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Thermal Convection in a Couple-Stress Fluid in the Presence of Horizontal Magnetic Field with Hall Currents

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Abstract

The thermal stability of a couple-stress fluid is considered to include the effects of uniform horizontal magnetic field and Hall currents. The analysis is carried out within the framework of linear stability theory and normal made technique. For the case of stationary convection, magnetic field has stabilizing effect whereas Hall currents are found to have destabilizing effect on the system. The couple stress, however, has dual character in contrast to its stabilizing effect in the absence of Hall currents. Oscillatory modes are not allowed in the system. Graphs in each case have been plotted by giving numerical values to the parameters, depicting the stability characteristics.

Keywords: Thermal convection, couple-stress fluid, magnetic field, Hall currents

MSC 2010 No .: 76E06, 76E07, 76E15, 76U05

1. Introduction

The growing importance of the use of non-Newtonian fluids in modern technology and industries has led various researchers to attempt diverse flow problems related to several non-Newtonian fluids. One such fluid that has attracted the attention of research workers during the last four

decades is the couple stress fluid. The theory of couple stress fluids initiated by Stokes (1966) is a generalization of the classical theory of viscous fluids, which allows for the presence of couple stresses and body couples in the fluid medium. The concept of couple stresses arises due to the way in which mechanical interactions in the fluid medium are modeled. In this theory the rotational field is defined in terms of velocity field itself and the rotation vector equals half the curl of the velocity vector. The stress tensor here is no longer symmetric. An excellent introduction to this theory is available in the monograph "Theories of Fluids with Microstructure - An Introduction" written by Stokes (1984) himself.

One of the applications of couple-stress fluid is its use in the study of the mechanisms of lubrication of synovial joints which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, hip, knee and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. The synovial fluid has been modeled as a couple-stress fluid in human joints by Walicki and Walicka (1999).

In the recent past instability in either a couple stress fluid layer or couple stress fluid saturated porous layer heated from below has been investigated including the external constraints such as magnetic field and/or rotation. Goel et al. (1999) have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and solute concentration gradients. A layer of couple-stress fluid saturating a porous medium heated from below in the presence of rotation has been studied by Sunil et al. (2000) and condition for the onset of convection is obtained. The stability of superposed couple-stress fluid in porous medium in hydromagnetics has been studied by Sunil et al. (2002a). Sunil et al. (2002b) have investigated the effect of magnetic field and rotation on a layer of couple-stress fluid heated from below in a porous medium, while Sunil et al. (2004) have considered the effect of suspended particles on the stability of a couple-stress fluid layer heated and soluted from below in a porous medium.

Sharma and Sharma (2004) have studied the onset of convection in a couple-stress fluid saturated porous layer in the presence of rotation and magnetic field. Siddheshwar and Pranesh (2004) have made an analytical study of linear and non-linear convection in a couple-stress fluid layer. Malashettty and Basavaraja (2005) have studied the effect of thermal and gravity modulation on the onset of Rayleigh-Benard convection in a couple stress fluid layers. Malashetty et al. (2006) have studied the Soret effect on double diffusive convection in a couple stress liquid using both linear and non-linear analyses. Gaikwad et al. (2007) have studied linear and non-linear double diffusive convection with Soret and Dufour effects in couple-stress liquids. Shivakumara (2009) has discussed the effect of various non-uniform basic temperature gradients on the onset of convection in a couple-stress fluid saturated porous layer. Malashetty et al. (2009) investigated the local thermal non-equilibrium effect on the onset of convection in a couple stress fluid saturated porous layers. Malashetty et al. (2010) analyzed the double diffusive instability of a couple stress fluid saturated porous layer using linear and weakly non-linear theories. Malashetty and Kollur (2011) investigated the double-diffusive convection in a horizontal couple stress fluid saturated anisotropic porous layer.

Effect of rotation on thermal convection in a couple-stress fluid saturated rotating rigid porous layer has been investigated by Shivakumara et al. (2011). Rudraiah et al. (2011) investigated linear stability of electro hydrodynamic purely conducting couple stress fluid flowing through a porous channel.

Hall currents are effects whereby a conductor carrying an electric current perpendicular to an applied magnetic field develops a voltage gradient which is transverse to both the current and the magnetic field. It was discovered by Hall in 1879 while he was working on his doctoral degree at Johns Hopkins University at Baltimore, Maryland. The Hall effect has again become an active area of research with the discovery of the quantized Hall effect by Klaus von Klitzing for which he was bestowed with Nobel Prize of Physics in 1985. In ionized gases (plasmas), where the magnetic field is very strong and effects the electrical conductivity, cannot be Hall currents.

The effect of rotation and Hall currents on free convection and mass transfer flow has been studied by Raptis and Ram (1984). Sharma and Rani (1988) investigated the Hall effects on thermosolutal instability of plasma. Sunil et al. (2005) analyzed the effect of Hall currents on thermosolutal instability of compressible Rivlin Ericksen fluid. Rani and Tomar (2010 a, b) investigated thermal and thermosolutal convection problem of micropolar fluid subjected to Hall current. Kumar (2011) examined the effect of Hall currents on thermal instability of compressible dusty viscoelastic fluid saturated in a porous medium subjected to vertical magnetic field.

