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Qualitative results on mixed problem of micropolar bodies with microtemperatures

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Abstract

The aim of our study is to transform the mixed initial boundary value problem considered in the context of micropolar thermoelastic bodies whose micro-particles possess microtemperatures in a temporal evolutionary equation on a Hilbert space. Then, with the help of some results from the theory of semigroups, the existence and the uniqueness of the solution for this equation is proved. Finally, we approach the continuous dependence of the solution upon initial data and loads, also with the help of the semigroup.

Keywords: Micro-particles; microtemperatures; micropolar; semigroup; existence result; uniqueness; continuous dependence

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1. Introduction

In the last period of time, the theory of bodies with microstructure became a subject of intensive study in the literature. For example, Eringen (1996) introduced the concept of micropolar continua, which is similar with Cosserat continua. Unlike Cosserat theory, he introduced, additionally, a conservation law for the microinertia tensor, as a special case of micromorphic continua. Some fundamental results on micropolar bodies can be found in Chirita-Ghiba (2012), Dyszlewicz

(2004), Iesan (2004), Marin (1996), Marin (1997), Marin (2010a) Marin (2010b) Marin (2010c). Classical elasticity ignores the fact that the response of the material to external stimuli depends heavily on the motions of its inner structure. It is not possible to consider this effect by ascribing only translation degrees of freedom to the material points of the body. In the micropolar continuum theory, we have six degrees of freedom, instead of the three considered in classical elasticity. The difference is the consideration of the rotational degrees of freedom which play a central role in this theory. Also, in order to characterize the force applied on the surface element, together with the classical stress tensor, a couple stress tensor is introduced.

There are a lot of materials, such as crystals, composites, polymers, suspensions, blood, grid and multibar systems, which can be considered as examples of media with microstructure, that is, which point out the necessity for considering micromotions into the mechanical studies. Many studies dedicated to the theory of microstretch elastic bodies were published (for instance see Eringen (1999)). This theory is a generalization of the micropolar theory and a special case of the micromorphic theory. In the context of this theory, each material point is endowed with three deformable directors. A body is a microstretch continuum if the directors are constrained to have only breathing-type microdeformations. Also, the material points of a microstretch solid can stretch and contract independently of their translations and rotations. Other materials with microstructure are studied in Marin (1996), Marin (1997), Marin (2010a), Ellahi et al. (2014a), Ellahi et al. (2014b). Some considerations on waves for micropolar bodies can be found in Marin (2010b), Marin (2010c), Straughan (2011) and Sharma-Marin (2013).

The purpose of these theories is to eliminate discrepancies between classical elasticity and experiments, since classical elasticity failed to present acceptable results when the effects of material microstructure were known to contribute significantly to the body's overall deformations, for example, in the case of granular bodies with large molecules (e.g. polymers), graphite, or human bones. Also, the classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wavelength, that is, vibrations generated by ultrasonic waves.

Other intended applications of this theory are to composite materials reinforced with chopped fibers and various porous materials. Grot (1969) is considered as the initiator of the theory of bodies with microtemperatures, who, on the basis of the theory of bodies with inner structure, established a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. In this case, the entropy production inequality is adapted to include microtemperatures. As a consequence, the first-order moment of the energy equations are added to the usual balance laws of a continuum with microstructure. The theory of thermoelasticity with microtemperatures has been investigated in various papers (for instance, see Chirita et al. (2013), Iesan and Quintanilla (2000)).

In the present study we consider the effect of microtemperatures on the main characteristics of the mixed initial boundary value problems for micropolar thermoelastic bodies. It is important to note that the presence of the microtemperatures allows the transmission of heat as thermal waves at finite speed. This mixed problem is transformed in an abstract evolutionary equation on a suitable Hilbert space. Then, by using some results from the theory of semi-groups of operators, we deduce the existence and the uniqueness of the solution. Also, the continuous dependence of the solution upon the initial data and loads is proved.

2. Basic equations and conditions

We assume that a bounded region B of the three-dimensional Euclidean space R^3 is occupied by a micropolar elastic body, referred to the reference configuration and a fixed system of rectangular Cartesian axes. Let $\bar{B}$ denote the closure of B and call ∂B the boundary of the domain B. We consider ∂B to be a piecewise smooth surface and designate by n_i the components of the outward unit normal to the surface ∂B. Letters in boldface stand for vector fields. We use the notation v_i to designate the components of the vector $\mathbf{v}$ in the underlying rectangular Cartesian coordinates frame. Superposed dots stand for the material time derivative. We shall employ the usual summation and differentiation conventions: the subscripts are understood to range over integers $(1, 2, 3)$. Summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate.

The spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion. We refer the motion of the body to a fixed system of rectangular Cartesian axes $Ox_i,\ i = 1, 2, 3$. Let us denote by u_i the components of the displacement vector and by φ_i the components of the microrotation vector. Also, we denote by ϕ the microstretch function and by θ the temperature measured from the constant absolute temperature T_0 of the body in its reference state.

As usual, we denote by t_{ij} the components of the stress tensor and by m_{ij} the components of the couple stress tensor over B. The equations of motion for micropolar thermoelastic bodies are (see Iesan and Nappa (2005))

$$t_{ji,j} + \varrho F_i = \varrho \ddot{u}_i,\ m_{ji,j} + \varepsilon_{ijk} t_{jk} + \varrho G_i = I_{ij} \ddot{\varphi}_j. \tag{1}$$

According to Iesan (2004), the balance of the first stress moment has the form

$$\lambda_{i,i} - \sigma + \varrho L = J\ddot{\phi}. \tag{2}$$

In these equations we have used the following notations: F_i are the components of the body force, G_i are the components of the body couple, L is the generalized external body load, ϱ is the reference constant mass density, and J and $I_{ij} = I_{ji}$ are the coefficients of microinertia.

If T is the temperature in the body, we will denote by θ the temperature measured from the constant absolute temperature T_0 in the body in its reference state, that is, $\theta = T - T_0$. We consider a generic microelement in the reference configuration and denote by $(X_i^{'})$ the coordinates of its center of mass. If (X_i) are the coordinates of an arbitary point in the body, then we can assume that the absolute temperature in the body is a sum of the form

$$\theta + T_i \left(X_i^{'} - X_i \right), \tag{3}$$

where the functions T_i are microtemperatures. We will denote by ϑ_i the microtemperatures measured from the microtemperatures T_i^0 in the reference state, namely, $\vartheta_i = T_i - T_i^0$.

The behavior of the micropolar thermoelastic bodies with microtemperatures can be characterized using the above - mentioned variables u_i, φ_i, ϕ and variables χ, τ_i, defined by

$$\chi = \int_{t_0}^{t} \theta dt,\ \tau_i = \int_{t_0}^{t} \vartheta_i dt, \tag{4}$$

in which, obviously, t_0 is the reference time. The components of the strain tensors ε_{ij}, μ_{ij} and γ_i are defined by means of the geometric equations

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{ijk}\varphi_k,\ \mu_{ij=}\varphi_{j,i},\ \gamma_i = \phi_{,i}, \tag{5}$$

where ε_{ijk} is the alternating symbol.

