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Multidimensional Inverse Boundary Value Problem for a System of Hyperbolic Equations

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Abstract

In the paper we investigate the solvability of the inverse multidimensional boundary value problem for the system of hyperbolic type equations. A method is proposed to reduce the considered problem to some non infinite system of differential equations. The proposed method allows one to prove the existence and uniqueness theorems for the multidimensional inverse boundary value problems in the class of the functions with bounded smoothness.

Keywords: Boundary Value Problem; Multidimensional System; Hyperbolic Equations; existence and uniqueness theorems

MSC (2010) No.: 35A05; 31A25; 35L20

1. Introduction

The solution of various problems of physics is often necessary for restoration of the basic characteristics of a studied phenomenon or process, if you know the mathematical formulation of the problem. These characteristics are coefficients or unknowns in the right side of the differential equation, which can determine under some additional information, are called inverse problems.

In the last twenty - thirty years, the theory of inverse problems has been enriched by considering the problem of determining the coefficients of partial differential equations as a function of one or several variables. The study of one-dimensional and multi-dimensional inverse problems associated with the restoration of the coefficients of partial differential equations involved in Lavrentev et al. (1980), Romanov (1984), Anikonov et al. (1988), Namazov (1984), Guliev (2004), Isakov (2006) and others. The book of Lavrentev et al. (1980) is described widely recent results on the uniqueness of integral geometry and inverse problems for differential equations in partial derivatives. In the book of Romanov (1984), the inverse problem for second-order equation and hyperbolic systems of first order equations, including kinematic problem of static, dynamic, Lamb problem for the equations of the theory of elasticity and the problem of electrodynamics.

In the paper of Anikonov et al. (1988), the example of a parabolic equation given way to explore the problem of solvability for some nonlinear inverse problems for differential equations. Considered here the inverse problem using the technique of Fourier transform can be reduced to quite acceptable for the study of boundary value problem for nonlinear integro-differential equation. In a study Bubnov (1987), author considered some inverse problems for hyperbolic equations. The method which used based on the reduction of the inverse problem of nonlinear infinite systems of differential equations, allows us to prove theorem existence and uniqueness for solutions of multidimensional inverse boundary value problems in classes of functions of finite smoothness.

In the book of Namazov (1984), author studied one-dimensional inverse problem for the parabolic and hyperbolic type. He was assumed that the unknown coefficients of the equation depend only on the argument t. There are theorems proved the existence and uniqueness of the problems. In paper of Guliev (2004) using the idea of work Bubnov (1987), author investigate the multidimensional inverse boundary value problem for integro-differential equation of hyperbolic type in a bounded domain. In the book of Isakov (2006) using the tool of Carleman estimates the author investigate the inverse problems for hyperbolic, parabolic and elliptic equations and then proved the existence and uniqueness theorems for the solutions of the considered problem.

In this paper, using the idea of work Bubnov (1987) discusses solvability of inverse boundary value problems for hyperbolic systems of equations and then proved the existence and uniqueness theorems for the solutions of the problem.

Note that the considered equation in this paper is called the Klein-Gordon equation. This relative invariant quantum equation describing spineless scalar or pseudo scalar particle, which plays one of the roles of the fundamental equation of quantum field theory. This system of equations cannot be written as a single equation.

2. Problem Statement and Preliminary Results

In the domain $D_T = \overline{\Omega} \times [0,T]$ consider the following problem

$$\frac{\partial^2 U(x,t)}{\partial t^2} - AU(x,t) = a(x)U(x,t) + b(x)V(x,t) + f(x,t) \ , \tag{1}$$

$$\frac{\partial^2 V(x,t)}{\partial t^2} - AV(x,t) = a_1(x)U(x,t) + b_1(x)V(x,t) + g(x,t) \quad (x,t) \in D_T \ , \tag{2}$$

$$U(x,0) = \varphi(x), \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{t=0} = 0, \qquad x \in \overline{\Omega}, \tag{3}$$

$$V(x,0) = \psi(x), \quad \left.\frac{\partial V(x,t)}{\partial t}\right|_{t=0} = 0, \qquad x \in \overline{\Omega}, \tag{4}$$

$$U(x',t) = F(x',t), \ \ V(x',t) = G(x',t), \quad (x',t) \in \Gamma = S \times [0,T] \ , \tag{5}$$

$$U(x,T) = h(x), \quad V(x,T) = q(x), \quad x \in \overline{\Omega}, \tag{6}$$

$$\left.\frac{\partial U(x,t)}{\partial t}\right|_{t=T} = 0, \quad \left.\frac{\partial V(x,t)}{\partial t}\right|_{t=T} = 0, \qquad x \in \overline{\Omega}, \tag{7}$$

where Ω is bounded domain in R^n, $S = \partial\Omega \in C^2$, $n \le 3$,

$$AU(x,t) = \sum_{i,j=1}^{n} (a_{ij}(x) U_{x_i}(x,t))_{x_j}, \ a_{ij}(x) = a_{ji}(x) \in C^4(\overline{\Omega}),$$

$$\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \ge \mu|\xi|^2, \ \mu > 0, \varphi(x), \ \ \psi(x), \ \ f(x,t), \ \ g(x,t), \ \ F(x,t), \ \ G(x,t), \ \ h(x), \ \ q(x)$$

are given functions, and $a(x), \ b(x), a_1(x), b_1(x), U(x,t), V(x,t)$ are unknown functions.

