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Hematocrit Level on Blood flow through a Stenosed Artery with Permeable Wall: A Theoretical Study

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Abstract

The paper deals with the hematocrit level on resistance of flow, wall shear stress in a stenosed artery of permeable wall. In the paper we have developed and solved some theoretical formulas based on stenosis and hematocrit effects. The results highlight that the resistance of flow increases for increasing of stenosis height where the hematocrit level (35%-45%) has significant effects. Moreover, the effects of slip parameter and Darcy number due to permeability of the wall on resistance of flow have been investigated. The effects of hematocrit level, slip parameter and Darcy number have been focused on wall shear stress of the permeable inner surface of the artery which has a fashion of downwardly concave for increasing of the *z*-axis of the artery.

Keywords: Hematocrit; Resistance; Stenosis; Permeable wall; Slip parameter; Darcy number; Wall shear stress

MSC (2010) No.: 76Z05, 92C35

1. Introduction

Blood flow in arteries is an important issue in physiological conditions and stenosis which causes cardiovascular diseases. Arteriosclerosis is a common disease due to narrowing of the arteries lumen – a stenosis. Fat and cholesterol can collect in the arterial wall and form plaque which makes it difficult for blood to flow through arteries.

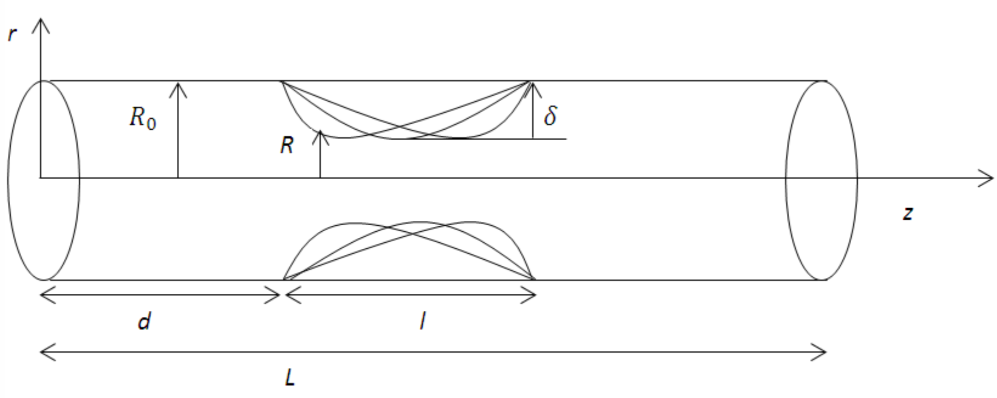

Figure 1. Geometry of the arterial segment with stenosis

The endothelium is semipermeable and it forms the inner cellular lining of blood vessels and plays an important role on permeability of the artery (Aird, 2007).
Some researchers (e.g. Mofrad et al. 2005; Nerem, 1992; Byoung Kwon et al. 2001) have studied this issue and is in the literature in various contexts. The wall shear stress is an important index of the blood flow through the arteries and plays an important role in remodeling the arterial wall (Mofrad et al. 2005). Nerem (1992) and Lee et al. (2001) highlighted that low wall shear stress can be regarded as risk factor in the development of atherosclerotic plaque. Pralhadet et al. (2004) compared the blood flow between stenosed arteries and normal arteries and found the resistance to flow was much higher in the case of stenosed arteries. Srivastav (2014) and Mishra et al. (2011) investigated the Newtonian blood flow through a stenosed artery with permeable wall and obtained resistance and wall shear stress graphically. They have shown that the resistance was increased for increasing the stenosis height and wall shear stress was downwardly concave. Misra et al. (2007) also observed that the wall shear stress increases for increasing height of stenosis when blood flowed through tapered artery with the permeable wall. He assumed blood as Newtonian fluid.

Taylor (1959) found that at low shear (less than $100s^{-1}$) rate blood behaves as a non-Newtonian pulsatile flow and at high shear rate ($1000s^{-1}$) blood exhibits Newtonian characteristic in large arteries like aorta. Low shear rate is observed in stenosis and blood flow through stenosed artery to behave like non-Newtonian characteristics (Leondes, 2000). Ellahi et al. (2014) have studied a mathematical model of non-Newtonian fluid to investigate the effect of composite stenosis through the permeable artery analytically. They observed that the flow impedance (resistance) increases by increasing the stenosis height and decreases by increasing the slip parameter and length.

