Example 1

GRAPHING FUNCTIONS OF THE FORM $f(x) = ax^n$
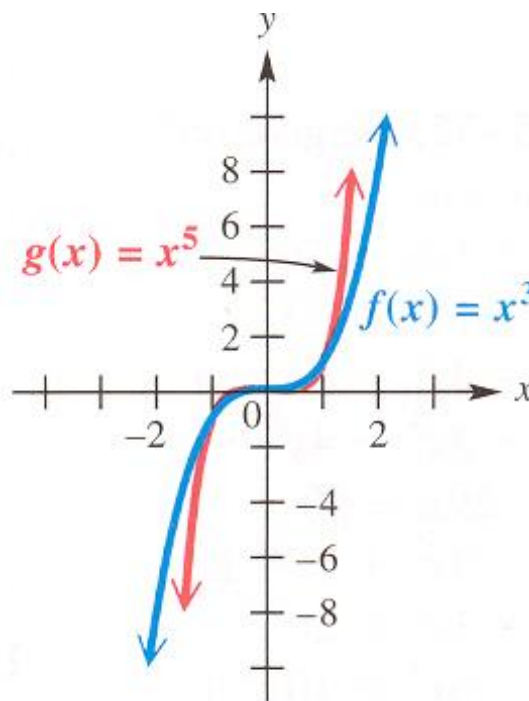
Graph the function.

a. $f(x) = x^3$
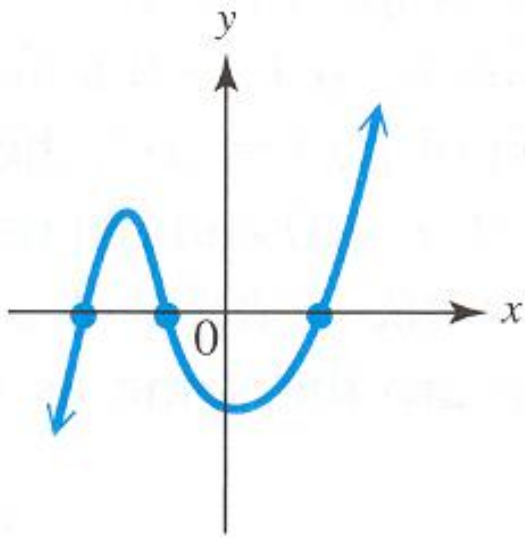
Solution Choose several values for x, and find the corresponding values of $f(x)$, or y.
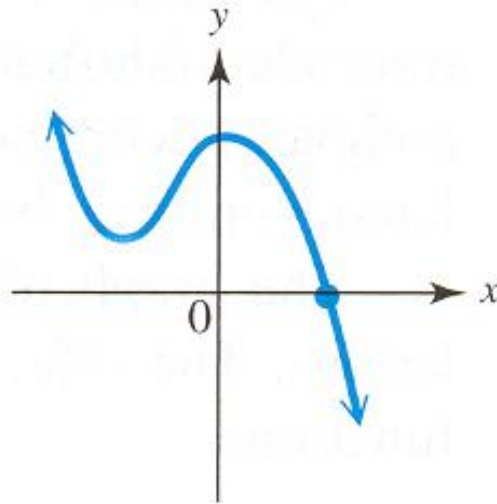
$f(x) = x^3$

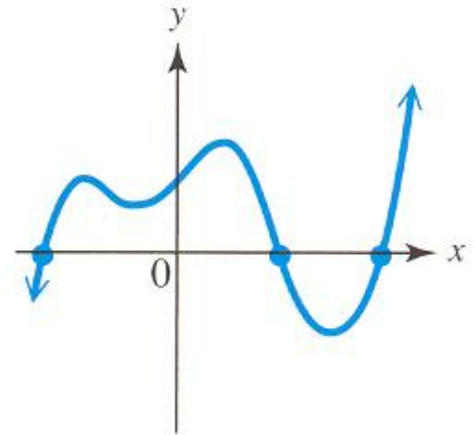| x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

