Peristaltic Induced Flow of a Particulate Suspension in a Non-Uniform Geometry

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Abstract

The flow induced by sinusoidal peristaltic waves of a particle-fluid suspension in a two-dimensional diverging channel under low Reynolds number and long wavelength approximation has been investigated. The analytical expression for the flow characteristics-the flow rate, pressure rise and friction force have been derived. Moreover, we present some results concerning the dependence of these quantities on the geometrical parameters.

1. Introduction

Peristalsis, a well known phenomena to the engineers and physiologists and responsible for fluid transport in many biological organs [Srivastava and Srivastava (1984)], has been the subject of

scientific research for about four and half decades since the first investigation of Latham (1966). It is a form of fluid transport induced by a progressive wave of area contraction or expansion along the walls of a distensible duct containing a liquid or mixture. Besides its various engineering applications (e.g., heart-lung machines, finger and roller pumps, etc.), it is known to be a significant mechanism responsible for fluid transport in many biological organs including in swallowing food through esophagus, urine transport from kidney to bladder through the ureter, movement of chyme in gastrointestinal tract, transport of spermatozoa in the ductus efferentes of the male reproductive tracts and in cervical canal, in movement of ovum in the female fallopian tubes, transport of lymph in lymphatic vessels, and in the vasomotion of small blood vessels such as arterioles, venules and capillaries.

Jaffrin and Shapiro (1971) explained the basic principles of peristaltic pumping in a two-dimensional channel and brought out clearly the significance of the various parameters governing the flow. A review of most of the theoretical and experimental investigations reported up to the year 1994 has been presented in Srivastava and coworkers (1984, 1995). Important contributions beyond this, and of recent years, include the studies of Srivastava and Srivastava (1997), Mekheimer et al. (1998), Hakeem et al. (2002), Srivastava (2002), Misra and Pandey (2002), Hayat and coworkers (2002; 2006a,b; 2008), Mekheimer (2003), Misra and Rao (2004), Srivastava (2007a), Ali and Hayat (2008), Medhavi and coworker (2008a,b, 2010), and a few others.

The theoretical study of particle-fluid mixture is very useful in understanding a number of diverse physical problems concerned with powder technology, fluidization, transport of solid particles in liquid and liquid slurries in chemical and nuclear processing, and metalized liquid fuel slurries for rocketry. The sedimentation of particles in a liquid is of interest in many chemical engineering processes, in medicine where erythrocytes sedimentation has become a standard clinical test, and in oceanography as well as in other fields. Recently, interest is developing in applying the particulate suspension theory to physiological flows as it provides improved understanding of the subjects such as diffusion of protein, the rheology of blood, the swimming of microorganism, the particle deposition on respiratory tract, etc.

Peristaltic transport of a particle-fluid mixture has been studied by Hung and Brown (1976), Srivastava and Srivastava (1989, 1997), Mekheimer et al. (1998), Srivastava (2002), Medhavi and Singh (2008b), and a few others. Barring a few [Gupta and Seshadri (1976); Srivastava and coworkers (1982, 1983, 1985, 1988); Mekheimer (2002); etc.)], most of the studies in the literature have been conducted in uniform geometry whereas it is known that in most of the practical applications, particularly, in physiological flows, the flow geometry is found to be non-uniform. With increasing interest in particulate suspension flow due to its direct applications to diverse physical problems, the present investigation is devoted to study the flow of a particle-fluid mixture in a non-uniform channel induced by sinusoidal peristaltic waves. In view of the theoretical model for blood flow proposed in Srivastava and Srivastava (1983) and discussed briefly in Srivastava and coworker (2007b, 2009), it is believed that the theoretical study presented here may be applied to study the peristaltic induced flow of blood in small vessels with varying cross-section.
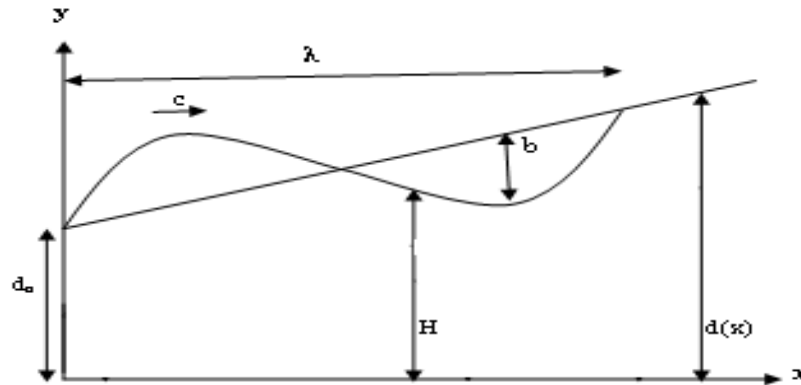
Figure 1. Peristaltic pumping in a diverging channel

