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Slip and chemical reaction effects on peristaltic transport of a couple stress fluid through a permeable medium with complaint wall

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Abstract

In the present article, the effects of slip and homogeneous-heterogeneous chemical reaction on peristaltic pumping of a couple stress fluid through a permeable medium with complaint wall is studied as a model for transport phenomena occurring in the small intestine of human beings during digestion process. The mean effective coefficient of dispersion on simultaneous homogeneous, heterogeneous chemical reactions has been derived through long wavelength assumption, and conditions of Taylor's limit. The behaviors of key parameters on the mean effective dispersion coefficient have been examined through the graphs. It is found that slip and wall parameters, and amplitude ratio favor the dispersion, while couples stress parameter resist the dispersion in the small intestine during pumping in digestive frame work.

Keywords: Chemical reaction; Dispersion; Couple stress fluid; Peristaltic transport; Slip condition; Complaint wall and Permeable medium

MSC 2010 No.: 76A05, 76S05, 76V05, 76Z05, 92C10

1. Introduction

Peristalsis plays a key role in the complete transportation of food bolus in the digestive process. The peristaltic mechanism is first described in clinical terms by Bayliss and Starling (1899). It is the most important method of transporting many physiological liquids. The digestive process commences in the mouth, which triggers the enzyme ptyalin by the salivary organs, and this serves to segregate starches into dextrose and maltose (glucose) by hydrogenation. After being chewed and swallowed, the food enters the esophagus, stomach. It is ousted from the stomach into the duodenum and moves through small intestine. The peristaltic flows in the small intestine and chemical reaction are important factors in composite physiological procedures. The chemical reaction takes place at every stage of digestive process. Further in the digestive framework, incompletely digested food mixed with stomach acids in stomach and is called chyme, which is a semi-liquid mass of incompletely digested food and reveals as non-Newtonian liquid. Reproduction of chyme flow in the small intestinal tract requires models, which can simulate fluid structure interaction (FSI), rheology and chemical response in addition to dispersion. Few studies have been reported in the literature (Robert (2005), Stoll el al. (2000), Taghipoor et al. (2012), Tharakan (2010)). In the fluid mechanics point of view, peristaltic creeping is characterized by dynamic interaction of liquid flow with the movement of flexible boundaries. In this connection it falls in the domain of moving boundary problems in applied mathematics or FSI problems in science and engineering. In view of its importance, some workers (Jaffrin et al. (1969), Fung et al. (1968), Misra and Gosh (1997), Shehawey and Sebaei (2000), Takagi and Balmforth (2011)) have explored the peristaltic transport of different liquids under various circumstances.

It is seen that couple stress fluids behavior are exceptionally useful in understanding dissimilar physiological and mechanical procedures. Such studies clarify the behavior of rheological complex liquids. It is a particular kind of non-Newtonian fluid, whose particle sizes are taken into account. Few studies on peristaltic transport of couple stress fluid have been reported in references (Srivastava (1986), Elshehawey et al. (1994), Mekheimer et al. (2008), Mekheimer (2004, 2008)). The effects of wall on Poiseuille flow with peristalsis have been examined by Mittra and Prasad (1973). After this study, a few investigators have studied the wall effects on different fluids with peristalsis (Sankad and Radhakrishnamacharya (2011), Pandey and Chaube (2011), Ellahi et al.(2016), Hina et al.(2015), Riaz et al.(2014), Akram et al.(2014)).

Dispersion plays a central role in chyme transport and other applications like environmental pollutant transportation, chromatographic separation, the mixing and transport of drugs or toxic substances in physiological structures (Ng, 2006). The dispersion of a solute in a solvent flowing in a channel has applications in many physiological fluid dynamics, biomedical and chemical engineering. The basic theory on dispersion was first proposed by Taylor (1953), who investigated theoretically and experimentally that the dispersion of a solute is miscible with a liquid flowing through a channel. Padma and Rao (1975), Gupta and Gupta (1972), Sobh (2013), and Sankad and Dhange (2016) explored the scattering of a substance in viscous liquid under various limitations. Afterward, these analyses have been extended to non-Newtonian fluids by (Chandra and Philip (1993), Dutta et al. (1974), Alemayehu and Radhakrishnamacharya (2012), Ravikiran and Radhakrishnamacharya (2015a, 2015b), Hayat et al. (2014), and 2016)). Flow through permeable medium has several applications in Geo-fluid, physiological fluid dynamics and Engineering field. The study of flow in permeable media is immensely used for understanding the transport process in kidneys, lungs, gallbladder with stones. Most of the tissues in the body are deformable permeable media. The proper functioning of such things depends on the stream of blood, nutrients, etc. Misra and Gosh (1997) have studied the blood flow in the micro-veins (or vessels) of the lungs which may be treated as channel constrained by two thin spongy layers. In many situations like physiology and engineering, the fluid slips at the walls of the channel The slip boundary condition was initially proposed by Beaver and Joseph (1967). Saffman (1971) modified the periphery condition of Beaver and Joseph. The presence of slip phenomenon at the boundaries and interfaces has been observed in physiological streams, flows through pipes in which chemical responses take place at the walls.

Motivated from the reported literature, a mathematical model for chyme transport in small intestine of human being during digestion process is prepared, the present study examine the slip and chemical reaction effects on the peristaltic stream of a couple stress fluid through a permeable media with complaint walls under the supposition of long wavelength, conditions of Taylor's limit and dynamic periphery conditions. The investigative expression for mean effective scattering coefficient has been obtained. Furthermore, mean scattering coefficient was calculated numerically. The effects of different values of penetrating parameters are discussed in detail through graphs.

2. Two-dimensional couple stress flow model

The flow of couple stress fluid model with peristalsis through a permeable medium and complaint wall is considered. The peristaltic wave produces the flow traveling along the channel walls. Figure 1 describes the geometry of the problem.

