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A Macroscopic Two-Phase Blood Flow through a Bell Shaped Stenosis in an Artery with Permeable Wall

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Abstract

The present work concerns the effects of the hematocrit and the permeability of the wall on blood flow characteristics due to the presence of a bell shaped stenosis in an artery. In this analysis, the flowing blood is represented by a macroscopic two-phase model, as a suspension of erythrocytes in plasma. The analytical expressions for the flow characteristics, namely, the flow resistance (impedance), the wall shear stress distribution in the stenotic region and the shearing stress at the stenosis throat have been derived. Results for the effects of permeability as well as of hematocrit on these flow characteristics are shown graphically and discussed briefly.

Keywords: Two-phase, erythrocytes, hematocrit, impedance, shear stress, stenosis throat

MSC 2010): 76Z05

1. Introduction

The generic medical term stenosis or arteriosclerosis, is the narrowing of anybody passage, tube or orifice, stems from the Greek words *arthero* (gruel or paste) and *sclerosis* (hardness). It is a frequently occurring cardiovascular disease (an abnormal and unnatural growth in the arterial wall thickness) develops at the various locations of the cardiovascular system under diseased conditions and occasionally results in to serious consequences (cerebral strokes, myocardial infarction, angina pectoris, cardiac arrests, etc). Although, the etiology of the initiation of stenosis is not completely understood, it is believed that the disease occurs due to the deposits of the cholesterol, fatty substances, cellular waste products, calcium and fibrin in the inner lining of an artery. It is further known that once the constriction has developed, it brings about the significant changes in the flow field. With the knowledge about cardiovascular disease, stenosis is closely associated with the flow conditions and other hemodynamic factors, a large number of researchers including Young (1968, 1979), Young and Tsai (1973), Caro et al. (1978), Shukla et al. (1980), Ahmed and Giddens (1983), Sarkar and Jayaraman (1998), Pralhad and Schultz (2004), Jung et al. (2004), Liu et al. (2004), Srivastava and coworkers (1996, 2009, 2010a,b,c), Mishra et al. (2006), Ponalagusamy (2007), Layek et al. (2009), Joshi et al. (2009), Mekheimer and El-Kot (2008), Tzirtzilakis (2008), Mandal and coworkers (2005, 2007), Politis et al. (2007, 2008), Singh et al. (2010), Medhavi (2011), and many others have addressed the stenotic development problems under various flow situations since the first investigation of Mann et al. (1938).

Barring a few, most of the studies conducted in the literature considered blood as a single-phase Newtonian or non-Newtonian fluid. However, the experimental observations of Cokelet (1972) and theoretical investigation of Haynes (1960) indicate that blood cannot be treated as a single-phase homogeneous viscous fluid while flowing through narrow arteries (of diameter $\leq 1000\ \mu m$). Srivastava and Srivastava (1983) observed that the individuality of red cells (of diameter $8\ \mu m$) is significant even in such large vessels with diameter up to hundred cells diameter and concluded that blood can be suitably represented by a macroscopic two-phase model (i.e., a suspension of red cells in plasma) in small vessels (of diameter $\leq 2400\ \mu m$). Recently Srivastava (2007) has presented a brief discussion and survey on suspension modeling of blood. In addition, the endothelial walls are known to be highly permeable with ultra microscopic pores through, which filtration occurs. Cholesterol is believed to increase the permeability of the arterial wall. Such increase in permeability results from dilated, damaged or inflamed arterial walls. It is also known that stenosis may develop in series (multiple stenosis), overlapping, bell shaped, of composite nature or of irregular shape (Srivastava et al., 2010). Assuming that the flowing blood is represented by a macroscopic two-phase model (i.e., a suspension of erythrocytes in plasma), the research reported here is devoted to study the flow of blood through a bell shaped stenosis in an artery with permeable wall. The flow in the permeable boundary is described by Darcy law. The wall in the vicinity of the stenosis is usually solid when stenosis develops in living vasculature. To neglect the entrance, end and special wall effects the artery length is considered large enough as compared to its radius.

2. Formulation of the Problem

Consider the axisymmetric flow of blood through a bell shaped stenosis, specified at the location as shown in Figure1, in an artery with permeable wall. The geometry of the stenosis, assumed to be manifested in the arterial wall segment, is described as

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{R_0} \exp\left(\frac{-m^2 \varepsilon^2 z^2}{R_0^2} \right), \tag{1}$$

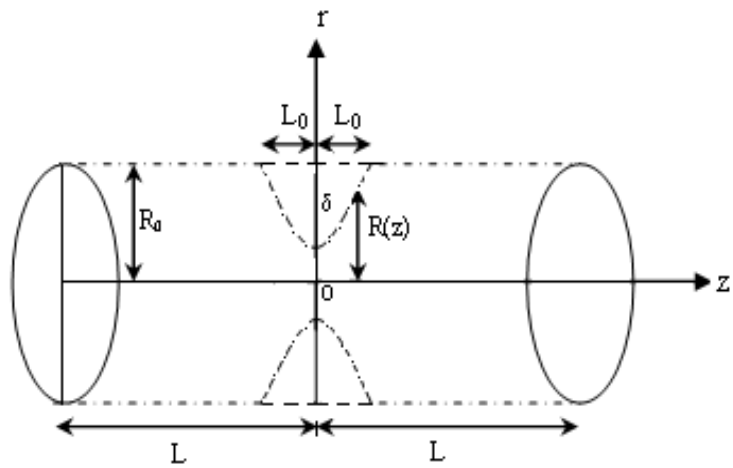

Figure 1. The geometry of a bell shaped stenosis with permeable wall.

