

(Course Binder)
PRAIRIE VIEW A&M UNIVERSITY
COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
MCEG 4309 FINITE ELEMENT ANALYSIS AND DESIGN
Fall Semester 2021

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Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

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Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

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Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

Course Description in ABET Format

MCEG 4309 Finite Element Analysis and Design

1.	Course number and name:	MCEG 4309 Finite Element Analysis and Design
2.	Credits and contact hours:	3 credit hours; 3 contact hours
3.	Instructor's or course coordinator's name:	Dr. Shield Lin
4.	Text book: Title:	Finite Element Modeling and Simulation with ANSYS Workbench
	Author:	Xiaolin Chen and Yijun Liu
	Publisher, Year & Edition:	CRC Press, Taylor & Francis Group, New York, 2018, 2 nd Edition
	a. Other Supplemental material	ANSYS Workbench User's Guide, Release 18.2, ANSYS, Inc., Canonsburg, Pennsylvania, August 2017.
5.	Specific course information	
	a. Brief description of the content of the course (catalog description)	An Introduction to finite element analysis as a modern computational tool to solve boundary value problems. Applications will be in structural mechanics, fluid flow, and heat transfer. Design and computer projects included.
	b. Prerequisites	CVEG2063 – Mechanics of Materials Prerequisite or Taking Concurrently: MCEG3013 – Heat Transfer.
	c. *Indicate whether a required, elective, or selected elective (as per definition provided) course in the program	Required.
6.	Specific goals for the course	Students will be able to:
	a. specific outcomes of instruction (e.g. The student will be able to explain the significance of current research about a particular topic.)	- understand the fundamental concepts of finite element analysis and the formulation of one and two-dimensional elements and the construction of the system equations; - solve engineering problems using ANSYS Workbench in solid mechanics, assembly of several solids, heat transfer, mechanical vibration, and fluid dynamics; and - conduct engineering design utilizing a finite element computer tool, ANSYS, and write a design report.
	b. Explicitly indicate which of the student outcomes listed in Criterion 3 or any other outcomes are addressed by the course.	This course addresses: Outcome 1 Ability to identify, formulate, and solve complex engineering problems (CEP) by applying principles of engineering, science, and mathematics.
7	Brief list of topics to be covered	1. Matrix algebra in finite element analysis (6 hours) 2. Finite element formulation in various analyses (18 hours) 3. Analysis using ANSYS Workbench for problems in structure, heat transfer, vibration, and fluid dynamics (15 hours) 4. Design project using finite element analysis and presentation (6 hours)

Detailed Course Syllabus
(Distributed to Students at the beginning of the Semester)
ABET Outcomes measured using course should be
explicitly listed in the syllabus



**MCEG 4309 Finite Element Analysis and Design
Fall Semester 2021**

Instructor: Dr. Shield Lin
Section # and CRN: Section P01, CRN # 16950
Office Location: S. R. Collins Building, Room 302A
Office Phone: (936) 261-9958
Email Address: shlin@pvamu.edu
Office Hours: 4:00 – 6:00 p.m. Tuesday, 11:00 a.m. – 3:00 p.m. Wednesday
Mode of Instruction: Face to Face

Course Location: Sam R. Collins Bldg. 114
Class Days & Times: 9:30 – 10:50 a.m. Tuesday and Thursday
Catalog Description: An Introduction to finite element analysis as a modern computational tool to solve boundary value problems. Applications will be in structural mechanics, fluid flow, and heat transfer. Design and computer projects included.

Prerequisites: CVEG2063 – Mechanics of Materials
Co-requisites: Prerequisite or Taking Concurrently: MCEG3013 – Heat Transfer.

Required Text(s): Finite Element Modeling and Simulation with ANSYS Workbench, Second Edition, by Xiaolin Chen and Yijun Liu, CRC Press, Taylor & Francis Group, New York, 2018; ISBN-13: 978-1-138-48629-4.

Recommended Text(s): ANSYS Workbench User's Guide, Release 18.2, ANSYS, Inc., Canonsburg, Pennsylvania, August 2017.
Finite Element Analysis - Theory and Application with ANSYS, Third Edition, by Saeed Moaveni, Pearson Education, Inc., Upper Saddle River, NJ, 2008.