Using a procedure analogous to that in Iesan and Quintanilla (2000), we obtain the constitutive equations

$$\begin{aligned}
t_{ij} &= A_{ijmn}\,\varepsilon_{mn} + B_{ijmn}\mu_{mn} + a_{ij}\phi - \alpha_{ij}\dot{\chi} + D_{ijmn}\tau_{m,n},\\
m_{ij} &= B_{ijmn}\,\varepsilon_{mn} + C_{ijmn}\mu_{mn} + b_{ij}\phi - \beta_{ij}\dot{\chi} + E_{ijmn}\tau_{m,n},\\
\lambda_i &= A_{ij}\,\gamma_j - d_{ij}\dot{\tau}_j + H_{ij}\chi_{,j},\\
\sigma &= a_{ij}\,\varepsilon_{ij} + b_{ij}\mu_{ij} + \zeta\phi - \kappa\dot{\chi} + F_{ij}\tau_{i,j},\\
\varrho\eta &= \alpha_{ij}\,\varepsilon_{ij} + \beta_{ij}\mu_{ij} + \kappa\phi + a\dot{\chi} + L_{ij}\tau_{i,j},\\
\varrho\eta_i &= d_{ji}\gamma_j + B_{ij}\dot{\tau}_j + C_{ij}\chi_{,j},\\
S_i &= H_{ji}\gamma_j - C_{ji}\dot{\tau}_j + K_{ij}\chi_{,j},\\
\Lambda_{ij} &= D_{ijmn}\,\varepsilon_{mn} + E_{ijmn}\mu_{ij} + F_{ji}\phi - L_{ji}\dot{\chi} + G_{ijmn}\tau_{m,n}.
\end{aligned} \tag{6}$$

In the above equations, the notations used have the following meanings: $t_{ij},\ m_{ij}$ and λ_i are the components of the stress, λ_i are the components of the internal hypertraction vector, σ is the generalized internal body load, η is the entropy per unit mass, η_i is the first entropy moment vector, S_i is the entropy flux vector and Λ_{ij} is the first entropy flux moment tensor.

Also, the quantities A_{ijmn}, B_{ijmn}, ..., L_{ji} and G_{ijmn} are characteristic constitutive coefficients and they obey the following symmetry relations

$$\begin{aligned}
&A_{ijmn} = A_{mnij},\ C_{ijmn} = C_{mnij},\ A_{ij} = A_{ji},\ B_{ij} = B_{ji},\ K_{ij} = K_{ji},\\
&a_{ij} = a_{ji},\ b_{ij} = b_{ji},\ D_{ijmn} = D_{jimn},\ E_{ijmn} = E_{jimn},\ G_{ijmn} = G_{mnij}.
\end{aligned} \tag{7}$$

If we denote by ξ_i the internal rate of production of entropy per unit mass and by H_i the mean entropy flux vector, then from the equation of energy we deduce the relation

$$\varrho\xi_i + S_i - H_i = 0, \tag{8}$$

wherein the meaning of S_i was exposed above. Also, if we denote by s the external rate of supply of entropy per unit mass and by Q_i the first moment of the external rate of supply of entropy, we can write two more equations of energy,

$$\varrho\dot{\eta} = S_{i,i} + \varrho s,\ \varrho\dot{\eta}_i = \Lambda_{ji,j} + \varrho Q_i. \tag{9}$$

We substitute now the geometric equations (5) and the constitutive equations (6) into the equations of motion (1), in the balance of the first stress moment (2) and into the equations of energy (9). As such, we get a system of partial differential equations in which the unknown functions are u_i, φ_i, ϕ, χ and τ_i, namely,

$$\begin{aligned}
&A_{ijmn}\left(u_{m,nj}+\varepsilon_{mnk}\varphi_{k,j}\right)+B_{ijmn}\varphi_{n,mj}+a_{ij}\phi_{,j}-\alpha_{ij}\dot{\chi}_{,j}+D_{ijmn}\tau_{m,nj}+\varrho F_i=\varrho\ddot{u}_i,\\
&B_{ijmn}\left(u_{m,nj}+\varepsilon_{mnk}\varphi_{k,j}\right)+C_{ijmn}\varphi_{n,mj}+b_{ij}\phi_{,j}-\beta_{ij}\dot{\chi}_{,j}+E_{ijmn}\tau_{m,nj}\\
&+\varepsilon_{ijk}[A_{jkmn}\left(u_{m,n}+\varepsilon_{mnk}\varphi_k\right)+B_{jkmn}\varphi_{n,m}+a_{jk}\phi-\alpha_{jk}\dot{\chi}+D_{jkmn}\tau_{m,n}]+\varrho G_i=I_{ij}\ddot{\varphi}_j,\\
&A_{ij}\phi_{,ij}-d_{ij}\dot{\tau}_{j,i}+H_{ij}\chi_{,ij}-a_{ij}\left(u_{j,i}+\varepsilon_{ijk}\varphi_k\right)-b_{ij}\varphi_{j,i}-\zeta\phi-\kappa\dot{\chi}-F_{ij}\tau_{i,j}+\varrho L=J\ddot{\phi},\\
&H_{ji}\phi_{,ij}-D_{ij}\dot{\tau}_{j,i}+K_{ij}\chi_{,ij}-\alpha_{ij}\left(\dot{u}_{j,i}+\varepsilon_{ijk}\dot{\varphi}_k\right)-\beta_{ij}\dot{\varphi}_{j,i}-\kappa\dot{\phi}-a\ddot{\chi}=-\varrho s,\\
&D_{ijmn}\left(u_{m,nj}+\varepsilon_{mnk}\varphi_{k,j}\right)+E_{ijmn}\varphi_{n,mj}+F_{ji}\phi_{,j}\\
&-D_{ji}\dot{\chi}_{,j}+G_{ijmn}\tau_{m,nj}-d_{ij}\dot{\phi}_{,j}-B_{ij}\ddot{\tau}_j=-\varrho Q_i.
\end{aligned} \tag{10}$$

Here we used the notation $D_{ij}=C_{ij}+L_{ij}$. Taking into account the Dirichlet problem associated to the system of equations (10), the boundary conditions have the form

$$u_i=\bar{u}_i,\ \varphi_i=\bar{\varphi}_i,\ \phi=\bar{\phi},\ \chi=\bar{\chi},\ \tau_i=\bar{\tau}_i,\ \text{ on }\ \partial B\times(0,\infty), \tag{11}$$

where $\bar{u}_i$, $\bar{\varphi}_i$, $\bar{\phi}$, $\bar{\chi}$, $\bar{\tau}_i$ are known functions. In the case of a boundary value problem of Neumann type, the boundary conditions (11) are replaced by the following,

$$t_{ji}n_j=\bar{t}_i,\ m_{ji}n_j=\bar{m}_i,\ \lambda_j n_j=\bar{\lambda},\ S_j n_j=\bar{S},\ \Lambda_{ji}n_j=\bar{\Lambda}_i,\ \text{ on }\ \partial B\times(0,\infty), \tag{12}$$

where also the functions $\bar{t}_i$, $\bar{m}_i$, $\bar{\lambda}$, $\bar{S}$ and $\bar{\Lambda}_i$ are given.