where R_0 is the radius of the arterial segment in the non-stenotic region, $R(z)$ is the radius of the stenosed portion located at the axial distance z from the left end of the segment, δ is the depth of stenosis at the throat and m is a parametric constant, ε the relative length of the constriction, defined as the ratio of the radius to the half length of the stenosis, i.e., $\varepsilon = R_0 / L_0$.

The equations describing the steady flow of a two-phase macroscopic model of blood may be expressed (Srivastava and Srivastava, 1983, 1989) as

$$(1\text{-}C)\ \rho_f \left\{ u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right\} = -\ (1\text{-}\ C)\ \frac{\partial p}{\partial z} + (1-C)\ \mu_s\ (C)\, \nabla^2 u_f\ + CS(u_p - u_f), \tag{2}$$

$$(1\text{-}C)\ \rho_f \left\{ u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right\} = -\ (1\text{-}C)\ \frac{\partial p}{\partial r} + (1-C)\ \mu_s(C)\ \left(\nabla^2 - \frac{1}{r^2}\right) v_f + CS\,(v_p - v_f), \tag{3}$$

$$\frac{\partial}{\partial r}\left[(1-C)v_f\right] + (1-C)\frac{v_f}{r} + \frac{\partial}{\partial z}\left[(1-C)u_f\right] = 0, \tag{4}$$

$$\rho_p \left\{ u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right\} = -\ C\ \frac{\partial p}{\partial z} +\ CS\,(u_f - u_p), \tag{5}$$

$$\rho_p \left\{ u_p \frac{\partial v_p}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right\} = - C \frac{\partial p}{\partial r} + CS\,(v_f - v_p), \tag{6}$$

$$\frac{\partial}{\partial r}[C\, v_p] + \frac{C v_p}{r} \frac{\partial [C\, u_p]}{\partial z} = 0, \tag{7}$$

with $\nabla^2 = \partial / \partial r^2 + (1/r)(\partial / \partial r) + \partial^2 / \partial z^2$ as a two-dimensional Laplacian operator, r is the radial coordinate measured perpendicular to the axis of the tube. (u_f, v_f) and (u_p, v_p) are the (axial, radial) components of the fluid and particle velocities, respectively, ρ_f and ρ_p are the actual density of the material constituting the fluid (plasma) and the particle (erythrocyte) phases, respectively, (1-C) ρ_f is the fluid phase and $C\rho_p$ is particle phase densities, C denotes the volume fraction density of the particles, p is the pressure, $\mu_s(C) \simeq \mu_s$ is the mixture viscosity (apparent or effective viscosity) , S is the drag coefficient of interaction for the force exerted by one phase on the other, and the subscripts f and p denote the quantities associated with the plasma (fluid) and erythrocyte (particle) phases, respectively. Others limitations of the present model are the same as discussed in Srivastava and Srivastava (2009). The expressions for drag coefficient of interaction, S and the viscosity of the suspension, μ_s for the present study are selected (Srivastava and Srivastava, 2009; Charm and Kurland, 1974) as

$$S = \frac{9}{2} \frac{\mu_o}{a_o^2} \quad \frac{4 + 3[8C - 3C^2]^{1/2} + 3C}{(2 - 3C)^2}, \tag{8}$$

$$\mu_s\,(C) = \frac{\mu_o}{1 - mC},$$

$$m = 0.070 \exp\,[2.49C + (1107/T) \exp\,(-1.69C)], \tag{9}$$

where T is the measure in absolute scale of temperature (K) , μ_0 is the constant plasma viscosity and a_0 is the radius of a red cell.

Now following the reports of Young (1968), Srivastava and Rastogi (2009), the equations governing the laminar, steady, one-dimensional flow of blood in an artery in the case of a mild stenosis (i.e., $\delta / R_0 << 1$) are derived from equations (2)-(7) as

$$(1\text{-}C)\, \frac{dp}{dz} = (1 - C) \frac{\mu_s}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r})\, u_f + CS\,(u_p - u_f), \tag{10}$$

$$C\, \frac{dp}{dz} = CS\,(u_f - u_p), \tag{11}$$

The appropriate boundary conditions (Beavers and Joseph, 1967) for the problem are stated as

$$\frac{\partial u_f}{\partial r} = 0 \; at \;\; r = 0, \tag{12}$$

$$u_f = u_B \text{ and } \frac{\partial u_f}{\partial r} = \frac{\alpha}{\sqrt{k}}(u_B - u_{porous}) \quad \text{at } r = R(z), \tag{13}$$

where $u_{\text{porous}} = -\frac{k}{\mu_0}\frac{dp}{dz}$, u_{porous} is the velocity in the permeable boundary, u_B is the slip velocity, μ_0 is the plasma (fluid viscosity), k is Darcy number and α (called the slip parameter) is a dimensionless quantity depending on the material parameters which characterize the structure of the permeable material within the boundary region.

