Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 13, Issue 1 (June 2018), pp. 97 - 113

System reliability using generalized intuitionistic fuzzy Rayleigh lifetime distribution

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Received: October 7, 2017; Accepted: March 22, 2018

Abstract

Reliability analysis as one of the important research topics in engineering has been researched by a number of authors. Reliability in classical distributions is based on precise parameters. It is usually assumed that parameters of distributions are precise real numbers. However, in the real world, the data sometimes cannot be measured and recorded precisely. In this paper, the concept of fuzzy reliability is extended by the idea of generalized intuitionistic fuzzy reliability. We investigate the reliability characteristics of systems using Rayleigh lifetime distribution, in which the lifetime parameter is assumed to be generalized intuitionistic fuzzy number. Generalized intuitionistic fuzzy reliability, generalized intuitionistic fuzzy hazard function, generalized intuitionistic fuzzy mean time to failure and their cut sets are discussed when the systems follow generalized intuitionistic fuzzy Rayleigh lifetime distribution. In this approach, for every special cut set, reliability curve and hazard curve are like a band with upper and lower bound. A numerical example is given to illustrate the proposed approach. Further, reliability analysis of the series and parallel systems are done.

Keywords: Generalized intuitionistic fuzzy numbers; (α_1, α_2) -cut set; generalized intuitionistic fuzzy distribution; generalized intuitionistic fuzzy reliability band; Generalized intuitionistic fuzzy hazard band; Generalized intuitionistic fuzzy mean time to failure; Rayleigh distribution

MSC 2010 No.: 03E72, 90C05, 90C70

1. Introduction

One of the major fields in engineering is reliability analysis. The problem of the reliability analysis in statistical distributions used to engineering is one of the significant problems constantly facing those who are interested in lifetime data analysis. The Rayleigh distribution is known to have wide applications in survival analysis and reliability theory. Reliability analysis for the Rayleigh distribution and its generalization has been discussed by several authors such as Dey (2009), Fernandez (2010), Azimi et al. (2012), Tarvirdizade and Kazemzadeh Garehchobogh (2014), Rastogi and Merovci (2017), Potdar and Shirke (2017) and Dey et al. (2017). However, the data sometimes cannot be measured and recorded precisely. Therefore, the researchers used fuzzy sets (FSs) to overcome this problem in the reliability analysis. Baloui Jamkhaneh (2011) considered the problem of the evaluation of system reliability, in which the lifetime of components is described using a fuzzy exponential distribution. Baloui Jamkhaneh (2014) investigated system reliability using fuzzy Weibull lifetime distribution. Pak et al. (2014) presented a Bayesian approach to estimate the parameter and reliability function of Rayleigh distribution from fuzzy lifetime data. Venkatesh et al. (2017) estimated the reliability and failure rate values of the fuzzy generalized Rayleigh distribution to compute the effects of oxytocin in cesarean segment underneath spinal anesthesia.

Intuitionistic fuzzy sets (IFSs) theory defined by Atanassov (1986) as a generalization of FSs is a useful tool in modeling real life problems such as transportation problems (Ebrahimnejad and Verdegay 2016, 2017), wherein hesitation between belongingness and non-belongingness cannot be ruled out. For this reason, IFSs were considered in order to analyze the systems reliability by many researchers. Burillo et al. (1994) proposed the definition of intuitionistic fuzzy number (IFN). Mahapatra and Roy (2009) presented triangular intuitionistic fuzzy number (TIFN) and used it for reliability evaluation. Mahapatra and Mahapatra (2010) presented intuitionistic fuzzy fault tree using arithmetic operation of trapezoidal intuitionistic fuzzy number (TrIFN) which are evaluated based on (α, β)-cuts method. They discussed fault-tree of failure of fire protection system with components having failure rates as TrIFNs. Pandey et al. (2011) describes a novel approach, based on IFS theory for reliability analysis of series and parallel network. Kumar et al. (2011) developed a new approach for analyzing the fuzzy system reliability of series and parallel systems using IFS theory. Kumar and Yadav (2012) presented a method for system reliability analysis based on arithmetic operations of different types of IFNs.

Sharma et al. (2012) presented the reliability of a system using IFS. Shaw and Roy (2012) presented some arithmetic operations on TIFN and its application on reliability evaluation. Garg et al. (2013) predicted the behavior of an industrial system under imprecise and vague environment. To handle the uncertainty in the data, they used IFS theory rather than FS theory. Also, Garg and Rani (2013) presented a technique for computing the membership functions of the IFS in reliability analyzed by utilizing imprecise, uncertain and vague data. Mahapatra and Roy (2013) proposed a definition of IFN according to the fuzzy number presentation approach. Also, they discussed the fault-tree of failure to start an automobile with components having failure rates as TrIFNs. Tyagi (2014) investigated the reliability analysis of a power loom plant by using interval valued intuitionistic fuzzy sets. He modeled a power loom plant as a gracefully degradable system having two units $A(n)$ and $B(m)$ connected in series. Kumar and Singh (2017) investigated the applications of generalized TrIFN in fuzzy reliability theory.

Classic lifetime distributions have crisp parameters. However, in real world, some collected lifetime data might be imprecise and are represented in the form of IFNs. Thus, there exists some uncertainty in the value of lifetime parameter obtained from estimation. So it is necessary to generalize classic lifetime distributions to intuitionistic fuzzy lifetime distribution. Bohra and Singh (2015) used intuitionistic fuzzy Rayleigh distribution in evaluating the systems reliability. Kumar and Singh (2015) presented fuzzy system reliability using intuitionistic fuzzy Weibull lifetime distribution, while Baloui Jamkhaneh and Nadarajah (2015) considered new generalized intuitionistic fuzzy sets ($\mathrm{GIFS_B}s$) and Baloui Jamkhnaeh (2017a), Baloui Jamkhaneh and Nadi Ghara (2017) and Baloui Jamkhaneh, and Nadarajah (2018) defined some operators over the $\mathrm{GIFS_B}$s. In 2017 Baloui Jamkhaneh and Garg considered some new operations over the generalized intuitionistic fuzzy sets and their application to decision making process.