In the following we restrict our considerations only on the Dirichlet problem.

The mixed initial boundary value problem associated to the system (10) is complete if we consider the initial conditions, namely,

$$\begin{aligned}
&u_i(x,0)=u_i^0(x), && \dot{u}_i(x,0)=u_i^1(x), && \varphi_i(x,0)=\varphi_i^0(x),\\
&\dot{\varphi}_i(x,0)=\varphi_i^1(x), && \phi(x,0)=\phi^0(x), && \dot{\phi}(x,0)=\phi^1(x),\\
&\chi(x,0)=\chi^0(x), && \dot{\chi}(x,0)=\chi^1(x), && \tau_i(x,0)=\tau_i^0(x), && \dot{\tau}_i(x,0)=\tau_i^1(x),
\end{aligned} \tag{13}$$

for any $x\in B$. Here the functions u_i^0, u_i^1, φ_i^0, φ_i^1, ϕ^0, ϕ^1, χ^0, χ^1, τ_i^0 and τ_i^1 are prescribed.

3. Qualitative results of the solutions

In this section we will study the existence and uniqueness of the solution of the mixed initial boundary value problem in our context. Also, we obtain the continuous dependence of the solution with regard to the initial data and charges.

In all that follows we will assume that the functions that appear in the equations and the conditions formulated in Section 2 are sufficiently regular on their domain of definition to allow mathematical operations that will be made later on them.

For the next result of uniqueness, we need the following auxiliary result.

Theorem 3.1.

Between the variables that characterize the deformation of a thermoelastic micropolar body with microtemperatures, the following equality takes place,

$$\begin{aligned} t_{ij}\varepsilon_{ij} + m_{ij}\mu_{ij} + \lambda_i\phi_{,i} + \sigma\phi + \varrho\eta\dot{\chi} + \varrho\eta_i\dot{\tau}_i + S_i\chi_{,i} + \Lambda_{ij}\tau_{i,j} \\ = A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\mu_{mn} + 2a_{ij}\varepsilon_{ij}\phi + 2D_{ijmn}\varepsilon_{ij}\tau_{m,n} \\ +C_{ijmn}\mu_{ij}\mu_{mn} + 2b_{ij}\mu_{ij}\phi + 2E_{ijmn}\mu_{ij}\tau_{m,n} + A_{ij}\phi_{,i}\phi_{,j} \\ +2H_{ij}\phi_{,i}\chi_{,j} + \zeta\phi^2 + 2F_{ij}\tau_{i,j}\phi + K_{ij}\chi_{,i}\chi_{,j} \\ +G_{ijmn}\tau_{m,n}\tau_{i,j} + a\dot{\chi}^2 + B_{ij}\dot{\tau}_i\dot{\tau}_j. \end{aligned} \tag{14}$$

Proof:

Multiply each equation in the system of the constitutive equations (6) as follows: $t_{ij}.\varepsilon_{ij}$, $m_{ij}.\mu_{ij}$, $\lambda_i.\phi_{,i}$, $\sigma.\phi$, $\varrho\eta.\dot{\chi}$, $\varrho\eta_i.\dot{\tau}_i$, $S_i.\chi_{,i}$ and $\Lambda_{ij}.\tau_{i,j}$. Then we add the equalities which are obtained, member with member, and by considering the relations of symmetry (7) we obtain the desired equality (14). ■

The following be useful for us in the following the quadratic form defined as follows

$$\begin{aligned} U = \frac{1}{2}\Big[A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\mu_{mn} + 2a_{ij}\varepsilon_{ij}\phi + 2D_{ijmn}\varepsilon_{ij}\tau_{m,n} \\ +C_{ijmn}\mu_{ij}\mu_{mn} + 2b_{ij}\mu_{ij}\phi + 2E_{ijmn}\mu_{ij}\tau_{m,n} + A_{ij}\phi_{,i}\phi_{,j} \\ +2H_{ij}\phi_{,i}\chi_{,j} + \zeta\phi^2 + 2F_{ij}\tau_{i,j}\phi + K_{ij}\chi_{,i}\chi_{,j} + G_{ijmn}\tau_{m,n}\tau_{i,j} \Big]. \end{aligned} \tag{15}$$

Now we can state and prove the uniqueness of the solution of the mixed initial boundary value problem considered in the previous section.

Theorem 3.2.

We assume that the following assumptions are met;

1. ϱ, I_{ij}, J and the constitutive coefficient a are strictly positive;

2. the symmetry relations (7) take place;

3. the quadratic form U defined in (15) is positive semi-definite; and

4. the constitutive coefficients B_{ij} are components of a positive definite tensor.

Then, the mixed initial boundary value problem that consists of equations (10), the initial conditions (13), and the boundary conditions (11) admits at most one solution.

Proof:

As in the proof of Theorem 1, we start by multiplying each equation in the system of the constitutive equations (6) as follows: $t_{ij}.\dot{\varepsilon}_{ij}$, $m_{ij}.\dot{\mu}_{ij}$, $\lambda_i.\dot{\phi}_{,i}$, $\sigma.\dot{\phi}$, $\varrho\dot{\eta}.\dot{\chi}$, $\varrho\dot{\eta}_i.\dot{\tau}_i$, $S_i.\dot{\chi}_{,i}$ and $\Lambda_{ij}.\dot{\tau}_{i,j}$. Then we add the equalities which are obtained, member with member, and considering the relations of symmetry (7) and the quadratic form U from (15) we obtain the following equality,

$$t_{ij}\dot{\varepsilon}_{ij}+m_{ij}\dot{\mu}_{ij}+\lambda_i\dot{\phi}_{,i}+\sigma\dot{\phi}+\varrho\dot{\eta}\dot{\chi}+\varrho\dot{\eta}_i\dot{\tau}_i+S_i\dot{\chi}_{,i}+\Lambda_{ij}\dot{\tau}_{i,j} = \frac{\partial}{\partial t}\left(U+\frac{1}{2}a\dot{\chi}^2+\frac{1}{2}B_{ij}\dot{\tau}_i\dot{\tau}_j\right). \tag{16}$$

