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Similarity Solutions for a Steady MHD Falkner-Skan Flow and Heat Transfer over a Wedge Considering the Effects of Variable Viscosity and Thermal Conductivity

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Abstract

An analysis is carried out to study the Falkner–Skan flow and heat transfer of an incompressible, electrically conducting fluid over a wedge in the presence of variable viscosity and thermal conductivity effects. The similarity solutions are obtained using scaling group of transformations. Furthermore the similarity equations are solved numerically by employing Kellr-Box method. Numerical results of the local skin friction coefficient and the local Nusselt number as well as the velocity and the temperature profiles are presented for different physical parameters.

Keywords: MHD, Similarity solutions, Falkner-Skan flow, Heat transfer, variable viscosity, thermal conductivity

MSC 2000 No.: 76R10

List of symbols

A	fluid viscosity variation parameter
a	constant
B_o	uniform magnetic field
b	constant
c	constant depending on the nature of fluid
f	similarity function
L	length of surface
M	magnetic parameter
m	Falkner-Skan power-law parameter
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
Re_x	local Reynolds number
S	thermal conductivity parameter
T	temperature
$\bar{u}_e$	the velocity over the wedge
U_∞	free stream velocity
u_e	dimensionless free stream velocity
$\bar{u}, \bar{v}$	velocity components along the $\bar{x}$ and $\bar{y}$ directions
v ,u	dimensionless velocity
$\bar{x}, \bar{y}$	Cartesian coordinates along the surface of the wedge and normal to it

Greek symbols

β	wedge angle
ε	infinitesimal Lie group parameter
η	similarity variable
κ	thermal diffusivity
κ_∞	ambient fluid thermal conductivity
μ	dynamic viscosity
μ_∞	constant value of the coefficient of viscosity far away from the surface
ν	Kinematic viscosity
θ	Dimensionless temperature,
ρ_∞	density of the fluid
σ	electric conductivity
τ_w	Local skin friction
ψ	stream function Subscripts
$\alpha_1, \alpha_2, \alpha_3, ..., \alpha_6$	constants

Subscripts

w	condition at the wall
∞	ambient condition Superscript

Superscript

'	differentiation with respect to η

1. Introduction

Within the field of aerodynamics, the analysis of boundary-layer problems for two-dimensional steady and incompressible laminar flow passing a wedge is a common area of interest. Falkner and Skan (1931) considered two-dimensional wedge flows. They developed a similarity transformation method in which the partial differential boundary-layer equation was reduced to a nonlinear third-order ordinary differential equation which could then be solved numerically. Hartree (1937) solved this problem and gave numerical results for the wall shear stress for different values of the wedge angle. Rajagopal et al. (1983) studied the Falkner–Skan boundary layer flow of a homogeneous incompressible second grade fluid past a wedge placed symmetrically with respect to the flow direction. Lin and Lin (1987) introduced a similarity solution method for the forced convection heat transfer from isothermal or uniform-flux surfaces to fluids of any Prandtl number. The solutions of the resulting similarity equations are given by the Runge–Kutta scheme.

Hsu et al. (1997) studied the temperature and flow fields of the flow past a wedge by the series expansion method, similarity transformation, Runge-Kutta integration and the shooting method. Harris et al. (2002) studied heat transfer problem for an impulsively started Falkner–Skan flow, and the majority of cases considered relate to the acute, semi-infinite wedge problem. In all these studies, the fluid properties were assumed to be constant. However, in many industrial applications this assumption is not obeyed and we have to consider such problems by assuming variable viscosities or variable conductivity. It is well known that viscosities of liquids change with temperature e. g. the viscosity of water decreases by about 24% when the temperature increases from 10^0C to 50^0C.

The first attempt to solve the Falkner-Skan problem including the variation of viscosity with temperature was made by Herwing et al. (1986). Seddeek et al. (2007) studied the effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media in the presence of radiation and magnetic field. The effect of temperature dependent viscosity on laminar mixed convection boundary layer flow and heat transfer on a continuously moving vertical surface is studied by Ali (2006), when the fluid viscosity is assumed to vary as an inverse linear function of temperature. Seddeek and Salem (2006) studied the effects of variable viscosity and magnetic field on flow and heat transfer to a continuously moving flat plate, when the fluid viscosity is assumed to vary as a linear function of temperature.

