Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Vol. 6, Issue 1 (June 2011) pp. 153 – 161
(Previously, Vol. 6, Issue 11, pp. 1893 – 1901)

Applications and Applied Mathematics:
An International Journal
(AAM)

Exact Travelling Wave Solutions for Konopelchenko-Dubrovsky Equation by the First Integral Method

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Received: August 17, 2010; Accepted: April 29, 2011

Abstract

In this paper, the first integral method is used to construct exact travelling wave solutions of Konopelchenko-Dubrovsky equation. The first integral method is algebraic direct method for obtaining exact solutions of nonlinear partial differential equations. This method can be applied to non-integrable equations as well as to integrable ones. This method is based on the theory of commutative algebra.

Keywords: First integral method; Konopelchenko-Dubrovsky equation.

MSC 2010 No.: 35Q53; 35Q80; 35Q55; 35G25.

1. Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of Mathematical-physical sciences such as physics, biology, and chemistry. The analytical solutions of such equations are of fundamental importance since a lot of mathematical physical models are described by NLEEs. Among the possible solutions to NLEEs, certain special form solutions may depend only on a single combination of variables such as travelling wave variables. In the literature, there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions, such as the Backlund transformation method [Miura (1978)], Hirota's direct method [Hirota (1971)), (2004)], tanh-sech method [Ma (1993), Malfliet (1992), Khater et al. (2002), Wazwaz (2006)], extended tanh method [Ma and Fuchssteiner (1996), El-Wakil et al. (2007), Fan (2000), Wazwaz (2005)], hyperbolic function method [Xia and Zhang (2001)], sine-cosine method [Wazwaz (2004), Yusufoglu and Bekir (2006)], Jacobi elliptic function expansion [Inc and Ergut (2005)], F-expansion method [Zhang (2006)], and the transformed rational function method [Ma and Lee (2009)].

The first integral method was first proposed by Feng (2002), in solving Burgers-KdV equation which is based on the ring theory of commutative algebra. Recently, this useful method is widely used by many such as in [Feng and Wang (2002), Raslan (2008), Abbasbandy and Shirzadi (2010), Tascan et al. (2009) and by the reference therein].

Raslan (2008) proposed the first integral method to solve the Fisher equation. Abbasbandy and Shirzadi (2010) solved the modified Benjamin-Bona-Mahoney equation by using the first integral method. Tascan et al. (2009) used the first integral method to obtain the exact solutions of the modified Zakharov-Kuznetsov equation and ZK-MEW equation. The aim of this paper is to find exact soliton solutions of the Konopelchenko-Dubrovsky equation [Wazwaz (2007)] by the first integral method.

2. First Integral Method

Consider the nonlinear partial differential equation in the form

$$F(u, u_x, u_y, u_t, u_{xx}, u_{xy}, \ldots) = 0, \tag{1}$$

where $u = u(x, y, t)$ is the solution of nonlinear partial differential equation (1). We use the transformations

$$u(x, y, t) = u(\xi), \qquad \xi = x + y - ct. \tag{2}$$

This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -c\frac{\partial}{\partial \xi}(.), \qquad \frac{\partial}{\partial x}(.) = \frac{\partial}{\partial \xi}(.), \quad \frac{\partial}{\partial y}(.) = \frac{\partial}{\partial \xi}(.), \quad \frac{\partial^2}{\partial x^2}(.) = \frac{\partial^2}{\partial \xi^2}(.), \ldots. \tag{3}$$

We use (3) to change the nonlinear partial differential equation (1) to nonlinear ordinary differential equation

$$G\left(u(\xi), \frac{\partial u(\xi)}{\partial \xi}, \frac{\partial^2 u(\xi)}{\partial \xi^2}, \ldots\right) = 0. \tag{4}$$

Next, we introduce a new independent variable

$$X(\xi) = u(\xi), \qquad Y = \frac{\partial u(\xi)}{\partial \xi}. \tag{5}$$

This leads to a system of nonlinear ordinary differential equations

$$\begin{aligned} \frac{\partial X(\xi)}{\partial \xi} &= Y(\xi), \\ \frac{\partial Y(\xi)}{\partial \xi} &= F_1(X(\xi), Y(\xi)). \end{aligned} \tag{6}$$

By the qualitative theory of ordinary differential equations [Ding and Li (1996)], if we can find the integrals to equation (6) under the same conditions, then the general solutions to equation (6) can be solved directly. However, in general, it is really difficult for us to realize this even for one first integral, because for a given plane autonomous system, there is no systematic theory that can tell us how to find its first integrals, nor is there a logical way for telling us what these first integrals are. We will apply the Division Theorem to obtain one first integral to equation (6) which reduces equation (4) to a first order integrable ordinary differential equation. An exact solution to equation (1) is then obtained by solving this equation.