Hematocrit measures the percentage of whole blood volume that is made up of red blood cells and affects blood viscosity and therefore resistance to flow. Srivastava (2010) investigated the resistance and wall shear stress with the stenosis height and hematocrit in the catheterized stenosed artery and concluded that the resistance is increased with stenosis height and hematocrit.

From the above discussion it is clear that the hematocrit level is likely to resist blood flow, but study in this area is not so significant. So the objective of this study is to calculate the resistance of flow and wall shear stress in presence of hematocrit level when the blood flow is considered as power law fluid using Walburn-Schnek model (1976).

2. Theoretical studies

One-dimensional model provides a simplified description of the flow in arteries. We consider the axisymmetric laminar flow in an artery for a circular tube since normal arterial flow is laminar (David N. Ku, 1997). To explain the physical problem, the governing equation is used for the blood flow in one dimensional form which was developed by Young (1968)

$$\frac{dp}{dz} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) = 0, \tag{1}$$

where p, μ, u represent the pressure, viscosity and axial velocity, respectively.

The non-Newtonian model of blood based on hematocrit level and shear rate is given by Walburn-Schneck (1976) as follows:

$$\mu = a_1 exp\left(a_2 H + \frac{a_3}{H^2}\right)\dot{\gamma}^{(-a_4 H)}, \tag{2}$$

where $\dot{\gamma}$ and H are the shear strain rate and hematocrit level on the blood, respectively and a_1, a_2, a_3 and a_4 are constant corresponding to 0.000797, 0.0608, 377.7515 and 0.00499 for whole blood analysis (Walburn-Schneck, 1976).

The geometry of an axially symmetric and radially non-symmetric stenosis and shape parameter $s \geq 2$ is the described as following:

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta}{R_0}\frac{s^{(s-1)}}{l^s(s-1)}\left[l^{s-1}(z-d) - (z-d)^s\right], & d \leq z \leq d+l, \\ 1, & \text{otherwise}, \end{cases} \tag{3}$$

where R_0, l, δ, d and s are the radius of the artery segment, length of the stenosis, maximum height of the stenosis, the location of the stenosis, respectively.

Boundary Conditions:

Formulation for flow through artery of permeable wall in the presence of slip velocity is given by Bavers *et .al*. (1967) as

$$\frac{\partial u}{\partial r} = 0 \quad at\ r = 0, \tag{4}$$

$$u = v_c \quad at\ r = R(z), \tag{5}$$

$$\frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{k}}\left(v_c - v_f\right) \quad at\ r = R(z), \tag{6}$$

where $v_f = -\frac{\alpha}{\mu}\frac{dp}{dz}$ is the velocity in the permeable boundary and v_c is the slip velocity. α and k are the dimensionless quantity depending on the slip parameter and Darcy number, respectively.

3. Solution

The constitutive equation of blood described by the Equation (2) can be interpreted in the artery using one dimensional flow model through the tube described by the Equation (1). The velocity of blood can be expressed of the form Equation (7) using the boundary conditions (5) into Equation (2) and is given by

$$u = \left\{a_2 e^{\frac{(a_2 H^3 + a_3)}{H^2}}\right\}^{\frac{1}{(a_4 H)}} \mu^{-\frac{1}{a_4 H}}(r - R) + v_c \, . \tag{7}$$

After integrating Equation (1) and using boundary condition (4), we have

$$\mu \frac{\partial u}{\partial r} = \frac{\partial p}{\partial z}\frac{r}{2} \quad \text{for } r = R(z). \tag{8}$$

The slip velocity v_c can be determined from Equation (8) using boundary condition (6) and it can be expressed as

$$v_c = \frac{\sqrt{k}}{2\mu\alpha}\frac{dp}{dz}\left\{R_0\left(\frac{R}{R_0}\right) - 2\alpha\sqrt{k}\right\}. \tag{9}$$

