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Reflection of Waves in Transversely Isotropic Thermoelastic Solid

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Abstract

The main objective of the present paper is to study the propagation of waves in the transversely isotropic medium in the context of thermoelasticity with GN theory of type-II and III. By imposing the boundary conditions on the components of displacement, stresses and temperature distribution, wave equation have been solved. Numerically simulated results have been plotted graphically with respect to frequency to evince the effect of anisotropy.

Keywords: Thermoelasticity, transversely isotropic, reflection, plane wave

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1. Introduction

In last three decades, non-classical theories involving finite speed of heat transportation in elastic solids have been developed to remove the paradox obtained in classical theory. These generalized theories involve a hyperbolic-type heat transport equation, in contrast to the conventional coupled thermo-elasticity theory (1967), which involves a parabolic-type heat transport equation, and are supported by experiments exhibiting the actual occurrence of wave-type heat transport in solids, called second sound effect. The extended thermo-elasticity theory proposed by Lord and Shulman (1967), incorporates a flux-rate term into Fourier's law of heat conduction, and formulates a generalized form that involves a hyperbolic-type heat transport admitting finite speed of thermal signals. Green and Lindsay (1972) developed temperature-rate-dependent thermo-elasticity theory by introducing relaxation time factors that does not violate the classical Fourier law of heat conduction and this theory also predicts a finite speed for heat propagation.

Chandrasekharaiah (1998), Hetnarski and Ignazack (1999) in their recent surveys, considered the theory proposed by Green and Naghdi (1991,1992,1993,1995) as an alternate way of formulating the propagation of heat. This theory is developed in a rational way to produce a fully consistent theory that is capable of incorporating thermal pulse transmission in a very logical manner. They make use of general entropy balance rather than an entropy inequality. The development is quite general and the characterization of material response for the thermal phenomena is based on three types of constitutive functions that are labeled as type I, type II, and type III. When theory of type I is linearized, the parabolic equation of heat conduction arises. Here, we are interested in the theory of type II (a limiting case of the type III), which does not admit energy dissipation. This theory is usually called "without energy dissipation". Following Green and Naghdi, the theory of thermoelasticity without energy dissipation is a good model to explain the heat conduction in continua.

Quintanilla (2002), proposed a model of the thermoelastic theory without energy dissipation for materials with affine microstructure. In this article he obtained the equations for linear theory and also obtained uniqueness theorem for materials with a centre of symmetry. Taheri et al. (2004) and Puri et al. (2004) employed the Green-Naghdi linear theory of thermoelasticity of types II and III to study the thermal and mechanical waves in a layer of homogeneous thermoelastic solid and plane waves in infinite medium respectively. Many researchers investigated different type of problems in the theory of thermoelasticity of type III [Lazzari and Nibbi (2008), Roychoudhuri and Bandyopadhyay (2007), Mukhopadhyay and Kumar (2008), Quintanilla and Racke (2003), Quintanilla (2009), Quintanilla (2001), Quintanilla (2004), Leseduarte and Quintanilla (2006)].

In the present paper, the propagation of waves in a transversely isotropic medium in the context of thermoelasticity with GN theory of type-II and III has been studied. This study has many applications in various field of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. The graphical representation in given for amplitude ratios of various reflected waves for different incident waves at different angle of incidence i.e., for $\theta = 30^o, 45^o$.

2. Basic Equations

The constitutive relations and balance laws in general anisotropic thermoelastic medium, possessing center of symmetry, in the absence of body forces following Green and Naghdi (1992) are given by

Constitutive relations

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

The deformation tensor is defined by

$$e_{ij} = \frac{(u_{i,j} + u_{j,i})}{2},$$

Balance law

$$t_{ij,j} = \rho\ddot{u}_i \tag{2}$$

Equation of heat conduction

$$K_{ij}\dot{T}_{,ij} + K^*_{ij}T_{,ij} = (T_0\beta_{ij}\ddot{u}_{i,j} + \rho c^*\ddot{T}), \quad i,j = 1,2,3, \tag{3}$$

where ρ is the mass density, t_{ij} are components of stress tensor, u_i the mechanical displacement, e_{ij} are components of infinitesimal strain, T the temperature change of a material particle, T_0 the reference uniform temperature of the body, K_{ij} is the thermal conductivity, K^*_{ij} are the characteristic constants of the theory, $\beta_{ij} = C_{ijkl}\alpha_{kl}$ are the thermal elastic coupling tensor, α_{kl} are the coefficient of linear thermal expansion, c the specific heat at constant strain, C_{ijkl} are characteristic constants of material following the symmetry properties $C_{ijkl} = C_{klij} = C_{jikl}$, $K^*_{ij} = K^*_{ji}, K_{ij} = K_{ji}, \beta_{ij} = \beta_{ji}$. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