Student Learning Outcomes:

	Upon successful completion of this course, students will be able to:	Program Learning Outcome # Alignment	Core Curriculum Outcome Alignment
1	understand the fundamental concepts of finite element analysis and the formulation of one and two-dimensional elements and the construction of the system.	#1	Develop critical thinking skills.
2	use finite element tool to solve engineering problems using ANSYS Workbench in solid mechanics, assembly of several solids, heat transfer, mechanical vibration, and fluid dynamics.	#1	Develop critical thinking skills.
3	conduct engineering design utilizing a finite element computer tool, ANSYS, and write a design report.	#2	Develop communication skills.

The Mechanical Engineering Program Objectives are to produce graduates who have:

1. the techniques and skills necessary for modern mechanical engineering practices, and an ability to function effectively in multi-disciplinary teams.
2. an ability in life-long learning skills, a knowledge of contemporary issues, and an ability to communicate effectively.
3. an understanding of global and societal context in the aspects of professional and ethical responsibility and the impact of engineering solutions on society.
4. the qualifications to be employed by major industries and government agencies in the State of Texas and the nation.
5. the qualifications to pursue graduate degrees.

Fall 2018-Spring 2022 Student Learning Outcomes Assessment Matrix for MCEG Courses
ENGINEERING ACCREDITATION COMMISSION OF ABET NEW STUDENT LEARNING OUTCOMES
(Please read the important information below the table)

MCEG Courses		ABET EAC Student Outcomes 1-7								Totals	Yearly Average
		Fall 2018	Spring 19	Fall 2019	Spring 2020	Fall 2020	Spring 2021	Fall 2021	Spring 2022		
1021 Mech. Drawing & Desg.		2	2	2	2	2SACS		2SACS			
2013 Thermodynamics I		8SACS	8SACS	8SACS	8SACS	8SACS		8SACS			
2023 Materials Sci/Engr.		8SACS	8SACS	8SACS	8SACS		8SACS		8SACS		
2053 Engineering Mechanics II		8SACS	8SACS	8SACS	8SACS	8SACS		8SACS			
3011 Measurement Lab		7	6	7	6	7	6	7	6		
3013 Heat Transfer		1		1		2	1	2	1		
3021 Thermal Sci. Lab		6	6	6	6		6		6		
3023 Thermodynamics II		1	1	1	1		2		2		
3031 Manufacturing Lab			2		4		2		4		
3033 Manufacturing Proc			2		4		2		4		
3043 Machine Design I		2		2		2		2			
3053 Kinematic Design		1		1		1		1			
3063 Fluid Mechanics		1	6	1	6	1	6	1	6		
4043 Machine Design II		2		2		2		2			
4063 Dynamic Systems			1		1		1		1		
4093 Finite Element		1		1		1		1			
4472 Senior Project I		4, 5		4, 5		4, 5		4, 5			
4482 Senior Project II			2,3,7		2,3,7		2,3,7		2,3,7		
Not Assessed											
Frequency											
Outcome Description & Breakdown into Performance criteria										Totals	Yearly Average
1	1.1: Identify & Formulate CEP (i-vi) based on E, S, M 1.2: Solve CEP (i-vi) based on E, S, M	5	2	5	2	3	2	3	2	24	6.0
2	2.1: Ability to Define/Understand the Problem and then Plan 2.2: Ability to Conduct a Review of the Literature, Generate 2.3: Ability to Perform Preliminary and Detailed Design	2	3	2	1	3	4	3	2	20	5.0
3 (Oral)	3.1: Ability to Organize, Plan, Design/Prepare and Use 3.2: Ability to Articulate Subject Knowledge (Technical 3.3: Appearance and Ability to Provide Good Oral Delivery										
3 (Written)	3.4: Ability to prepare an executive summary 3.5: Ability to organize, plan and properly format a written 3.6: Ability to compose original texts and properly apply the 3.7: Ability to provide appropriate discussion, conclusions	0	1	0	1	0	1	0	1	4	1.0
4	4.1: Recognize ethical and professional responsibilities in engineering 4.2: Make informed judgments on ethical and professional responsibilities	1		1	2	1		1	2	8	2.0
5	5.1: Function effectively on a team whose members together 5.2: Establish goals, plan tasks and meet objectives	1	0	1	0	1	0	1	0	4	1.0
6	6.1: Ability to develop appropriate experiments 6.2: Ability to conduct experiments 6.3: Ability to analyze and Interpret experimental data 6.4: Draw appropriate conclusions and recommendations	0	3	0	3	0	3	0	3	12	3.0
7	7.1: Students are able to acquire new knowledge 7.2: Apply new knowledge to solve problems	0	1	0	1	0	1	0	1	4	1.0
8	8.1: Identify & Formulate EP (i-vi) based on E, S, M 8.2: Solve EP based on E, S, M	3	3	3	3	3	3	3	3	24	6