Furthermore, the volumetric flow rate Q is defined as

$$Q = 2\pi \int_0^R u r dr, \tag{10}$$

which is modified by Equations (7) and (9). Finally, the modified volumetric flow rate becomes

$$Q = -\frac{1}{6}TR^3 + \frac{\sqrt{k}}{2\mu\alpha}\frac{dp}{dz}\left\{R_0\left(\frac{R}{R_0}\right) - 2\alpha\sqrt{k}\right\}R, \tag{11}$$

where

$$T = \left\{a_2 e^{(a_2 H^3 + a_3)/H^2}\right\}^{1/(a_4 H)} \mu^{-1/(a_4 H)}.$$

We have, after simplifying the Equation (11),

$$\frac{dp}{dz} = \frac{Q + \frac{1}{6}TR_0^3\left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2\left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k}R_0\left(\frac{R}{R_0}\right)\right]}. \tag{12}$$

Equation (12) represents the pressure gradient of blood due to stenosis in the artery. To obtain the resistance of the flow we integrate Equation (12) over the artery described in Figure 1; that is, integrating from $p = p_1$ at $z = 0$ and $p = p_2$ at $z = L$, then we get

$$p_2 - p_1 = \int_0^L \frac{Q + \frac{1}{6} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz. \tag{13}$$

The resistance to flow λ defined by Somchai Sriyab (2014) can be written as

$$\lambda = \frac{p_2 - p_1}{Q}.$$

Thus,

$$\lambda = \int_0^L \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz,$$

and

$$\lambda = \int_0^d \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz + \int_d^{d+l} \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz + \int_{d+l}^{L} \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz\ . \tag{14}$$

But in the absence of stenosis (R/R_0) is 1 from Equation (3) and stenosis is present in the region $d \le z \le d + l$. Therefore, Equation (14) becomes

$$\lambda = \frac{1 + \frac{1}{6Q} T R_0^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 - 2\alpha\sqrt{k} R_0\right]}(L - l) + \int_d^{d+l} \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz\ . \tag{15}$$

If there is no stenosis, the resistance to flow λ_N is given by

$$\lambda_N = \frac{1 + \frac{1}{6Q} T R_0^3}{\frac{\sqrt{k}}{2\mu\alpha}\left[R_0^2 - 2\alpha\sqrt{k} R_0\right]}(L). \tag{16}$$

In dimensionless form, the flow resistance to flow may be expressed as

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{l}{L} + \frac{\left[R_0^2 - 2\alpha\sqrt{k} R_0\right]}{L\left(1 + \frac{1}{6Q} T R_0^3\right)} \int_d^{d+l} \frac{1 + \frac{1}{6Q} T R_0^3 \left(\frac{R}{R_0}\right)^3}{\left[R_0^2 \left(\frac{R}{R_0}\right)^2 - 2\alpha\sqrt{k} R_0 \left(\frac{R}{R_0}\right)\right]} dz. \tag{17}$$

The wall shear stress is obtained as

$$\tau_R = -\frac{R}{2}\frac{dp}{dz} = -\frac{Q + \frac{1}{6}TR_0^3\left(\frac{R}{R_0}\right)^3}{\frac{\sqrt{k}}{\mu\alpha}\left[R_0\left(\frac{R}{R_0}\right) - 2\alpha\sqrt{k}\right]}. \tag{18}$$

If there is no stenosis in the artery ($R = R_0$), then Equation (18) can be expressed as

$$\tau_R = -\frac{Q + \frac{1}{6}TR_0^3}{\frac{\sqrt{k}}{\mu\alpha}\left[R_0 - 2\alpha\sqrt{k}\right]}\,. \tag{19}$$

In dimensionless form, the wall shear stress can be expressed as

$$\bar{\tau} = \frac{\tau_R}{\tau_N}\,. \tag{20}$$

The resistance to flow and wall shear stress in the stenosed artery for different parameters such as hematocrit, viscosity of blood, slip parameter etc. can be determined analytically through Equations (15-17).

4. Results and discussion

In the present study, the effect of several blood flow parameters has been calculated. The values of these parameters have been taken from Srivastav (2014); Mishra et al. (2011) etc. The values of the parameters are considered with its range as H = 35%-45%, Q = 0.1, L = 1, l =. 5, d = .25, R_0 = .5, δ/R_0 = 0.1-0.5, $\alpha = 0.1 - 0.2, \sqrt{k} = 0.1 - 0.2$. Blood viscosity μ is defined as the inherent resistance of blood to flow. Normal adult blood viscosity is 0.3 and reported in units of millipoise.