Now we take into account the geometric equations (5), the equations of motion (1), the balance of the first stress moment (2) and the equations of energy (9) so that we are led to the equality

$$\begin{aligned} t_{ij}\dot{\varepsilon}_{ij}&+m_{ij}\dot{\mu}_{ij}+\lambda_i\dot{\phi}_{,i}+\sigma\dot{\phi}+\varrho\dot{\eta}\dot{\chi}+\varrho\dot{\eta}_i\dot{\tau}_i+S_i\dot{\chi}_{,i}+\Lambda_{ij}\dot{\tau}_{i,j}\\ &=\left(t_{ij}\dot{u}_i+m_{ij}\dot{\varphi}_i+\lambda_j\dot{\phi}+S_j\dot{\chi}+\Lambda_{ij}\dot{\tau}_i\right)_{,j}\\ &+\varrho\left(F_i\dot{u}_i+G_i\dot{\varphi}_i+L\dot{\phi}+s\dot{\chi}+Q_i\dot{\tau}_i\right)-\varrho\ddot{u}_i\dot{u}_i-I_{ij}\ddot{\varphi}_i\dot{\varphi}_j-J\ddot{\phi}\dot{\phi}. \end{aligned} \tag{17}$$

It is easy to see that equalities (16) and (17) provide the equality

$$\begin{aligned} \frac{1}{2}\frac{\partial}{\partial t}&\left(2U+\varrho\dot{u}_i\dot{u}_i+I_{ij}\dot{\varphi}_i\dot{\varphi}_j+J\dot{\phi}^2+a\dot{\chi}^2+B_{ij}\dot{\tau}_i\dot{\tau}_j\right)\\ &=\left(t_{ij}\dot{u}_i+m_{ij}\dot{\varphi}_i+\lambda_j\dot{\phi}+S_j\dot{\chi}+\Lambda_{ij}\dot{\tau}_i\right)_{,j}+\varrho\left(F_i\dot{u}_i+G_i\dot{\varphi}_i+L\dot{\phi}+s\dot{\chi}+Q_i\dot{\tau}_i\right). \end{aligned} \tag{18}$$

Equality (18) is integrated over the domain B such that with the help of the divergence theorem we obtain

$$\begin{aligned} \frac{1}{2}\frac{\partial}{\partial t}&\int_B\left(2U+\varrho\dot{u}_i\dot{u}_i+I_{ij}\dot{\varphi}_i\dot{\varphi}_j+J\dot{\phi}^2+a\dot{\chi}^2+B_{ij}\dot{\tau}_i\dot{\tau}_j\right)dV\\ &=\int_{\partial B}\left(t_{ji}\dot{u}_i+m_i\dot{\varphi}_i+\lambda_j\dot{\phi}+S_j\dot{\chi}+\Lambda_{ij}\dot{\tau}_i\right)n_j dA+\int_B\varrho\left(F_i\dot{u}_i+G_i\dot{\varphi}_i+L\dot{\phi}+s\dot{\chi}+Q_i\dot{\tau}_i\right)dV, \end{aligned} \tag{19}$$

where n_i are the components of the outward unit normal of the surface ∂B.

We will mark with "*" the difference of two solutions of the mixed problem that consists of (10), (13) and (11), that is,

$$u_i^*=u_i^2-u_i^1,\ \varphi_i^*=\varphi_i^2-\varphi_i^1,\ \phi^*=\phi^2-\phi^1,\ \chi^*=\chi^2-\chi^1,\ \tau_i^*=\tau_i^2-\tau_i^1.$$

Also, we will mark with "*" the other quantities which correspond to the above differences. Because of linearity, these differences also satisfy the equations of motion (1), the balance of the first stress moment (2) and the energy equations (9), but with null body loads. Also, the initial conditions become homogeneous, that is, for any $x\in B$,

$$\begin{aligned} &u_i^*(x,0)=0, && \dot{u}_i^*(x,0)=0, && \varphi_i^*(x,0)=0, && \dot{\varphi}_i^*(x,0)=0, && \phi^*(x,0)=0,\\ &\dot{\phi}^*(x,0)=0, && \chi^*(x,0)=0, && \dot{\chi}^*(x,0)=0, && \tau_i^*(x,0)=0, && \dot{\tau}_i^*(x,0)=0, \end{aligned} \tag{20}$$

and, certainly, the boundary conditions become null,

$$u_i^*=0,\ \varphi_i^*=0,\ \phi^*=0,\ \chi^*=0,\ \tau_i^*=0,\ \text{ on }\ \partial B\times(0,\infty), \tag{21}$$

$$\varepsilon_{ij}^*(x,0)=0,\ \mu_{ij}^*(x,0)=0,\ \phi_{,i}^*(x,0)=0,\ \chi_{,i}^*(x,0)=0,\ \tau_{i,j}^*(x,0)=0,\ \ x\in B. \tag{22}$$

Taking into account these considerations, the relation (19) written for these differences, becomes

$$\int_B\left(2U^*+\varrho\dot{u}_i^*\dot{u}_i^*+I_{ij}\dot{\varphi}_i^*\dot{\varphi}_j^*+J\left(\dot{\phi}^*\right)^2+a\left(\dot{\chi}^*\right)^2+B_{ij}\dot{\tau}_i^*\dot{\tau}_j^*\right)dV=0,\ t\geq 0. \tag{23}$$

Based on hypothesis 3 of the theorem and using (22), we deduce that the quadratic form U written for the differences becomes null and then from (23) we deduce that

$$\int_B \left[\varrho \dot{u}_i^* \dot{u}_i^* + I_{ij} \dot{\varphi}_i^* \dot{\varphi}_j^* + J \left(\dot{\phi}^*\right)^2 + a \left(\dot{\chi}^*\right)^2 + B_{ij} \dot{\tau}_i^* \dot{\tau}_j^*\right] dV = 0. \tag{24}$$

Considering the hypothesis 1 of the theorem regarding the amounts ϱ, I_{ij}, J and a and the hypothesis 4 regarding the tensor B_{ij}, from (24) we must have

$$\dot{u}_i^* = 0,\ \dot{\varphi}_i^* = 0,\ \dot{\phi}^* = 0,\ \dot{\chi}^* = 0,\ \dot{\tau}_i^* = 0, \text{ on } B \times (0, \infty),$$

so that, if we take into account (20), we deduce that

$$u_i^* = 0,\ \varphi_i^* = 0,\ \phi^* = 0,\ \chi^* = 0,\ \tau_i^* = 0, \text{ on } B \times (0, \infty),$$

so the proof of the theorem is complete. ■

We shall prove now a result of existence of the solution for the mixed initial boundary value problem, mentioned above, but in the case in which the boundary conditions are homogeneous, that is,

$$u_i = \varphi_i = \phi = \chi = \tau_i = 0, \text{ on } \partial B \times (0, \infty). \tag{25}$$

Because the system of governing equations and conditions for our problem is more complicated, it is necessary a new approach for the existence of the solution in this context. To this end we will transform the problem in an abstract evolutionary equation on a Hilbert space, suitably chosen.