Now, let us recall the Division Theorem:

Division Theorem:

Suppose that $P(w,z)$ and $Q(w,z)$ are polynomials in $C(w,z)$, and $P(w,z)$ is irreducible in $C(w,z)$. If $Q(w,z)$ vanishes at all zero points of $P(w,z)$, then there exists a polynomial $F_2(w,z)$ in $C(w,z)$ such that $Q(w,z) = P(w,z)F_2(w,z)$.

3. Konopelchenko-Dubrovsky Equation

Konopelchenko and Dubrovsky (1984) presented the Konopelchenko-Dubrovsky (KD) equation

$$\begin{aligned} &u_t - u_{xxx} - 6buu_x + \frac{3}{2}a^2u^2u_x - 3v_y + 3au_xv = 0, \\ &u_y = v_x, \end{aligned} \tag{7}$$

where a and b are real parameters. Equation (7) is a new nonlinear integrable evolution equation on two spatial dimensions and one temporal. In Konopelchenko and Dubrovsky (1984), this equation was investigated by the inverse scattering transform method. The F-expansion method is used in Wang and Zhang (2005) to investigate the KD equation.

By making the transformations $u(x,y,t) = u(\xi)$, $v(x,y,t) = v(\xi)$, and $\xi = x + y - ct$, equation (7) becomes

$$-cu' - u''' - 6buu' + \frac{3}{2}a^2u^2u' - 3v' + 3au'v = 0, \tag{8}$$
$$u' = v',$$

where by integrating the second equation we find:

$$u = v. \tag{9}$$

Substituting (9) into the first equation of (8) and integrating the resulting equation we obtain

$$u'' - \frac{a^2}{2}u^3 + 3(b - \frac{a}{2})u^2 + (c+3)u = -R, \tag{10}$$

where R is an integration constant that is to be determined later.

Using (5) and (6), we can get

$$\dot{X}(\xi) = Y(\xi), \tag{11}$$

$$\dot{Y}(\xi) = \frac{a^2}{2}X^3(\xi) + 3(\frac{a}{2} - b)X^2(\xi) - (c+3)X(\xi) - R. \tag{12}$$

According to the first integral method, we suppose the $X(\xi)$ and $Y(\xi)$, are the nontrivial solutions of (11) and (12) also

$$Q(X,Y) = \sum_{i=0}^{N} a_i(X)Y^i = 0,$$

is an irreducible polynomial in the complex domain $C(X,Y)$, such that

$$Q(X(\xi),Y(\xi)) = \sum_{i=0}^{N} a_i(X(\xi))Y^i(\xi) = 0, \tag{13}$$

where $a_i(X)$, $i = 0,1,\ldots,N$, are polynomials of X and $a_N(X) \neq 0$. Equation (13) is called the first integral to (11), (12). Due to the Division Theorem, there exists a polynomial $g(X) + h(X)Y$, in the complex domain $C(X,Y)$, such that

$$\frac{dQ}{d\xi} = \frac{dQ}{dX}.\frac{dX}{d\xi} + \frac{dQ}{dY}.\frac{dY}{d\xi} = (g(X) + h(X)Y)\sum_{i=0}^{N} a_i(X)Y^i. \tag{14}$$

In this example, we take two different cases, assuming that $N = 1$, and $N = 2$, in (13).

