Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 12, Issue 1 (June 2017), pp. 201 - 216

EFFECT OF DAMPING AND THERMAL GRADIENT ON VIBRATIONS OF ORTHOTROPIC RECTANGULAR PLATE OF VARIABLE THICKNESS

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Received June 2, 2016; Accepted January 21, 2017

Abstract

In this present paper, damped vibrations of an orthotropic rectangular plate resting on elastic foundation with thermal gradient is modeled, considering variable thickness of plate. Following Le`vy approach, the governed equation of motion is solved numerically using quintic spline technique with clamped and simply supported edges. The effect of damping parameter and thermal gradient together with taper constant, density parameter and elastic foundation parameter on the natural frequencies of vibration for the first three modes of vibration are depicted through Tables and Figures, and mode shapes have been computed for fixed value of plate parameter. It has been observed that the rate of decrease of frequency parameter with damping parameter D_k for C-SS plate is higher than that for C-C plate keeping all other parameter fixed.

Keywords: Damping; Thermal gradient; Taperness; Elastic foundation; Orthotropy

MSC 2010 No.: 41A15, 81V99

1. Introduction

Structural damping is essential for design engineering as damping materials enhance the performance of system by increasing structural stiffness and thermal stability and the effect of temperature on mechanics of solid bodies has acquired great interest because of rapid development in space technology, high speed atmospheric flights and nuclear energy applications. Numerous studies on free vibration for isotropic/orthotropic plates of uniform/non-uniform thickness with or without temperature effect have been reported. Among these, Laura and Gutierrez (1980) discussed vibration analysis of a rectangular plate subjected to a thermal gradient. Tomar and Gupta (1983, 1985) analyzed the effect of thermal gradient on frequencies of an orthotropic rectangular plate with variable thickness. In this series, Gupta et al. (2007) observed the thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Effect of non-homogeneity on thermally induced vibration of orthotropic visco-elastic rectangular plate of linearly varying thickness studied by Gupta and Singhal (2010). Gupta et al. (2013) had evaluated the effect of thermal gradient on the vibration of parallelogram plate with linearly varying thickness in both directions and thermal effect in linear form only. Effect of thermal gradient on vibration of non-homogeneous orthotropic trapezoidal plate of varying thickness has been studied by Gupta and Sharma (2013). Khanna and Singhal (2014) analyzed the thermal effect on vibration of tapered rectangular plate. Gupta and Jain (2014) analyzed exponential temperature effect on frequencies of a rectangular plate of non-linear varying thickness using quintic spline technique.

Plates resting on elastic foundation have applications in pressure vessels technology such as petrochemical, marine and aerospace industry, building activities in cold regions and aircraft landing in arctic operations discussed by Mcfadden and Bennett (1991), and Civalek and Acar (2007). In a series of papers, Lal et al., (2001) have studied the transverse vibrations of a rectangular plate of exponentially varying thickness resting on an elastic foundation. Transverse vibration of non-homogeneous orthotropic rectangular plate with variable thickness was discussed by Lal and Dhanpati (2007). In many applications of vibration and wave theory the magnitudes of the damping forces are small in comparison with the elastic and inertia forces but these small forces may have very great influence under some special situations. Recently, Robin and Rana (2013) analyzed the damped vibrations of isotropic/orthotropic rectangular plate with varying thickness resting on elastic foundation. The study of damped vibration of infinite plate with variable thickness resting on elastic foundation was carried out by Rana and Robin (2015).

In reality all the vibrations are damped vibration and foundations of underlying vibrational problems are elastic in nature, and thermal effects on the damped vibrations of orthotropic rectangular plates of variable thickness with elastic foundations is not considered by researchers but it plays a key role in vibrational problems. Keeping this in view thermally induced damped vibrations of orthotropic rectangular plates with exponentially thickness variation resting on Winkler foundation is presented here using quintic spline method on the basis of classical plate theory. Effect of damping parameter and thermal gradient together with thickness variation and foundation parameter on the frequencies has been illustrated for the first three modes of vibration for two different combinations of clamped, and simply supported correct to four decimal places.

2. Mathematical Formulation

Consider a non- homogeneous orthotropic rectangular plate of length 'a', breath 'b', thickness '$h(x,y)$' and density ' ρ', with resting on a winkler- type elastic foundation 'k_f' occupying the domain $0 \le x \le a, 0 \le y \le b$ in the xy plane. The x-and y axes are taken along the principal directions and z –axes is perpendicular to the xy plane. The middle surface being $z = 0$ and the origin is at one of the corners of the plate. The differential equation which governs the damped transverse vibration of such plates is given by

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 w}{\partial y \partial x^2} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3}$$
$$+\frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$$
$$+4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 w}{\partial y \partial x} + \rho h \frac{\partial^2 w}{\partial t^2} + K\frac{\partial w}{\partial t} + K_f w = 0, \tag{1}$$

where

$$D_x = E_x h^3/(1-\upsilon_x\upsilon_y), D_y = E_y h^3/(1-\upsilon_x\upsilon_y), D_1 = \upsilon_y E_x h^3/12(1-\upsilon_x\upsilon_y), H = D_1 + 2D_{xy},$$
$$D_{xy} = G_{xy}h^3/12, \upsilon_y E_x = \upsilon_x E_y,$$

$w(x,y,t)$ is the transverse deflection, t is the time, and $E_x, E_y, \upsilon_x, \upsilon_y$ and G_{xy} are material constants in proper directions defined by an orthotropic stress-strain law.

Let the two opposite edges $y = 0$ and $y = b$ of the plate be simply supported and thickness $h = h(x,y)$ varies exponentially along the length i.e., in the direction of x-axis. Thus, 'h' is independent of y i.e., $h = h(x)$.