2. Formulation of the Problem

Consider the flow of a particulate suspension through a two-dimensional channel of non-uniform width with a sinusoidal wave traveling down its wall. The geometry of the wall surface is described as (Figure 1)

$$H(x,t) = d(x) + b\ sin\frac{2\pi}{\lambda}(x - ct),\qquad(1)$$

with

$$d(x) = d_o + kx,\qquad(2)$$

where $d(x)$ is the half width of the channel at any axial distance x from inlet, d_o is the half width of the channel at inlet, $k\ (\ll 1)$ is a constant whose magnitude depends on the length of the channel and exit and inlet dimensions, b is the amplitude of the wave, λ is the wavelength, c is the wave propagation velocity and t is the time.

We introduce the following dimensionless variables as

$$x' = x/\lambda,\ y' = y/d_o,\ t' = ct/\lambda,\ (u'_f, u'_p) = (u_f, u_p)/c,\ (v'_f, v'_p) = \lambda(v_f, v_p)/cd_o,$$

$$p' = pd_0^2/\lambda c\mu_o,\ \mu = \mu_s/\mu_o,\ S' = S\ d_o^2/\mu_o,$$

where (x, y) are Cartesian coordinates with x measured in the direction of the wave and y measured in the direction normal to the mean position of the channel walls; (u_f, v_f) denotes the fluid phase and (u_p, v_p) particle phase velocity components along (x, y) directions respectively; ρ_f, ρ_p be the actual densities of the material constituting fluid and particulate phases respectively; $(1-C)\rho_f$ is the fluid phase density, $C\rho_f$ the particulate phase density; p denotes the pressure and C denotes the volume fraction of the particles; $\mu_s(C) \cong \mu_s$ is the suspension viscosity and S being the drag coefficient of interaction for the force exerted by one phase on the

other. The limitations of present theoretical model are well stated in Srivastava and coworkers (1983, 2009). An applications of the above mentioned non-dimensional variables and using a continuum approach, the equations governing the linear momentum and the mass for both the fluid and particle phases [Srivastava and Srivastava (1989)] after dropping the primes, are obtained as

$$(1\text{-}C)\ \delta\ R_e \left\{ \frac{\partial u_f}{\partial t} + u_f\ \frac{\partial u_f}{\partial x} + v_f\ \frac{\partial u_f}{\partial y} \right\} = -\ (1\text{-}C)$$

$$\times \frac{\partial p}{\partial x} + (1-C)\ \mu \left(\delta^2\ \frac{\delta^2 u_f}{\partial x^2} + \frac{\delta^2 u_f}{\partial y^2} \right) + CS\ (u_p - u_f), \tag{3}$$

$$(1\text{-}C)\delta^3 R_e \left\{ \frac{\partial v_f}{\partial t} + u_f\ \frac{\partial v_f}{\partial x} + v_f\ \frac{\partial v_f}{\partial y} \right\} = -(1-C)$$

$$\times \frac{\partial p}{\partial y} + (1-C)\ \mu \delta^2 \left(\delta^2\ \frac{\delta^2 v_f}{\partial x^2} + \frac{\delta^2 v_f}{\partial y^2} \right) + C S\ \delta^2\ (v_p - v_f), \tag{4}$$

$$\frac{\partial}{\partial x} \left[(1-C)\ u_f \right] + \frac{\partial}{\partial y} \left[(1-C) v_f \right] = 0, \tag{5}$$

$$C\ (\rho_f/\rho_p)\delta\ R_e \left\{ \frac{\partial u_p}{\partial t} + u_p\ \frac{\partial u_p}{\partial x} + v_p\ \frac{\partial u_p}{\partial y} \right\} = -\ C\ \frac{\partial p}{\partial x} +\ C S (u_f - u_p), \tag{6}$$

$$C\ (\rho_f/\rho_p)\delta^3\ R_e \left\{ \frac{\partial v_p}{\partial t} + u_p\ \frac{\partial v_p}{\partial x} + v_p\ \frac{\partial v_p}{\partial y} \right\} = -\ C\ \frac{\partial p}{\partial y} + C S\ \delta^2\ (v_f - v_p), \tag{7}$$

$$\frac{\partial}{\partial x} \left[C\ u_p \right] + \frac{\partial}{\partial y} \left[C v_p \right] = 0, \tag{8}$$

$R_e = c\ \rho_f\ d_o/\mu_o$ and $\delta = d_o/\lambda$ are Reynolds number and the wave number respectively.