3. Analysis

The expressions for velocities, u_f and u_p obtained as the solutions of equations (10)-(11), subject to the boundary conditions (12)-(13), are obtained as

$$u_f = -\frac{R_o^2}{4(1-C)\,\mu_s}\frac{dp}{dz}\left\{(R/R_0)^2 - (r/R_0)^2\right\} + \frac{\sqrt{k}R_0}{2\alpha\;(1\text{-}C)\,\mu_s}\frac{dp}{dz}\left\{(R/R_0) - \frac{2\sqrt{k}\alpha(1-C)}{R_0}\mu\right\}, \tag{14}$$

$$\begin{aligned} u_p = &-\frac{R_o^2}{4(1-C)\,\mu_s}\frac{dp}{dz}\left\{(R/R_0)^2 - (r/R_0)^2 + \frac{4(1-C)\mu_s}{SR_0^2}\right\} \\ &+ \frac{\sqrt{k}R_0}{2\alpha\;(1\text{-}C)\,\mu_s}\frac{dp}{dz}\left\{(R/R_0) - \frac{2\sqrt{k}\alpha(1-C)}{R_0}\mu\right\}, \end{aligned} \tag{15}$$

where $\mu = \mu_s / \mu_0$.

The volumetric flow rate, Q is now calculated as

$$Q = 2\pi\,(1\text{-}C)\int_0^R r\,u_f\;dr + 2\pi\,C\int_0^R r\,u_p\;dr$$

$$= -\frac{\pi R_o^4}{8(1-C)\,\mu_s}\frac{dp}{dz}\left[(R/R_0)^4 + \beta(R/R_0)^2 - \frac{4(R/R_0)^2\sqrt{k}}{\alpha R_0^2}\left\{R_0(R/R_0) - 2\sqrt{k}\,\alpha\,\mu(1-C)\right\}\right], \tag{16}$$

with $\beta = 8C(1-C)\mu_s / SR_0^2$, a non-dimensional suspension parameter.

From equation (16), one now obtains

$$\frac{dp}{dz} = -\frac{8(1-C)\,\mu_s\,Q}{\pi\,R_o^4}\;\varphi(z), \tag{17}$$

where

$$\varphi(z) = 1/F(z), \quad F(z) = \left[\left(R/R_0\right)^4 + \beta\left(R/R_0\right)^2 - \frac{4(R/R_0)^2\sqrt{k}}{\alpha\,R_0^2}\left\{R_0(R/R_0) - 2\sqrt{k}\,\alpha\,\mu(1-C)\right\}\right].$$

The pressure drop, Δp (= p at $z = -L, -p$ at $z = L$) across the stenosis in the tube of length, $2L$ is obtained as

$$\Delta p = \int_{-L}^{L}\left(-\frac{dp}{dz}\right)dz = \frac{8(1-C)\,\mu_s\,Q}{\pi\,R_o^4}\psi\,, \tag{18}$$

where

$$\psi = \int_{-L}^{-L_0}\left[\varphi(z)\right]_{R/R_0=1}dz + \int_{-L_0}^{L_0}\varphi(z)\,dz + \int_{L_0}^{L}\left[\varphi(z)\right]_{R/R_0=1}dz\,. \tag{19}$$

The first and the third integrals in the expression for ψ obtained above are straight forward whereas the analytical evaluation of second integral is a formidable task and therefore shall be evaluated numerically. Following now the reports (Young, 1968; Srivastava and Rastogi, 2009), one derives the expressions for the impedance (flow resistance), λ, the wall shear stress distribution in the stenotic region, τ_w and shearing stress at the stenosis throat, τ_s in their non-dimensional form as

$$\lambda = (1-C)\,\mu\left\{1 - L_o/L + \frac{\eta}{L}\int_{-L_0}^{L_0}\varphi(z)\,dz\right\}, \tag{20}$$

$$\tau_w = \frac{\eta\,(1-C)\,\mu}{\left[\left(R/R_0\right)^3 + \beta\left(R/R_0\right) - \frac{4(R/R_0)\sqrt{k}}{\alpha\,R_0^2}\left\{R_0(R/R_0) - 2\sqrt{k}\,\alpha\,\mu(1-C)\right\}\right]}, \tag{21}$$

$$\tau_s = \frac{\eta\,(1-C)\,\mu}{\left[a^3 + \beta a - \frac{4a\sqrt{k}}{\alpha\,R_0^2}\left\{R_0 a - 2\sqrt{k}\,\alpha\,\mu(1-C)\right\}\right]}, \tag{22}$$

where

$$\eta = 1 + \beta - \frac{4\sqrt{k}}{\alpha R_0^2}\left\{R_0 - 2\sqrt{k}\,\alpha\,\mu(1-C)\right\}, a = 1 - \delta / R_0, \lambda = \bar{\lambda} / \lambda_0, \left(\tau_w, \tau_s\right) = \left(\bar{\tau}_w, \bar{\tau}_s\right) / \tau_0,$$