Case A:

Suppose that $N=1$, by comparing with the coefficients of $Y^i\,(i=2,1,0)$ of both sides of (14), we have

$$\dot{a}_1(X)=h(X)a_1(X), \tag{15}$$

$$\dot{a}_0(X)=g(X)a_1(X)+h(X)a_0(X), \tag{16}$$

$$a_1(X)[\frac{a^2}{2}X^3(\xi)+3(\frac{a}{2}-b)X^2(\xi)-(c+3)X(\xi)-R]=g(X)a_0(X). \tag{17}$$

We obtain that $a_1(X)$, is constant and $h(X)=0$, take $a_1(X)=1$, and balancing the degrees of $g(X)$, $a_1(X)$ and $a_0(X)$, we conclude that $\deg(g(X))=1$, only.

Suppose that $g(X)=A_1X+B_0$, then we find $a_0(X)$.

$$a_0(X)=A_0+B_0X+\frac{1}{2}A_1X^2, \tag{18}$$

where A_0 is arbitrary integration constant. Substituting $a_0(X), a_1(X)$ and $g(X)$ in the last equation in (17) and setting all the coefficients of X to be zero, then we obtain a system of nonlinear equations and by solving it, we obtain

$$A_0=-\frac{4a^2-4ab+4b^2+ca^2}{a^3}, \qquad A_1=a, \quad B_0=\frac{a-2b}{a}, \tag{19}$$

$$R=\frac{(a-2b)(4a^2-4ab+4b^2+ca^2)}{a^4}.$$

$$A_0=\frac{4a^2-4ab+4b^2+ca^2}{a^3}, \qquad A_1=-a, \quad B_0=-\frac{a-2b}{a}, \tag{20}$$

$$R=\frac{(a-2b)(4a^2-4ab+4b^2+ca^2)}{a^4}.$$

Using the conditions (19) in (13), we obtain

$$Y(\xi)=-\frac{a}{2}(X(\xi))^2-\frac{(a-2b)}{a}X(\xi)+\frac{4a^2-4ab+4b^2+ca^2}{a^3}. \tag{21}$$

Combining (21) with (11), we obtain the exact solution to equation (10) and then the exact solution to Konopelchenko-Dubrovsky equation can be written as

$$u(x,y,t)=\frac{(2b-a)}{a^2}+\frac{\sqrt{a^2(9+2c)+12b(b-a)}}{a^2}\times\tanh[\frac{\sqrt{a^2(9+2c)+12b(b-a)}}{2a}((x+y-ct)+\xi_0)], \tag{22}$$

where ξ_0 is an arbitrary constant.

Similarly, in the case of (20), from (13), we obtain

$$Y(\xi)=\frac{a}{2}(X(\xi))^2+\frac{(a-2b)}{a}X(\xi)-\frac{4a^2-4ab+4b^2+ca^2}{a^3}, \tag{23}$$

and then the exact solution of Konopelchenko-Dubrovsky equation can be written as

$$u(x,y,t)=\frac{(2b-a)}{a^2}-\frac{\sqrt{a^2(9+2c)+12b(b-a)}}{a^2}\times\tanh[\frac{\sqrt{a^2(9+2c)+12b(b-a)}}{2a}((x+y-ct)+\xi_0)], \tag{24}$$

where ξ_0 is an arbitrary constant.

Case B:

Suppose that $N=2,$ by equating with the coefficients of $Y^i\,(i=3,2,1,0)$ of both sides of (14), we have

$$\dot{a}_2(X)=h(X)a_2(X), \tag{25}$$

$$\dot{a}_1(X)=g(X)a_2(X)+h(X)a_1(X), \tag{26}$$

$$\dot{a}_0(X)=-2a_2(X)[\frac{a^2}{2}X^3(\xi)+3(\frac{a}{2}-b)X^2(\xi)-(c+3)X(\xi)-R]+g(X)a_1(X)+h(X)a_0(X), \tag{27}$$

$$a_1(X)[\frac{a^2}{2}X^3(\xi)+3(\frac{a}{2}-b)X^2(\xi)-(c+3)X(\xi)-R]=g(X)a_0(X). \tag{28}$$