The empirical relation for the viscosity of suspension, μ_s and the expression for the drag coefficient of interaction, S are selected as [Srivastava and Srivastava (1989); Srivastava (2007b)]

$$\mu_s\ (C) = \mu_o\ / \ (1-mC),$$

$$m = 0.070\ exp\ [2.49C + (1107/T)\ exp\ (-1.69C)], \tag{9}$$

$$S = \frac{9}{2} \frac{\mu_o}{a_o^2} \frac{4+3[8C-3C^2]^{1/2}+3C}{(2-3C)^2},$$ (10)

where μ_o is the fluid viscosity (suspending medium), a_o is the radius of a particle and T is measured in absolute temperature ($^o K$). The viscosity of the suspension expressed by the formula (9) is found to be reasonably accurate up to C=0.6 (i.e., 60% hematocrit in the case of blood).

Jaffrin and Shapiro (1971) observed that the Reynolds number is quite small when the wavelength is large, and therefore, the inertial convective acceleration terms may be neglected in comparison to viscous terms. Under the long wavelength approximation of Jaffrin and Shapiro (1971), the equations (3)-(8) governing the flow in its non-dimensional form in the laboratory frame of reference reduce to

$$(1-C) \frac{dp}{dx} = (1-C) \mu \frac{d^2 u_f}{dy^2} + C S (u_p - u_f),$$ (11)

$$C \frac{dp}{dx} = C S (u_f - u_p),$$ (12)

The non-dimensional boundary conditions are

$$u_f = 0, \text{ at } \quad y = h = H/d_o = 1 + \frac{k \lambda x}{d_o} + \phi \sin 2\pi \ (x\text{-}t),$$ (13)

$$\frac{\partial u_f}{\partial y} = \frac{\partial u_p}{\partial y} = 0 \quad at \quad y = 0.$$ (14)

with

$$\phi = b/d_o.$$

The boundary conditions given in equations (13) and (14) are respectively standard no-slip condition at the channel walls and the symmetric condition at the axis.

3. Analysis

The expressions for the velocity profiles, u_f and u_p, obtained as the solution of the equations (11) and (12) subject to the boundary conditions (13) and (14) are given as

$$u_f = -\frac{1}{2(1-C)\mu} \frac{dp}{dx} (h^2 - y^2),$$ (15)

$$u_p = -\frac{1}{2(1-C)\,\mu}\,\frac{dp}{dx}\left\{h^2 - y^2 + \frac{2(1-C)\,\mu}{S}\right\}, \tag{16}$$

The flow flux, $q(x, t)$ is now calculated as

$$q(x, t) = \int_o^h \left[(1-C)u_f + C\,u_p\right]dy = -\frac{1}{3(1-C)\,\mu}\,\frac{dp}{dx}\left\{h^3 + \frac{3C(1-C)\,\mu}{S}\,h\right\}, \tag{17}$$

or

$$-\frac{dp}{dx} = \frac{3(1-C)\,\mu\,q(x, t)}{h^3 + \eta h}, \tag{18}$$

with $\eta = 3C(1-C)\,\mu/S$, a non-dimensional suspension parameter.

The pressure rise $\Delta p_L(t)$ and the friction force at the wall, $F_L(t)$ in the channel of length L in their non-dimensional form are obtained as

$$\Delta p_L(t) = \int_o^{L/\lambda} \frac{dp}{dx}\,dx = -3(1-C)\,\mu \int_o^{L/\lambda} \frac{q(x, t)}{h(h^2 + \beta)}\,dx, \tag{19}$$