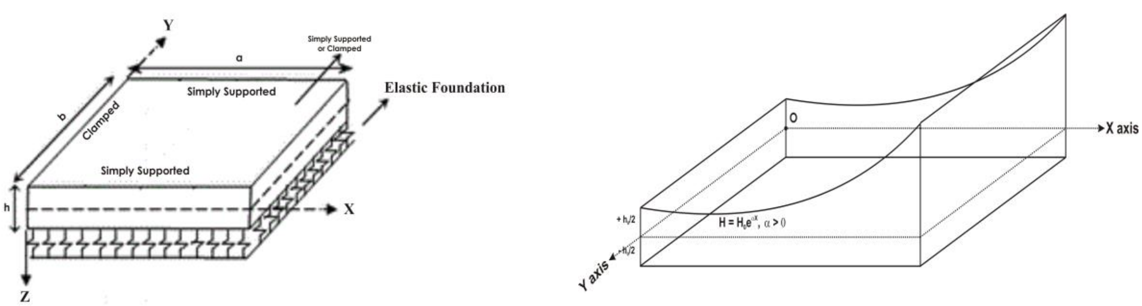


Figure 1. Boundary conditions and vertical cross-section of the plate

For a harmonic solution, the deflection function w satisfying the condition at $y = 0$ and $y = b$, is assumed

$$w(x,y,t) = W(x)\sin\frac{m\pi y}{b}e^{-\gamma t}\cos pt, \tag{2}$$

where 'p' is the circular frequency of vibration and 'm' is a positive integer. Furthermore, for elastically non-homogeneous material, it is assumed that Young's moduli E_x, E_y and density ρ are functions of space variable x only, and shear modulus is

$$G_{xy} = \sqrt{E_x E_y}/2(1+\sqrt{\upsilon_x\upsilon_y}).$$

Thus, Equation (1) becomes

$$\left[\begin{array}{l} E_x h^3 W^{(iv)} + \left\{6E_x h^2 h' + 2h^3 E_x'\right\} W''' + \left\{\begin{array}{l} 6E_x hh'^2 + 3E_x h^2 h'' + 6h^2 h' E_x' + h^3 E_x'' \\ -2\dfrac{m^2\pi^2}{b^2}(\upsilon_y E_x h^3 \\ +2G_{xy}(1+\upsilon_x\upsilon_y)h^3) \end{array}\right\} W'' \\ -2\dfrac{m^2\pi^2}{b^2}\left\{\upsilon_y\left(3E_x h^2 h' + h^3 E_x'\right) + 2(1-\upsilon_x\upsilon_y)(3h^2 G_{xy} h' + h^3 G_{xy}')\right\} W' \end{array}\right] \cos pt$$

$$+\left[\frac{m^4\pi^4}{b^4} E_y h^3 - \upsilon_y \frac{m^2\pi^2}{b^2}\left\{3E_x h^2 h'' + 6E_x hh'^2 + 6h^2 h' E_x' + h^3 E_x''\right\}\right] W \cos pt \tag{3}$$

$$+12(1-\upsilon_x\upsilon_y)K_f W \cos pt + \left[\begin{array}{l} 12(1-\upsilon_x\upsilon_y)\rho h \left\{\begin{array}{l} (\gamma^2 - p^2)\cos pt \\ +2p\gamma \sin pt \end{array}\right\} \\ +12(1-\upsilon_x\upsilon_y)k\left\{-p\sin pt - \gamma\cos pt\right\} \end{array}\right] W = 0.$$

Introducing the non-dimensional variables

$$H = \frac{h}{a}; X = \frac{x}{a}; \overline{E} = \frac{E}{a}; \overline{W} = \frac{W}{a}; \overline{\rho} = \frac{\rho}{a}; \lambda^2 = m^2\pi^2\left(\frac{a}{b}\right)^2,$$

and equating the coefficient of sin*(pt)* and cos*(pt)* independently to zero, Equation (3) reduces to

$$\overline{E}_x H^3 \overline{W}^{(iv)} + 2\left\{3\overline{E}_x H^2 H' + H^3 \overline{E}_x'\right\} \overline{W}''' + \left\{\begin{array}{l} 3\overline{E}_x H^2 H'' + 6\overline{E}_x HH'^2 + 6H^2 H' \overline{E}_x + H^3 \overline{E}_x'' \\ -2\lambda^2(\upsilon_y \overline{E}_x H^3 + 2G_{xy}(1+\upsilon_x\upsilon_y)H^3) \end{array}\right\} \overline{W}''$$

$$-2\lambda^2\left\{\upsilon_y\left(3\overline{E}_x H^2 H' + H^3 \overline{E}_x'\right) + 2(1-\upsilon_x\upsilon_y)(3H^2 G_{xy} H' + H^3 G_{xy}')\right\} \overline{W}'$$

$$+\left[\begin{array}{l} \lambda^4 \overline{E}_y H^3 - \lambda^2 \upsilon_y \left\{3\overline{E}_x H^2 H'' + 6\overline{E}_x HH'^2 + 6H^2 H' \overline{E}_x' + H^3 \overline{E}_x''\right\} + \\ 12(1-\upsilon_x\upsilon_y)\left\{\dfrac{-k^2}{4H\overline{\rho}} - a^2 p^2 \overline{\rho} H + aK_f\right\} \end{array}\right] \overline{W} = 0. \tag{4}$$

The exponential temperature variation for the plate is assumed in the following form

$$T = T_0\left(\frac{e-e^x}{e-1}\right), \tag{5}$$

where T is the temperature excess above the reference temperature at any point x and T_0 is the temperature excess above the reference temperature at the end $x = 0$. Furthermore, most of the engineering materials are found to have a linear relationship between the modulus of elasticity and temperature as discussed by Nowacki (1962).
Therefore, we get

$$\overline{E} = \overline{E}_0(1-\xi T), \tag{6}$$

where $\overline{E}_0$ is the modulus of elasticity of the material at the reference temperature and ξ is a constant. If the temperature at $x = 1$ is assumed as the reference temperature, the modulus variable becomes

$$\overline{E}(x) = \overline{E}_0\left\{1-\eta\left(\frac{e-e^x}{e-1}\right)\right\}, \tag{7}$$

where

$\eta = \xi T_0, (0 \le \eta \le 1)$, is thermal gradient.