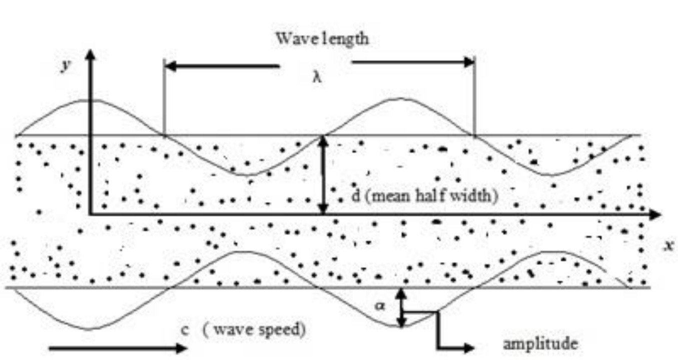


Figure 1. Geometry of the problem

The relevant equation of sinusoidal wave is:

$$y = \pm h = \pm\left[d + a\sin\frac{2\pi}{\lambda}\left(x - ct\right)\right], \tag{1}$$

where a is the amplitude, λ is the wavelength and c is the speed of the peristaltic wave.

The corresponding flow equations of the present issue are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\rho\left[\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]u=-\frac{\partial p}{\partial x}+\mu\nabla^2 u-\eta'\nabla^4 u-\frac{\mu}{\bar{\bar{k}}}u, \tag{3}$$

$$\rho\left[\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right]v=-\frac{\partial p}{\partial y}+\mu\nabla^2 v-\eta'\nabla^4 v-\frac{\mu}{\bar{\bar{k}}}v, \tag{4}$$

where $\nabla^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}$, $\nabla^4=\nabla^2\nabla^2$, ρ is the density of the fluid, η' is the constant associated with couple stress fluid, p is the pressure, μ is the viscosity coefficient, and u, v are the velocity components in the x and y directions.

The equation of the bendable wall movement (Mittra-Prasad, 1973) is given as:

$$L(h)=p-p_0\,, \tag{5}$$

where L is the movement of an expanded membrane by the damping forces and is given by the equation:

$$L=-T\frac{\partial^2}{\partial \mathrm{x}^2}+m\frac{\partial^2}{\partial t^2}+C\frac{\partial}{\partial t}. \tag{6}$$

Here, m is the mass/unit area, T is the tension in the membrane, and C is the viscous damping force coefficient.

After solving the Equations (2) to (4) under long - wavelength hypothesis, neglecting body couples and body forces, the flow equations of the present issue are reduced as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\eta'\frac{\partial^4 u}{\partial y^4}-\frac{\mu}{\bar{\bar{k}}}u=0, \tag{8}$$

$$-\frac{\partial p}{\partial y}=0. \tag{9}$$

The related periphery conditions are

$$u=-d\,\frac{\sqrt{D_a}}{\gamma_1}\frac{\partial u}{\partial y},\qquad \frac{\partial^2 u}{\partial y^2}=0,\qquad \text{at}\ \ y=\pm h. \tag{10}$$

It is presumed that $p_0=0$ and the channel walls are inextensible; therefore, the straight displacement of the wall is nil and only lateral movement takes place, and

$$\frac{\partial}{\partial x}L(h)=\mu\frac{\partial^2 u}{\partial y^2}-\eta'\frac{\partial^4 u}{\partial y^4}-\frac{\mu}{\bar{k}}u=0,\quad \text{at}\quad y=\pm h, \tag{11}$$

where

$$\frac{\partial}{\partial x}L(h)=\frac{\partial p}{\partial x}=P'=-T\,\frac{\partial^3 h}{\partial x^3}+m\frac{\partial^3 h}{\partial x\,\partial t^2}+C\,\frac{\partial^2 h}{\partial x\,\partial t}. \tag{12}$$

After solving Equations (8) and (9) with conditions (10) and (11); we get

$$u(y)=-\frac{\bar{k}}{\mu}\frac{\partial p}{\partial x}\left[A'_1\ \cosh(m'_1 y)+A'_2\ \cosh(m'_2 y)+1\right], \tag{13}$$

$$m'_1=\sqrt{\frac{\mu}{2\eta'}\left(1+\sqrt{1-\frac{4\eta'}{\mu\bar{k}}}\right)}\ ,\qquad m'_2=\sqrt{\frac{\mu}{2\eta'}\left(1-\sqrt{1-\frac{4\eta'}{\mu\bar{k}}}\right)}.$$

The mean velocity is given as

$$\bar{u}=\frac{1}{2h}\int_{-h}^{h}u(y)dy. \tag{14}$$

From Equations (13) and (14); we get

$$\bar{u}=-\frac{\bar{k}}{\mu}\frac{\partial p}{\partial x}\left[\frac{A'_1}{m'_1\,h}\sinh(m'_1 h)+\frac{A'_2}{m'_2\,h}\ \sinh(m'_2 h)+1\right]. \tag{15}$$

Employing idea of Alemayehu and Radhakrishnamacharya (2012), the fluid velocity is given by the equation:

$$u_x=u-\bar{u}. \tag{16}$$

From Equations (13), (15) and (16); we obtain

$$u_x = -\frac{\bar{k}}{\mu}\frac{\partial p}{\partial x}\left[\begin{array}{c} A'_1 \cosh(m'_1 y) + A'_2 \cosh(m'_2 y) \\ -\frac{A'_1}{m'_1 h}\sinh(m'_1 h) - \frac{A'_2}{m'_2 h}\sinh(m'_2 h)\end{array}\right], \tag{17}$$

where

$$A'_1 = \frac{(m'_2)^2 \cosh(m'_2 h)}{(a'_1 - a'_2)}, \; A'_2 = \frac{-(m'_1)^2 \cosh(m'_1 h)}{(a'_1 - a'_2)},$$

$$a'_1 = (m'_1)^2 \cosh(m'_1 h)\left(\cosh(m'_2 h) + d\frac{\sqrt{D_a}}{\gamma_1} m'_2 \sinh(m'_2 h)\right),$$

$$a'_2 = (m'_2)^2 \cosh(m'_2 h)\left(\cosh(m'_1 h) + d\frac{\sqrt{D_a}}{\gamma_1} m'_1 \sinh(m'_1 h)\right),$$

$$P' = \frac{\partial p}{\partial x} = m\frac{\partial^3 h}{\partial x\,\partial t^2} + C\frac{\partial^2 h}{\partial x\,\partial t} - T\frac{\partial^3 h}{\partial x^3}.$$

2.1. Diffusion with simultaneous homogeneous and heterogeneous chemical reactions

Referring Gupta-Gupta (1972), the dispersion equation for the concentration C of the substance for the present issue under isothermal conditions:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2} - k_1 C. \tag{18}$$

In the above equation, C is the concentration of the fluid, D is the diffusion coefficient for chemical reactions, and k_1 is the rate constant of chemical reaction.

