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# Inventory Model with Ramp-type Demand and Price Discount on Back Order for Deteriorating Items under Partial Backlogging

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## Abstract

Modeling of inventory problems provides a good insight to retailers and distributors to maintain stock of different items such as seasonal products, perishable goods and daily useable goods etc. The deterioration of all these items exists to a certain extent due to several reasons like mishandling, evaporation, decay, environmental conditions, transportation etc. It is found from the literature that previously many of the researchers have developed inventory model ignoring deterioration and drawn conclusion. In the absence of deterioration parameter, an inventory model cannot be completely realistic. In this paper, we have made an attempt to extend an inventory model with ramp-type demand and price discount on back order where deterioration was not taken into account. In our study, deterioration and constant holding cost are taken into consideration keeping all other parameters same. As a result, the inventory cost function is newly constructed in the presence of deterioration. The objective of this investigation is to obtain optimal cycle length, time of occurrence of shortages and corresponding inventory cost. This extended model is solved for minimum value of average inventory cost analytically. A theorem is framed to characterize the optimal solution. To validate the proposed model, a numerical example is taken and convexity of the cost function is verified. In order to study the effect of changes of different parameters of the inventory system on optimal cycle length, time of occurrence of shortages and average inventory cost, sensitivity analyses have been performed. Also, the numerical result and sensitivity analyses are graphically presented in the respective section of this paper to demonstrate the model. This study reveals that a better solution can be obtained in the presence of our newly introduced assumptions in the existing model.

**Keywords:** Deterioration; Ramp-type demand; Back order ratio; Marginal profit; Planning horizon; Cycle length; Seasonal goods

**MSC 2010 No.:** 90B05, 93A30

## 1. Introduction

Mathematical modeling of real life problems and physical problems have drawn considerable attention of researchers in recent years. Starting from biological problems to industrial problems, researchers have given their efforts to develop models using different mathematical techniques. Ahmed et al. (2016), Sengupta (2015) designed a mathematical model in their respective study.

The inventory modeling with ramp-type demand was first discussed by Hill (1995). His work was further extended by Mandal and Pal (1998) with ramp-type demand and shortages. Wu and Ouyang (2000) extended the inventory model with ramp-type demand for two different inventory policies. Deng et al. (2007) pointed out some questionable results which appeared in Mandal and Pal (1998) and Wu and Ouyang (2000). Darzanou and Skouri (2011) considered inventory system for deteriorating products, with ramp-type demand rate, under two-level trade credit policy. Giri et al. (2003) considered economic order quantity model for deteriorating items with weibull deterioration rate and ramp type demand in the presence of shortages. Jain and Kumar (2010) developed inventory model with ramp-type demand and three-parameter weibull distribution deterioration rate. Skouri et al. (2009) studied inventory model with general ramp-type demand, time dependent weibull deterioration and partial backlogging under different replenishment policies. Ahmed et al. (2013) attempted to formulate single-item inventory model with ramp type demand and general deterioration rate. Skouri et al. (2011) extended the work of Manna and Choudhuri (2006) taking a general function of time for the variable part of the demand rate. Tripati and Kumar (2011) developed economic ordered quantity model with ramp-type demand and time dependent weibull deterioration where production is time dependent.

Kumar and Rajput (2015) presented an inventory model with ramp-type demand, constant deterioration and shortages under complete backlogging. Manna et al. (2016) developed an order level inventory system for deteriorating items with demand rate as a ramp type function of time. In their work finite production rate is proportional to the demand rate, the deterioration rate is independent of time, and the unit production cost is inversely proportional to the demand rate. Teng et al. (2011) revisited the work of Skouri et al. (2009) by taking inventory related cost as higher than shortage related cost. Recently, Chandra (2017) discussed inventory model with ramp -type demand with time varying holding cost. Apart from the aforesaid studies, the work of Bhunia et al. (2017), Jaggi et al. (2017), Pacheco-Velázquez and Cárdenas-Barrón (2016), Cárdenas-Barrón et al. (2014), Cárdenas-Barrón and Sana (2014) are significant contributions in the domain of inventory modeling.