Substituting

$$H = H_0 e^{\alpha X}; \overline{E}_x = E_1\left\{1-\eta\left(\frac{e-e^x}{e-1}\right)\right\}; \overline{E}_y = E_2\left\{1-\eta\left(\frac{e-e^x}{e-1}\right)\right\}; \overline{\rho} = \overline{\rho_0} e^{\beta X},$$

where

$$H_0 = (H)_{X=0}, \overline{E}_1 = \left(\overline{E}_x\right)_{X=0}, \overline{E}_2 = \left(\overline{E}_y\right)_{y=0}, \overline{\rho}_0 = \left(\overline{\rho}\right)_{X=0},$$

and 'α' is the taper constant due to exponentially varying thickness of plate, and equating the coefficients, the following equation is formed

$$A_0\frac{d^4\overline{W}}{dX^4} + A_1\frac{d^3\overline{W}}{dX^3} + A_2\frac{d^2\overline{W}}{dX^2} + A_3\frac{d\overline{W}}{dX} + A_4\overline{W} = 0, \tag{8}$$

where

$$A_0 = 1-\eta\left(\frac{e-e^X}{e-1}\right),$$

$$A_1 = 2\left[\frac{\eta e^X}{e-1}+3\alpha\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\}\right],$$

$$A_2 = 9\alpha^2\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\}+\left(6\alpha+1\right)\frac{\eta e^X}{e-1}-2\lambda^2\left(\upsilon_y+\sqrt{E_2/E_1}(1-\sqrt{\upsilon_x\upsilon_y})\right)\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\},$$

$$A_3 = -2\lambda^2\left[\left(\upsilon_y+\sqrt{E_2/E_1}(1-\sqrt{\upsilon_x\upsilon_y})\right)\left(\frac{\eta e^X}{e-1}+3\alpha\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\}\right)\right],$$

$$A_4 = \lambda^4\,{E_2}/{E_1}\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\}-\lambda^2\upsilon_y\left(9\alpha^2\left\{1-\eta\left(\frac{e-e^X}{e-1}\right)\right\}+(6\alpha+1)\frac{\eta e^X}{e-1}\right)$$

$$-\left\{3D_K I^{*2}\,e^{(-4\alpha-\beta)X}+\Omega^2 I^* e^{(-2\alpha+\beta)X}-12E_f e^{-3\alpha X}C^*\right\},$$

$$D_K = \frac{(1-\upsilon_x\upsilon_y)K^2}{E_1\overline{\rho_0}}, I^* = \frac{1}{H_0^{\,2}}, C^* = \frac{1}{H_0^{\,3}}, E_f = \frac{a(1-\upsilon_x\upsilon_y)K_f}{E_1}, \Omega^2 = \frac{12(1-\upsilon_x\upsilon_y)a^2\overline{\rho_0}p^2}{E_1},$$

and Ω, D_K, E_f are frequency parameter, damping parameter and elastic foundation parameter, respectively. $\overline{\rho_0}$ is the density of the plate and E_1, E_2 the Young's moduli in proper directions at $x = 0$ .The solution of Equation (8) together with boundary conditions at the edge $x = 0$ and $x = 1$ constitutes a two-point boundary value problem. As the PDE has several plate parameters, it becomes quite difficult to find its exact solution. Keeping this in mind which is complex for the purpose of computation, the quintic spline interpolation technique is used.

According to the spline technique, suppose *W(x)* be a function with continuous derivatives in [0, 1] and interval [0, 1] be divided into '*n*' subintervals by means of points X_i such that

$$0 = X_0 < X_1 < X_2 < \ldots < X_n = 1,$$

where

$$\Delta X = 1/n, X_i = i\Delta X\,(i = 0,1,2,\ldots,n).$$

Let the approximating function $\overline{W}(X)$ for the *W(x)* be a quintic spline with the following properties:

(i) $\overline{W}(X)$ is a quintic polynomial in each interval (X_k, X_{k+1}).
(ii) $\overline{W}(X) = W(X_k), k = 0,1,2,\ldots,n.$

(iii) $\frac{d\overline{W}}{dX}, \frac{d^2\overline{W}}{dX^2}, \frac{d^3\overline{W}}{dX^3}$ *and* $\frac{d^4\overline{W}}{dX^4}$ are continuous.

In view of the above axioms, the quintic spline takes the form

$$\overline{W}(X) = a_0 + \sum_{i=0}^{4} a_i (X - X_0)^i + \sum_{j=0}^{n-1} b_j (X - X_j)_+^5 \ , \tag{9}$$

where

$$(X - X_j)_+ = \begin{cases} 0, & if \ X \le X_j \\ (X - X_j), & if \ X > X_j \end{cases},$$

and a_i's, b_j's are constants.

Thus, for the satisfaction at the n^{th} knot, Equation (8) reduces to

$$\begin{aligned}
&A_4 a_0 + [A_4 (X_m - X_0) + A_3] a_1 + [A_4 (X_m - X_0)^2 + 2A_3 (X_m - X_0) + 2A_2] a_2 \\
&+ [A_4 (X_m - X_0)^3 + 3A_3 (X_m - X_0)^2 + 6A_2 (X_r - X_0) + 6A_1] a_3 \\
&+ [A_4 (X_m - X_0)^4 + 4A_3 (X_m - X_0)^3 + 12A_2 (X_m - X_0)^2 + 24A_1 (X_m - X_0) + 24A_0] a_4 \\
&+ \sum_{j=0}^{n-1} b_j [A_4 (X_m - X_j)_*^5 + 5A_3 (X_m - X_j)_*^4 + 20A_2 (X_m - X_j)_*^3 + 60A_1 (X_m - X_j)_*^2 \\
&+ 120A_0 (X_m - X_j)_*] = 0.
\end{aligned} \tag{10}$$

For $m = 0, 1, \cdots, n,$ the system (10) contains (n + 1) homogeneous equation with (n + 5) unknowns, a_i, $i = 0, 1, \cdots, 4,$ and b_j, j = 0, 1, 2, … , (n - 1), can be represented in matrix form as

$$[A][B] = [0], \tag{11}$$

where [A] is a matrix of order (n+1) $\times$ (n+5), while [B] and [0] are column matrices of order (n×5).