Definition:

The system $\{U(x,t), V(x,t), a(x), b(x), a_1(x), b_1(x)\}$ is called the solution of the problem (1) - (7), if it satisfies to following conditions

1. $a(x), \ b(x), a_1(x), b_1(x) \in W_2^2(\Omega)$.
2. Functions $U(x,t)$, and $V(x,t)$ are continuous in closed domain D_T together with all

derivatives in the equations (1) and (2), respectively.

3. Conditions (1) - (7) are satisfied in usual sense.

Let's designate through $Z_l, g_l(x)$ eigenvalues and eigenfunctions of the problem

$$Ag_l(x) = -Z_l^2 g_l(x), \qquad g_l(x)\big|_S = 0. \tag{8}$$

Then, from the general theory of elliptic equations it follows that $Z_l \to \infty, l \to \infty$ and $g_l(x)$ form a complete set in $\overset{\circ}{W}{}_2^1(\Omega) \cap W_2^2(\Omega)$.

Condition I.

Let for T the domain Ω and for any integers n and l the following inequality be satisfied

$$\left| Z_l^2 - \frac{\pi^2 n^2}{T^2} \right| \ge \frac{C_\circ(\alpha)}{Td} = \delta_1,$$

where $\alpha = diam\ \Omega$, Z_l - eigenvalues of a problem (8) [see Bubnov (1987)].

Note. [See Bubnov (1987)]. Suppose that, for an operator $Au = u_{xx}$, $\Omega = \{0 < x < R_0\}$, the eigenfunction g_l of the problem $Ag_l^{(x)} = -z_l^2 g_l(x),\ g_l(x)\big|_s = 0$ will be the function $g_l(x) = \sin\frac{l\pi x}{R_0}$, $z_l = \frac{\pi l}{R_0}$. Then, the condition I can be written in the form

$$\pi^2 \left| \frac{l^2}{R_0^2} - \frac{k^2}{T^2} \right| = \frac{\pi^2 l^2}{T^2} \left| \frac{T^2}{R_0^2} - \frac{k^2}{l^2} \right| = \frac{\pi^2 l^2}{T^2} \left| \frac{T}{R_0} - \frac{k}{l} \right| \left| \frac{T}{R_0} + \frac{k}{l} \right| \ge \frac{\pi^2 l^2}{T^2} \cdot \frac{T}{R_0} \left| \frac{T}{R_0} - \frac{k}{l} \right| \ge \frac{C_0(\alpha)}{TR_0}$$

The last inequality is written on the basis of Liouville's theorem, provided that the $\frac{T}{R_0}$ is quadratic irrationality.

Lemma 1:

Let the condition I be satisfied and

$$\Phi_k(x) \in L_2(\Omega), \quad \lambda_k = \frac{k\pi}{T}, \quad (k = 1,2,...).$$

Then, for $\forall k$ there exists a solution of the Dirichlet problem

$$Ab_k(x) + \lambda_k^2 b_k(x) = \Phi_k(x), \quad b_k(x)\big|_S = 0,$$

for which the estimation

$$\int_\Omega b_k^2(x)dx \le \frac{1}{\delta_1^2}\int_\Omega \Phi_k^2(x)dx$$

is true.

3. Reduction of the Inverse Problems to the Nonlinear Infinite System of Differential Equations

Let us assume that operator $\boldsymbol{A}$ and functions $\phi(x),\ \psi(x),\ f(x,t),\ g(x,t\ ), F(x,t),$ $G(x,t),\ h(x), q(x)$ satisfy the following conditions

1. $\forall U \in \overset{\circ}{W}{}_2^1(\Omega) \bigcap W_2^2(\Omega)$

$$\int_\Omega |AU(x,t)|^2\, dx \ge \mu_1 \int_\Omega \left[\sum_{i,j=1}^n U_{x_1 x_j}^2(x,t) + U^2(x,t)\right] dx, \qquad \mu_1 > 0;$$

2. $f(x,t),\ g(x,t), \quad D_t^6 f(x,t),\ D_t^6 g(x,t) \in L^2\big((0,T);\ W_2^2(\Omega)\big),$

$$f(x,T),\ g(x,T), \in W_2^2(\Omega), \quad D_t^k f(x,t)\big|_{t=0} = D_t^k f(x,t)\big|_{t=T} = 0,$$

$$D_t^k g(x,t)\big|_{t=0} = D_t^k g(x,t)\big|_{t=T} = 0, \quad (k = 1,3,5);$$

3. $F(x,t), G(x,t), \quad D_t^8 F(x,t),\ D_t^8 G(x,t) \in L^2\big((0,T);\ W_2^{7/2}(S)\big),$

$$D_t^k F(x,t)\big|_{t=0} = D_t^k F(x,t)\big|_{t=T} = 0, \quad D_t^k G(x,t)\big|_{t=0} = D_t^k G(x,t)\big|_{t=T} = 0 \quad (k = 1,3,5,7);$$

4. $\varphi(x), \psi(x) \in W_2^4(\Omega), \quad A(Ah(x)), \ A(Ag(x)) \in L_2(\Omega),$

$$\forall x \in \Omega \quad \varphi(x) \ne \psi(x), \quad h(x) \ne q(x);$$

$$F(x,t)\big|_{t=o} = \varphi(x)\big|_S,\ G(x,t)\big|_{t=o} = \psi(x)\big|_S,\ F(x,t)\big|_{t=T} = h(x)\big|_S,\ G(x,t)\big|_{t=T} = q(x)\big|_S;$$

5. $|\Delta(x)| = |\varphi(x)q(x) - \psi(x)h(x)| \ge \delta > 0$.