In our present study, keeping some of the assumptions and notations of Chandra (2017) the same, we have revisited the model for deterioration items to obtain optimal cycle length, time of occurrence of shortages and corresponding inventory cost.

## 2. Notations and Assumptions

The following notations are used to develop the model.

- $I(t)$: Inventory level at any time, $t$.
- $b$ : Fraction of demand back ordered during the shortage period.
- $b_0$ : Upper bound of back order ratio.
- $A$ : Ordering cost per order.

$s_1$ : Back order cost per unit back order per unit time.
$s_2$ : Cost due to lost sale.
$a$ : Price discount on unit back order offered.
$a_o$ : Marginal profit per unit.
$T$ : Length of replenishment cycle.
$T_1$: Time at which shortage starts.
$Io$ : Inventory level at the beginning of each cycle of length $T$.
$s$ : Shortage at the end of a replenishment cycle.
$\theta$ : Deterioration rate.

The proposed model is formulated on the basis of the following assumptions:

i. The inventory model involve single item.
ii. Replenishment is instantaneous.
iii. Shortages are allowed and a fraction $b$ of unmet demands during ($T_1$, $T$) is back ordered.
iv. Holding cost '$h$' per unit is constant.
v. The demand rate D (t) is assumed to be ramp-type and function of time $t$ which is given by D (t) =$D_0$ [t-(t-μ) H (t-μ)], where $D_0$ ,μ are positive constants and $H$ is Heaviside function defined as $H(t-\mu)=\begin{cases}1, & t \geq \mu, \\ 0, & t < \mu.\end{cases}$.
vi. $T_1 > \mu$ and $T_1 < T$.
vii. Deterioration is constant.
viii. Planning horizon is of length $L$.
ix. Back order fraction is directly proportional to the price discount and so $b = \dfrac{b_0}{a_0} a$, where $0 \leq b_0 \leq 1, 0 \leq a \leq a_0$.

In the next section, the formulation of mathematical model is done.

## 3. Model Formulation

Inventory level changes due to demand and deterioration. So the governing equation of the above mentioned inventory system is given by

$$\frac{d}{dt} I(t) + \theta I(t) = \begin{cases} -D_0 t, & 0 \leq t \leq \mu, \\ -D_0 \mu, & \mu \leq t \leq T_1, \\ -bD_0 \mu, & T_1 \leq t \leq T, \end{cases}$$

with boundary condition $I(0)=I_0, I(T_1)=0$.

The solutions of the above differential equation for three different cases (I, II, III), are as follows.

Case I: $0 \leq t \leq \mu$,

$$I(t)=\frac{D_0}{\theta^2}(1-\theta t)+(I_0-\frac{D_0}{\theta^2})e^{-\theta t}. \tag{1}$$

Case II: $\mu \le t \le T$,

$$I(t)=\frac{D_0\mu}{\theta}\left\{e^{\theta(T_1-t)}-1\right\}. \tag{2}$$

Case III: $T_1 \le t \le T$,

$$I(t)=bD_0\mu\left(T_1-t\right) . \tag{3}$$

At $t=\mu$, from (1) and (2), we find

$$\left(\mathrm{I}_0-\frac{D_0}{\theta^2}\right)=\frac{D_0}{\theta^2}\left(\theta\mu e^{\theta T_1}-e^{\theta\mu}\right). \tag{4}$$

$$\mathrm{I}_0=\frac{D_0}{\theta^2}\left\{1+\theta\mu e^{\theta T_1}-e^{\theta\mu}\right\}. \tag{5}$$

Rewriting Equation (1), we obtain

$$I(t)=\frac{D_0}{\theta^2}\left\{1-\theta t+\theta\mu e^{\theta(T_1-t)}-e^{\theta(\mu-t)}\right\}. \tag{6}$$

Also, we have $S=bD_0\mu(T-T_1)$.

