-e^2}, +\sqrt{1-x^2-e^2}\right],\ x^2+e^2 \leq 1.$$

Then T is a soft set-valued map. The three dimensional (3D) representation of the soft set-valued map in Example 2.4 is given by Figure 1.

Example 2.5.

Let $X = \{1,2,3\}$ and $E = \{1,2\}$. Define $T : X \longrightarrow [P(X)]^E$ as follows:

$$(T_e x) = \begin{cases} \{1,3\}, & \text{if } e = 1, \\ \{2,3\}, & \text{if } e = 2. \end{cases}$$

Then T is a soft set-valued map. Notice that $1 \in (T_e 1)$ for $e = 1$ and $2 \in (T_e 2)$ for $e = 2$; hence, 1 and 2 are e-soft fixed points of T. But, $2 \notin (T_e 2)$ and $1 \notin (T_e 1)$ for $e = 1$ and $e = 2$, respectively. If follows that 1 and 2 are not E-soft fixed points of T. On the other hand, $3 \in (T_e 3)$ for all $e \in E$; thus, the set of all E-soft fixed points of T is given by $E_{Fix(T)} = \{3\}$. The map T can be represented as in Figure 2. Notice that in Figure 2, the dots represent other subsets of X.

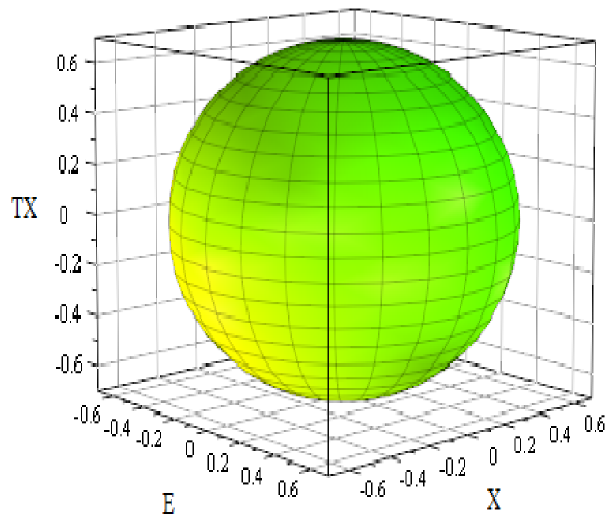

Figure 1. 3D Representation of the Soft Set-Valued Map in Example 2.4

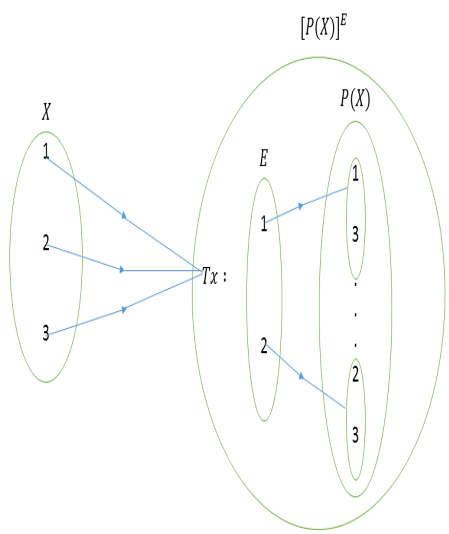

Figure 2. Graphical representation of the soft set-valued map in Example 2.5

3. Main Results

We start this section by presenting a few new definitions as follows.

Definition 3.1.

Let $T : X \longrightarrow [P(X)]^E$ be a soft set-valued map. A sequence $\{x_n\}_{n\in\mathbb{N}}$ of points of X is said to be an iterative E-sequence of T at x if and only if $x_n \in (T_e x_{n-1})$ for each $n \in \mathbb{N}$ and for some $e \in E$.

Definition 3.2.

A soft set-valued map $T : X \longrightarrow [P(X)]^E$ and a single-valued mapping $\Lambda : X \longrightarrow X$ are said to be E-compatible if whenever an iterative E-sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq X$ satisfies $\lim_{n\longrightarrow\infty} \Lambda x_n \in \lim_{n\longrightarrow\infty}(T_e x_n)$ (provided $\lim_{n\longrightarrow\infty} \Lambda x_n$ and $\lim_{n\longrightarrow\infty}(T_e x_n)$ exist in (X, σ) and $(\mathcal{X}^*, \mathrm{S}^{\infty}_{\mathrm{EX}})$, respectively and $(T_e\Lambda x_n) \in \mathcal{X}^*$), then

$$\lim_{n\longrightarrow\infty} S_{EX}^{(e(x_n),e(x_n))}(\Lambda(T_e x_n),(T_e\Lambda x_n)) = 0.$$

Theorem 3.3.