The following assertion holds:

Lemma 2:

Let $\{U(x,t),V(x,t),a(x),b(x),a_1(x),b_1(x)\}$ be a solution of the inverse problem (1) - (7). Then, following relations are satisfied:

$$a(x)=\frac{1}{\Delta(x)}\left\{q(x)U_{tt}(x,t)\big|_{t=0}-\psi(x)U_{tt}(x,t)\big|_{t=T}+\Phi_0(x)\right\},$$
$$b(x)=\frac{1}{\Delta(x)}\left\{\varphi(x)U_{tt}(x,t)\big|_{t=T}-h(x)U_{tt}(x,t)\big|_{t=0}+\Phi_1(x)\right\},$$
$$a_1(x)=\frac{1}{\Delta(x)}\left\{q(x)V_{tt}(x,t)\big|_{t=0}-\psi(x)V_{tt}(x,t)\big|_{t=T}+\Phi_2(x)\right\},$$
$$b_1(x)=\frac{1}{\Delta(x)}\left\{\varphi(x)V_{tt}(x,t)\big|_{t=T}-h(x)V_{tt}(x,t)\big|_{t=0}+\Phi_3(x)\right\},$$

where

$$\Phi_0(x)=\left(-A\varphi(x)-f(x,o)\right)q(x)+\left(Ah(x)+f(x,T)\right)\psi(x),$$
$$\Phi_1(x)=\left(A\varphi(x)+f(x,o)\right)h(x)-\varphi(x)\left(Ah(x)+f(x,T)\right),$$
$$\Phi_2(x)=\left(Aq(x)+g(x,T)\right)\psi(x)-q(x)\left(A\psi(x)+g(x,0)\right),$$
$$\Phi_3(x)=h(x)\left(A\psi(x)+g(x,o)\right)-\varphi(x)\left(Aq(x)+g(x,T)\right).$$

The proof of a Lemma 2 follows from (1) - (7).

Lemma 3:

If the functions $\tilde{U}_k(x),\ \tilde{V}_k(x),\ k=0,1,2,\cdots,$ are solutions of the following problem in the domain Ω

$$-\lambda_k^2\tilde{U}_k(x)-A\tilde{U}_k(x)=\frac{\tilde{U}_k(x)}{\Delta(x)}\left\{-q(x)\sum_{m=0}^{\infty}\lambda_m^2\tilde{U}_m(x)+\psi(x)\sum_{m=0}^{\infty}(-1)^m\lambda_m^2\tilde{U}_m(x)+\Phi_0(x)\right\}$$
$$+\frac{\tilde{V}_k(x)}{\Delta(x)}\left\{-\varphi(x)\sum_{m=0}^{\infty}(-1)^m\lambda_m^2\tilde{U}_m(x)+h(x)\sum_{m=0}^{\infty}\lambda_m^2\tilde{U}_m(x)+\Phi_1(x)\right\}+f_k(x),$$

$$-\lambda_k^2\tilde{V}_k(x)-A\tilde{V}_k(x)=\frac{\tilde{U}_k(x)}{\Delta(x)}\left\{-q(x)\sum_{m=0}^{\infty}\lambda_m^2\tilde{V}_m(x)+\psi(x)\sum_{m=0}^{\infty}(-1)^m\lambda_m^2\tilde{V}_m(x)+\Phi_2(x)\right\}$$
$$+\frac{\tilde{V}_k(x)}{\Delta(x)}\left\{-\varphi(x)\sum_{m=0}^{\infty}(-1)^m\lambda_m\tilde{V}_m(x)+h(x)\sum_{m=0}^{\infty}\lambda_m^2\tilde{V}_m(x)+\Phi_3(x)\right\}+g_k(x) \quad (9)$$

$$\tilde{U}_k(x)\big|_S=F_k(x),\quad \tilde{V}_k(x)\big|_S=G_k(x),\quad (k=0,1,2,...) \qquad (10)$$

from the class

$$\sum_{m=0}^{\infty}\lambda_m^8\int_{\Omega}\left[A\tilde{U}_m(x)\right]^2 dx+\sum_{m=0}^{\infty}\int_{\Omega}\lambda_m^{12}\left|\tilde{U}_m(x)\right|^2 dx+\sum_{m=0}^{\infty}\lambda_m^6\left\|A\tilde{U}_m(x)\right\|_{W_2^1(\Omega)}^2 +\sum_{m=0}^{\infty}\lambda_m^2\left\|A\left(A\tilde{U}_m(x)\right)\right\|_{L_2(\Omega)}^2<+\infty,$$

$$\sum_{m=0}^{\infty}\lambda_m^{12}\int_{\Omega}\left|\tilde{V}_m(x)\right|^2 dx+\sum_{m=0}^{\infty}\lambda_m^8\int_{\Omega}\left[A\tilde{V}_m(x)\right]^2 dx+\sum_{m=0}^{\infty}\lambda_m^6\left\|A\tilde{V}_m(x)\right\|_{W_2^1(\Omega)}^2 +\sum_{m=0}^{\infty}\lambda_m^2\left\|A\left(A\tilde{V}_m(x)\right)\right\|_{L_2(\Omega)}^2<+\infty, \quad (11)$$

