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New structure for exact solutions of nonlinear time fractional Sharma-Tasso-Olver equation via conformable fractional derivative

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Abstract

In this paper new fractional derivative and direct algebraic method are used to construct exact solutions of the nonlinear time fractional Sharma-Tasso-Olver equation. As a result, three families of exact analytical solutions are obtained. The results reveal that the proposed method is very effective and simple for obtaining approximate solutions of nonlinear fractional partial differential equations.

Keywords: Nonlinear time fractional Sharma-Tasso-Olver equation; Conformable fractional derivative; Direct algebraic method

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1. Introduction

There are several definitions to the generalization of the notion of fractional differentiation. The Riemann-Liouville derivative, Caputo's derivative, Hilfer derivative and Grunwald-Letnikov derivative are the most popular definitions (Podlubny, 1998; Hilfer, 2000). Most of the existing fractional operators are defined via the fractional integrals with singular kernels which is due to their nonlocal structures. In addition, most of the nonlocal fractional derivatives do not obey the basic chain, quotient and product rules. Recently, to overcome these and other difficulties, Khalil et al. (Khalil et al., 2014) introduced a new well-behaved definition of local fractional (non-integer order) derivative, called the conformable fractional derivative. The conformable fractional derivative is theoretically very easier to handle. The conformable calculus is very fascinating and is gaining interest; see (Abdeljawad, 2015; Chung, 2015; Kurt et al., 2015; Zheng et al., 2015; Rezazadeh et al., 2016; Benkhettou et al., 2016; Ünal and Gökdoğan, 2017) and reference therein.

Definition 1.1.

Let $f:(0,\infty)\to\mathbb{R}$ be a function. Then the conformable fractional derivative of f of order α is defined by

$$\frac{\partial^\alpha f}{\partial t^\alpha}=\lim_{\varepsilon\to 0^+}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon},\quad 0<\alpha<1,\qquad t>0. \tag{1}$$

If the above limit exists then we say f is α-differentiable.

Some useful properties for the suggested conformable fractional derivative given in (Khalil et al., 2014) are as follows:

Theorem 1.1.

Suppose $\alpha\in(0,1)$ and $f,g:(0,\infty)\to\mathbb{R}$ be α-differentiable at a point $t>0$.Then

(1) $\dfrac{\partial^\alpha}{\partial t^\alpha}(af+bg)=a\dfrac{\partial^\alpha f}{\partial t^\alpha}+b\dfrac{\partial^\alpha g}{\partial t^\alpha},\quad \forall a,b\in\mathbb{R}.$

(2) $\dfrac{\partial^\alpha t^\beta}{\partial t^\alpha}=pt^{\beta-\alpha},\quad \forall\beta\in\mathbb{R}.$

(3) $\dfrac{\partial^\alpha}{\partial t^\alpha}(\lambda)=0,\quad \lambda=$ constant.

(4) $\dfrac{\partial^\alpha fg}{\partial t^\alpha}=g\dfrac{\partial^\alpha f}{\partial t^\alpha}+f\dfrac{\partial^\alpha g}{\partial t^\alpha}.$

(5) $\dfrac{\partial^\alpha u}{\partial t^\alpha}(\dfrac{f}{g})=\dfrac{g\dfrac{\partial^\alpha f}{\partial t^\alpha}-f\dfrac{\partial^\alpha g}{\partial t^\alpha}}{g^2}.$

(6) If f is differentiable, then $\dfrac{\partial^\alpha f}{\partial t^\alpha}=t^{1-\alpha}\dfrac{df}{dt}.$

Theorem 1.2.

Suppose $f:(0,\infty)\to\mathbb{R}$ is a function such that f is differentiable and also $\alpha-$differentiable. Let g be a function defined in the range of f and also differentiable and for all $t\neq 0, g(t)\neq 0$; then, one has the following rule (Abdeljawad, 2015)

$$\frac{\partial^\alpha}{\partial t^\alpha}(fog)(t)=(T_\alpha f)(g(t))(T_\alpha g)(t)g(t)^{\alpha-1}. \tag{2}$$

Corollary 1.1.

From Theorem 1.2 we obtain

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}}(fog)(t) = t^{1-\alpha} f'(g(t))g'(t).$$
(3)

Definition 1.2.