Shabani and Baloui Jamkhaneh (2014) introduced a new generalized intuitionistic fuzzy number ($\mathrm{GIFN_B}$) based on generalization of the IFS. Then Baloui Jamkhaneh (2017b) presented system reliability using generalized intuitionistic fuzzy exponential lifetime distribution. He defined generalized intuitionistic fuzzy reliability, generalized intuitionistic fuzzy hazard function, generalized intuitionistic fuzzy mean time to failure and their (α_1, α_2)-cut when systems follow generalized intuitionistic fuzzy lifetime parameter. Using the intuitionistic fuzzy set in reliability analysis is more powerful than the fuzzy sets at handling vagueness and uncertain information in practice. The main objective of this paper is to evaluate systems reliability using generalized intuitionistic fuzzy Rayleigh distribution, in which the lifetime parameter is taken as a $\mathrm{GIFN_B}$ for handling the randomness, vagueness and incompleteness in information. Intuitionistic fuzzy system reliability is based on the concept of intuitionistic fuzzy set and intuitionistic fuzzy probability theory in our method.

This paper is organized as follows: Section 2 presents basic concepts of $\mathrm{GIFN_B}$s. Section 3 presents generalized intuitionistic fuzzy Rayleigh distribution. Section 4 gives generalized intuitionistic fuzzy reliability characteristics. In Section 5, generalized intuitionistic fuzzy reliability of series and parallel systems are calculated. The paper is concluded in Section 6.

2. Preliminaries

In this section, we review some basic concepts of $\mathrm{GIFN_B}$s.

2.1. Generalized Intuitionistic Fuzzy Number

Definition 2.1.1. (Baloui Jamkhaneh, Nadarajah (2015))

Let X be a non-empty set. A generalized intuitionistic fuzzy sets $(GIFS_B(X))$ A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, denote the degree of membership and degree of non-membership functions of x in A, respectively, and $0 \leq \mu_A(x)^\delta + \ \nu_A(x)^\delta \leq 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}, n = 1,2, \dots, N$.

Definition 2.1.2. (Shabani, Baloui Jamkhaneh (2014))

In special case, generalized *L-R* type intuitionistic fuzzy number A can be described as any $GIFS_B(X)$ of the real line $\mathbb{R}$ whose membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ are defined as follows:

$$\mu_A(x)=\begin{cases}(\frac{x-a}{b-a})^{\frac{1}{\delta}} & , \quad a\le x\le b\\ 1 & , \quad b\le x\le c\\ (\frac{d-x}{d-c})^{\frac{1}{\delta}} & , \quad c\le x\le d\\ 0 & , \quad o.w.\end{cases},\qquad \nu_A(x)=\begin{cases}(\frac{b-x}{b-a_1})^{\frac{1}{\delta}} & , \quad a_1\le x\le b\\ 0 & , \quad b\le x\le c\\ (\frac{x-c}{d_1-c})^{\frac{1}{\delta}} & , \quad c\le x\le d_1\\ 1 & , \quad o.w.\end{cases},$$

where $a_1\le a\le b\le c\le d\le d_1$ and $0\le \mu_A(x)^{\delta}+\ \nu_A(x)^{\delta}\le 1,\forall x\in X$. The $GIFN_B\,A$ is denoted as $A=(a_1,a,b,c,d,d_1,\delta)$.

Definition 2.1.3.

A $GIFN_B$ is said to be symmetric $GIFN_B$ if $b-a=d-c$ and $b-a_1=d_1-c$.

2.2. Cut sets on $\mathrm{GIFN_B}$

In this subsection, we explore the concept of cut set on $\mathrm{GIFN_B}$ (Baloui Jamkhaneh 2016).
Let $\alpha_1,\alpha_2\in[0,1]$ be fixed numbers such that $0\le{\alpha_1}^{\delta}+{\alpha_2}^{\delta}\le 1$. A set of (α_1,α_2)-cut generated by a $GIFN_B\ A$ is defined by

$$A[\alpha_1,\alpha_2,\delta]=\{\langle x,\mu_A(x)\ge\alpha_1,\nu_A(x)\le\alpha_2\rangle : x\in X\}.$$

The α_1- cut set of a $GIFN_B\ A$ is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\alpha_1,\delta]=\{\langle x,\mu_A(x)\ge\alpha_1,\rangle : x\in X\}\qquad ,\ 0\le\alpha_1\le 1.$$

According to the definition of $GIFN_B$, it can be easily shown that

$$A[\alpha_1,\delta]=[L_1(\alpha_1),U_1(\alpha_1)]\qquad ,\ 0\le\alpha_1\le 1,$$
$$L_1(\alpha_1)=a+(b-a){\alpha_1}^{\delta}\quad ,\qquad U_1(\alpha)=d-(d-c){\alpha_1}^{\delta}.$$

Similarity a α_2 - cut set of a $GIFN_B\ A$ is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\alpha_2,\delta]=\{\langle x,\nu_A(x)\le\alpha_2\rangle : x\in X\}\quad ,\quad 0\le\alpha_2\le 1.$$

According to the definition of $GIFN_B$, it can be easily shown that

$$A[\alpha_2,\delta]=[L_2(\alpha_2),U_2(\alpha_2)]\quad ,\quad 0\le\alpha_2\le 1,$$
$$\mathrm{L_2(\alpha_2)=b\big(1-{\alpha_2}^{\delta}\big)+a_1{\alpha_2}^{\delta}}\quad ,\qquad \mathrm{U_2(\beta)=c\big(1-{\alpha_2}^{\delta}\big)+d_1{\alpha_2}^{\delta}}.$$

Therefore, the (α_1,α_2)-cut set of a $GIFN_B$ is given by

$$A[\alpha_1,\alpha_2,\delta]=\{x,x\in[L_1(\alpha_1),U_1(\alpha_1)]\cap[L_2(\alpha_2),U_2(\alpha_2)]\}.$$