$\lambda_0 = 16\mu_0\, L / \pi\, \eta R_0^4$, $\tau_0 = 4\mu_0 Q / \pi\, \eta R_0^3$ are the flow resistance and shear stress, respectively for a Newtonian fluid in a normal artery (no stenosis), and $\bar{\lambda}$, $\bar{\tau}_w$ and $\bar{\tau}_s$ are the impedance, wall shear stress and shearing stress at stenosis throat, respectively in their dimensional form obtained from the definitions: $\bar{\lambda} = \Delta p / Q$, $\bar{\tau}_w = (-R/2)dp/dz$, $\bar{\tau}_s = \left(\bar{\tau}_w\right)_{R/R_0 = a}$.

4. Numerical Results and Discussion

To discuss the results of the study quantitatively, computer codes are developed to evaluate the analytical results obtained in equations (20)-(21) numerically in a tube of radius 0.01cm at the temperature of 37^0C. The values of the parameters are selected (Young, 1968; Srivastava, 1996; Beavers and Joseph, 1967) as $d = 0; L_0(cm) = 1$.

$L(cm) = 1, 2, 5; C = 0, 0.2, 0.4, 0.6; \alpha = 0.1, 0.2, 0.3, 0.4, 0.5$; $\sqrt{k}$ (square root of Darcy number, k and hereafter referred as Darcy number) $= 0.1, 0.2, 0.3, 0.4, 0.5$; $\delta/R_0 = 0, 0.05, 0.10, 0.15, 0.20$.

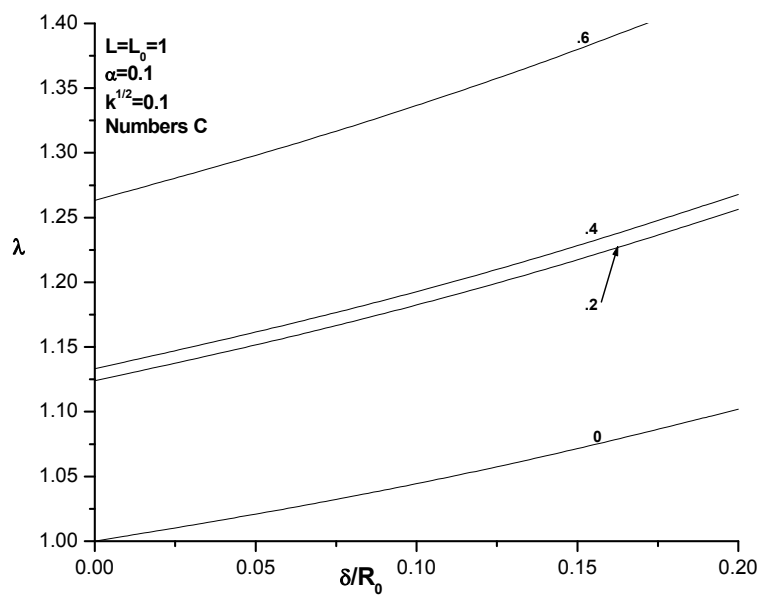

Fig. 2 Impedance, λ versus δ/R_0 for different C.

For any given set of other parameters, the impedance (resistance to flow), λ increases with hematocrit, C as well as with the stenosis height, δ / R_0 (Figure 2). The flow characteristic, λ increases with increasing Darcy number, $\sqrt{k}$ for any given set of other parameters (Figure 3). One observes that the blood flow characteristic, λ increases with the slip parameter, α for other given parameters (Figure 4). The impedance, λ decreases with the increasing value of the

parameter, L (tube length/2) which in turn implies that λ increases with stenosis length, L_0 (Figure 5). The resistance to flow, λ steeply increases with the hematocrit, C for any given value of α, $\sqrt{k}$ and δ / R_0 (Figure 6). For other given parameters, that the blood flow characteristic, λ decreases from its maximal magnitude at $\sqrt{k} = 0$ to its asymptotic value at $\sqrt{k} = 0.15$ (Figure 7). One notices that flow resistance, λ increases with the slip parameter, α from its minimal value at $\alpha = 0.1$ and achieves an asymptotic magnitude at about $\alpha = 0.5$ (Figure 8).