Suppose $a \geq 0, t \geq 0,$ and f is defined on $(0,\ t],$ then the $\alpha-$fractional integral is given by

$$I_{\alpha} f(t) = I_1(t^{\alpha-1} f)(t) = \int_0^t \frac{f(x)}{x^{1-\alpha}} dx. \tag{4}$$

Nonlinear partial differential equations with integer or fractional order have played a very important role in various fields of science and engineering, such as mechanics, electricity, chemistry and so on. In all these scientific fields, it is important to obtain exact solutions of partial differential equations with integer or fractional order (Biswas, 2008; Biswas and Kara, 2010; Taghizadeh et al., 2010; Kudriashov, 2012; Bhrawy et al., 2013; Bhrawy et al., 2016; Ebadi et al., 2013; Mirzazadeh et al., 2014a; Mirzazadeh et al., 2014b; Mirzazadeh et al., 2014c; Eslami et al., 2014a; Eslami et al., 2014b; Savescu et al., 2014; Aminikhah et al., 2015; Ekici et al., 2016; Arnous et al., 2017a; Arnous et al., 2017b; Bekir et al., 2016; Biswas et al., 2014; Ekici et al., 2016a; Ekici et al., 2016b; Ekici et al., 2017a; Ekici et al., 2017b; Mirzazadeh et al., 2016a; Mirzazadeh et al., 2016b; Mirzazadeh et al., 2016c; Ullah et al., 2017). In recent months, many powerful methods for obtaining exact solutions of nonlinear conformable fractional partial differential equations have been presented such as, the First integral method (Eslami and Rezazadeh, 2015; Eslami, 2016), the tanh-function method (Tariq, and Akram, 2016), the (G'/G)-expansion method (Taghizadeh et al., 2016), the sub equation method (Aminikhah et al., 2016; Rezazadeh and Ziabarya, 2016) and Modified Kudryashov method (Hosseiniet al., 2016).

In this paper, we will apply the direct algebraic method for solving nonlinear time fractional Sharma-Tasso-Olver equation in the sense of Conformable fractional derivative by Khalil et al. The article is organized as follows: In Section 2, direct algebraic method is discussed. In Section 3, we exert this method to the nonlinear time conformable fractional Sharma-Tasso-Olver equation, and in Section 5 conclusions are given.

2. An Analysis of the Method

For a given time conformable fractional partial differential equation

$$G\left(u, \frac{\partial^{\alpha} u}{\partial t^{\alpha}}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^{2} u}{\partial x^{2}}, \ldots\right) = 0, \qquad t \geq 0, \qquad 0 < \alpha \leq 1, \tag{5}$$

our method mainly consists of four steps:

Step 1: We seek solutions of Equation (5) in the following form

$$u = u(\xi), \qquad \xi = kx + c\frac{t^{\alpha}}{\alpha}, \tag{6}$$

where k and c are real constants. Under the transformation (6), Equation (5) becomes an ordinary differential equation

$$N(u, cu', ku', c^2u'', k^2u''.....) = 0, \tag{7}$$

where

$$u' = \frac{du}{d\xi}.$$

Step 2: We assume that the solution of Equation (7) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i \varphi^i(\xi), \tag{8}$$

where $a_i\,(i = 1, 2, .., n)$ are real constants to be determined later. $\varphi(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi) = b + \varphi^2(\xi), \tag{9}$$

Equation (9) admits the following solutions

$$\varphi(\xi) = \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b < 0, \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b < 0, \\ \sqrt{b}\tan(\sqrt{b}\xi), & b > 0, \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b > 0, \\ -\dfrac{\alpha}{\xi}, & b = 0. \end{cases} \tag{10}$$

Integer n in (8) can be determined by considering homogeneous balance between the nonlinear terms and the highest derivatives of $u(\xi)$ in Equation (7).

Step 3: Substituting (8) into (7) with (9), then the left hand side of Equation (7) is converted into a polynomial in $\varphi(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c.

Step 4: Solving the algebraic equations obtained in step 3, and substituting the results into (8), then we obtain the exact traveling wave solutions for Equation (5).

