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Numerical Solution of Interval and Fuzzy System of Linear Equations

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Abstract

A system of linear equations, in general is solved in open literature for crisp unknowns, but in actual case the parameters (coefficients) of the system of linear equations contain uncertainty and are less crisp. The uncertainties may be considered in term of interval or fuzzy number. In this paper, a detail of study of linear simultaneous equations with interval and fuzzy parameter (triangular and trapezoidal) has been performed. New methods have been proposed for solving such systems. First, the methods have been tested for known problems viz. a circuit analysis solved in the literature and the results are found to be in good agreement with the present. Next more example problems are solved using the proposed methods to strengthen confidence on these new methods. The solutions of the example problems clearly show the efficacy and reliability of the proposed method(s).

Keywords: Interval, Fuzzy, linear simultaneous equation, Fuzzy number

MSC 2010: 03E72, 15B15, 94D05

1. Introduction

The system of linear equations has applications in various areas of science such as operational research, physics, statistics, engineering, and social sciences. Equations of this type are necessary to solve for the involved parameters. It is simple and straight forward when the variables involving the system of equations are crisp number. However, in actual case the system variables cannot be obtained as crisp. Those are found by some experiment in general with experimental uncertainties associated with the measurement. Therefore, these variables will either be an interval or a fuzzy number. For example, if we want to measure the length of a wire by a ruler, we do not get the crisp value from the measurement. In particular, the measured value depends on the viewing angle of the ruler as well as on the person measuring it. As such there will be vagueness in the result of the experiment. Hence, to overcome the vagueness we may use the interval and fuzzy numbers [Zadeh (1965)] instead of a crisp number.

Fuzzy linear systems have recently been studied by a good number of authors but only a few of them are mentioned here. A fuzzy linear system $Ax=b$ where A is a crisp matrix and b is a fuzzy number vector has been studied by Friedman et al. (1998), Ma et al. (2000) and Allahviranloo (2004, 2005). In particular, Friedman et al. (1998), investigated a general $n \times n$ fuzzy system of linear equation using the embedding approach and deduced the conditions for the existence of a unique fuzzy solution. Allahviranloo (2004, 2005) has used the iterative Jacobi and Gauss Siedel method, the Adomian method and the Successive over-relaxation method, respectively. Some iterative methods to solve fuzzy system of linear equations have been extended by Dehghan and Hashemi (2006). Ma et al. (2000) have discussed the existence of a solution of duality in fuzzy linear equation systems. Two necessary and sufficient conditions for the existence of solution are given. Furthermore, Muzziolia et al. (2006) discussed fuzzy linear systems in the form of $A_1x + b_1 = A_2x + b_2$ with A_1, A_2 square matrices of fuzzy coefficients and b_1, b_2 fuzzy number vectors. Abbasbandy and Jafarian (2006) proposed the steepest descent method for solving fuzzy system of linear equation. Solution of system of linear equations involving fuzzy input parameters for the engineering systems has been proposed by Rao and Chen (1998). Methods for solving fuzzy equations in economics and finance have been proposed as well [Buckley (1992)].

Dehgan et al. (2007, 2006) have proposed the Adomian decomposition method, iterative methods and some computational methods [Dehghan et al. (2006)] such as Cramer's rule, Gauss elimination method, LU decomposition method and linear programming approach for finding the solutions of $n\times n$ fully fuzzy system of linear systems where all the parameters are fuzzy numbers.

In the present article, we mainly concentrate on solving fuzzy system of linear equation for the problem of circuit analysis. There are many books having circuit analysis such as by Badrinarayanan and Nandini (2004) which gives system of linear equations, where the resistive networks may include fuzzy or interval number. Thus we need to solve interval/fuzzy system of linear equations. Recently Rahgooy et al. (2009) has investigated this type of system for the static response of a resistive network. Transient response in a circuit has been discussed by Yazdi et al. (2008) using a fuzzy differential equation with fuzzy variables. Recently Das and Chakraverty (2011) studied the Fuzzy system of linear equation and its application to circuit analysis.

The rest of the paper is organized as follows. In Section 2, we will give some basic preliminaries of interval and its arithmetic, fuzzy number, α-cut of a fuzzy number, various types of fuzzy numbers and its arithmetic. The concepts of these have been used for the numerical solution of system of linear equation. In Section 3, we propose new methods to solve fully fuzzy system of linear equations and interval problems in $\Re^+$. In section 4, we apply our proposed method in the case of circuit analysis. Investigation of circuit analysis has been performed for interval/fuzzy input and interval/fuzzy source which are governed by corresponding linear simultaneous equation. The solutions are also compared with the known results that are found in literature. In Section 5, we test our method for solving linear simultaneous equations with more unknown variables than in the case of circuit analysis. In Section 6, we present our conclusions.

2. Interval and Fuzzy Arithmetic

In this section we will discuss preliminaries of interval and its arithmetic and fuzzy number, *α*-cut of a fuzzy number, various types of fuzzy numbers and its arithmetic.

2.1. System of Linear Equations

The $n \times n$ linear systems of equations (crisp) may be written as

$$\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + \cdots + & a_{1n}x_n & = & y_1 \\ a_{21}x_1 & + & a_{22}x_2 & + \cdots + & a_{2n}x_n & = & y_2 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1}x_1 & + & a_{n2}x_2 & + \cdots + & a_{nn}x_n & = & y_n, \end{array} \tag{1}$$

where the coefficient matrix $A = (a_{ij}), 1 \le i, j \le n$, is a crisp $n \times n$ matrix.

Our aim is here to have the above system as either in interval or fuzzy variables and constants and their solution. In the following paragraphs we will first discuss preliminaries of interval arithmetic and then about fuzzy set and numbers.

2.2. Interval Arithmetic

An interval is a subset of $\Re$ such that $A = [a_1, a_2] = \{t \mid a_1 \le t \le a_2, a_1, a_2 \in \Re\}$.
If $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are two intervals, then the arithmetic operations are:

Addition:

$$[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$$

Subtraction

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

Product

$$[a_1, a_2] \bullet [b_1, b_2] = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)]$$

Division

$$[a_1, a_2]/[b_1, b_2] = [\min(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)] \quad b_1, b_2 \neq 0$$

2.3. Definition of a Fuzzy Set

Let X be a universal set. Then a fuzzy set A can be defined as the set of ordered pairs such that $A = \{(x, \mu_A(x)) / x \in X, \mu_A(x) \in [0,1]\}$, where $\mu_A(x)$ is called the membership function or grade of membership of x [Zimmermann (1996)].

2.4. Definition of a Fuzzy Number

Definition 1:

A fuzzy number is a convex normalized fuzzy set of the crisp set such that for only one $x \in X$, $\mu_A(x) = 1$ and $\mu_A(x)$ is piecewise continuous [Zimmermann (1996)]. Alternatively a fuzzy number A with the membership function $\mu_A(x), x \in X$, can be defined as [Goetschel and Voxman (1986)]:

$$\mu_{\bar{A}}(x) = \begin{cases} l_{\bar{A}}(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ r_{\bar{A}}(x), & c \le x \le d, \\ 0, & \text{otherwise}, \end{cases}$$

where $l_{\bar{A}}(x)$ is the left membership function that is an increasing function on [a, b] and $r_{\bar{A}}(x)$ is the right membership function that is decreasing function on [c, d] such that $l_{\bar{A}}(a) = r_{\bar{A}}(d) = 0$ and $l_{\bar{A}}(b) = r_{\bar{A}}(c) = 1$. In addition, if $l_{\bar{A}}(x)$ and $r_{\bar{A}}(x)$ are linear, then A is a trapezoidal fuzzy number discussed in the following paragraph.