For the common standards of physiologically important parameters of this issue, it is expected that $\bar{u} \approx C$ (Alemayehu-Radhakrishnamacharya (2012)).

Utilizing the clause $\bar{u} \approx C$, and consequent non-dimensional quantities,

$$\theta = \frac{t}{\bar{t}}, \; \bar{t} = \frac{\lambda}{\bar{u}}, \; \eta = \frac{y}{d}, \; \xi = \frac{(x - \bar{u}\,t)}{\lambda}, \; P = \frac{d^2}{\mu C\lambda}P', \\ H = \frac{h}{d}, \; D_a = \frac{\bar{k}}{d^2}. \tag{19}$$

Equations (12), (17), and (18) reduce to

$$P = -\in\left[(2\pi)^3 (E_1 + E_2)\cos(2\pi\xi) - (2\pi)^2 E_3 \sin(2\pi\xi)\right], \tag{20}$$

$$u_x = -\frac{D_a d^2}{\mu}\frac{\partial p}{\partial x}\left[A_1 \cosh(m_1\eta) + A_2 \cosh(m_2\eta) + A_3\right], \tag{21}$$

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D} C = \frac{d^2}{\lambda D} u_x \frac{\partial C}{\partial \xi} \ , \tag{22}$$

where

$$a_1 = (m_1)^2 \cosh(m_1 H)\left(\cosh(m_2 H) + \frac{\sqrt{D_a}}{\gamma_1} m_2 \sinh(m_2 H)\right),$$

$$a_2 = (m_2)^2 \cosh(m_2 H)\left(\cosh(m_1 H) + \frac{\sqrt{D_a}}{\gamma_1} m_1 \sinh(m_1 H)\right),$$

$$A_1 = \frac{(m_2)^2 \cosh(m_2 H)}{(a_1 - a_2)},\ A_2 = \frac{-(m_1)^2 \cosh(m_1 H)}{(a_1 - a_2)},$$

$$A_3 = -A_1 \sinh(m_1 H) - A_2 \sinh(m_2 H),$$

$$m_1 = m'_1 d = \sqrt{\frac{\gamma^2}{2}\left(1 + \sqrt{1 - \frac{4}{\gamma^2 D_a}}\right)}\ ,\ m_2 = m'_2 d = \sqrt{\frac{\gamma^2}{2}\left(1 - \sqrt{1 - \frac{4}{\gamma^2 D_a}}\right)},$$

$E_1\left(= -\frac{Td^3}{\lambda^3 \mu C}\right)$ is the rigidity, $E_2\left(= \frac{mCd^3}{\lambda^3 \mu}\right)$ is the stiffness, $E_3\left(= \frac{Cd^3}{\lambda^2 \mu}\right)$ is the damping characteristic of the wall and $\in\left(= -\frac{a}{d}\right)$ is an amplitude ratio, $\gamma\left(= d\sqrt{\frac{\mu}{\eta'}}\right)$ is the couple stress constraint, and $D_a\left(= \frac{\bar{k}}{d^2}\right)$ is the permeability parameter.

Below, we have discussed the diffusion with first-order irreversible chemical reaction taking place in the mass of the fluid medium and at the walls of the channel; the walls are catalytic to chemical reaction. Hence, the periphery conditions at the walls (Chandra-Philip, 1993) are given by the following equations:

$$\frac{\partial C}{\partial y} + f\,C = 0 \quad \text{at} \ \ y = h = \left[d + a\sin\frac{2\pi}{\lambda}\left(x - \bar{u}\,\text{t}\right)\right]\ , \tag{23}$$

$$\frac{\partial C}{\partial y} - f\,C = 0 \quad \text{at} \ \ y = -h = -\left[d + a\sin\frac{2\pi}{\lambda}\left(x - \bar{u}\,\text{t}\right)\right]\ . \tag{24}$$

From Equations (19), (23), and (24); we get

$$\frac{\partial C}{\partial \eta} + \beta\, C = 0 \quad \text{at } \eta = H = \left[1 + \in \sin 2(\pi \xi)\right], \tag{25}$$

$$\frac{\partial C}{\partial \eta} - \beta\, C = 0 \quad \text{at } \eta = -H = -\left[1 + \in \sin 2(\pi \xi)\right], \tag{26}$$

where $\beta = fd$ is the heterogeneous response rate corresponding to the catalytic response at the walls.

From the Equations (25) and (26), we obtain the primitive of Equation (22) as follows:

$$C(\eta) = -\frac{D_a\, d^4}{\lambda\, \mu\, D}\left(\frac{\partial C}{\partial \xi}\right)\left(\frac{\partial p}{\partial x}\right)\begin{bmatrix} A_4 \cosh(\mathrm{m}_1\eta) + A_5 \cosh(\mathrm{m}_2\eta) \\ + A_6 \cosh(\alpha\eta) + A_7 \end{bmatrix}. \tag{27}$$

The volumetric rate Q is defined as the rate in which the solute matter is pumping across a section of channel per unit breadth.