3. Generalized Intuitionistic Fuzzy Distribution

Let the lifetime random variable of a component (*X*) is modeled by $f(x,\tilde{\lambda})$, where $\tilde{\lambda}$ is a $GIFN_B$. In this case, the generalized intuitionitic fuzzy probability of obtaining a value in B is $\tilde{P}(B)$ which computes its cut sets as follows.

$$P(B)[\alpha_i] = \{\int_B f(x,\lambda)dx \big| \lambda \in \lambda[\alpha_i,\delta] = [P^L[\alpha_i], P^U[\alpha_i]], \quad i = 1,2,$$

for all $0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq {\alpha_1}^{\delta} + {\alpha_2}^{\delta} \leq 1$, where

$$P^L[\alpha_i] = min\{\int_B f(x,\lambda)dx \big| \lambda \in \lambda[\alpha_i,\delta], i = 1,2.$$

$$P^U[\alpha_i] = max\{\int_B f(x,\lambda)dx \big| \lambda \in \lambda[\alpha_i,\delta] , i = 1,2.$$

Based on this definition, $\tilde{P}(B)$ is a $GIFN_B$, where $[P^L[\alpha_1], P^U[\alpha_1]]$ and $[P^L[\alpha_2], P^U[\alpha_2]]$ are α_1 - cut set of membership function, and α_2 - cut set of non-membership function, respectively. Also (α_1, α_2)- cut set is as follows

$$P(B)[\alpha_1, \alpha_2] = [P^L[\alpha_1], P^U[\alpha_1]] \cap [P^L[\alpha_2], P^U[\alpha_2]].$$

Let lifetime random variable of a component (*X*) be modeled by a Rayleigh distribution. Then

$$f(x,\tilde{\lambda}) = \frac{2x}{\tilde{\lambda}} e^{-\frac{x^2}{\tilde{\lambda}}}, \quad x > 0,$$

where $\tilde{\lambda}$ is a $GIFN_B$. In this case we have

$$\tilde{P}(n \leq X \leq m)[\alpha_i] = \{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \,|\lambda \in \lambda[\alpha_i, \delta]\} = [P^L[\alpha_i], P^U[\alpha_i]], \quad i = 1,2,$$

for all $0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq {\alpha_1}^{\delta} + {\alpha_2}^{\delta} \leq 1$, where

$$P^L[\alpha_i] = \min\{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \,|\lambda \in \lambda[\alpha_i, \delta]\} = \min\{e^{-\frac{n^2}{\lambda}} - e^{-\frac{m^2}{\lambda}} |\lambda \in \lambda[\alpha_i, \delta]\}, \quad i = 1,2,$$

$$P^U[\alpha_i] = \max\{\int_n^m \frac{2x}{\lambda} e^{-\frac{x^2}{\lambda}} dx \,|\lambda \in \lambda[\alpha_i, \delta]\} = \max\left\{e^{-\frac{n^2}{\lambda}} - e^{-\frac{m^2}{\lambda}} |\lambda \in \lambda[\alpha_i, \delta]\right\}, \quad i = 1,2.$$

4. Reliability Characteristics

In this section, we present generalized intuitionistic fuzzy reliability characteristics.

4.1. Generalized Intuitionistic Fuzzy Reliability Band

Generalized intuitionistic fuzzy reliability (GIFR) is based on $GIFN_B$ defined by Shabani and Baloui Jamkhaneh (2014). *GIFR* is the generalized intuitionistic fuzzy probability unit which survives beyond time *t*. Let the lifetime random variable *X* have a Rayleigh distribution with generalized intuitionistic fuzzy lifetime parameter $\tilde{\lambda}=(a_1,a,b,c,d,d_1,\delta)$. In this case, *GIFR* of component is $\tilde{S}(t)$ and compute its cut sets as follows:

$$S(t)[\alpha_i]=P(X>t)[\alpha_i]=\{\int_t^{\infty}\frac{2x}{\lambda}e^{-\frac{x^2}{\lambda}}dx\ |\lambda\in\lambda[\alpha_i,\delta]\}=\{e^{-\frac{t^2}{\lambda}}|\lambda\in\lambda[\alpha_i,\delta]\},\quad i=1,2.$$

Since $e^{-\frac{t^2}{\lambda}}$ is a monotonically increasing function with respect to λ, then α_1-cut set of membership function and α_2-cut set of non-membership function are as follow:

$$S(t)[\alpha_1]=\left[S^L[\alpha_1],S^U[\alpha_1]\right]=\left[e^{-\frac{t^2}{\left(a+(b-a)\alpha_1^{\delta}\right)}},e^{-\frac{t^2}{\left(d-(d-c)\alpha_1^{\delta}\right)}}\right],$$
$$S(t)[\alpha_2]=\left[S^L[\alpha_2],S^U[\alpha_2]\right]=\left[e^{-\frac{t^2}{\left(b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}\right)}},e^{-\frac{t^2}{\left(c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}\right)}}\right],$$

where $S(t)[\alpha_i], i=1,2$ are two dimensional functions in terms of α_i and t ($0\le\alpha_1\le1, 0\le\alpha_2\le1, 0\le{\alpha_1}^{\delta}+{\alpha_2}^{\delta}\le1$ and $t>0$). For t_0, $\tilde{S}(t_0)$ is a $GIFN_B$ and membership function and non-membership functions of $\tilde{S}(t_0)$ are as follows:

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\frac{-\frac{{t_0}^2}{\ln x}-a}{b-a}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{a}}\le x\le e^{-\frac{{t_0}^2}{b}}\\ 1, & e^{-\frac{{t_0}^2}{b}}\le x\le e^{-\frac{{t_0}^2}{c}},\\ \left(\frac{\frac{{t_0}^2}{\ln x}+d}{d-c}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{c}}\le x\le e^{-\frac{{t_0}^2}{d}}\\ 0, & o.w.\end{cases}$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\frac{\frac{{t_0}^2}{\ln x}+b}{b-a_1}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{a_1}}\le x\le e^{-\frac{{t_0}^2}{b}}\\ 0, & e^{-\frac{{t_0}^2}{b}}\le x\le e^{-\frac{{t_0}^2}{c}}.\\ \left(\frac{-\frac{{t_0}^2}{\ln x}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & e^{-\frac{{t_0}^2}{c}}\le x\le e^{-\frac{{t_0}^2}{d_1}}\\ 1, & o.w.\end{cases}$$