Odd Degree

Degree 3;
three real zeros

Degree 3;
one real zero

Degree 5;
three real zeros

End Behavior

The **end behavior** of a polynomial graph is determined by the **dominating term**, that is, the term of greatest degree. A polynomial of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

has the same end behavior as $f(x) = a_n x^n$.
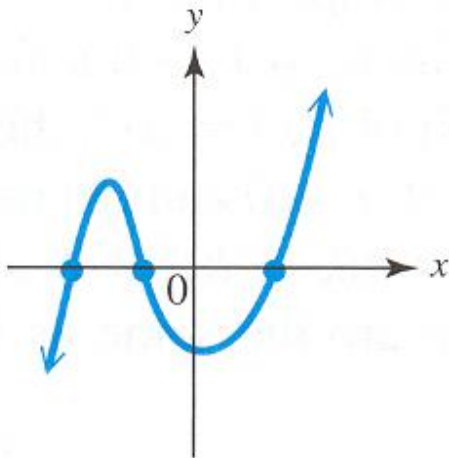
End Behavior

For instance,

$$f(x) = 2x^3 - 8x^2 + 9$$

has the same end behavior as $f(x) = 2x^3$. It is large and positive for large positive values of *x* and large and negative for negative values of *x* with large absolute value.

End Behavior

The arrows at the ends of the graph look like those of the graph shown here; the right arrow points up and the left arrow points down.

The graph shows that as x takes on larger and larger positive values, y does also. This is symbolized as $x \rightarrow \infty, \quad y \rightarrow \infty,$
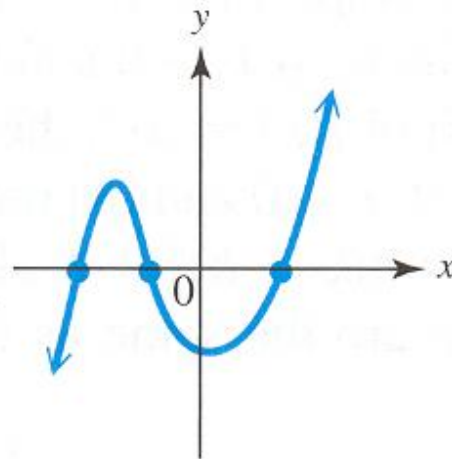
read "as x approaches infinity, y approaches infinity."

End Behavior

For the same graph, as *x* takes on negative values of larger and larger absolute value, *y* does also:
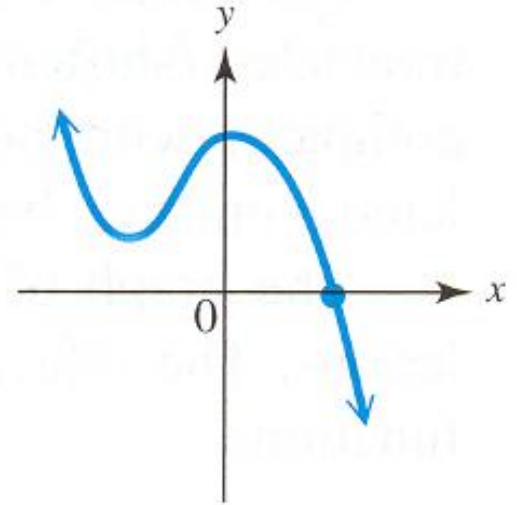
as $x \to -\infty, \quad y \to -\infty,$

End Behavior

For this graph, we have

as $x \rightarrow \infty$, $y \rightarrow -\infty$,
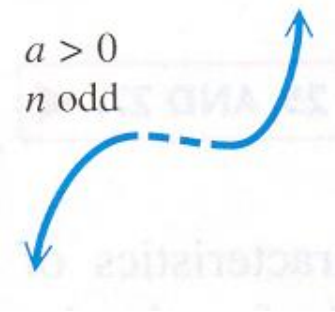
and as $x \rightarrow -\infty$, $y \rightarrow -\infty$.