Let (X, σ) be a complete metric space and let $T : X \longrightarrow [P(X)]^E$, $\Lambda : X \longrightarrow X$ be E-compatible mappings. Suppose that for each $x \in X$, there exists $e \in E$ such that $(T_e x) \in \mathcal{X}^*$, $\bigcup_{x\in X}(T_e x) \subseteq \Lambda X$ and the following conditions hold:
for each $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\epsilon \leq \sigma(\Lambda x, \Lambda y) < \epsilon + \delta \text{ implies } \sigma(r, w) < \epsilon, \tag{1}$$

$$r \in (T_e x),\ w \in (T_e y),\ \text{and}, \tag{2}$$

$$(T_e x) = (T_e y) \text{ when } \Lambda x = \Lambda y.$$

If Λ is continuous, then there exists $\eta \in X$ such that $\Lambda\eta = \eta$ and $\eta \in (T_e\eta)$, for some $e \in E$.

Proof:

Let $x_0 \in X$, and consider the sequences $\{x_j\}_{j\in\mathbb{N}}$, $\{y_j\}_{j\in\mathbb{N}}$ in X and Ω_j in $\mathcal{X}^*$, $y_j = \Lambda x_j \in (T_e x_{j-1}) = \Omega_{j-1},\ j \in \mathbb{N}$ (which is possible due to the assumption $\bigcup_{x\in X}(T_e x) \subseteq \Lambda X$). Then, for each $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon \leq \sigma(\Lambda x_i, \Lambda x_j) < \epsilon + \delta \text{ implies } \sigma(\Lambda x_{i+1}, \Lambda x_{j+1}) < \epsilon.$$

This means $0 < \sigma(y_j, y_{j+1}) < \sigma(y_{j-1}, y_j)$. Hence, the sequence $\{\sigma(y_j, y_{j+1})\}_{j\in\mathbb{N}}$ is nonincreasing, and thus converges to its infimum. Let $\inf_j d(y_j, y_{j+1}) = \tau \geq 0$. In fact, $\tau = 0$. To see this, assume that $\tau > 0$, then choose $n_0 \in \mathbb{N}$ so that $j \geq n_0$ implies $\tau \leq \sigma(y_j, y_{j+1}) < \tau + \delta$. This gives $\sigma(y_{j+1}, y_{j+2}) < \tau$, a contradiction, since τ is the greatest lower bound of $\{\sigma(y_j, y_{j+1})\}_{j\in\mathbb{N}}$. Consequently,

$$\sigma(y_j, y_{j+1}) = \sigma(\Lambda x_j, (T_e x_j)) \leq \sigma(\Lambda x_j, \Lambda x_{j+1}) \longrightarrow 0.$$

Now, we show that the sequence $\{y_j\}_{j\in\mathbb{N}}$ is Cauchy. Assume that $\sigma(y_j, y_{j+1}) = 0$, for some $j > 0$. Then $\sigma(y_i, y_{i+1}) = 0$ for all $i > j$; if not, $\sigma(y_j, y_{j+1}) = 0 < \sigma(y_{j+1}, y_{j+2})$, yields a contradiction. It follows that $\{y_j\}_{j\in\mathbb{N}}$ is a Cauchy sequence of elements of X.

On the other hand, suppose that $\sigma(y_j, y_{j+1}) \neq 0$ for each $j \in \mathbb{N}$. Define $p = 2\epsilon$ and choose $\delta \in (0, \epsilon)$ such that (1) hold. Since $\sigma(y_j, y_{j+1}) \longrightarrow 0$, there exists an $n_0 \in \mathbb{N}$ such that $\sigma(y_t, y_{t+1}) < \frac{\delta}{6}$ for $t \geq n_0$. Let $a > b > n_0$, then, we shall show that $\sigma(y_a, y_b) \leq p$, to prove that $\{y_j\}_{j\in\mathbb{N}}$ is in fact, a Cauchy sequence in X. Assume that

$$\sigma(y_a, y_b) \geq p. \tag{3}$$

First, we prove that there exist some $i, a \in \mathbb{N}$ with $i > a$ such that

$$\epsilon + \frac{\delta}{3} < \sigma(y_a, y_i) < \epsilon + \delta, \tag{4}$$

where a and i are of opposite parity. Let q be the smallest integer greater than a such that

$$\sigma(y_a, y_q) > \epsilon + \frac{\delta}{2} \text{ (because } \delta \in (0, \epsilon)). \tag{5}$$