Using the usual Hilbert spaces $W_0^{1,2}$ and L^2, we consider the Hilbert space $\mathcal{H}$ defined by

$$\mathcal{H} = \mathbf{W}_0^{1,2} \times \mathbf{L}^2 \times \mathbf{W}_0^{1,2} \times \mathbf{L}^2 \times W_0^{1,2} \times L^2 \times W_0^{1,2} \times L^2 \times \mathbf{W}_0^{1,2} \times \mathbf{L}^2,$$

where we used the notation $\mathbf{W}_0^{1,2} = W_0^{1,2} \times W_0^{1,2} \times W_0^{1,2}$, or, shorter, $\mathbf{W}_0^{1,2} = \left[W_0^{1,2}\right]^3$. Also, $\mathbf{L}^2 = \left[L^2\right]^3$. For Hilbert and Sobolev spaces see the basic book by Adams (1975).

On the space $\mathcal{H}$ we define the following scalar product

$$\begin{aligned}
&\langle (u_i, U_i, \varphi_i, \Psi_i, \phi, \Phi, \chi, \mu, \tau_i, \nu_i), (u_i^*, U_i^*, \varphi_i^*, \Psi_i^*, \phi^*, \Phi^*, \chi^*, \mu^*, \tau_i^*, \nu_i^*) \rangle \\
&= \frac{1}{2} \int_B (\varrho U_i U_i^* + I_{ij} \Psi_i \Psi_i^* + J \Phi \Phi^* + a \mu \mu^* + B_{ij} \nu_i \nu_i^*) \, dV \\
&+ \frac{1}{2} \int_B \left[A_{ijmn} \varepsilon_{ij} \varepsilon_{mn}^* + B_{ijmn} \left(\varepsilon_{ij} \mu_{mn}^* + \varepsilon_{ij}^* \mu_{mn}\right) + a_{ij} \left(\varepsilon_{ij} \phi^* + \varepsilon_{ij}^* \phi\right)\right. \\
&+ D_{ijmn} \left(\varepsilon_{ij} \tau_{m,n}^* + \varepsilon_{ij}^* \tau_{m,n}\right) + C_{ijmn} \mu_{ij} \mu_{mn}^* + b_{ij} \left(\mu_{ij} \phi^* + \mu_{ij}^* \phi\right) \\
&+ E_{ijmn} \left(\mu_{ij} \tau_{m,n}^* + \mu_{ij}^* \tau_{m,n}\right) + A_{ij} \phi_{,i} \phi_{,j}^* + H_{ij} \left(\phi_{,i} \chi_{,j}^* + \phi_{,i}^* \chi_{,j}\right) \\
&\left.+ \zeta \phi \phi^* + F_{ij} \left(\tau_{i,j} \phi^* + \tau_{i,j}^* \phi\right) + K_{ij} \chi_{,i} \chi_{,j}^* + G_{ijmn} \tau_{m,n} \tau_{i,j}^*\right] dV.
\end{aligned} \tag{26}$$

We can prove that the norm induced by this scalar product is equivalent to the original norm on the Hilbert space $\mathcal{H}$.

Now, with a suggestion given by the operators which appear in the left-hand side of equations (10), we introduce the operators

$$
\begin{aligned}
&A_i^1\mathbf{u}=\frac{1}{\varrho}A_{ijmn}u_{m,nj},\ A_i^2\boldsymbol{\varphi}=\frac{1}{\varrho}\left[A_{ijmn}\varepsilon_{mnk}\varphi_{k,j}+B_{ijmn}\varphi_{n,mj}\right],\ B_i^1\phi=\frac{1}{\varrho}a_{ij}\phi_{,j},\\
&C_i^1\mu=-\frac{1}{\varrho}\alpha_{ij}\mu_{,j},\, D_i^1\boldsymbol{\tau}=\frac{1}{\varrho}D_{ijmn}\tau_{m,nj},\, A_i^3\mathbf{u}=\frac{1}{I_{ij}}\left(B_{ijmn}u_{m,nj}+\varepsilon_{ijk}A_{jkmn}u_{m,n}\right),\\
&A_s^4\boldsymbol{\varphi}=W_{si}\left[A_{ijmn}\varepsilon_{jmn}\varphi_j+B_{ijmn}\varepsilon_{jmn}\varphi_{n,m}+C_{ijmn}\varphi_{n,mj}\right],\, B_s^2\phi=W_{si}\left(b_{ij}\phi_{,j}+a_{jk}\varepsilon_{ijk}\phi\right),\\
&C_s^2\mu=-W_{si}\left(\beta_{ij}\mu_{,j}+\varepsilon_{ijk}\alpha_{jk}\mu\right),\ D_s^2\boldsymbol{\tau}=W_{si}\left(E_{ijmn}\tau_{m,nj}+\varepsilon_{ijk}D_{jkmn}\tau_{m,n}\right),\\
&E\phi=\frac{1}{J}\left(A_{ij}\phi_{,ij}-\zeta\phi\right),\ F\boldsymbol{\nu}=-\frac{1}{J}d_{ij}\nu_{j,i},\ G\chi=\frac{1}{J}H_{ij}\chi_{,ij},\ H\mathbf{u}=-\frac{1}{J}a_{ij}u_{j,i},\\
&K\boldsymbol{\varphi}=-\frac{1}{J}\left(a_{ij}\varepsilon_{ijk}\varphi_k+b_{ij}\varphi_{j,i}\right),\ L\mu=\frac{1}{J}\kappa\mu,\ M\boldsymbol{\tau}=-\frac{1}{J}F_{ij}\tau_{i,j},\ N\chi=\frac{1}{a}K_{ij}\chi_{,ij},\\
&P\phi=\frac{1}{a}H_{ij}\phi_{,ij},\, Q\boldsymbol{\nu}=-\frac{1}{a}D_{ij}\nu_{j,i},\, R^1\mathbf{v}=-\frac{1}{a}\alpha_{ij}v_{i,j},\, R^2\boldsymbol{\Psi}=-\frac{1}{a}\left(\alpha_{ij}\varepsilon_{ijk}\Psi_k+\beta_{ij}\Psi_{j,i}\right),\\
&S\Phi=-\frac{1}{a}\kappa\Phi,\, A_s^5\mathbf{u}=\Gamma_{si}D_{ijmn}u_{m,nj},\, A_s^6\boldsymbol{\varphi}=\Gamma_{si}\left(D_{ijmn}\varepsilon_{mnk}\varphi_{k,j}+E_{ijmn}\varphi_{n,mj}\right),\\
&W_s\phi=\Gamma_{si}F_{ij}\phi_{,j},\ X_s\mu=-\Gamma_{si}D_{ij}\mu_{,j},\ Y_s\boldsymbol{\tau}=\Gamma_{si}G_{ijmn}\tau_{m,nj},\ Z_s\Phi=-\Gamma_{si}d_{ji}\Phi_{,j},
\end{aligned}
\tag{27}
$$

in which the matrices W_{si} and Γ_{si} are defined by means of the equations $W_{si}J_{ir}=\delta_{sr}, \Gamma_{si}B_{ir}=\delta_{sr}$. If we denote by $\mathcal{T}$ the matrix operator which has as components the operators defined in (27), then the mixed initial boundary value problem is transformed in a Cauchy problem associated to an evolutionary equation, namely

$$\frac{d\mathcal{U}}{dt}=\mathcal{T}\mathcal{U}(t)+\mathcal{F}(t),\ \mathcal{U}(0)=\mathcal{U}_0. \tag{28}$$