Then, the functions

$$\tilde{U}(x,t)=\sum_{k=o}^{\infty}\tilde{U}_k(x)\cos\lambda_k t, \qquad \tilde{V}(x,t)=\sum_{k=o}^{\infty}\tilde{V}_k(x)\cos\lambda_k t,$$

$$a(x)=\frac{1}{\Delta(x)}\left\{-q(x)\sum_{k=0}^{\infty}\lambda_k^2\tilde{U}_k(x)+\psi(x)\sum_{k=0}^{\infty}(-1)^k\lambda_k^2\tilde{U}_k(x)+\Phi_0(x)\right\},$$

$$b(x)=\frac{1}{\Delta(x)}\left\{-\varphi(x)\sum_{k=0}^{\infty}(-1)^k\lambda_k^2\tilde{U}_k(x)+h(x)\sum_{k=0}^{\infty}\lambda_k^2\tilde{U}_k(x)+\Phi_1(x)\right\},$$

$$a_1(x)=\frac{1}{\Delta(x)}\left\{-q(x)\sum_{k=0}^{\infty}\lambda_k^2\tilde{V}_k(x)+\psi(x)\sum_{k=0}^{\infty}(-1)^k\lambda_k^2\tilde{V}_k(x)+\Phi_2(x)\right\}, \text{and}$$

$$b_1(x)=\frac{1}{\Delta(x)}\left\{-\varphi(x)\sum_{k=0}^{\infty}(-1)^k\lambda_k^2\tilde{V}_k(x)+h(x)\sum_{k=0}^{\infty}\lambda_k^2\tilde{V}_k(x)+\Phi_3(x)\right\}$$

are the solution of an inverse problem (1) - (7), where

$$\lambda_k=\frac{k\pi}{T}, \quad f_k(x)=\frac{2}{T}\int_0^T f(x,t)\cos\lambda_k t dt,$$
$$g_k(x)=\frac{2}{T}\int_0^T g(x,t)\cos\lambda_k t dt, \quad F_k(x)=\frac{2}{T}\int_0^T F(x,t)\cos\lambda_k t dt,$$
$$G_k(x)=\frac{2}{T}\int_0^T G(x,t)\cos\lambda_k t dt, \quad (k=0,1,2,....).$$

Proof:

From (9) - (11) we obtain, (1), (2), (5) and

$$\widetilde{U}_t(x,t)\Big|_{t=0}=0,\ \ \widetilde{U}_t(x,t)\Big|_{t=T}=0,\ \ \widetilde{V}_t(x,t)\Big|_{t=0}=0\ \ \ \widetilde{V}_t(x,t)\Big|_{t=T}=0.$$

Therefore, to prove Lemma 3, it is required to establish that

$$\widetilde{U}(x,t)\Big|_{t=0}=\varphi(x)\ \ \ \widetilde{U}(x,t)\Big|_{t=T}=h(x),\ \ \widetilde{V}(x,t)\Big|_{t=0}=\psi(x)\ \ \ \widetilde{V}(x,t)\Big|_{t=T}=q(x).$$

Let

$$\widetilde{U}(x,t)\Big|_{t=0}=\widetilde{\varphi}(x),\ \ \widetilde{V}(x,t)\Big|_{t=0}=\widetilde{\psi}(x)\,.$$

Then, for the function $\widetilde{\varphi}(x)-\varphi(x)=Z(x),\ \ \widetilde{\psi}(x)-\psi(x)=\widetilde{Z}(x)$ from (8), (9) we have

$$AZ(x)=\frac{Z(x)}{\Delta(x)}\left\{-\psi(x)\tilde{U}_{tt}(x,t)\Big|_{t=T}+q(x)\tilde{U}_{tt}(x,t)\Big|_{t=0}-\Phi_0(x)\right\}$$
$$+\frac{\widetilde{Z}(x)}{\Delta(x)}\left\{-h(x)\widetilde{U}_{tt}(x,t)\Big|_{t=0}+\varphi(x)\widetilde{U}_{tt}(x,t)\Big|_{t=T}-\Phi_1(x)\right\}, \tag{12}$$

$$A\tilde{Z}(x)=\frac{Z(x)}{\Delta(x)}\left\{-\psi(x)\tilde{V}_{tt}(x,t)\Big|_{t=T}+q(x)\tilde{V}_{tt}(x,t)\Big|_{t=0}-\Phi_2(x)\right\}$$
$$+\frac{\widetilde{Z}(x)}{\Delta(x)}\left\{-h(x)\widetilde{V}_{tt}(x,t)\Big|_{t=0}+\varphi(x)\widetilde{V}_{tt}(x,t)\Big|_{t=T}-\Phi_3(x)\right\},$$

$$Z(x)\Big|_S=0,\qquad \widetilde{Z}(x)\Big|_S=0. \tag{13}$$

The uniqueness of the solution of the problem (12), (13) will be established below. Then, we prove the solvability of (9), (10).

It will be similarly shown that $U(x,t)\Big|_{t=T}=h(x),\quad V(x,t)\Big|_{t=T}=q(x).$

We investigate the solvability of (9), (10).