Further,

$$\sigma(y_a, y_b) < \epsilon + \frac{2\delta}{3}. \tag{6}$$

Otherwise, we get

$$\epsilon + \frac{2\delta}{3} \leq \sigma(y_a, y_{q-1}) + \sigma(y_{q-1}, y_q). \tag{7}$$

Since $n_0 \leq a \leq q-1$, therefore, $\sigma(y_{q-1}, y_q) < \frac{\delta}{6}$. This yields

$$\sigma(y_a, y_{q-1}) > \epsilon + \frac{\delta}{2}, \tag{8}$$

a contradiction to the fact that q is the smallest integer such that (5) holds. Hence,

$$\epsilon + \frac{\delta}{2} < \sigma(y_a, y_q) < \epsilon + \frac{2\delta}{2}. \tag{9}$$

If a and q are of opposite parity, then taking $q = i$ in (9) gives (4). If a and q are of the same parity, then a and $q+1$ are of opposite parity. For this, we have

$$\begin{aligned} \sigma(y_a, y_{q+1}) \leq & \sigma(y_a, y_q) + \sigma(y_q, y_{q+1}) \\ \leq & \epsilon + \frac{2\delta}{3} + \frac{\delta}{6} = \epsilon + \frac{5\delta}{6}. \end{aligned} \tag{10}$$

Moreover,

$$\sigma(y_a, y_q) \leq \sigma(y_a, y_{q+1}) + \sigma(y_{q+1}, y_q).$$

That is,

$$\sigma(y_a, y_q) - \sigma(y_{q+1}, y_q) \leq \sigma(y_a, y_{q+1}),$$

yields

$$\epsilon + \frac{\delta}{2} - \frac{\delta}{6} < \sigma(y_a, y_{q+1}),$$

$$\epsilon + \frac{\delta}{3} < \sigma(y_a, y_{q+1}). \tag{11}$$

Therefore,

$$\epsilon + \frac{\delta}{3} < \sigma(y_a, y_{q+1}) < \epsilon + \frac{5\delta}{6}. \tag{12}$$

Setting $i = q+1$ in (12) yields (4). Now,

$$\begin{aligned} \epsilon + \frac{\delta}{3} < & \sigma(y_a, y_i) \\ \leq & \sigma(y_a, y_{a+1}) + \sigma(y_{a+1}, y_{i+1}) + \sigma(y_{i+1}, y_i) \\ < & \frac{\delta}{6} + \epsilon + \frac{\delta}{6} = \epsilon + \frac{\delta}{6}, \end{aligned} \tag{13}$$

gives a contradiction. Thus, $\{y_j\}_{j\in\mathbb{N}} = \{\Lambda x_j\}_{j\in\mathbb{N}}$ is a Cauchy sequence of points of X. The completeness of X implies that there exists $u \in X$ such that $\sigma(y_j, u) \longrightarrow 0$ as $j \longrightarrow \infty$, and since Λ is continuous, it follows that $\sigma(\Lambda y_j, \Lambda u) \longrightarrow 0$ as $j \longrightarrow \infty$. Consequently,

$$\begin{aligned} S_{EX}^{(e(y_j),e(u))}\left((T_e y_j),(T_e u)\right) &\leq S_{EX}^{\infty}\left((T_e y_j),(T_e u)\right) \\ &\leq \sup\left\{\sigma(r,w) : r \in (T_e y_j), w \in (T_e u)\right\} \\ &< \sigma(\Lambda y_j, \Lambda u) \longrightarrow 0 \text{ as } j \longrightarrow \infty. \end{aligned}$$

Since $\{\Lambda x_j\}_{j\in\mathbb{N}}$ is a Cauchy sequence of elements of X and

$$\begin{aligned} \inf E^{\sigma}_{(\Omega_i,\Omega_j)} =& S_{EX}^{(e(x_i),e(x_j))}\left((T_e x_i),(T_e x_j)\right) \\ \leq& S_{EX}^{\infty}\left((T_e x_i),(T_e x_j)\right) \\ \leq& \sup\left\{\sigma(r,w) : r \in (T_e x_i), w \in (T_e x_j)\right\} \\ <& \sigma(\Lambda x_i, \Lambda x_j) \longrightarrow 0 \text{ as } j \longrightarrow \infty, \end{aligned} \tag{14}$$

it follows that $\{\Omega_j\}_{j\in\mathbb{N}}$ is a Cauchy sequence in $\mathcal{X}^*$. The completeness of $(\mathcal{X}^*, S_{EX}^{\infty})$ implies that there exists $\Omega \in \mathcal{X}^*$ such that

$$\inf E^{\sigma}_{(\Omega_j,\Omega)} \longrightarrow 0 \text{ as } j \longrightarrow \infty.$$