End Behavior of Polynomials

Suppose that ax^n is the dominating term of a polynomial function f of **odd degree.**

1. If a > 0, then as $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

   Therefore, the end behavior of the graph is of the type that looks like the figure shown here.

   We symbolize it as $\nearrow$.
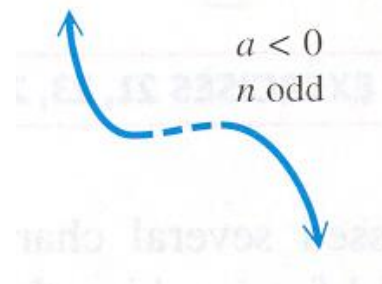
$a > 0$
n odd

End Behavior of Polynomials

Suppose that ax^n is the dominating term of a polynomial function f of **odd degree.**

2. If $a < 0$, then as $x \to \infty, f(x) \to -\infty,$ and as $x \to -\infty, f(x) \to \infty.$

Therefore, the end behavior of the graph looks like the graph shown here.

We symbolize it as ⌇.

$a < 0$
n odd
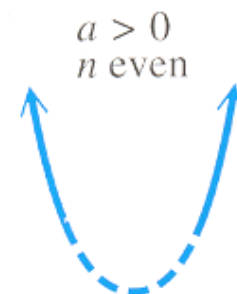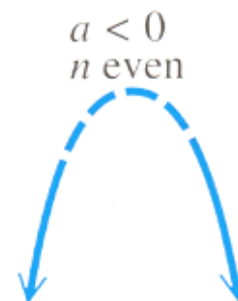
End Behavior of Polynomials

Suppose that ax^n is the dominating term of a polynomial function f of **even degree.**

1. If $a > 0$, then as $|x| \to \infty, f(x) \to \infty$. Therefore, the end behavior of the graph looks like the graph shown here.

$a > 0$
n even

We symbolize it as ⌣ ⌣.

End Behavior of Polynomials

Suppose that is the dominating term of a polynomial function f of **even degree.**

2. If $a < 0$, then as $|x| \to \infty, f(x) \to -\infty$.
   Therefore, the end behavior of the graph looks like the graph shown here.

$a < 0$
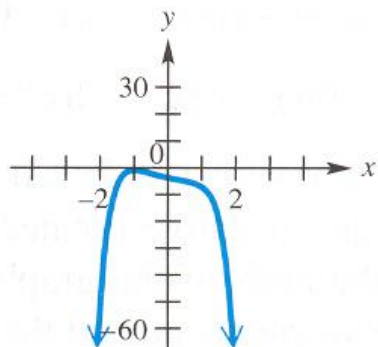n even

We symbolize it as $\curvearrowright$.

Example 3

DETERMINING END BEHAVIOR GIVEN THE DEFINING POLYNOMIAL

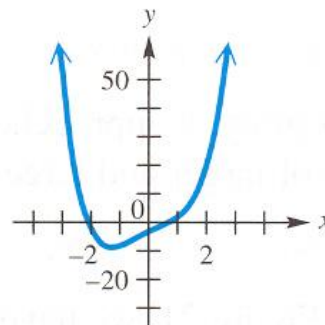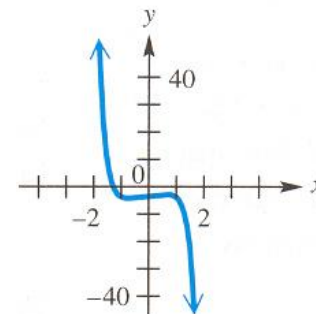Match each function with its graph.

$$f(x) = x^4 - x^2 + 5x - 4$$

A.

B.

C.