During the survey it has been noticed that Hall effects are completely neglected from the studies of couple stress fluid. Keeping in mind the importance of couple-stress fluid, convection in fluid layer heated from below, magnetic field and Hall current effects, the present paper attempts to studied the effect of Hall current on thermal instability of a couple-stress fluid in the presence of horizontal magnetic field.

2. Formulation of the Problem

Notations

a Dimensionless wave number

c Speed of light

d Depth of layer

D Derivative with respect to $z\ (= d/dz)$

e Charge of an electron

F Couple stress parameter $\left(= \dfrac{\mu'}{\rho_o d^2}\right)$

$\mathbf{g}(0,0,-g)$ Acceleration due to gravity field

$\mathbf{H}\ (H,0,0)$ Uniform magnetic field

(h_x, h_y, h_z) Perturbations in magnetic field

k_x Wave number in x-direction

k_y Wave number in y-direction

k Resultant wave number $k = \sqrt{k_x^2 + k_y^2}$

k_T Thermal diffusivity

M Hall current parameter $\left[= \left(\frac{cH}{4\pi Ne\eta} \right)^2 \right]$

N Electron number density

n Growth rate

p Fluid pressure

p_1 Prandtl number $\left(= \frac{\nu}{k_T} \right)$

p_2 Magnetic Prandtl number $\left(= \frac{\nu}{\eta} \right)$

$\mathbf{q}$ Velocity of fluid

(u, v, w) Perturbations in fluid velocity

Q Chandrasekhar number $\left(= \frac{\mu_e H^2 d^2}{4\pi\rho_n \nu\eta} \right)$

R Rayleigh number $\left(= \frac{g\alpha\beta d^4}{\nu k_T} \right)$

R_1 Critical Rayleigh number

t Time coordinate

T Temperature

$\boldsymbol{x}(x, y, z)$ Space coordinates

Greek Symbols

η Electrical resistivity

α Coefficient of thermal expansion

β Uniform temperature gradient

θ Perturbation in temperature

δp Perturbation in pressure p

ρ Fluid density

$\delta\rho$ Perturbation in density ρ

ν Kinematic viscosity

μ' Couple stress viscoelasticity

μ_e Magnetic permeability

∇ Del operator

∂ Curly operator

Consider an infinite layer of an incompressible, finitely conducting (electrically and thermally both) couple-stress fluid, confined between two horizontal planes situated at $z = 0$ and $z = d$, acted upon by a uniform horizontal magnetic field $\mathbf{H}\,(H, 0, 0)$ and gravity field $\mathbf{g}(0, 0, -g)$. The fluid layer is heated from below such that a steady adverse temperature gradient $\beta = \dfrac{T_0 - T_1}{d}$, where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ are maintained.

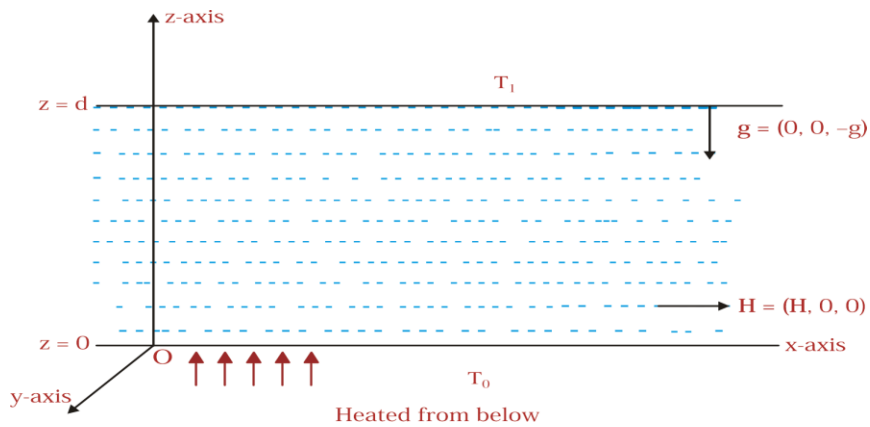

Figure 1. Geometrical configuration

The hydromagnetic equations [Stokes (1966), Chandrasekhar (1981)] relevant to the physical model are

$$\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\mathbf{q}\right] = -\frac{1}{\rho_0}\nabla p + \mathbf{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}\ (\nabla \times \mathbf{H}) \times \mathbf{H}\ , \tag{1}$$

$$\nabla.\mathbf{q} = 0\,, \tag{2}$$

$$\frac{\partial \mathbf{H}}{\partial t} + (q.\nabla)\mathbf{H} = (\mathbf{H}.\nabla)\mathbf{q} + \eta\nabla^2\mathbf{H} - \frac{c}{4\pi Ne}\nabla \times\ (\nabla \times \mathbf{H}) \times \mathbf{H}\ , \tag{3}$$

$$\nabla\,.\,\mathbf{H} = 0, \tag{4}$$

$$\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = k_T\nabla^2 T\,, \tag{5}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{6}$$

where $\mathbf{q}$, ρ, p, and T denote, respectively the fluid velocity, density, pressure and temperature and ν, ν', k_T, α, μ_e, N, e, c, and η stand for the kinematic viscosity, couple stress viscoelasticity, thermal diffusivity, coefficient of thermal expansion, magnetic permeability, electron number density, charge of an electron, speed of light and electrical resistivity. The suffix zero refers to the values at the reference level $z = 0$.