A (α_1,α_2)- cut set of $\tilde{S}(t)$ is as follows

$$S(t)[\alpha_1,\alpha_2]=S(t)[\alpha_1]\cap S(t)[\alpha_2].$$

In this method, for every special α_{10} and α_{20} reliability curve is like a band with upper and lower bounds. In this case, it is called *GIFR* band. This reliability band has the following properties:

(i) $S(0)[\alpha_{10}, \alpha_{20}] = [1,1]$, i.e. no one starts off dead,
(ii) $S(\infty)[\alpha_{10}, \alpha_{20}] = [0,0]$, i.e. everyone dies eventually,
(iii) $S(t_1)[\alpha_{10}, \alpha_{20}] \geq S(t_2)[\alpha_{10}, \alpha_{20}] \Leftrightarrow t_1 \leq t_2$, i.e. band of $S(t)[\alpha_{10}, \alpha_{20}]$ declines monotonically.

Corollary4.1.1.

Let $\eta = \frac{\alpha_2^\delta}{1-\alpha_1^\delta}$, $k_1 = \frac{b-a}{b-a_1}$ and $k_2 = \frac{d-c}{d_1-c}$, then we have

if $k_1 < k_2$

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} \left[S^L[\alpha_2], S^U[\alpha_2]\right], & \eta < k_1 \\ [S^L[\alpha_1], S^U[\alpha_2]], & k_1 \leq \eta \leq k_2, \\ \left[S^L[\alpha_1], S^U[\alpha_1]\right], & k_2 < \eta. \end{cases}$$

if $k_1 > k_2$

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} \left[S^L[\alpha_2], S^U[\alpha_2]\right], & \eta < k_2 \\ [S^L[\alpha_2], S^U[\alpha_1]], & k_2 \leq \eta \leq k_1, \\ \left[S^L[\alpha_1], S^U[\alpha_1]\right], & k_1 < \eta. \end{cases}$$

if $k_1 = k_2 = k$ (i.e. $\tilde{\lambda}$ is symmetric $GIFN_B$)

$$S(t_0)[\alpha_1, \alpha_2] = \begin{cases} \left[S^L[\alpha_2], S^U[\alpha_2]\right], & \eta < k \\ \left[S^L[\alpha_1], S^U[\alpha_1]\right], & \eta \geq k. \end{cases},$$

if $\eta = 1$(i.e. $\alpha_2^\delta = 1 - \alpha_1^\delta$), then $S(t_0)[\alpha_1, \alpha_2] = \left[S^L[\alpha_1], S^U[\alpha_1]\right]$.

4.2. Generalized Intuitionistic Fuzzy Hazard Band

Let the lifetime random variable *X* have a Rayleigh distribution with generalized intuitionistic fuzzy lifetime parameter $\tilde{\lambda} = (a_1, a, b, c, d, d_1, \delta)$. In this case, generalized intuitionistic fuzzy hazard function (*GIFHF*) of component ($\tilde{h}(t)$) is as follows:

$$h(t)[\alpha_i] = \left\{\frac{f(t)}{s(t)} | \lambda \in \lambda[\alpha_i, \delta]\right\} = \left\{\frac{2t}{\lambda} | \lambda \in \lambda[\alpha_i, \delta]\right\}, i = 1,2,$$
$$h(t)[\alpha_i] = \left[h(t)^L[\alpha_i], h(t)^U[\alpha_i]\right], \qquad i = 1,2,$$

for all $0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq {\alpha_1}^\delta + {\alpha_2}^\delta \leq 1$, where

$$h(t)^L[\alpha_i] = \min\{\tfrac{2t}{\lambda} | \lambda \in \lambda[\alpha_i, \delta]\} \quad , \quad h(t)^U[\alpha_i] = \max\{\tfrac{2t}{\lambda} | \lambda \in \lambda[\alpha_i, \delta]\}, i = 1,2.$$

Therefore,

$$h(t)[\alpha_1] = [\frac{2t}{d-(d-c){\alpha_1}^{\delta}}, \frac{2t}{a+(b-a){\alpha_1}^{\delta}}] \ , \ h(t)[\alpha_2] = [\frac{2t}{c(1-{\alpha_2}^{\delta})+d_1{\alpha_2}^{\delta}}, \frac{2t}{b(1-{\alpha_2}^{\delta})+a_1{\alpha_2}^{\delta}}].$$

The (α_1, α_2)- cut set of $\tilde{h}(t)$ is as follows

$$h(t)[\alpha_1, \alpha_2] = h(t)[\alpha_1] \cap h(t)[\alpha_2].$$

Thus, for a generalized intuitionistic fuzzy Rayleigh distribution, the GIFH function is increasing with respect to time. For t_0, $\tilde{h}(t_0)$ is a generalized intuitionistic fuzzy number and membership function and non-membership function of $\tilde{h}(t_0)$ are as follows:

$$\mu_{h(t_0)}(x) = \begin{cases} \left(\frac{d-\frac{2t_0}{x}}{d-c}\right)^{\frac{1}{\delta}} & , \quad \frac{2t_0}{d} \le x \le \frac{2t_0}{c} \\ 1 & , \quad \frac{2t_0}{c} \le x \le \frac{2t_0}{b} \\ (\frac{\frac{2t_0}{x}-a}{b-a})^{\frac{1}{\delta}} & , \quad \frac{2t_0}{b} \le x \le \frac{2t_0}{a} \\ 0 & , \quad o.w. \end{cases}, \nu_{h(t_0)}(x) = \begin{cases} (\frac{\frac{2t_0}{x}-c}{d_1-c})^{\frac{1}{\delta}} & , \quad \frac{2t_0}{d_1} \le x \le \frac{2t_0}{c} \\ 0 & , \quad \frac{2t_0}{c} \le x \le \frac{2t_0}{b} \\ \left(\frac{b-\frac{2t_0}{x}}{b-a_1}\right)^{\frac{1}{\delta}} & , \quad \frac{2t_0}{b} \le x \le \frac{2t_0}{a_1} \\ 1 & , \quad o.w. \end{cases}.$$