3. Nonlinear time conformable fractional Sharma-Tasso-Olver equation in the form

In this section, we will exert the direct algebraic method to find the exact solutions of nonlinear time conformable fractional Sharma-Tasso-Olver equation. Let us consider the nonlinear time conformable fractional Sharma-Tasso-Olver equation (Song et al,. 2009; Lu., 2012)

$$\frac{\partial^\alpha u}{\partial t^\alpha}+3\beta\left(\frac{\partial u}{\partial x}\right)^2+3\beta u^2\frac{\partial u}{\partial x}+3\beta u\frac{\partial^2 u}{\partial x^2}+\beta\frac{\partial^3 u}{\partial x^3}=0,\quad t\geq 0,\qquad 0<\alpha\leq 1. \tag{11}$$

The nonlinear time fractional Sharma-Tasso-Olever Equation (11) is a KdV-like equation, and plays an important role in describing the nonlinear wave phenomena. Exact solutions for it with different forms can describe different nonlinear waves phenomena, such as the solitary wave phenomenon and the periodic wave phenomenon and so on.

To solve Equation (11), we consider the following traveling wave transformation

$$u(x,t)=u(\xi),\qquad \xi=kx+c\frac{t^\alpha}{\alpha}, \tag{12}$$

then Equation (11) can be reduced to the following nonlinear differential equation,

$$cu'+3k^2\beta(u')^2+3\beta ku^2u'+3k^2\beta uu''+k^3\beta u'''=0. \tag{13}$$

Now we suppose that the Equation (13) has a solution in the form

$$u(\xi)=\sum_{i=0}^{m}a_i\varphi^i, \tag{14}$$

where a_i are constants to be determined later and the new variable $\varphi=\varphi(\xi)$ satisfies the following fractional Riccati equation:

$$\varphi'=b+\varphi^2. \tag{15}$$

Balancing the highest order derivative with nonlinear term in Equation (13) gives $m=1$, from which we have

$$u(\xi)=a_0+a_1\varphi. \tag{16}$$

Substituting Equation (16) along with Equation (15) into Equation (13) and setting the coefficients of φ^j to zero, we finally obtain a system of algebraic equations, and solving these algebraic equations we have

$$a_0=\pm\sqrt{\frac{\beta bk^3-c}{3\beta k}},\qquad a_1=-k, \tag{17}$$

$$b = \frac{1}{4}\frac{c}{\beta k^3}, \quad a_0 = 0, \quad a_1 = -2k. \tag{18}$$

***Case* 1:** Substituting Equation (17) into (16) along with (10), we have the solutions of Equation (11) as follows:

When $b < 0$

$$u_{1,2}(x,t) = \pm\sqrt{\frac{\beta b k^3 - c}{3\beta k}} + k\left(\sqrt{-b}\tanh\left(\sqrt{-b}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{19}$$

$$u_{3,4}(x,t) = \pm\sqrt{\frac{\beta b k^3 - c}{3\beta k}} + k\left(\sqrt{-b}\coth\left(\sqrt{-b}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right). \tag{20}$$

When $b > 0$

$$u_{5,6}(x,t) = \pm\sqrt{\frac{\beta b k^3 - c}{3\beta k}} - k\left(\sqrt{b}\tan\left(\sqrt{b}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{21}$$

$$u_{7,8}(x,t) = \pm\sqrt{\frac{\beta b k^3 - c}{3\beta k}} + k\left(\sqrt{b}\cot\left(\sqrt{b}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right). \tag{22}$$

When $b = 0$

$$u_{9,10}(x,t) = \pm\sqrt{\frac{-c}{3\beta k}} + \frac{k\alpha}{k\alpha x + ct^\alpha}. \tag{23}$$

Solutions (19) and (20) are topological soliton and singular soliton solution respectively while (21) and (22) are singular periodic solutions.

Case 2: Substituting Equation (18) into (16) along with (10), we have the solutions of Equation (11) as follows:

When $\frac{c}{\beta k^3} < 0$

$$u_{11}(x,t) = k\left(\sqrt{-\frac{c}{\beta k^3}}\tanh\left(\frac{1}{2}\sqrt{-\frac{c}{\beta k^3}}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{24}$$

$$u_{12}(x,t) = k\left(\sqrt{-\frac{c}{\beta k^3}}\coth\left(\frac{1}{2}\sqrt{-\frac{c}{\beta k^3}}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{25}$$

which respectively represent topological soliton solution and singular soliton solution to the equation.