3. Problem Formulation

Following Slaughter (2002), the appropriate transformation is used on the set of equations given by equation (1), to derive equations for transversely isotropic medium and restricted our analysis to the two dimensional problem. The origin of the coordinate system (x_1, x_2, x_3) is taken at the free surface of the half space. The $x_1 - x_2$ plane is chosen to coincide with the free surface and x_3 axis pointing normally into the half-space, which is thus represented by $x_3 \geq 0$. We consider plane waves in plane such that all particles on a line parallel to x_2-axis are equally displaced. Therefore, all the field quantities will be independent of x_2 coordinate. So, the component of the displacement vector is taken in the form

$$\vec{U} = (u_1, 0, u_3) \tag{4}$$

and assume that the solutions are explicitly independent of x_2, i.e. $\partial / \partial x_2 \equiv 0$. Thus the field equations and constitutive relations for such a medium reduces to:

$$C_{11}\frac{\partial^2 u_1}{\partial x_1{}^2} + C_{55}\frac{\partial^2 u_1}{\partial x_3{}^2} + (C_{13}+C_{55})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{5}$$

$$C_{55}\frac{\partial^2 u_3}{\partial x_1{}^2} + C_{33}\frac{\partial^2 u_3}{\partial x_3{}^2} + (C_{13}+C_{55})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} - \beta_3 \frac{\partial T}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{6}$$

$$K_1\frac{\partial^2 \dot{T}}{\partial x_1{}^2} + K_3\frac{\partial^2 \dot{T}}{\partial x_3{}^2} + K_1^*\frac{\partial^2 T}{\partial x_1{}^2} + K_3^*\frac{\partial^2 T}{\partial x_3{}^2} = \rho c^* \frac{\partial^2 T}{\partial t^2} + T_o(\beta_1 \frac{\partial \ddot{u}_1}{\partial x_3} + \beta_3 \frac{\partial \ddot{u}_3}{\partial x_1}), \tag{7}$$

where $\beta_1 = C_{11}\alpha_1 + C_{13}\alpha_3, \beta_3 = C_{31}\alpha_1 + C_{33}\alpha_3$ and we have used the notations $11 \to 1, 13 \to 5, 33 \to 3,$ for the material constants.

For further considerations, it is convenient to introduce the non-dimensional quantities defined by

$$x_i^{'} = \frac{x_i}{L}, u_i^{'} = \frac{u_i}{L}, t_{ij}^{'} = \frac{t_{ij}}{C_{11}}, t' = \frac{t}{t_o}, T' = \frac{T}{T_o}, \tag{8}$$

where L, t_o, T_o are parameters having dimension of length, time and temperature respectively.

Solution of the Problem

Let $\vec{p} = (p_1, 0, p_3)$ denote the unit propagation vector, c and ξ are respectively the phase velocity and the wave number of the plane waves propagating in $x_1 - x_3$ plane.

For plane wave solution of the equations of motion of the form

$$(u_1, u_3, T) = (\overline{u}_1, \overline{u}_3, \overline{T}) e^{i\xi(p_1 x_1 + p_3 x_3 - ct)} . \tag{9}$$

With the help of equations (8) and (9) in equations (5)-(7), three homogeneous equations in three unknowns are obtained. Solving the resulting system of equations for non-trivial solution results in

$$Ac^6 + Bc^4 + Cc^2 + D = 0, \tag{10}$$

where

$$A = g_6, B = g_5 + \omega g_7 + g_8\omega^{-2}, C = \omega g_4 - g_2, D = g_1 + \omega g_3, g_1 = (a_5 a_1 - d_2 d_6 p_1^2 p_3^2) a_{10},$$

$$g_2 = a_1[a_{10}(d_8 + d_9) - p_3^2 d_7 d_{14}] + d_4 a_5 a_{10} - p_1^2 p_3^2 d_2 (d_6 d_9 + d_7 d_{13}) + p_1^2 d_3 (p_1^2 d_5 d_{13} + p_3^2 (d_{13} + d_6 d_{14})),$$

$$g_3 = -i[a_5 (p_1^4 d_1 d_{11} + p_3^2 a_1 d_{12})] + p_1^2 p_3^2 (d_{11}(p_1^2 + p_3^2 - d_2 d_6 p_1^2) - d_2 d_6 d_{12} p_3^2)], g_6 = d_4 d_8 d_9,$$

$$g_4 = i[p_1^2 d_{11}(d_8 a_1 + d_4 a_5) + p_3^2 d_{12}(d_4 a_5 + d_8 a_4)], g_7 = -i d_4 d_8 (p_3^2 d_{12} + p_1^2 d_{11}), g_8 = p_1^2 d_4 d_8 d_{10},$$

$$g_5 = d_4 d_9 a_{10} + p_1^2 d_8 (d_1 d_9 + d_3 d_{13}) + p_3^2 [d_8 (d_4 - d_9) - d_4 d_7 d_{14}], a_i = (d_i p_1^2 + p_3^2), i = 1,5,10,$$

