Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466
Vol. 12, Issue 1 (June 2017), pp. 23 – 34

Applications and Applied Mathematics:
An International Journal
(AAM)

Conditional full support for Fractional Brownian Motion

[1]Soumia Dani, [1]Abdeldjebbar Kandouci, & [2]Amina Angelika Bouchentouf

[1]Laboratory of Stochastic Models, Statistic and Applications
Tahar Moulay University of Saida
P.O. Box 138
En-Nasr, 20000 Saida, Algeria
dani.cusmath@yahoo.fr; kandouci1974@yahoo.fr
&
[2]Department of Mathematics
Djillali Liabes University of Sidi Bel Abbes
B. P. 89
Sidi Bel Abbes 22000, Algeria
bouchentouf_amina@yahoo.fr

Received: October 18, 2015; Accepted: December 15, 2016

Abstract

In this paper we establish conditions that imply the conditional full support (CFS) property, introduced by Guasoni et al. (2008), for the processes $S_t = R_t + \int_0^t \phi_s dB_s^H$, where B^H is a Fractional Brownian motion, R is a continuous process, and the processes R and ϕ are either progressive or independent of B^H. Moreover we build the absence of arbitrage opportunities without calculating the risk-neutral probability.

Keywords: Conditional Full Support; Fractional Brownian Motion; Stochastic Integral

MSC 2010 No.: 34A08, 34K50, 93B05, 58J65

1. Introduction

Condition full support (CFS) is a straightforward condition on asset prices stating that from any time, the asset price path can continue arbitrarily close to any given path with positive conditional probability. The CFS's notion was introduced by Guasoni et al. (2008), where it was proved that the fractional Brownian motion with arbitrary Hurst parameter has a desired property. Then, this later was generalized by Cherny (2008), who showed that any Brownian moving average satisfies the CFS. After that, this property was established for Gaussian processes with stationary increments by Gasbarra (2011).

Let's note that, by the main result of Guasoni et al. (2008), the CFS generates the consistent price systems which admits a martingale measure.

Pakkanan (2009) established conditions that imply the conditional full support for the process $Z := R + \phi * W$, where W is a Brownian motion and R is a continuous process.

In this paper, we enjoy this property by thinking of the problems of no arbitrage for asset prices driven by a fractional Brownian motion process.

The layout of the paper is as follows. Section 2 contains results on consistent price system and conditions that imply the conditional full support. In Section 3 we present some useful basic concepts on fractional Brownian motion needed for our main resul. In Section 4 we provide our main result on conditional full support for the process $S_t = R_t + \int_0^t \phi_s dB_s^H$, where B^H is a fractional Brownian motion and build the absence of arbitrage opportunities without calculating the risk-neutral probability by the existence of the consistent price systems. Finally, in Section 5 we finish the paper by a small conclusion.

2. Basic Definitions and Results on Stochastic Integral and Conditional Full Support

2.1. Conditional Full Support

2.1.1. Definitions

Recall first that when $\mathbf{E}$ is separable metric space and $\mu : \mathfrak{B}(\mathbf{E}) \to [0,1]$ is a Borel probability measure, the support of μ, denoted by supp(μ), is the unique minimal closed set $A \subset \mathbf{E}$ such that $\mu(A) = 1$. Let $(X_t)_{t\in[0,T]}$ be a continuous stochastic process taking values in an open interval $\mathbf{I} \subset \mathbb{R}$, defined on a complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$, and let $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ be a filtration on this space. Moreover, let $\mathcal{C}_x([u,v], \mathbf{I})$ be the space of function $f \in \mathcal{C}([u,v], \mathbf{I})$ such that $f(u) = x \in \mathbf{I}$. As usual, we equip the space $\mathcal{C}([u,v], \mathbf{I})$ and $\mathcal{C}_x([u,v], \mathbf{I})$, $x \in \mathbf{I}$, with the uniform topologies.

The two next definitions given in Guasoni et al. (2008) present the notion of Conditional Full Support (CFS) as well as that of consistent price system.

Definition.

