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Thermal Instability of Rivlin-Ericksen Elastico-Viscous Rotating Fluid in Porous Medium in Hydromagnetics

S. K. Kango
Department of Mathematics
Government College, Haripur (Manali)
Himachal Pradesh – 175136 (INDIA)
drsanjaykanungo@in.com

Vikram Singh
Department of Mathematics
Jwalaji Degree College, Jwalamukhi
Himachal Pradesh – 176031 (INDIA)

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Abstract

The thermal instability of a layer of Rivlin-Ericksen elastico-viscous rotating fluid in a porous medium in hydromagnetics is considered. For stationary convection, the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary (Newtonian) fluid. The magnetic field is found to have a stabilizing effect on the thermal instability of a layer of Rivlin-Ericksen fluid in the absence of rotation whereas the medium permeability has a destabilizing effect on thermal instability of Rivlin-Ericksen fluid in the absence of rotation. Rotation always has a stabilizing effect. The magnetic field, medium permeability and rotation introduce oscillatory modes in the system, which were non-existent in their absence. The case of over stability is also considered and the sufficient conditions for the non-existence of over stability are obtained in the process. The study finds applications in geophysics, chemical technology and engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals and rotating machinery.

Keywords: Thermal Instability; Rivlin-Ericksen Fluid; Viscoelasticity

MSC 2010: 76A10, 76D50, 76E25, 76S05

1. Introduction

Thermal convection in a Newtonian fluid layer in the presence of magnetic field and rotation was discussed in detail by Chandrasekhar (1981). Bhatia and Steiner (1972) studied the problem of thermal instability of a Maxwell fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary (Newtonian) fluid. Bhatia and Steiner (1973) also studied the thermal instability of a Maxwell fluid in the presence of a magnetic field while the thermal convection in an Oldroyd fluid in hydromagnetics was studied by Sharma (1975).

In the physical world, the investigation of the flow of the Rivlin-Ericksen fluid through a porous medium has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. Flows in porous regions are a creeping flow. When a fluid permeates a porous material, the actual path of the individual particles cannot be followed analytically. When the density of a stratified layer of a single-component fluid decreases upwards, the configuration is stable. This is not necessarily the case for a fluid consisting of two or more components which can diffuse relative to each other. The reason lies in the fact that the diffusivity of heat is usually much greater than the diffusivity of a solute. A displaced particle of fluid thus loses excess heat, if any, more rapidly than the excess solute. The resulting buoyancy force may tend to increase the displacement of the particle from its original position and thus cause instability.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. Two such classes of elastico-viscous fluids are the Rivlin-Ericksen fluid (1955) and the Walters' B' fluid (1960). Walters proposed the constitutive equations of such elastico-viscous fluids. Walters (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25˚C containing 30.5g of polymer per litre behaves very nearly as the Walters' B' elastico-viscous fluid. Rivlin-Ericksen (1955) proposed a theoretical model for yet another elastico-viscous fluid. Such types of polymers are used in agriculture, communication appliances and in bio-medical applications. Specific examples of these include the filtration process, packed bed reactor, insulation system, ceramic processing, enhanced oil recovery, chromatography etc. These polymers are also used in the manufacture of parts of space-crafts, aero plane parts, tires, belt conveyers, ropes, cushions, seat foams, plastics, engineering equipments, adhesives and contact lens. Sharma and Kumar (1996) studied the effect of rotation on thermal instability in the Rivlin-Ericksen elastico-viscous fluid whereas the thermal convection in electrically conducting Rivlin-Ericksen fluid in the presence of magnetic field was studied by Sharma and Kumar (1997).Thermal convection in the Rivlin-Ericksen elastico-viscous fluid in a porous medium in hydromagnetics was studied by Sharma and Kango (1999).

Thermal convection in a rotating layer of a porous medium saturated by a homogeneous fluid is a subject of practical interest for its applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals and rotating machinery. An application of the result of flow through a porous medium in the presence of a magnetic field is in the geothermal regions. The rotation of the Earth distorts the boundaries of a hexagonal convection cells in fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions.

Keeping in mind the relevance and growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and industry; the present paper attempts to study the thermal instability of the Rivlin-Ericksen elastico-viscous rotating fluid in a porous medium in hydromagnetics.