Definition 2:

A fuzzy number A is a pair $(\underline{u}(r), \bar{u}(r))$ of functions $\underline{u}(r)$ and $\bar{u}(r)$, $0 \le r \le 1$ which satisfy the following requirements [Kaleva (1987)]:

(i) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,

(ii) $\bar{u}(r)$ is a bounded monotonic decreasing left continuous function, and

(iii) $\underline{u}(r) \le \bar{u}(r)$ in $0 \le r \le 1$

This is known as parametric form of a fuzzy number.

2.5. Types of a Fuzzy Number

Here we will discuss two types of fuzzy numbers, namely,

1) Triangular Fuzzy Number

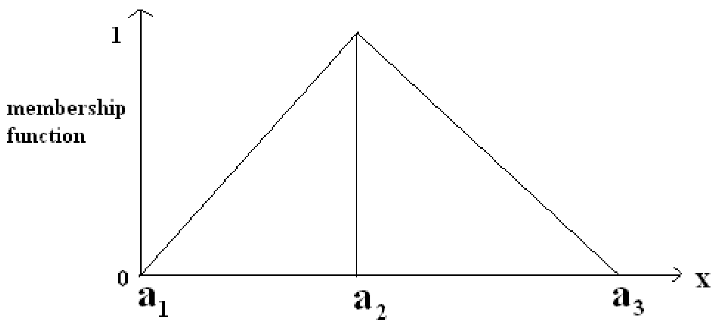

Figure 1. Triangular Fuzzy Number

A triangular fuzzy number (TFN) as shown in Figure 1 is a special type of fuzzy number and its membership function $\mu_{\bar{A}}(x)$ is given by:

$$\mu_{\bar{A}}(x) = \begin{cases} 0, & x \le a, \\ \dfrac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2], \\ \dfrac{a_3-x}{a_3-a_2}, & x \in [a_2, a_3], \\ 0, & x \ge a_3. \end{cases}$$

2) Trapezoidal Fuzzy Number

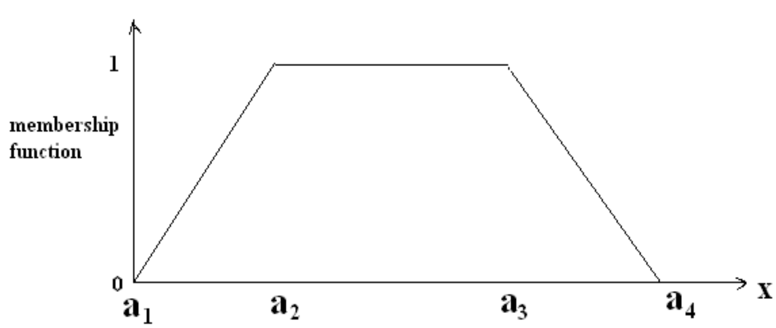

Figure 2. Trapezoidal fuzzy number

A trapezoidal fuzzy number (Tr F N) as shown in Figure 2 is a special type of fuzzy number and its membership function $\mu_{\overline{A}}(x)$ is given by

$$\mu_{\overline{A}}(x)=\begin{cases}0, & x\le a_1,\\ \dfrac{x-a_1}{a_2-a_1}, & x\in[a_1,a_2],\\ 1, & x\in[a_2,a_3],\\ \dfrac{a_4-x}{a_4-a_3}, & x\in[a_3,a_4],\\ 0, & x\ge a_4.\end{cases}$$

2.6. Definition of Alpha Cut

The crisp set of elements that belong to the fuzzy set $\overline{A}$ at least to the degree α is called the α-level set

$A_\alpha=\{x\in X:\mu_{\overline{A}}(x)\ge\alpha\}$ [Zimmermann (1996)] where $\alpha\in[0,1]$.

2.7. Conversion from Fuzzy Number to Interval Using Alpha Cut

1) Triangular Fuzzy Number to Interval

Let, a triangular fuzzy number defined as $\overline{A}=(a_1,a_2,a_3)$, then to find α-cut of $\overline{A}$ we first set α equal to the left and right membership function of $\overline{A}$. That is,

$$\alpha=\frac{x-a_1}{a_2-a_1} \quad \text{and} \quad \alpha=\frac{a_3-x}{a_3-a_2}.$$

Expressing x in terms of α we have, $x=\alpha(a_2\text{-}a_1)+a_1$ and $x=-\alpha(a_3\text{-}a_2)+a_3$.
Therefore, we can write the fuzzy interval in terms of α -cut interval as:

$$\overline{A_\alpha}=[\alpha(a_2-a_1)+a_1,-\alpha(a_3-a_2)+a_3]$$

2) Trapezoidal Fuzzy Number to Interval

Let, a trapezoidal fuzzy number defined as $\overline{A}=(a_1,a_2,a_3,a_4)$
Following the similar procedure as above, we can write the fuzzy interval in terms of α -cut interval as:

$$\overline{A_\alpha}=[\alpha(a_2-a_1)+a_1,-\alpha(a_4-a_3)+a_4].$$

2.8. Fuzzy Arithmetic

As $\overline{A_\alpha}$ is now interval, so fuzzy addition, subtraction, multiplication and division are same as interval arithmetic.

3. Interval/Fuzzy Solution of $n\times n$ System of Linear Equations

Here we will outline the proposed method of solving fully fuzzy linear system of the form $\overline{A}\overline{x}=\overline{b}$ where $\overline{A}$ and $\overline{b}$ are a fuzzy matrix and a fuzzy vector respectively. We will solve for the unknown fuzzy vector $\overline{x}$. We will first convert fuzzy numbers in the form of intervals using α-cut. For example, a fuzzy number a_{ij} (which is an element of the matrix $\overline{A}$) can be written in the form $(\underline{a_{ij}},\overline{a_{ij}})$ using α-cut. Similarly, the nth element of the fuzzy vector can be written in the form of interval using α-cut. Converting the fuzzy numbers in the form of intervals, we can write $n\times n$ system of linear equations as follows:

$$\begin{aligned}
&(\underline{a_{11}},\overline{a_{11}})(\underline{x_1},\overline{x_1})+(\underline{a_{12}},\overline{a_{12}})(\underline{x_2},\overline{x_2})+\ldots.+(\underline{a_{1n}},\overline{a_{1n}})(\underline{x_n},\overline{x_n})=(\underline{r_1},\overline{r_1})\\
&(\underline{a_{21}},\overline{a_{21}})(\underline{x_1},\overline{x_1})+(\underline{a_{22}},\overline{a_{22}})(\underline{x_2},\overline{x_2})+\ldots.+(\underline{a_{2n}},\overline{a_{2n}})(\underline{x_n},\overline{x_n})=(\underline{r_2},\overline{r_2})\\
&\vdots\\
&(\underline{a_{n1}},\overline{a_{n1}})(\underline{x_1},\overline{x_1})+(\underline{a_{n2}},\overline{a_{n2}})(\underline{x_2},\overline{x_2})+\ldots.+(\underline{a_{nn}},\overline{a_{nn}})(\underline{x_n},\overline{x_n})=(\underline{r_n},\overline{r_n}),
\end{aligned} \tag{2}$$

where all a_{ij} are in $\Re^+$.