Seddeek and Salama (2007) studied the effects of variable viscosity and thermal conductivity on an unsteady two-dimensional laminar flow of a viscous incompressible conducting fluid past a semi-infinite vertical porous moving plate taking into account the effect of magnetic field. Afify (2007) examined the effects of non-Darcian flow phenomena, variable viscosity, Hartmann-Darcy number and thermal stratification on free convective transport and demonstrates the variation in heat transfer prediction based on three different flow models. In this analysis, we discuss MHD Falkner-Skan flow and heat transfer over a wedge.

The effects of variable viscosity and thermal conductivity are investigated. By using scaling transformations, the set of governing equations and the boundary conditions are reduced to nonlinear ordinary differential equations with appropriate boundary conditions. Furthermore the similarity equations are solved numerically by employing Kellr-Box method. Numerical results of the local skin friction coefficient, the local Nusselt number and the local Sherwood numbers as well as the velocity and the temperature profiles are presented for different parameters.

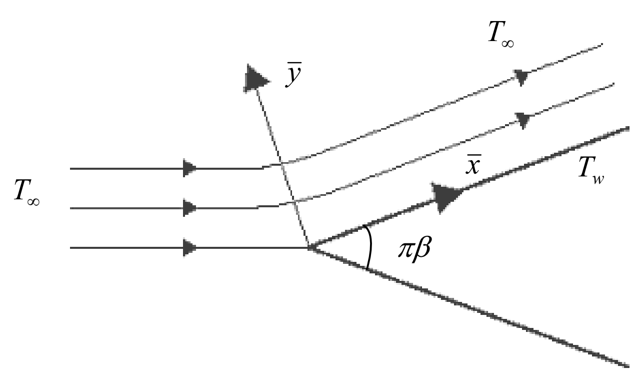

Figure1. The flow configuration and coordinate system

2. Mathematical Analysis

We consider the boundary layer flow of an electrically conducting viscous fluid over a wedge, as shown in Figure 1, A magnetic field B_o acts transversely to the flow. The induced magnetic field is neglected by choosing small magnetic Reynolds number assumption. Furthermore the electric field is absent. The relevant problem is

$$\begin{aligned}
&\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0\\
&\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}}+\frac{1}{\rho_\infty}\frac{\partial}{\partial \bar{y}}\left[\mu(\bar{T})\frac{\partial \bar{u}}{\partial \bar{y}}\right]-\frac{\sigma B_0^2}{\rho_\infty}(\bar{u}-\bar{u}_e)\\
&\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}=\frac{1}{\rho_\infty C_p}\frac{\partial}{\partial \bar{y}}\left[k(\bar{T})\frac{\partial \bar{T}}{\partial \bar{y}}\right].
\end{aligned} \tag{2.1}$$

With the boundary conditions:

$$\begin{cases}\bar{u}=0\,,\bar{v}=0\,,\bar{T}=\bar{T}_w \quad at \quad \bar{y}=0\\ \bar{u}=\bar{u}_e(\bar{x})\;\;,\bar{T}=\bar{T}_\infty \quad as \quad \bar{y}\to\infty,\end{cases} \tag{2.2}$$

where $\bar{u}_e$ is the velocity over the wedge. The exponent *m*, which is called the Falkner-Skan power –law parameter, is related to the wedge angle β by $m=\dfrac{\beta}{2-\beta}$. ρ_∞ is the density of the fluid (assumed constant), μ and κ are the viscosity coefficient and thermal diffusivity respectively which are variation as a function of temperature T, σ is the electrical conductivity, B_o the magnetic field of constant strength, T_∞ is the fluid free stream temperature, T_w is the wall temperature. The temperature-dependent fluid viscosity is given by Batchelor (1987) and the fluid thermal conductivity is given by Slattery (1972), respectively.

$$\mu=\mu_\infty[a+b(T_w-T)],\quad \kappa=\kappa_\infty[1+c(T-T_\infty)], \tag{2.3}$$

where μ_∞ is the constant value of the coefficient of viscosity far away from the surface, κ_∞ the ambient fluid thermal conductivity, *a*, *b* are constants and *b* > 0, and *c* is a constant depending on the nature of the fluid.