Since $y_{j+1} \in \Omega_j$ and $\sigma(y_{j+1}, u) \longrightarrow 0$ as $j \longrightarrow \infty$, then $u \in \Omega$; that is, $\lim_{j\longrightarrow\infty} \Lambda x_j \in \lim_{j\longrightarrow\infty}(T_e x_j)$. Given that T and Λ are compatible, we have

$$\lim_{j\longrightarrow\infty} S_{EX}^{(e(x_j),e(x_j))}\left(\Lambda(T_e x_j),(T_e\Lambda x_j)\right) = 0. \tag{15}$$

Since $\sigma(\Lambda y_{j+1}, (T_e y_j)) \leq S_{EX}^{(e(x_j),e(x_j))}\left(\Lambda(T_e x_j),(T_e \Lambda x_j)\right)$, therefore, $\Lambda u \in (T_e u)$, in other words, $\lim_{j\longrightarrow\infty} \Lambda y_j \in \lim_{j\longrightarrow\infty}(T_e y_j)$ and

$$\lim_{j\longrightarrow\infty} S_{EX}^{(e(x_j),e(x_j))}\left(\Lambda(T_e x_j),(T_e\Lambda x_j)\right) = S_{EX}^{(e(u),e(u))}\left(\Lambda(T_e u),(T_e\Lambda u)\right) = 0.$$

Let $\eta = \Lambda u$, then using (1), we have

$$\begin{aligned} \sigma(\eta, \Lambda\eta) &\leq S_{EX}^{(e(u),e(u))}\left((T_e u),\Lambda(T_e u)\right) \\ &\leq S_{EX}^{\infty}\left((T_e u),\Lambda(T_e u)\right) \\ &\leq \sup\{\sigma(r,w) : r \in (T_e u), w \in \Lambda(T_e u)\} \\ &< \sigma(\Lambda u, \Lambda\Lambda u) = \sigma(\eta, \Lambda\eta). \end{aligned}$$

Hence, $\Lambda\eta = \eta$. Again, consider

$$\begin{aligned} \sigma(\eta, (T_e\eta)) &\leq S_{EX}^{(e(u),e(u))}\left((T_e u),(T_e\Lambda u)\right) \\ &\leq S_{EX}^{\infty}\left((T_e u),(T_e\Lambda u)\right) \\ &\leq \sup\{\sigma(r,w) : r \in (T_e u), w \in (T_e\Lambda u)\} \\ &< \sigma(\Lambda u, \Lambda\Lambda u) = \sigma(\eta, \Lambda\eta) = 0. \end{aligned}$$

Consequently, $\eta \in (T_e\eta)$. ■

Definition 3.4.

Let (X, σ) be a metric space and $T : X \longrightarrow \mathcal{X}^*$ be a multi-valued mapping. A single-valued mapping $\Lambda : X \longrightarrow X$ is said to be a selection of T, if $\Lambda x \in Tx,\ x \in X$.

Definition 3.5.

Let (X, σ) be a metric space and $T : X \longrightarrow [P(X)]^E$ be a soft set-valued map. A single valued mapping $\Lambda : X \longrightarrow X$ is said to be a selection of T, if there exist some $e \in E$ such that $\Lambda x \in (T_e x)$, for all $x \in X$.

Theorem 3.6.

Let $\mathbb{M}$ be a compact subset of a complete metric space (X, σ) and let $T : \mathbb{M} \longrightarrow [P(\mathbb{M})]^E$ be a soft set-valued map satisfying the following conditions:
for every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in \mathbb{M}$,

$$\epsilon \leq \max\{\sigma(x, (T_e x)), \sigma(y, (T_e y))\} < \epsilon + \delta, \tag{16}$$

implies

$$S_{E\mathbb{M}}^{(e(x),e(y))}((T_e x), (T_e y)) < \epsilon.$$

Then, there exists a subset $\mathbb{A}$ of $\mathbb{M}$ such that $(T_e \eta) = \mathbb{A}$, for each $\eta \in \mathbb{A}$. Furthermore, for each $\eta \in \mathbb{A}$, there exists a selection of T having η as a unique fixed point.