We say that the process X has Conditional Full Support (CFS) with respect to the filtration $\mathbb{F}$, or briefly $\mathbb{F}$-CFS, if

- X is adapted to $\mathbb{F}$,

- for all $t \in [0, T)$ and $\mathbf{P}$-almost all $\omega \in \Omega$,
$$\text{supp}(\text{law}[(X_u)_{u\in[t,T]}|\mathcal{F}_t](\omega)) = \mathcal{C}_{X_t(\omega)}([t, T], \mathbf{I}).$$

Definition.

Let $\varepsilon > 0$. An ε-consistent price system to X is a pair $(\widetilde{X}, \mathbf{Q})$, where $\mathbf{Q}$ is a probability measure equivalent to $\mathbf{P}$ and $\widetilde{X}$ is a $\mathbf{Q}$-martingale in the filtration $\mathcal{F}$, such that

$$\frac{1}{1+\varepsilon} \leq \frac{\widetilde{X}_i(t)}{X_i(t)} \leq 1+\varepsilon, \text{ almost surely for all } t \in [0, T] \text{ and } i = 1, \ldots, n.$$

2.1.2. Stochastic integral and Conditional Full Support

Recall the results about CFS for processes of the form

$$Z_t := R_t + \int_0^t \phi_s dW_s, \qquad t \in [0, T],$$

where R is a continuous process, the integrator W is a Brownian motion, and the integrand ϕ satisfies some varying assumptions. We focus on two cases.

(1) Independent integrands and Brownian integrators

Theorem 2.1. (Pakkanan, 2009)

Let us define

$$Z_t := R_t + \int_0^t \phi_s dW_s, \qquad t \in [0, T].$$

Suppose that

- $(R_t)_{t\in[0,T]}$ is a continuous process,
- $(\phi_t)_{t\in[0,T]}$ is a measurable process s.t.$\int_0^T \phi_s^2 ds < \infty$ a.s,
- $(W_t)_{t\in[0,T]}$ is a standard Brownian motion independent of R and ϕ.

If we have

$$\text{meas}(t \in [0, T] : \phi_t = 0) = 0 \qquad \mathbf{P} - a.s, \quad \text{(meas: Lebesgue measure)},$$

then Z has CFS.

As an application of this theorem, several popular stochastic volatility models which have the CFS property can be presented.

Application to stochastic volatility model

Let us consider the price process $(P_t)_{t\in[0,T]}$ in $\mathbb{R}_+$ given by:

$$dP_t = P_t(f(t, V_t)dt + \rho g(t, V_t)dB_t + \sqrt{1-\rho^2} g(t, V_t)dW_t,$$

$P_0 = p_0 \in \mathbb{R}_+$, where

(1) $f, g \in C([0, T] \times \mathbb{R}^d, \mathbb{R})$,
(2) (B,W) is a planar Brownian motion,

(3) $\rho \in (-1, 1)$,
(4) V is a (measurable) process in $\mathbb{R}^d$ s.t. $g(t, V_t) \neq 0$ a.s. for all $t \in [0, T]$,
(5) (B, V) is independent of W.

Using Itôs formula, we have

$$\log P_t = \underbrace{\log P_0 + \int_0^t (f(s, V_s) - \frac{1}{2} g(s, V_s)^2) ds + \rho \int_0^t g(s, V_s) dB_s}_{=R_t}.$$

$$\underbrace{+\sqrt{1-\rho^2} \int_0^t g(s, V_s) dW_s}_{=\phi_s}.$$

Since W is independent of B and V, the previous theorem implies that $\log P$ has CFS and from the next remark which entails that P has CFS.

Recall that if $I \subset \mathbb{R}$ is an open interval and $f : \mathbb{R} \longrightarrow I$ is a homeomorphism, then $g \longmapsto f \circ g$ is a homeomorphism between $C_x([0, T])$ and $C_{f(x)}([0, T], I)$. Hence, for $f(X)$, understood as a process in I, we have

$$f(X) \quad \text{has} \quad \mathbb{F} - CFS \Longleftrightarrow X \quad \text{has} \quad \mathbb{F} - CFS. \tag{1}$$

Next, we relax the assumption about independence, and consider the second case:

(2) Progressive integrands and Brownian integrators

Let's note that the assumption about independence between W and (R,ϕ) cannot be dispensed with in general without imposing additional conditions. Namely, if,

$$R_t = 1; \phi_t := e^{W_t - \frac{1}{2}t}; t \in [0, T],$$

then $Z = \phi = \xi(W)$, the Doléans exponential of W, which is strictly positive does not have CFS, if the process is considered in $\mathbb{R}$.