3. Basic State and Perturbation Equations

In the undisturbed state, let the fluid be at rest. Constants temperatures are maintained in the fluid and a constant horizontal magnetic field is applied, therefore, the steady state solution is given by

$$\mathbf{q} = (0, 0, 0),\ \ \mathbf{H} = (H,\ 0,\ 0),\ \ T = T(z),\ \rho = \rho(z),\ \ \ p = p(z)$$

with

$$T(z) = T_0 - \beta z \ \text{ and } \ \rho = \rho_0\,[\,1 - \alpha(T - T_o)]. \tag{7}$$

To use linearized stability theory and normal mode technique let $\mathbf{q}(u, v, w), \mathbf{h}(h_x, h_y, h_z)$, $\delta\rho$, δp and θ are respectively the perturbations in the fluid velocity, magnetic field, density, pressure and temperature and are functions of space as well as time. Then linearized hydromagnetic perturbation equations for couple-stress fluid become

$$\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_0}\nabla p + \mathbf{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) + \left(v - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}\ \ (\nabla \times \mathbf{h}) \times \mathbf{H}\ , \tag{9}$$

$$\nabla.\mathbf{q} = 0\,, \tag{10}$$

$$\frac{\partial \mathbf{H}}{\partial t}\ =\ (\mathbf{H}.\nabla)\mathbf{q} + \eta\nabla^2\mathbf{h} - \frac{c}{4\pi Ne}\nabla \times\ (\nabla \times \mathbf{h}) \times \mathbf{H}\ , \tag{11}$$

$$\nabla\,.\,\mathbf{h} = 0, \tag{12}$$

$$\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = k_T\nabla^2 T\ . \tag{13}$$

The change in density $\delta\rho$ caused by the perturbation θ in temperature is given by

$$\delta\rho = -\rho_0\alpha\theta. \tag{14}$$

Analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$w, \theta, h_z, \zeta, \xi\ =\ W(z), \Theta(z), L(z), Z(z), X(z)\ \ \mathrm{e}^{ik_x x + ik_y y + nt}, \tag{15}$$

where k_x and k_y are the wave numbers in x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of the vorticity and current density respectively.

We eliminate the physical quantities after using the non-dimensional parameters $a = kd$, $\sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{k_T}$, $p_2 = \frac{\nu}{\eta}$, $D = \frac{D^*}{d}$, and dropping (*) is for convenience, equations (9) to (14) give

$$\left[(D^2 - a^2) - F(D^2 - a^2)^2 - \sigma\right](D^2 - a^2)W - \frac{ga^2d^2\alpha}{\nu}\Theta = ik_x \frac{\mu_e H d^2}{4\pi\rho_0\nu}(D^2 - a^2)L, \tag{16}$$

$$\left[(D^2 - a^2) - F(D^2 - a^2)^2 - \sigma\right]Z = -ik_x \frac{\mu_e H d^2}{4\pi\rho_0\nu} X\ , \tag{17}$$

$$D^2 - a^2 - \sigma p_2 \quad X = -\frac{ik_x H d^2}{\eta} Z - \frac{ik_x cH}{4\pi Ne\eta} \quad D^2 - a^2 \quad L\ , \tag{18}$$

$$D^2 - a^2 - \sigma p_2 \quad L = -\frac{ik_x H d^2}{\eta} W + \frac{ik_x cHd^2}{4\pi Ne\eta} X, \tag{19}$$

$$D^2 - a^2 - \sigma p_1 \quad \Theta = -\frac{\beta d^2}{k_T} W, \tag{20}$$

where $F = \frac{\mu'}{\rho_0 d^2}$ is the couple stress parameter. On eliminating various physical parameters from equations (16) to (20), we obtain the final stability governing equation as

$$\begin{aligned}
&\Big[\ (D^2 - a^2) - F(D^2 - a^2)^2 - \sigma \quad D^2 - a^2 - \sigma p_2 \quad {}^2 + Qk_x^2 d^2 \quad D^2 - a^2 - \sigma p_2 \ \Big] \\
&\qquad - Mk_x^2 d^2 \quad (D^2 - a^2) - F(D^2 - a^2)^2 - \sigma \quad (D^2 - a^2) \\
&\qquad \times \quad (D^2 - a^2) - F(D^2 - a^2)^2 - \sigma \quad D^2 - a^2 - \sigma p_1 \quad D^2 - a^2 \quad W + Ra^2 W \\
&\qquad + Qk_x^2 d^2 \Big[\ (D^2 - a^2) - F(D^2 - a^2)^2 - \sigma \quad D^2 - a^2 - \sigma p_2 \quad + Qk_x^2 d^2 \Big] \\
&\qquad \times \ D^2 - a^2 - \sigma p_1 \quad D^2 - a^2 \quad W = 0,
\end{aligned} \tag{21}$$

where $R = \frac{g\alpha\beta d^4}{\nu k_T}$ is the Rayleigh number, $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number and $M = \left(\frac{cH}{4\pi Ne\eta}\right)^2$ is the non-dimensional number accounting for Hall currents.