By using the non-dimensional variables:

$$x=\frac{\bar{x}}{L},y=\frac{\bar{y}\sqrt{\mathrm{Re}}}{L},u=\frac{\bar{u}}{U_\infty},v=\frac{\bar{v}\sqrt{\mathrm{Re}}}{U_\infty},u_e=\frac{\bar{u}_e}{U_\infty},\theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty} \tag{2.4}$$

where $\mathrm{Re} = \dfrac{U_\infty L \rho_\infty}{\mu_\infty}$ is the Reynolds number, and U_∞ is the free stream velocity.

Equation (2.3) can be rewritten as follows:

$$\mu = \mu_\infty [a + b(\overline{T}_w - \overline{T}_\infty)(1-\theta)] = \mu_\infty [a + A(1-\theta)] \text{ and } \kappa = \kappa_\infty [1 + S\theta], \tag{2.5}$$

where A fluid viscosity variation parameter, and S is the thermal conductivity parameter. Equation (2.1) can be rewritten as follows:

$$\begin{aligned}
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
&u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + [a + A(1-\theta)]\frac{\partial^2 u}{\partial y^2} - A\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} - M(u - u_e) \\
&u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}[\ (1+S\theta)\frac{\partial^2 \theta}{\partial y^2} + S(\frac{\partial \theta}{\partial y})^2],
\end{aligned} \tag{2.6}$$

where $M = \dfrac{\sigma B_o{}^2 L}{\rho_\infty U_\infty}$ is the magnetic parameter, $\mathrm{Pr} = \dfrac{C_p \mu_\infty}{\kappa_\infty}$ is the Prandtl number.

The boundary conditions (2.2) become:

$$\begin{cases} u = 0,\ \ v = 0,\ \ \theta = 1 \ \text{ at } \ y = 0, \\ u = u_e,\ \ \theta = 0 \ \text{ as } \ y \to \infty. \end{cases} \tag{2.7}$$

By using the stream function $\left(u = \dfrac{\partial \psi}{\partial y},\ v = -\dfrac{\partial \psi}{\partial x} \right)$ and the velocity over the wedge $u_e = x^m$ we have:

$$\begin{aligned}
&\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = m\, x^{2\,m-1} + [a + A(1-\theta)]\frac{\partial^3 \psi}{\partial y^3} - A\frac{\partial^2 \psi}{\partial y^2}\frac{\partial \theta}{\partial y} - M\left(\frac{\partial \psi}{\partial y} - x^m\right) \\
&\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\left[(1+S\theta)\frac{\partial^2 \theta}{\partial y^2} + S\left(\frac{\partial \theta}{\partial y}\right)^2\right]
\end{aligned} \tag{2.8}$$

The boundary conditions are

$$\begin{cases} \dfrac{\partial \psi}{\partial y} = 0,\ \dfrac{\partial \psi}{\partial x} = 0,\ \ \theta = 1 \text{ at } y = 0 \\ \dfrac{\partial \psi}{\partial y} = x^m,\ \theta = 0 \text{ as } y \to \infty. \end{cases} \tag{2.9}$$

3. Scaling group of Transformations

We now introduce the simplified form of Lie group transformations namely, the scaling group of transformations Tapanidis et al. (2003):

$$\Gamma : x^* = e^{\varepsilon\alpha_1}x,\ y^* = e^{\varepsilon\alpha_2}y,\ \psi^* = e^{\varepsilon\alpha_3}\psi,\ u^* = e^{\varepsilon\alpha_4}u,\ v^* = e^{\varepsilon\alpha_5}v,\ \theta^* = e^{\varepsilon\alpha_6}\theta \qquad (3.1)$$