The different costs associated with this inventory system are given in below.

1. Ordering cost OC =A.
2. Back order cost BC= $-s_1\int_{T_1}^{T} I(t)dt=\frac{s_1bD_0\mu}{2}(T-T_1)^2$.
3. Purchase cost PC= $P(I_0+s)=\frac{PD_0}{\theta^2}\left\{1+\theta\mu e^{\theta T_1}-e^{\theta\mu}\right\}+PbD_0\mu(T-T_1)$.
4. Lost sale cost LC= $s_2(1-b)D_0\mu(T-T_1)$.
5. Holding cost per cycle HC= $h\int_0^{T_1} I(t)dt=\frac{hD_0}{\theta^2}\left\{\frac{\theta\mu^2}{2}+\frac{1}{\theta}+\mu e^{\theta T_1}-\frac{1}{\theta}e^{\theta\mu}-\theta\mu T_1\right\}$.
6. Deterioration cost DC= $P[I_0-\int_0^{T_1} D(t)dt]=\frac{PD_0}{\theta^2}[1+\theta\mu e^{\theta T_1}-e^{\theta\mu}-\theta^2\mu T_1+\frac{\theta^2\mu^2}{2}]$.

Therefore, cost per unit cycle length is

$$C(T,T_1,b)=\frac{1}{T}[OC+HC+BC+LC+PC+DC]=\frac{1}{T}N(T,T_1,b), \tag{7}$$

where,

$$N(T,T_1,b)=OC+HC+BC+LC+PC+DC.$$

$$=A+\frac{hD_0}{\theta^2}\left\{\frac{\theta\mu^2}{2}+\frac{1}{\theta}+\mu e^{\theta T_1}-\frac{1}{\theta}e^{\theta\mu}-\theta\mu T\right\}+\frac{s_1bD_0\mu}{2}(T-T_1)^2+s_2(1-b)D_0\mu(T-T_1)$$

$$+\frac{PD_0}{\theta^2}\left\{1+\theta\mu e^{\theta T_1}-e^{\theta\mu}\right\}+PbD_0\mu(T-T_1)+\frac{PD_0}{\theta^2}\{1+\theta\mu e^{\theta T_1}-e^{\theta\mu}-\theta^2\mu T_1+\frac{\theta^2\mu}{2}\}.$$

The first and second order partial derivatives of $N$ with respect to $T$ and $T_1$ are obtained as

$$\frac{\partial N}{\partial T}=s_1bD_0\mu(T-T_1)+s_2(1-b)D_0\mu+PbD_0\mu. \tag{8}$$

$$\frac{\partial^2 N}{\partial T^2}=s_1bD_0\mu>0. \tag{8/}$$

$$\frac{\partial N}{\partial T_1}=\frac{hD_0\mu}{\theta}(e^{\theta T_1}-1)-s_1bD_0\mu(T-T_1)-s_2(1-b)D_0\mu+2D_0\mu e^{\theta T_1}-PD_0\mu(1+b). \tag{9}$$

$$\frac{\partial^2 N}{\partial T^2{}_1}=D_0\mu[h+2P\theta)e^{\theta T_1}+s_1b]>0. \tag{9/}$$

$$\frac{\partial^2 N}{\partial T\partial T_1}=-s_1bD_0\mu.$$

Clearly $N$ has second order continuous partial derivatives and also $N$ is continuous. Therefore,

$$\frac{\partial^2 N}{\partial T\partial T_1}=\frac{\partial^2 N}{\partial T_1\partial T}=-s_1bD_0\mu,$$

$$\frac{\partial^2 N}{\partial^2 T}\frac{\partial^2 N}{\partial^2 T_1}-\left(\frac{\partial^2 N}{\partial T_1\partial T}\right)^2=D^2{}_0\mu^2(h+2P\theta)e^{\theta T_1}s_1b>0. \tag{9//}$$