Resistance to blood flow through the vessels is determined by the size of the vessels. This resistance is simply due to the width of the vessels – it is hard to push a lot of blood through thin vessels. Thus, there is an inverse relationship between blood vessel resistance and the blood flow rate described by Poiseuille's equation. That is, the higher the resistance becomes the slower the flow rate. Resistance to flow depends on the rheological behavior of blood flowing through microvascular network. Non-Newtonian model Equation (2) of blood to represent the resistance of blood flow in the presence of stenosis in the artery with permeable wall is given by Equation (15) analytically in account of hematocrit level. Graphical presentations of resistance determined by Equation (15) are plotted in Figures 2 and 4 for different values of hematocrit level, slip parameter and Darcy number respectively. In Figure 2, it is shown that that resistance of flow is increased for increasing stenosis height and hematocrit level H = 35%-45% produces no significant changes. In Figures 3 and 4 resistance is increased for different values of slip parameter ($\alpha = 0.1 - 0.2$) and Darcy number ($\sqrt{k} = 0.1 - .02$) for increasing stenosis height. In dimensionless form, resistance of flow with stenosis height is represented in Figures 5 and 7 which have the same similarity as described in Figures 2 and 4, respectively for different values of hematocrit level, slip parameter and Darcy number.

Wall shear stress is the tangential force at the endothelial surface (arterial wall) produced by blood moving through artery. The magnitude of wall shear stress is directly proportional to blood flow and blood viscosity and inversely proportional to the cube of the radius Masuda et al. (1999). Thus, a small change in the radius of a vessel will have a large effect on wall shear stress.

Wall shear stress plays a vital role in the generation, progression, and destabilization of atherosclerotic plaques. The effects of stenosis height, hematocrit level, slip parameter and Darcy number on wall shear stress of the permeable stenosed arterial wall along the horizontal axis z is described by Equations (18) and (20). The results have been presented in Figures 8 to 11 against the horizontal axis z. It is observed that the hematocrit level, slip parameter and Darcy number have a small effect on wall shear stress in artery which is downwardly concave. Furthermore, it is also observed that wall shear stress significantly depends on stenosis height along the increasing of axis of the stenosis and it is increased with the increasing of stenosis height.

Finally, the present study is compared with the results of Ellahi et al. (2014) in Figure 12 and it is observed from Figure 12 that for the value of δ/R_0 from [0 - 0.1] the resistance is at most the same with that of Ellahi et al. (2014). Then it (the resistance) had some difference gradually and at the final value of δ/R_0 it varies significantly. The reason may be due to different stenosis, different values of the parameters or different model of non-Newtonian fluid. Walburn-Schnek model is considered in the comparison for power law fluid as non-Newtonian fluid and slip parameter is considered as $\alpha = 0.24$ and Ellahi et al. (2014) was taken N-S equation for non-Newtonian micropolar fluid.

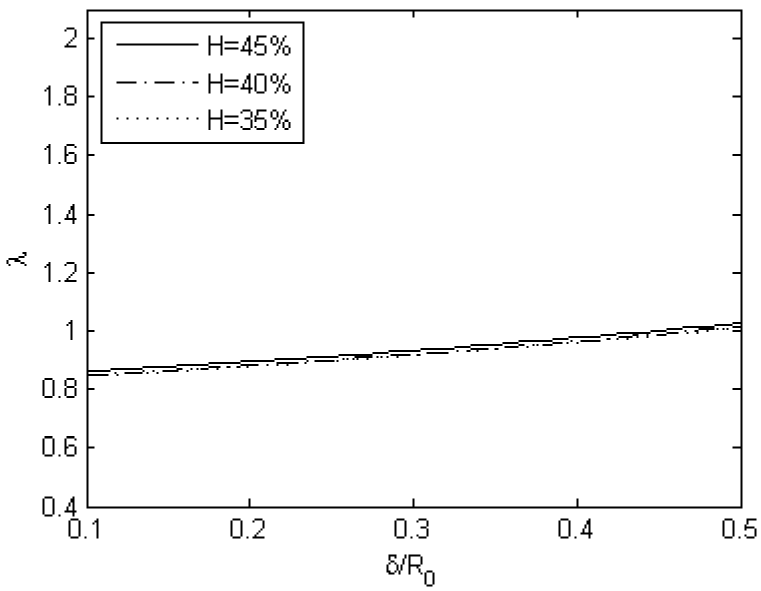

Figure 2. Variation of resistance of flow along the stenosis height for different value of hematocrit level H