Equation (3.1) may be considered as a point-transformation which transforms coordinates $(x, y, \psi, u, v, \theta)$ to the coordinates $(x^*, y^*, \psi^*, u^*, v^*, \theta^*)$, substituting (3.1) in Equations (2.8) and boundary conditions (2.9), we get

$$\begin{cases} e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\dfrac{\partial\psi^*}{\partial y^*}\dfrac{\partial^2\psi^*}{\partial x^*\partial y^*}-\dfrac{\partial\psi^*}{\partial x^*}\dfrac{\partial^2\psi^*}{\partial y^{*2}}\right) \\ \qquad = me^{-\varepsilon\alpha_1(2m-1)}x^{*(2m-1)}+[a+A(1-\theta^* e^{-\varepsilon\alpha_6})]\times e^{\varepsilon(3\alpha_2-\alpha_3)}\dfrac{\partial^3\psi^*}{\partial y^{*3}} \\ \qquad\quad -Ae^{\varepsilon(3\alpha_2-\alpha_3-\alpha_6)}\dfrac{\partial^2\psi^*}{\partial y^{*2}}\dfrac{\partial\theta^*}{\partial y^*}-M\left(e^{\varepsilon(\alpha_2-\alpha_3)}\dfrac{\partial\psi^*}{\partial y^*}-e^{-\varepsilon\alpha_1 m}x^{*m}\right) \\ \qquad\qquad e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\dfrac{\partial\psi^*}{\partial y^*}\dfrac{\partial\theta^*}{\partial x^*}-\dfrac{\partial\psi^*}{\partial x^*}\dfrac{\partial\theta^*}{\partial y^*}\right) \\ \qquad = \dfrac{1}{\Pr}[(e^{\varepsilon(2\alpha_2-\alpha_6)}+S\,\theta^* e^{2\varepsilon(\alpha_2-\alpha_6)})\dfrac{\partial^2\theta^*}{\partial y^{*2}}+Se^{2\varepsilon\,(\alpha_2-\alpha_6)}\left(\dfrac{\partial\theta^*}{\partial y^*}\right)^2. \end{cases} \qquad (3.2)$$

The boundary conditions (2.9) become:

$$\begin{cases} e^{\varepsilon(\alpha_2-\alpha_3)}\dfrac{\partial\psi^*}{\partial y^*}=0\,,\ e^{\varepsilon(\alpha_1-\alpha_3)}\dfrac{\partial\psi^*}{\partial x^*}=0\,,\ \theta^*=1 \quad at \quad y^*=0 \\ e^{\varepsilon(\alpha_2-\alpha_3)}\dfrac{\partial\psi^*}{\partial y^*}=e^{-\varepsilon\alpha_1 m}x^{*m}\ ,\ \theta^*=0 \quad as \quad y^*\to\infty. \end{cases} \qquad (3.3)$$

The system will remain invariant under the group of transformations Γ, we would have the following relations among the parameters, namely

$$\begin{aligned} \alpha_1+2\alpha_2-2\alpha_3 &= -\alpha_1(2m-1)=3\alpha_2-\alpha_3=3\alpha_2-\alpha_3-\alpha_6 \\ &= 3\alpha_2-\alpha_3-\alpha_6=\alpha_2-\alpha_3 \\ &= -\alpha_1 m \end{aligned}$$

and

$$\alpha_1+\alpha_2-\alpha_3-\alpha_6=2\alpha_2-\alpha_6=2(\alpha_2-\alpha_6)\,.$$

By solving the previous conditions with boundary conditions, we obtain

$\alpha_1 = \alpha_3 = \alpha_4$, $\alpha_2 = \alpha_5 = \alpha_6 = 0$ and $m = 1$.

The set of transformations Γ reduces to

$$x^* = xe^{\varepsilon\,\alpha_1}, y^* = y, \psi^* = e^{\varepsilon\,\alpha_1}\psi, u^* = e^{\varepsilon\,\alpha_1}u, v^* = v, \theta^* = \theta. \tag{3.4}$$

Expanding by Taylor's method in powers of ε and keeping terms up to the order ε, we get

$$\begin{aligned} &x^* = x + x\varepsilon\alpha_1,\ \ y^* = y,\ \ \psi^* = \psi + \psi\varepsilon\alpha_1 \\ &u^* = u + u\varepsilon\alpha_1,\ v^* = v,\ \ \theta^* = \theta. \end{aligned} \tag{3.5}$$

In terms of differentials these yield

$$\frac{dx}{\alpha_1 x} = \frac{dy}{0} = \frac{d\psi}{\alpha_1\psi} = \frac{du}{\alpha_1 u} = \frac{dv}{0} = \frac{d\theta}{0} \tag{3.6}$$