In this method, for every special α_{10} and α_{20} hazard function ($\tilde{h}(t)$) is like a band with upper and lower bounds. In this case, it is called GIFH band. The bandwidth depends on the value of δ parameter and the uncertainty value of the lifetime parameter. When the value of δ increases, the bandwidth of $S(t)[\alpha_{10}]$ and $h(t)[\alpha_{10}]$ becomes wider, and the bandwidth of $S(t)[\alpha_{20}]$ and $h(t)[\alpha_{20}]$ narrows down. Moreover, more uncertainty value results in a more bandwidth.

Corollary 4.2.1.

For every δ, $S(t)[1,0] = \left[e^{-\frac{t^2}{b}}, e^{-\frac{t^2}{c}}\right]$, $h(t)[1,0] = \left[\frac{2t}{c}, \frac{2t}{b}\right]$, $S(t)[0,1] = \left[e^{-\frac{t^2}{a}}, e^{-\frac{t^2}{d}}\right]$, $h(t)[1,0] = \left[\frac{2t}{d}, \frac{2t}{a}\right]$.

Corollary4.2.2.

Let $\eta = \frac{\alpha_2^{\delta}}{1-\alpha_1^{\delta}}$, $k_1 = \frac{b-a}{b-a_1}$and $k_2 = \frac{d-c}{d_1-c}$, then we have
if $k_1 < k_2$

$$h(t_0)[\alpha_1, \alpha_2] = \begin{cases} \left[h^L[\alpha_2], h^U[\alpha_2]\right] & , \quad \eta < k_1 \\ [h^L[\alpha_2], h^U[\alpha_1]] & , \quad k_1 \le \eta \le k_2 \\ \left[h^L[\alpha_1], h^U[\alpha_1]\right] & , \quad k_2 < \eta. \end{cases},$$

if $k_1 > k_2$

$$h(t_0)[\alpha_1,\alpha_2]=\begin{cases}\left[h^L[\alpha_2],h^U[\alpha_2]\right], & \eta<k_2\\ [h^L[\alpha_1],h^U[\alpha_2]], & k_2\le\eta\le k_1,\\ \left[h^L[\alpha_1],h^U[\alpha_1]\right], & k_1<\eta.\end{cases}$$

if $k_1=k_2=k$

$$h(t_0)[\alpha_1,\alpha_2]=\begin{cases}\left[h^L[\alpha_2],h^U[\alpha_2]\right], & \eta<k\\ \left[h^L[\alpha_1],h^U[\alpha_1]\right], & \eta\ge k.\end{cases},$$

if $\eta=1$(i.e. $\alpha_2^\delta=1-\alpha_1^\delta$), then $h(t_0)[\alpha_1,\alpha_2]=\left[h^L[\alpha_1],h^U[\alpha_1]\right]$.

4.3. Generalized Intuitionistic Fuzzy Mean Time to Failure

Generalized intuitionistic fuzzy mean time to failure (GIFMTTF) is the expected the mean time to failure (MTTF). According definition, MTTF of any component with generalized intuitionistic fuzzy Rayleigh distribution is a $GIFN_B$ defined as follows:

$$GIFMTTF[\alpha_i]=\{\int_0^\infty s(x)dx\,|\lambda\in\lambda[\alpha_i,\delta]\}$$
$$=\{\int_0^\infty e^{-\frac{x^2}{\lambda}}dx\,|\lambda\in\lambda[\alpha_i,\delta]\}=\{\frac{\sqrt{\pi\lambda}}{2}|\lambda\in\lambda[\alpha_i,\delta]\},\quad i=1,2,$$
$$GIFMTTF[\alpha_1]=\left[\frac{\sqrt{\pi\left(a+(b-a)\alpha_1^\delta\right)}}{2},\frac{\sqrt{\pi\left(d-(d-c)\alpha_1^\delta\right)}}{2}\right],$$
$$\mathrm{GIFMTTF}[\alpha_2]=\left[\frac{\sqrt{\pi\left(b\left(1-\alpha_2^\delta\right)+a_1\alpha_2^\delta\right)}}{2},\frac{\sqrt{\pi\left(c\left(1-\alpha_2^\delta\right)+d_1\alpha_2^\delta\right)}}{2}\right],$$

where membership function and non-membership function of $GIFMTTF$ are defined as follows:

$$\mu_G(x)=\begin{cases}\left(\frac{\frac{4x^2}{\pi}-a}{b-a}\right)^{\frac{1}{\delta}}, & \frac{1}{2}\sqrt{\pi a}\le x\le\frac{1}{2}\sqrt{\pi b}\\ 1, & \frac{1}{2}\sqrt{\pi b}\le x\le\frac{1}{2}\sqrt{\pi c}\\ \left(\frac{d-\frac{4x^2}{\pi}}{d-c}\right)^{\frac{1}{\delta}}, & \frac{1}{2}\sqrt{\pi c}\le x\le\frac{1}{2}\sqrt{\pi d}\\ 0, & o.w.\end{cases},$$

$$\nu_G(x) = \begin{cases} \left(\frac{-\frac{4x^2}{\pi}+b}{b-a_1}\right)^{\frac{1}{\delta}} & , \quad \frac{1}{2}\sqrt{\pi a_1} \le x \le \frac{1}{2}\sqrt{\pi b} \\ 0 & , \quad \frac{1}{2}\sqrt{\pi b} \le x \le \frac{1}{2}\sqrt{\pi c} \\ \left(\frac{\frac{4x^2}{\pi}-c}{d_1-c}\right)^{\frac{1}{\delta}} & , \quad \frac{1}{2}\sqrt{\pi c} \le x \le \frac{1}{2}\sqrt{\pi d_1} \\ 1 & , \quad o.w. \end{cases}.$$