The above equations may be written equivalently as

$$\begin{aligned}
&\underline{a_{11}}\ \underline{x_1}+\underline{a_{12}}\ \underline{x_2}+\ldots+\underline{a_{1n}}\ \underline{x_n}=\underline{r_1}\\
&\overline{a_{11}}\ \overline{x_1}+\overline{a_{12}}\ \overline{x_2}+\ldots+\overline{a_{1n}}\ \overline{x_n}=\overline{r_1}\\
&\underline{a_{21}}\ \underline{x_1}+\underline{a_{22}}\ \underline{x_2}+\ldots+\underline{a_{2n}}\ \underline{x_n}=\underline{r_2}\\
&\overline{a_{21}}\ \overline{x_1}+\overline{a_{22}}\ \overline{x_2}+\ldots+\overline{a_{2n}}\ \overline{x_n}=\overline{r_2}\\
&.\\
&.\\
&\underline{a_{n1}}\ \underline{x_1}+\underline{a_{n2}}\ \underline{x_2}+\ldots+\underline{a_{nn}}\ \underline{x_n}=\underline{r_n}\\
&\overline{a_{n1}}\ \overline{x_1}+\overline{a_{n2}}\ \overline{x_2}+\ldots+\overline{a_{nn}}\ \overline{x_n}=\overline{r_n}
\end{aligned} \tag{3}$$

Equations (3) can now be written in matrix form as:

$$\begin{pmatrix} \underline{a_{11}} & \underline{a_{12}} & . & . & . & \underline{a_{1n}} & 0 & 0 & . & . & . & 0 \\ 0 & 0 & . & . & . & 0 & \overline{a_{11}} & \overline{a_{12}} & . & . & . & \overline{a_{1n}} \\ \underline{a_{21}} & \underline{a_{22}} & . & . & . & \underline{a_{2n}} & 0 & 0 & . & . & . & 0 \\ 0 & 0 & . & . & . & 0 & \overline{a_{21}} & \overline{a_{22}} & . & . & . & \overline{a_{2n}} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ \underline{a_{n1}} & \underline{a_{n2}} & . & . & . & \underline{a_{nn}} & 0 & 0 & . & . & . & 0 \\ 0 & 0 & . & . & . & 0 & \overline{a_{n1}} & \overline{a_{n2}} & . & . & . & \overline{a_{nn}} \end{pmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ . \\ . \\ . \\ \underline{x_n} \\ \overline{x_1} \\ \overline{x_2} \\ . \\ . \\ . \\ \overline{x_n} \end{pmatrix} = \begin{pmatrix} \underline{r_1} \\ \overline{r_1} \\ \underline{r_2} \\ \overline{r_2} \\ . \\ . \\ . \\ \underline{r_n} \\ \overline{r_n} \end{pmatrix} \tag{4}$$

For clear understanding we now give the procedure with two equations and two unknowns. Therefore, for two equations and two unknowns we have:

$$(\underline{a_{11}}, \overline{a_{11}})(\underline{x_1}, \overline{x_1}) + (\underline{a_{12}}, \overline{a_{12}})(\underline{x_2}, \overline{x_2}) = (\underline{r_1}, \overline{r_1}), \tag{5}$$

$$(\underline{a_{21}}, \overline{a_{21}})(\underline{x_1}, \overline{x_1}) + (\underline{a_{22}}, \overline{a_{22}})(\underline{x_2}, \overline{x_2}) = (\underline{r_2}, \overline{r_2}). \tag{6}$$

Similar to Equation (4) one may write (5) and (6) as

$$\begin{bmatrix} \underline{a_{11}} & \underline{a_{12}} & 0 & 0 \\ 0 & 0 & \overline{a_{11}} & \overline{a_{12}} \\ \underline{a_{21}} & \underline{a_{22}} & 0 & 0 \\ 0 & 0 & \overline{a_{21}} & \overline{a_{22}} \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \overline{x_1} \\ \overline{x_2} \end{pmatrix} = \begin{pmatrix} \underline{r_1} \\ \overline{r_1} \\ \underline{r_2} \\ \overline{r_2} \end{pmatrix}.$$

This is a crisp system of equation and the above matrix equation may be easily solved with the following solutions,

$$\underline{x_1} = \frac{\underline{a_{22}}\,\underline{r_1} - \underline{a_{12}}\,\underline{r_2}}{\underline{a_{11}}\,\underline{a_{22}} - \underline{a_{12}}\,\underline{a_{21}}}$$

$$\underline{x_2} = \frac{\underline{a_{11}}\,\underline{r_2} - \underline{a_{21}}\,\underline{r_1}}{\underline{a_{11}}\,\underline{a_{22}} - \underline{a_{12}}\,\underline{a_{21}}}$$

$$\overline{x_1} = \frac{\overline{a_{22}}\,\overline{r_1} - \overline{a_{12}}\,\overline{r_2}}{\overline{a_{11}}\,\overline{a_{22}} - \overline{a_{12}}\,\overline{a_{21}}}$$

$$\overline{x_2} = \frac{\overline{a_{11}}\,\overline{r_2} - \overline{a_{21}}\,\overline{r_1}}{\overline{a_{11}}\,\overline{a_{22}} - \overline{a_{12}}\,\overline{a_{21}}}.$$

Otherwise, we may also write equations (5) and (6) as left and right as

LEFT:

$$\begin{aligned} \underline{a}_{11}\,\underline{x}_1 + \underline{a}_{12}\,\underline{x}_2 &= \underline{r}_1 \\ \overline{a}_{11}\,\overline{x}_1 + \overline{a}_{12}\,\overline{x}_2 &= \overline{r}_1 \end{aligned} \qquad (7) - (8)$$

RIGHT:

$$\begin{aligned} \underline{a}_{21}\,\underline{x}_1 + \underline{a}_{22}\,\underline{x}_2 &= \underline{r}_2 \\ \overline{a}_{21}\,\overline{x}_1 + \overline{a}_{22}\,\overline{x}_2 &= \overline{r}_2 \end{aligned} \qquad (9) - (10)$$

Solving (7) and (9); (8) and (10) we get,

$$\underline{x}_1 = \frac{\underline{a}_{22}\,\underline{r}_1 - \underline{a}_{12}\,\underline{r}_2}{\underline{a}_{11}\,\underline{a}_{22} - \underline{a}_{12}\,\underline{a}_{21}}$$

$$\underline{x}_2 = \frac{\underline{a}_{11}\,\underline{r}_2 - \underline{a}_{21}\,\underline{r}_1}{\underline{a}_{11}\,\underline{a}_{22} - \underline{a}_{12}\,\underline{a}_{21}}$$

$$\overline{x}_1 = \frac{\overline{a}_{22}\,\overline{r}_1 - \overline{a}_{12}\,\overline{r}_2}{\overline{a}_{11}\,\overline{a}_{22} - \overline{a}_{12}\,\overline{a}_{21}}$$

$$\overline{x}_2 = \frac{\overline{a}_{11}\,\overline{r}_2 - \overline{a}_{21}\,\overline{r}_1}{\overline{a}_{11}\,\overline{a}_{22} - \overline{a}_{12}\,\overline{a}_{21}}$$

As is shown in the above methods that we may solve for the left and right individually too although both the procedures are the same.

Since the elements of the matrix and vectors are now crisp, we can alternatively solve the system of linear equations in (4) by taking the inverse of the coefficient matrix (if the inverse of the coefficient matrix exists). One may also use the inverse of the coefficient matrices in (7)-(8) (Left) and (9)-(10) (Right) to get the final solution.

4. Numerical Example by Known and Proposed Method

4.1. Interval and Fuzzy Equations

Let us first consider the following two interval equations with two unknowns. First of all we will start with an example given in Rahgooy et al. (2009).

Here three different cases taking source and resistance as crisp or interval or fuzzy are considered in the circuit analysis. These cases are named as case I to case III in the following paragraph. In the Case I, we will consider the same circuit as presented by Rahgooy et al. (2009) where resistance is considered as crisp and both the source and current is taken as fuzzy. The results of our method are compared with Rahgooy et al. (2009). In the Case II, we will consider all the elements in the circuit as intervals. In Case III, we will further generalize the circuit elements and represent them in the form of triangular fuzzy numbers. In the Case II and Case III, we will solve for the unknown currents in the circuit in the form of intervals and triangular fuzzy numbers, respectively.

Case-I

4.1.1. Method of Rahgooy et al. (2009)

First, we will consider a circuit (Rahgooy et al. (2009)), having source as well as current as fuzzy and resistance as crisp. Related circuit is shown in Figure 3.