Let us define by $r_k(x)$ and $æ_k(x)$ $(k=0,1,2,....)$ the solutions of the following problems

$$A\,r_k(x)=0,\qquad r_k(x)\Big|_S=F_k(x)\qquad (k=0,1,2,...),$$

$$\left\|r_k(x)\right\|^2_{C(\overline{\Omega})}\le C_1\left\|F_k(x)\right\|^2_{W_2^{7/2}(S)}, \tag{14}$$

$$A\,æ_k(x)=0,\qquad æ_k(x)\Big|_S=G_k(x)\qquad (k=0,1,2,...),$$

$$\|æ_k(x)\|^2_{C(\bar{\Omega})} \le C_1 \|G_k(x)\|^2_{W_2^{7/2}(S)} \quad . \tag{15}$$

Let us designate $\tilde{U}_k(x) = U_k(x) + r_k(x), \quad \tilde{V}_k(x) = V_k(x) + æ_k(x)$.

Then, from (9) and (10), we have

$$\begin{aligned}
-\lambda_k^2 U_k(x) - AU_k(x) = {} & \frac{U_k(x)+r_k(x)}{\Delta(x)} \Bigg\{ -q(x)\sum_{m=0}^{\infty} \lambda_m^2 U_m(x) + \psi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 U_m(x) \\
& + \Phi_0(x) - q(x)\sum_{m=0}^{\infty}\lambda_m^2 r_m(x) + \psi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 r_m(x) \Bigg\} \\
& + \frac{V_k(x)+æ_k(x)}{\Delta(x)} \Bigg\{ -\phi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 U_m(x) + h(x)\sum_{m=0}^{\infty}\lambda_m^2 U_m(x) + \Phi_1(x) - \phi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 r_m(x) \\
& + h(x)\sum_{m=0}^{\infty}\lambda_m^2 r_m(x) \Bigg\} + f_k(x) + \lambda_k^2 r_k(x), \\
-\lambda_k^2 V_k(x) - AV_k(x) = {} & \frac{U_k(x)+r_k(x)}{\Delta(x)} \Bigg\{ -q(x)\sum_{m=0}^{\infty} \lambda_m^2 V_m(x) + \psi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 V_m(x) \\
& + \Phi_2(x) - q(x)\sum_{m=0}^{\infty}\lambda_m^2 æ_m(x) + \psi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 æ_m(x) \Bigg\} \\
& + \frac{V_k(x)+æ_k(x)}{\Delta(x)} \Bigg\{ -\phi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 V_m(x) + h(x)\sum_{m=0}^{\infty}\lambda_m^2 V_m(x) + \Phi_3(x) - \phi(x)\sum_{m=0}^{\infty}(-1)^m \lambda_m^2 æ_m(x) \\
& + h(x)\sum_{m=0}^{\infty}\lambda_m^2 æ_m(x) \Bigg\} + g_k(x) + \lambda_k^2 æ_k(x),
\end{aligned} \tag{16}$$

$$U_k(x)\big|_S = 0, \quad V_k(x)\big|_S = 0 \quad (k = 0,1,2,\ldots). \tag{17}$$

4. Existence and Uniqueness for the Problem (1) - (7)

Now we investigate problem (16), (17). We consider the following set of functions

$$B = \{ (U_0(x), U_1(x), \ldots, U_k(x), \ldots), (V_0(x), V_1(x), \ldots, V_k(x), \ldots), \quad U_k(x)\big|_S = 0, \quad V_k(x)\big|_S = 0$$

$$(k = 0,1,2,\ldots), \ \sum_{k=1}^{\infty} \lambda_k^{12} \|U_k(x)\|^2_{L_2(\Omega)} + \|U_0(x)\|^2_{L_2(\Omega)} \le \frac{R}{4}, \ \sum_{k=1}^{\infty} \lambda_k^{12} \|V_k(x)\|^2_{L_2(\Omega)} + \|V_0(x)\|^2_{L_2(\Omega)} \le \frac{R}{4},$$

$$\sum_{k=1}^{\infty}\lambda_k^8\|AU_k(x)\|^2_{L_2(\Omega)}+\|AU_0(x)\|^2_{L_2(\Omega)}\le R,\ \sum_{k=1}^{\infty}\lambda_k^8\|AV_k(x)\|^2_{L_2(\Omega)}+\|AV_0(x)\|^2_{L_2(\Omega)}\le R\Big\}.$$

Let us show that at suitable choice $\boldsymbol{R}$, the problem (16), (17) is solvable in B. Let $(\omega_0(x),\omega_1(x),...,\omega_k(x),...)$, $(\tilde\omega_0(x),\tilde\omega_1(x),...,\tilde\omega_k(x),...)$ be an arbitrary elements from B and substitute it in a right hand side of (16). Then, we will obtain a function, which belongs to the space $L_2(\Omega)$. Then, from Lemma 1, we have

$$\begin{aligned}\int_\Omega U_k^2(x)dx\le\frac{22}{\delta_1^2}\int_\Omega\Bigg[&2N_0^2\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\omega_m(x)\right)^2+2N_0^2\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 r_m(x)\right)^2\\
&+2N_0^2r_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\omega_m(x)\right)^2+2r_k^2(x)N_0^2\left(\sum_{m=1}^{\infty}\lambda_m^2 r_m(x)\right)^2+\omega_k^2(x)\left(\frac{\Phi_0(x)}{\Delta(x)}\right)^2\\
&+r_k^2(x)\left(\frac{\Phi_0(x)}{\Delta(x)}\right)^2+2N_0^2\tilde\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\omega_m(x)\right)^2+2N_0^2\tilde\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 r_m(x)\right)^2\\
&+2N_0^2æ_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\omega_m(x)\right)^2+2N_0^2æ_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 r_m(x)\right)^2+f_k^2(x)+\lambda_k^4r_k^2(x)s\\
&+\tilde\omega_k^2(x)\left(\frac{\Phi_1(x)}{\Delta(x)}\right)^2+æ_k^2(x)\left(\frac{\Phi_1(x)}{\Delta(x)}\right)^2\Bigg]dx\ ,\end{aligned}\tag{18}$$