3. Boundary Conditions and Frequency Equation

The following two cases of boundary conditions have been considered:

(i) (C-C): clamped at both the edge X = 0 and X = 1.
(ii) (C-SS): clamped at X = 0 and simply supported at X = 1.

The relations that should be satisfied at clamped and simply supported, respectively, are

$$W = \frac{dW}{dX} = 0 \ , \ W = \frac{d^2W}{dX^2} = 0 \, . \tag{12}$$

Applying the boundary conditions C-C to the displacement function by Equation (9) one obtains a set of four homogeneous equations in terms of (n+5) unknown constants which can be written as

$$[B^{cc}][B] = [0], \tag{13}$$

where B^{cc} is a matrix of order 4×(n+5).

Therefore, Equation (11) together with the Equation (13) gives a complete set of (n+5) homogeneous equations having (n+5) unknowns which can be written as

$$\left[\frac{A}{B^{cc}}\right][B] = [0] \, . \tag{14}$$

For a non-trivial solution of Equation (11), the characteristic determinant must vanish, i.e.,

$$\left|\frac{A}{B^{cc}}\right| = 0 \, . \tag{15}$$

Similarly, for (C-SS) plate the frequency determinant can be written as

$$\left|\frac{A}{B^{ss}}\right| = 0 \, , \tag{16}$$

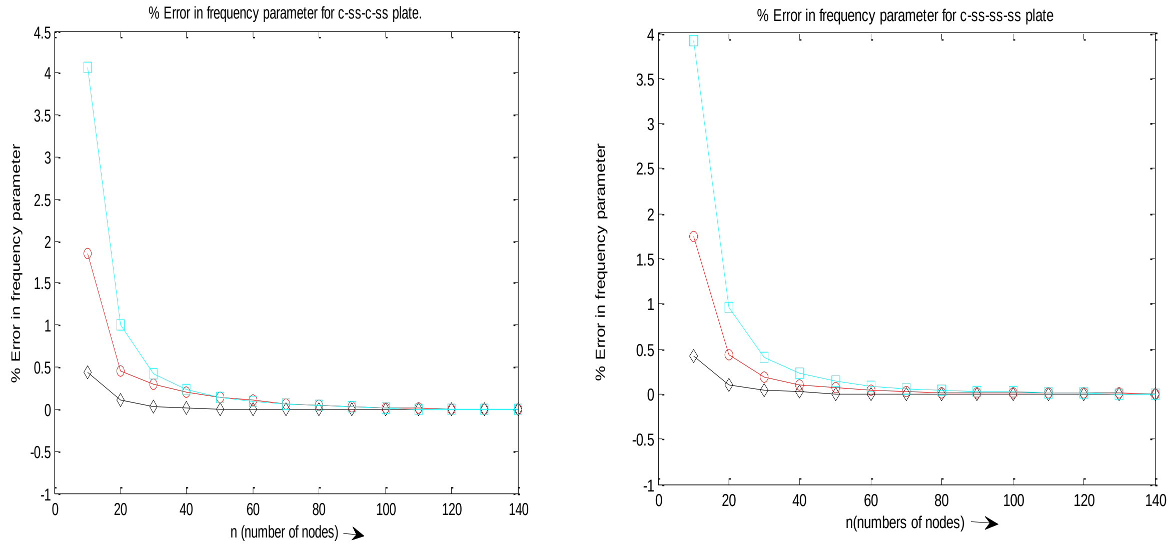
where B^{ss} is a matrix of order 4×(n+5).

4. Numerical Results and Discussions

The frequency Equations (15) and (16) have been solved to get the values of the frequency parameter Ω for various values of plate parameters. By putting m = 1 in frequency equations, the numerical results have been computed for first three modes of vibrations. The elastic constants for the plate material (‘ORTHOI’) are taken as

$$E_1 = 1\times 10^{10}\, Mpa, \ E_2 = 5\times 10^{10}\, Mpa \, , \nu_x = 0.2, \nu_y = 0.1.$$

At the edge X = 0, the thickness h_0 have been considered as 0.1. To choose the appropriate interval ΔX, a computer program was developed for the evaluation of frequency parameter Ω, and run for $n = 10, 20, 30, \cdots, 150,$ and for different sets of the values of parameters for both the boundary conditions. Figure 2 represents the percentage error in the numerical values of frequency parameter Ω up to fourth decimal places with the increase in the number of nodes. In all the computation n = 140 has been fixed to achieve the decimal accuracy.



(a) **(b)**

Figure 2. Percentage error in frequency parameter Ω: (a) for c-c plate (b) c-ss plate for a/b = 0.25, α = 0.04, β = 0.4, E_f = 0.02, D_k = 0.01; ◊, First mode; ○, Second mode; □,Third mode ; Percentage error = $[(\Omega_n - \Omega_{140})/\ \Omega_{140}]$x100; $n = 10, 20, 30, \cdots, 140$

The results are presented in Tables 1-3 and Figures 3-5, for different values of damping parameter D_k = 0.0, 0.001, 0.002, 0.003, 0.004, and 0.005, thermal gradient η = 0.0, 0.2, 0.4, 0.6, foundation parameter E_f = 0.0, 0.2, density parameter $\beta = \pm 0.5$ and taper paremeter $\alpha = \pm 0.5$ for both the boundary conditions C-C and C-SS, by taking the fixed values of aspect ratio a/b=1.0. It is found that the values of frequency parameter Ω for a C-C plate are greater than that for C-SS plate for the same set of values of plate parameters.