IMPORTANT INFORMATION

8 Assessment is based on the new outcome 1 but for regular problems, NOT CEP type problems. It is for SACS reporting only and will not be reported to ABET. However, the 1 page assessment summary and grade sheet will be needed for SACS assessment report.

MCEG 1021 assessment is based on NX design project. Since ability to design using NX is important, it will be reported to ABET and SACS, and thus an assessment report will be needed.

Major Course Requirements

Method of Determining Final Course Grade

Course Grade Requirement	Value	Total
1. attendance, discussion, homework, quiz, etc.	Various points	20%
2. Test 1	100	15%
3. Test 2	100	15%
4. Test 3	100	15%
5. Test 4	100	15%
6. Design Project	100	20%
Total:		100%

Grading Criteria and Conversion: 100

A = 90-100

B = 80-89

C = 70-79

D = 60-69

F = 0-59

Detailed Description of Major Assignments:

Assignment Title or Grade Requirement	Description
1. Test 1	Matrix Operations, Formulation of 1D finite element system, Solving for solutions.
2. Test 2	Formulation of 2D finite element system, Solving for solutions.
3. Test 3	Use of ANSYS program to solve a structure problem on computer.
4. Test 4	Use of ANSYS program to solve heat transfer or vibration problem on computer.
5. Design Project	Design an engineering device using finite element method and write a report.

Course Procedures or Additional Instructor Policies

Taskstream

Taskstream is a tool that Prairie View A&M University uses for assessment purposes. One of your assignments may be required to be submitted as an "artifact," an item of coursework that serves as evidence that course objectives are met. If applicable, more information will be provided during the semester by your department, but for general information, you can visit Taskstream via the link in eCourses.

Assignments & Due Dates

All assignments must be submitted to Canvas before the due day and time, **30% of grade deduction** for the late submission between "Due" time and "Available Until" time. After "Available Until" time, the grade for the assignment **will be 0**.

Email Messages

For non-urgent issues, the instructor encourages the students to use emails to communicate with the instructor. The instructor will reply email messages within 24 hours.

Office Hours

The students, as a group or an individual, are welcome to schedule zoom meetings with the instructor during the office hours. Scheduling can be done by an email request.

Grading Feedback

Other than the final design project, the instructor will grade assignments, discussions, and tests within one week. The results of grading and comments will be posted on Canvas.

Professionalism of On-line Discussions

When responding to discussion assignments, all responders shall be professional during the entire communication. We shall be considerate to others and use encouraging words even you do not agree with other's opinions. We also must adhere to the privacy of personal data.

Semester Calendar for Fall Semester 2021

Week	Description
Week One:	
08/24	Introduction of Finite Element Method and Software Access
08/26	Matrix Algebra and MATLAB
Week Two:	
08/31	Finite Element Basics
09/02	Finite Element Formulation – Spring
Week Three:	
09/07	Finite Element Formulation – Bars
09/09	Test 1
Week Four:	
09/14	Finite Element Formulation – Trusses
09/16	Examples of Truss Problems
Week Five:	
09/21	Formulation of Beam Element
09/23	Modeling of Beams and Frames
Week Six:	
09/28	Examples of Beam and Frame Problems
09/30	Solution Process of Beam and Frame Problems
Week Seven:	
10/05	Modeling and Solution Techniques
10/07	Test 2
Week Eight:	
10/12	ANSYS Workbench – Trusses
10/14	ANSYS Workbench – Wrench
Week Nine:	
10/19	ANSYS Workbench – Rotating Fountain
10/21	ANSYS Workbench – Stand Assembly
Week Ten:	
10/26	ANSYS Workbench – Heat Transfer Analysis
10/28	Test 3
Week Eleven:	
11/02	Design Project
11/04	ANSYS Workbench – Guitar Vibration Modes
Week Twelve:	
11/09	ANSYS Workbench – Aerodynamics of Car
11/11	Design Optimization with Finite Element
Week Thirteen:	
11/16	Test 4
11/18	Design Project Presentation/Discussions
Week Fourteen:	
11/23	Design Project Presentation/Discussions
11/25	(Thanksgiving Holiday)
Week Fifteen:	
11/30	Guest Speaker/Industry Applications