$$F_L(t) = \int_o^{L/\lambda} h\left(-\frac{dp}{dx}\right)dx = 3(1-C)\,\mu \int_o^{L/\lambda} \frac{q(x, t)}{h^2 + \beta}. \tag{20}$$
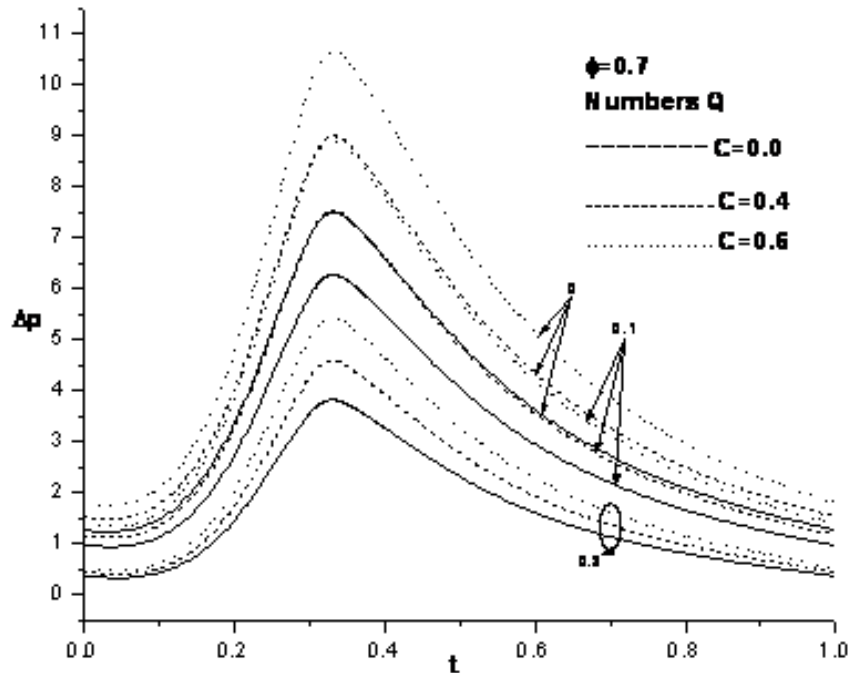

Fig. 2 Variation of pressure rise, Δp with time, t for different Q and C in a non-uniform channel.

Setting $k = 0$ in equations (19) and (20), one obtains the expressions for pressure rise and friction force for a particle-fluid suspension in a uniform channel. In the absence of the particulate phase (i.e., $C = 0$), the expressions given in equations (19) and (20) reduce to the results obtained in Gupta and Seshadri (1976) for a Newtonian fluid in a non-uniform channel. Also, with $k = 0$ and $C = 0$ in equations (19) and (20), one derives the results obtained in Jaffrin and Shapiro (1971) in the laboratory frame of reference.

4. Results and Discussion

Computer codes are now developed to evaluate the analytical results obtained in equations (19) and (20) at the temperature of 25.5° C in order to observe the quantitative effects of the various parameters on the flow characteristics-the pressure rise and the friction force. We assume the form of instantaneous flow rate $q(x, t)$, periodic in (x-t) as [Gupta and Seshadri (1976)]

$$q\,(x,\,t)\ = Q + \phi\,\sin\,2\pi\,(x\text{-}t)\,, \tag{22}$$

where Q is the time average of the flow over one period of the wave. This form of $q(x, t)$ has been assumed in view of the fact that the constant value of $q(x, t)$ gives $\Delta p_L(t)$ always negative and hence there will be no pumping action. Using the form of $q(x, t)$ given in equation (22), we compute the dimensionless pressure rise $\Delta p_L(t)$ and friction for $F_L(t)$ over the channel length for various values of the dimensionless flow average Q, amplitude ratio ϕ and particle concentration, C. The average pressure rise Δp_L and the friction force F_L are then evaluated by averaging $\Delta p_L(t)$ and $F_L(t)$, respectively, over one period of wave. Some of the critical results are displayed graphically in Figures 2-11.

The various parameters values selected [Srivastava and Srivastava (1984); Mekheimer (2002)] are

$$d_o = 0.01 \text{ cm}, \ L = \lambda = 10 \text{ cm}, \quad k = \frac{0.5\,d_o}{L} = 0.0005.$$