We now consider the case where both the boundaries are free as well as perfect conductors of heat. Since both the boundaries are maintained at constant temperature, the perturbation in the temperature are zero at the boundaries therefore, the appropriate boundary conditions are

$$W = 0,\ D^2W = 0,\ DZ = 0,\ \Theta = 0 \text{ at } z = 0 \text{ and } z = 1$$

with

$$DX = 0,\ \ K = 0\,. \tag{22}$$

Therefore, the proper solution of equation (21) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{23}$$

where W_0 is constant. Using equation (23), equation (21) gives

$$\begin{aligned} R_1 = (1+x)\Big[\frac{1}{x}\ (1+x) + F_1(1+x)^2 + i\sigma_1\ \ (1+x+i\sigma_1 p_1)\Big] \times + Q_1 \cos^2\theta \\ \times\Big[\ (1+x) + F_1(1+x)^2 + i\sigma_1\ \ (1+x+i\sigma_1 p_2) + Q_1 x\cos^2\theta\Big](1+x+i\sigma_1 p_1)(1+x) \\ \times\Big[\ \ (1+x) + F_1(1+x)^2 + i\sigma_1\ \ (1+x+i\sigma_1 p_2)^2 + Q_1 x\cos^2\theta(1+x+i\sigma_1 p_2) \\ + Mx\cos^2\theta\ \ (1+x) + F_1(1+x)^2 + i\sigma_1\ \ (1+x)\Big]^{-1}, \end{aligned} \tag{24}$$

where $k_x = k\cos\theta$ [Chandrasekhar (1981)], $R_1 = \dfrac{R}{\pi^4}$, $i\sigma_1 = \dfrac{\sigma}{\pi^2}$, $x = \dfrac{a^2}{\pi^2}$, $F_1 = \pi^2 F$ and $Q_1 = \dfrac{Q}{\pi^2}$. Equation (24) is the required dispersion relation including the parameters characterizing the Hall currents, magnetic field and couple-stress.

4. Results and Discussion

4.1 . Stationary Convection

When the instability sets in as stationary convection $(\sigma = 0)$ equation (24), reduces to

$$\begin{aligned} R_1 = (1+x)\Bigg[\frac{1+F_1(1+x)\ \ (1+x)^2}{x} + Q_1\cos^2\theta\Big[\ 1+F_1(1+x)\ \ (1+x)^2 + Q_1 x\cos^2\theta\Big] \\ \times\ \ 1+F_1(1+x)\ \ (1+x)^2 + Q_1 x\cos^2\theta + Mx\cos^2\theta\ \ 1+F_1(1+x)\ \ (1+x)\ \ ^{-1}\Bigg]. \end{aligned} \tag{25}$$

The above relation expresses the modified Rayleigh number R_1 as a function of the parameters M, Q_1, F_1 and dimensionless wave number x. For the case of stationary convection, to study the effect of Hall currents, magnetic field and couple-stress, we examine the nature of $\dfrac{dR_1}{dM}$, $\dfrac{dR_1}{dQ_1}$ and $\dfrac{dR_1}{dF_1}$ analytically.

From equation (25), we have

$$\frac{dR_1}{dM} = -Q_1\left[\frac{x(1+x)^2\cos^4\theta\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta\ \ 1+F_1(1+x)\ \right]}{\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx(1+x)\cos^2\theta\ \ 1+F_1(1+x)\ \right]^2}\right], \tag{26}$$

which is negative, therefore Hall currents have a destabilizing effect on the thermal convection in a couple-stress fluid.

From equation (25), we have

$$\begin{aligned}\frac{dR_1}{dQ_1} = &\left[\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx\cos^2\theta(1+x)\ \ 1+F_1(1+x)\ \right]\right.\\ &\times\left[\ 1+F_1(1+x)\ \ \cos^2\theta(1+x)^3+Q_1x(1+x)\cos^4\theta\right]+Q_1M\cos^6\theta x^2(1+x)^2\ \ 1+F_1(1+x)\ \Big]\\ &\times\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx\cos^2\theta(1+x)\ \ 1+F_1(1+x)\ \right]^{-2},\end{aligned} \tag{27}$$

which shows that magnetic field has a stabilizing effect on the thermal convection in a couple-stress fluid. Furthermore, equation (25) yields

$$\begin{aligned}\frac{dR_1}{dF_1} = &\frac{(1+x)^3}{x}\left[(1+x)\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx\cos^2\theta(1+x)\ \ 1+F_1(1+x)\ \right]^2\right.\\ &\left.-Q_1^2Mx^3\cos^6\theta\right]\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx\cos^2\theta(1+x)\ \ 1+F_1(1+x)\ \right]^{-2},\end{aligned} \tag{28}$$

which shows that couple-stress has stabilizing or destabilizing effect according as

$$\left[\ 1+F_1(1+x)\ \ (1+x)^2+Q_1x\cos^2\theta+Mx\cos^2\theta(1+x)\ \ 1+F_1(1+x)\ \right]^2 > \text{or} < \frac{Q_1^2Mx^3\cos^6\theta}{(1+x)}.$$

It is to be noted that in the absence of Hall currents, equation (28) provides

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{x}, \tag{29}$$

which clearly shows that couple-stress has a stabilizing effect. Thus couple stress has a dual character, in the absence of Hall currents it has stabilizing effect while in their presence it may destabilize the system.