$$d_1 = \frac{C_{11}}{C_{55}}, d_2 = \frac{(C_{13} + C_{55})}{C_{55}}, d_3 = \frac{\beta_1 T_o}{C_{55}}, d_4 = \frac{\rho L^2}{C_{55} t_o^2}, d_5 = \frac{C_{55}}{C_{33}}, d_6 = d_2, d_7 = \frac{\beta_1 T_o}{C_{33}}, d_8 = \frac{\rho L^2}{C_{33} t_o^2},$$

$$d_9 = \frac{\rho c^* L^2}{K_3^* t_o^2}, d_{10} = \frac{K_1^*}{K_3^*}, d_{11} = \frac{K_1}{K_3^* t_o}, d_{12} = \frac{K_3}{K_3^* t_o}, d_{13} = \frac{\beta_1 L^2}{K_3^* t_o^2}, d_{14} = \frac{\beta_3 L^2}{K_3^* t_o^2}.$$

The roots of this equation give three values of c^2. Three positive values of c will be the velocities of propagation of three possible waves. The waves with velocities c_1, c_2, c_3 correspond to three types of quasi waves. We name these waves as quasi-longitudinal displacement (qLD) wave, quasi thermal wave (qT) and quasi transverse displacement (qTD) wave.

4. Reflection of Waves

Consider a homogeneous generalized thermoelastic transversely isotropic half-space occupying the region $x_3 > 0$. Incident qLD or qT or qTD wave at the interface will generate reflected qLD, qT and qTD waves in the half space $x_3 > 0$. The total displacements and temperature distribution are given by

$$(u_1, u_3, T) = \sum_{j=1}^{6} A_j (1, r_j, s_j) e^{iB_j}, \tag{11}$$

where

$$B_j = \begin{cases} \omega(t - x_1 \sin e_j - x_3 \cos e_j)/c_j, & j = 1,2,3, \\ \omega(t - x_1 \sin e_j + x_3 \cos e_j)/c_j, & j = 4,5,6, \end{cases} \tag{12}$$

ω is the angular frequency. Here subscripts 1,2,3 respectively denote the quantities corresponding to incident qLD, qT and qTD wave whereas the subscripts 4,5 and 6 respectively denote the corresponding reflected waves and

$$r_j = \frac{\wedge_{1j}}{\wedge_j}, s_j = \frac{\wedge_{2j}}{\wedge_j},$$

$$\wedge_j = \begin{vmatrix} a_5 - d_8 c_j^2 & i d_7 p_3 \xi^{-1} \\ i c_j^2 \xi^2 p_3 d_{14} & \xi(a_{10} - i\omega a_2 - d_9 c_j^2) \end{vmatrix}, \wedge_{1j} = \begin{vmatrix} d_8 p_1 p_3 & i d_7 p_3 \xi^{-1} \\ -i c_j^2 \xi^2 p_1 d_{13} & \xi(a_{10} - i\omega a_2 - d_9 c_j^2) \end{vmatrix},$$

$$\wedge_{2j} = \begin{vmatrix} d_8 p_1 p_3 & a_5 - d_8 c_j^2 \\ -i c_j^2 \xi^2 p_1 d_{13} & i c_j^2 \xi^2 p_3 d_{14} \end{vmatrix}, a_2 = p_1^2 d_{11} + p_3^2 d_{12}.$$

For incident

qLD-wave: $p_1 = \sin e_1, p_3 = \cos e_1,$

qT-wave: $p_1 = \sin e_2, p_3 = \cos e_2,$

qTD-wave: $p_1 = \sin e_3, p_3 = \cos e_3,$

for reflected

qLD-wave: $p_1 = \sin e_4, p_3 = \cos e_4,$

qT-wave: $p_1 = \sin e_5, p_3 = \cos e_5,$

qTD-wave: $p_1 = \sin e_6, p_3 = \cos e_6.$

Here $e_1 = e_4, e_2 = e_5, e_3 = e_6$, i.e. the angle of incidence is equal to the angle of reflection in generalized thermoelastic transversely isotropic, so that the velocities of reflected waves are equal to their corresponding to their corresponding incident wave's i.e. $c_1 = c_4, c_2 = c_5, c_3 = c_6$.