D.
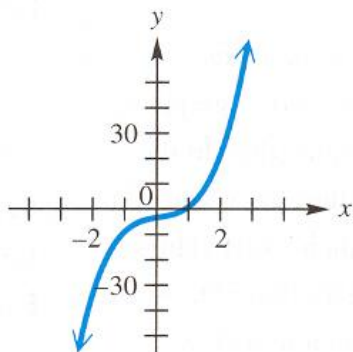
Solution Because f is of even degree with positive leading coefficient, its graph is C.
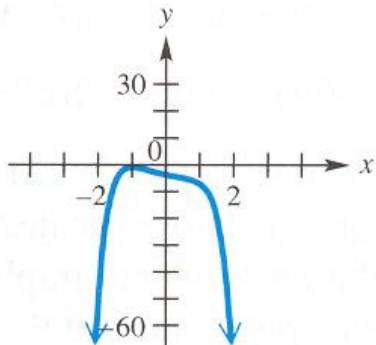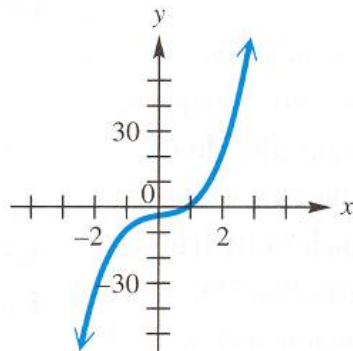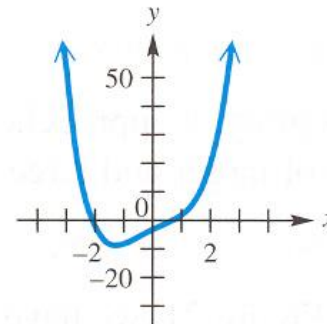
Example 3

Match each function with its graph.

$$g(x) = -x^6 - x^2 - 3x - 4$$

A.

B.

C.

D.

Solution Because g is of even degree with negative leading coefficient, its graph is A.

Match each function with its graph.

$$h(x) = 3x^3 - x^2 + 2x - 4$$

A.

B.

C.

D.

Solution Because function h has odd degree and the dominating term is positive, its graph is in B.

Example 3

DETERMINING END BEHAVIOR GIVEN THE DEFINING POLYNOMIAL

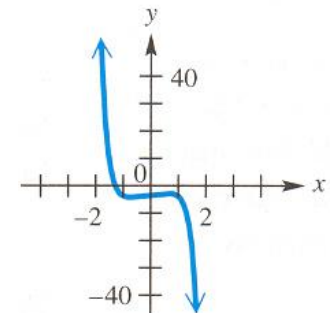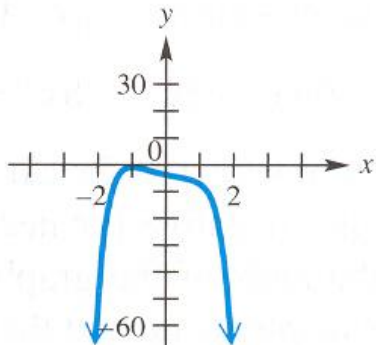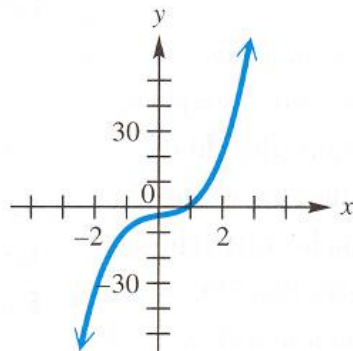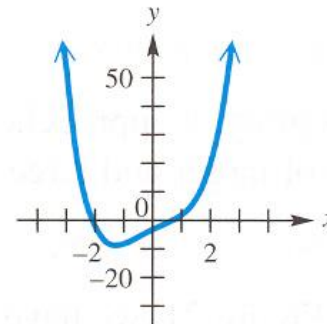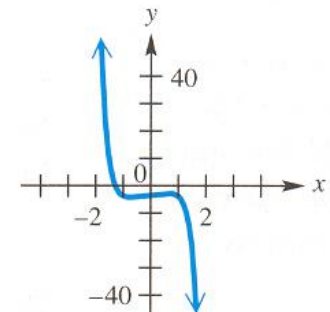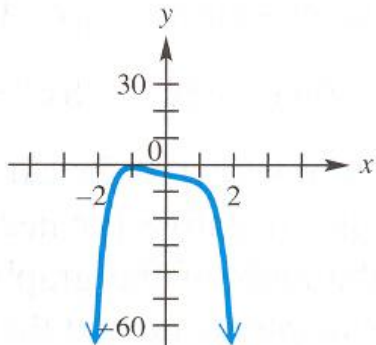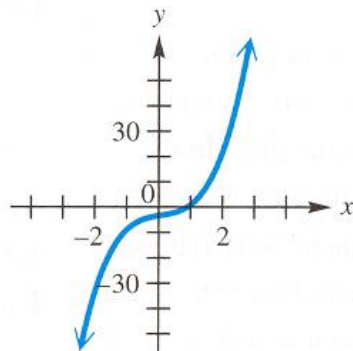Match each function with its graph.
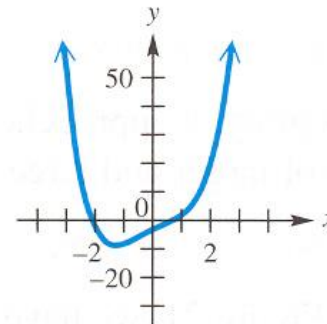
$$k(x) = -x^7 + x - 4$$