4.4. Numerical Example

Let the lifetime of electronic product be modeled by a Rayleigh distribution with generalized intuitionistic fuzzy parameter $\tilde{\lambda} = (0.2,0.25,0.3,0.35,0.4,0.45,0.5)$. Then α_i -cut of generalized intuitionistic fuzzy probability of $0 \le X \le 2$ is as follows:

$$P(0 \le X \le 2)[\alpha_i] = \{1 - e^{-\frac{4}{\lambda}} | \lambda \in \lambda[\alpha_i, 0.5]\}, \quad i = 1,2.$$
$$P(0 \le X \le 2)[\alpha_1] = \left[1 - e^{-\frac{4}{0.4-0.05\alpha_1{}^{0.5}}}, 1 - e^{-\frac{4}{0.25+0.05\alpha_1{}^{0.5}}}\right].$$
$$P(0 \le X \le 2)[\alpha_2] = [1 - e^{-\frac{4}{0.35+0.1\alpha_2^{0.5}}}, 1 - e^{-\frac{4}{0.3-0.1\alpha_2{}^{0.5}}}].$$

The membership function and non-membership function of $\tilde{P}(0 \le X \le 2)$ are given as follows:

$$\mu_P(x) = \begin{cases} (8 + 80(ln(1-x))^{-1})^2 & , \quad 1 - e^{-10} \le x \le 1 - e^{-\frac{4}{0.35}} \\ 1 & , \quad 1 - e^{-\frac{4}{0.35}} \le x \le 1 - e^{-\frac{4}{0.3}} \\ (-5 - 80(ln(1-x))^{-1})^2 & , \quad 1 - e^{-\frac{4}{0.3}} \le x \le 1 - e^{-16} \\ 0 & , \quad o.w. \end{cases},$$

$$\nu_p(x) = \begin{cases} (-3.5 - 40(ln(1-x))^{-1})^2 & , \quad 1 - e^{-\frac{4}{0.45}} \le x \le 1 - e^{-\frac{4}{0.35}} \\ 0 & , \quad 1 - e^{-\frac{4}{0.35}} \le x \le 1 - e^{-\frac{4}{0.3}} \\ (3 + 40(ln(1-x))^{-1})^2 & , \quad 1 - e^{-\frac{4}{0.3}} \le x \le 1 - e^{-\frac{4}{0.2}} \\ 1 & , \quad o.w. \end{cases}.$$

Table 1. Values of $\tilde{P}(0 \le X \le 2)$ for different (α_1, α_2) –cuts

α_1	α_2	$\tilde{P}(0 \le X \le 2)$
1	0	$[1 - e^{-11.4256}, 1 - e^{-13.3333}]$
0.5	0.5	$[1 - e^{-10.9696}, 1 - e^{-14.0176}]$
0	1	$[1 - e^{-10}, 1 - e^{-16}]$

Table 1 shows (α_1, α_2)-cuts of generalized intuitionistic fuzzy probability of $0 \le X \le 2$. According to this table we see that, with increasing $\alpha_2$and decreasing α_1, ambiguity increases in probability. The α_i-cut and (α_1, α_2) -cut of *GIFR* of component is given by

$$S(t)[\alpha_1] = \left[e^{-\frac{t^2}{0.25+0.05\alpha_1{}^{0.5}}}, e^{-\frac{t^2}{0.4-0.05\alpha_1{}^{0.5}}}\right], \qquad S(t)[\alpha_2] = \left[e^{-\frac{t^2}{0.3-0.1\alpha_2{}^{0.5}}}, e^{-\frac{t^2}{0.35+0.1\alpha_2^{0.5}}}\right].$$

$$S(t)[\alpha_1, \alpha_2] = \left[e^{-\frac{t^2}{0.25+0.05\alpha_1{}^{0.5}}}, e^{-\frac{t^2}{0.4-0.05\alpha_1{}^{0.5}}}\right] \cap \left[e^{-\frac{t^2}{0.3-0.1\alpha_2{}^{0.5}}}, e^{-\frac{t^2}{0.35+0.1\alpha_2^{0.5}}}\right].$$

If $t = 2$ then α_i-cut set of *GIFR* are

$$S(2)[\alpha_1] = \left[S^L[\alpha_1], S^U[\alpha_1]\right] = \left[e^{-\frac{4}{0.25+0.05\alpha_1{}^{0.5}}}, e^{-\frac{4}{0.4-0.05\alpha_1{}^{0.5}}}\right].$$

$$S(2)[\alpha_2] = \left[S^L[\alpha_2], S^U[\alpha_2]\right] = \left[e^{-\frac{4}{0.3-0.1\alpha_2{}^{0.5}}}, e^{-\frac{4}{0.35+0.1\alpha_2^{0.5}}}\right].$$

The membership function and non-membership function of $\tilde{S}(2)$ are as follows:

$$\mu_{S(2)}(x) = \begin{cases} \left(\frac{-\frac{4}{\ln x}-0.25}{0.05}\right)^2 , & e^{-\frac{4}{0.25}} \le x \le e^{-\frac{4}{0.3}} \\ 1 , & e^{-\frac{4}{0.3}} \le x \le e^{-\frac{4}{0.35}}, \\ (\frac{\frac{4}{\ln x}+0.4}{0.05})^2 , & e^{-\frac{4}{0.35}} \le x \le e^{-\frac{4}{0.4}} \\ 0 , & o.w. \end{cases}$$

$$\nu_{S(2)}(x) = \begin{cases} \left(\frac{\frac{4}{\ln x}+0.3}{0.1}\right)^2 , & e^{-\frac{4}{0.2}} \le x \le e^{-\frac{4}{0.3}} \\ 0 , & e^{-\frac{4}{0.3}} \le x \le e^{-\frac{4}{0.35}}. \\ (\frac{-\frac{4}{\ln x}-0.35}{0.1})^2 , & e^{-\frac{4}{0.35}} \le x \le e^{-\frac{4}{0.45}} \\ 1 , & o.w. \end{cases}$$