5. Boundary Conditions

The boundary conditions at the thermally insulated surface $x_3 = 0$ are given by

$$t_{33} = 0,\ t_{31} = 0, \frac{\partial T}{\partial x_3} = 0, \tag{13}$$

where

$$t_{33} = C_{13}\frac{\partial u_1}{\partial x_1} + C_{33}\frac{\partial u_3}{\partial x_3} - \beta_3 T, t_{31} = C_{55}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}). \tag{14}$$

The wave numbers $\xi_j, j = 1,2,.....6$ and the apparent velocity $c_j, j = 1,2,.....6$ are connected by the relation

$$c_1\xi_1 = c_2\xi_2 = ... = c_6\xi_6 = \omega, \tag{15}$$

at the surface $x_3 = 0$. Relation (14) may also be written in order to satisfy the boundary conditions (12) as

$$\frac{\sin e_1}{c_1} = \frac{\sin e_2}{c_2} = ... = \frac{\sin e_6}{c_6} = \frac{1}{c}. \tag{16}$$

Making use of equations (7), (10), (13) and (15) into thermally insulated boundary conditions (12), we obtain

$$\sum_{j=1}^{6} A_{ij} A_j = 0, \; i = 1,2,3, \tag{17}$$

where

$$A_{1j} = \begin{cases} \frac{C_{13}}{C_{11}}\frac{\sin e_j}{c_j} + r_j\frac{C_{33}}{C_{11}}\frac{\cos e_j}{c_j} - \frac{\beta_3 s_j}{C_{11}}, & j = 1,2,3, \\ \frac{C_{13}}{C_{11}}\frac{\sin e_j}{c_j} - r_j\frac{C_{33}}{C_{11}}\frac{\cos e_j}{c_j} - \frac{\beta_3 s_j}{C_{11}}, & j = 4,5,6, \end{cases}, \quad A_{2j} = \begin{cases} \frac{C_{55}}{C_{11}}[\frac{\cos e_j}{c_j} - r_j\frac{\sin e_j}{c_j}, & j = 1,2,3, \\ \frac{C_{55}}{C_{11}}[\frac{\cos e_j}{c_j} + r_j\frac{\sin e_j}{c_j}, & j = 4,5,6, \end{cases},$$

$$A_{3j} = \begin{cases} s_j\frac{\cos e_j}{c_j}, & j = 1,2,3, \\ -s_j\frac{\cos e_j}{c_j}, & j = 4,5,6, \end{cases}.$$

Incident qLD-wave:

In case of incident qLD- wave, $A_2 = A_3 = 0$. Dividing set of eqns. (16) throughout by A_1, we obtain a system of three non-homogeneous equations in three unknowns which can be solved by Gauss elimination method and we have

$$Z_i = \frac{A_{i+3}}{A_1} = \frac{\Delta_i^1}{\Delta},\ i = 1,2,3. \tag{18}$$

Incident qT-wave:

In case of incident qT- wave, $A_1 = A_2 = 0$ and thus we have

$$Z_i = \frac{A_{i+3}}{A_2} = \frac{\Delta_i^2}{\Delta},\ i = 1,2,3. \tag{19}$$

Incident qTD-wave:

In case of incident qTD- wave, $A_1 = A_2 = 0$ and thus we have

$$Z_i = \frac{A_{i+3}}{A_3} = \frac{\Delta_i^3}{\Delta},\ i = 1,2,3, \tag{20}$$

where $\Delta = \left|A_{i\,i+3}\right|_{3\times 3}$ and Δ_i^p $(i = 1,2,3,\ p = 1,2,3,)$ can be obtained by replacing, respectively, the 1^{st}, 2^{nd}, 3^{rd} column of Δ by $\left[-A_{1p}\ \ -A_{2p}\ \ -A_{3p}\right]^T$.

6. Numerical Results and Discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. The following relevant physical constants for Cobalt material are taken from Dhaliwal et. al. (1980) for a thermoelastic transversely isotopic material,

$C_{11} = 3.071\times 10^{11}\,Nm^{-2}, C_{12} = 1.650\times 10^{11}\,Nm^{-2}, C_{13} = 1.027\times 10^{11}\,Nm^{-2}, C_{33} = 3.581\times 10^{11}\,Nm^{-2}$,

$C_{55} = 1.51\times 10^{11}\,Nm^{-2}, \beta_1 = 7.04\times 10^{6}\,Nm^{-2}K,\ \beta_1 = 6.98\times 10^{6}\,Nm^{-2}K,\ \rho = 8.836\times 10^{3}\,Kgm^{-3}$,

$K_1 = 6.90\times 10^{2}\,Wm^{-1}K,\ K_3 = 7.01\times 10^{2}\,Wm^{-1}K, K_1^* = 1.313\times 10^{2}\,W\sec, K_3^* = 1.54\times 10^{2}\,W\sec$,

$c^* = 4.27\times 10^{2}\,J\,Kg\,K,\ T = 298K.$

The variations of amplitude ratio of reflected qLD, qT and qTD waves, for incident qLD, qT and qTD waves at the free surface are represented graphically to compare the results obtained in two cases, viz., for the waves incident from transversely isotropic medium in the context of thermoelasticity with energy dissipation model (TIWED) and other from isotropic thermoelastic (IWED) half space. In figures 1-3, the graphical representation is given for the variation of amplitude ratios $|Z_1|,|Z_2|$ and $|Z_3|$ for incident qLD wave. Figures 4-6 and 7-9, respectively represent the similar situation, when qT and qTD waves are incident.