Theorem 2.2. (Pakkanan, 2009)

Suppose that

- $(X_t)_{t \in [0,T]}$ is a continuous process,
- R and ϕ are progressive $[0, T] \times C([0, T])^2 \longrightarrow \mathbb{R}$,
- ε is a random variable,
- and $\mathcal{F}_t = \sigma\{\varepsilon, X_s, W_s : s \in [0, t]\}, t \in [0, T]$.

If W is an $\mathcal{F}_{t \in [0,T]}-$Brownian motion and

- $E[e^{\lambda \int_0^T \phi_s^{-2} ds}] < \infty$ for all $\lambda > 0$,
- $E[e^{2 \int_0^T \phi_s^{-2} h_s^2 ds}] < \infty$, and
- $\int_0^T \phi_s^2 ds \leq \overline{K}$ a.s for some constant $\overline{K} \in (0, \infty)$,

then the process

$$Z_t = \varepsilon + \int_0^t R_s ds + \int_0^t \phi_s dW_s, \qquad t \in [0, T]$$

has CFS.

3. Fractional Brownian Motion

In this part of paper, we give some definitions and existing results on fractional Brownian motion presented in Nahmani (2009). First let's note that the name fractional Brownian motion (fBm) was given by Mandelbrot and Van Ness (1968). However, Kolmogorov studied it first within the Hilbert Space framework.

The fBm is a H-sssi process (self-similar process with stationary increments) that has stationary increments. It is the unique Gaussian H-sssi process. It has very important and powerful properties for many applications. One of them is its memory. It has many applications in telecommunications as well as in finance.

Definition.

For $H \in (0, 1)$, a standard fractional Brownian motion of Hurst parameter H is a centered and continuous Gaussian process, denoted by $(B_t^{(H)})_{t \in T}$, with covariance function

$$E(B_t^{(H)} B_s^{(H)}) = \frac{1}{2}(\mid t \mid^{2H} + \mid s \mid^{2H} - \mid t - s \mid^{2H} := R_H(t, s).$$

There is another classic definition of the fBm using self-similar properties, which is given in the following theorem.

Theorem 3.1. (Nahmani, 2009)

For $H \in (0, 1)$, the fBm $(B_t^{(H)})_{t \in \mathbb{R}_+}$ is a Gaussian H-sssi process.

Next, we present two different representations of fBm given in Nahmani (2009); representions of fBm on a finite interval and Lévy-Hida representation.

A. Representions of fBm on a finite interval

The fBm can be presented as a Wiener integral but defined on an interval, e.g. commonly taken as $[0, T]$. We shall still use fractional analysis.

For a one-sided fBm $(B_t^{(H)})_{0 \le t \le T}$, a general formula is given as

$$B_t^{(H)} = \int_0^t K_H(t, s) dB_s, \quad t \in [0, T], \tag{2}$$

where $(B_t)_{0 \le t \le T}$ is a one-sided standard Brownian motion.

B. Lévy-Hida Representation

Note that the fractional Brownian motion is a particular case of Volterra processes. Following Decreusfond and Üstünel (1999) we have this kernel

$$K_H(t,s) = \frac{(t-s)_+^{H-\frac{1}{2}}}{\Gamma(H+\frac{1}{2})} F\left(\frac{1}{2}-H, H-\frac{1}{2}, H+\frac{1}{2}, 1-\frac{t}{s}\right), \quad 0 < s < t < \infty,$$

where F is the Gauss hypergeometric function. Remark that, generally, the covariance $R_H(t,s)$ of B^H is given by

$$R_H(t,s) = \int_0^{t\wedge s} K_H(t,u)K_H(s,u)du.$$

Indeed, by (2), it follows that

$$R_H(t,s) = E(B_t^{(H)}B_s^{(H)}) = E\left(\int_0^{t\wedge s} K_H(t,u)K_H(s,u)dB_u\right) = \int_0^{t\wedge s} K_H(t,u)K_H(s,u)du.$$

- **Case** $H \in \left(\frac{1}{2}, 1\right)$.