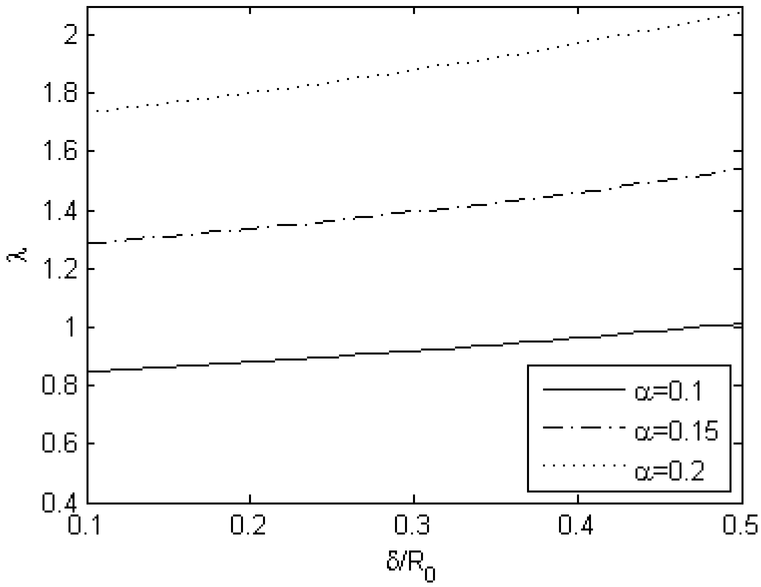

Figure 3. Variation of resistance of flow along the stenosis height for different value of slip parameter α

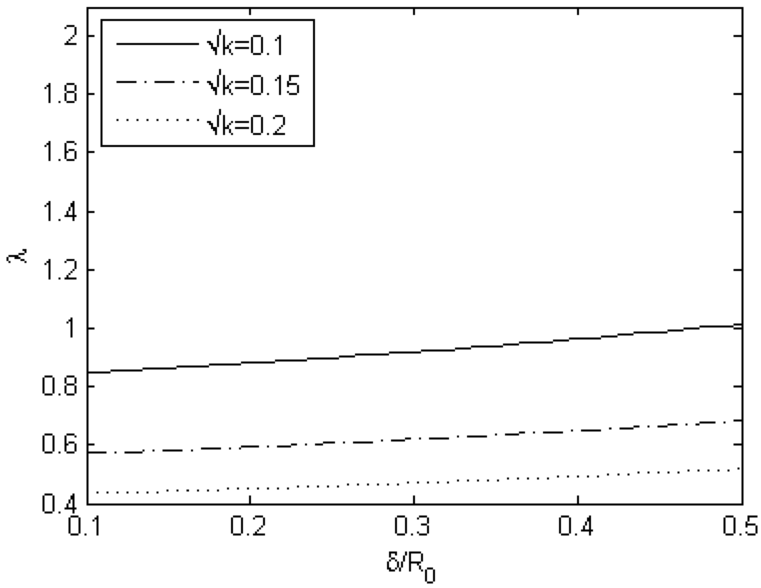

Figure 4. Variation of resistance of flow along the stenosis height for different value of Darcy number k