$$Q = \int_{-H}^{H} C\, u_x \, d\eta \,. \tag{28}$$

Using Equations (21) and (27) in Equation (28); we obtain

$$Q = -2\frac{d^6}{\lambda D \mu^2}\left(\frac{\partial C}{\partial \xi}\right) G\left(\xi, \alpha, \beta, \in, E_1, E_2, E_3,\ D_a, \gamma, \gamma_1\right), \tag{29}$$

where

$$G = \left[-D^2{}_a P^2 \begin{pmatrix} \frac{A_1\, A_4}{2} B_1 + \frac{A_2\, A_5}{2} B_2 + \left(A_1\, A_5 + A_2\, A_4\right) B_3 \\ + A_1\, A_6\, B_4 + A_2\, A_6\, B_5 + \left(A_1\, A_7 + A_3\, A_4\right) B_6 \\ + \left(A_2\, A_7 + A_3\, A_5\right) B_7 + A_3\, A_6\, A_8 + A_3\, A_7\, H \end{pmatrix} \right], \tag{30}$$

$$A_4 = \frac{(m_2)^2 \cosh(m_2 H)}{\left((m_1)^2 - \alpha^2\right)(a_1 - a_2)},\ A_2 = \frac{-(m_1)^2 \cosh(m_1 H)}{\left((m_2)^2 - \alpha^2\right)(a_1 - a_2)},$$

$$A_6 = A_3\, L_1 - A_4\, L_2 - A_5\, L_3,\ \mathrm{A}_7 = -\frac{A_3}{\alpha^2},$$

$$L_1 = \frac{\beta}{\alpha^2(\alpha \sinh \alpha H + \beta \cosh \alpha H)}, \quad L_2 = \frac{(m_1 \sinh m_1 H + \beta \cosh m_1 H)}{(\alpha \sinh \alpha H + \beta \cosh \alpha H)},$$

$$L_3 = \frac{(m_2 \sinh m_2 H + \beta \cosh m_2 H)}{(\alpha \sinh \alpha H + \beta \cosh \alpha H)}, \quad B_1 = \frac{2m_1 H + \sinh 2m_1 H}{2m_1},$$

$$B_2 = \frac{2m_2 H + \sinh 2m_2 H}{2m_2}, \quad \alpha = \sqrt{\frac{\mathrm{k}_1}{D}}d,$$

$$B_3 = \frac{m_1 \sinh(m_1 H)\cosh(m_2 H)}{\left[(m_1)^2 - (m_2)^2\right]} - \frac{m_2 \cosh(m_1 H)\sinh(m_2 H)}{\left[(m_1)^2 - (m_2)^2\right]},$$

$$B_4 = \frac{m_1 \sinh(m_1 H)\cosh(\alpha H)}{\left[(m_1)^2 - (\alpha)^2\right]} - \frac{\alpha \cosh(m_1 H)\sinh(\alpha H)}{\left[(m_1)^2 - (\alpha)^2\right]},$$

$$B_5 = \frac{m_2 \sinh(m_2 H)\cosh(\alpha H)}{\left[(m_2)^2 - (\alpha)^2\right]} - \frac{\alpha \cosh(m_2 H)\sinh(\alpha H)}{\left[(m_2)^2 - (\alpha)^2\right]},$$

$$B_6 = \frac{\sinh m_1 H}{m_1}, \quad \mathrm{B}_7 = \frac{\sinh m_2 H}{m_2}, \quad \mathrm{B}_8 = \frac{\sinh \alpha H}{\alpha}.$$

Glancing at Equation (30) with Fick's law of diffusion, the scattering coefficient D^* was calclated such that the solute diffuses comparative to the plane moving with the average speed of the flow and is given as:

$$D^* = 2\frac{d^6}{D\mu^2}G(\xi, \alpha, \beta, \in, E_1, E_2, E_3, D_a, \gamma, \gamma_1). \tag{31}$$

Let $\overline{G}$ be the average of G, and is obtained by the following equation:

$$\overline{G} == \int_0^1 G(\xi, \alpha, \beta, \in, E_1, E_2, E_3, D_a, \gamma, \gamma_1)\, d\xi. \tag{32}$$

Introduce stream function ψ by using the relation $u = \frac{\partial \psi}{\partial y}$. From Equations (17), (19), and (20), the stream function is

$$\psi(\xi, \eta) = \in \left[(2\pi)^3 (E_1 + E_2)\cos(2\pi\xi) - (2\pi)^2 E_3 \sin(2\pi\xi) \right] \times \left[\frac{D_a A_1 \sinh(m_1\eta)}{m_1} + \frac{D_a A_2 \sinh(m_2\eta)}{m_2} + D_a A_3 \eta \right]. \tag{33}$$

Special cases: This is worth pointing out that the expression for u_x is similar to that of Tripathi et al. (2013), when there are no chemical reactions and no wall properties. Further, it is noticed that the solution expression for u_x and G are in agreement with Alemayehu and Radhakrishnamacharya (2012), if there are no wall properties.

3. Numerical computations and discussion

The expression for $\overline{G}(\xi, \alpha, \beta, \in, E_1, E_2, E_3, D_a, \gamma, \gamma_1)$ as shown in Equation (32) has been obtained by numerical integration using the software MATHEMATICA and the domino effects are presented through graphs. The pertinent parameters present in this argument are couple stress parameter (γ), an amplitude ratio $(\in)$ the homogeneous response rate (α), the permeability parameter (D_a), the heterogeneous response rate (β), slip parameter (γ_1), the rigidity (E_1), the stiffness (E_2), and the viscous damping force (E_3). We may ensure that E_1, E_2 and E_3 cannot be zero all together.