Proposition 3.1. (Nahmani, 2009)

For the case $H \in \left(\frac{1}{2}, 1\right)$, the kernel K_H can be written as

$$K_H(t,s) = c_H s^{\frac{1}{2}-H} \int_s^t \mid u-s \mid^{H-\frac{3}{2}} u^{H-\frac{1}{2}} du, \quad t > s,$$

where

$$c_H = \left(\frac{H(2H-1)}{\mathbf{B}(2-2H, H-\frac{1}{2})}\right)^{\frac{1}{2}},$$

and

$$\mathbf{B} \text{ the Beta function, i.e. } \mathbf{B}(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt.$$

Corollary 3.1. (Nahmani, 2009)

Besides, we have

$$R_H(t,s) = (\varpi_1(H))^2 \int_0^T \left(r^{\frac{1}{2}-H}(I_{T^-}^{H-\frac{1}{2}} u^{H-\frac{1}{2}} \mathbf{1}_{[0,t)}(u))(r)\right)\left(r^{\frac{1}{2}-H}(I_{T^-}^{H-\frac{1}{2}} u^{H-\frac{1}{2}} \mathbf{1}_{[0,s)}(u))(r)\right)dr,$$

with $\varpi_1(H) = \left(\frac{\Gamma(H-\frac{1}{2})^2 H(2H-1)}{\mathbf{B}(2-2H, H6\frac{1}{2})}\right)^{\frac{1}{2}}$.

Theorem 3.2. (Nahmani, 2009)

The representation of a fBm for $H \in \left(\frac{1}{2}, 1\right)$ over a finite interval is

$$B_t^{(H)} = \int_0^t K_H(t,s)dW_s, \qquad s,t \in [0,T],$$

where $(W_t)_{t\in[0,T]}$ is a particular Wiener process.

- **Case** $H \in \left(0, \frac{1}{2}\right)$.

Corollary 3.2. (Nahmani, 2009)

For the case $H \in \left(0, \frac{1}{2}\right)$ we have that the kernel is given by

$$K_H(t,s) = b_H\left(\left(\frac{t}{s}\right)^{H-\frac{1}{2}}(t-s)^{H-\frac{1}{2}} - \left(H-\frac{1}{2}\right)s^{\frac{1}{2}-H}\int_s^t (u-s)^{H-\frac{1}{2}}u^{H-\frac{1}{2}}du\right),$$

where

$$b_H = \left(\frac{2H}{(1-2H)\mathbf{B}(1-2H, H+\frac{1}{2})}\right)^{\frac{1}{2}}.$$

4. Main Result

The main aim of this part of paper is to enjoy the CFS property by thinking of the problems of no arbitrage for stochastic integral in which the fractional Brownian motion is the integrator.

So, in this paper we extend the work of Guasoni et al. (2008) by incorporating an application using the concept of fractional Brownain motion which is the integrator. The presentation of the process for which we shall study the Wiener integration with respect to it is based on Embrechts et al. (2002), Samorodnitsky et al. (1994), and Mandelbrot and Van Ness (1968).

First let us present a theorem which will be useful to show the absence of arbitration without calculating the risk-neutral probability.

Theorem 4.1. (Guasoni et al., 2008)

Let X_t be an $\mathbb{R}^d_+$-valued, continuous adapted process satisfying CFS. Then X admits an ε-consistent pricing system for all $\varepsilon > 0$.

Our main result is given in the following theorem.

Theorem 4.2.

Let us consider the process

$$S_t = R_t + \int_0^t \phi_s dB_s^H,$$

where

- $(R_t)_{t\in[0,T]}$ is a continuous adapted process,
- $(\phi_t)_{t\in[0,T]}$ is elementary predictable s.t. $\int_0^T \phi_s^2 ds < \infty$ a.s.,
- $(B_t^H)_{t\in[0,T]}$ is a fractional Brownian motion independent of R and ϕ.

