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Non-Inflationary Bianchi Type VI_0 Model in Rosen's Bimetric Gravity

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Abstract

In this paper, we have present the solution of Bianchi type VI_0 space-time by solving the Rosen's field equations with massless scalar field ϕ and with constant scalar potential $V(\phi)$ for flat region. It is observed that the scalar field ϕ is an increasing function of time and affects the physical parameters of the model and leads to non-inflationary type solution of model, which contradicts the inflationary scenario. Other geometrical and physical properties of the model in relation to this non-inflationary model are also studied.

Keywords: Bianchi type VI_0 space-time; Rosen's field equations; gravitation; cosmology; non-inflationary model

MSC 2010 No.: 83D05, 83Fxx, 83F05

1. Introduction

Inflationary universe resolves several problems in the big bang cosmology like homogeneity, the isotropy and flatness of the observed universe. Inflation was first discovered by Guth (1981) in the context of grand unification theories. There are two basic types of inflationary cosmological models. One is due to appearance of a flat potential and the other is due to a scalar curvature-squared term. In general relativity, a positive-energy scalar field would generate an exponential expansion of space. It was very quickly realized that such an expansion would resolve many other long-standing problems. Several versions of inflationary scenario exist as investigated by Linde (1982, 1983), Abrecht and Steinhards (1982), Abbott and Wise (1984), Mathizhagan and Johri (1984), Mataresse and Luechin (1985), Mijic et al. (1986) and La and Steinhardt (1989) in general relativity. The role of self-interacting scalar fields in inflationary cosmology has been discussed by Chakraborty (1991), Rahman et al. (2003), Reddy and Naidu (2008) in general relativity.

The Higgs field is an energy field that exists everywhere in the universe. The field is accompanied by a fundamental particle called the Higgs boson (1964), and the field continues to interact with other particles. Although there was initially no experimental confirmation for the theory, over time it came to be seen as the only explanation for mass that was widely viewed as consistent with the rest of the Standard Model. One consequence of the theory was that the Higgs field could manifest itself as a particle, much in the way that other fields in quantum physics manifest as particles. Detecting the Higgs boson became a major goal of experimental physics, but the problem is that the theory did not actually predict the mass of the Higgs boson. If we caused particle collisions in a particle accelerator with enough energy, the Higgs boson should manifest but without knowing the mass that they were looking for, physicists were not sure how much energy would need to go into the collisions.

Using the concept of Higgs field ϕ with potential $V(\phi)$, inflation will take place if $V(\phi)$ has a flat region and the ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Most of the researchers like Bali and Jain (2002), Singh and Kumar (2007) and Bali and Poonia (2011) evaluated the universe by considering the mass less Higgs scalar field ϕ with flat region.

Friedmann-Robertson-Walker described the homogeneous and isotropic models. But universe is neither homogeneous and nor isotropic in smaller scale and therefore homogeneous and anisotropic models have been studied in General Relativity by many researchers like Wainwright et al. (1979), Collins and Hawking (1973), Ellis and MacCallum (1969), Dunn and Tupper (1978), MacCullam (1969), Roy and Bali (1984), Bali (1986), Roy and Banerjee (1992), Bali and Singh (2005). Barrow (1984) discussed the Bianchi type VI0 cosmological model that give a better explanation of some of cosmological problem like primordial helium abundance and the models isotopize in special case. Krori et al. (1990) have investigated massive string cosmological models for Bianchi type VI0 space time. Bali et al. (2009) have studied LRS Bianchi type VI_0 cosmological models with special free gravitational field. Recently, Bali (2012), Bali and Singh (2014) investigated chaotic and inflationary cosmological models in Bianchi Type I and Bianchi Type IX space-times, respectively.