Also,

$$\frac{\partial C}{\partial T}=-\frac{1}{T^2}N+\frac{1}{T}\frac{\partial N}{\partial T}. \tag{10}$$

$$\frac{\partial C}{\partial T_1}=\frac{1}{T}\frac{\partial N}{\partial T_1}. \tag{11}$$

Suppose, $\frac{\partial C}{\partial T_1}=0=\frac{\partial C}{\partial T}$, which gives

$$\frac{N}{T}=\frac{\partial N}{\partial T}.$$

$$\Rightarrow C(T,T_1,b)=s_1bD_0\mu(T-T_1)+s_2(1-b)D_0\mu+PbD_0\mu, \tag{12}$$

and

$$\frac{1}{T}\frac{\partial N}{\partial T_1}=0.$$

$$\Rightarrow h(e^{\theta T_1}-1)+2\theta Pe^{\theta T_1}=P\theta(1+b)+s_1b\theta(T-T_1)+s_2(1-b)\theta. \tag{13}$$

Now we are to minimize total inventory cost. For minimization of the inventory cost, we recall the following result.

**Lemma 3.1**.

If a function $Z(T,T_1)=\frac{1}{T}F(T,T_1)$ where $F$ possesses second order partial derivatives and $F$ is continuous then $Z(T,T_1)$ is minimum if

$$\frac{\partial^2 F}{\partial T^2}>0, \frac{\partial^2 F}{\partial T_1^2}>0 \text{ and } \frac{\partial^2 F}{\partial T^2}.\frac{\partial^2 F}{\partial T_1^2}-(\frac{\partial^2 F}{\partial T\partial T_1})^2>0 \text{ at } T=T^* \text{and } T_1=T_1^*,$$

which are obtained by solving

$$\frac{\partial Z}{\partial T}=0 \text{ and } \frac{\partial Z}{\partial T_1}=0.$$

***Proof*** :

See Theorem 1 in Khanra et al. (2013).

Let $T=T^*$ and $T_1=T_1^*$ be obtained by solving (12) and (13) for given *b*. Then by the above lemma and from conditions ($8^{/}$), ($9^{/}$) and ($9^{//}$), we have the following theorem for minimality of C

**Theorem 3.1.**

For a given *b*, *C*, the total cost per unit cycle length, is minimum at $T=T^*$ and $T_1=T_1^*$ obtained by $\frac{\partial C}{\partial T}=0=\frac{\partial C}{\partial T_1}$ provided $\frac{\partial^2 N}{\partial T^2}>0, \frac{\partial^2 N}{\partial T_1^2}>0$ & $\frac{\partial^2 N}{\partial T^2}.\frac{\partial^2 N}{\partial T_1^2}-(\frac{\partial^2 N}{\partial T\partial T_1})^2>0$ at $T=T^*$ & $T={T^*}_1$.

## 4. Numerical result, sensitivity analysis, and Graphical Representation

To illustrate the model numerically, we consider the values of following model parameters in appropriate units as $A = 500, D_0 = 100, \theta = 0.01, \mu = 0.25, s_1 = 6, s_2 = 7, b = 0.6, h = 3$. Then, we obtain optimal solution as T=7, $T_1$=2.92 and C=335.7. It is worth mentioning that *b* lies between 0 and 1 as $b_0$ lies between 0 and 1.

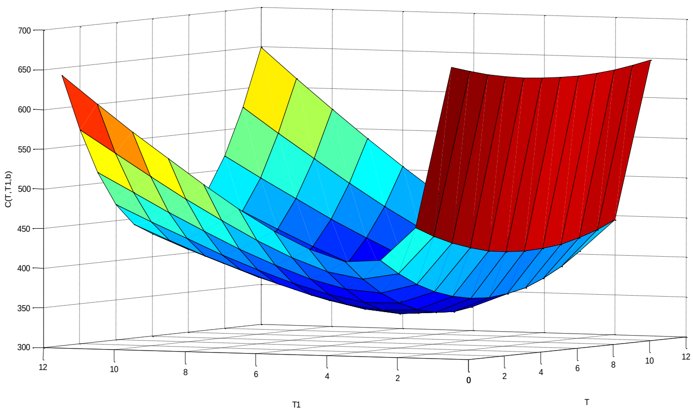


**Figure 1.** Graphical representation of optimal result

### 4.1. Sensitivity

Sensitivity analysis of some of the model parameters are discussed along with graphical representation of them in the following tables**.**