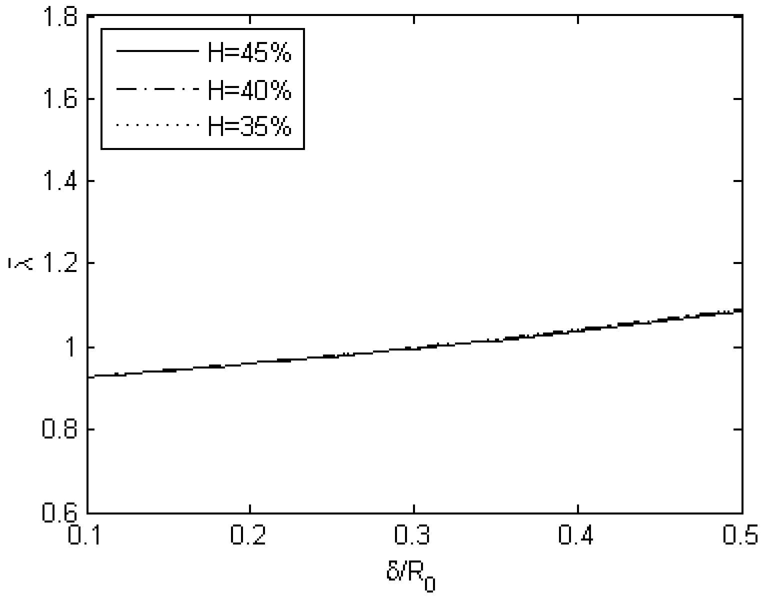

Figure 5. Variation of resistance of flow in dimensionless form along the stenosis height for different value of hematocrit level H

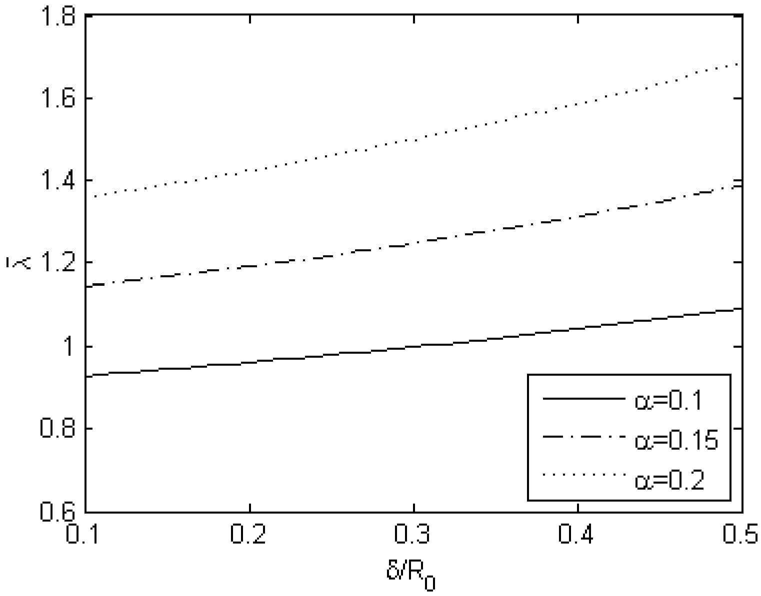

Figure 6. Variation of resistance of flow in dimensionless form along the stenosis height for different value of slip parameter α

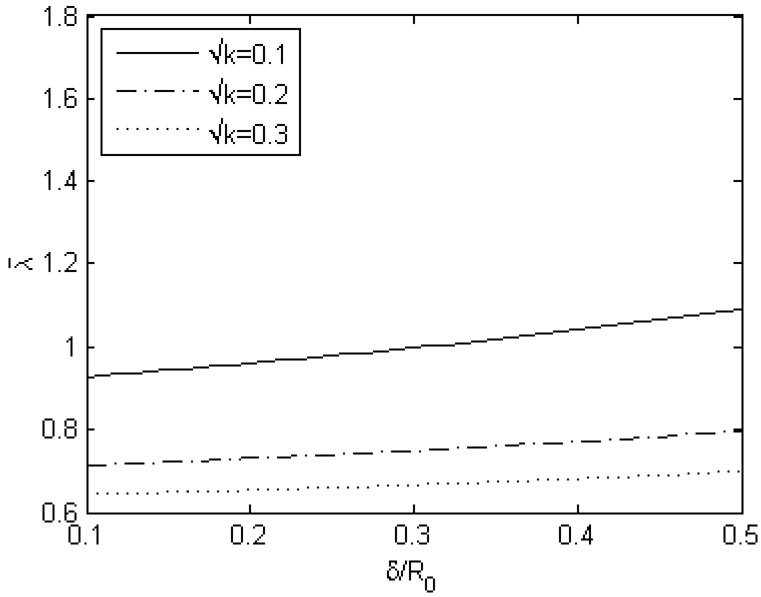

Figure 7. Variation of resistance of flow in dimensionless form along the stenosis height for different value of Darcy number *k*