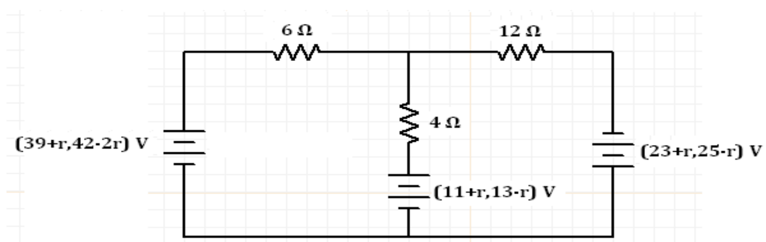

Figure 3. A circuit with fuzzy current, fuzzy source and crisp resistance

From the first and second loop, Kirchhoff's second law gives,

$$6I_1 + 4(I_1 - I_2) + (11+r, 13-r) - (39+r, 42-2r) = 0 \tag{11}$$

$$-4(I_1 - I_2) + 12I_2 - (23+r, 25-r) - (11+r, 13-r) = 0 \tag{12}$$

$$\Rightarrow 10I_1 - 4I_2 = (39+r, 42-2r) - (11+r, 13-r) - 4I_1 + 16I_2$$
$$= (23+r, 25-r) + (11+r, 13-r).$$

The above may be written as

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & -4 \\ 0 & 16 & -4 & 0 \\ 0 & -4 & 10 & 0 \\ -4 & 0 & 0 & 16 \end{bmatrix}, \quad \hat{Y} = \begin{bmatrix} 26+2r \\ 34+2r \\ 31-3r \\ 38-2r \end{bmatrix}, \quad I = \begin{bmatrix} \underline{I_1} \\ \underline{I_2} \\ \overline{I_1} \\ \overline{I_2} \end{bmatrix}.$$

Now writing $SI = \hat{Y}$, as in (3), we have

$$\begin{aligned} 10\underline{I_1} - 4\overline{I_2} &= 26 + 2r \\ 16\underline{I_2} - 4\overline{I_1} &= 34 + 2r \\ -4\underline{I_2} + 10\overline{I_1} &= 31 - 3r \\ -4\underline{I_1} + 16\overline{I_2} &= 38 - 2r \end{aligned} \qquad (13-16)$$

So, the solution in interval form may be written as

$$I_1 = \left(\frac{71}{18} + \frac{1}{6}r, \frac{79}{18} - \frac{5}{18}r\right) = (3.944 + 0.167r, 4.389 - 0.278r)$$

$$I_2 = \left(\frac{29}{9} + \frac{1}{18}r, \frac{121}{36} - \frac{1}{12}r\right) = (3.22 + 0.056r, 3.361 - 0.833r)$$

Corresponding plot of the solutions are given in Figure 4.

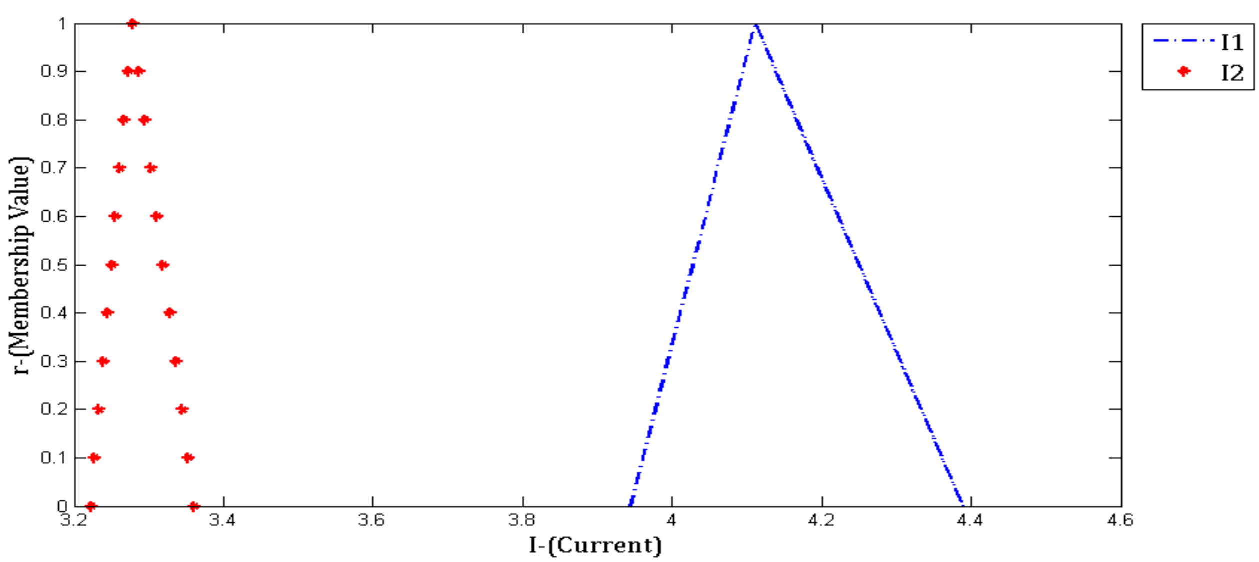

Figure 4. The solution of the system for case I

4.1.2. Proposed Method

Now, the above problem is solved using the proposed methods. The problem is given as,

$$6I_1 + 4(I_1 - I_2) + (11 + r, 13 - r) - (39 + r, 42 - 2r) = 0$$
$$-4(I_1 - I_2) + 12I_2 - (23 + r, 25 - r) - (11 + r, 13 - r) = 0 \qquad (17)-(18)$$

This can be written as

$$6(\underline{I}_1, \overline{I}_1) + 4((\underline{I}_1, \overline{I}_1) - (\underline{I}_2, \overline{I}_2)) = (26 + 2r, 31 - 3r)$$
$$-4((\underline{I}_1, \overline{I}_1) - (\underline{I}_2, \overline{I}_2)) + 12(\underline{I}_2, \overline{I}_2) = (34 + 2r, 38 - 2r) \qquad (19)-(20)$$

Simplifying the above equations we can get,

$$10\underline{I}_1 - 4\overline{I}_2 = 26 + 2r$$
$$10\overline{I}_1 - 4\underline{I}_2 = 31 - 2r$$
$$-4\underline{I}_1 - 16\overline{I}_2 = 38 - 2r$$
$$4\overline{I}_1 - 16\underline{I}_2 = 34 + 2r \qquad (21)-(24)$$

First we can get the matrix directly as

$$\begin{bmatrix} 10 & 0 & 0 & -4 \\ 0 & 10 & -4 & 0 \\ 0 & -4 & 16 & 0 \\ -4 & 0 & 0 & 16 \end{bmatrix} \begin{pmatrix} \underline{I}_1 \\ \overline{I}_1 \\ \underline{I}_2 \\ \overline{I}_2 \end{pmatrix} = \begin{pmatrix} 26+2r \\ 31-3r \\ 34+2r \\ 38-2r \end{pmatrix}$$

So, the solution in interval form may be written as

$$I_1 = (\underline{I}_1, \overline{I}_1) = \left(\frac{71}{18} + \frac{1}{6}r, \frac{79}{18} - \frac{5}{18}r\right) = (3.944 + 0.167r, 4.389 - 0.278r)$$

$$I_2 = (\underline{I}_2, \overline{I}_2) = \left(\frac{29}{9} + \frac{1}{18}r, \frac{121}{36} - \frac{1}{12}r\right) = (3.22 + 0.056r, 3.361 - 0.833r)$$

Otherwise we may write as left and write and we may get the same solution as before. Using this procedure from (21); (23) and (22); (24) we get the results as:

$$(\underline{I}_1, \overline{I}_2) = \left(\frac{71}{18} + \frac{1}{6} r, \frac{121}{36} - \frac{1}{12} r \right) = (3.944 + 0.167 r, 3.361 - 0.833 r)$$

$$(\overline{I}_1, \underline{I}_2) = \left(\frac{79}{18} - \frac{5}{18} r, \frac{29}{9} + \frac{1}{18} r \right) = (4.389 - 0.278 r, 3.22 + 0.056 r)$$

$$\text{So, } I_1 = (\underline{I}_1, \overline{I}_1) = \left(\frac{71}{18} + \frac{1}{6} r, \frac{79}{18} - \frac{5}{18} r \right) = (3.944 + 0.167 r, 4.389 - 0.278 r)$$

$$I_2 = (\underline{I}_2, \overline{I}_2) = \left(\frac{29}{9} + \frac{1}{18} r, \frac{121}{36} - \frac{1}{12} r \right) = (3.22 + 0.056 r, 3.361 - 0.833 r)$$

As mentioned both the procedures turns out to be same and so the results are also exactly same.