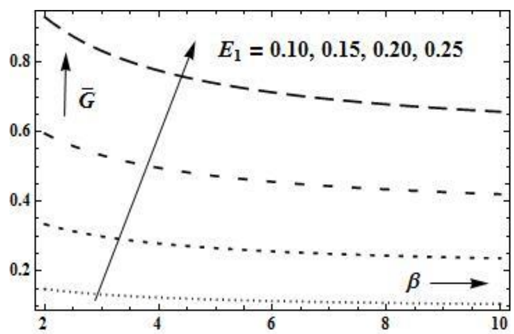


Figure 2. Plot of $\overline{G}$ for E_1 with $\in$ = 0.2, α = 1.0, D_a =0.002, γ=2.0, γ_1=0.02, E_2 = 0.0, E_3 = 0.00

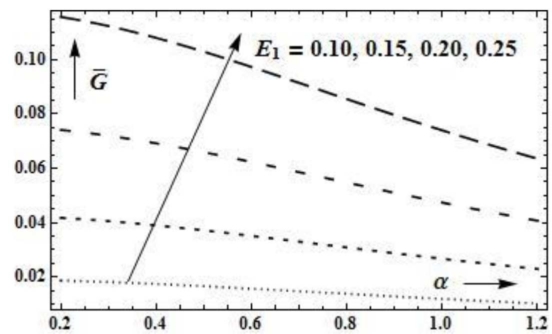


Figure 3. Plot of $\overline{G}$ for E_1 with $\in$ =0.2, β = 5.0, D_a =0.002, γ=2.0, γ_1=0.02, E_2 = 0.0, E_3 = 0.06

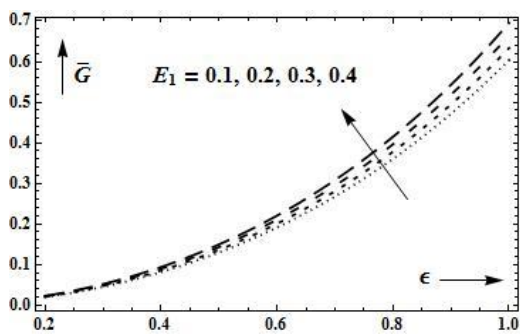


Figure 4. Plot of $\overline{G}$ for E_1 with α=1.0, β=5.0, D_a =0.002, γ=2.0, γ_1=0.02, E_2 = 4.0, E_3 = 0.00

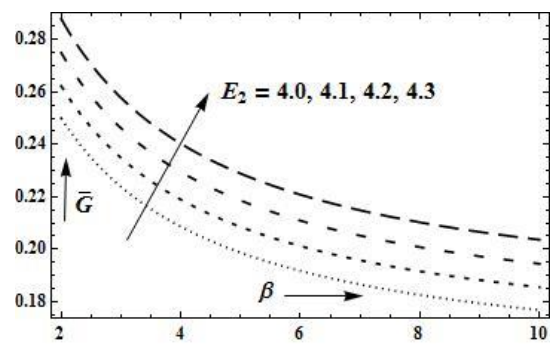


Figure 5. Plot of $\overline{G}$ for E_2 with $\in$ =0.2, α= 1.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_3 = 0.00

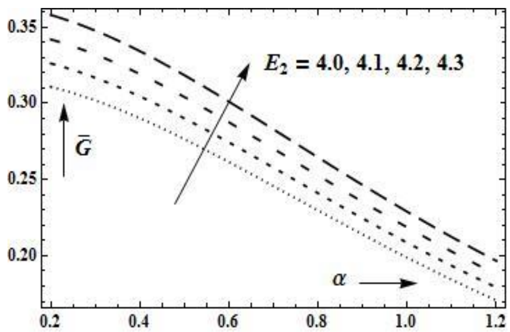


Figure 6. Plot of $\overline{G}$ for E_2 with $\in$ =0.2, β=5.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_3 = 0.06

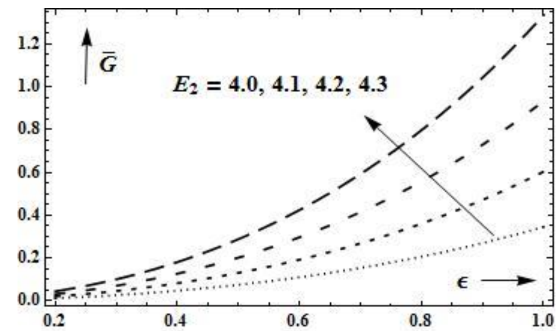


Figure 7. Plot of $\overline{G}$ for E_2 with β = 5.0, α = 1.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_3 = 0.06

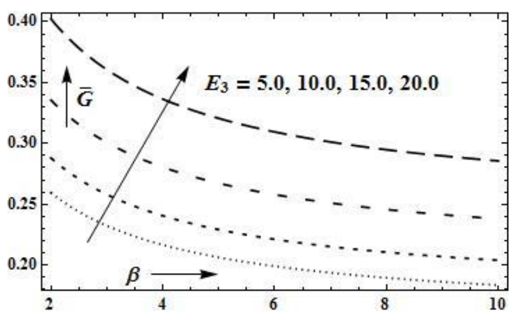


Figure 8. Plot of $\overline{G}$ for E_3 with $\in$ =0.2, α=1.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_2 = 4.0

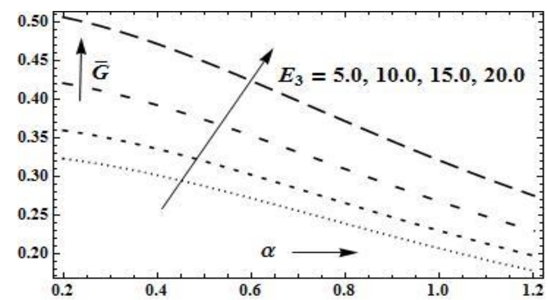


Figure 9. Plot of $\overline{G}$ for E_3 with $\in$ =0.2, β = 5.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_2= 4.0

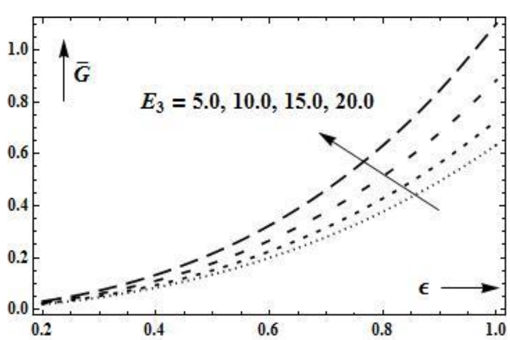


Figure 10. Plot of $\overline{G}$ for E_3 with β = 5.0, α = 1.0, D_a =0.002, γ=2.0, γ_1=0.02, E_1 = 0.1, E_2 = 4.0