4.2 . Oscillatory Convection

Multiplying equation (16) with W^* (the complex conjugate of W), integrating over the range of z and making use of equations (17) to (20) together with the boundary condition (22), we get

$$I_1 + FI_2 + \sigma I_3 - \frac{g_0 \alpha k_T a^2}{\nu\beta}[I_4 + \sigma^* p_1 I_5] - \frac{\mu_e \eta}{4\pi\rho_0 \nu}[I_6 + \sigma^* p_2 I_7] + \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} \; I_8 + \sigma p_2 I_9$$
$$- d^2 \left[I_{10} + FI_{11} + \sigma^* I_{12} \right] = 0, \tag{30}$$

where

$$I_1 = \int |D^2 W|^2 + 2a^2 |DW|^2 + a^4 |W|^2 \; dz,$$
$$I_2 = \int |D^3 W|^2 + 3a^2 |D^2 W|^2 + 3a^4 |DW|^2 + a^6 |W|^2 \; dz,$$
$$I_3 = \int |DW|^2 + a^2 |W|^2 \; dz, \; I_4 = \int |D\Theta|^2 + a^2 |\Theta|^2 \; dz, \; I_5 = \int |\Theta|^2 \, dz,$$
$$I_6 = \int |D^2 K|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \; dz, \; I_7 = \int |DK|^2 + a^2 |K|^2 \; dz,$$
$$I_8 = \int |DX|^2 + a^2 |X|^2 \; dz, \; I_9 = \int |X|^2 \, dz, \; I_{10} = \int |DZ|^2 + a^2 |Z|^2 \; dz,$$
$$I_{11} = \int |D^2 Z|^2 + a^4 |Z|^2 + 2a^2 |DZ|^2 \; dz, \; I_{12} = \int |Z|^2 \, dz.$$

All the integrals I_1 - I_{12} are positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ in equation (30) and then equating the real and imaginary parts, we get

$$\sigma_r \left[I_3 - \frac{g_0 \alpha k_T a^2}{\nu\beta} p_1 I_5 - \frac{\mu_e \eta}{4\pi\rho_0 \nu} p_2 I_7 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} p_2 I_9 - d^2 I_{12} \right]$$
$$= - \left[I_1 + FI_2 - \frac{g \alpha k_T a^2}{\nu\beta} I_4 + \frac{\mu_e \eta}{4\pi\rho_0 \nu} I_6 - \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} I_8 + d^2 \; I_{10} + FI_{11} \right] \tag{31}$$

and

$$\sigma_i \left[I_3 + \frac{g_0 \alpha k_T a^2}{\nu\beta} p_1 I_5 + \frac{\mu_e \eta}{4\pi\rho_0 \nu} p_2 I_7 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} p_2 I_9 + d^2 I_{12} \right] = 0. \tag{32}$$

It may be inferred from equation (31) that σ_r may be positive or negative which means that the system may be unstable or stable while equation (32) predicts that $\sigma_i = 0$ necessarily because all the terms in the bracket are positive definite, which implies that oscillatory modes are not allowed in the system.

5. Numerical Computations

For the stationary convection critical thermal Rayleigh number for the onset of instability is determined for critical wave number obtained by the condition $\frac{dR_1}{dx} = 0$ and analyzed numerically using Newton-Raphson method.

Critical Rayleigh number R_1 is plotted against Hall currents parameter M for fixed values of $\theta = 45°$, $Q_1 = 50$ and $F_1 = 10, 15, 20$. The critical Rayleigh number R_1 decreases with increase in Hall currents parameter M which shows that Hall currents have a destabilizing effect on the system. Further R_1 increases with increasing couple stress which shows the stabilizing character of couple stress **(**see Figure 2).

Critical Rayleigh number R_1 is plotted against Hall currents parameter M for fixed value of $\theta = 45°$, $F_1 = 10$ and $Q_1 = 100, 200, 300$. The critical Rayleigh number R_1 decreases with increase in Hall currents parameter M and increases with increase in Q_1 which shows that Hall currents have a destabilizing effect on the system while magnetic field has a stabilizing effect on it **(**see Figure 3).

Critical Rayleigh number R_1 is plotted against magnetic field parameter Q_1 for fixed value of $\theta = 45°$, $F_1 = 10$ and $M = 5, 15, 25$. The critical Rayleigh number R_1 increases with increase in magnetic field parameter Q_1 as well as Hall current parameter M which shows that magnetic field and Hall currents both have stabilizing effect on the system **(**see Figure 4).

Critical Rayleigh number R_1 is plotted against magnetic field parameter Q_1 for fixed value of $\theta = 45°$, $M = 10$ and $F_1 = 5, 15, 25$. The critical Rayleigh number R_1 increases with increase in magnetic field parameter Q_1 and couple stress parameter F_1 which shows that magnetic field and couple stress both have stabilizing effect on the system **(**see Figure 5).

Critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for fixed value of $\theta = 45°$, $M = 10$ and $Q_1 = 100, 200, 300$. The critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1 and magnetic field parameter Q_1 which shows that couple-stress and magnetic field both have stabilizing effect on the system **(**see Figure 6).

Critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for fixed value of $\theta = 45°$, $M = 0$ and $Q_1 = 100, 200, 300$. The critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1 and rotation parameter Q_1 which shows that couple-stress and rotation both have stabilizing effect on the system in the absence of Hall currents **(**see Figure 7).

Critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for fixed value of $\theta = 45°, Q_1 = 500$ and $M = 10, 30, 50$. The critical Rayleigh number R_1 decreases up to certain values of F_1 and gradually which shows that couple-stress has both destabilizing and stabilizing effect on the system **(**see Figure 8).