4.2. Case-II

The next example gives the solution for a circuit with source, current and resistance all in interval. Corresponding circuit is shown in Figure 5.

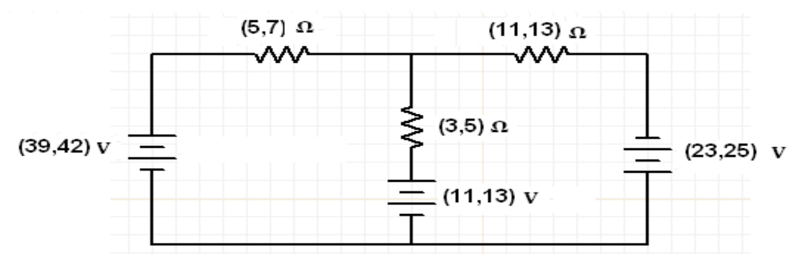

Figure 5. A circuit with current, source and resistance in interval

From the above circuit using Kirchhoff's second law and interval arithmetic we can write:

$$(5,7)(\underline{I}_1, \overline{I}_1) + (3,5)((\underline{I}_1, \overline{I}_1) - (\underline{I}_2, \overline{I}_2)) - (26,31) = 0$$
$$-(3,5)((\underline{I}_1, \overline{I}_1) - (\underline{I}_2, \overline{I}_2)) + (11,13)(\underline{I}_2, \overline{I}_2) - (34,38) = 0$$

Simplifying the above two equations we obtain:

$$\begin{aligned} 8\underline{I}_1 - 3\overline{I}_2 &= 26 \\ 12\overline{I}_1 - 5\underline{I}_2 &= 31 \\ 5\overline{I}_1 + 16\underline{I}_2 &= 34 \\ -3\underline{I}_1 + 16\overline{I}_2 &= 38. \end{aligned} \qquad (25) – (28)$$

Solution of the equations (25) to (28) are:

$$\underline{I_1} = \frac{530}{119} = 4.454$$
$$\overline{I_1} = \frac{666}{167} = 3.99$$
$$\underline{I_2} = \frac{563}{167} = 3.37$$
$$\overline{I_2} = \frac{382}{119} = 3.21.$$

Here lower nodes are larger than the upper nodes and it is called a weak solution. If the right hand side '0' is considered as interval which includes zero for example as [-3, 3] then the equations after some calculation can be written as

$$\begin{aligned} 8\underline{I_1} - 3\overline{I_2} &= 23 \\ 12\overline{I_1} - 5\underline{I_2} &= 34 \\ 5\overline{I_1} + 16\underline{I_2} &= 31 \\ -3\underline{I_1} + 16\overline{I_2} &= 41. \end{aligned} \qquad (29)-(32)$$

Solution of (29) to (32) gives,

$$\underline{I_1} = \frac{491}{119} = 4.12$$
$$\overline{I_1} = \frac{699}{167} = 4.18$$
$$\underline{I_2} = \frac{542}{167} = 3.24$$
$$\overline{I_2} = \frac{397}{119} = 3.33.$$

In this case first we get a weak solution. Then a strong solution is obtained when the right hand side '0' is arbitrarily changed to interval which includes '0'.

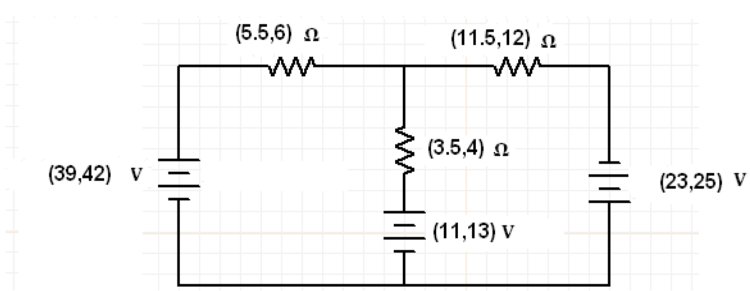

Figure 6. A circuit with interval current, source and resistance

This case is simulated by reducing the interval width for the resistance as shown in Figure 6. The corresponding solutions are given as follows:

$$\overline{I_1} = \frac{1233}{278} = 4.435$$
$$\underline{I_1} = \frac{2144}{509} = 4.21$$
$$\underline{I_2} = \frac{464}{139} = 3.34$$
$$\overline{I_2} = \frac{1732}{509} = 3.40.$$

4.3. Case-III

In this case the above example is considered taking resistance and voltage both as fuzzy and the corresponding circuit is shown in Figure 7. The solution is easily obtained following the procedure similar to Case I and II.

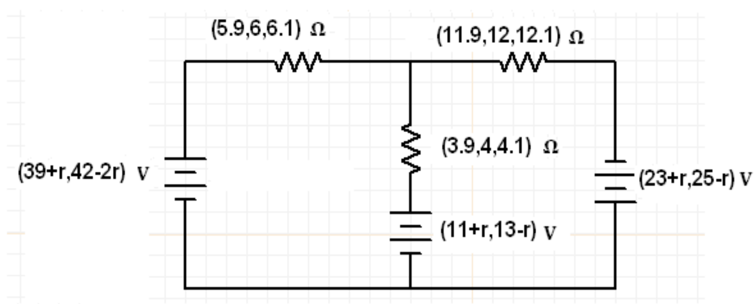

Figure 7. A circuit with fuzzy source, fuzzy current and fuzzy resistance

From the above circuit using Kirchhoff's second law and interval arithmetic we can write

$$(5.9,6,6.1)(\underline{I_1},\overline{I_1}) + (3.9,4,4.1)((\underline{I_1},\overline{I_1}) - (\underline{I_2},\overline{I_2})) = (26 + 2r, 31 - 3r)$$

$$-(3.9,4,4.1)((\underline{I_1},\overline{I_1}) - (\underline{I_2},\overline{I_2})) + (11.9,12,12.1)(\underline{I_2},\overline{I_2}) = (34 + 2r, 38 - 2r)$$

In the above circuit resistances are represented by triangular fuzzy numbers in the triplet (a_1, a_2, a_3) form and the voltage sources are already converted to interval form using α-cut (r is equivalent to α). Transforming the resistances also in the interval form using α-cut and simplifying, we get:

$$(0.1\alpha+5.9,-0.1\alpha+6.1)(\underline{I}_1,\overline{I}_1)+(0.1\alpha+3.9,-0.1\alpha+4.1)(\underline{I}_1-\overline{I}_2,\overline{I}_1-\underline{I}_2)=(26+2r,31-3r) \quad (33)$$

$$-(0.1\alpha+3.9,-0.1\alpha+4.1)(\underline{I}_1-\overline{I}_2,\overline{I}_1-\underline{I}_2)+(0.1\alpha+11.9,-0.1\alpha+12.1)(\underline{I}_2,\overline{I}_2)=(34+2r,38-2r) \quad (34)$$

$$\begin{aligned}
\Rightarrow\ & (0.2\alpha+9.8)\underline{I}_1-(0.1\alpha+3.9)\overline{I}_2=26+2r \\
& (-0.2\alpha+10.2)\overline{I}_1-(-0.1\alpha+4.1)\underline{I}_2=31-3r \\
& -(-0.1\alpha+4.1)\overline{I}_1+16\underline{I}_2=34+2r \\
& -(0.1\alpha+3.9)\underline{I}_1+16\overline{I}_2=38-2r
\end{aligned}$$