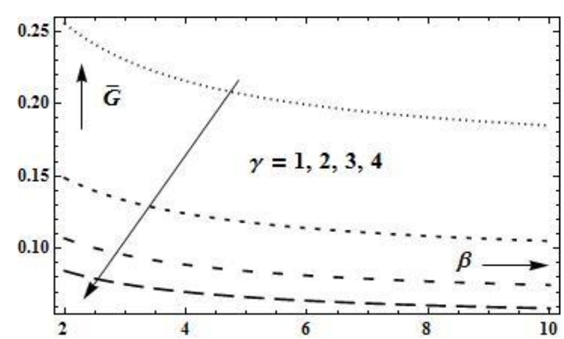


Figure 11. Plot of $\overline{G}$ for γ with $\in$ = 0.2, α = 1.0, D_a =0.002, γ_1=0.02, E_1 = 0.1, E_2 = 0.0, E_3=0.06

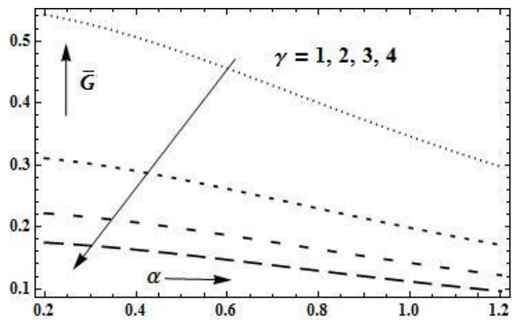


Figure 12. Plot of $\overline{G}$ for γ with $\in$ =0.2, β=5.0, D_a =0.002, γ_1=0.02, E_1 = 0.1, E_2 = 4.0, E_3=0.06

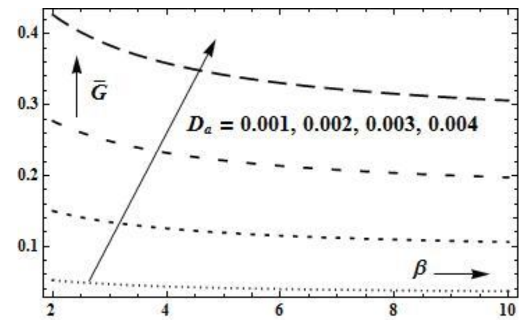


Figure 13. Plot of $\overline{G}$ for D_a with $\in$ =0.2, α = 1.0, γ=2.0, γ_1=0.02, E_1 = 0.1, E_2 = 0.0, E_3=0.06

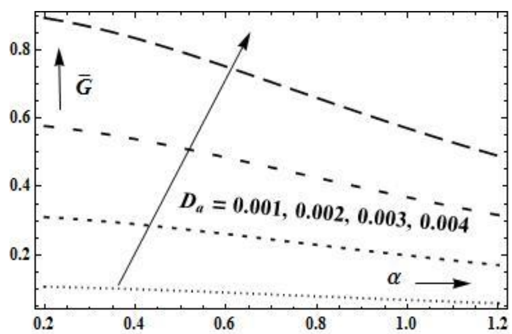


Figure 14. Plot of $\overline{G}$ for D_a with $\in$ = 0.2, β = 5.0, γ=2.0, γ_1=0.02, E_1 = 0.1, E_2 = 4.0, E_3=0.06

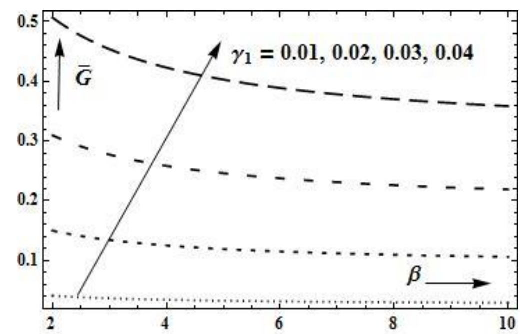


Figure 15. Plot of $\overline{G}$ for γ_1 with $\in$ =0.2, α=1.0, D_a =0.002, γ=2.0, E_1 = 0.1, E_2 = 0.0, E_3=0.06

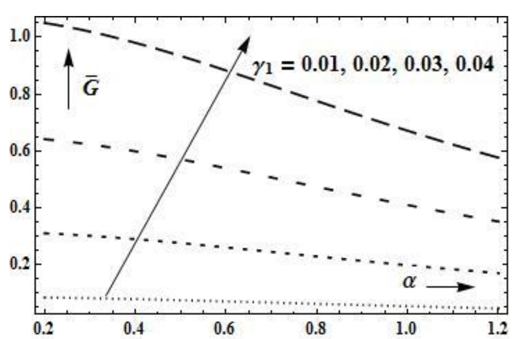


Figure 16. Plot of $\overline{G}$ for γ_1 with $\in$ = 0.2, β = 5.0, D_a =0.002, γ=2.0, E_1 = 0.1, E_2 = 4.0, E_3=0.06

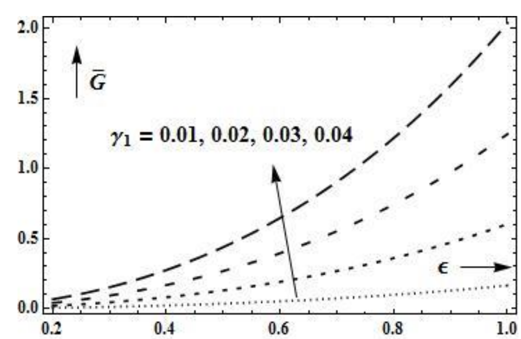


Figure 17. Plot of $\overline{G}$ for γ_1 with β =5.0, α = 1.0, D_a =0.002, γ=2.0, E_1 = 0.1, E_2 = 4.0, E_3=0.00

The effects of the rigidity parameter (E_1), stiffness (E_2) and viscous damping force (E_3) on the dispersion coefficient $(\overline{G})$ are depicted in Figures 2-10. It is observed that $\overline{G}$ ascends monotonically with an increase in E_1, E_2 and E_3. This understanding supports the fact that increment in the flexibility of the channel walls help the stream moment which enhances the scattering. This result is in agreement with the results of Ravikiran and Radhakrishnamacharya (2015a).