$$\begin{aligned}\int_\Omega V_k^2(x)dx\le\frac{22}{\delta_1^2}\int_\Omega\Bigg[&2N_0^2\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_k^2\tilde\omega_m(x)\right)^2+2N_0^2\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_k^2 æ_m(x)\right)^2\\
&+2N_0^2r_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\tilde\omega_m(x)\right)^2+2N_0^2r_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 æ_m(x)\right)^2+\omega_k^2(x)\left(\frac{\Phi_2(x)}{\Delta(x)}\right)^2\\
&+r_k^2(x)\left(\frac{\Phi_2(x)}{\Delta(x)}\right)^2+2N_0^2\tilde\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\tilde\omega_m(x)\right)^2+2N_0^2\tilde\omega_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 æ_m(x)\right)^2\\
&+2N_0^2æ_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2 æ_m(x)\right)^2+2N_0^2æ_k^2(x)\left(\sum_{m=1}^{\infty}\lambda_m^2\tilde\omega_m(x)\right)^2+\tilde\omega_k^2(x)\left(\frac{\Phi_3(x)}{\Delta(x)}\right)^2\\
&+æ_k^2(x)\left(\frac{\Phi_3(x)}{\Delta(x)}\right)^2+g_k^2(x)+\lambda_k^4æ_k^2(x)\Bigg]dx,\quad(k=0,1,2,...)\,,\end{aligned}\tag{19}$$

where

$$N_0=\max\left\{\left\|\frac{q(x)}{\Delta(x)}\right\|_{c(\bar\Omega)},\left\|\frac{\psi(x)}{\Delta(x)}\right\|_{c(\bar\Omega)},\left\|\frac{\varphi(x)}{\Delta(x)}\right\|_{c(\bar\Omega)},\left\|\frac{h(x)}{\Delta(x)}\right\|_{c(\bar\Omega)}\right\}.$$

Now, multiplying the relation (18) by $\bar\lambda_k^{12}$ and summing over k from 0 to ∞, we obtain

$$\begin{aligned}\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} U_k^2(x)dx \le \frac{44}{\delta_1^2} N_0^2 C_1 C_2 \Bigg[& \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\omega_k(x)\right)^2 dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} \\
& + \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} \cdot \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} r_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\omega_k(x)\right)^2 dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} æ_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} \Bigg] + \frac{22}{\delta_1^2} \left\|\frac{\Phi_0(x)}{\Delta(x)}\right\|^2_{C(\overline{\Omega})} \Bigg[\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} r_k^2(x)dx \Bigg] + \frac{22}{\delta_1^2} \left\|\frac{\Phi_1(x)}{\Delta(x)}\right\|^2_{C(\Omega)} \Bigg[\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} æ_k^2(x)dx \Bigg] \\
& + \frac{22}{\delta_1^2} \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} f_k^2(x)dx + \frac{22}{\delta_1^2} \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{16} r_k^2(x)dx, \qquad (20)\end{aligned}$$

where

$$\overline{\lambda}_k = \begin{cases} 1, & k = 0, \\ \lambda_k, & k = 1, 2, \cdots. \end{cases}$$

Similarly from (19) we obtain

$$\begin{aligned}\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} V_k^2(x)dx \le \frac{44 N_0^2 C_1 C_2}{\delta_1^2} \Bigg[& \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\tilde{\omega}_k(x)\right)^2 dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\|F_k(x)\right\|^2_{W_{(S)}^{7/2}} \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} r_k^2(x)dx \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\tilde{\omega}_k(x)\right)^2 dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} æ_k^2(x)dx \cdot \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\|G_k(x)\right\|^2_{W_{(S)}^{7/2}} \Bigg] + \frac{22}{\delta_1^2} \left\|\frac{\Phi_2(x)}{\Delta(x)}\right\|^2_{C(\Omega)} \Bigg[\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx \\
& + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} r_k^2(x)dx \Bigg] + \frac{22}{\delta_1^2} \left\|\frac{\Phi_3(x)}{\Delta(x)}\right\|^2_{C(\overline{\Omega})} \Bigg[\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} æ_k^2(x)dx \Bigg]\end{aligned}$$

$$+\frac{22}{\delta_1^2}\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{12} g_k^2(x)dx+\frac{22}{\delta_1^2}\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{16} æ_k^2(x)dx\,. \tag{21}$$