For any given flow rate Q, the pressure rise, $\Delta p_L(t)$ (hereafter denoted by Δp) increases with particle concentration, C but for a given particle concentration, C the pressure rise, Δp decreases with increasing flow rate, Q (Figure 2). The flow characteristic, Δp assumes much smaller magnitude in non-uniform channel than its corresponding magnitude in the uniform channel (Figures 2 and 3). The average pressure rise Δp_L versus time average flow rate, Q has been plotted in Figure 4 in a non-uniform channel which indicates a linear relationship between Δp_L and Q and thus the maximum flow rate is achieved at zero pressure rise and maximum pressure occurs at zero flow rate. The flow characteristic, Δp_L is found to be indefinitely increasing with increasing amplitude ratio, ϕ for any given flow rate, Q and the particle

concentration C (Figure 5). The magnitude of Δp_L assumes a very high asymptotic value at about $\phi = 0.6$ in both the uniform and non-uniform channels (Figures 5 and 6).
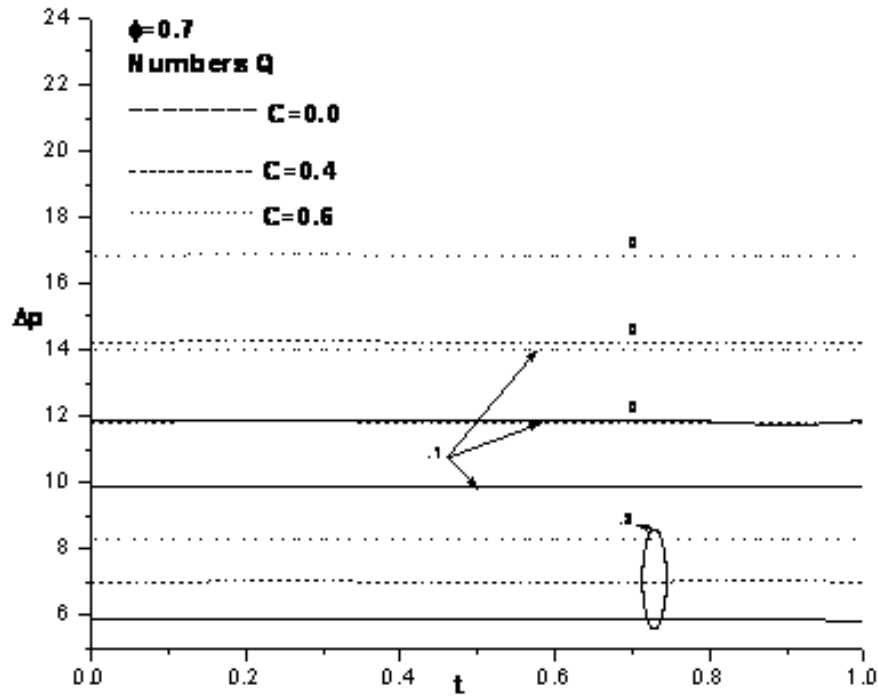

Fig. 3 Variation of pressure rise, Δp with time, t for different Q and C in a uniform channel.
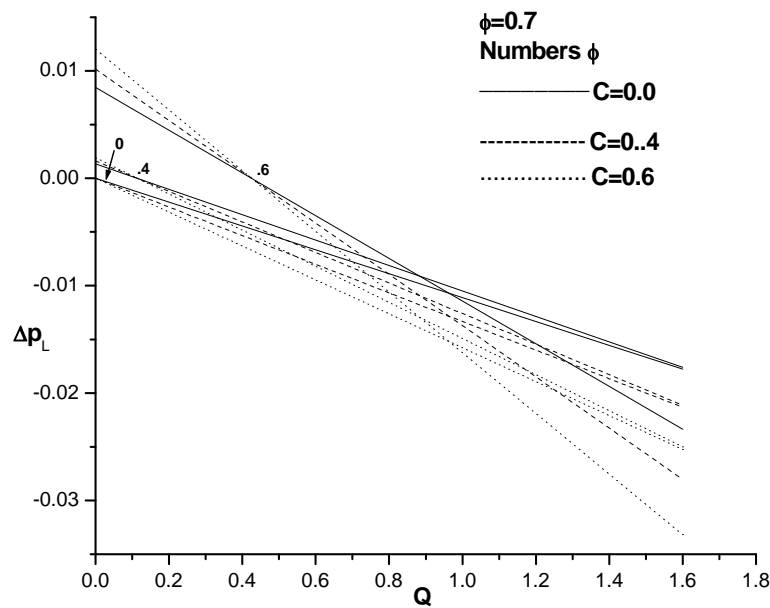