**Table 1.** Sensitivity analysis for θ

| % | $\theta$ | $T$ | $T_1$ | $C(T, T_1, b)$ |
|---|---|---|---|---|
| +50% | 0.0150 | 7 | 2.92 | 337.1 |
| +25% | 0.0125 | 7 | 2.92 | 336.4 |
| 0% | 0.01 | 7 | 2.92 | 335.7 |
| -25% | 0.0075 | 7 | 2.92 | 335 |
| -50% | 0.0050 | 7 | 2.92 | 334.3 |

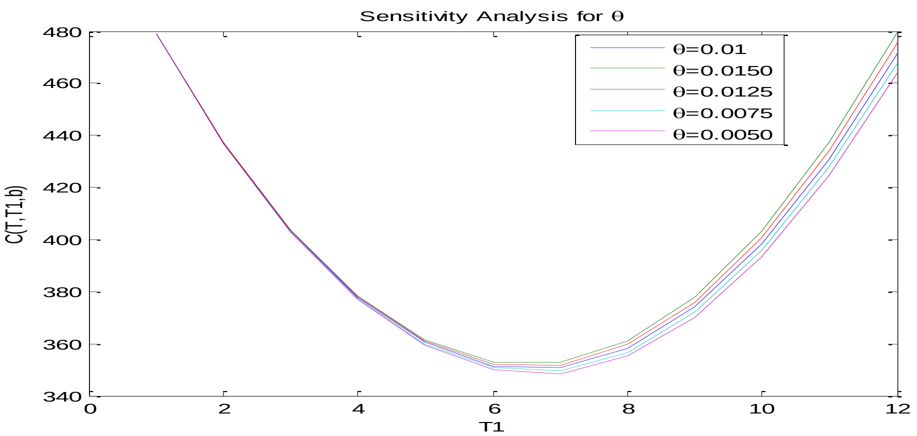


**Figure 2**. Graphical Representation of sensitivity analysis for θ

**Table 2**. Sensitivity analysis for *b*

| % | $b$ | $T$ | $T_1$ | $C(T,T_1,b)$ |
|---|---|---|---|---|
| +50% | 0.9000 | 8 | 2.67 | 347.1 |
| +25% | 0.7500 | 7 | 2.92 | 342.3 |
| 0% | 0.6 | 7 | 2.92 | 335.7 |
| -25% | 0.4500 | 6 | 2.5 | 326.5 |
| -50% | 0.3000 | 5 | 2.5 | 312.7 |