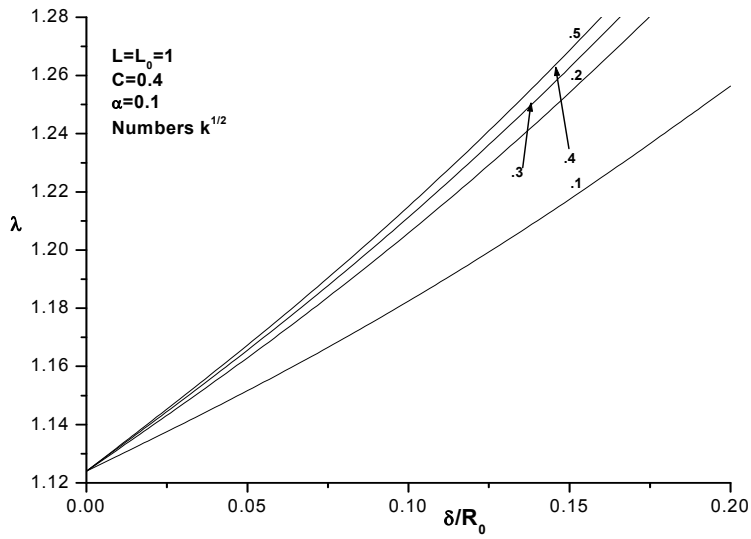

Fig. 3 Impedance, λ versus δ/R_0 for different $k^{1/2}$.

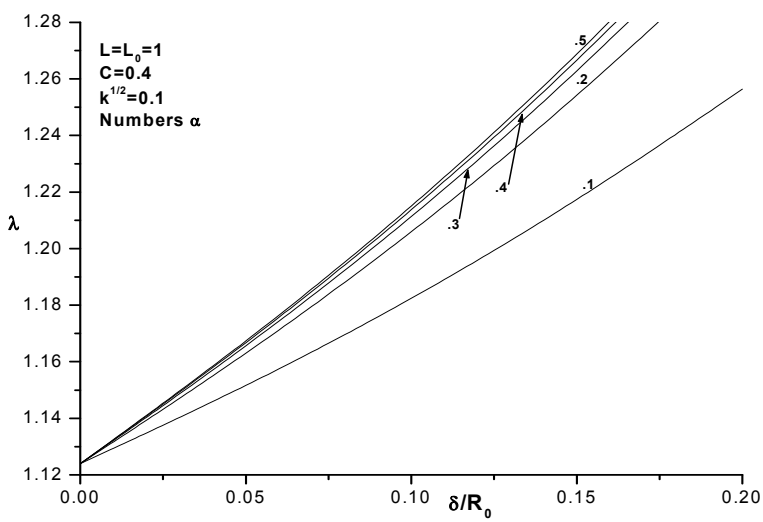

Fig. 4 Impedance, λ versus δ/R_0 for different α.

The wall shear stress in the stenotic region, τ_w increases rapidly in the upstream of the stenosis throat from its approached value at $z/L_0 = -1$ and achieves its maximal magnitude at stenosis throat (i.e., at $z/L_0 = 0$), it then decreases rapidly in the downstream of the throat to its approached value at the end point of the constriction profile (at $z/L_0 = 1$). The blood flow characteristic, τ_w increases with the hematocrit, C and the stenosis height, δ/R_0 at any axial location in the stenotic region (Figure 9). At any axial location, the wall shear stress τ_w increases with the Darcy number, $\sqrt{k}$ (Figure 10) and also with the slip parameter, α (Figure 11).

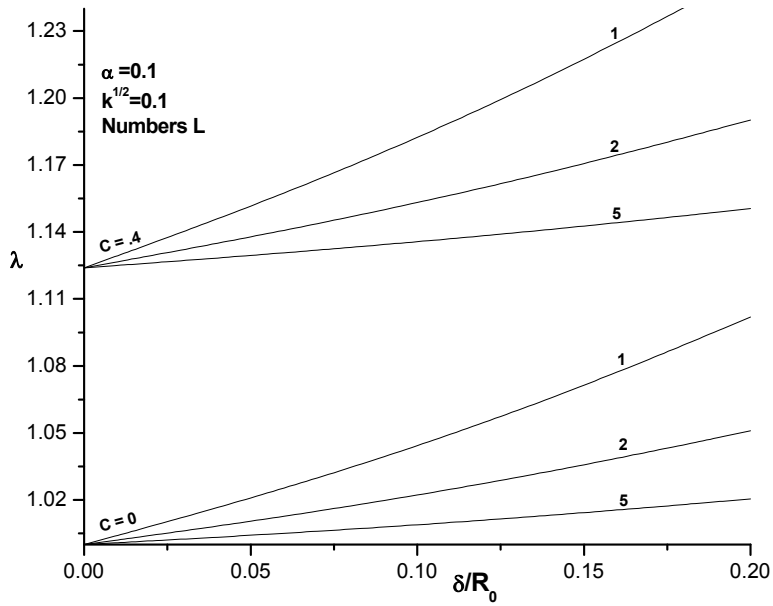

Fig. 5 Impedance, λ versus δ/R_0 for different L.