6. Conclusion

In the present paper, we have investigated the effect of Hall currents on an electrically conducting couple-stress fluid layer heated from below in the presence of horizontal magnetic field. Dispersion relation governing the effects of Hall currents, magnetic field and couple-stress is derived. The main results obtained are as follows:

(i) For the case of stationary convection, Hall currents have a destabilizing effect on the thermal convection as can be seen from equation (26) and graphically from Figure 2 and Figure 3.

(ii) In the absence or presence of Hall currents, magnetic field has a stabilizing effect on the thermal convection as can be seen from equation (27) and graphically from Figure 4 and Figure 5.

(iii) In the presence of Hall currents, couple-stress has destabilizing or stabilizing effect on the thermal convection under certain conditions as can be seen from equation (28) and graphically from Figure 6 and Figure 8, while in their absence couple-stress has a stabilizing effect on the thermal convection as can be seen from equation (29) and graphically from Figure 7, in this way dual character of couple-stress is recognized.

(iv) Oscillatory modes are not allowed in the system and Principle of Exchange of Stabilities is satisfied in the present problem.

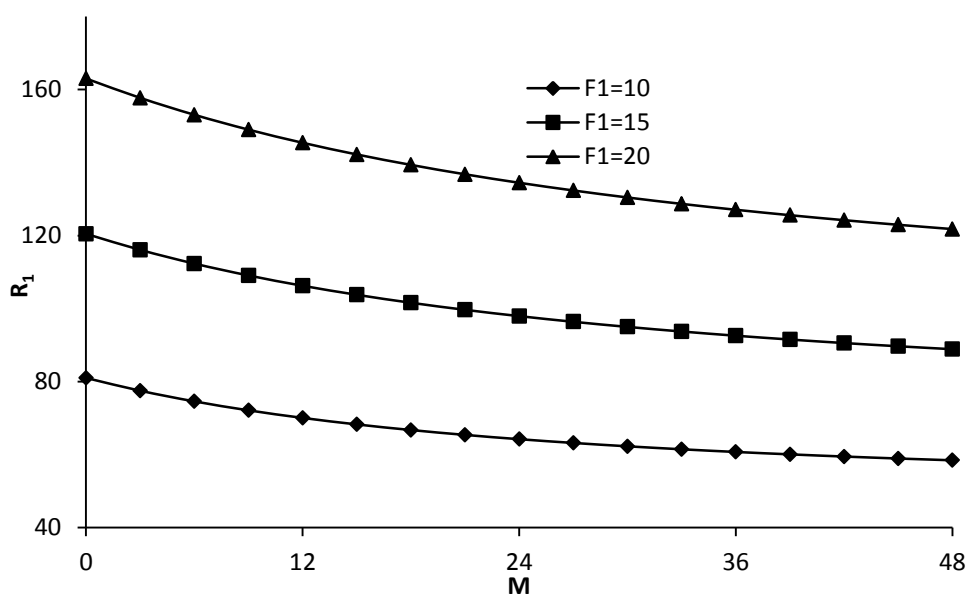

Figure 2. Variations of critical Rayleigh number R_1 with M for fixed value of $\theta = 45°$, $Q_1 = 50$ and $F_1 = 10, 15, 20$

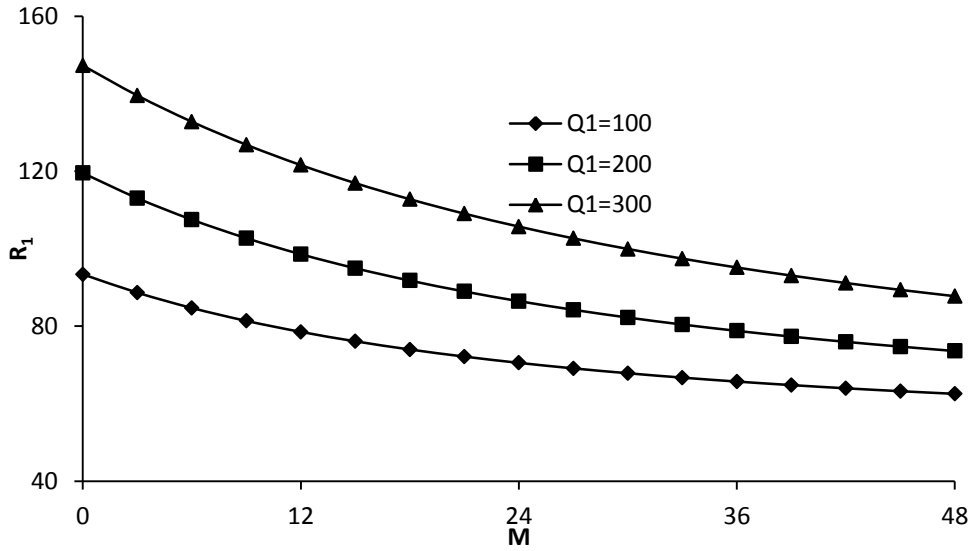

Figure 3: Variations of critical Rayleigh number R_1 with *M* for fixed value of $\theta = 45^0$, $F_1 = 10$ and $Q_1 = 100, 200, 300$