The $(0.5,0.5)$-cut set of $\tilde{S}(t)$ is as follows

$$S(t)[\alpha_1] = \left[e^{-3.5044t^2}, e^{-2.7424t^2}\right] \quad , \qquad S(t)[\alpha_2] = \left[e^{-4.3613t^2}, e^{-2.3769t^2}\right].$$

$$S(t)[0.5,0.5] = S(t)[\alpha_1] \cap S(t)[\alpha_2] = \left[e^{-3.5044t^2}, e^{-2.7424t^2}\right].$$

The(1,0)-cut set of $\tilde{S}(t)$ is as follows

$$S(t)[\alpha_1] = \left[e^{-3.3333t^2}, e^{-2.8571t^2}\right] \quad , \quad S(t)[\alpha_2] = \left[e^{-3.3333t^2}, e^{-2.8571t^2}\right].$$

$$S(t)[1,0] = \left[e^{-3.3333t^2}, e^{-2.8571t^2}\right].$$

The α_i-cut and (α_1,α_2) -cut of *GIFHF* are given by

$$h(t)[\alpha_1]=[\frac{2t}{0.4-0.05{\alpha_1}^{0.5}},\frac{2t}{0.25+0.05{\alpha_1}^{0.5}}]\ ,\ h(t)[\alpha_2]=\left[\frac{2t}{0.35+0.1\alpha_2^{0.5}},\frac{2t}{0.3-0.1{\alpha_2}^{0.5}}\right].$$

$$h(t)(\alpha_1,\alpha_2)=[\frac{2t}{0.4-0.05{\alpha_1}^{0.5}},\frac{2t}{0.25+0.05{\alpha_1}^{0.5}}]\cap[\frac{2t}{0.35+0.1\alpha_2^{0.5}},\frac{2t}{0.3-0.1{\alpha_2}^{0.5}}].$$

The membership function and non-membership function of $\tilde{h}(2)$ are given as follows:

$$\mu_{h(2)}(x)=\begin{cases}\left(\frac{0.4-\frac{4}{x}}{0.05}\right)^2 & , \quad 10\le x\le 11.4286\\ 1 & , \quad 11.4286\le x\le 13.3333\\ (\frac{\frac{4}{x}-0.25}{0.05})^2 & , \quad 13.3333\le x\le 16\\ 0 & , \quad o.w.\end{cases},$$

$$\nu_{h(2)}(x)=\begin{cases}(\frac{\frac{4}{x}-0.35}{0.1})^2 & , \quad 8.8888\le x\le 11.4286\\ 0 & , \quad 11.4286\le x\le 13.3333\\ \left(\frac{0.3-\frac{4}{x}}{0.1}\right)^2 & , \quad 13.3333\le x\le 20\\ 1 & , \quad o.w.\end{cases}.$$

The (0.5,0.5)- cut sets of $\tilde{h}(t)$ is as follows

$$h(t)[\alpha_1]=[5.4848t,7.0088t]\ ,\ h(t)[\alpha_2]=[4.7539t,8.7226t],$$

$$h(t)(0.5,0.5)=h(t)[\alpha_1]\cap h(t)[\alpha_2]=[5.4847t,7.0088t].$$

The α_i-cut of GIFMTTF is given by

$$\text{GIFMTTF}[\alpha_1]=\left[\frac{\sqrt{\pi\left(0.25+0.05\alpha_1^{0.5}\right)}}{2},\frac{\sqrt{\pi\left(0.4-0.05\alpha_1^{0.5}\right)}}{2}\right],$$

$$\text{GIFMTTF}[\alpha_2]=\left[\frac{\sqrt{\pi\left(0.3-0.1\alpha_2^{0.5}\right)}}{2},\frac{\sqrt{\pi\left(0.35+0.1\alpha_2^{0.5}\right)}}{2}\right].$$

Membership function and non-membership function of $GIFMTTF$ are as follows:

$$\mu_G(x)=\begin{cases}(\frac{\frac{4x^2}{\pi}-0.25}{0.05})^2 & , \quad \frac{1}{2}\sqrt{0.25\pi}\le x\le\frac{1}{2}\sqrt{0.3\pi}\\ 1 & , \quad \frac{1}{2}\sqrt{0.3\pi}\le x\le\frac{1}{2}\sqrt{0.35\pi}\\ \left(\frac{0.4-\frac{4x^2}{\pi}}{0.05}\right)^2 & , \quad \frac{1}{2}\sqrt{0.35\pi}\le x\le\frac{1}{2}\sqrt{0.4\pi}\\ 0 & , \quad o.w.\end{cases},$$

$$\nu_G(x)=\begin{cases}\left(\frac{-\frac{4x^2}{\pi}+0.3}{0.1}\right)^2 & , \quad \frac{1}{2}\sqrt{0.2\pi}\le x\le\frac{1}{2}\sqrt{0.3\pi}\\ 0 & , \quad \frac{1}{2}\sqrt{0.3\pi}\le x\le\frac{1}{2}\sqrt{0.35\pi}\\ \left(\frac{\frac{4x^2}{\pi}-0.35}{0.1}\right)^2 & , \quad \frac{1}{2}\sqrt{0.35\pi}\le x\le\frac{1}{2}\sqrt{0.45\pi}\\ 1 & , \quad o.w.\end{cases}.$$

5. GIFR of Series and Parallel Systems

In this section, we evolve a generalized intuitionistic fuzzy reliability evaluation technique for series and parallel systems.