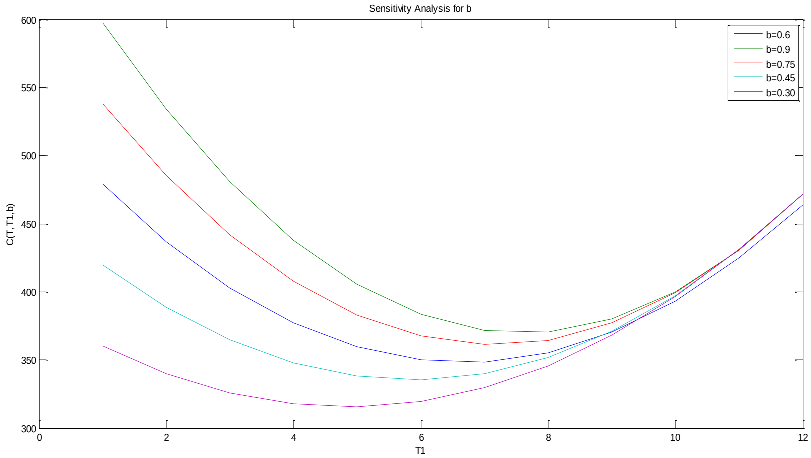


**Figure 3.** Graphical representation of sensitivity analysis for *b*

**Table 3.** Sensitivity analysis for $s_1$

| % | $s_1$ | $T$ | $T_1$ | $C(T,T_1,b)$ |
|---|---|---|---|---|
| +50% | 9 | 8 | 2.67 | 352.1 |
| +25% | 7.50 | 7 | 2.92 | 345.4 |
| 0% | 6 | 7 | 2.92 | 335.7 |
| -25% | 4.5 | 6 | 2.5 | 322.7 |
| -50% | 3 | 5 | 2.5 | 303.9 |

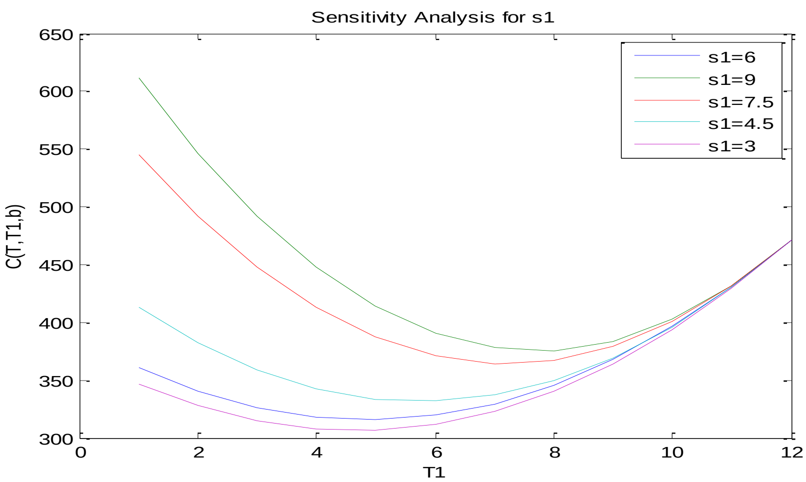


**Figure 4**. Graphical representation of sensitivity analysis for $s_1$

**Table 4.** Sensitivity analysis for s2

| % | $s_2$ | $T$ | $T_1$ | $C(T,T_1,b)$ |
|---|---|---|---|---|
| +50% | 10.50 | 7 | 2.92 | 350.3 |
| +25% | 8.75 | 7 | 2.92 | 343 |
| 0% | 7 | 7 | 2.92 | 335.7 |
| -25% | 5.25 | 6 | 2.5 | 328.1 |
| -50% | 3.50 | 6 | 2.5 | 319.3 |