A.

B.

C.

D.
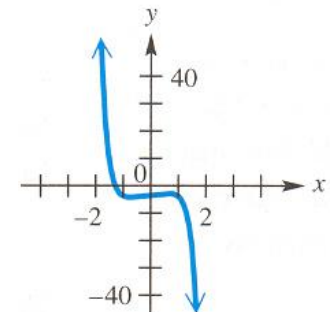
Solution Because function k has odd degree and a negative dominating term, its graph is in D.

Example 4   **GRAPHING A POLYNOMIAL FUNCTION**

Graph $f(x) = 2x^3 + 5x - x - 6.$

Solution

Step 1 Thus,

$$f(x) = (x-1)(2x^2 + 7x + 6)$$

$$= (x-1)(2x+3)(x+2).$$ Factor $2x^2 + 7x + 6$.

Set each linear factor equal to 0, then solve for *x* to find real zeros. The three real zeros of f are 1, $-3/2$, and -2.
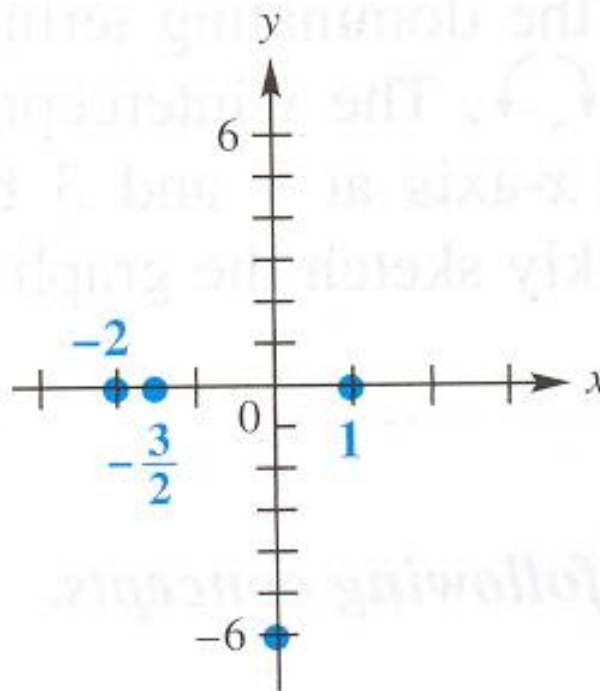
Example 4