2. Formulation of the Problem and Perturbation Equations

Consider an infinite, horizontal, incompressible layer of an electrically conducting Rivlin-Ericksen elastico-viscous fluid of depth d in a porous medium which is acted on by a uniform horizontal magnetic field $\mathbf{H} = (H, 0, 0)$, gravity force $\mathbf{g} = (0, 0, -g)$ and uniform rotation $\mathbf{\Omega} = (0, 0, \Omega)$. This layer is heated from below such that a steady adverse temperature gradient β (= dT/dz) is maintained.

Let p, ρ, T, α, g, ε, η, μ_e, k_1 and $\mathbf{q} = (u, v, w)$ denote, respectively, the fluid pressure, density, temperature, thermal coefficient of expansion, gravitational acceleration, medium porosity, electrical resistivity, magnetic permeability, medium permeability and fluid velocity. The hydromagnetic equations [Chandrasekhar (1981), Joseph (1976), Rivlin-Ericksen (1955)] are

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\mathbf{q}\right] = -\left(\frac{1}{\rho_0}\right)\nabla p + \frac{\mathbf{g}\rho}{\rho_0} + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \mathbf{H}) \times \mathbf{H}$$

$$-\frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\mathbf{q} + 2\left(\mathbf{q} \times \mathbf{\Omega}\right), \tag{1}$$

$$\nabla.\mathbf{q} = 0, \tag{2}$$

$$E\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \kappa\nabla^2 T, \tag{3}$$

$$\nabla.\mathbf{H} = 0, \tag{4}$$

$$\varepsilon\frac{\partial \mathbf{H}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2\mathbf{H}. \tag{5}$$

Here, E = ε + (1- ε) $\rho_s C_s/(\rho_0$ C) is a constant ; κ is the thermal diffusivity ; ρ_s, C_s and ρ_0, C stand for the density and heat capacity for solid and fluid, respectively. The equation of state is

$$\rho = \rho_0\left[1 - \alpha\left(T - T_0\right)\right], \tag{6}$$

where the suffix zero refers to values at the reference level z = 0 and in writing eqn.(1), use has been made of the Boussinesq approximation.

The steady state solution is

$$\mathbf{q} = (0,0,0), \quad T = T_0 - \beta z, \quad \rho = \rho_0 (1 + \alpha \beta z), \tag{7}$$

where $\beta = (T_0 - T_1)/d$ is the magnitude of the uniform temperature gradient and is positive as temperature decreases upwards.

Here, we use the linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let $\mathbf{q} = (u, v, w)$, δp, $\delta\rho$, θ and $\mathbf{h} = (h_x, h_y, h_z)$ denote the perturbations in velocity (0,0,0), pressure p, density ρ, temperature T and magnetic field $\mathbf{H} = (H, 0, 0)$ respectively. The change in the density $\delta\rho$, caused by the perturbation θ in temperature, is given by

$$\delta\rho = -\rho_0 \alpha \theta \ . \tag{8}$$

Then the linearized hydromagnetic perturbation eqns. become

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\left(\frac{1}{\rho_0}\right)\nabla \delta p - \mathbf{g}\alpha\theta + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \mathbf{h}) \times \mathbf{H} - \frac{1}{k_1}\left(\nu + \nu' \frac{\partial}{\partial t}\right)\mathbf{q} + 2(\mathbf{q} \times \mathbf{\Omega}), \tag{9}$$

$$\nabla.\mathbf{q} = 0, \tag{10}$$

$$E\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta, \tag{11}$$

$$\nabla.\mathbf{h} = 0, \tag{12}$$

$$\varepsilon \frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2 \mathbf{h}. \tag{13}$$

Writing Equations (9)-(13) in scalar form, using (8) and eliminating $u, v, h_x, h_y, \delta p$ between them, we obtain

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w = g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\alpha\theta + \left(\frac{\mu_e H}{4\pi\rho_0}\right)\frac{\partial}{\partial z}\nabla^2 h_z - \frac{1}{k_1}\left(\nu + \nu' \frac{\partial}{\partial t}\right)\nabla^2 w - 2\Omega \frac{\partial}{\partial z}\zeta, \tag{14}$$

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t} + \frac{1}{k_1}\left(\nu + \nu' \frac{\partial}{\partial t}\right)\right]\zeta = \left(\frac{\mu_e H}{4\pi\rho_0}\right)\frac{\partial}{\partial z}\xi + 2\Omega \frac{\partial w}{\partial z}, \tag{15}$$

$$\left(E\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\beta w, \tag{16}$$

$$\varepsilon\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)\xi=H\frac{\partial\zeta}{\partial z}, \tag{17}$$

$$\varepsilon\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z=H\frac{\partial w}{\partial z}, \tag{18}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ stands for the z-components of the vorticity and current density, respectively.