In order to use the theoretical results that follow, we have to take as domain for the operator $\mathcal{T}$, that is, $D(\mathcal{T})$, the next set

$$
\begin{aligned}
&\left(\mathbf{W}_0^{1,2}\cap\mathbf{W}^{2,2}\right)\times\mathbf{W}_0^{1,2}\times\left(\mathbf{W}_0^{1,2}\cap\mathbf{W}^{2,2}\right)\times\mathbf{W}_0^{1,2}\times\left(W_0^{1,2}\cap W^{2,2}\right)\times W_0^{1,2}\times\left(W_0^{1,2}\cap W^{2,2}\right)\times W_0^{1,2}\\
&\times\left(W_0^{1,2}\cap W^{2,2}\right)\times W_0^{1,2}\times\left(W_0^{1,2}\cap W^{2,2}\right)\times W_0^{1,2}\times\left(\mathbf{W}_0^{1,2}\cap\mathbf{W}^{2,2}\right)\times\mathbf{W}_0^{1,2}\times\left(\mathbf{W}_0^{1,2}\cap\mathbf{W}^{2,2}\right)\times\mathbf{W}_0^{1,2}.
\end{aligned}
$$

Also, the unknown matrix function $\mathcal{U}$, the initial data $\mathcal{U}_0$ and the matrix of charges $\mathcal{F}$ are defined by

$$
\begin{aligned}
\mathcal{U}&=\left(u_i,v_i,\varphi_i,\Psi_i,\phi,\Phi,\chi,\mu,\tau_i,\nu_i\right),\\
\mathcal{U}_0&=\left(u_i^0,v_i^0,\varphi_i^0,\Psi_i^0,\phi^0,\Phi^0,\chi^0,\mu^0,\tau_i^0,\nu_i^0\right),\\
\mathcal{F}&=\left(\mathbf{0},F_i,\mathbf{0},G_i,0,L,0,s,\mathbf{0},Q_i\right).
\end{aligned}
$$

We will prove, in the next theorem, a property of the operator $\mathcal{T}$ which is needed to prove the existence of the solution of the abstract problem (28).

Theorem 3.3.

We assume that the following assumptions are met;

1. ϱ, I_{ij}, J and the constitutive coefficient a are strictly positive;

2. the symmetry relations (7) take place;

3. the quadratic form U defined in (15) is positive definite; and

4. the constitutive coefficients B_{ij} are components of a positive definite tensor.

Then, the operator $\mathcal{T}$ is dissipative.

Proof:

In fact, we have to prove that

$$\langle \mathcal{T}\mathcal{U},\mathcal{U}\rangle \leq 0, \ \forall\, \mathcal{U} \in D(\mathcal{T}). \tag{29}$$

Let us consider $\mathcal{U}$, an arbitrary element in the domain of the operator $\mathcal{T}$. Taking into account the definition of the scalar product (26) and the expressions of the operators defined in (27), we obtain

$$\begin{aligned}\langle \mathcal{T}\mathcal{U},\mathcal{U}\rangle = & -\int_{\partial B} \left(t_{ji}U_i + m_{ij}\Psi_i + \lambda_j \Phi + S_j \mu + \Lambda_{ij}\nu_i\right) n_j dA \\ & + \int_B \big[A_{ijmn}\varepsilon_{ij}\varepsilon^*_{mn} + B_{ijmn}\left(\varepsilon_{ij}\mu^*_{mn} + \varepsilon^*_{ij}\mu_{mn}\right) + a_{ij}\left(\varepsilon_{ij}\phi^* + \varepsilon^*_{ij}\phi\right) \\ & + D_{ijmn}\left(\varepsilon_{ij}\tau^*_{m,n} + \varepsilon^*_{ij}\tau_{m,n}\right) + C_{ijmn}\mu_{ij}\mu^*_{mn} + b_{ij}\left(\mu_{ij}\phi^* + \mu^*_{ij}\phi\right) \\ & + E_{ijmn}\left(\mu_{ij}\tau^*_{m,n} + \mu^*_{ij}\tau_{m,n}\right) + A_{ij}\phi_{,i}\phi^*_{,j} + H_{ij}\left(\phi_{,i}\chi^*_{,j} + \phi^*_{,i}\chi_{,j}\right) \\ & + \zeta\phi\phi^* + F_{ij}\left(\tau_{i,j}\phi^* + \tau^*_{i,j}\phi\right) + K_{ij}\chi_{,i}\chi^*_{,j} + G_{ijmn}\tau_{m,n}\tau^*_{i,j}\big]\, dV.\end{aligned} \tag{30}$$

The integrand in the last integral from (30) is a quadratic form which corresponds to the elements $\boldsymbol{\omega} = (u_i, \varphi_i, \phi, \chi, \tau_i)$ and $\boldsymbol{\omega}^* = (U_i, \Psi_i, \Phi, \mu, \nu_i)$, that is, this integral is of the form

$$\int_B W(\boldsymbol{\omega},\boldsymbol{\omega}^*)dV = \int_B W\left(\left(u_i, \varphi_i, \phi, \chi, \tau_i\right), \left(U_i, \Psi_i, \Phi, \mu, \nu_i\right)\right) dV.$$

Keep in mind this observation and apply the divergence theorem in the first integral in (30) so that we get

$$\begin{aligned}\langle \mathcal{T}\mathcal{U},\mathcal{U}\rangle = & -\int_B \left(t_{ji}U_{i,j} + m_{ji}\Psi_i + \lambda_j \Phi_{,j} + S_j \mu_{,j} + \Lambda_{ij}\nu_{i,j}\right) dV \\ & + \int_B W\left(\left(u_i, \varphi_i, \phi, \chi, \tau_i\right), \left(U_i, \Psi_i, \Phi, \mu, \nu_i\right)\right) dV = 0,\end{aligned}$$

which concludes the proof of the theorem. ■

The property of the operator $\mathcal{T}$ which will be proved in the following theorem is essential to characterize the solution of the problem (28).

Theorem 3.4.

Suppose that the conditions of the Theorem 3 are satisfied. Then, the operator $\mathcal{T}$ satisfies the range condition.