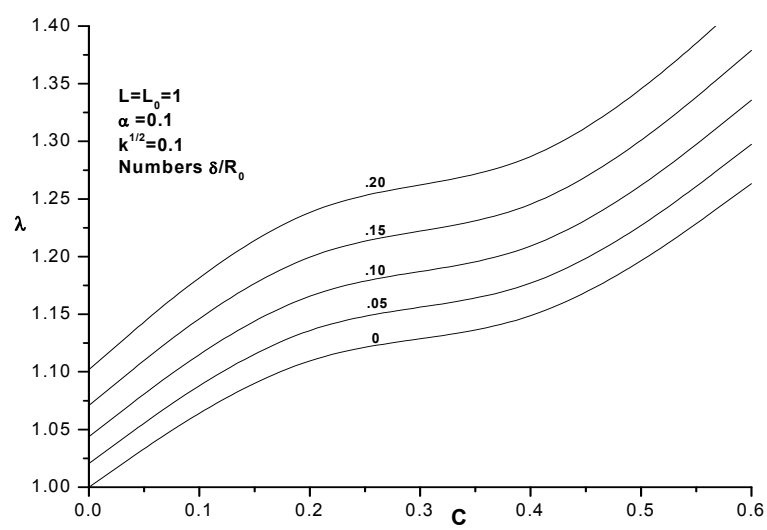

Fig. 6 Impedance, λ versus C for different δ/R_0.

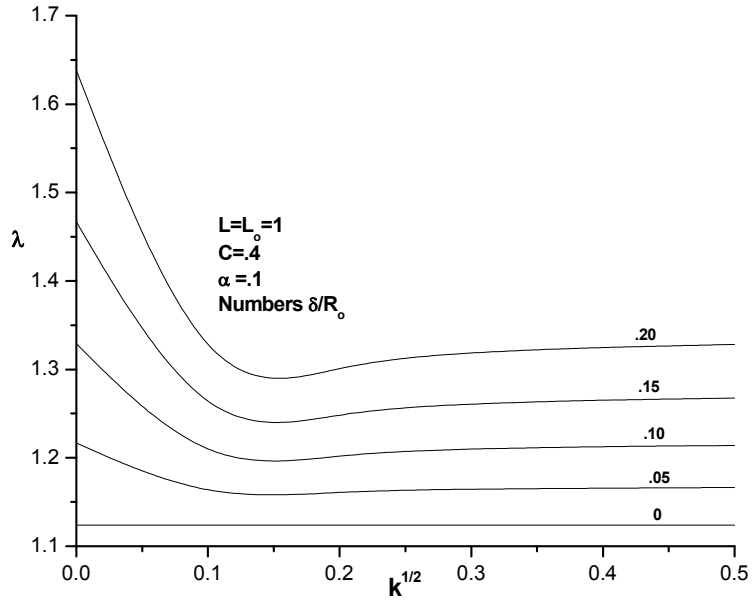

Fig.7 Impedance, λ versus Darcy number, $k^{1/2}$ for different stenosis height, δ/R_o.

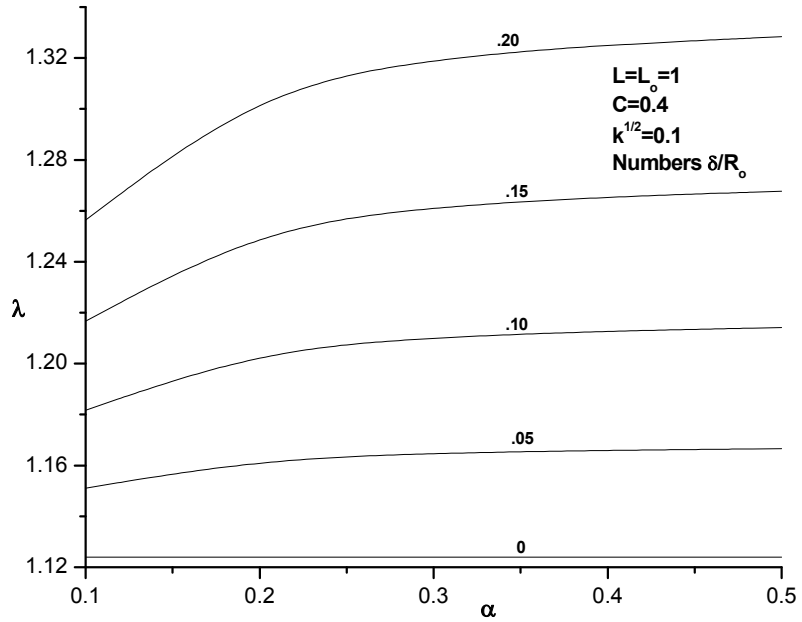

Fig.8 Impedance, λ versus slip parameter, α for different stenosis height, δ/R_o.