If we have

$$\mathrm{meas(t \in [0,T] : \phi_t = 0) = 0, \quad P-a.s.,} \text{ (meas: Lebesgue measure)},$$

then S has CFS.

***Proof*:**

We adapt the proof of proposition 4.2 (Guasoni et al., 2008).

Let

$$J(t) = \int_0^t \phi_s dB_s^H.$$

By considering the restriction of S on an interval $[v, \mu]$, $v < \mu < T$, it is sufficient to prove that the conditional law $P(J_{|_{[v,T]}} \mid \mathcal{F}_v)$ has full support on $C_{J_v}([v, \mu], \mathbb{R})$ almost surely. And it is enough to show this property on an interval, where ϕ is constant with respect to time (and thus continuous). Thus, we can take T small enough such that ϕ has the form $\phi(t) = \xi$ on $[v, T]$, where $\xi \neq 0$ and it is $\mathcal{F}_v$-measurable.

So, we have to prove that

$$J(t) = \int_v^t \phi_s K_H(t, s) dB_s, \quad s \in [v, T]$$

has full support on $C_0([v, T], \mathbb{R})$.

Theorem 3 in Decreusefond and Üstünel (1999) states that the topological support of a continuous Gaussian process is equal to the norm closure of its reproducing kernel Hilbert space.

In our case, the support of $J(t)$ is

$$\mathbb{H} := \left\{ f \in C_0([v, T], \mathbb{R}) : f(t) = \int_v^t \phi(s) K_H(t, s) g(s) ds, \text{ for some } g \in L^2[v, T] \right\}.$$

Thus, it is sufficient to show that $\mathbb{H}$ is norm-dense in $C_0([v, T], \mathbb{R})$.

To achieve this, we need to recall the Liouville fractional integral operator for any $f \in L^1[a, b]$ and $\alpha > 0$,

$$(I_{a^+}^{\alpha} g)(t) := \frac{1}{\Gamma(\alpha)} \int_a^t g(s)(t - s)^{\alpha - 1} ds, \qquad a \leq t \leq b,$$

and to introduce the kernel operator K_H,

$$(K_H g)(t) := \int_0^t K_H(t, s) g(s) ds, \qquad f \in L^2[0, T], \quad t \in [0, T].$$

(1) We first treat the case $H < \frac{1}{2}$.

$$(K_H(g\phi))(t) := \xi \int_v^t K_H(t, s) g(s) ds, \qquad g \in L^2[0, T], \quad t \in [0, T].$$

In this case, we have via Theorem 2.1 (Decreusefond and Üstünel, 1999), that

$$(K_H(g\phi)) = I_{0^+}^{2H}(s^{\frac{1}{2} - H} I_{0^+}^{\frac{1}{2} - H}(s^{H - \frac{1}{2}}(g\phi)(s)))$$

The argument needs to be split into two steps.

• **Step 1.** Via the following Lemma.

Lemma 4.1. (Guasoni et al., 2008)

If $g \in C_0[v,T]$, then $L_1 g \in C_0([v,T]$, where

$$(L_1 g)(t) = (I_{0^+}^{\frac{1}{2}-H}(s^{H-\frac{1}{2}}(g)(s)))(t).$$

Moreover, $L_1 : C_0[v,T] \longrightarrow C_0[v,T]$ is continuous and has dense range (with respect to the uniform norm).

We have $\varphi \in C_0[v,T]$. Then $L_1 \varphi\phi \in C_0([v,T])$, where

$$(L_1 \varphi\phi)(t) = (I_{0^+}^{\frac{1}{2}-H}(s^{H-\frac{1}{2}}(g\phi)(s)))(t).$$