Table 1 shows the values of frequency parameter Ω with the increasing value of damping parameter D_k for the fixed value of density parameter β = 0.5 and foundation parameter E_f = 0.02, for first three modes of vibration of C-C and C-SS plates. Figure 3a shows the behavior of frequency parameter Ω. It decreases with the increasing values of damping parameter D_k for two different values of taper parameter α = ±0.5, thermal gradient η = 0.0, 0.5 and density parameter β = 0.5 for both the plates. The rate of decrease of Ω with damping parameter D_k for C-SS is higher than that for C-C plate keeping all other plate parameters fixed. This rate increases with the increase in the value of non taper parameter α and thermal gradient η. A similar inference can be drawn from Figures 3b and 3c, when the plate is vibrating in the second mode as well as in the third mode of vibration except that the rate of decrease of Ω with D_k is lesser as compared to the first mode.

Tables 2 and 3, provide the inference of thermal gradient η on frequency parameter Ω for two values of damping parameter D_k = 0.0, and 0.001, taper parameter α = ±0.5 density parameter β = ±0.5 and foundation parameter E_f = 0.0, 0.02 for the fixed value of aspect ratio a/b = 1.0. It is noticed that the frequency parameter Ω decreases continuously with the increasing value of thermal gradient η for C-C and C-SS plates, whatever be the value of other plate parameters. It is found that the rate of decrease of frequency parameter Ω for C-C plate is higher than C-SS plate for three modes. Figure 4a gives the inference of thermal gradient η on frequency parameter Ω for the first mode of vibration for fixed values of $\beta = 0.5$ and $\alpha = 0.5$. This rate decreases with the increase in the value of foundation parameter E_f, and also increases with the increase in the value of damping parameter D_k and the number of modes, as is clear from Tables 3b and 3c when the plate is vibrating in the second and third mode of vibration.

Table 1. Values of frequency parameter Ω for C-C and C-SS plate for a/b= 1.0 E_f= 0.02, β= 0.5

		α=-0.5,η=0.0		α=-0.5,η=0.5		α=0.5,η=0.0		α=0.5,η=0.5	
	MODE	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**
	I	24.0249	21.2478	21.8226	19.6711	33.0882	26.1283	28.6233	22.9014
Dk=0.0	**II**	48.2836	41.4958	41.3611	35.7113	77.3121	63.8223	64.8674	53.4586
	III	87.6634	77.2	74.064	65.2198	144.4087	124.9076	120.3263	103.814
	I	23.9408	21.1701	21.7216	19.5798	31.8089	24.2056	27.1969	20.7586
Dk=0.001	**II**	48.23	41.4371	41.2941	35.6382	76.7399	63.0842	64.2083	52.6032
	III	87.6308	77.1639	74.0228	65.1743	144.0946	124.5323	119.9612	103.3756
	I	23.8562	21.0919	21.6199	19.4879	30.4718	22.1038	25.685	18.3465
Dk=0.002	**II**	48.1764	41.3784	41.2271	35.565	76.1642	62.3401	63.5437	51.7382
	III	87.5981	77.1278	73.9817	65.1288	143.78	124.1563	119.5952	102.9361
	I	23.7712	21.0132	21.5174	19.3951	29.0688	19.7658	24.0713	15.54
Dk=0.003	**II**	48.1228	41.3197	41.16	35.4918	75.585	61.5899	62.8734	50.8634
	III	87.5654	77.0917	73.9406	65.0833	143.465	123.7797	119.2285	102.4954
	I	23.6857	20.9341	21.4142	19.3016	27.59	17.0951	22.3337	12.0671
Dk=0.004	**II**	48.0691	41.2609	41.0929	35.4185	75.0022	60.8333	62.1973	49.9781
	III	87.5327	77.0555	73.8995	65.0378	143.1495	123.4023	118.861	102.0537
	I	23.5998	20.8544	21.3103	19.2073	26.0224	13.9011	20.4405	6.9974
Dk=0.005	**II**	48.0154	41.202	41.0257	35.3451	74.4157	60.0701	61.5151	49.0819
	III	87.5	77.0194	73.8583	64.9923	142.8335	123.0242	118.4927	101.6108

Table 2. Values of frequency parameter Ω for C-C and C-SS plate for a/b=1.0, α=0.5, β=0.5

		***Dk*=0.0 , *E*$_f$=0.0**		***Dk*=0.001, Ef=0.0**		***Dk*=0.0, Ef=0.02**		***Dk*=0.001, Ef=0.02**	
	MODE	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**
	I	30.7478	23.3055	29.3705	21.1373	33.0882	26.1283	31.8089	24.2056
η=0.0	**II**	76.3268	62.6477	75.7463	61.8934	77.3121	63.8223	76.7399	63.0842
	III	143.8809	124.3019	143.5654	123.9244	144.4087	124.9076	144.0946	124.5323
	I	28.9646	21.9316	27.5188	19.6399	31.4543	24.9275	30.1241	22.9267
η=0.2	**II**	71.7401	58.7801	71.1292	57.9834	72.7931	60.0369	72.192	59.2596
	III	135.1045	116.5815	134.772	116.1828	135.6692	117.2303	135.3383	116.8342
	I	26.9497	20.3998	25.4181	17.9528	29.6319	23.6147	28.2409	21.5233
η=0.4	**II**	66.5659	54.4427	65.9166	53.5925	67.7073	55.8063	67.0702	54.9804
	III	125.2135	107.9092	124.8594	107.4836	125.8265	108.6142	125.4744	108.1919
	I	24.5758	18.6252	22.9292	15.9702	27.525	22.1365	26.0588	19.9373
η=0.6	**II**	60.4844	49.3811	59.7835	48.4588	61.7507	50.8948	61.0658	50.0046
	III	113.6045	97.7699	113.2213	97.308	114.2859	98.5546	113.9053	98.097