5.1. Series System

If n-components are connected in series, then the α_i-cut ($i=1,2$) of $GIFR$ with generalized intuitionistic fuzzy Rayleigh distribution is given by

$$S(t)[\alpha_i]=\{P(Y_1>t)\,|\lambda\in\lambda[\alpha_i,\delta]\}=\{e^{-\frac{nt^2}{\lambda}}|\lambda\in\lambda[\alpha_i,\delta]\},\quad i=1,2.$$

$$S(t)[\alpha_1]=\left[e^{-\frac{nt^2}{a+(b-a)\alpha_1^{\delta}}},e^{-\frac{nt^2}{d-(d-c)\alpha_1^{\delta}}}\right],\ S(t)[\alpha_2]=\left[e^{-\frac{nt^2}{b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}}},e^{-\frac{nt^2}{c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}}}\right],$$

For t_0, $GIFR$ is a $GIFN_B$ and membership function and non-membership function of $\tilde{S}(t)$ are as follows:

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\frac{-\frac{nt_0^2}{\ln x}-a}{b-a}\right)^{\frac{1}{\delta}}, & e^{-\frac{nt_0^2}{a}}\le x\le e^{-\frac{nt_0^2}{b}}\\ 1 & , \ e^{-\frac{nt_0^2}{b}}\le x\le e^{-\frac{nt_0^2}{c}}\\ \left(\frac{\frac{nt_0^2}{\ln x}+d}{d-c}\right)^{\frac{1}{\delta}} & , \ e^{-\frac{nt_0^2}{c}}\le x\le e^{-\frac{nt_0^2}{d}}\\ 0 & , \quad o.w.\end{cases},\nu_{S(t_0)}(x)=\begin{cases}\left(\frac{\frac{nt_0^2}{\ln x}+b}{b-a_1}\right)^{\frac{1}{\delta}} & , \ e^{-\frac{nt_0^2}{a_1}}\le x\le e^{-\frac{nt_0^2}{b}}\\ 0 & , \ e^{-\frac{nt_0^2}{b}}\le x\le e^{-\frac{nt_0^2}{c}}\\ \left(\frac{-\frac{nt_0^2}{\ln x}-c}{d_1-c}\right)^{\frac{1}{\delta}} & , \ e^{-\frac{nt_0^2}{c}}\le x\le e^{-\frac{nt_0^2}{d_1}}\\ 1 & , \quad o.w.\end{cases}.$$

5.2. Parallel System

If n-components are connected in parallel, then the α_i-cut ($i=1,2$) of $GIFR$ with generalized intuitionistic fuzzy Rayleigh distribution is given by

$$S(t)[\alpha_i]=\{P(Y_n>t)\,|\lambda\in\lambda[\alpha_i,\delta]\}=\{1-\left(1-e^{-\frac{t^2}{\lambda}}\right)^n|\lambda\in\lambda[\alpha_i,\delta]\},\quad i=1,2.$$

$$S(t)[\alpha_1]=\left[1-(1-e^{-\frac{t^2}{a+(b-a)\alpha_1^{\delta}}})^n, 1-(1-e^{-\frac{t^2}{d-(d-c)\alpha_1^{\delta}}})^n\right].$$

$$\mathrm{S(t)}[\alpha_2]=\left[1-(1-\mathrm{e}^{-\frac{\mathrm{t}^2}{b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}}})^{\mathrm{n}}, 1-(1-\mathrm{e}^{-\frac{\mathrm{t}^2}{c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}}})^{\mathrm{n}}\right].$$

For t_0, this is a $GIFN_B$ and membership function and non-membership function of $\tilde{S}(t_0)$ are as follows

$$\mu_{S(t_0)}(x)=\begin{cases}\left(\frac{-a-t_0{}^2\left(ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{b-a}\right)^{\frac{1}{\delta}}, & 1-(1-e^{-\frac{t_0{}^2}{a}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{b}})^n\\ 1, & 1-(1-e^{-\frac{t_0{}^2}{b}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{c}})^n\\ \left(\frac{d+t_0{}^2\left(ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{d-c}\right)^{\frac{1}{\delta}}, & 1-(1-e^{-\frac{t_0{}^2}{c}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{d}})^n\\ 0, & o.w.\end{cases},$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\frac{b+t_0{}^2\left(ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{b-a_1}\right)^{\frac{1}{\delta}}, & 1-(1-e^{-\frac{t_0{}^2}{a_1}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{b}})^n\\ 0, & 1-(1-e^{-\frac{t_0{}^2}{b}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{c}})^n\\ \left(\frac{-c-t_0{}^2\left(ln\left(1-(1-x)^{\frac{1}{n}}\right)\right)^{-1}}{d_1-c}\right)^{\frac{1}{\delta}}, & 1-(1-e^{-\frac{t_0{}^2}{c}})^n\le x\le 1-(1-e^{-\frac{t_0{}^2}{d_1}})^n\\ 1, & o.w.\end{cases}.$$

6. Conclusion

In this paper, we have presented a method for analyzing system reliability of different types of systems using generalized intuitionistic fuzzy sets theory, where the lifetime parameter of a component is represented by $GIFN_B$. We investigated the generalized intuitionistic fuzzy reliability function and generalized intuitionistic fuzzy hazard function and then constructed their (α_1,α_2)- cut. In this approach, $S(t)[\alpha_i]$ and $h(t)[\alpha_i]$ are two dimensional functions in terms of α_i and t. For t_0, $\tilde{S}(t_0)$ and $\tilde{h}(t_0)$ are generalized intuitionistic fuzzy numbers and for every especially α_{10} and α_{20}, reliability curve and hazard curve are like a band with upper and lower bound. Any increase in the value of δ can result in an increase in bandwidth of $S(t)[\alpha_{10}]$ and $h(t)[\alpha_{10}]$ and can also decrease bandwidth of $S(t)[\alpha_{20}]$ and $h(t)[\alpha_{20}]$. The GIFH band is increasing with respect to time. Finally, we described reliability analysis of series and parallel system based on generalized intuitionistic fuzzy lifetime parameter. Our method is more comprehensive than any previous methods such as Bohra and Singh's (2015). For the future research, one can define conditional reliability, mean residual lifetime function, mean past lifetime function, cumulative hazard function and reversed hazard function under the generalized intuitionistic fuzzy lifetime distribution and study their properties.

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor in chief for their insightful comments and suggestions.

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