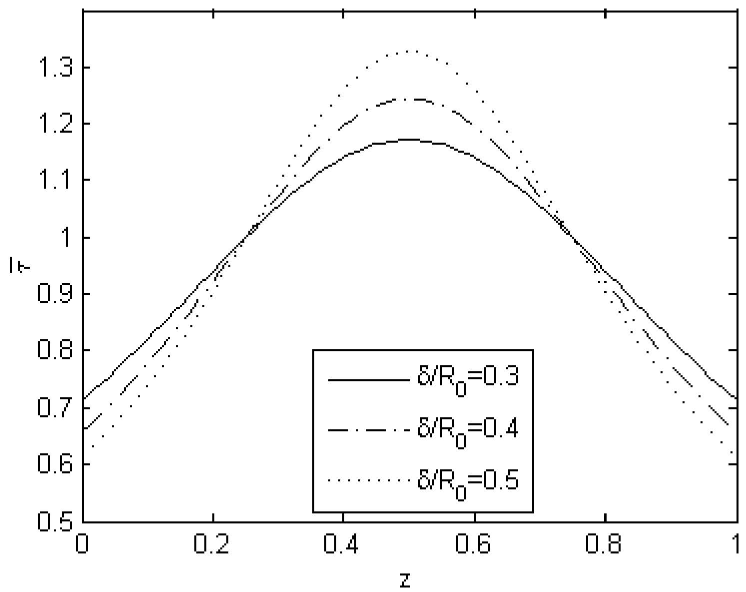

Figure 8. Variation of wall shear stress in dimensionless form along z axis for different value of stenosis height

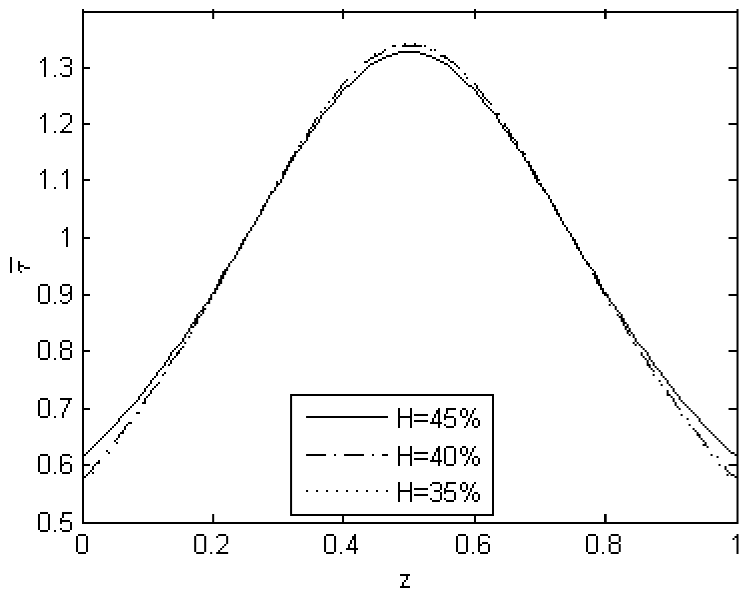

Figure 9. Variation of wall shear stress in dimensionless form along z axis for different value of hematocrit level H

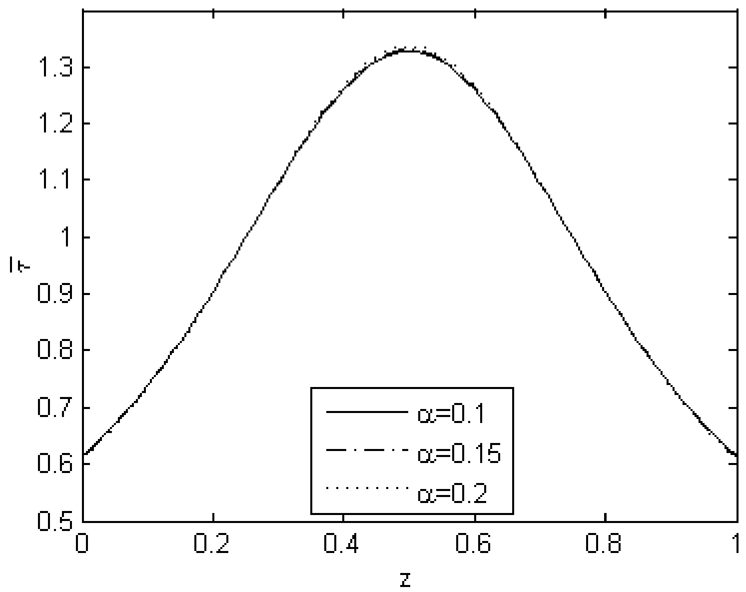

Figure 10. Variation of wall shear stress in dimensionless form along z axis for different value of slip parameter α