It is observed that most of the work in relation to inflationary cosmological scenario have been carried out by researchers in general relativity and not looked at in other gravitational theories, in particular Rosen's bimetric theory of gravitation even though it is physically important to explain the behavior of universe. Therefore, an attempt has been made to study the nature of the inflationary solution in one of the modified Rosen's (1973, 1975) bimetric theory of gravitation. Rosen developed the bimetric theory of gravitation by attaching flat metric, with Riemannian metric in order to have singularity free space-time, since the presence of singularity means break-down of the universe and any satisfactorily physical theory should be free from singularity. In this regards, Rosen developed the field equations in his bimetric theory of gravitation as

$$N_i^j - \frac{1}{2} N \delta_i^j = -T_i^j, \tag{1}$$

where

$$N_i^j = \frac{1}{2} \gamma^{\alpha\beta} \left(g^{sj} g_{si\,|\,\alpha} \right)_{|\beta} \tag{2}$$

is the Rosen's Ricci tensor and $N = g^{ij} N_{ij}$, $k = \sqrt{\frac{g}{\gamma}}$ together with $g = \det\left(g_{ij}\right)$ and $\gamma = \det\left(\gamma_{ij}\right)$. Here, the vertical bar $(\,|\,)$ stands for γ-covariant differentiation and T_i^j is the energy–momentum tensor of matter field. Right from Rosen, several aspects of this theory have been studied by many researchers like Yilmiz (1975), Isrelit (1976, 1981), Goldman (1977), Karade (1980), Kryzier and Kryzier (1983), Reddy and Rao (1998), Mohanty and Sahoo (2002), Reddy (2003), Khadekar and Kandalkar (2004), Katore and Rane (2006), Borkar and Charjan (2010), Sahoo and Mishra (2010), Gaikwad et al. (2011), Berg (2012), Borkar et al. (2013, 2014) and many other researchers contributed their work in the development of models in bimetric theory of gravitation.

Bali and Poonia (2013) have deduced Bianchi type VI_0 cosmological model in general relativity and observed inflationary solution of the model. Further, they observed anisotropic nature of the universe starting with decelerating phase and expanding with acceleration at late time and matched with inflationary scenario. In this paper, an attempt has been made to extend this work in Rosen's bimetric gravity and tried to see the nature of the model in regards to its geometrical and physical aspects. It is seen that the rate of Higg's field ϕ is increasing with time, affects the physical parameters and contributing non-inflationary type solution of the model which contradicts the cosmological scenario of Bali et al. (2012, 2013) and others.

2. Metric and Field Equations

We consider Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dy^2, \tag{3}$$

where A, B and C are functions of cosmic time t only.

The flat metric corresponding to metric (3) is

$$ds^2 = -dt^2 + dx^2 + e^{2x} dy^2 + e^{-2x} dy^2. \tag{4}$$

When the gravitational field g_{ij} minimally coupled to a scalar field ϕ with potential $V(\phi)$, then its action S is given by

$$S = \int \sqrt{-g}\left(R - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi) \right) d^4 x, \tag{5}$$

and from variation of this action S, (Equation (5)) with respect to dynamical fields, we get the Einstein's field equations (in case of massless scalar field ϕ with a potential $V(\phi)$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \tag{6}$$

in which the energy momentum tensor T_i^j is

$$T_i^j = \partial_i \phi \partial^j \phi - \left[\frac{1}{2} \partial_l \phi \partial^l \phi + V(\phi) \right]. \tag{7}$$

We write the components of energy momentum tensor T_i^j

$$T_1^1 = T_2^2 = T_3^3 = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad T_4^4 = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{8}$$

The law of conservation of energy momentum tensor (Equation (7)) yields

$$\frac{1}{\sqrt{-g}} \partial_i \left[\sqrt{-g}\, \partial_i \phi \right] = -\frac{\mathrm{dV}}{\mathrm{d}\phi}. \tag{9}$$

where $\partial_i \phi = \frac{\partial \phi}{\partial x^i}$, $\partial_j \phi = \frac{\partial \phi}{\partial x^j}$ and $\partial^l \phi = g^{lv} \partial_v \phi = g^{lv} \frac{\partial \phi}{\partial x^v}$.

We have applied this theory to the Rosen field equations and developed the behavior of massless scalar field ϕ with potential $V(\phi)$ in Rosen's bimetric theory of gravitation. The Rosen field Equations (1) and (2) for the metric (3) and flat metric (4), together with the components of energy momentum tensor (8) yield the differential equations,

$$\frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} - \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = -16\pi ABC\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) \right], \tag{10}$$

$$-\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} + \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = -16\pi ABC\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) \right], \tag{11}$$

$$-\frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = -16\pi ABC\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) \right], \tag{12}$$

$$\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}=16\pi ABC\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right], \tag{13}$$

where

$$\dot{A}=\frac{\partial A}{\partial t},\quad \dot{B}=\frac{\partial B}{\partial t}, \dot{C}=\frac{\partial C}{\partial t},\quad \ddot{A}=\frac{\partial^2 A}{\partial t^2},\quad \ddot{B}=\frac{\partial^2 B}{\partial t^2},\quad \ddot{C}=\frac{\partial^2 C}{\partial t^2}.$$