3. Dispersion Relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,h_z,\zeta,\xi]=[W(z),\Theta(z),K(z),Z(z),X(z)]\exp\left(ik_x x+ik_y y+nt\right), \tag{19}$$

where k_x, k_y are the wave numbers along the x and y directions, respectively; k = $\sqrt{k_x{}^2+k_y{}^2}$ is the resultant wave number and n is, in general, a complex constant.
Using expression (19), Equations (14) – (18), in non-dimensional form, become

$$\left[\sigma+\frac{\varepsilon}{P_l}(1+F\sigma)\right]\left(D^2-a^2\right)W+\left(\frac{g\varepsilon\alpha a^2d^2}{\nu}\right)\Theta-\left(\frac{ik_x\varepsilon\mu_e Hd^2}{4\pi\rho_0\nu}\right)\left(D^2-a^2\right)K-\frac{2i\varepsilon k_x\Omega d^4}{\nu}Z=0, \tag{20}$$

$$\left[\sigma+\frac{\varepsilon}{P_l}(1+F\sigma)\right]Z=\left(\frac{ik_x\varepsilon\mu_e Hd^2}{4\pi\rho_0\nu}\right)X+\frac{2i\varepsilon k_x\Omega d^2}{\nu}W, \tag{21}$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \tag{22}$$

$$\left(D^2-a^2-p_2\sigma\right)X=-\left(\frac{ik_x Hd^2}{\varepsilon\eta}\right)Z, \tag{23}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -\left(\frac{ik_x H d^2}{\varepsilon\eta}\right)W, \tag{24}$$

where we have put $a = kd$, $\sigma = nd^2/\nu$, $x^* = x/d$, $y^* = y/d$, $z^* = z/d$ and $D = \left(d/dz^*\right)$. $p_1 = \nu/\kappa$ is Prandtl number, $p_2 = \nu/\eta$ is magnetic Prandtl number, $P_l = k_1/d^2$ is the dimensionless medium permeability and $F = \nu'/d^2$ is the dimensionless kinematic viscoelasticity.

Elimination Θ, K, Z and X between Equations (20)-(24), we obtain

$$\begin{bmatrix} \left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)\left(D^2-a^2-p_2\sigma\right)\left\{\sigma+\frac{\varepsilon}{P_l}\left(1+F\sigma\right)\right\} \\ -R\varepsilon a^2\left(D^2-a^2-p_2\sigma\right)-k_x^2 Qd^2\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right) \end{bmatrix}$$

$$\times\left[\left(D^2-a^2-p_2\sigma\right)\left\{\sigma+\frac{\varepsilon}{P_l}\left(1+F\sigma\right)\right\}-k_x^2 Qd^2\right]W$$

$$-T_A\varepsilon^2 k_x^2 d^2\left(D^2-a^2-Ep_1\sigma\right)\left(D^2-a^2-p_2\sigma\right)^2 W = 0, \tag{25}$$

where Q = $\mu_e H^2 d^2/\left(4\pi\rho_0\nu\eta\right)$ is the Chandrasekhar number, R = $g\alpha\beta d^4/\left(\nu\kappa\right)$ is the Rayleigh number and $T_A = 4\Omega^2 d^4/\nu^2$ is the Taylor number.

Consider the case where both boundaries are free as well as maintained at constant temperatures while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Equations (20)-(24) must be solved, are (Chandrasekhar (1981)):

$$W = D^2W = 0, \Theta = 0, DZ = 0, \quad \text{at } z = 0 \text{ and } z = 1$$

and $DX = 0$, $K = 0$ on the perfectly conducting boundaries. (26)

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres (Spiegel (1965)).Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of (25) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{27}$$

where W_0 is a constant. Substituting (27) in (25) and letting $a^2 = \pi^2 x,\ R_1 = R/\pi^4$,