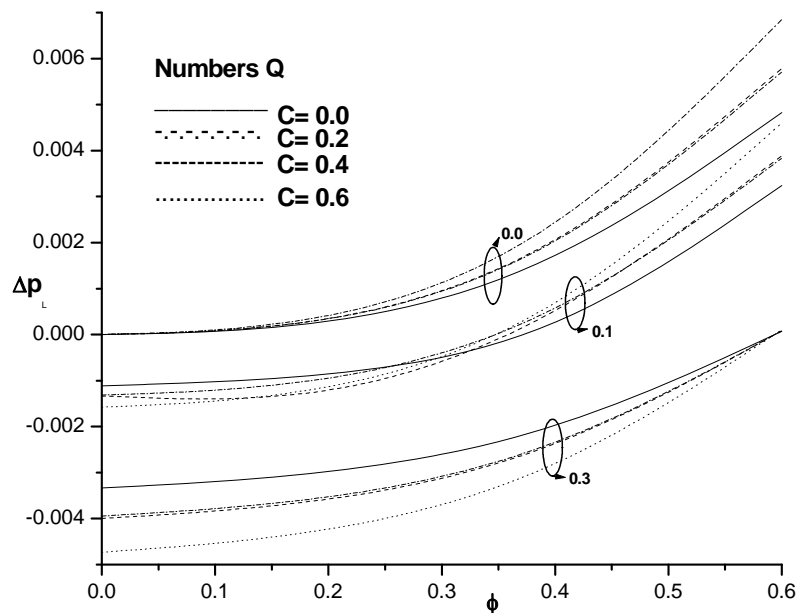Fig.4 Pressure-flow rate relationship for different ϕ and C in a non-unioform channel.

Fig. 5 Variation of pressure rise, Δp_L with amplitude ratio, ϕ for different Q and C non-uniform channel.
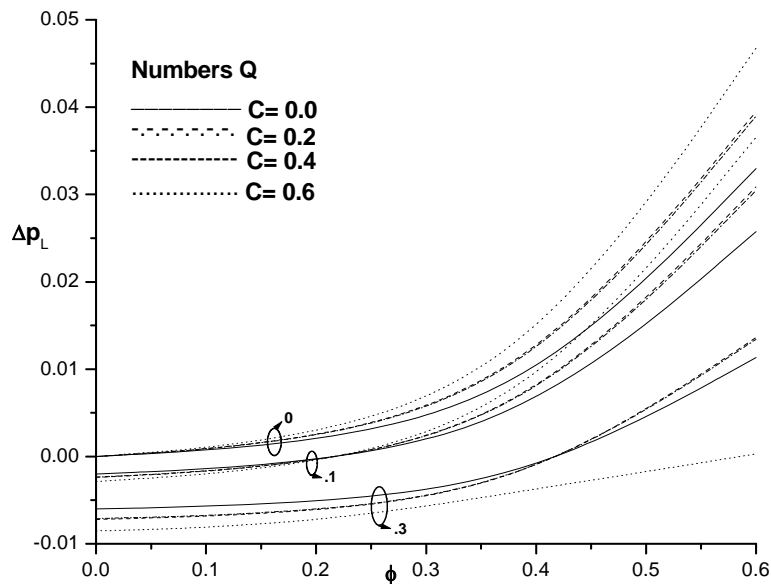

Fig. 6 Variation of pressure rise, Δp_L with amplitude ratio, ϕ for different Q and C uniform channel.

However, as expected Δp_L assumes reasonably smaller value in non-uniform channel than its corresponding magnitude in uniform channel. The average pressure rise, Δp_L versus C has been displayed in Figures 7 and 8 for non-uniform and uniform channels, respectively. The flow characteristic, Δp_L increases with C at zero flow rate for any given amplitude ratio, ϕ, however,

the nature in the variation of Δp_L with C is highly influenced with decreasing values of the amplitude ratio, ϕ for any given non-zero value of the average flow rate, Q. It is noticed that for any given flow rate, Q, the flow characteristic, Δp_L increases with amplitude ratio, ϕ (Figures 7 and 8).