Recall the identity for $a, b > 0$,

$$\int_0^t (t-u)^{a-1} u^{b-1} du = \mathcal{C}(a,b) t^{a+b-1},$$

where $\mathcal{C}(a,b) \neq 0$ is a constant. Defining, for a fixed $\alpha > 0$,

$$\varphi(s) := \frac{(s-v)^{\alpha}}{\xi s^{H-\frac{1}{2}}},$$

we obtain, for $t \in [v,T]$,

$$\begin{aligned}
(L_1 \varphi\phi)(t) &= \frac{\xi}{\Gamma(\frac{1}{2}-H)} \int_v^t (t-s)^{-H-\frac{1}{2}} \varphi(s) s^{H-\frac{1}{2}}\, ds \\
&= \frac{1}{\Gamma(\frac{1}{2}-H)} \int_v^t (t-s)^{-H-\frac{1}{2}} (s-v)^{\alpha}\, ds \\
&\leq \int_0^{t-v} u^{\alpha} (t-v-u)^{-H-\frac{1}{2}}\, du \\
&= \mathcal{C}\left(\frac{1}{2}-H, \alpha+1\right)(t-v)^{\alpha-H+\frac{1}{2}}.
\end{aligned} \tag{3}$$

Varying α, we find that $(t-v)^n \in \mathrm{Im}(L_1)$ for $n \geq 1$ and the Stone-Weierstrass theorem guarantees that $\mathrm{Im}(L_1)$ is dense in $\mathcal{C}_0[v,T]$.

• **Step 2.** Via the following Lemma.

Lemma 4.2. (Guasoni et al., 2008)

If $g \in C_0[v,T]$, then $L_2 g \in C_0([v,T]$, where

$$(L_2 g)(t) = (I_{O^+}^{2H}(s^{\frac{1}{2}-H} g(s)))(t)$$

and $L_2 : C_0[v,T] \longrightarrow C_0[v,T]$ is continuous and has dense range.

We have $g \in C_0[v,T]$, then $L_1 g\phi \in C_0([v,T])$, where

$$(L_2 g\phi)(t) = (I_{O^+}^{2H}(s^{\frac{1}{2}-H}(g\phi)(s)))(t)$$

and $L_2 : C_0[v,T] \longrightarrow C_0[v,T]$ is continuous and has dense range.

Since the restriction of K_H to $C_0[v,T]$ is exactly $L_2 \circ L_1$, we may conclude that $K_H : C_0[v,T] \longrightarrow C_0[v,T]$ has dense range and, a fortiori, $\mathbb{H}$ is norm-dense in $C_0[v,T]$.

(2) In the case $H \geq \frac{1}{2}$, a similar representation holds as seen in Theorem 2.1 (Decreusefond and Üstünel, 1999),

$$K_H(g\phi) = I^1_{0^+}(s^{H-\frac{1}{2}} I^{H-\frac{1}{2}}_{0^+}(s^{\frac{1}{2}-H}(g\phi))),$$

This argument also needs to be split into two steps.

- **Step 1.** We have $g \in C_0[v,T]$. Then $L_3 g\phi \in C_0([v,T]$, where

$$(L_3(g\phi))(t) = \xi(I^1_{0^+}(s^{H-\frac{1}{2}} g(s)))(t).$$

Defining, for a fixed $\alpha > 0$,

$$g(s) := \frac{(s-v)^\alpha}{\xi s^{\frac{1}{2}-H}},$$

we obtain, for $t \in [v,T]$,

$$\begin{aligned}(L_3 g\phi)(t) &= \frac{\xi}{\Gamma(H-\frac{1}{2})} \int_v^t (t-s)^{H-\frac{3}{2}} g(s) s^{\frac{1}{2}-H}\, ds \\ &= \frac{1}{\Gamma(H-\frac{1}{2})} \int_v^t (t-s)^{H-\frac{3}{2}} (s-v)^\alpha\, ds \\ &\leq \int_0^{t-v} u^\alpha (t-v-u)^{H-\frac{3}{2}}\, du \\ &= \mathcal{C}(H-\frac{1}{2}, \alpha+1)(t-v)^{\alpha+H-\frac{1}{2}}.\end{aligned} \tag{4}$$

Varying α, we find that $(t-v)^n \in \mathrm{Im}(L_3)$ for $n \geq 1$ and the Stone-Weierstrass theorem guarantees that $\mathrm{Im}(L_3)$ is dense in $\mathcal{C}_0[v,T]$.