$T_{A_1} = T_A/\pi^4, Q_1 = Q/\pi^2, k_x = k\cos\theta, i\sigma_1 = \sigma/\pi^2, P = \pi^2 P_l$, we obtain the dispersion relation

$$R_1 = \frac{1}{\varepsilon}\left[\left(\frac{1+x}{x}\right)\left(1+x+Ep_1 i\sigma_1\right)\left\{\frac{\varepsilon}{P}+i\sigma_1\left(1+\frac{F\pi^2\varepsilon}{P}\right)\right\}+\frac{(1+x)(1+x+Ep_1 i\sigma_1)}{(1+x+ip_2\sigma_1)}Q_1\cos^2\theta\right.$$
$$+\varepsilon T_{A_1}\left(1+x+Ep_1 i\sigma_1\right)\left(1+x+ip_2\sigma_1\right)\cos^2\theta$$
$$\left.\times\left[\left(1+x+ip_2\sigma_1\right)\left\{\frac{\varepsilon}{P}+i\sigma_1\left(1+\frac{F\pi^2\varepsilon}{P}\right)\right\}+Q_1 x\cos^2\theta\right]^{-1}\right] \quad (28)$$

Equation (28) is the required dispersion relation studying the effects of magnetic field, kinematic viscoelasticity, medium permeability and rotation on thermal instability of Rivlin-Ericksen fluid.

4. The Stationary Convection

For the case of stationary convection, $\sigma = 0$ and Equation (28) reduces to

$$R_1 = \frac{1}{\varepsilon}\left[\frac{(1+x)^2\varepsilon}{xP}+Q_1(1+x)\cos^2\theta+\frac{\varepsilon^2 T_{A_1}(1+x)^2\cos^2\theta}{\frac{(1+x)\varepsilon}{P}+Q_1 x\cos^2\theta}\right], \quad (29)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters Q_1, P and T_{A_1}. For stationary convection the parameter F accounting for the kinematic viscoelasticity effect vanishes and thus the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid.

To investigate the effects of magnetic field, medium permeability and rotation, we examine the behavior of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dP}$ and $\frac{dR_1}{dT_{A_1}}$ analytically. Equation (29) yields

$$\frac{dR_1}{dQ_1} = \frac{1}{\varepsilon}\left[(1+x)\cos^2\theta - \frac{\varepsilon^2 T_{A_1} x(1+x)^2\cos^4\theta}{\left\{\frac{(1+x)\varepsilon}{P}+Q_1 x\cos^2\theta\right\}^2}\right], \quad (30)$$

In the absence of rotation, $\frac{dR_1}{dQ_1} = \frac{(1+x)}{\varepsilon}\cos^2\theta$, which is always positive. The magnetic field, therefore, has a stabilizing effect on thermal instability of Rivlin-Ericksen fluid in the absence of rotation. This stabilizing effect of magnetic field is in good agreement with earlier work of Sharma and Kango (1999). In the presence of rotation, the system is stable if

$$(1+x)\cos^2\theta > \frac{\varepsilon^2 T_{A_1} x(1+x)^2\cos^4\theta}{\left\{\frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta\right\}^2} .$$

Similarly, it can be shown from eqn. (29) that

$$\frac{dR_1}{dP} = \frac{1}{P^2}\left[\frac{-(1+x)^2}{x} + \frac{\varepsilon^2 T_{A_1}(1+x)^3\cos^2\theta}{\left\{\frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta\right\}^2}\right]. \tag{31}$$

In the absence of rotation, $\frac{dR_1}{dP} = \frac{-(1+x)^2}{xP^2}$, which is always negative. There is an analogous relation for thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium in hydromagnetics as derived by Sharma and Kango (1999). The medium permeability, therefore, has a destabilizing effect on thermal instability of Rivlin-Ericksen fluid in the absence of rotation. In the presence of rotation, the system is stable if

$$\frac{(1+x)^2}{x} < \frac{\varepsilon^2 T_{A_1}(1+x)^3\cos^2\theta}{\left\{\frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta\right\}^2} .$$

Similarly, it can be shown from eqn. (29) that

$$\frac{dR_1}{dT_{A_1}} = \frac{\varepsilon(1+x)^2\cos^2\theta}{\left[\frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta\right]^2} , \tag{32}$$

which is always positive. The rotation, therefore, always has a stabilizing effect on thermal instability of Rivlin-Ericksen fluid.

The kinematic viscoelasticity has no effect for stationary convection.

5. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, in the stability problem due to the presence of magnetic field, kinematic viscoelasticity and rotation. Multiplying Equation (20) by W*, the complex conjugate of W, integrating over the range of z and making use of Equations (21) - (24) together with the boundary conditions (26), we obtain

$$\left[\sigma+\frac{\varepsilon}{P_l}\left(1+F\sigma\right)\right]I_1-\left(\frac{g\alpha\varepsilon\kappa a^2}{\nu\beta}\right)\left[I_2+Ep_1\sigma^* I_3\right]+\left(\frac{\mu_e\eta\varepsilon^2}{4\pi\rho_0\nu}\right)\left[I_4+p_2\sigma^* I_5\right]$$

$$+d^2\left[\sigma^*+\frac{\varepsilon}{P_l}\left(1+F\sigma^*\right)\right]I_6+\left(\frac{\mu_e\eta\varepsilon^2 d^2}{4\pi\rho_0\nu}\right)\left[I_7+p_2\sigma I_8\right], \quad (33)$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\ I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_3=\int_0^1|\Theta|^2dz,$$

$$I_4=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz,\quad I_5=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_6=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,\ I_7=\int_0^1|X|^2dz,\ I_8=\int_0^1|Z|^2dz\ . \quad (34)$$

The integrals $I_1,\cdots,I_8$ are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating the real and imaginary parts of Equation (33), we obtain

$$\sigma_r\left[\left(1+\frac{\varepsilon F}{P_l}\right)I_1+\left(\frac{g\alpha\varepsilon\kappa a^2}{\nu\beta}\right)Ep_1I_3-\left(\frac{\mu_e\eta\varepsilon^2}{4\pi\rho_0\nu}\right)p_2I_5-\left(\frac{\mu_e\eta\varepsilon^2d^2}{4\pi\rho_0\nu}\right)p_2I_7+d^2\left(1+\frac{\varepsilon F}{P_l}\right)I_8\right]$$

$$=-\left[\frac{\varepsilon}{P_l}I_1+\left(\frac{g\alpha\varepsilon\kappa a^2}{\nu\beta}\right)I_2-\left(\frac{\mu_e\eta\varepsilon^2}{4\pi\rho_0\nu}\right)I_4-\left(\frac{\mu_e\eta\varepsilon^2d^2}{4\pi\rho_0\nu}\right)I_6+\left(\frac{\varepsilon d^2}{P_l}\right)I_8\right], \quad (35)$$

$$\sigma_i\left[\begin{matrix}\left(1+\frac{\varepsilon F}{P_l}\right)I_1-\left(\frac{g\alpha\varepsilon\kappa a^2}{\nu\beta}\right)Ep_1I_3-\left(\frac{\mu_e\eta\varepsilon^2}{4\pi\rho_0\nu}\right)p_2I_5\\ -d^2\left(1+\frac{\varepsilon F}{P_l}\right)I_8+\left(\frac{\mu_e\eta\varepsilon^2d^2}{4\pi\rho_0\nu}\right)p_2I_7\end{matrix}\right]=0\,. \quad (36)$$

It follows from Equation (35) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (36) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, magnetic field and rotation which were non-existent in their absence.

6. The Case of Over Stability

Here we discuss the possibility of whether instability may occur as over stability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (28) will admit of solutions with σ_1 real.

If we equate real and imaginary parts of (28) and eliminate R_1 between them, we obtain

$$A_3c_1^3 + A_2c_1^2 + A_1c_1 + A_0 = 0, \tag{37}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_3 = b^2 p_2^4\left(1+\frac{F\pi^2\varepsilon}{P}\right)^3 + b\frac{\varepsilon E p_1 p_2^4}{P}\left(1+\frac{F\pi^2\varepsilon}{P}\right)^2, \tag{38}$$

$$\begin{aligned}A_0 = \; & b^6\frac{\varepsilon^2}{P^2}\left(1+\frac{F\pi^2\varepsilon}{P}\right) + b^5\left(\frac{\varepsilon^3 E p_1}{P^3}\right) + b^5(b-1)\varepsilon\left(\frac{2Q_1\cos^2\theta}{P} - \varepsilon T_{A_1}\right)\left(1+\frac{F\pi^2\varepsilon}{P}\right)\\ & + b^4(b-1)\left[\frac{2\varepsilon^2 E p_1 Q_1\cos^2\theta}{P} + \frac{\varepsilon^3 E p_1 T_{A_1}}{P} + \frac{\varepsilon^2 Q_1\cos^2\theta}{P^2}(Ep_1 - p_2)\right]\\ & + b^4(b-1)^2 Q_1^2\cos^4\theta\left(1+\frac{F\pi^2\varepsilon}{P}\right)\\ & + b^3(b-1)^2\left[\frac{\varepsilon E p_1 Q_1^2\cos^4\theta}{P} + \frac{2\varepsilon Q_1^2\cos^4\theta}{P}(Ep_1 - p_2) + \varepsilon^2 T_{A_1} Q_1\cos^2\theta(Ep_1 + p_2)\right]\\ & + b^2(b-1)^3 Q_1^3\cos^6\theta(Ep_1 - p_2).\end{aligned} \tag{39}$$