Proof:

Let x_0 be an arbitrary but fixed element of X. We shall construct two sequences $\{x_j\}_{j\in\mathbb{N}}$ and $\{h_j\}_{j\in\mathbb{N}}$ of points of X and $\mathbb{M}$, respectively. $(T_e x_0)$ is a closed subset of $\mathbb{M}$ and hence compact. Therefore, there exists $x_1 \in (T_e x_0)$ such that $\sigma(x_0, x_1) = \sigma(x_0, (T_e x_0))$. Similarly, there exists $x_2 \in (T_e x_1)$ such that $\sigma(x_1, x_2) = \sigma(x_1, (T_e x_1)) = h_1$. By continuing this process repeatedly, we can generate two sequences $\{x_j\}_{j\in\mathbb{N}}$ and $\{h_j\}_{j\in\mathbb{N}}$ such that $x_j \in (T_e x_{j-1})$, $\sigma(x_j, x_{j+1}) = \sigma(x_j, (T_e x_j)) = h_j,\ j \in \mathbb{N}$. From (16), we have

$$\begin{aligned}\sigma(x_j, (T_e x_j)) \leq & S_{E\mathbb{M}}^{(e(x_{j-1}),e(x_j))}((T_e x_{j-1}), (T_e x_j)) \\ \leq & S_{E\mathbb{M}}^{\infty}((T_e x_{j-1}), (T_e x_j)) \\ < & \max\{\sigma(x_{j-1}, (T_e x_{j-1})), \sigma(x_j, (T_e x_j))\}.\end{aligned} \tag{17}$$

If $\sigma(x_{j-1}, (T_e x_{j-1})) < \sigma(x_j, (T_e x_j))$, then from (17), we have

$$\sigma(x_j, (T_e x_j)) < \sigma(x_j, (T_e x_j)),$$

a contradiction. Therefore, $h_j = \sigma(x_j, (T_e x_j)) < \sigma(x_{j-1}, (T_e x_{j-1}))$. Hence, $\{h_j\}_{j\in\mathbb{N}}$ is a monotone nonincreasing sequence of positive reals, and thus converges to its infimum, say τ. Assume that $\inf\{h_j : j \in \mathbb{N}\} = \tau > 0$. Then, choose $i \in \mathbb{N}$ so that $i \geq n_0 \in \mathbb{N}$ implies

$$\tau \leq h_j < \tau + \delta. \tag{18}$$

From (18) and (16), we have

$$h_{j+1} \leq S_{E\mathbb{M}}^{(e(x_j),e(x_{j+1}))}((T_e x_j), (T_e x_{j+1})) < \tau, \tag{19}$$

a contradiction to the supposition that $\tau = \inf\{h_j : j \in \mathbb{N}\}$. Thus, $h_j = \sigma(x_j, (T_e x_j)) \longrightarrow 0$ as $j \longrightarrow \infty$. Therefore, $S_{E\mathbb{M}}^{(e(x_j),e(x_i))}((T_e x_j), (T_e x_i)) \longrightarrow 0$. Since $P(\mathbb{M})$ is closed in X, and $(P(\mathbb{M}), S_{E\mathbb{M}}^{\infty})$ is complete, hence, there exists an $\mathbb{A} \in P(\mathbb{M})$ such that $\inf E^{\sigma}_{((T_e x_j),\mathbb{A})} \longrightarrow 0$ as

$j \longrightarrow \infty$. Take $\eta \in \mathbb{A}$, then $\eta \in (T_e\eta)$. Otherwise, assume that $\sigma(\eta, (T_e\eta)) = \lambda > 0$, then

$$\begin{aligned}\lambda =&\sigma(\eta, (T_e\eta)) \leq \inf E^{\sigma}_{((T_e\eta),\mathbb{A})} \\ \leq& S^{(e(\eta),e(x_j))}_{E\mathbb{M}}((T_e\eta), (T_e x_j)) + \inf E^{\sigma}_{((T_e x_j),\mathbb{A})} \\ <& \max\{\sigma(\eta, (T_e\eta)), \sigma(x_j, (T_e x_j))\} \\ &+ \inf E^{\sigma}_{((T_e x_j),\mathbb{A})}.\end{aligned} \tag{20}$$

Letting $j \longrightarrow \infty$ in (20), yields $\lambda < \lambda$, a contradiction. Therefore, $\eta \in (T_e\eta)$. Now,

$$\begin{aligned}\inf E^{\sigma}_{((T_e\eta),\mathbb{A})} &= \lim_{j\longrightarrow\infty} S^{(e(\eta),e(x_j))}_{E\mathbb{M}}((T_e\eta), (T_e x_j)) \\ &< \lim_{j\longrightarrow\infty} \max\{\sigma(\eta, (T_e\eta)), \sigma(x_j, (T_e x_j))\} = 0.\end{aligned}$$

It follows that $(T_e\eta) = \mathbb{A}$.