Table 3. Values of frequency parameter Ω for C-C and C-SS plate for a/b=1.0, E_f=0.02, D_k=0.001

		α=-0.5,β=-0.5		**α=-0.5,β=0.5**		**α=0.5,β=-0.5**		**α=0.5,β=0.5**	
	MODE	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**	**C-C**	**C-SS**
	I	31.101	28.2429	23.9408	21.1701	39.0993	29.4179	31.8089	24.2056
η=0.0	**II**	62.7187	54.5053	48.23	41.4371	96.5973	79.8367	76.7399	63.0842
	III	113.8324	100.8062	87.6308	77.1639	182.4646	158.3541	144.0946	124.5323
	I	29.917	27.3433	23.1179	20.5718	36.8136	27.5746	30.1241	22.9267
η=0.2	**II**	59.2707	51.5817	45.7001	39.3095	90.5569	74.6896	72.192	59.2596
	III	107.1392	94.8687	82.6838	72.7941	170.9029	148.1364	135.3383	116.8342
	I	28.6047	26.3559	22.2145	19.9265	34.2566	25.5436	28.2409	21.5233
η=0.4	**II**	55.4056	48.327	42.8678	36.9442	83.7495	68.9231	67.0702	54.9804
	III	99.612	88.2124	77.1238	67.8979	157.8728	136.6598	125.4744	108.1919
	I	27.0944	25.2306	21.1907	19.2109	31.2905	23.2392	26.0588	19.9373
η=0.6	**II**	50.9022	44.5677	39.5743	34.2179	75.7616	62.2063	61.0658	50.0046
	III	90.803	80.452	70.6227	62.1942	142.5807	123.2446	113.9053	98.097

From Figure 4b, the effect of thermal gradient η is found to decrease the frequency parameter Ω; however, the rate of decrease gets increased to more than twice of the first mode for both the boundary conditions. In case of third mode, this rate of decrease further increases and becomes nearly twice of the second mode as is evident from Figure 4c.

Figure 5a provides graphs of frequency parameter Ω verses thermal gradient η for the first mode of vibration. There is a continuous decrease in the value of frequency parameter Ω for fixed values of E_f = 0.02 and damping parameter D_k = 0.001 for both the boundary conditions. This rate decreases with the increase in the value of density parameter β; it increases with the increase in the value of taper parameter α and in increases the number of modes, as is clear from Tables 3b and 3c, when the plate is vibrating in the second and third mode of vibration. From Figure 5b, the effect of thermal gradient η is found to decrease the frequency parameter Ω; however, the rate of decrease gets increased to more than twice of the first mode for both the boundary conditions. In case of third mode, this rate of decrease further increases and becomes nearly twice of the second mode as is evident from Figure 5c.The normalized displacements for the two boundary conditions C-C and C-SS are plotted in Figures 6 and 7, respectively. The plate thickness varies parabolically in X-direction and the plate is considered resting on elastic foundations E_f = 0.02 with damping parameter D_k = 0.01. Mode shapes for a rectangular plate i.e, a/b = 0.25 have been computed and observed that the nodal lines are seen to shift towards the edge, i.e., X = 1 as the edge X = 0 increases in thickness for both the plates. No special change was seen in the pattern of nodal lines by taking different values of β and E_f. The normalized displacements were differing only at the third or fourth place after decimal for both the boundary conditions.