From estimations (20), (21) and (16) we obtain

$$\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{12}U_k^2(x)dx\le\frac{44N_0^2C_1C_2}{\delta_1^2}\left[\frac{R^2}{2}+L_0R+2L_0^2\right]+\frac{11}{\delta_1^2}KR+\frac{44}{\delta_1^2}KL_0+\frac{44}{\delta_1^2}L_0, \tag{22}$$

$$\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{12}V_k^2(x)dx\le\frac{44N_0^2C_1C_2}{\delta_1^2}\left[\frac{R^2}{2}+L_0R+2L_0^2\right]+\frac{11}{\delta_1^2}KR+\frac{44}{\delta_1^2}KL_0+\frac{44}{\delta_1^2}L_0, \tag{23}$$

$$\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{8}\left|AU_k(x)\right|^2dx\le2\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{12}U_k^2(x)dx+44N_0^2C_1C_2\left[\frac{R^2}{2}+L_0R+2L_0^2\right]$$
$$+11KR+44KL_0+44L_0, \tag{24}$$

$$\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{8}\left|AV_k(x)\right|^2dx\le2\int_\Omega\sum_{k=0}^{\infty}\overline{\lambda}_k^{12}V_k^2(x)dx+44N_0^2C_1C_2\left[\frac{R^2}{2}+L_0R+2L_0^2\right]$$
$$+11KR+44KL_0+44L_0, \tag{25}$$

where

$$K=\max\left\{\left\|\frac{\Phi_0(x)}{\Delta(x)}\right\|_{c(\overline{\Omega})},\left\|\frac{\Phi_1(x)}{\Delta(x)}\right\|_{c(\overline{\Omega})},\left\|\frac{\Phi_2(x)}{\Delta(x)}\right\|_{c(\overline{\Omega})},\left\|\frac{\Phi_3(x)}{\Delta(x)}\right\|_{c(\overline{\Omega})}\right\},$$

$$L_0=\frac{2}{T}\int_0^T\left(\|F(x,t)\|^2_{W_2^{7/2}(S)}+\left\|D_t^8F(x,t)\right\|^2_{W_2^{7/2}(S)}\right)dt+\frac{2}{T}\int_0^T\left(\|G(x,t)\|^2_{W_2^{7/2}(S)}\right.$$

$$\left.+\left\|D_t^8G(x,t)\right\|^2_{W_2^{7/2}(S)}+\|f(x,t)\|^2_{L_2(\Omega)}+\left\|D_t^6f(x,t)\right\|^2_{L_2(\Omega)}+\|g(x,t)\|^2_{L_2(\Omega)}+\left\|D_t^6g(x,t)\right\|^2_{L_2(\Omega)}\right)dt\,.$$

Lemma 4:

Let conditions 1-5, I and $\frac{1}{\delta_1^2}\le\frac{22}{35}$ be satisfied. Then,

$$44N_0^2C_1C_2\left[\frac{R}{2}+L_0\right]+11KR=\frac{R}{8},$$

$$88N_0^2C_1C_2L_0^2 + 44KL_0 + 44L_0 \le \frac{3}{11}, \text{ and}$$

$$\left(\frac{1}{\delta_1^2}+1\right)\left[204N_0^2C_1C_2\left(\frac{3R}{2}+2L_0\right)+204K(C_1+1)\right]<1.$$

Then, there exists a solution of the problem (16), (17) in B and

$$\sum_{k=0}^{\infty}\overline{\lambda}_k^{12}\left\|U_k(x)\right\|^2_{L_2(\Omega)} \le \frac{R}{4}, \quad \sum_{k=0}^{\infty}\overline{\lambda}_k^{12}\left\|V_k(x)\right\|^2_{L_2(\Omega)} \le \frac{R}{4},$$

$$\sum_{k=0}^{\infty}\overline{\lambda}^8\left\|AU_k(x)\right\|^2_{L_2(\Omega)} \le R, \quad ,\sum_{k=0}^{\infty}\overline{\lambda}^8\left\|AV_k(x)\right\|^2_{L_2(\Omega)} \le R.$$

Proof:

System (16) has the form

$$EZ_k = \widetilde{L}(Z_k),$$

where $\tilde{\boldsymbol{L}}$ is determined by the right-hand sides of the system (16), and $\boldsymbol{E}$ an elliptic operator of the left hand sides of (16), $Z_k(x)=\{U_k(x),V_k(x)\}$, $(k=0,1,2,...)$. But since $\boldsymbol{E}$ has a bounded inverse, the system (16) can be written as:

$$Z_k = L(Z_k), \tag{26}$$

where

$$L = E^{-1}\widetilde{L}, \quad L(Z_k)=\{L_1(U_k(x),\ V_k(x)),\ L_2(U_k(x),\ V_k(x))\} \quad (k=0,1,2,...).$$

Take an arbitrary vector function $(\omega_0(x),...,\omega_k(x),...)$, $(\widetilde{\omega}_0(x),...,\widetilde{\omega}_k(x),...)\in B$ and substitute into the right hand side of (16). Then, for the solutions $U_k(x)=L_1(\omega_k(x),\tilde{\omega}_k(x))$, $V_k(x)=L_2(\omega_k(x),\tilde{\omega}_k(x))$ we obtain estimations (22) - (25). Hence, under the conditions of Lemma 4, we get that $((U_k(x),...,U_k(x)...),(V_0(x),...,V_k(x)...))\in B$. This proves that, operator $\boldsymbol{L}$ is contracting.