Student Support and Success

John B. Coleman Library

The John B. Coleman Library's mission is to enhance the scholarly pursuit of knowledge, to foster intellectual curiosity, and to promote life-long learning and research through our innovative services, resources, and cultural programs, which support the Prairie View A&M University's global mission of teaching, service, and research. It maintains library collections and access both on campus, online, and through local agreements to further the educational goals of students and faculty. Website: <https://www.pvamu.edu/library/>; Phone: 936-261-1500

Academic Advising Services

Academic Advising Services offers students a variety of services that contributes to student success and leads towards graduation. We assist students with understanding university policies and procedures that affect academic progress. We support the early alert program to help students get connected to success early in the semester. We help refer students to the appropriate academic support services when they are unsure of the best resource for their needs. Faculty advisors support some students in their respective colleges. Your faculty advisor can be identified in PantherTracks. Advisors with Academic Advising Services are available to all students. We are located across campus. Find your advisor's location by academic major at www.pvamu.edu/advising. Phone: 936-261-5911

The University Tutoring Center

The University Tutoring Center (UTC) offers free tutoring and academic support to all registered PVAMU students. The mission of the UTC is to help provide a solid academic foundation that enables students to become confident, capable, independent learners. Competent and caring staff and peer tutors guide students in identifying, acquiring, and enhancing the knowledge, skills, and attitudes needed to reach their desired goals. Tutoring and academic support are offered face-to-face in the UTC, in virtual face-to-face sessions (<https://www.pvamu.edu/student-success/sass/university-tutoring-center/>), and through online sessions (<https://www.pvamu.edu/pvplace/>). Other support services available for students include Supplemental Instruction, Study Break, Academic Success Workshops, and Algebra Study Jam. Location: J. B. Coleman Library, Rm. 307; Phone: 936-261-1561; Email: pvtutoring@pvamu.edu; Website: <https://www.pvamu.edu/student-success/sass/university-tutoring-center/>

Writing Center

The Writing Center provides well-trained peer tutors to assist students with writing assignments at any stage of the writing process. Tutors help students with various writing tasks from understanding assignments, brainstorming, drafting, revising, editing, researching, and integrating sources. Students have free access to Grammarly online writing assistance. Grammarly is an automated proofreading and plagiarism detection tool. Students must register for Grammarly by using their student email address. In addition, students have access to face-to-face and virtual tutoring services either asynchronously via email or synchronously via Zoom. Location: J. B. Coleman Library, Rm. 209; Phone: 936-261-3724; Website: <https://www.pvamu.edu/student-success/writing-center/>; Grammarly Registration: <https://www.grammarly.com/enterprise/signup>

Academic Early Alert

Academic Early Alert is a proactive system of communication and collaboration between faculty, academic advisors, and PVAMU students that is designed to support student success by promptly identifying issues and allowing for intervention. Academic Early Alerts help students by providing a central location to schedule advising appointments, view advisor contact information, and request assistance. Students who recognize that they have a problem that is negatively affecting their academic performance or ability to continue school may self-refer an Academic Early Alert. To do so, students will log in to PV Place and click on Academic Early Alert on the left sidebar. Phone: 936-261-5902; Website: <https://www.pvamu.edu/student-success/early-alert/>

Student Counseling Services

The Student Counseling Services unit offers a range of services and programs to assist students in maximizing their potential for success: short-term individual, couples, and group counseling, as well as crisis intervention, outreach, consultation, and referral services. The staff is licensed by the State of Texas and assists students who are dealing with academic skills concerns, situational crises, adjustment problems, and emotional difficulties. Information shared with the staff is treated confidentially and in

accordance with Texas State Law. Location: Hobart Taylor, 2nd floor; Phone: 936-261-3564; Website: <https://www.pvamu.edu/healthservices/student-counseling-services/>