Here $|Z_1|,|Z_2|$ and $|Z_3|$ are, respectively, the amplitude ratios of reflected qLD, qT and qTD waves. These variation are shown for two angles of incidence viz, $\theta = 30^o, 45^o$. In these figures the solid curves lines correspond to the case of TIWED, while broken curves correspond to the case of IWED. Also, the curves without centre symbol correspond to the case, when $\theta = 30^o$ and with centre symbol (-o-o-) represents the variation corresponding to the case of $\theta = 45^o$.

Incident qLD-wave:

It is evident from figure 1 that the amplitude ratio $|Z_1|$ of reflected qLD-wave, for $\theta = 30^o$ and for TIWED, first decreases within the interval (0, 2), then sharply increases to attain a peak value, then decreases and flatten to become steady at the end. However, for the case of $\theta = 45^o$, its value remains steady at initial stage, then decreases over the interval (10, 30) to become constant at the end with increase in frequency. For IWED, its value initially oscillates and then become steady with increase in frequency for both values of theta.

Figures 2 and 3 indicates the variation of amplitude ratio $|Z_2|$ and $|Z_3|$ of reflected qT and qTD-waves, which shows that for the case of TIWED, their value represents the similar behavior as depicted in the case of $|Z_1|$, with difference in their initial region of decrease. In the present case, its value decrease for a small interval and then attain a peak value within the intervals (7, 30) and (5, 30) respectively. However, for IWED the variation is almost similar as depicted for $|Z_1|$ with difference in their amplitude.

Incident qT-wave:

The variation in the amplitude ratio of various reflected wave for incident qT-wave is shown in figures 4-6. It is depicted from figure 4 that the value of amplitude ratio of $|Z_1|$ decreases with increase in frequency to attain a constant value at the end. However, as the angle of inclination gets increased, its value gets increased within the interval $0 \le \omega \le 50$, and then decreases with further increase in frequency. For IWED and $\theta = 30^o$, the value increase within the range (0, 30) and then decrease, while for $\theta = 45^o$ its value decrease with increase in frequency. Figure 5 shows that the value of amplitude ratio of $|Z_2|$, represent the similar behavior as depicted in figure 4 for all the cases, except for the case TIWED at $\theta = 45^o$, where its value decrease with increase in frequency. It is illustrated from figure 6 that the value of amplitude of $|Z_3|$ for IWED, its value sharply increases over the interval (0, 30) and then decreases with increase in frequency.

While at $\theta = 45^o$ the interval of sharp increase changes to (0, 50) and remaining have the same variation.

Incident qTD-wave:

Figures 7-9 illustrates the variation of amplitude ratios of $|Z_i|, i = 1,2,3,$ with frequency for incident qTD-wave. It can be seen from these figures that the variation pattern of the amplitudes ratios for the case of IWED and for both angle of inclination almost similar with slight difference in their amplitude, while for the case of TIWED, its value represent reverse behavior. The amplitude ratio of $|Z_1|$, for $\theta = 30^o$ initially decreases within the interval $0 \le \omega \le 70$, and then increase. While, for $\theta = 45^o$ it decreases with increase in frequency. However, the value of amplitude ratio $|Z_3|$ goes on increasing with increase in frequency for both angle of inclination. It is evident from figure 8 that the value of amplitude ratio of $|Z_2|$ represents similar behavior for all the cases, with slight difference in their amplitudes.

7. Conclusion

The importance of thermal stresses in causing structural damages and changes in functioning of the structure is well recognized whenever thermal stress environments are involved. Plane wave reflection from the free surface of transversely isotropic medium in the context of thermoelasticity with GN theory of type-II and III has been discussed. It is concluded from the graphs that the value of amplitude ratio $|Z_1|$ shows oscillation at initial frequencies for incident qLD wave, as compared to qT and qLD incident waves. An appreciable effect of anisotropy and angle of incidence is observed on amplitude ratios of various reflected waves.