From the law of conservation of energy momentum tensor, (Equation (9)), we have

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi}=-\frac{dV}{d\phi}. \tag{14}$$

3. Solutions of field Equations

From Equations (10) and (11), we have

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0, \tag{15}$$

and Equations (11) and (12) gives

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0. \tag{16}$$

After integrating Equations (15) and (16), we have

$$A=n\,B, \tag{17}$$
$$C=m\,B. \tag{18}$$

For simplicity, we consider constant, $m=n=1$ so that

$$A=B=C. \tag{19}$$

With this $A=B=C$, we rewrite the differential equations (10-14) as

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-V(\phi)\right], \tag{20}$$

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-V(\phi)\right], \tag{21}$$

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-V(\phi)\right], \tag{22}$$

$$-3\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=16\pi ABC\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right], \tag{23}$$

$$\ddot{\phi}+\left(\frac{3\dot{A}}{A}\right)\dot{\phi}=-\frac{dV}{d\phi}. \tag{24}$$

We are solving these differential equations (20 - 24) for metric components and scalar field φ, by assuming the flat region in which ϕ is massless scalar field and the potential $V(\phi)=\mathrm{a}$ (constant)(see Stein-Schabes (1987)). With this assumption, the differential equations (20) – (24) reduce to

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-a\right], \tag{25}$$

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-a\right], \tag{26}$$

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=-16\pi ABC\left[\frac{1}{2}\dot{\phi}^2-a\right], \tag{27}$$

$$-3\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=16\pi ABC\left[\frac{1}{2}\dot{\phi}^2+a\right], \tag{28}$$

$$\ddot{\phi}+\left(\frac{3\dot{A}}{A}\right)\dot{\phi}=0. \tag{29}$$

Equation (29) yields

$$\dot{\phi}=\frac{b}{A^3}, \tag{30}$$

where b is constant of integration. From Equation (28) and (30), we have

$$-3\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=\frac{8\pi b^2}{A^3}+k\,A^3, \tag{31}$$

where $k=16\pi a$, which on integrating, we get

$$-3\frac{\dot{A}}{A}=-\frac{8\pi b^2}{2A^2}+k\frac{A^4}{4}+c_1, \tag{32}$$

where c_1 is the constant of integration which is taken to be zero. So

$$\dot{A}=\frac{l}{6A}-\frac{A^5}{12}k, \tag{33}$$

(when $l=8\pi b^2$), and it has a solution

$$A = \left\{ \sqrt{\frac{2l}{k}} \tanh\left[\sqrt{\frac{2l}{k}} \left(\frac{k}{6} t + L \right) \right] \right\}^{1/3}, \qquad L(>0), \text{constant}. \tag{34}$$

In view of $A = B = C$, we write

$$A = B = C = \left\{ \sqrt{\frac{2l}{k}} \tanh\left[\sqrt{\frac{2l}{k}} \left(\frac{k}{6} t + L \right) \right] \right\}^{1/3}. \tag{35}$$

Thus, the required metric is

$$ds^2 = -dt^2 + \left\{ \sqrt{\frac{2l}{k}} \tanh\left[\sqrt{\frac{2l}{k}} \left(\frac{k}{6} t + L \right) \right] \right\}^{2/3} \left(dx^2 + e^{2x} dy^2 + e^{-2x} dz^2 \right). \tag{36}$$

Using the transformations,

$x = X,\ y = Y,\ z = Z$ and $\left(\frac{k}{6} t + L \right) = T$, with $\sqrt{\frac{2l}{k}} = \alpha$, our model (36) will be

$$ds^2 = -\frac{36}{k^2} dT^2 + \{\alpha \tanh \alpha T\}^{2/3} \left(dX^2 + e^{2x} dY^2 + e^{-2x} dZ^2 \right). \tag{37}$$

Equation (37) represents Bianchi type VI_0 space-time in bimetric theory of gravitation.