- **Step 2.** We have $g \in C_0[v,T]$, then $L_4 g\phi \in C_0([v,T]$, where

$$(L_4(g\phi))(t) = \xi(I^1_{0^+}(s^{H-\frac{1}{2}}(g\phi)(s)))(t)$$

and $\mathrm{Im}(L_4)$ is dense in $\mathcal{C}_0[v,T]$.

Since the restriction of K_H to $C_0[v,T]$ is exactly $L_4 \circ L_3$, we may conclude that $K_H : C_0[v,T] \longrightarrow C_0[v,T]$ has dense range and, a fortiori, $\mathbb{H}$ is norm-dense in $C_0[v,T]$.

As a result, the (S_t) has the property of CFS and there is the consistent price systems which can be seen as generalization of equivalent martingale measures.

Fundamentally on this observation we conclude that this price process doesn't admit arbitrage opportunities under arbitrary small transaction. With it we ensure no-arbitrage without calculating the risk-neutral probability. In this paper we have investigated the conditional full support by the process when the fractional Brownian motion is the integrator, and we have also built the absence of arbitrage opportunities without calculating the risk-neutral probability by the existence of the consistent price systems which admits a martingale measure.

5. Conclusion

In this paper we have investigated the conditional full support by the process when the fractional Brownian motion is the integrator, and we have also built the absence of arbitrage opportunities without calculating the risk-neutral probability by the existence of the consistent price systems which admits a martingale measure.

Acknowledgments

The authors are thankful to the anonymous referees and the Editor-in-Chief, Professor Aliakbar Montazer Haghighi, for useful comments and suggestions towards the improvement of this paper.

REFERENCES

Cherny, A. (2008). Brownian moving average have conditional full support, The Annals of Applied Probability, Vol. 18, No. 5, pp. 1825-1830.

Decreusefond, L. and Üstünel, A. S. (1999). Stochastic analysis of the fractional Brownian motion, Potential Analysis, An International Journal Devoted to the Interactions between Potential Theory, Probability Theory, Geometry and Functional Analysis, Vol. 10, No. 2, pp. 177-214.

Embrechts, P. and Maejima, M. (2002). *Selfsimilar processes*, Princeton Series in Applied Mathematics.

Fernholz, R., Karatzas, I. and Kardaras, C. (2005). Diversity and relative arbitrage in equity markets, Finance and Stochastics, Vol. 9, pp. 1-27.

Gasbarra, D., Sottinen, T. and Van Zanten, H. (2011). Conditional full support of Gaussian processes with stationary increments, Journal of Appl. Prob., Vol. 48, No. 2, pp. 561-568.

Guasoni, P., Rásonyi, M. and Schachermayer, W. (2008). Consistent price systems and face-lifting pricing under transaction costs, The Annals of Applied Probability, Vol. 18, No. 2, pp. 491-520.

Herczegh, A., Prokaj, V. and Rasonyi, M. (2014). Diversity and no arbitrage, Stochastic Analysis and Applications, Vol. 32, No. 5, pp. 876-888.

Kallianpur, G. (1971). *Abstract Wiener processes and their reproducing kernel Hilbert spaces*, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete.

Mandelbrot, B. B. and Van Ness, J. W. (1968). Fractional Brownian motions, Fractional noises and applications, SIAM Rev.

Millet, A. and Sanz-Solé, M. (1994). A simple proof of the support theorem for diffusion processes, Lecture Notes in Math, pp. 36-48.

Nahmani, Y. J. (2009). Introduction to Stochastic Integration with respect to Fractional Brownian Motion, Stochastic Processes, http://ssrn.com/abstract=2087921.

Pakkanen, M. S. (2010). Stochastic integrals and conditional full support, Journal of Applied Probability, Vol. 47, No. 3, pp. 650-667.

Samorodnitsky, G. and Taqqu, M. S. (1994). Stable non-Gaussian random processes, Stochastic

Modeling.
Stroock, D. W. and Varadhan, S. R. S. (1972). On the support of diffusion processes with applications to the strong maximum principle, Univ. California Press, Berkeley, Calif., pp. 333-359.