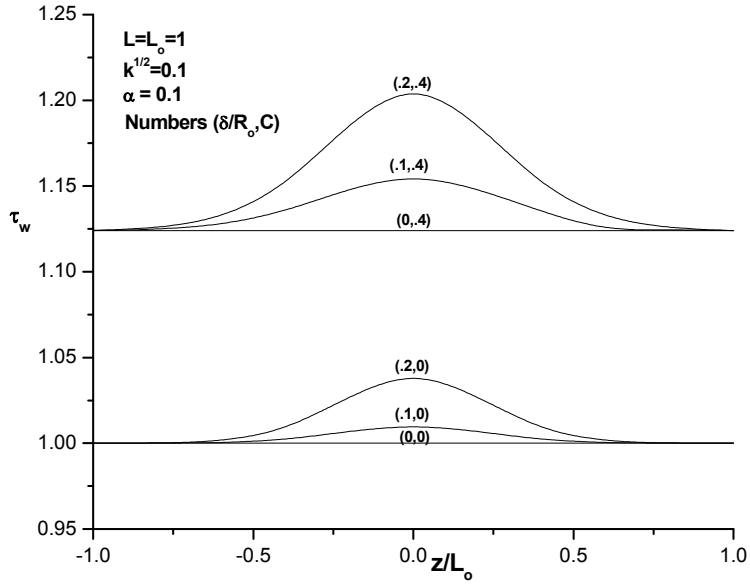

Fig. 9 Wall Shear stress distribution, τ_w in stenotic region for different δ/R_o and C.

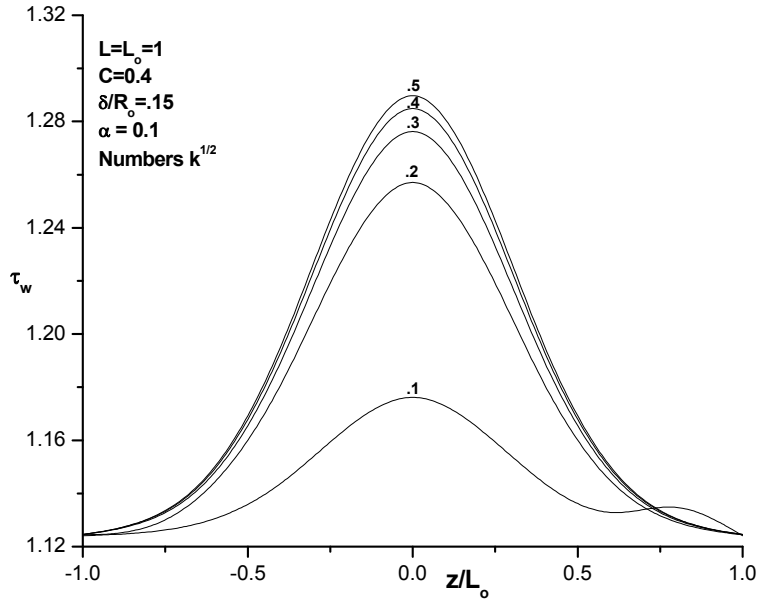

Fig.10 Wall shear stress distribution, τ_w in stenotic region for different $k^{1/2}$.

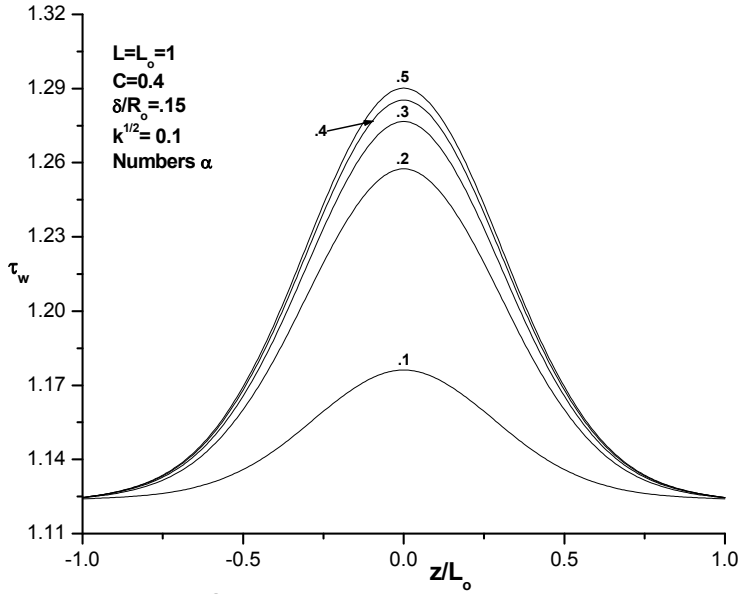

Fig.11 Wall Shear stress distribution, τ_w in stenotic region for different α.