For any particle concentration, C the friction force, $F_L(t)$ increases with flow rate, Q but decreases with increasing particle concentration, C for a given flow rate, Q (Figure 9). The flow characteristic, $F_L(t)$ assumes higher magnitude in uniform channel than its corresponding magnitude in non-uniform channel (Figures 9 and 10). One notices that the average friction force F_L increases with the flow rate, Q for any given particle concentration, C and the amplitude ratio, ϕ retaining the linear relationships between the average friction force F_L and the flow rate, Q (Figure11).The inspection of Figures 2 and 9 reveals that the flow characteristics, $F_L(t)$ possesses characteristics opposite to that of the pressure rise, Δp for any given set of the parameters. The similar conclusion may be drawn in the case of their averaged values, F_L and Δp_L from Figures 4 and 11.
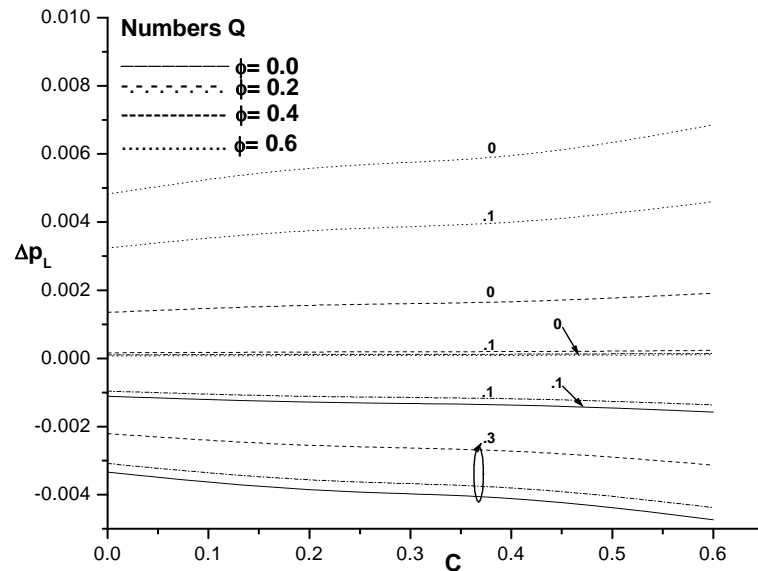

Fig. 7 Variation of pressure rise, Δp_L with particle concentration, C for different Q and ϕ non-uniform channel.
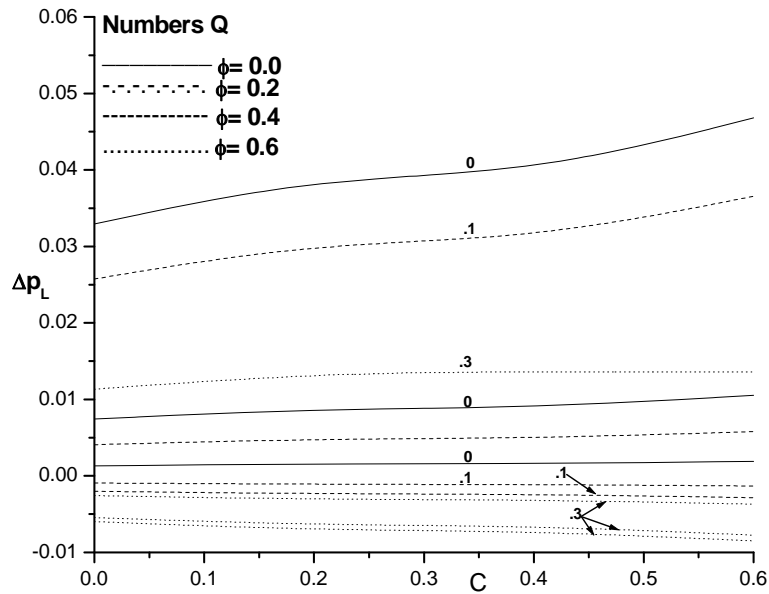
Fig. 8 Variation of pressure rise, Δp_L with particle concentration, C for different Q and ϕ uniform channel.
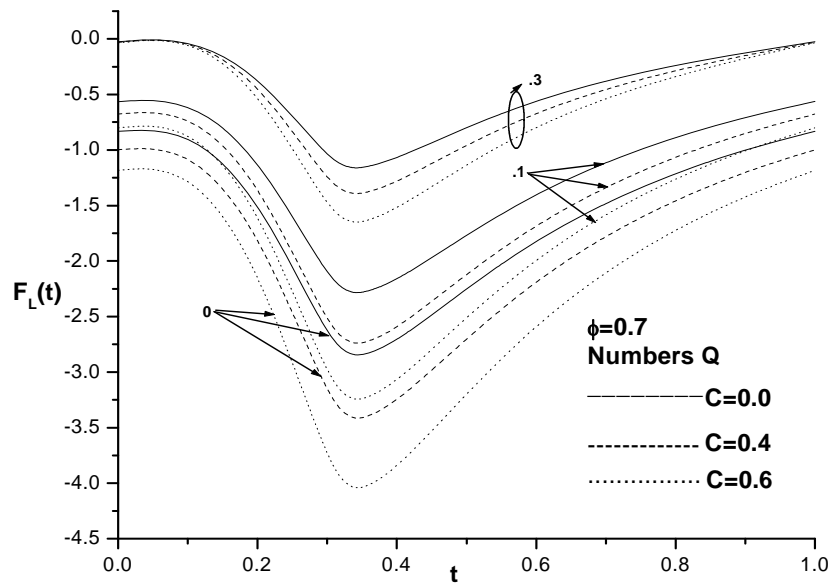