Next, we shall show that the soft set-valued map T admits a selection which has a unique fixed point in X. First, notice that for each $\xi \in \mathbb{M}$, $(T_e\xi)$ is a compact subset of $\mathbb{M}$. Hence, for each $\eta \in \mathbb{M}$, there exists $\xi_\eta \in (T_e\xi)$ such that

$$\sigma(\eta, \xi_\eta) = \sigma(\eta, (T_e\xi)). \tag{21}$$

Let $\Lambda : \mathbb{M} \longrightarrow \mathbb{M}$ defined as $\Lambda\xi = \xi_\eta$ be a selection of $T : \mathbb{M} \longrightarrow [P(\mathbb{M})]^E$. Then, for each $\xi \in \mathbb{M}$, we have $\Lambda\xi = \xi_\eta \in (T_e\xi)$. Let $\Lambda_\eta = l$. Then

$$\sigma(\eta, l) = \sigma(\eta, (T_e\eta)) = 0.$$

It follows that $l = \eta = \Lambda\eta$. Now,

$$\begin{aligned}\sigma(\Lambda\xi, \Lambda l) &\leq \sigma(\Lambda\xi, \eta) + \sigma(\eta, \Lambda l) \\ &\leq \sigma(\xi_\eta, \eta) + \sigma(\eta, l_\eta) \\ &\leq \sigma(\eta, (T_e\xi)) + \sigma(\eta, (T_e l)) \\ &\leq S^{(e(\eta),e(\xi))}_{E\mathbb{M}}((T_e\eta), (T_e\xi)) \\ &\quad + S^{(e(\eta),e(l))}_{E\mathbb{M}}((T_e\eta), (T_e l)) \\ &< \sigma(\xi, (T_e\xi)) + \sigma(l, (T_e l)) \\ &< \sigma(\xi, \Lambda\xi) + \sigma(l, \Lambda l).\end{aligned}$$

Consequently, the fixed point of Λ is unique. ∎

In what follows, we provide examples to support the hypotheses of Theorems 3.3 and 3.6.

Example 3.7.

Let $X = [0, \infty) = E$ and $\sigma : X \times X \longrightarrow \mathbb{R}$ be defined as $\sigma(x, y) = |x - y|$, for all $x, y \in X$, so that $([0, \infty), \sigma)$ is a complete metric space. Let $\Lambda : X \longrightarrow X$ be defined as

$$\Lambda x = \frac{x}{n+1},\ n \geq 0,\ \text{for all } x \in X.$$

Then, it is easy to see that Λ is continuous, and $\Lambda x_n \longrightarrow 0$ as $n \longrightarrow \infty$. Consider a soft set-valued map $T : X \longrightarrow [P(X)]^E$ defined as:

$$(T_e x) = \{t \in \Lambda X : t \leq x \text{ and } 0 \leq e < \infty\}.$$

Notice that $\lim_{n \longrightarrow \infty} \Lambda x_n \in \lim_{n \longrightarrow \infty}(T_e x_n)$ and

$$\lim_{n \longrightarrow \infty} S_{EX}^{(e(x_n),e(x_n))}(\Lambda(T_e x_n), (T_e \Lambda x_n)) = 0,$$

so that T and Λ are E-compatible. Moreover, it can be verified that given any $\epsilon > 0$, there exists a $\delta = (n+1)\epsilon$ such that all the conditions of Theorem 3.3 are satisfied. In this case, there exists $\eta = 0 \in X$ such that $\Lambda 0 = 0$ and $0 \in (T_e 0)$.

Example 3.8.