4. Physical significance of the model

From Equations (30) and (34), we have

$$\dot{\phi} = \frac{b}{\alpha \tanh \alpha T}, \tag{38}$$

and then after integrating, we get massless scalar field ϕ as

$$\phi = \frac{b}{\alpha^2} \log\{\sinh \alpha T\}. \tag{39}$$

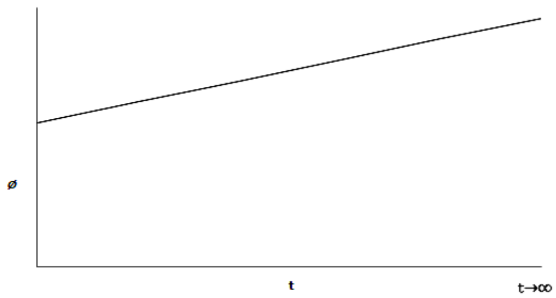

Figure 1. Higg's Field ϕ Vs Time t

The massless scalar field ϕ is an increasing function of time t. Initially at $t=0$, i.e., $T=L\,(\geq 3)$, ϕ attains the non-zero positive value and it is gradually increasing, when t increasing. Its derivative $\dot{\phi}$ admits the constant value, for the whole range of time t. Thus, the scalar field ϕ is continuously increasing (slow), and its derivative is approximately constant (≈ 1), in the evolution of the universe. In view of the behavior of ϕ and $\dot{\phi}$, we put the nature of the model and its physical parameters.
The spatial volume V is given by

$$V=\alpha\tanh\alpha T=\frac{b}{\dot{\phi}}. \tag{40}$$

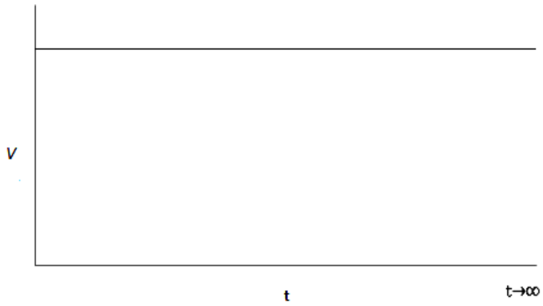

Figure 2. Volume *V* Vs Time *t*

It is observed that for the whole range of time $t=0$, i.e., $T=L\,(\geq 3)$, the volume V appeared approximately with constant value (≈ 1) and it is $b/\dot{\phi}$ with constant $\dot{\phi}$. This suggested that the volume of the model is described by $\dot{\phi}$ and the model starts with non-zero volume and volume is approximately constant in the evolution of the universe, due to the presence of massless scalar field ϕ.

We introduced the perfect fluid with pressure p and density ρ like Singh and Kumar (2007) as

$$p=\frac{1}{2}\dot{\phi}^2-V(\phi),$$

and

$$\rho=\frac{1}{2}\dot{\phi}^2+V(\phi).$$

and in our model, these are

$$p=\frac{b^2}{2\,\alpha^2\tanh^2\alpha T}-a, \tag{41}$$

$$\rho=\frac{b^2}{2\,\alpha^2\tanh^2\alpha T}+a\,. \tag{42}$$

The perfect fluid with this pressure (Equation (41)) and density (Equation (42)) described the behavior of the model, in terms of scalar field ϕ in vacuum fluid as well as in Zel'dovich fluid. In vacuum fluid model, we have $\rho = -p = V(\phi) = a$ (constant), and ρ is the cosmological constant with value a, measure energy of vacuum model, and it is constant in flat potential in the beginning of vacuum fluid model. Zel'dovich fluid with pressure p and density ρ is $\rho = p = b^2/(2\alpha^2 \tanh^2 \alpha T)$ which is equal to $\dot{\phi}^2/2$ and $\dot{\phi}^2/2$ is approximately constant. Therefore, it is clear that at the ending stage of Zel'dovich fluid model, the scalar field ϕ contributes constant pressure and density and the influence of $V(\phi)$ is negligible in the evolution.
The Hubble parameter H and its directional H_1, H_2, H_3 is given by

$$H = H_1 = H_2 = H_3 = \frac{k\alpha}{18\, e^{\phi\alpha^2/k} \cosh \alpha T}. \tag{43}$$