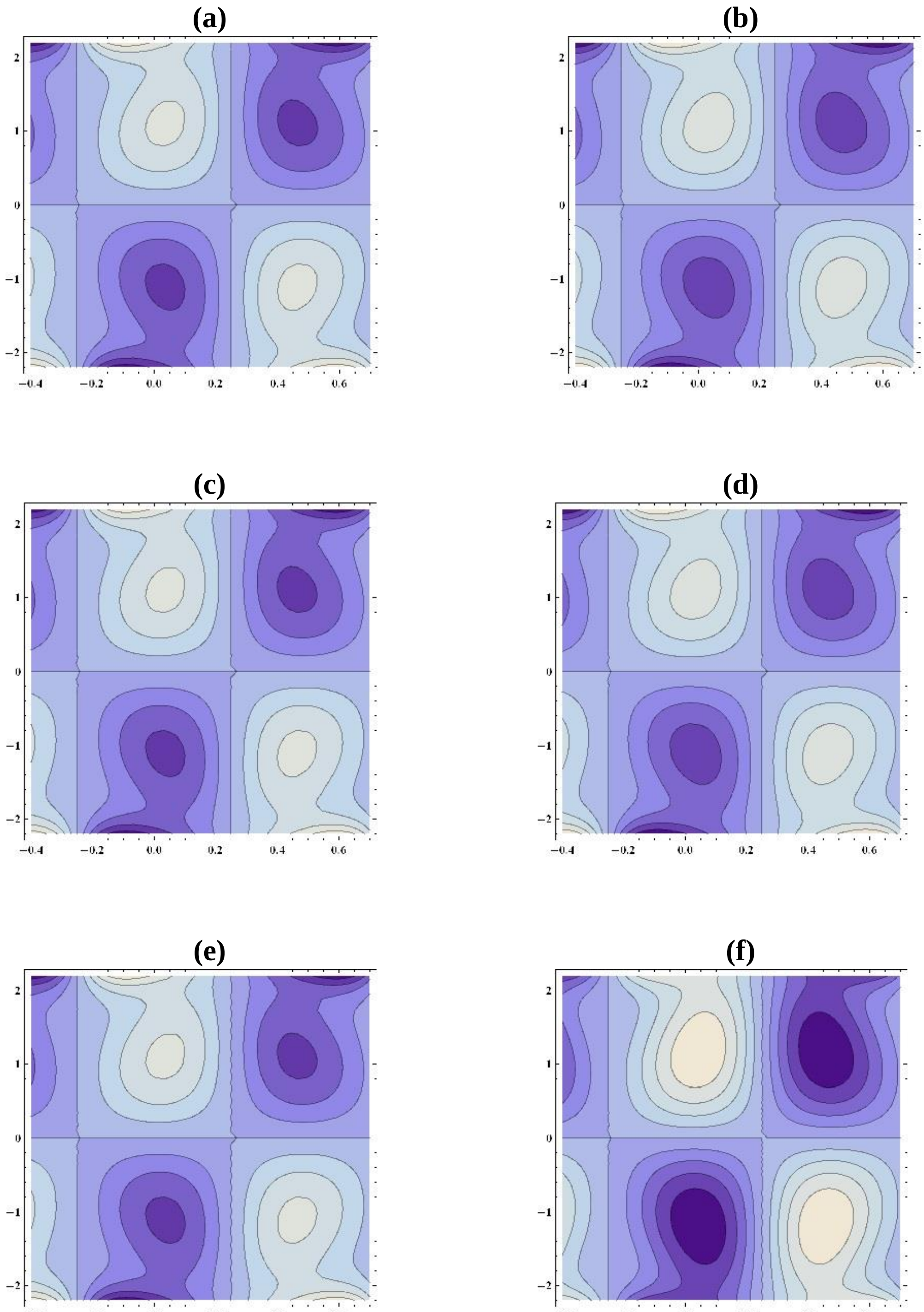
In Figures 11-12, it is observed that $\overline{G}$ descends with an increase in couple stress parameter (γ). Increase in couple stress parameter leads to drop in the fluid velocity, and as a result scattering may reduce. This finding agrees with the conclusion of Alemayehu and Radhakrishnamacharya (2012), Mekheimer (2008), and Sankad et al. (2011). Figures 13-14 indicates that $\overline{G}$ is enhanced with an increase in the permeability parameter (D_a). This is a direct result of the way that growing porosity in a channel generates the fluid speed and causes it to ascend the dispersion. This result agrees with the results of Alemayehu and Radhakrishnamacharya (2012). The impact of slip parameter (γ_1) on $\overline{G}$ is depicted in Figures 15-17. It is noticed that $\overline{G}$ enhances with an increase in slip parameter. This end result agrees with that of Ravikiran and Radhakrishnamacharya (2015b). Furthermore, $\overline{G}$ ascends with an increment in the amplitude ratio $(\in)$ (Figures 4, 7, 10, and 17). As already known, increment in the amplitude ratio is the expansion in the amplitude of the wave across the channel and this increases the fluid velocity within the channel and consequently dispersion may be enhanced. This outcome concurs with that of Sobh (2013) and Alemayehu and Radhakrishnamacharya (2012).

Diffusion reduces with homogeneous response rate parameter (α) (Figures 3, 6, 9, 12, 14 and 17) and heterogeneous response rate (β) (Figures 2, 5, 8, 11, 13 and 15), whereas scattering diminishing with β is less significant. This outcome is normal since expansion in α prompts and expansion in number of moles of solute experiences chemical response. This result is consistent with the arguments of Padma and Rao (1975) and Hayat et al. (2014).