Proof:

Let $\mathcal{U}^*$ be an element in the Hilbert space $\mathcal{H}$, defined above, that is, it has the form $\mathcal{U}^* = (u_i^*, U_i^*, \varphi_i^*, \Psi_i^*, \phi^*, \Phi^*, \chi^*, \mu^*, \tau_i^*, \nu_i^*)$. The affirmation of the statement of the theorem is equivalent to showing that equation $\mathcal{T}\mathcal{U} = \mathcal{U}^*$ has a solution $\mathcal{U} \in D(\mathcal{T})$. In view of operators (27), we will use the vector notations

$$\begin{array}{cccccc} \mathbf{A}^1 = (A_i^1), & \mathbf{A}^2 = (A_i^2), & \mathbf{A}^3 = (A_i^3), & \mathbf{A}^4 = (A_s^4), & \mathbf{A}^5 = (A_s^5), & \mathbf{A}^6 = (A_s^6), \\ \mathbf{B}^1 = (B_i^1), & \mathbf{B}^2 = (B_s^2), & \mathbf{C}^1 = (C_i^1), & \mathbf{C}^2 = (C_s^2), & \mathbf{D}^1 = (D_i^1), & \mathbf{D}^2 = (D_s^2), \\ & \mathbf{W} = (W_s), & \mathbf{X} = (X_s), & \mathbf{Y} = (Y_s), & \mathbf{Z} = (Z_s). & \end{array} \tag{31}$$

Taking into account the operators from (27) and the notations (31), the system of equations (10) can be rewritten in the form

$$\begin{aligned} &\mathbf{U} = \mathbf{u}^*, \\ &\quad \mathbf{A}^1\mathbf{u} + \mathbf{A}^2\boldsymbol{\varphi} + \mathbf{B}^1\phi + \mathbf{C}^1\mu + \mathbf{D}^1\boldsymbol{\tau} = \mathbf{U}^*, \\ &\boldsymbol{\Psi} = \boldsymbol{\varphi}^*, \\ &\quad \mathbf{A}^3\mathbf{u} + \mathbf{A}^4\boldsymbol{\varphi} + \mathbf{B}^2\phi + \mathbf{C}^2\mu + \mathbf{D}^2\boldsymbol{\tau} = \boldsymbol{\Psi}^*, \\ &\Phi = \phi^*, \\ &\quad H\mathbf{u} + E\phi + G\chi + L\mu + M\boldsymbol{\tau} + F\boldsymbol{\nu} = \Phi^*, \\ &\mu = \chi^*, \\ &\quad R\mathbf{U} + P\phi + S\Phi + N\chi + Q\boldsymbol{\nu} = \mu^*, \\ &\boldsymbol{\nu} = \boldsymbol{\tau}^*, \\ &\quad \mathbf{A}^5\mathbf{u} + \mathbf{A}^6\boldsymbol{\varphi} + \mathbf{W}\phi + \mathbf{Z}\Phi + \mathbf{X}\mu + \mathbf{Y}\boldsymbol{\tau} = \boldsymbol{\nu}^*. \end{aligned} \tag{32}$$

In the next step, from the system (32) we get a new system of equations in which the main unknowns are $(\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau})$ and the other variables pass on the right-hand side, in the role of "free terms". The resulting system is

$$\begin{aligned} &\mathbf{A}^1\mathbf{u} + \mathbf{A}^2\boldsymbol{\varphi} + \mathbf{B}^1\phi + \mathbf{D}^1\boldsymbol{\tau} = \mathbf{U}^* - \mathbf{C}^1\chi^*, \\ &\mathbf{A}^3\mathbf{u} + \mathbf{A}^4\boldsymbol{\varphi} + \mathbf{B}^2\phi + \mathbf{D}^2\boldsymbol{\tau} = \boldsymbol{\Psi}^* - \mathbf{C}^2\chi^*, \\ &H\mathbf{u} + E\phi + G\chi + M\boldsymbol{\tau} = \Phi^* - L\chi^* - F\boldsymbol{\tau}^*, \\ &P\phi + N\chi = \mu^* - R\mathbf{u}^* - S\phi^* - Q\boldsymbol{\tau}^*, \\ &\mathbf{A}^5\mathbf{u} + \mathbf{A}^6\boldsymbol{\varphi} + \mathbf{W}\phi + \mathbf{Y}\boldsymbol{\tau} = \boldsymbol{\nu}^* - \mathbf{Z}\phi^* - \mathbf{X}\chi^*. \end{aligned} \tag{33}$$

Now we introduce the notations

$$\begin{aligned} &\tilde{\mathbf{u}} = \mathbf{A}^1\mathbf{u} + \mathbf{A}^2\boldsymbol{\varphi} + \mathbf{B}^1\phi + \mathbf{D}^1\boldsymbol{\tau}, \\ &\tilde{\boldsymbol{\varphi}} = \mathbf{A}^3\mathbf{u} + \mathbf{A}^4\boldsymbol{\varphi} + \mathbf{B}^2\phi + \mathbf{D}^2\boldsymbol{\tau}, \\ &\tilde{\phi} = H\mathbf{u} + E\phi + G\chi + M\boldsymbol{\tau}, \\ &\tilde{\chi} = P\phi + N\chi, \\ &\tilde{\boldsymbol{\tau}} = \mathbf{A}^5\mathbf{u} + \mathbf{A}^6\boldsymbol{\varphi} + \mathbf{W}\phi + \mathbf{Y}\boldsymbol{\tau}, \end{aligned} \tag{34}$$

such that the scalar product $\left\langle \left(\tilde{\mathbf{u}}, \tilde{\boldsymbol{\varphi}}, \tilde{\phi}, \tilde{\chi}, \tilde{\boldsymbol{\tau}}\right), (\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau})\right\rangle$ is a bounded bilinear form on $W_0^{1,2}$.

Moreover, by direct calculations we obtain

$$\begin{aligned}
&\langle (\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau}), (\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau})\rangle \\
&\quad = \int_B [A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ijmn}\varepsilon_{ij}\mu_{mn} + 2a_{ij}\varepsilon_{ij}\phi + 2D_{ijmn}\varepsilon_{ij}\tau_{m,n} \\
&\quad +C_{ijmn}\mu_{ij}\mu_{mn} + 2b_{ij}\mu_{ij}\phi + 2E_{ijmn}\mu_{ij}\tau_{m,n} + A_{ij}\phi_{,i}\phi_{,j} \\
&\quad +2H_{ij}\phi_{,i}\chi_{,j} + \zeta\phi^2 + 2F_{ij}\tau_{i,j}\phi + K_{ij}\chi_{,i}\chi_{,j} + G_{ijmn}\tau_{m,n}\tau_{i,j}\Big]\, dV,
\end{aligned} \tag{35}$$

such that, based on the assumptions of the theorem, we infer that this bilinear form is coercive on the space $W_0^{1,2}$. Clearly, the functions from the right-hand side of the system (33), namely, $\mathbf{U}^* - \mathbf{C}^1\chi^*$, $\boldsymbol{\Psi}^* - \mathbf{C}^2\chi^*$, $\Phi^* - L\chi^* - F\boldsymbol{\tau}^*$, $\mu^* - R\mathbf{u}^* - S\phi^* - Q\boldsymbol{\tau}^*$, and $\boldsymbol{\nu}^* - \mathbf{Z}\phi^* - \mathbf{X}\chi^*$, are functions which belong to the space $W^{1,2}$. So, we met the conditions to apply the Lax-Milgram theorem, which ensures the existence of the functions $\mathcal{U} = (\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau})$ as a solution of the system (33), and this, in turn, ensure the existence of the solution for the system (32). Thus, the proof of the theorem is complete. ■