When $\dfrac{c}{\beta k^3} > 0$

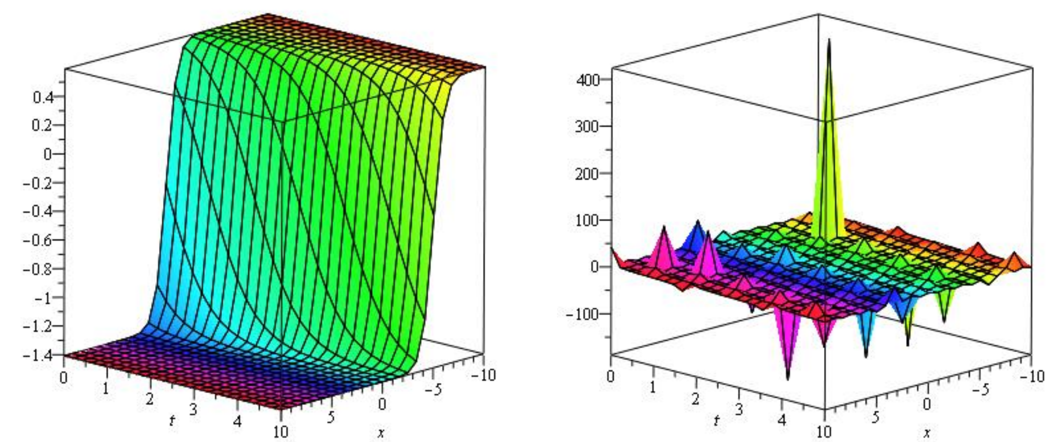
$$u_{13}(x,t) = -k\left(\sqrt{\frac{c}{\beta k^3}}\tan\left(\frac{1}{2}\sqrt{\frac{c}{\beta k^3}}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{26}$$

$$u_{14}(x,t) = k\left(\sqrt{\frac{c}{\beta k^3}}\cot\left(\frac{1}{2}\sqrt{\frac{c}{\beta k^3}}\left(kx + c\frac{t^\alpha}{\alpha}\right)\right)\right), \tag{27}$$

which are singular periodic solutions to the equations.

When $c = 0$.

$$u_{15}(x,t) = -\frac{2\alpha}{x}. \tag{28}$$



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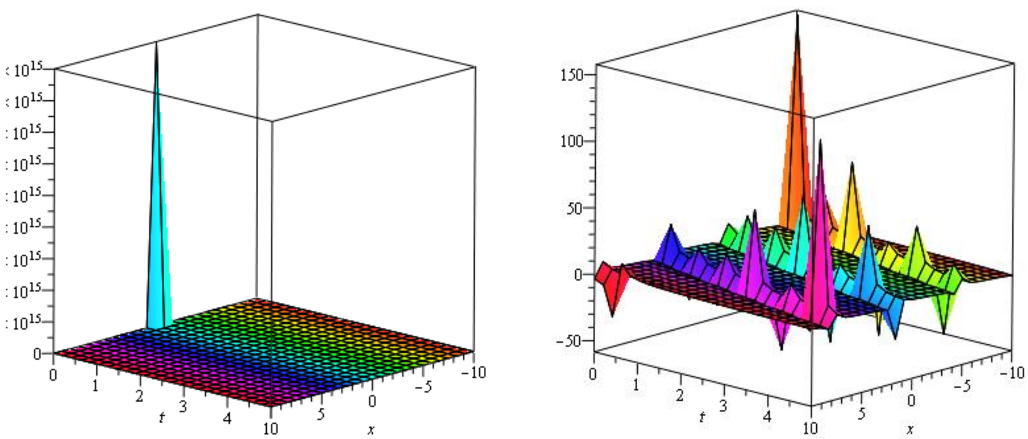

c d

Figure 1. The graphs of exact travelling wave solutions of Equation (7) **(a):** $u_1(x,t)$ with $b=-1, k=1, c=1, \beta=2$ and order $\alpha=0.98$ **(b):** $u_6(x,t)$ with $b=3, k=1, c=1, \beta=1.5$ and order $\alpha=0.95$ **(c):** $u_{12}(x,t)$ with order $\alpha=0.95$ and $k=-1, c=-2, \beta=-2.$ **(d):** $u_{13}(x,t)$ with order $\alpha=0.9$ and $k=2, c=-2, \beta=-1$

4. Conclusion

In this paper, based on a conformable fractional derivative and direct algebraic method, we obtained many new types of the exact solutions of the nonlinear time conformable fractional Sharma-Tasso-Olver equation. The results show that direct algebraic method is accurate and effective. These solutions may be useful for describing certain nonlinear physical phenomena. Maple has been used for computations and programming in this paper.

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