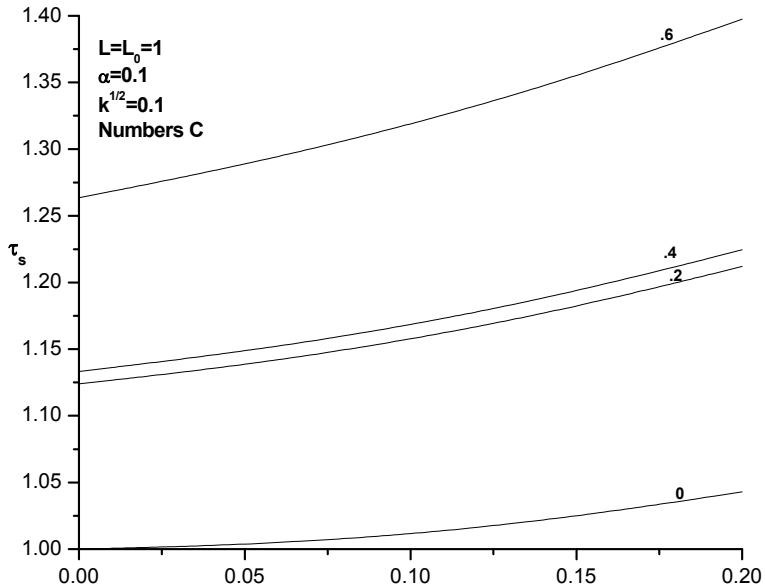

Fig. 12 Shear stress, τ_s versus δ/R_0 for different C.

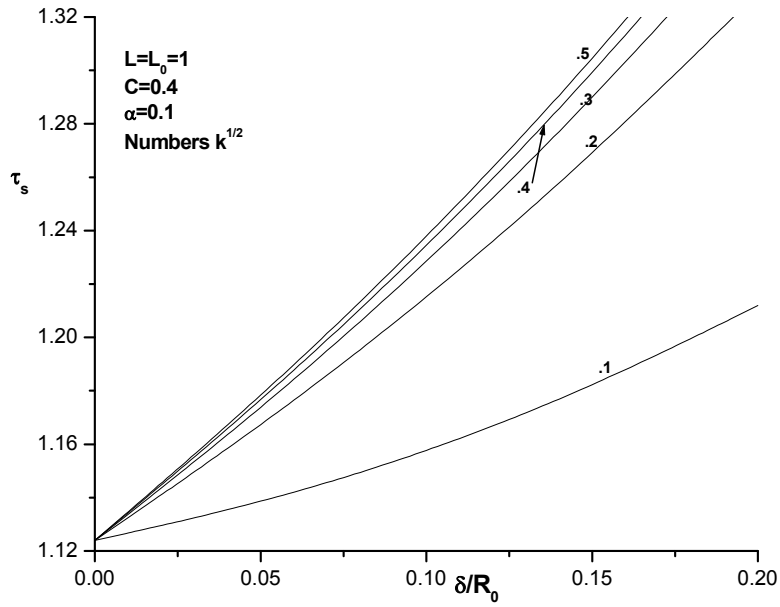

Fig. 13 Shear stress, τ_s versus δ/R_0 for different $k^{1/2}$.

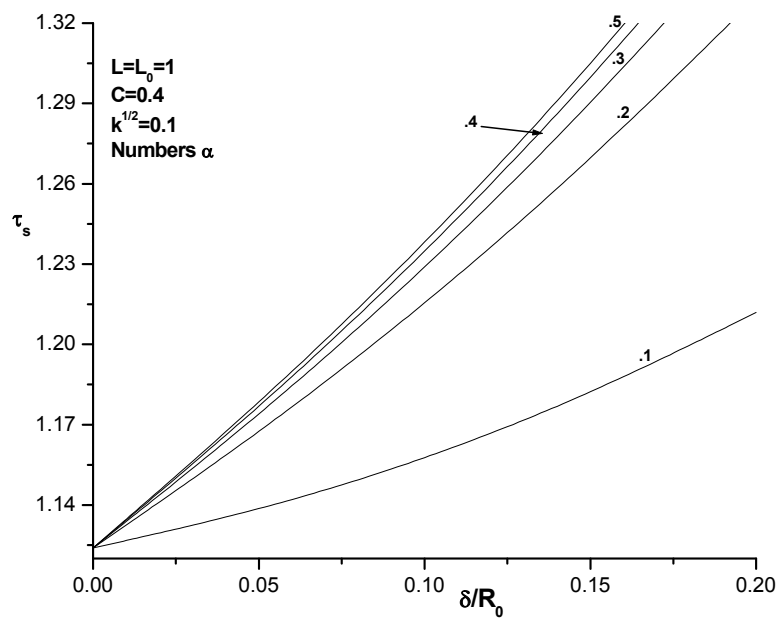

Fig. 14 Shear stress, τ_s versus δ/R_0 for different α.

The shear stress at stenosis throat τ_s increases with the hematocrit, C as well as with the stenosis height, δ / R_0 (Figure 12). τ_s also increases with Darcy number, $\sqrt{k}$ (Figure 13) and with the slip parameter, α (Figure 14). The blood flow characteristic τ_s possesses characteristic similar to that of the flow resistance, λ with respect to any parameter (Figures 2 and 13).

5. Conclusions

To discuss the effects of the wall permeability and the hematocrit, the flow through a bell shaped stenosis in an artery with permeable wall; a macroscopic two-phase blood model has been used. The flow resistance increases with the hematocrit, stenosis size (height and length both), Darcy number and as well as with the slip parameter. The wall shear stress at any axial location in the stenotic region possesses variations similar to that of the impedance with respect to any parameter. The shear stress at stenosis throat possesses the characteristic similar to that of the impedance.

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