Solving the above equations, we get

$$y^* = \eta, \psi^* = x^* f(\eta),\ \ \theta^* = \theta(\eta). \tag{3.7}$$

With the help of these relations, Equation (3.2) and boundary conditions (3.3) become

$$\begin{aligned} &[a + A(1-\theta)]f''' + (f - A\theta'\,)f'' - M\ \ (f'-1) - f'^2 + 1 = 0 \\ &(1 + S\ \theta)\theta'' + S\ \theta'^2 + \Pr f\ \ \theta' = 0 \end{aligned} \tag{3.8}$$

with boundary conditions:

$$\begin{aligned} &f = 0, \quad f' = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \\ &f' = 1, \quad \theta = 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{3.9}$$

The quantities of physical interest in this problem are the local skin friction coefficient and the local Nusselt number, which are defined by

$$C_f \operatorname{Re}_x = [1 + A(1-\theta) f''(0)], \tag{3.10}$$

$$Nu = -\theta'(0) \tag{3.11}$$

4. Results and Discussions

By applying one-parameter group theory to the analysis of the governing equations and the boundary conditions, the two independent variables are reduced by one; consequently, the governing equations reduce to a system of non-linear ordinary differential equations with the appropriate boundary conditions. Finally, the systems of similarity equations (3.8) with boundary conditions (3.9) are solved numerically by employing Kellr-Box method Perot and Subramanian (2007).

The computations have been carried out for various values of variable viscosity parameter A, magnetic filed parameter M, variable thermal conductivity S when Prandtl number $Pr = 0.7$ (air) and $a = 1$. The edge of the boundary layer $\eta_\infty = 8$ depending on the values of parameters. In this work, we only discussed the similarity equation with $m = 1$, where the primary stream-wise boundary layer is the famous stagnation point flow. See White (1991), Schlichting and Gersten (2000) and Rosehead (1963).

From Table 1, we see that the local skin friction coefficient at the surface increases by increasing of the variable viscosity parameter but the local Nusselt number decreases by increasing of the variable viscosity parameter. The results presented demonstrate quite clearly that A, which is an indicator of the variation of viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics. From Table 2, we see that the local skin friction coefficient at the surface and the local Nusselt number decrease by increasing of variable thermal conductivity. From Table 3, we see that the local skin friction coefficient and the local Nusselt number increase by increasing of the magnetic parameter. This result qualitatively agrees with the expectations, since the magnetic field exerts retarding force on the free convection flow.

Figure 2 shows that the velocity profiles decrease by increasing the temperature-dependent fluid viscosity parameter A. Figure 3 show that the temperature profiles increase slightly with an increase in the temperature-dependent fluid viscosity parameter A. Figure 4 shows that the velocity profiles increase by increasing the magnetic parameter. Figure 5 show that the temperature profiles decrease by increasing the magnetic parameter. Figures (6) and (7) depict the velocity and the temperature profiles for different values the thermal conductivity parameter S. It is also observed that the velocity profiles increase slightly and the temperature profiles increase by increasing the thermal conductivity parameter S.