Fig. 9 Variation of friction force, $F_L(t)$ with time, t for different Q and C in a non-uniform channel
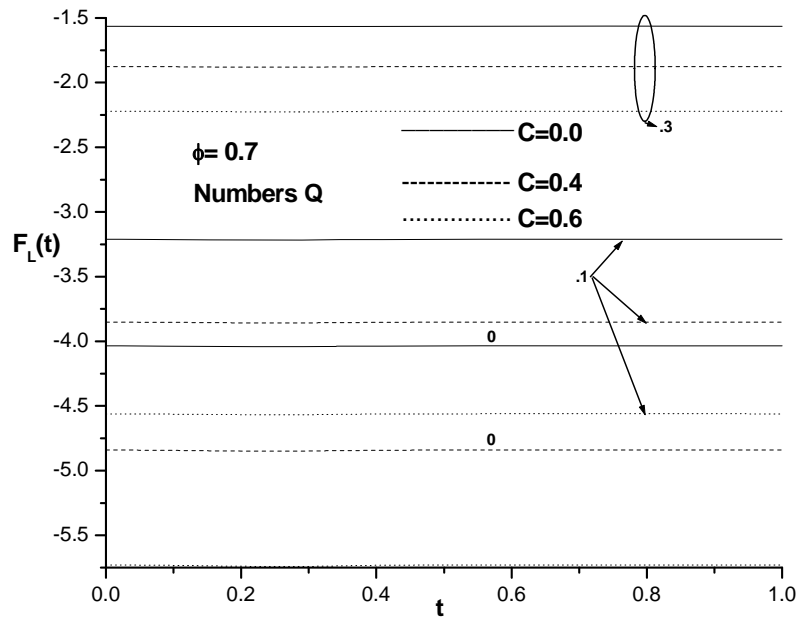
Fig. 10 Variation of friction force, $F_L(t)$ with time, t for different Q and C in a non-uniform channel
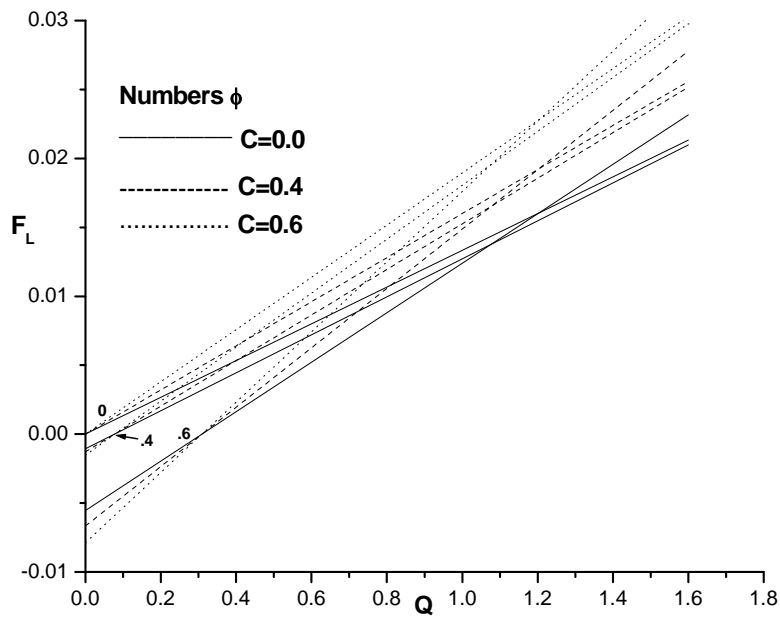

Fig.11 Friction force-flow rate relationship for different ϕ and C in a non-unioform channel.

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