Let $X = \mathbb{R}$ be endowed with the usual metric and $\mathbb{M} = [3, 40] = E$. Define a soft set-valued map $T : \mathbb{M} \longrightarrow [P(\mathbb{M})]^E$ as follows:

$$(T_e x) = \begin{cases} \left\{t \in \mathbb{M} : 3 - \frac{1}{x} \leq t \leq 5 - \frac{1}{x}\right\}, & \text{if } e \in [3, 27), \\ \{t \in \mathbb{M} : 7 \leq t \leq 18\}, & \text{if } e \in (27, 40]. \end{cases}$$

In this case, $\mathbb{M} \supseteq \mathbb{A} = \{t \in \mathbb{M} : 7 \leq t \leq 18\} \in \mathcal{X}^*$ such that $(T_e \eta) \in \mathbb{A}$, for each $\eta \in \mathbb{A}$ and for some $e \in E$. Moreover, corresponding to each $\eta \in \mathbb{A}$, the mapping $\Lambda : \mathbb{M} \longrightarrow \mathbb{M}$ defined as

$$\Lambda x = \begin{cases} \eta, & \text{if } \eta \in (T_e x), \\ 5 - \frac{1}{x}, & \text{otherwise}, \end{cases}$$

is a selection of T. Consequently, for any given $\epsilon > 0$, we can choose a $\delta = 32\epsilon$ such that all the hypotheses of Theorem 3.6 are satisfied.

4. Decision making problems involving soft set-valued maps

In this section, we propose a decision making algorithm involving the notion of E-soft fixed points of soft set-valued maps. Thereafter, an example is provided to illustrate the usability of the techniques.

Assume that a company wants to fill some vacant positions. There are n numbers of candidates who apply for the positions. There is a committee of decision makers to interview the applicants. Each of these candidates meets one criterion or the other of the positions. We propose herein that given the large number of candidates, the idea of soft set-valued maps can be incorporated in the decision process to reduce the applicants to a small optimal numbers which corresponds to the E-soft fixed points of a soft set-valued map. In other words, the objective is to filter out the candidates that meet all the criteria based on the parameters set by the decision makers. This standard operating procedures can be adopted by the following algorithm.

Algorithm:

- Provide the set of all candidates, say X.
- Choose feasible subsets of the set of parameters, say $A_i \subseteq E$.
- Input some soft sets (ρ, A_1), (σ, A_2), and so on.
- Define a soft set-valued map $T : X \longrightarrow [P(X)]^{\cup A_i}$.
- Construct the E-soft fixed points of T, that is $E_{Fix(T)}$.

Then, the decision is $x_k \in X$ is an optimal candidate if $x_k \in E_{Fix(T)}$.

We illustrate the above algorithm using the following example.

Example 4.1.

Assume that a company wants to fill certain vacant positions. There are 50 applicants who applied for the jobs. There is a committee of decision makers, some of them are from the department of human resources and others are from the board of directors. The interview is divided into two stages. To solve this decision making problem, we apply the above algorithm as follows:

- Let $X = \{x_1, x_2, \cdots, x_{50}\}$ be the set of all applicants.
- Let $E = \{e_1, e_2, \cdots, e_6\}$ be the set of all parameters. For $i = 1, 2, \cdots, 6$, the parameters e_i stand for " experience", "computer knowledge", "training", "age limit", "higher education", and "good health", respectively. For the two stages of the interview, the decision makers consider the subsets of E, given by $A = \{e_1, e_2, e_4, e_6\}$ and $B = \{e_1, e_2, e_3, e_5\}$.
- The decision makers critically investigate the CV of the candidates, and on the basis of the constraint of the parameters $A, B \subseteq E$, the following soft sets are constructed.

$$
\begin{aligned}
(\rho, A) = \Big\{ &(e_1, \{c_4, x_7, x_{11}, x_{13}, x_{21}, x_{28}, x_{31}, x_{32}, x_{36}, x_{39}, x_{41}, x_{43}, x_{44}, x_{45}, x_{48}, x_{49}, x_{50}\}),\\
&(e_2, \{x_1, x_3, x_4, x_7, x_{11}, x_{18}, x_{19}, x_{21}, x_{22}, x_{24}, x_{28}, x_{32}, x_{36}, x_{41}, x_{42}, x_{44},\\
&x_{45}, x_{46}, x_{48}, x_{50}\}),\\
&(e_4, \{x_2, x_3, x_4, x_7, x_{11}, x_{13}, x_{15}, x_{18}, x_{21}, x_{23}, x_{25}, x_{28}, x_{30}, x_{33}, x_{36}, x_{38},\\
&x_{41}, x_{43}, x_{45}, x_{48}, x_{50}\}),\\
&(e_6, \{x_1, x_4, x_5, x_7, x_{11}, x_{12}, x_{13}, x_{17}, x_{20}, x_{21}, x_{24}, x_{28}, x_{29}, x_{34}, x_{36}, x_{41},\\
&x_{45}, x_{48}, x_{50}\}) \Big\}.
\end{aligned}
$$