GRAPHING A POLYNOMIAL FUNCTION

Graph $f(x) = 2x^3 + 5x - x - 6.$

Solution

Step 2 $f(0) = -6,$ so plot $(0, -6).$
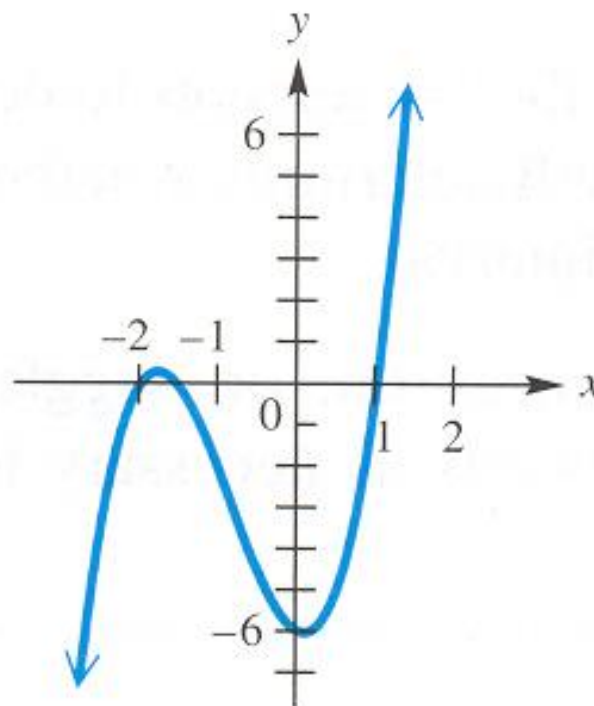
Example 4 **GRAPHING A POLYNOMIAL FUNCTION**

Graph $f(x) = 2x^3 + 5x - x - 6.$

Solution

Step 3

Plot the test points and join the *x*-intercepts, *y*-intercept, and test points with a smooth curve to get the graph.

$$f(x) = 2x^3 + 5x^2 - x - 6$$
$$= (x - 1)(2x + 3)(x + 2)$$

Graph $f(x) = 2x^3 + 5x - x - 6.$

Solution

Step 3

The sketch could be improved by plotting the points found in each interval in the table. Notice that the left arrow points down and the right arrow points up. This end behavior is correct since the dominating term of the polynomial is $2x^3$.
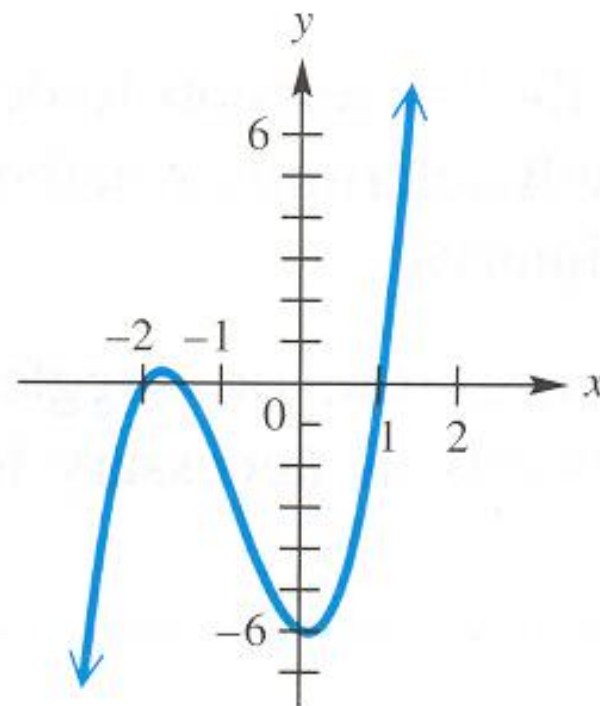
$f(x) = 2x^3 + 5x^2 - x - 6$
$= (x - 1)(2x + 3)(x + 2)$