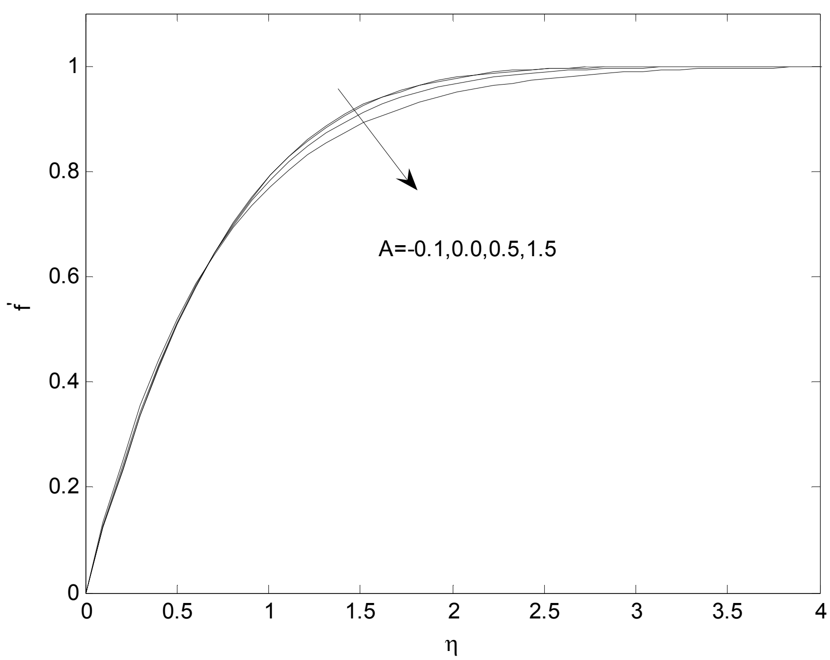

Figure 2. Velocity profiles for $f'(\eta)$: $Pr = 0.7, M = 0.1, S = 0.5$ with different values of A

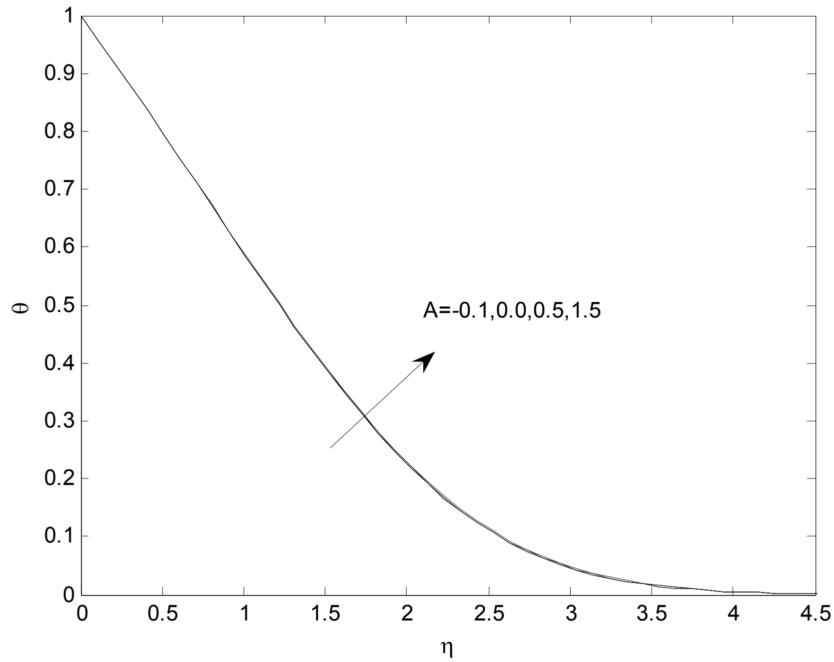

Figure 3. Temperature profiles $\theta(\eta)$ for: $\Pr = 0.7, M = 0.1, S = 0.5$ with different values of A