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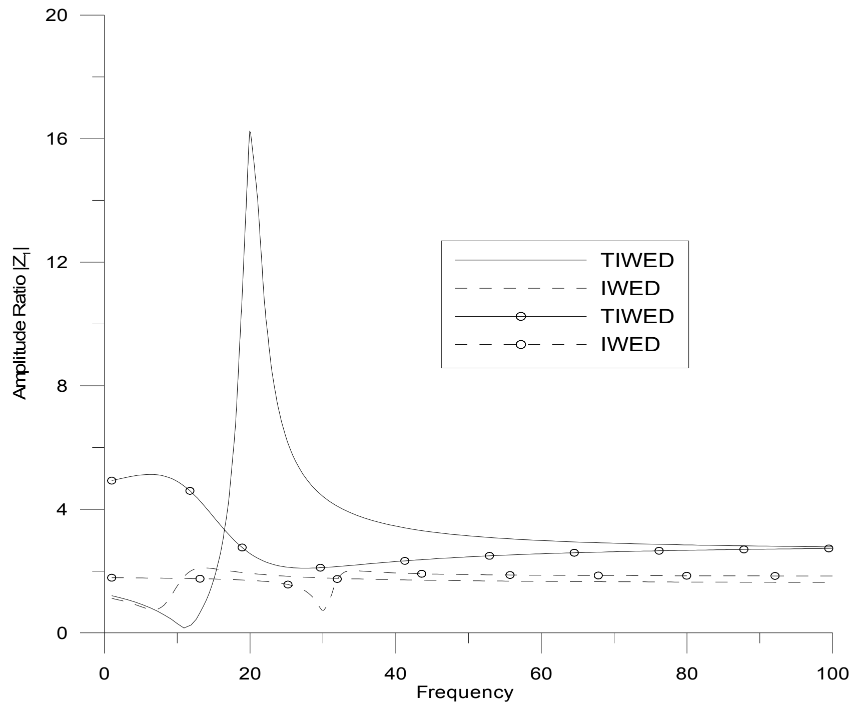