We obtain that $a_2(X)$ is constant and $h(X)=0$. Taking $a_2(X)=1$ and balancing the degrees $g(X)$, $a_2(X)$ and $a_0(X)$, we conclude that $\deg(g(X))=1$, only. Suppose that $g(X)=A_1X+B_0$, then we find $a_0(X)$, and $a_1(X)$ as

$$a_1(X)=A_0+B_0X+\frac{1}{2}A_1X^2, \tag{29}$$

$$a_0(X)=d+(A_0B_0+2R)X+(\frac{B_0^2}{2}+\frac{A_1A_0}{2}+c+3)X^2+(\frac{A_1B_0}{2}+2b-a)X^3+(\frac{A_1^2}{8}-\frac{a^2}{4})X^4, \tag{30}$$

where d is arbitrary integration constant. Substituting $a_0(X), a_1(X), a_2(X)$ and $g(X)$, in the last equation in (28) and setting all the coefficients of X to be zero, then we obtain a system of nonlinear equations and by solving it with aid Maple, we obtain

$$R=-\frac{A_0(a-2b)}{2a},\ A_1=2a,\ B_0=\frac{2(a-2b)}{a},\ d=\frac{A_0^2}{4},\ c=-\frac{8a^2-8ab+8b^2+a^3A_0}{2a^2}, \tag{30}$$

$$R=\frac{A_0(a-2b)}{2a},\ A_1=-2a,\ B_0=-\frac{2(a-2b)}{a},\ d=\frac{A_0^2}{4},\ c=-\frac{8a^2-8ab+8b^2-a^3A_0}{2a^2}, \tag{32}$$

where A_0 is arbitrary constant.

Using the condition (31) into (13), we get

$$Y(\xi)=-\frac{a}{2}X^2(\xi)+\frac{(2b-a)}{a}X(\xi)-\frac{A_0}{2}. \tag{33}$$

Combining (33) with (11), we obtain the exact solution to equation (10) and the exact solution to Konopelchenko-Dubrovsky equation can be written as

$$\begin{aligned} u(x,y,t)=&\frac{2b-a}{a^2}-\frac{\sqrt{a^2(aA_0-1)+4b(a-b)}}{a^2}\\ &\times\tan[\frac{\sqrt{a^2(aA_0-1)+4b(a-b)}}{2a}(x+y+(\frac{8a^2-8ab+8b^2+a^3A_0}{2a^2})t+\xi_0)], \end{aligned} \tag{34}$$

where ξ_0 is an arbitrary constant.

Similarly, in the case of (32), from (13), we obtain

$$Y(\xi)=\frac{a}{2}X^2(\xi)-\frac{(2b-a)}{a}X(\xi)-\frac{A_0}{2}. \tag{35}$$

Then, the exact solution to Konopelchenko-Dubrovsky equation can be written as:

$$u(x,y,t)=\frac{2b-a}{a^2}-\frac{\sqrt{a^2(aA_0+1)+4b(b-a)}}{a^2}\times\tanh\left[\frac{\sqrt{a^2(aA_0+1)+4b(b-a)}}{2a}\left(x+y+\left(\frac{8a^2-8ab+8b^2-a^3A_0}{2a^2}\right)t+\xi_0\right)\right], \tag{36}$$

where ξ_0 is an arbitrary constant.

In (36), if $A_0=\xi_0=0,$ we have

$$u(x,y,t)=\frac{2b-a}{a^2}\left(1-\tanh\left[\frac{2b-a}{2a}\left(x+y-\left(\frac{4(ab-a^2-b^2)}{a^2}\right)t\right)\right]\right). \tag{37}$$

4. Conclusion

The first integral method is applied successfully for solving the system of nonlinear partial differential equations. Thus, we deduce that the proposed method can be extended to solve many systems of nonlinear partial differential equations which are arising in the theory of solitons and other areas. The exact solution of the general system of nonlinear partial differential equations using the first integral method is still an open point of research.

REFERENCES

Abbasbandy, S. and Shirzadi, A. (2010). The first integral method for modified Benjamin-Bona-Mahony equation. Commun Nonlinear Sci Numer Simulat. Vol. 15, pp. 1759 – 1764.

Ding, T.R. and Li, C. Z. (1996). Ordinary differential equations. Peking University Press, Peking.

El-Wakil, S.A. and Abdou, M.A. (2007). New exact travelling wave solutions using modified extended tanh-function method, Chaos Solitons Fractals, Vol. 31, No. 4, pp. 840-852.

Fan, E. (2000). Extended tanh-function method and its applications to nonlinear equations. Phys Lett A, Vol. 277, No.4, pp. 212-218.