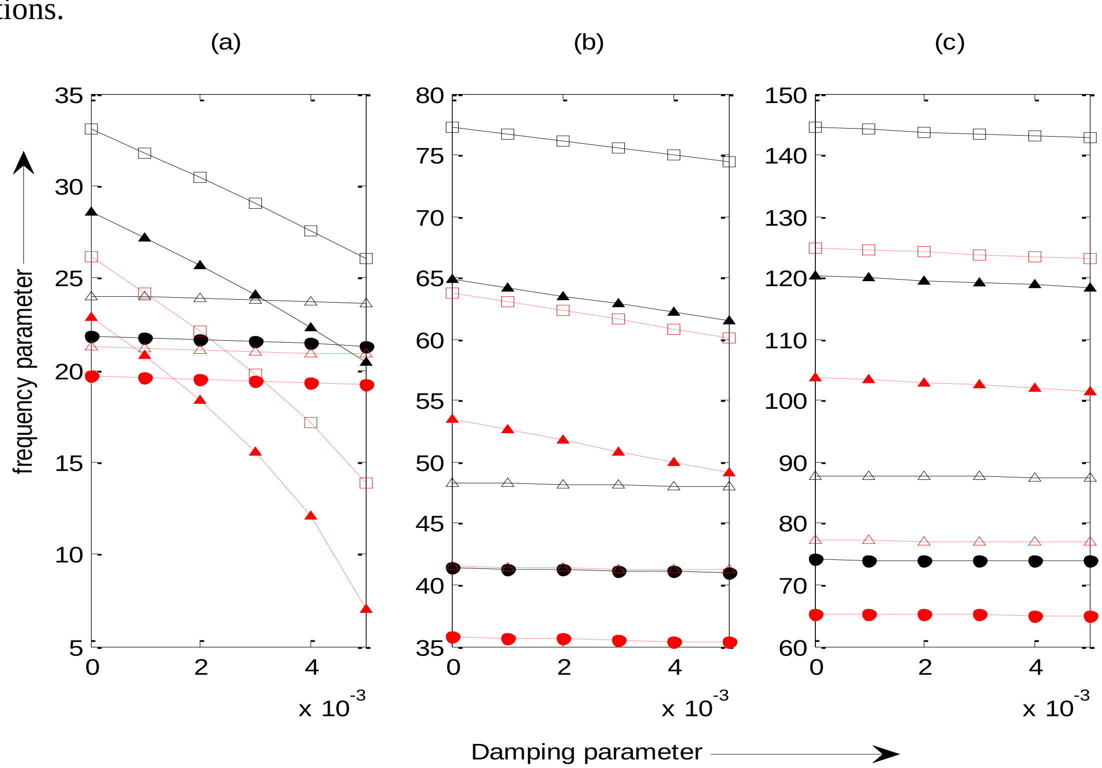


Figure 3. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=1.0,β=0.5, E_f=0.02 , —,C-C; --,C-SS; Δ, α=-0.5,η=0.0, ●, α=-0.5,η=0.5; □, α=0.5,η=0.0; ▲ , α=0.5,η=0.5

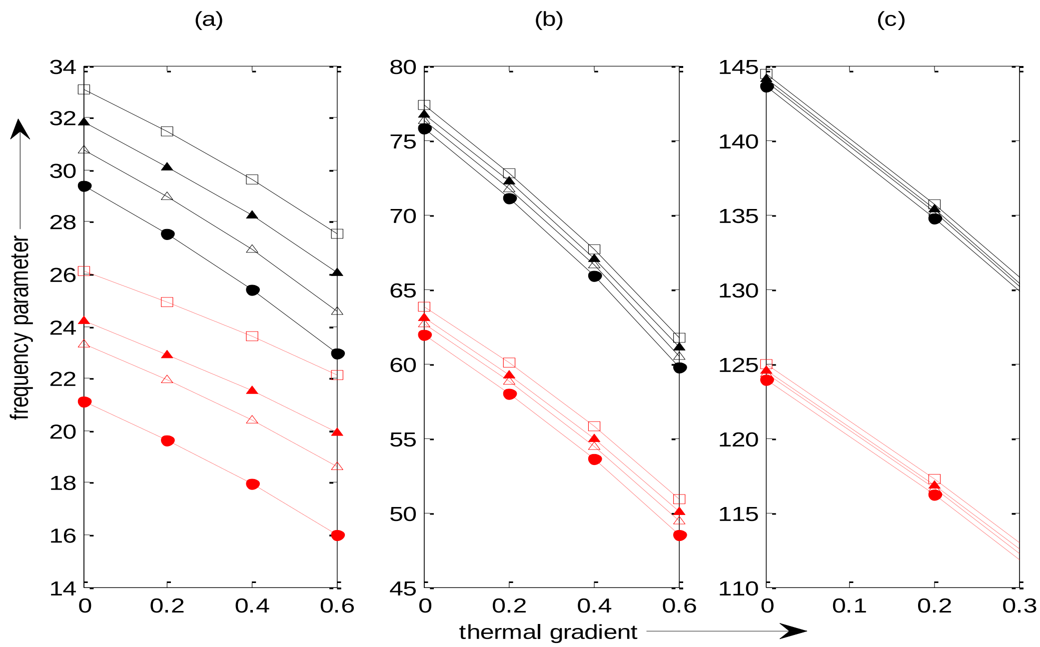


Figure 4. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=1.0,β=0.5, α=0.5 , —,C-C; --,C-SS; Δ,D_k=0.0 , E_f=0.0, , ●,D_k =0.001 , E_f =0.0; □,D_k =0.0 , E_f =0.02; ▲ ,D_k =0.001 , E_f =0.02

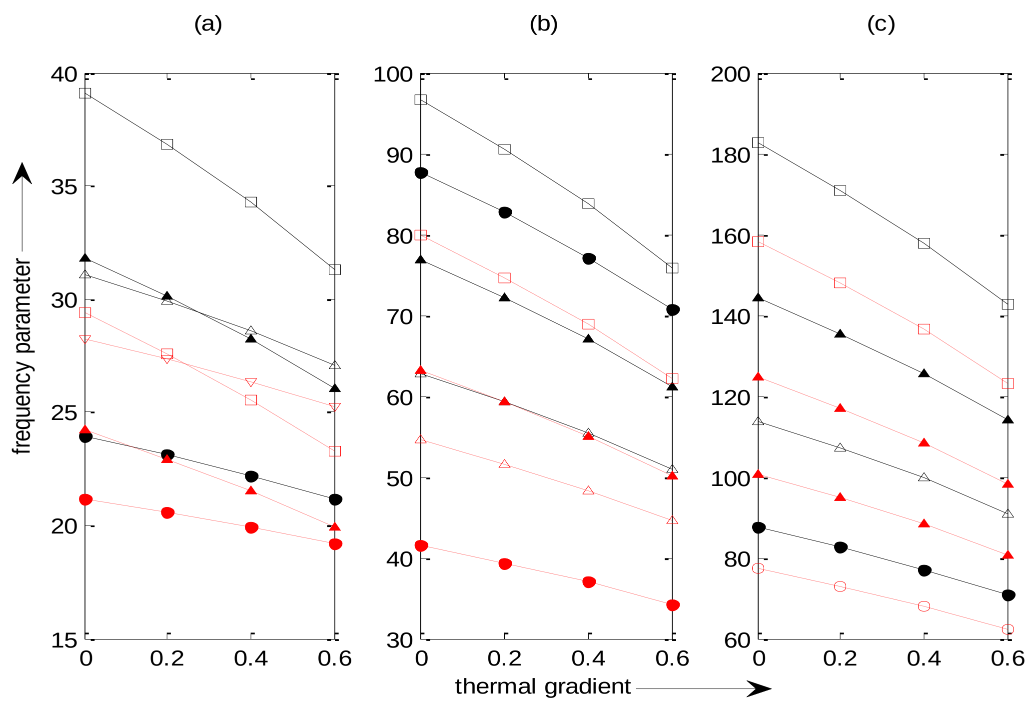


Figure 5. Natural frequencies for c-c and c-s plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=1.0, $Ef=0.02$, $D_k=0.001$, —,C-C; --,C-SS; Δ, $\alpha=-0.5,\beta=-0.5$; ●, $\alpha=-0.5,\beta=0.5$; □, $\alpha=0.5,\beta=-0.5$; ▲ , $\alpha=0.5,\beta=0.5$