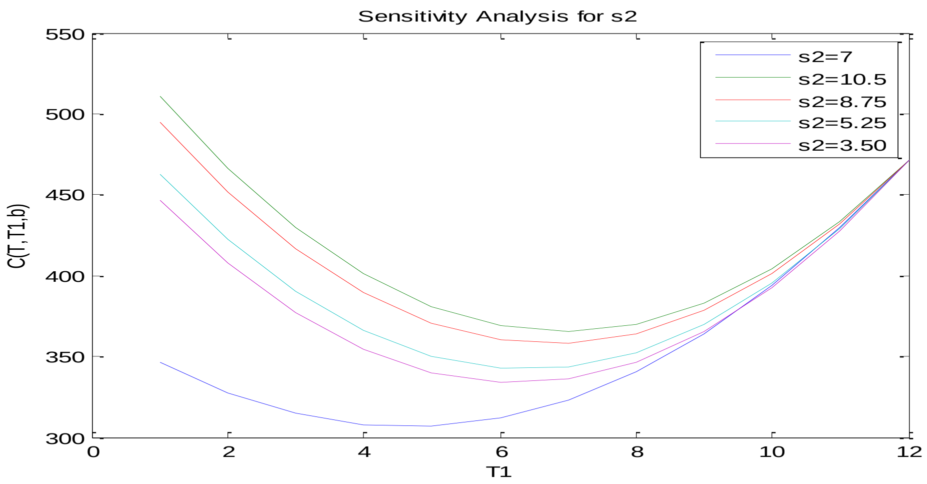


**Figure 5.** Graphical representation of sensitivity analysis for s2

**Table 5.** Sensitivity analysis for *P*

| % | $P$ | $T$ | $T_1$ | $C(T,T_1,b)$ |
|---|---|---|---|---|
| +50% | 7.50 | 6 | 2.5 | 386 |
| +25% | 6.25 | 6 | 2.5 | 361.4 |
| 0% | 5 | 7 | 2.92 | 335.7 |
| -25% | 3.75 | 7 | 2.92 | 309.9 |
| -50% | 2.5 | 7 | 2.92 | 284.1 |

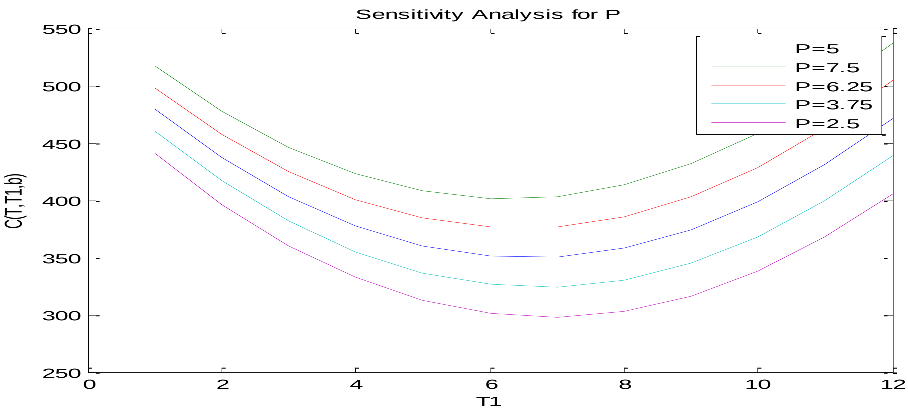


**Figure 6**. Graphical representation of sensitivity analysis for P

## 5. Conclusion

This paper discusses inventory model for deteriorating item with ramp-type demand. In this paper extension of existing model as mentioned in the introduction is done with introduction of deterioration and constant holding cost. In practice, deterioration of any product (may be seasonal product) cannot be ignored. So, in this study we have observed the optimality of decision variables and associated function in the presence of deterioration and constant holding cost. The graphical representation of cost function shows its convexity corresponding to the values of $T$ and $T_1$. Sensitivity analysis with respect to deterioration, fraction of demand back ordered, back order cost and cost due to lost sale optimizes the value of inventory cost. The positive variation in the value of deterioration produces changes in cost function in increasing direction and with negative variation of it, inventory cost decreases without affecting replenishment cycle and time of occurrence of shortages. From Table 2, it may be noted that 25% and 50% decrease in parametric values give the same result for time of occurrence of shortages whereas cycle length is directly proportional. From Table 3, it is evident that with 25% increase in parametric value, cycle length and time of shortage occurrence remain unaffected but 50% increase on it produces change in the above said variables. From Table 4, it is seen that 25% and 50% increase in parametric value, cycle length and shortage occurrence period remain unchanged; however, if there is a decrease by same percentage, these two variables vary from its original value with the same magnitude. Table 5 shows there is no change in cycle length if the parametric value decreases and cycle length changes by same magnitude if parametric value increases. The analysis of the results presented in different tables give us a clear indication to make comparison among the optimal results. There remains the possibility of extension of this model by considering variable deterioration, cost component, fuzzy back order cost, none zero lead time and multi-items.

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