From the above equations one may write:

$$\begin{bmatrix} 0.2\alpha+9.8 & -(0.1\alpha+3.9) & 0 & 0 \\ 0 & 0 & (-0.2\alpha+10.2) & -(-0.1\alpha+4.1) \\ 0 & 0 & -(-0.1\alpha+4.1) & 16 \\ -(0.1\alpha+3.9) & 16 & 0 & 0 \end{bmatrix}\begin{pmatrix}\underline{I}_1 \\ \overline{I}_2 \\ \overline{I}_1 \\ \underline{I}_2\end{pmatrix}=\begin{pmatrix}26+2r \\ 31-3r \\ 34+2r \\ 38-2r\end{pmatrix}.$$

The obtained solution with $r=\alpha$ is given below

$$\underline{I}_1=\frac{20(\alpha^2-140\alpha-2821)}{\alpha^2-242\alpha-14159}, \qquad \overline{I}_2=\frac{20(\alpha^2+8\alpha-2369)}{\alpha^2-242\alpha-14159}$$

$$\overline{I}_1=\frac{20(\alpha^2+216\alpha-3177)}{\alpha^2+238\alpha-14639}, \qquad \underline{I}_2=\frac{10(\alpha^2+18\alpha-4739)}{\alpha^2+238\alpha-14639}.$$

Corresponding graph is shown in Figure 8.

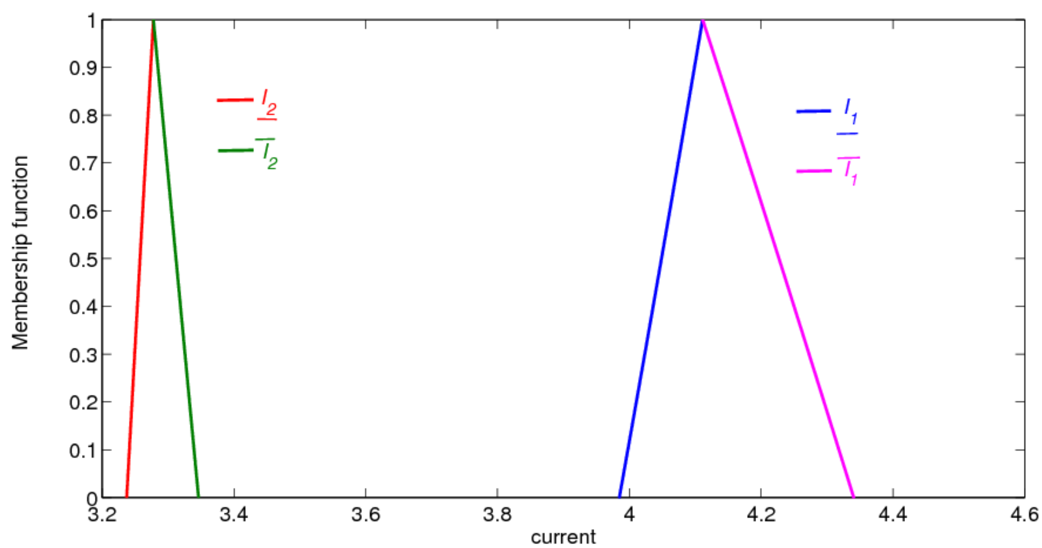

Figure 8. The solution of the system from the above example

5. Some More Examples of Crisp, Interval and Fuzzy Equation by the Proposed Method

In this section we will show the reliability of our proposed method by solving more example problems.

Example 1.

In this example, we will solve three equations with three unknowns with the crisp coefficients.

$$0.4(\underline{x_1},\overline{x_1}) + 1.4(\underline{x_2},\overline{x_2}) + 0.3(\underline{x_3},\overline{x_3}) = 0.1$$

$$0.15(\underline{x_1},\overline{x_1}) + 0.14(\underline{x_2},\overline{x_2}) + 6.1(\underline{x_3},\overline{x_3}) = 0.14$$

$$5.1(\underline{x_1},\overline{x_1}) + 0.3(\underline{x_2},\overline{x_2}) + 0.2(\underline{x_3},\overline{x_3}) = 0.14$$

These equations can be written in the interval form as

$$\begin{aligned}
&(0.4,0.4)(\underline{x_1},\overline{x_1}) + (1.4,1.4)(\underline{x_2},\overline{x_2}) + (0.3,0.3)(\underline{x_3},\overline{x_3}) = (0.1,0.1)\\
&(0.15,0.15)(\underline{x_1},\overline{x_1}) + (0.14,0.14)(\underline{x_2},\overline{x_2}) + (6.1,6.1)(\underline{x_3},\overline{x_3}) = (0.14,0.14)\\
&(5.1,5.1)(\underline{x_1},\overline{x_1}) + (0.3,0.3)(\underline{x_2},\overline{x_2}) + (0.2,0.2)(\underline{x_3},\overline{x_3}) = (0.14,0.14)
\end{aligned} \qquad (35)\text{-}(37)$$

From (35)-(37) we get,

$$\begin{bmatrix} 0.4 & 1.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.14 & 6.1 \\ 5.1 & 0.3 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.1 & 0.3 & 0.2 \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \\ \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.14 \\ 0.14 \\ 0.14 \\ 01.4 \end{pmatrix}$$

Solving the above crisp matrix equation we get the solution as

$$\underline{x_1} = 0.0231$$
$$\underline{x_2} = 0.0603$$
$$\underline{x_3} = 0.0210$$

$$\overline{x_1} = 0.0231$$
$$\overline{x_2} = 0.0603$$
$$\overline{x_3} = 0.0210$$

Example 2.

In this example again three equations in three unknowns are considered but here the coefficients are in interval. The system considered is as below.

$$(0.1,0.9)(\underline{x}_1,\overline{x_1}) + (1,1.9)(\underline{x}_2,\overline{x_2}) + (0.11,0.9)(\underline{x}_3,\overline{x_3}) = (0.01,0.2)$$

$$(0.1,0.2)(\underline{x}_1,\overline{x_1}) + (0.11,0.2)(\underline{x}_2,\overline{x_2}) + (6,6.2)(\underline{x}_3,\overline{x_3}) = (0.11,0.2) \qquad (38)\text{-}(40)$$

$$(5,5.4)(\underline{x}_1,\overline{x_1}) + (0.1,0.4)(\underline{x}_2,\overline{x_2}) + (0.11,0.4)(\underline{x}_3,\overline{x_3}) = (0.1,0.2)$$

Equation (38)-(40) can be written in matrix form as:

$$\begin{bmatrix} 0.1 & 1 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 1.9 & 0.9 \\ 0.1 & 0.11 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.2 & 6.2 \\ 5 & 0.1 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.4 & 0.4 & 0.4 \end{bmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_4 \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.2 \\ 0.11 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$$

Solution of the above matrix equation is obtained as,

$$\underline{x}_1 = 0.0195$$
$$\underline{x}_2 = 0.0061$$
$$\underline{x}_3 = 0.0179$$

$$\overline{x}_1 = 0.0291$$
$$\overline{x}_2 = 0.0778$$
$$\overline{x}_3 = 0.0288$$

Example 3.