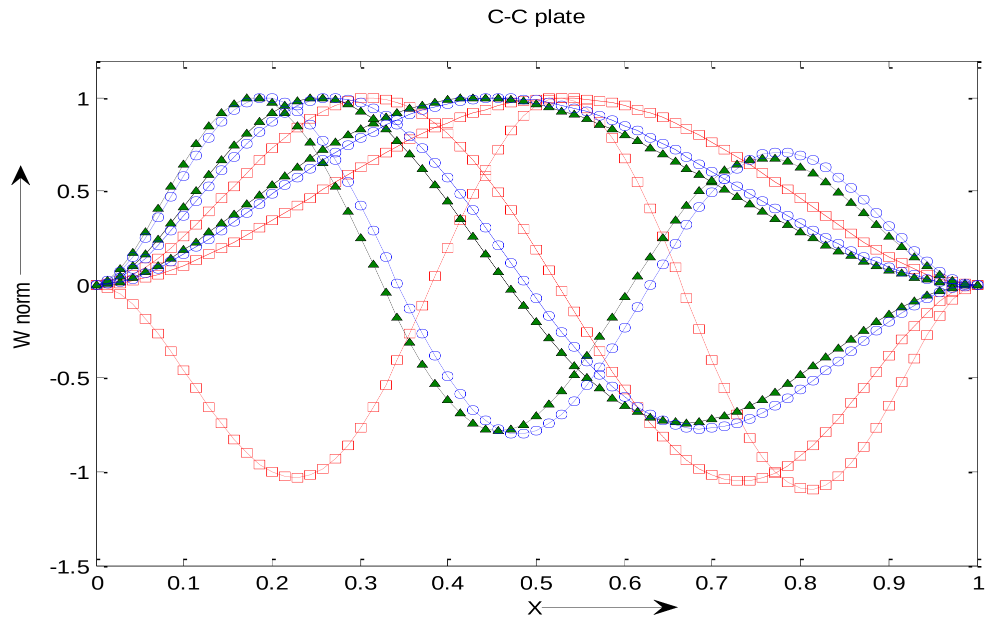


Figure 6. Normalized displacements for c-ss-c-ss plate, for a/b=1; h=0.1, β=0.5,D_k=0.005, E_f=0.02; —,First mode; ——, Second mode ;……Third mode , □, *α=-0.5,η=0.5;* ▲ , α=0.5,η=0.5; o, α=0.5,η=0.0

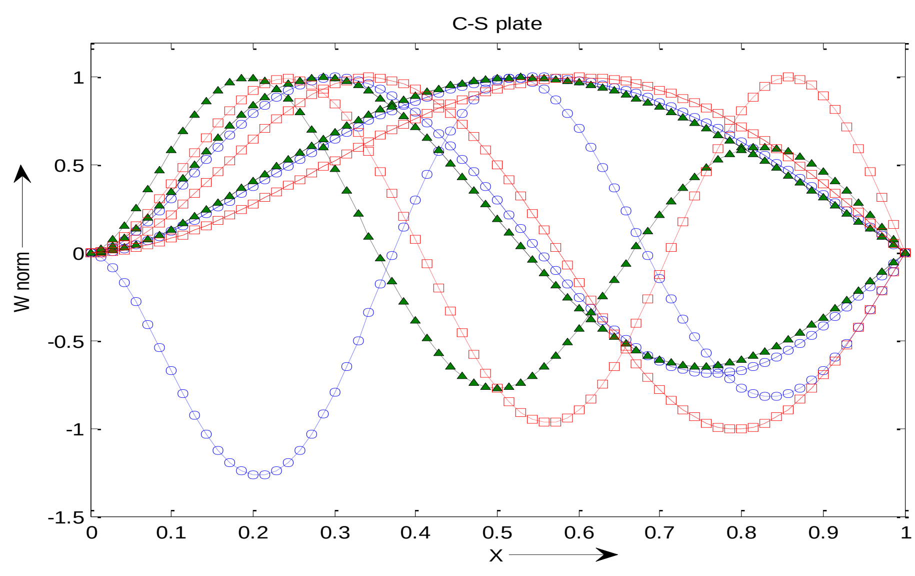


Figure 7. Normalized displacements for c-ss-ss-ss plate, for a/b=1; h=0.1, β=0.5,D_k=0.005, E_f=0.02; —,First mode; ——, Second mode ;……Third mode , □, *α=-0.5,η=0.5;* ▲ , α=0.5,η=0.5; o, α=0.5,η=0.0

5. Conclusion

In the present study results are computed using MATLAB within the permissible range of parameters up to the desired accuracy (10^{-8}), which validates the actual phenomenon of vibrational problem. The rate of decrease of frequency parameter Ω with thermal gradient η, decreases with the increase in the value of foundation parameter and non-homogeneity parameter and this rate increases with the increase in the value of taper parameter and damping parameter. Temperature effect, variation in thickness, elastic foundation, damping parameter and non-homogeneity parameter are of great interest because engineers often try to know natural frequency and modes of vibration before finalizing the design of a structure or machine. Our endeavor is to provide a mathematical model for analyzing the vibrational behavior of orthotropic rectangular plate resting on elastic foundation for different values of thermal gradient, taperness and damping parameter. Thus, the present study may be helpful in the determination of natural frequencies and mode shapes by proper choice of plate parameters so that robustness of structure can be determined for improved structural design.

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