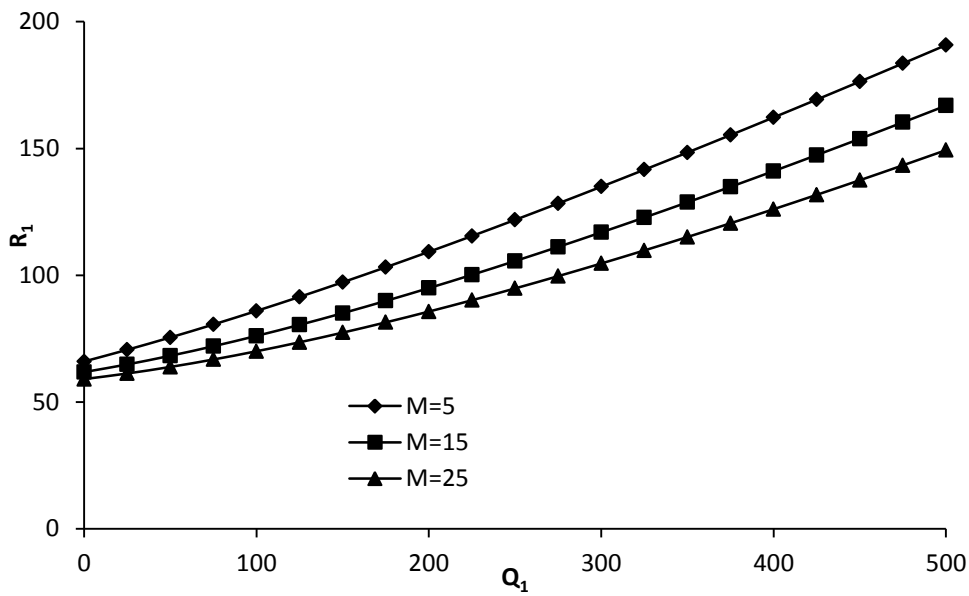

Figure 4. Variations of critical Rayleigh number R_1 with Q_1 for fixed value of $\theta = 45^0$, $F_1 = 10$ and $M = 5, 15, 25$

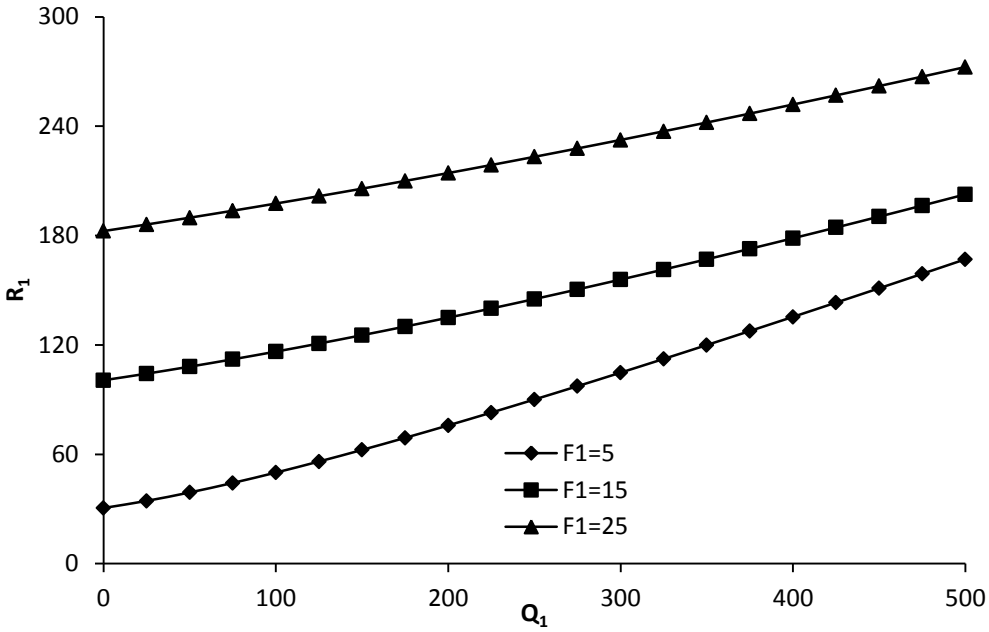

Figure 5. Variations of critical Rayleigh number R_1 with Q_1 for fixed value of $\theta = 45^0$, $M = 10$ and $F_1 = 5, 15, 25$

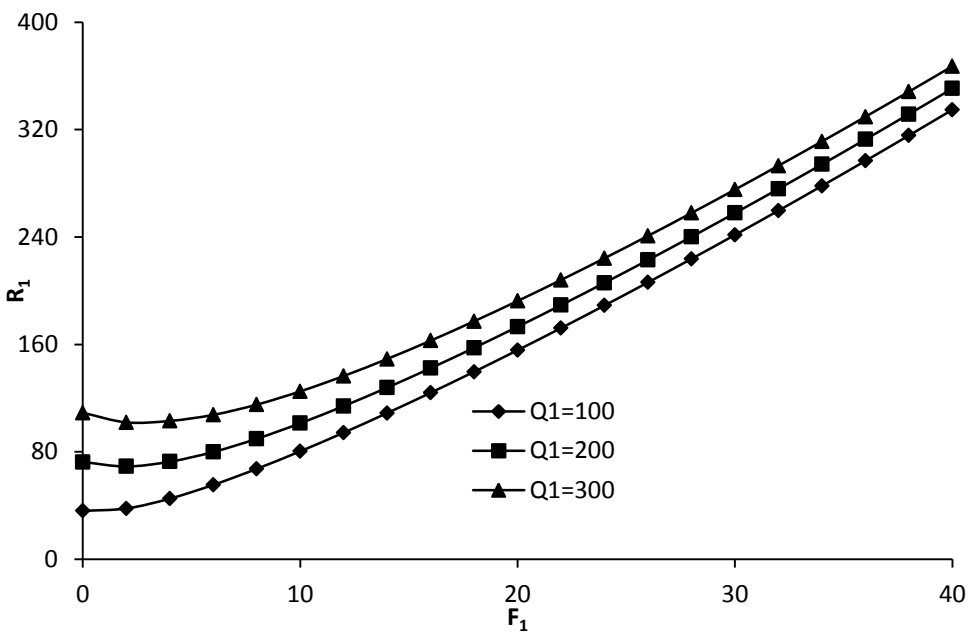

Figure 6. Variations of critical Rayleigh number R_1 with F_1 for fixed value of $\theta = 45^0$, $M = 10$ and $Q_1 = 100, 200, 300$