The coefficients of the following system of equations are TFN and can be written as,

$$(0.1,0.4,0.9)(\underline{x}_1,\overline{x_1}) + (1,1.4,1.9)(\underline{x}_2,\overline{x_2}) + (0.11,0.3,0.9)(\underline{x}_3,\overline{x_3}) = (0.01,0.1,0.2)$$

$$(0.1,0.15,0.2)(\underline{x}_1,\overline{x_1}) + (0.11,0.14,0.2)(\underline{x}_2,\overline{x_2}) + (6,6.1,6.2)(\underline{x}_3,\overline{x_3}) = (0.11,0.14,0.2)$$

$$(5,5.1,5.4)(\underline{x}_1,\overline{x_1}) + (0.1,0.3,0.4)(\underline{x}_2,\overline{x_2}) + (0.11,0.2,0.4)(\underline{x}_3,\overline{x_3}) = (0.1,0.14,0.2)$$

Transforming the fuzzy equations into interval form we obtain:

$$(0.3\alpha+0.1,-0.5\alpha+0.9)(\underline{x}_1,\overline{x}_1)+(0.4\alpha+1,-0.5\alpha+1.9)(\underline{x}_2,\overline{x}_2)+(0.19\alpha+0.11,-0.6\alpha+0.9)(\underline{x}_3,\overline{x}_3)=(0.09\alpha+0.01,-0.1\alpha+0.2)$$
$$(0.05\alpha+0.1,-0.05\alpha+0.2)(\underline{x}_1,\overline{x}_1)+(0.03\alpha+0.11,-0.06\alpha+0.2)(\underline{x}_2,\overline{x}_2)+(0.1\alpha+6,-0.1\alpha+6.2)(\underline{x}_3,\overline{x}_3)=(0.03\alpha+0.11,-0.06\alpha+0.2)$$
$$(0.1\alpha+5,-0.3\alpha+5.4)(\underline{x}_1,\overline{x}_1)+(0.2\alpha+0.1,-0.1\alpha+0.4)(\underline{x}_2,\overline{x}_2)+(0.09\alpha+0.11,-0.2\alpha+0.4)(\underline{x}_3,\overline{x}_3)=(0.04\alpha+0.1,-0.06\alpha+0.2)$$

(41) - (43)

Equation (41)-(43) may be written in matrix form as,

$$\begin{bmatrix} 0.3\alpha+0.1 & 0.4\alpha+1 & 0.19\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5\alpha+0.9 & -0.5\alpha+1.9 & -0.6\alpha+0.9 \\ 0.05\alpha+0.1 & 0.03\alpha+0.11 & 0.1\alpha+6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.05\alpha+0.2 & -0.06\alpha+0.2 & -0.1\alpha+6.2 \\ 0.1\alpha+5 & 0.2\alpha+0.1 & 0.09\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3\alpha+5.4 & -0.1\alpha+0.4 & -0.2\alpha+0.4 \end{bmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_3 \end{pmatrix} = \begin{pmatrix} 0.09\alpha+0.01 \\ -0.1\alpha+0.2 \\ 0.03\alpha+0.11 \\ -0.06\alpha+0.2 \\ 0.04\alpha+0.1 \\ -0.06\alpha+0.2 \end{pmatrix}$$

$$\underline{x}_1=\frac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x}_2=\frac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x}_3=\frac{(41\alpha^3-4387\alpha^2-31628\alpha-53460)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\overline{x}_1=\frac{-2/5(34\alpha^3+2689\alpha^2-22150\alpha+44000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$
$$\overline{x}_2=\frac{-(40\alpha^3-1513\alpha^2-19830\alpha+47000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$
$$\overline{x}_3=\frac{1/5(250\alpha^3-8017\alpha^2+50050\alpha-87000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$

Corresponding plots are shown in Figures 9, 10 and 11 respectively.

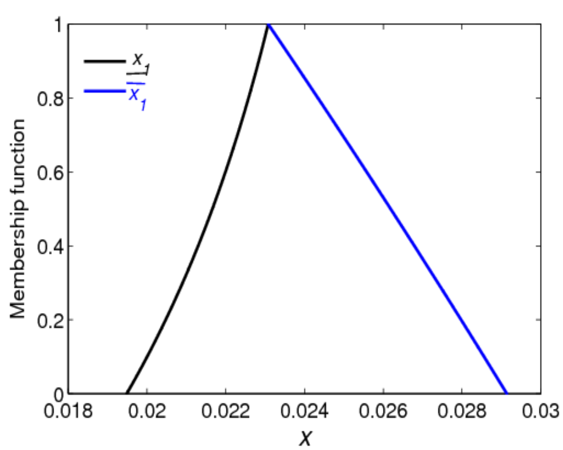

Figure 9. Fuzzy solution for x_1

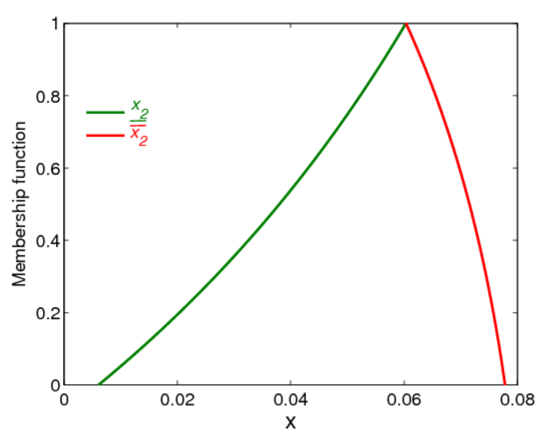

Figure 10. Fuzzy solution for x_2

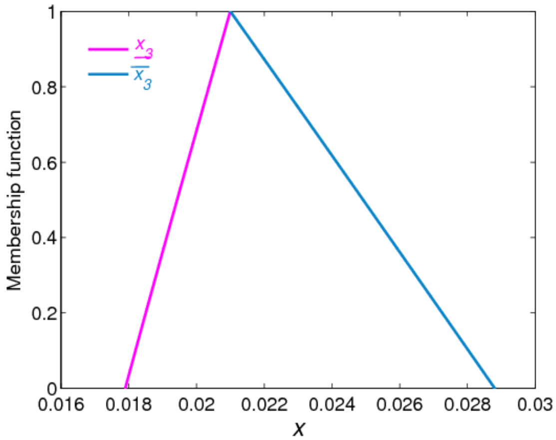

Figure 11. Fuzzy solution for x_3

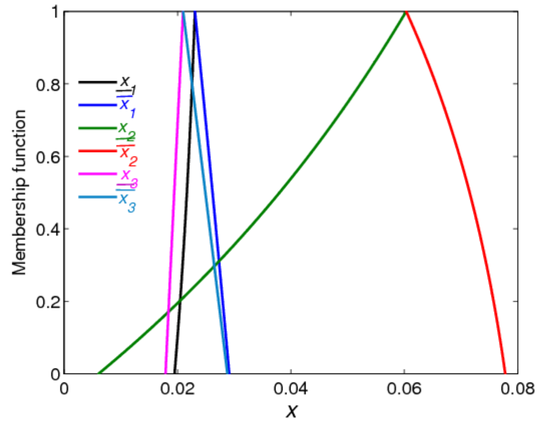

Figure 12. Fuzzy solution for x_1, x_2, x_3

Plots for x_1, x_2 and x_3 are given all together in Figure 12.

Here we note that the obtained solutions are also triangular fuzzy numbers as the elements of the coefficient matrix.

Special Case:

One may note that the result for crisp case can be obtained simply by putting $\alpha = 1$ in the fuzzy case. Moreover, by substituting $\alpha = 0$ in the fuzzy case, we may obtain the results for the interval case.

Example 4.