$$
\begin{aligned}
(\sigma, B) = \Big\{ &(e_1, \{x_3, x_4, x_5, x_7, x_8, x_{11}, x_{14}, x_{21}, x_{22}, x_{26}, x_{27}, x_{28}, x_{34},\\
&x_{35}, x_{36}, x_{37}, x_{40}, x_{41}, x_{36}, x_{37}, x_{40}, x_{41}, x_{42}, x_{45}, x_{48}, x_{50}\}),\\
&(e_2, \{x_1, x_4, x_5, x_7, x_{11}, x_{13}, x_{15}, x_{21}, x_{28}, x_{29}, x_{30}, x_{32}, x_{36},\\
&x_{41}, x_{45}, x_{46}, x_{48}, x_{50}\}),\\
&(e_3, \{x_1, x_4, x_7, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{15}, x_{19}, x_{21}, x_{28}, x_{33},\\
&x_{36}, x_{40}, x_{41}, x_{42}, x_{45}, x_{48}, x_{50}\}),\\
&(e_5, \{x_2, x_4, x_5, x_6, x_7, x_8, x_9, x_{11}, x_{12}, x_{13}, x_{14}, x_{16}, x_{17}, x_{21},\\
&x_{23}, x_{28}, x_{36}, x_{41}, x_{42}, x_{45}, x_{48}, x_{50}\}) \Big\}.
\end{aligned}
$$

- Then define a soft set-valued map $T : X \longrightarrow [P(X)]^{A\cup B}$ as follows:

$$(T_e x) = \begin{cases} \rho(e), \text{ if } x \in \Big\{ x_1, x_3, x_4, x_5, x_8, x_9, x_{12}, x_{15}, x_{20}, \\ x_{22}, x_{23}, x_{25}, x_{30}, x_{31}, x_{36}, x_{38}, x_{40}, x_{42}, x_{43}, \\ x_{44}, x_{45}, x_{46}, x_{49}, x_{50} \Big\} \\ \sigma(e), \text{ if } x \in \Big\{ x_2, x_7, x_{10}, x_{11}, x_{13}, x_{14}, x_{16}, x_{17}, x_{18}, \\ x_{19}, x_{21}, x_{24}, x_{26}, x_{27}, x_{28}, x_{29}, x_{32}, x_{33}, x_{34}, x_{35}, \\ x_{37}, x_{39}, x_{41}, x_{47}, x_{48} \Big\}. \end{cases}$$

- The set of all E-soft fixed points of the soft set-valued map T in Step (iv) is given by

$$E_{Fix(T)} = \{x_4, x_7, x_{11}, x_{21}, x_{28}, x_{36}, x_{41}, x_{45}, x_{48}, x_{50}\}.$$

  Hence, the decision makers conclude there are 10 optimal candidates for the available positions.

5. Conclusion

Two classical theorems involving fixed points are Banach and Brouwer's Theorems. Banach fixed point theorem states that if X is a complete metric space and T is a contraction on X, then T has a unique fixed point in X. In Brouwer's fixed point theorem, X is required to be a closed unit ball in a Euclidean space. Then, any contraction T on X has a fixed point. But in this case, uniqueness of fixed point is not guaranteed. In Banach theorem, a metric on X is used with the assumption that T is a contraction. The unit ball in a Euclidean space is also a metric space and the metric topology determines the continuity of continuous functions. However, the main idea of Brouwer's theorem is a topological property of the unit ball, namely, the unit ball is compact and contractible. Banach theorem and Brouwer theorem tell us a difference between two main branches of fixed point theory, metric fixed point theory and topological fixed point theory. It is not easy to differentiate two fixed point theories in an exact way, or to determine a certain topics belonging to which branch. Generally, fixed point theory is regarded as a branch of topology. But due to deep influence on topics related to nonlinear analysis or dynamical systems, many areas of fixed point theory can be thought of as a branch of analysis.

In the setting of metric fixed point theory, here in this paper, two variants of Meir-Keeler fixed point theorem are presented by using the recently introduced notions of soft set-valued maps. Secondly, a theoretic approach towards decision making problems via the idea of E-soft fixed points of soft set-valued maps is proposed. Hopefully, the presented ideas herein will motivate the interested researcher(s) and thereby, bringing about its extension to other related areas such as topological fixed point theory. Moreover, the soft set component of this work can also be studied in other hybrid models such as N-soft set, fuzzy soft set, intuitionistic fuzzy soft set, intuitionistic neutrosophic soft set, rough sets, and so on.

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