Figure 1. Amplitude Ratio $|Z_1|$

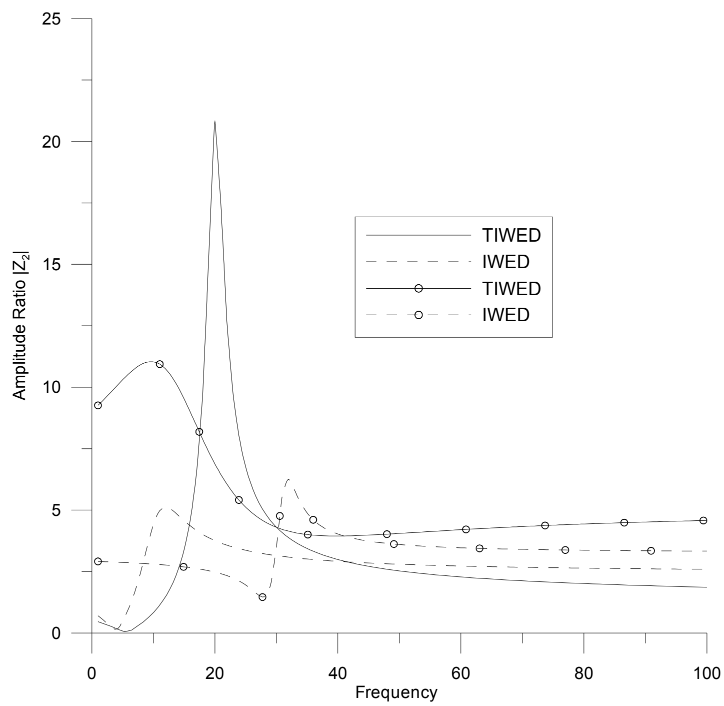

Figure 2. Amplitude Ratio $|Z_2|$

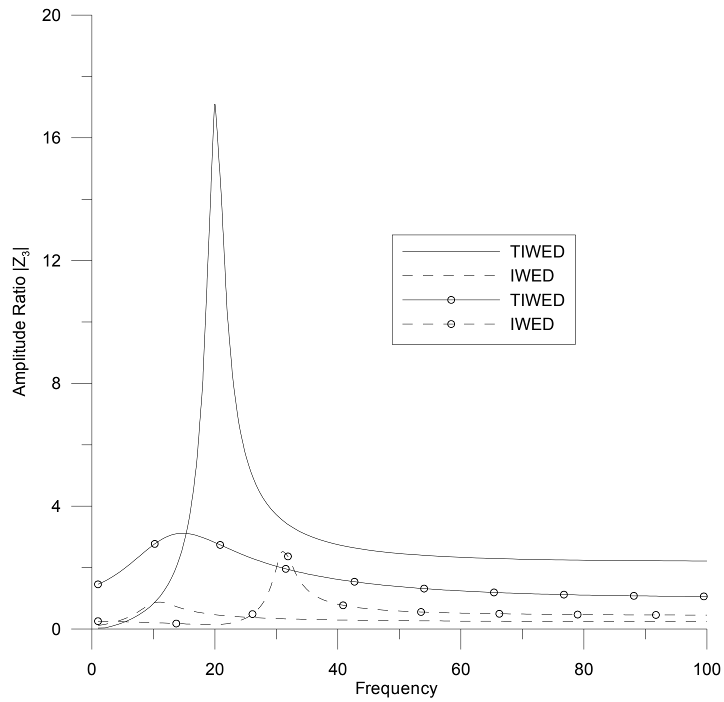

Figure 3. Amplitude Ratio $|Z_3|$

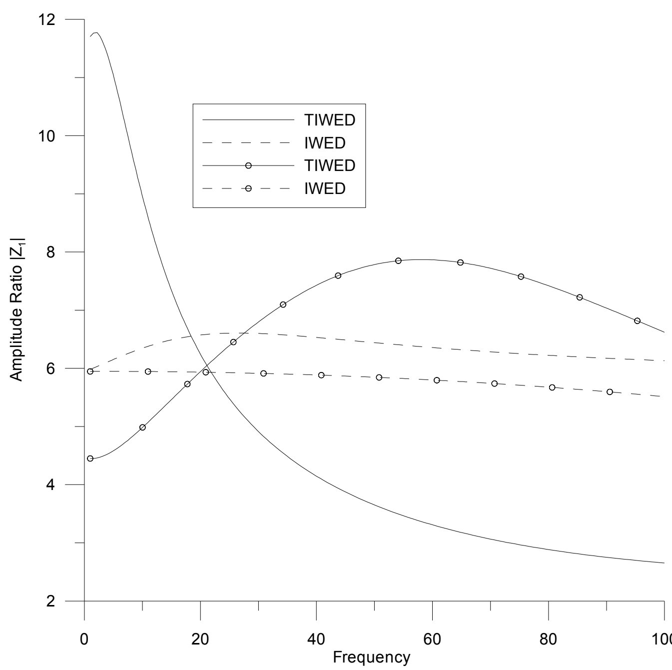

Figure 4. Amplitude Ratio $|Z_1|$

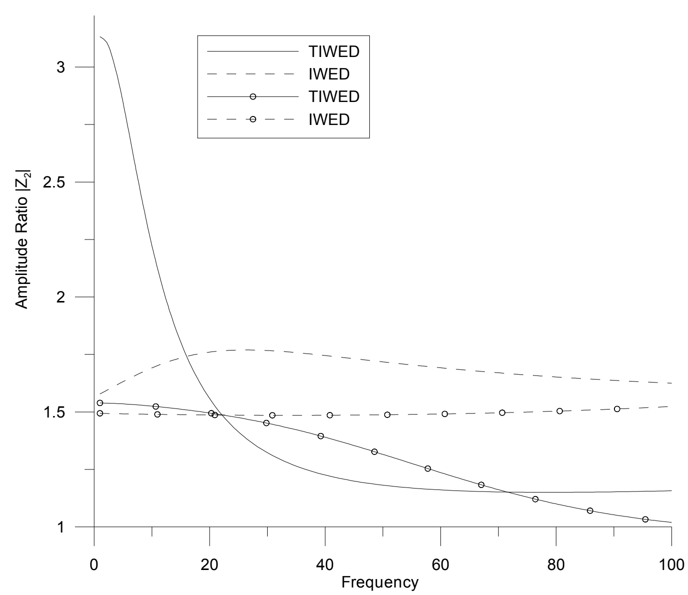