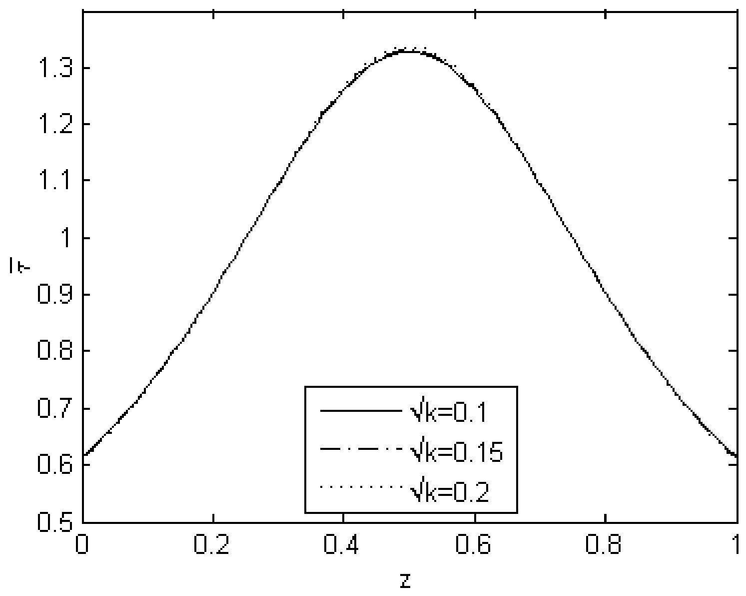

Figure 11. Variation of wall shear stress in dimensionless form along z axis for different value of Darcy number *k*

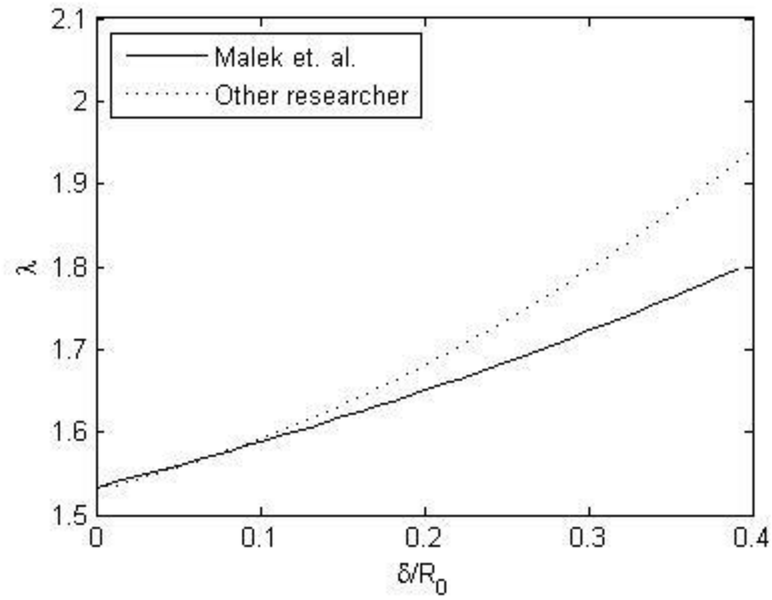

Figure 12. Comparison of resistance of our result with other researcher

5. Conclusion

The effect of hematocrit level on resistance of flow through a stenosed artery and wall shear stress of permeable wall was determined in the present study. It was found that resistance of flow increases with increasing of stenosis height and it has small effects of the hematocrit level (35%-45%). It is also observed that the resistances of flow increases with increasing slip parameter and decreases with the increasing of Darcy number due to permeable wall.

Moreover, wall shear stresses were increased downwardly concave in a fashion with the increase of stenosis height and hematocrit level whereas, slip parameter and Darcy number has no change.

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