Feng, Z. S. and Wang, X. H. (2002). The first integral method to the two-dimensional Burgers-KdV equation, Phys. Lett. A. Vol. 308, pp. 173-178.

Feng, Z.S. (2002). The first integral method to study the Burgers-Korteweg-de Vries equation, J Phys. A. Math. Gen, Vol. 35, No.2, pp. 343-349.

Hirota, R. (1971). Exact solution of the Korteweg-de Vries equation for multiple collisions of solitons, Phys. Rev. Lett. Vol. 27, pp. 1192 – 1194.

Hirota, R. (2004). The Direct Method in Soliton Theory, Cambridge University Press.

Inc, M. and Ergut, M. (2005). Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method, Appl. Math. E-Notes, Vol. 5, pp. 89-96.

Khater A. H, Malfliet, W, Callebaut, D. K. and Kamel, E.S. (2002). The tanh method, a simple transformation and exact analytical solutions for nonlinear reaction–diffusion equations, Chaos Solitons Fractals, Vol. 14, No. 3, pp. 513-522.

Konopelchenko, B.G. and Dubrovsky, V.G. (1984). Some new integrable nonlinear evolution equations in (2+1) dimensions, Physics Letters A. Vol. 102; No. (1-2); pp. 15-17.

Ma, W. X. and Lee, J. H. (2009). A transformed rational function method and exact solutions to the (3 + 1)-dimensional Jimbo-Miwa equation, Chaos Solitons Fractals, Vol.42; pp. 1356 – 1363.

Ma, W. X. and Fuchssteiner, B. (1996). Explicit and exact solutions to a Kolmogorov-Petrovskii-Piskunov equation, Int. J. Non-Linear Mech. Vol. 31; pp. 329 – 338.

Ma, W. X. (1993). Travelling wave solutions to a seventh order generalized KdV equation, Phys. Lett. A. Vol. 180; PP. 221 – 224.

Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations, Am. J. Phys, Vol. 60, No. 7, pp. 650-654.

Miura, M. R. (1978). Backlund Transformation, Springer-Verlag, Berlin.

Raslan, K. R. (2008). The first integral method for solving some important nonlinear partial differential equations, Nonlinear Dynam. Vol. 53, pp. 281.

Tascan, F., Bekir, A. and Koparan, M. (2009). Travelling wave solutions of nonlinear evolutions by using the first integral method, Commun. Non. Sci. Numer. Simul. Vol. 14; pp. 1810 – 1815.

Wang, D. and Zhang, H. Q. (2005). Further improved F-expansion method and new exact solutions of Konopelchenko-Dubrovsky equation, Chaos, Solitons and Fractals. Vol. 25, pp. 601-610.

Wazwaz, A.M. (2004). A sine-cosine method for handling nonlinear wave equations, Math. Comput. Modelling, Vol. 40, No.5, pp. 499-508.

Wazwaz, A.M. (2004). The sine-cosine method for obtaining solutions with compact and noncompact structures, Appl. Math. Comput**,** Vol. 159, No.2, pp. 559-576.

Wazwaz, A.M. (2005). The tanh-function method: Solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos Solitons and Fractals, Vol. 25, No. 1, pp. 55-63.

Wazwaz, A.M. (2006). Two reliable methods for solving variants of the KdV equation with compact and noncompact structures, Chaos Solitons Fractals, Vol. 28, No. 2, pp. 454-462.

Wazwaz, A.M. (2007). New kinks and solitons solutions to the (2 + 1)-dimensional Konopelchenko-Dubrovsky equation, Mathematical and Computer Modelling, Vol. 45; pp. 473-479.

Xia, T.C., Li, B. and Zhang, H.Q. (2001). New explicit and exact solutions for the Nizhnik-Novikov-Vesselov equation. Appl. Math. E-Notes, Vol. 1, pp. 139-142.

Yusufoglu, E. and Bekir, A. (2006). Solitons and periodic solutions of coupled nonlinear evolution equations by using Sine-Cosine method**,** Internat. J. Comput. Math, Vol. 83, No. 12, pp. 915-924.

Zhang, Sheng (2006). The periodic wave solutions for the (2+1)-dimensional Konopelchenko Dubrovsky equations, Chaos Solitons Fractals, Vol. 30, pp. 1213-1220.