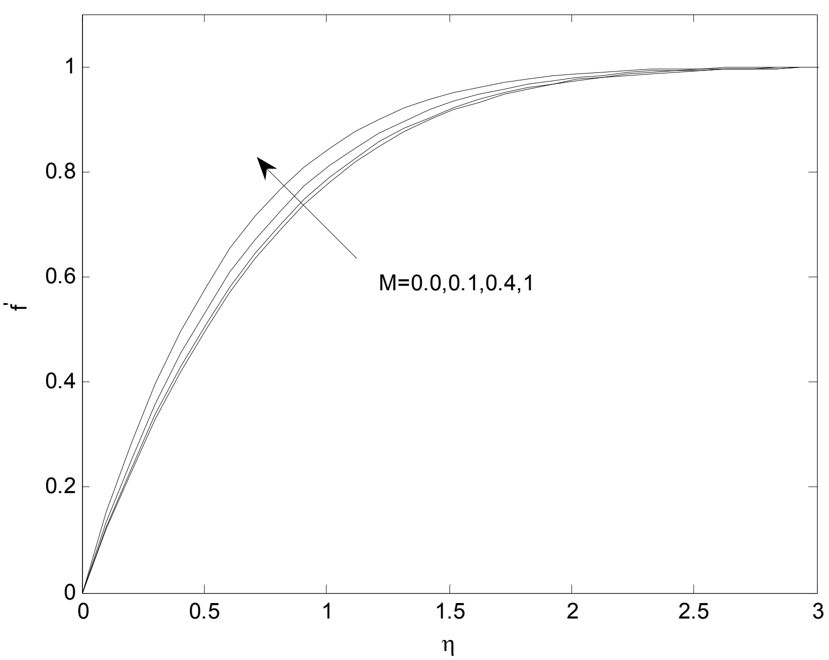

Figure 4. Velocity profiles for: $\Pr = 0.7, A = 0.1, S = 0.5$ with different values of M

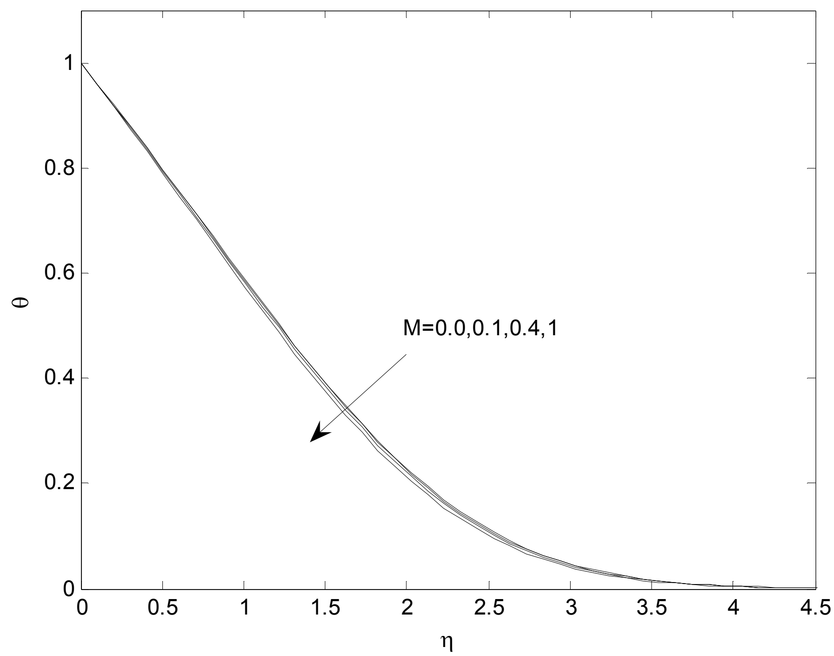

Figure 5. Temperature profiles $\theta(\eta)$ for: $\Pr = 0.7, A = 0.1, S = 0.5$ with different values of M

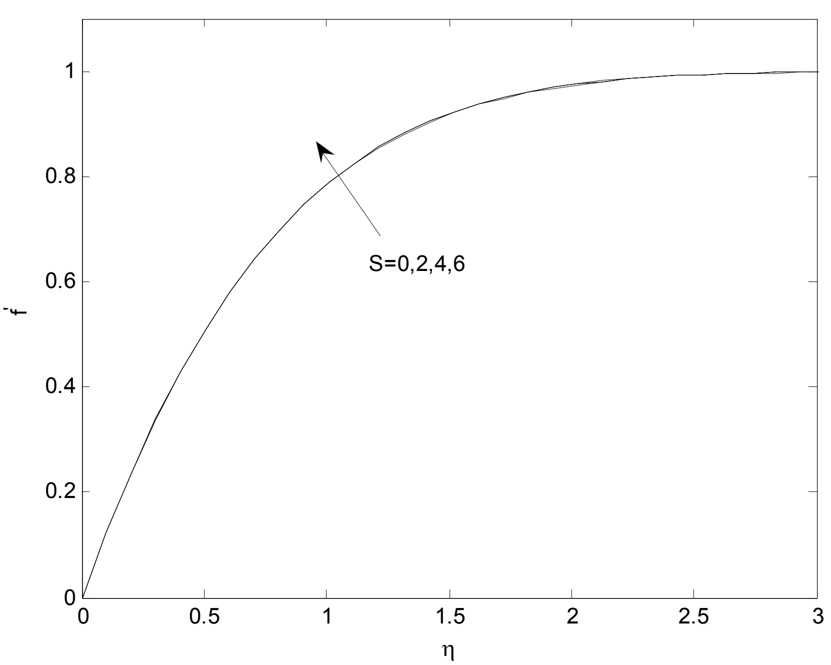

Figure 6. Velocity profiles for: for: for $\Pr = 0.7, M = 0.1, A = 0.1$ .with different values of S