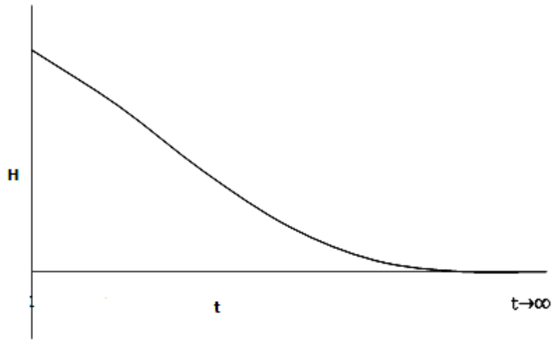

Figure 3. Hubble Parameter *H* Vs Time *t*

It is observed that the Hubble parameter H and its directionals H_1, H_2, H_3 along x, y and z directions respectively are the same and decreasing functions of time t, since scalar fields do not favor a particular direction. Initially at $t = 0$, i.e., $T = L\,(\geq 3)$, they attained the maximum values, gradually decreasing with time t increasing and goes to zero, when t tends to infinity.

The scalar expansion θ is given by

$$\theta = 3H = \frac{k\alpha}{6\, e^{\phi\alpha^2/k} \cosh \alpha T}. \tag{44}$$

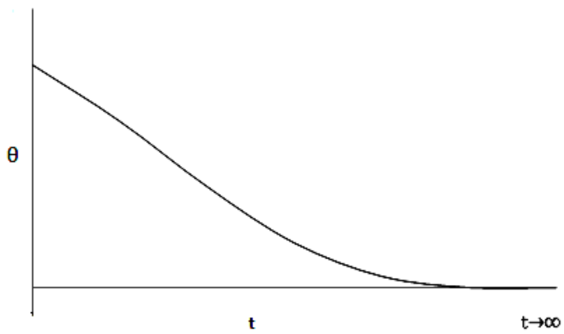

Figure 4. Scalar expansion θ Vs Time t

The scalar expansion θ also has similar nature, since $\theta = 3H$. This suggested that the model starts with maximum rate of expansion (rate is same in all directions) and rate of expansion is gradually decreasing when time t is increasing and approaches to zero at the ending stage, due to the presence of ϕ.

The shear σ is given by

$$\sigma = 0. \tag{45}$$

The deceleration parameter q for the model is

$$q = 2(\cosh \alpha T)^2. \tag{46}$$

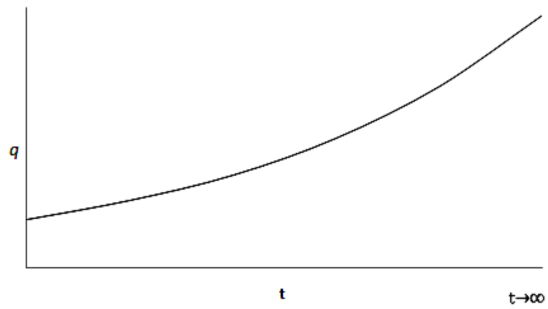

Figure 5. Deceleration Parameter *q* Vs Time *t*

The shear σ in the model admit zero value and the decelerating parameter q is always positive in the evolution of the model. This suggested that the model has decelerating phase always and the rate of expansion of the universe is slowing down more and more and goes to zero. Thus, the universe is pulled back by gravity. The model is shearless.

5. Summary

1. We have presented the solution of Bianchi type VI_0 space-time by solving Rosen's field equations for flat region in case of massless scalar field ϕ and constant scalar potential $V(\phi)$. It is observed that the solutions are of the non-inflationary type.
2. The model appeared with constant volume in vacuum fluid as well as in Zel'dovich fluid due to the presence of scalar field ϕ.
3. The expansion of the model is maximum in the beginning. It is slowing down and approaches to zero at the end.
4. The deceleration parameter q is always positive and goes on increasing with time t. This suggests the non-inflationary model due to increasing rate of Higgs field ϕ, with time.. The model is shearless and an isotropized.

6. Conclusion

In this paper, we have presented the solution of Bianchi type VI_0 space-time by solving the Rosen's field equations with massless scalar field ϕ and with constant scalar potential $V(\phi)$ for flat region. It is observed that the scalar field (Higgs field) ϕ is increasing with time and affects the physical parameters of the model and lead to non-inflationary type solution which contradicts the inflationary scenario of Bali et al. (2012, 2013) and others. This happened due to the increasing rate of Higgs field ϕ with time.

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