Based on Theorem 3 and Theorem 4 we deduce that the operator $\mathcal{T}$ satisfies the requirements of the Lumer-Phillips corollary of the known Hille-Yosida theorem (see Pazy (1983)). That is, we have the following result.

Theorem 3.5.

We assume that the following assumptions are met;

1. ϱ, I_{ij}, J and the constitutive coefficient a are strictly positive;

2. the symmetry relations (7) take place;

3. the quadratic form U defined in (15) is positive definite; and

4. the constitutive coefficients B_{ij} are components of a positive definite tensor.

Then, the operator $\mathcal{T}$ generates a semigroup of contracting operators on the Hilbert space $\mathcal{H}$. Also, with the help of the same corollary, we deduce the following result of uniqueness.

Theorem 3.6.

Suppose that the conditions of Theorem 5 are satisfied. Moreover, we assume that $F_i, G_i, L, s, Q_i \in C^1([0,\infty), L^2) \cap C^0([0,\infty), W_0^{1,2})$ and the initial data $\mathcal{U}_0$ belongs to the domain of the operator $\mathcal{T}$. Then the abstract problem (28) admits the only one solution $\mathcal{U}(t) \in C^1([0,\infty), \mathcal{H})$. A final result to characterize the solution of the abstract problem (28) is a result regarding the continuous dependence of the solutions with respect to the initial data and loads.

Theorem 3.7.

Suppose that the conditions of Theorem 5 are satisfied. Then the solution $\mathcal{U} = (\mathbf{u}, \boldsymbol{\varphi}, \phi, \chi, \boldsymbol{\tau})$ of problem (28) depends continuously with regard to the initial data $\mathcal{U}_0$ and the loads F_i, G_i, L, s, Q_i, that is,

$$|\mathcal{U}(t)| \leq |\mathcal{U}_0| + \int_0^t \|(F_i, G_i, L, s, Q_i)\| \, ds.$$

4. Conclusion

To get a more faithful behavior of modern materials both the consideration of the intimate structure of those materials and the fact that the microtemperatures are important for microparticles were proposed. Consequently, the number of unknown functions and the number of differential equations, of the boundary conditions and of the initial data have been increased. Due to the suppleness of the theory of semigroups of operators, these complications do not affect the qualitative results of the mixed initial boundary value problem considered in the context of micropolar thermoelastic bodies with microtemperatures.

REFERENCES

Adams, R.A. (1975). *Sobolev Spaces*, Academic Press, New York.

Chirita, S, and Ghiba, I.D. (2012). Rayleigh waves in Cosserat elastic materials, Int. J. Eng. Sci., Vol. 51, pp. 117–127.

Chirita, S., Ciarletta, M. and D'Apice, C. (2013). On a theory of thermoelasticity with microtemperatures, J. Math. Anal. Appl., Vol. 397, pp. 349–361.

Dyszlewicz, J. (2004). *Micropolar Theory of Elasticity*, Lect. Notes Appl. Comput. Mech., Vol. 15, Springer, Berlin/Heidelberg/New York.

Ellahi, R., Rahman, S.U., Gulzar, M.M., Nadeem, S. and Vafai, K. (2014a). A Mathematical Study of Non-Newtonian Micropolar Fluid in Arterial Blood Flow through Composite Stenosis, Appl. Math. Inf. Sci. Vol. 8, No. 4, pp. 1567–1573.

Ellahi, R., Rahman, S.U., Nadeem, S. and Akbar, N.S. (2014b). Influence of Heat and Mass Transfer on Micropolar Fluid of Blood Flow Through a Tapered Stenosed Arteries with Permeable Walls, J Comput Theor Nanos, Vol. 11, No. 4, pp. 1156–1163.

Eringen, A.C. (1996). Linear theory of micropolar elasticity, Journal of Mathematics and Mechanics, Vol. 15, pp. 909–924.

Eringen, A.C. (1999). *Microcontinuum Field Theories I: Foundations and Solids*, Springer-Verlag, New York/Berlin/Heidelber.

Grot, R.A. (1969). Thermodynamics of a continuum with microstructure, Int. J. Eng. Sci., Vol. 7, pp. 801–814.

Iesan, D. (2004). *Thermoelastic Models of Continua*, Kluwer Academic Publishers, Dordrecht.

Iesan, D. and Quintanilla, R. (2000). On a theory of thermoelasticity with microtemperatures, J. Thermal Stresses, Vol. 23, pp. 199–215.

Iesan, D. and Nappa, L. (2005). On the theory of heat for micromorphic bodies, Int. J. Eng. Sci., Vol. 43, pp. 17–32.

Marin, M. (1996). Generalized solutions in elasticity of micropolar bodies with voids, Revista de la Academia Canaria de Ciencias, Vol. 8, No. 1, pp. 101–106.

Marin, M. (1997). On weak solutions in elasticity of dipolar bodies with voids, J. Comp. Appl. Math., Vol. 82, No. 1-2, pp. 291–297.

Marin, M. (2010a). A partition of energy in thermoelasticity of microstretch bodies, Nonlinear Analysis: R.W.A., Vol. 11, No. 4, pp. 2436–2447.

Marin, M. (2010b). Harmonic vibrations in thermoelasticity of microstretch materials, J. Vibr. Acoust., ASME, Vol. 132, No. 4, pp. 044501 (6 pages).

Marin, M. (2010c). Some estimates on vibrations in thermoelasticity of dipolar bodies, Journal of Vibration and Control, Vol. 16, No. 1, pp. 33–47.

Pazy, A. (1983). *Semigroups of operators of linear operators and applications*, Springer, New York, Berlin.

Sharma, K. and Marin, M. (2013). Effect of distinct conductive and thermodynamic temperatures on the reflection of plane waves in micropolar elastic half-space, U.P.B. Sci. Bull., Series A-Appl. Math. Phys., Vol. 75, No. 2, pp. 121–132.

Straughan, B. (2011). *Heat waves, in: Applied Mathematical Sciences*, Vol. 177, Springer, New York.