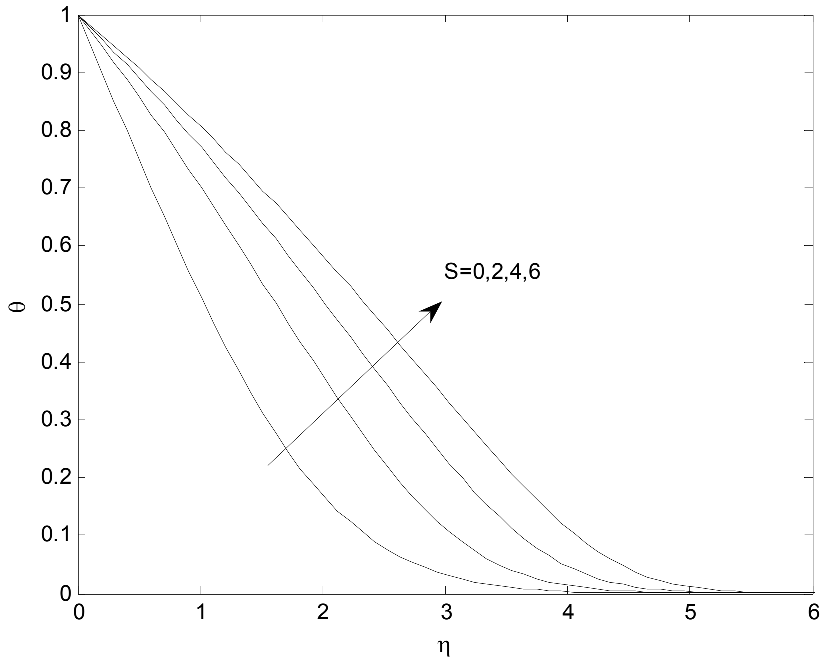

Figure 7. Temperature profiles $\theta(\eta)$ for $\Pr = 0.7, M = 0.1, A = 0.1$ with different values of S

Table 1. Values of $f''(0)$ and $-\theta'(0)$ for $M = 0.1, S = 0.5$

	$A = -0.1$	$A = 0.0$	$A = 0.5$	$A = 1.5$
$f''(0)$	**1.261131**	**1.27218**	**1.325965**	**1.427501**
$-\theta'(0)$	0.39135	0.39127	0.39086	0.39015

Table 2. Values of $f''(0)$ and $-\theta'(0)$ for $M = 0.1, A = 0.1$

	$S = 0$	$S = 2$	$S = 4$	$S = 6$
$f''(0)$	**1.285337**	**1.279977**	**1.278218**	**1.277287**
$-\theta'(0)$	0.49821	0.26897	0.20839	0.17713

Table 3. Values of $f''(0)$ and $-\theta'(0)$ for $A = 0.1, S = 0.5$

	$M = 0.0$	$M = 0.1$	$M = 0.4$	$M = 1.0$
$f''(0)$	**1.243536**	**1.283126**	**1.395266**	**1.596315**
$-\theta'(0)$	0.38936	0.39118	0.39606	0.40392

5. Conclusions

In this paper, we discuss a steady MHD Falkner-Skan flow and heat transfer over a wedge. The effects of variable viscosity parameter, magnetic parameter, and variable thermal conductivity are investigated. The similarity solutions are obtained using scale group of transformations. The set of governing equations and the boundary conditions are reduced to ordinary differential equations with appropriate boundary conditions. Furthermore, the differential equations are solved numerically by employing Kellr-Box method. We observed that, the local skin friction coefficient at the surface increase by increasing of the variable viscosity parameter and magnetic parameter. On the contrary it is decrease by increasing of variable thermal conductivity. The Nusselt number decrease by increasing of the variable viscosity parameter and variable thermal conductivity parameter. On the contrary it is increase by increasing of the magnetic parameter.

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