Graphing Polynomial Functions

> ▶ **Note** If a polynomial function is given in factored form, such as
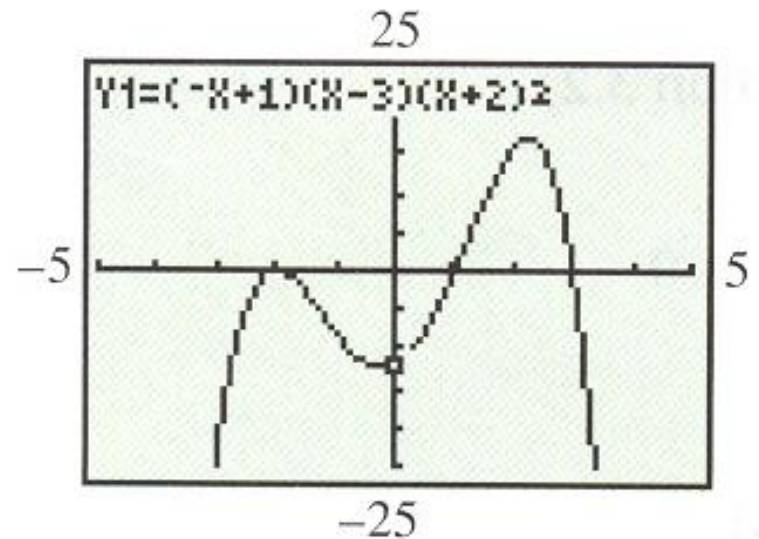> $$f(x) = (-x + 1)(x - 3)(x + 2)^2,$$
> Step 1 of the guidelines is easier to perform, since real zeros can be determined by inspection. For this function, we see that 1 and 3 are zeros of multiplicity 1, and –2 is a zero of multiplicity 2.

Graphing Polynomial Functions

▶ **Note** Since the dominating term is $-x(x)(x^2) = -x^4$, the end behavior of the graph is ⌢ ⌟.
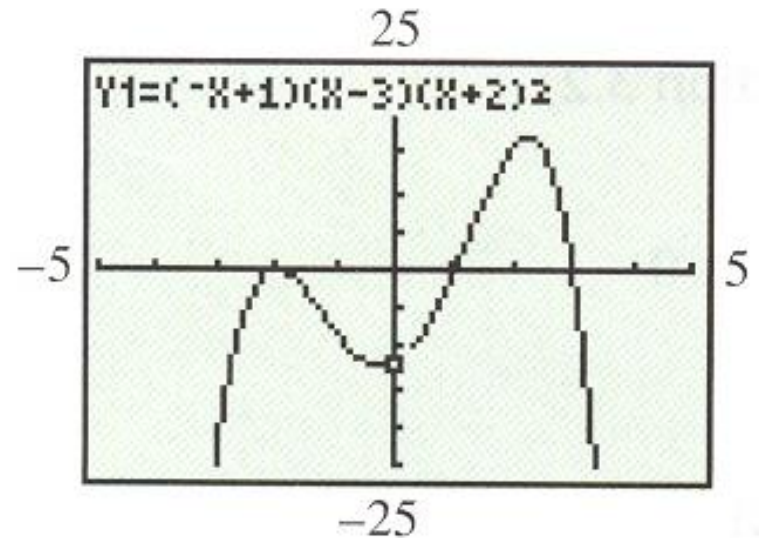
The y-intercept is $f(0) = 1(-3)(2)^2 = -12$.

Here is a calculator graph of the function $f(x) = (-x + 1)(x - 3)(x + 2)^2$.

Graphing Polynomial Functions

▶ **Note**

The graph intersects the *x*-axis at 1 and 3 but bounces at –2.

This information is sufficient to quickly sketch the graph of $f(x)$.

Here is a calculator graph of the function $f(x) = (-x + 1)(x - 3)(x + 2)^2$.

x-Intercepts, Zeros, Solutions, and Factors

If a is an x-intercept of the graph of $y = f(x)$, then a is a zero of f, a is a solution of $f(x) = 0$, and $x - a$ is a factor of $f(x)$.