Figure 5. Amplitude Ratio $|Z_2|$

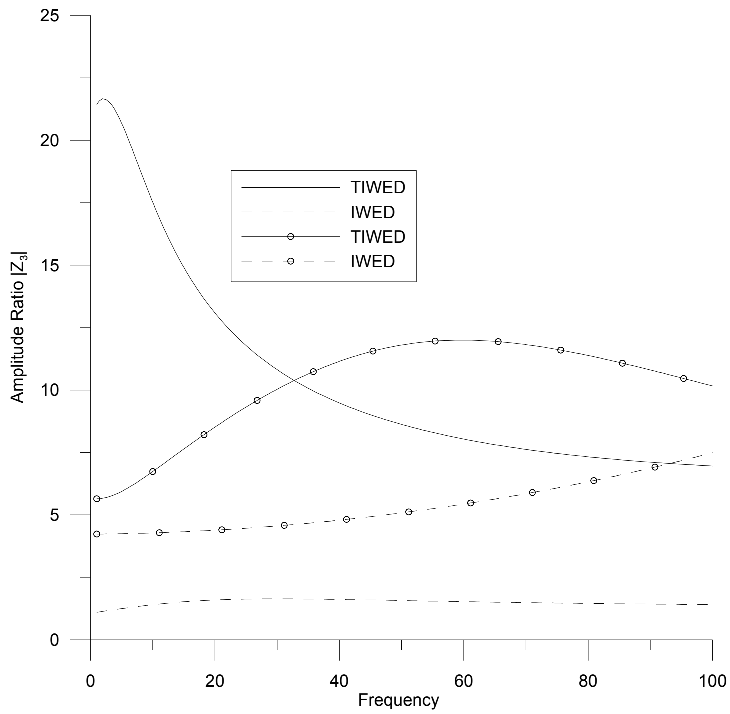

Figure 6. Amplitude Ratio $|Z_3|$

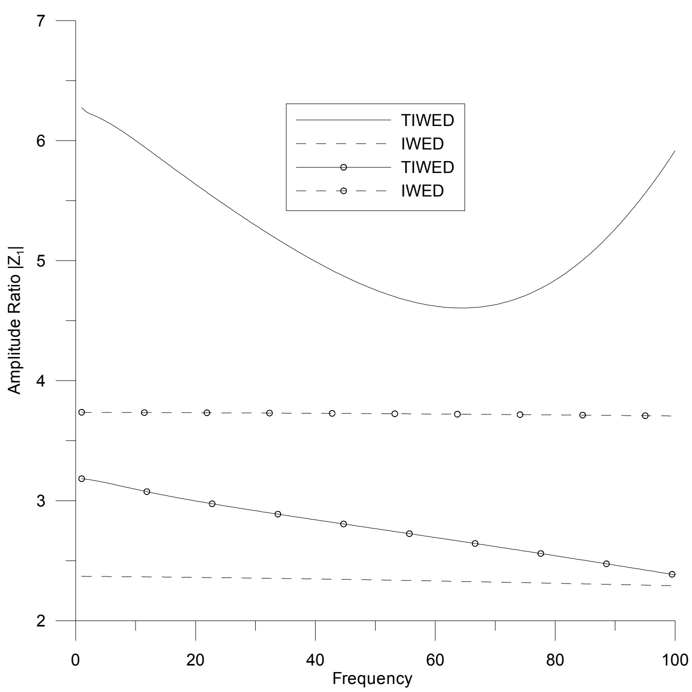

Figure 7. Amplitude Ratio $|Z_1|$

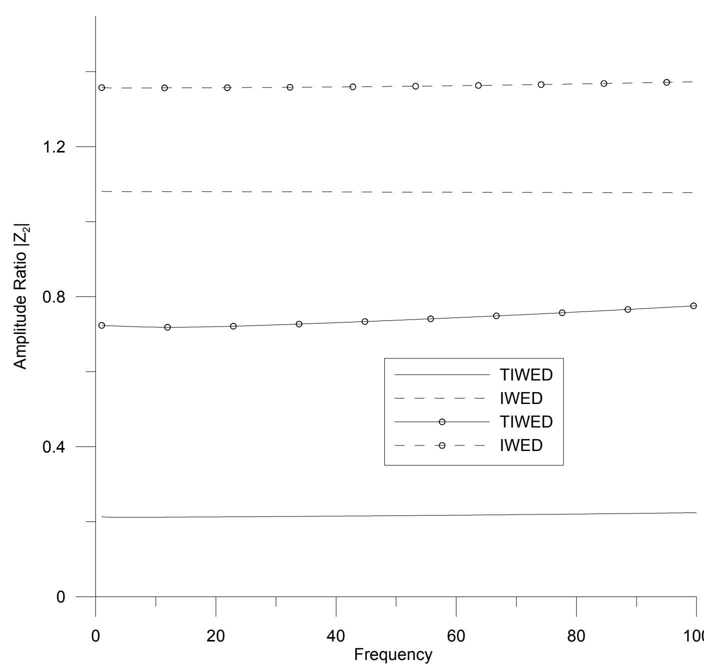

Figure 8. Amplitude Ratio $|Z_2|$

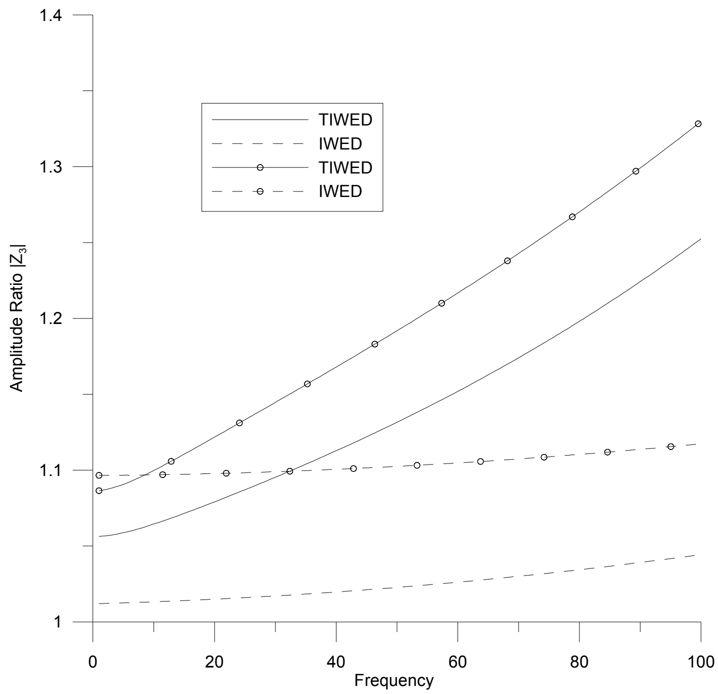

Figure 9. Amplitude Ratio $|Z_3|$