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The effects of wall parameters (E_1 and E_2), porosity parameter (D_a) and couple stress parameter (γ) on trapping are shown in Figure 18. It is observed that the size of the trapped bolus enlarges with increase in E_1 (Figures a, b), E_2 (Figures c, d), D_a (Figures e, f) and γ_1 (Figures i, j) while size of the bolus shrinks with increase in γ (Figures g, h).

4. Conclusions

This work analyzes the effects of couple stress parameter (γ), slip parameter (γ_1), amplitude ratio $(\in)$, homogeneous response rate (α), heterogeneous response rate (β), rigidity (E_1), stiffness (E_2), damping characteristic of the wall (E_3) on dispersion coefficient $\overline{G}$ for the couple stress fluid in a uniform channel with complaint walls and homogeneous-heterogeneous reactions. It is observed that the concentration profile $(\overline{G})$ increases with an increase in parameters E_1, E_2, E_3, γ_1 and $\in$. It is also noticed that $\overline{G}$ decreases with boost in homogeneous response rate parameter (α), couple stress parameter (γ), and heterogeneous response rate parameter (β). Streamline pattern shows that trapping phenomenon occurs and size of trapped bolus decreases with increase in couple stress parameter. Finally, it concludes that wall parameters, slip parameter, and amplitude ratio favor dispersion, while couple stress parameter resist the dispersion in the transport phenomena occurring in the small intestine of human being during digestion process. This authenticates that creeping sinusoidal stream assists the absorption of active components in small intestine. This model may help in understanding the transport phenomena occurring in the small intestine leading to absorption of nutrients and drugs.



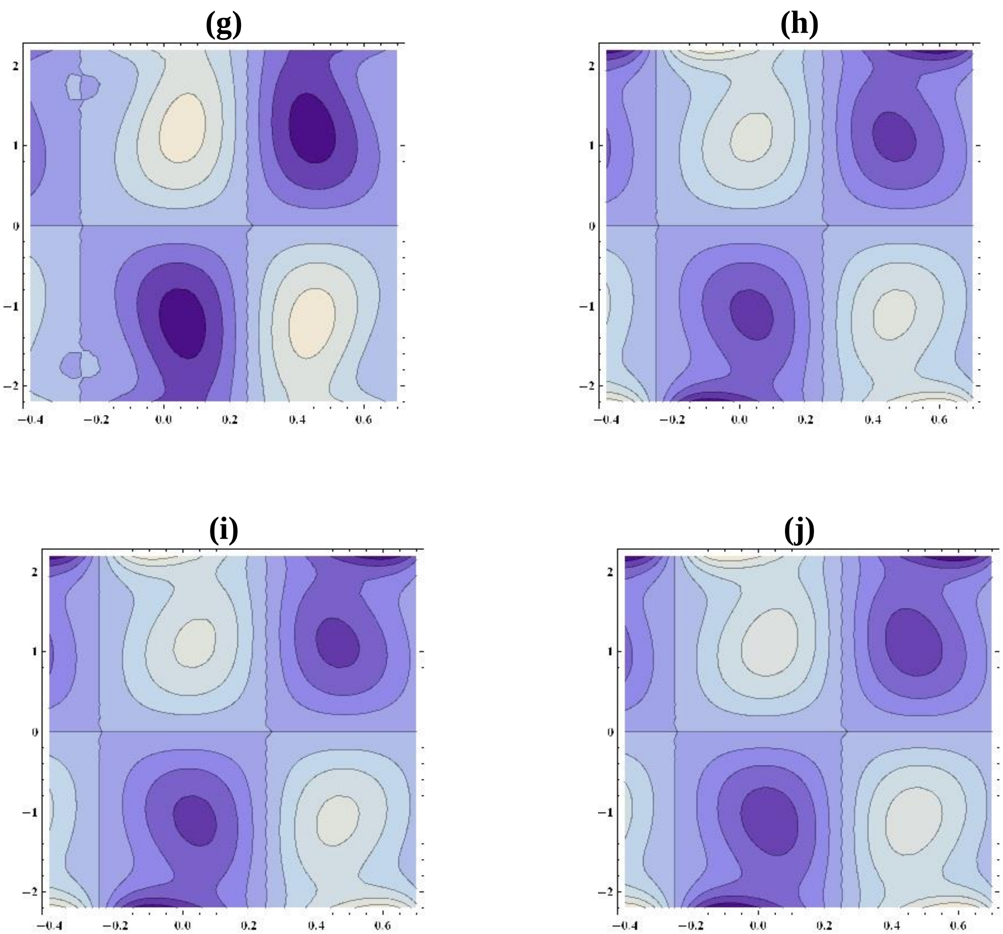


Figure 18. Stream lines for different value of E_1 (a) E_1 =0.1 (b) E_1=0.4 with ($\in$ = 0.2, α=1.0, β = 5.0, D_a =0.002, γ=0.2, γ_1=0.02, E_2 = 4.0, E_3=0.06).

Stream lines for different value of E_2 (c) E_2 =4.0 (d) E_2=4.3 with ($\in$ = 0.2, α=1.0, β = 5.0, D_a =0.002, γ=0.2, γ_1=0.02, E_1 = 0.1, E_3=0.06).

Stream lines for different value porosity parameter D_a (e) $D_a = 0.002$ (f) $D_a = 0.005$ with ($\in$ = 0.2, α=1.0, β = 5.0, γ=0.2, γ_1=0.02, E_1 = 0.1, E_2 = 4.0, E_3=0.06).

Stream lines for different value couple stress parameter γ (g) γ = 0.1 (h) γ = 0.2 *with* ($\in$ = 0.2, α=1.0, β = 5.0, $D_a = 0.002$, γ_1=0.02, E_1 = 0.1, E_2 = 4.0, E_3=0.06).

Stream lines for different value slip parameter γ_1 (i) γ_1 = 0.01 (j) γ_1 = 0.04 with ($\in$ = 0.2, α=1.0, β = 5.0, $D_a = 0.002$, γ=0.2, E_1 = 0.1, E_2 = 4.0, E_3=0.06).

Acknowledgments

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