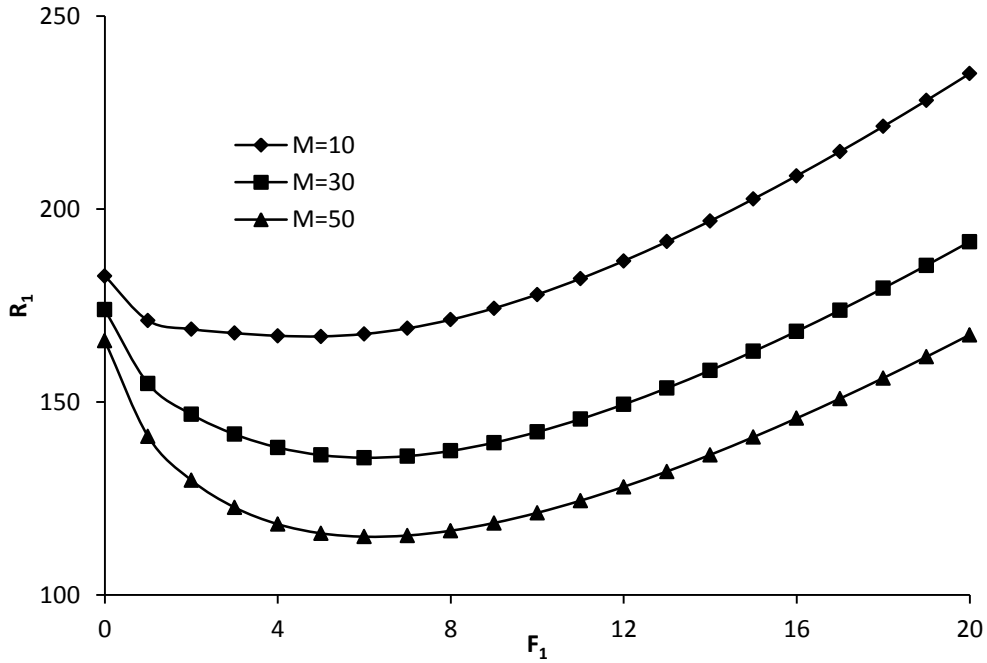

Figure 7. Variations of critical Rayleigh number R_1 with F_1 for fixed value of $\theta = 45^0$, $Q_1 = 500$ and $M = 10, 30, 50$

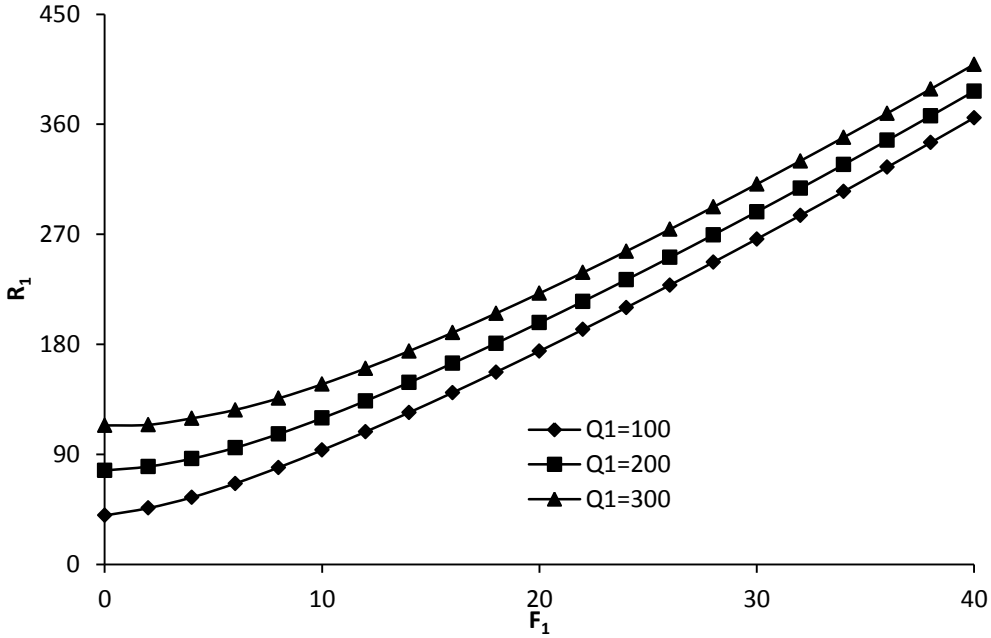

Figure 8. Variations of critical Rayleigh number R_1 with F_1 for fixed value of $\theta = 45^0$, $M = 0$ and $Q_1 = 100, 200, 300$

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