In this example, the coefficients of the following system of equations are considered as trapezoidal fuzzy numbers:

$$(0.1, 0.4, 0.6, 0.9)(\underline{x}_1, \overline{x}_1) + (1, 1.4, 1.6, 1.9)(\underline{x}_2, \overline{x}_2) + (0.11, 0.3, 0.5, 0.9)(\underline{x}_3, \overline{x}_3) = (0.01, 0.1, 0.15, 0.2)$$
$$(0.1, 0.15, 0.18, 0.2)(\underline{x}_1, \overline{x}_1) + (0.11, 0.14, 0.18, 0.2)(\underline{x}_2, \overline{x}_2) + (6, 6.1, 6.15, 6.2)(\underline{x}_3, \overline{x}_3) = (0.11, 0.14, 0.16, 0.2)$$
$$(5, 5.1, 5.2, 5.4)(\underline{x}_1, \overline{x}_1) + (0.1, 0.3, 0.35, 0.4)(\underline{x}_2, \overline{x}_2) + (0.11, 0.2, 0.3, 0.4)(\underline{x}_3, \overline{x}_3) = (0.1, 0.14, 0.16, 0.2)$$

(44)-(46)

Transforming the trapezoidal fuzzy numbers into interval we have

$$(0.3\alpha + 0.1, -0.3\alpha + 0.9)(\underline{x}_1, \overline{x}_1) + (0.4\alpha + 1, -0.3\alpha + 1.9)(\underline{x}_2, \overline{x}_2) + (0.19\alpha + 0.11, -0.4\alpha + 0.9)(\underline{x}_3, \overline{x}_3) = (0.09\alpha + 0.01, -0.05\alpha + 0.2)$$
$$(0.05\alpha + 0.1, -0.02\alpha + 0.2)(\underline{x}_1, \overline{x}_1) + (0.03\alpha + 0.11, -0.02\alpha + 0.2)(\underline{x}_2, \overline{x}_2) + (0.1\alpha + 6, -0.05\alpha + 6.2)(\underline{x}_3, \overline{x}_3) = (0.03\alpha + 0.11, -0.04\alpha + 0.2)$$
$$(0.1\alpha + 5, -0.2\alpha + 5.4)(\underline{x}_1, \overline{x}_1) + (0.2\alpha + 0.1, -0.05\alpha + 0.4)(\underline{x}_2, \overline{x}_2) + (0.09\alpha + 0.11, -0.1\alpha + 0.4)(\underline{x}_3, \overline{x}_3) = (0.04\alpha + 0.1, -0.04\alpha + 0.2)$$

(47)-(49)

We can write the above equations in matrix form as:

$$\begin{bmatrix} 0.3\alpha+0.1 & 0.4\alpha+1 & 0.19\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3\alpha+0.9 & -0.3\alpha+1.9 & -0.4\alpha+0.9 \\ 0.05\alpha+0.1 & 0.03\alpha+0.11 & 0.1\alpha+6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.02\alpha+0.2 & -0.02\alpha+0.2 & -0.05\alpha+6.2 \\ 0.1\alpha+5 & 0.2\alpha+0.1 & 0.09\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2\alpha+5.4 & -0.05\alpha+0.4 & -0.1\alpha+0.4 \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \\ \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.09\alpha+0.01 \\ -0.05\alpha+0.2 \\ 0.03\alpha+0.11 \\ -0.04\alpha+0.2 \\ 0.04\alpha+0.1 \\ -0.04\alpha+0.2 \end{pmatrix}$$

$$\underline{x_1} = \frac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\underline{x_2} = \frac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\underline{x_3} = \frac{41\alpha^3-4387\alpha^2-31628\alpha-53460}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\overline{x_1} = \frac{-1/10(29\alpha^3+10856\alpha^2-123600\alpha+352000)}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$

$$\overline{x_2} = \frac{-2/5(94\alpha^3-4659\alpha^2-39150\alpha+235000}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$

$$\overline{x_3} = \frac{(33\alpha^3-1490\alpha^2+13500\alpha-34800)}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$

Corresponding individual graph is shown in Figures 13, 14 and 15 respectively.

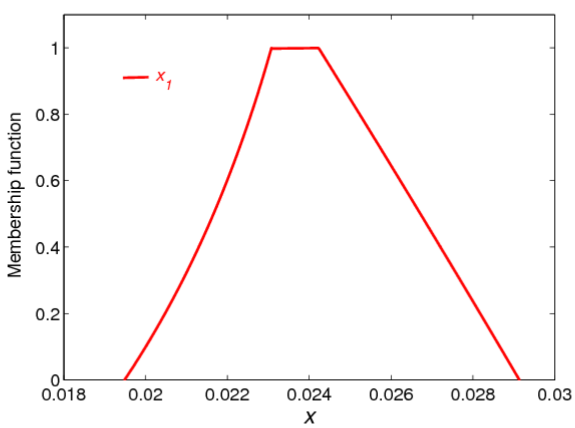

Figure 13. Fuzzy solution for x_1

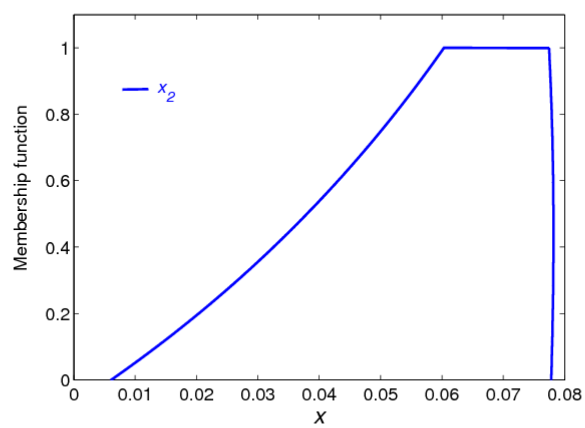

Figure 14. Fuzzy solution for x_2

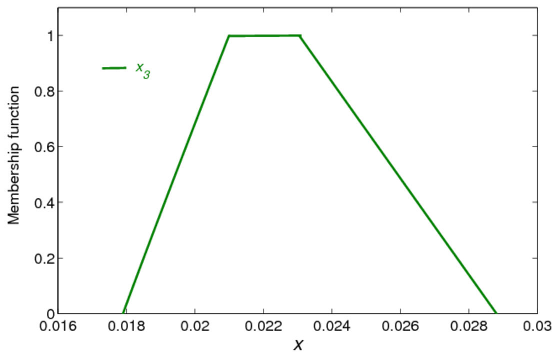

Figure 15. Fuzzy solution for x_3

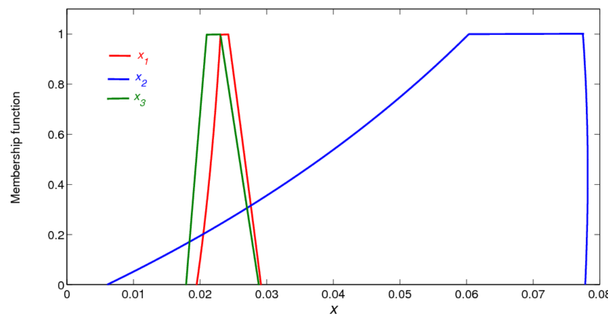

Figure 16. Fuzzy solution for x_1, x_2, x_3

Plots for x_1, x_2, x_3 are shown all together in Figure 16.

Special Cases:

Substituting $\alpha = 0$ in fuzzy case we can obtain the results for the interval case and at the point $\alpha_2 = \alpha_3$ we get the similar results for triangular fuzzy number.

6. Conclusion

The present work demonstrates a new method for interval and fuzzy solution of a fuzzy system of linear equations. The concept of fuzzy number (that is, triangular fuzzy number and trapezoidal fuzzy number) with $\alpha - cut$ has been used to solve the numerical problems of system of linear equations. This method is applied first in a known problem of circuit analysis where the resistance is crisp and voltage source is fuzzy. The same solutions were obtained. We have applied the proposed method for the general case of circuit analysis where both the resistance and source are in interval or fuzzy numbers. To test the feasibility of the method in general, we have extended the problem from two unknowns (as in the case of circuit analysis) to three unknowns. The solutions are obtained for four different cases where coefficients are crisp, intervals, triangular fuzzy numbers and trapezoidal fuzzy numbers, respectively. We believe that due to the simplicity of the proposed method, it can be applied to the various physical and engineering problems where uncertainty aspects are inherent.

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