Let $(\omega_{1,k}(x),\ \widetilde{\omega}_{1,k}(x))$ and $(\omega_{2,k}(x),\ (\tilde{\omega}_{2,k}(x)\ )$ $(k=0,1,2,\cdots)\in B$. Then, the corresponding solutions will be

$$U_{1,k}(x) = L_1(\omega_{1,k}(x), \tilde{\omega}_{1,k}(x)),$$

$$V_{1,k}(x) = L_2(\omega_{1,k}(x), \tilde{\omega}_{1,k}(x)),$$

$$U_{2,k}(x) = L_1(\omega_{2,k}(x), \tilde{\omega}_{2,k}(x)),$$

$$V_{2,k}(x) = L_2(\omega_{2,k}(x), \tilde{\omega}_{2,k}(x)).$$

Let us denote that

$$U_k(x) = U_{1,k}(x) - U_{2,k}(x),\ V_k(x) = V_{1,k}(x) - V_{2,k}(x),$$

$$\omega_k(x) = \omega_{1,k}(x) - \omega_{2,k}(x),\ \tilde{\omega}_k(x) = \tilde{\omega}_{1,k}(x) - \tilde{\omega}_{2,k}(x).$$

Let's rewrite the relation (16) for $U_{1,k}(x), V_{1,k}(x), \omega_{1,k}(x), \tilde{\omega}_{1,k}(x)$ and $U_{2,k}(x), V_{2,k}(x), \omega_{2,k}(x), \tilde{\omega}_{2,k}(x)$. Further, subtracting the first relation to the second, under conditions of Lemma 4, we obtain the following estimation

$$\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} U_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} V_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(AU_k(x)\right)^2 dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(AV_k(x)\right)^2 dx$$

$$\le \alpha \left[\int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \omega_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \tilde{\omega}_k^2(x)dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\omega_k(x)\right)^2 dx + \int_\Omega \sum_{k=0}^{\infty} \overline{\lambda}_k^{8} \left(A\tilde{\omega}_k(x)\right)^2 dx \right], \quad (27)$$

where

$$\alpha = \left(\frac{1}{\delta_1^2} + 1\right)\left[204 N_0^2 C_1 C_2 \left(\frac{3R}{2} + 2L_0\right) + 204 K C_1\right].$$

From this we obtain the Lemma 4.

Theorem:

Let conditions of Lemma 4 be satisfied. Then, there exists unique solution of problem (16), (17) (an inverse problem (1) - (7)) for which the following estimations are true

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\| U_k(x) \right\|_{L_2(\Omega)}^2 \le \frac{R}{4}, \quad \sum_{k=0}^{\infty} \overline{\lambda}_k^{12} \left\| V_k(x) \right\|_{L_2(\Omega)}^2 \le \frac{R}{4},$$

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^8 \|AU_k(x)\|_{L_2(\Omega)}^2 \le R, \quad , \sum_{k=0}^{\infty} \overline{\lambda}_k^8 \|AV_k(x)\|_{L_2(\Omega)}^2 \le R,$$

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^6 \|AU_k(x)\|_{W_2^1(\Omega)}^2 + \sum_{k=0}^{\infty} \overline{\lambda}_k^2 \|A(AU_k(x))\|_{L_2(\Omega)}^2 \le RC, \text{ and}$$

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^6 \|AV_k(x)\|_{W_2^1(\Omega)}^2 + \sum_{k=0}^{\infty} \overline{\lambda}_k^2 \|A(AV_k(x))\|_{L_2(\Omega)}^2 \le RC.$$

Proof:

Lemma 4 implies the solvability of the problem (16), (17) in B. Thus, the first estimate of the Theorem holds. Multiplying (16), by $\overline{\lambda}_k^{10} U_k(x)$ and $\overline{\lambda}_k^{10} V_k(x)$, accordingly, and take the sum over k from 0 to ∞, then, integrating in Ω and using the first estimation, we obtain

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^{10} \|U_k(x)\|_{W_2^1(\Omega)}^2 \le constR,$$

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^{10} \|V_k(x)\|_{W_2^1(\Omega)}^2 \le constR.$$

Then, from (10) and this estimation, we get

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^6 \|AU_k(x)\|_{W_2^1(\Omega)}^2 \le constR,$$

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^6 \|AV_k(x)\|_{W_2^1(\Omega)}^2 \le constR.$$

Similarly, we can prove the following estimations

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^2 \|A(AU_k(x))\|_{L_2(\Omega)}^2 \le constR,$$

and

$$\sum_{k=0}^{\infty} \overline{\lambda}_k^2 \|A(AV_k(x))\|_{L_2(\Omega)}^2 \le constR.$$

To complete the proof of the theorem, it remains to prove that the solution of (12), (13) is equal to zero. Indeed, under the conditions of the Theorem we have

$$\int_{\Omega} |AZ(x)|^2 dx + \int_{\Omega} \left| A\overline{Z}(x) \right|^2 dx \le 12 \left[N_0^2 C_1 C_2 \frac{R}{2} + KC_1 \right] \cdot \left[\int_{\Omega} |AZ(x)|^2 dx + \int_{\Omega} \left| A\widetilde{Z}(x) \right|^2 dx \right].$$

Then, $AZ(x) = 0, \quad Z(x)\big|_S = 0$ and $A\widetilde{Z}(x) = 0, \quad Z(x)\big|_S = 0,$ hence, $Z(x) = 0, \ \widetilde{Z}(x) = 0$.

4. Conclusion

In the paper we investigate the solvability for the inverse boundary value problems for the system of hyperbolic equations. The method based on the reducing of inverse boundary value problem to some nonlinear infinite systems of differential equations is proposed. Given method allows one to prove existence and uniqueness theorems of multidimensional inverse boundary value problems in the class of finite smoothness functions.

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