As σ_1 is real for over stability, the three values of $c_1\left(=\sigma_1^2\right)$ must be positive. The product of the roots of (37) is $-\frac{A_0}{A_3}$ and this has to be positive.

It is clear from Equations (38) and (39) that A_0 and A_3 are always positive if

$$Ep_1 > p_2 \text{ and } \frac{2Q_1 \cos^2\theta}{P} > \varepsilon T_{A_1},$$

which implies that $\kappa < \eta\left[\varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C}\right]$ (40)

and

$$\eta < \frac{\nu\mu_e H^2 \cos^2\theta}{8\pi\varepsilon\rho_0 k_1 \Omega^2}. \tag{41}$$

Equations (40) and (41) are, therefore, the sufficient conditions for non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability.

7. Nomenclature

p = pressure	ρ = fluid density
δp = the perturbation in pressure	$\delta\rho$ = the perturbation in density
ν = kinematic viscosity	ν' = kinematic viscoelasticity
ε = medium porosity	k_1 = medium permeability
g = acceleration due to gravity	H = magnetic field
η = electrical resistivity	μ_e = magnetic permeability
Ω = rotation vector	κ = thermal diffusivity

$\vec{q}(u,v,w)$ = perturbation in fluid velocity $\vec{q}$ (0, 0, 0)

k_x, k_y = wave numbers in the x and y directions, respectively

$k = \left(k_x^2 + k_y^2\right)^{1/2}$ = wave number of the disturbance

8. Conclusion

The study of viscoelastic fluids finds applications in geophysics and chemical technology. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. Rivlin-Ericksen is one such class of elastico-viscous fluids.

A layer of electrically conducting Rivlin-Ericksen elastico-viscous fluid heated from below has been considered in the presence of a uniform horizontal magnetic field and uniform rotation in a porous medium. For stationary convection, the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary (Newtonian) fluid. The magnetic field is found to have a stabilizing effect on the thermal instability of the layer of Rivlin-Ericksen fluid in the absence of rotation whereas the medium permeability has a destabilizing effect on thermal instability of Rivlin-Ericksen fluid in the absence of rotation. Rotation always has a stabilizing effect. The magnetic field, medium permeability and rotation generate oscillatory modes in the system that were non-existent in their absence. The case of over stability is also considered and the sufficient conditions for the non-existence of over stability are determined.

The sufficient conditions for the non-existence of over stability for thermal instability in Rivlin-Ericksen elastico-viscous fluid in the presence of magnetic field and rotation in porous medium are, respectively,

$$\kappa < \eta\left[\varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C}\right] \text{ and } \eta < \frac{\nu \mu_e H^2 \cos^2\theta}{8\pi\varepsilon\rho_0 k_1 \Omega^2}.$$

$$\kappa < \eta\left[\varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C}\right] \text{ and } \eta < \frac{\nu \mu_e H^2 \cos^2\theta}{8\pi\varepsilon\rho_0 k_1 \Omega^2}$$

are, therefore, the sufficient conditions for nonexistence of over stability, the violation of which does not necessarily imply the occurrence of over stability.

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Autobiography of the author (S. K. Kango)

I, Dr. S.K. Kango, am 41 years old and working as Assistant Professor in the subject Mathematics in Government College Haripur (Manali) of Himachal Pradesh in India. I have done Ph.D. in Mathematics in the faculty of Physical Sciences of HP University, Shimla (India) in 1999 under the supervision of Late Professor R.C. Sharma. I have attended a 4-week orientation programme, 3-week refresher course in Mathematics, presented research papers in an international conference and national seminar on Mathematics.

Biography of the second author (Vikram Singh)

He is 38 years old and working as Lecturer in Mathematics in Jwalaji Degree College Jwlamukhi of Himachal Pradesh in India. He passed his M.Phil Degree in the subject of Mathematics from Madurai Kamraj University, Madurai (India) in 2008. He is doing Ph.D. in Mathematics under my supervision.