Office of Testing Services

Testing Services serves to create opportunities by offering a suite of exams that aid in the students' academic and professional success. Currently, we administer entrance (HESI A2), college readiness (TSI assessment), Prior Learning (CLEP, DSST), and proctored exams. Location: Wilhelmina Delco, 3rd Floor, Rm. 305; Phone: 936-261-3627; Email: aetesting@pvamu.edu; Website: www.pvamu.edu/testing

Office of Diagnostic Testing and Disability Services

The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, contact the Office of Disability Services. As a federally-mandated educational support unit, the Office of Disability Services serves as the repository for confidential disability files for faculty, staff, and students. For persons with a disability, the Office develops individualized ADA letters of request for accommodations. Other services include learning style inventories, awareness workshops, accessibility pathways, webinars, computer laboratory with adapted hard and software, adapted furniture, proctoring non-standardized test administrations, ASL interpreters, ALDs, digital recorders, Livescribe, and a comprehensive referral network across campus and the broader community. Location: Hobart Taylor, Rm. 1D128; Phone: 936-261-3583; Website: <https://www.pvamu.edu/disabilityservices/>

Center for Instructional Innovation and Technology Services (CIITS)

Distance Learning, also referred to as Distance Education, is the employment of alternative instructional delivery methods to extend programs and services to persons unable to attend college in the traditional manner. The Center for Instructional Innovation and Technology Services (CIITS) supports student learning through online, hybrid, web-assist, and 2-way video course delivery. For more details and contact information, visit:

<https://www.pvamu.edu/dlearning/distance-learning-2-2/students-2/>; Phone: 936-261-3283

Veteran Affairs

Veterans Services works with student veterans, current military and military dependents to support their transition to the college environment and continued persistence to graduation. The Office coordinates and certifies benefits for both the G.I. Bill and the Texas Hazlewood Act. Location: Evans Hall, Rm. 102; Phone: 936-261-3563; Website: <https://www.pvamu.edu/sa/departments/veteranaffairs/>

Office for Student Engagement

The Office for Student Engagement delivers comprehensive programs and services designed to meet the co-curricular needs of students. The Office implements inclusive and accessible programs and services that enhance student development through exposure to and participation in diverse and relevant social, cultural, intellectual, recreational, community service, leadership development, and campus governance. Location: Memorial Student Center, Rm. 221; Phone: 936-261-1340; Website: <https://www.pvamu.edu/studentengagement/>

Career Services

Career Services supports students through professional development, career readiness, and placement and employment assistance. The Office provides one-on-one career coaching, interview preparation, resume and letter writing, and career exploration workshops and seminars. Services are provided for students at the Northwest Houston Center and College of Nursing in the Medical Center twice a month or on a requested basis. Distance Learning students are encouraged to visit the Career Services website for information regarding services provided. Location: Anderson Hall, 2nd floor; Phone: 936-261-3570; Website: <https://www.pvamu.edu/careerservices/>

University Rules and Procedures

Academic Misconduct

Academic dishonesty is defined as any form of cheating or dishonesty that has the effect or intent of interfering with any academic exercise or fair evaluation of a student's performance. The college faculty can provide additional information, particularly related to a specific course, laboratory, or assignment.

You are expected to practice academic honesty in every aspect of this course and all other courses. Make sure you are familiar with the *University Administrative Guidelines on Academic Integrity*, which can be found on the [Academic Integrity webpage](#). Students who engage in academic misconduct are subject to university disciplinary procedures. As listed in the *University Administrative Guidelines on Academic Integrity*, the University Online Catalog, and the Student Code of Conduct, the following are examples of prohibited conduct. This list is not designed to be all-inclusive or exhaustive. In addition to academic sanctions, any student found to have committed academic misconduct that is also a violation of criminal law may also be subject to disciplinary review and action by the Office of Student Conduct (as outlined in the Student Code of Conduct).

Forms of Academic Dishonesty:

1. Cheating: Deception in which a student misrepresents that he/she has mastered information on an academic exercise that he/she has not learned, giving or receiving aid unauthorized by the instructor on assignments or examinations. Examples: unauthorized use of notes for a test; using a "cheat sheet" on a quiz or exam; any alteration made on a graded test or exam which is then resubmitted to the teacher;
2. Plagiarism: Careless or deliberate use of the work or the ideas of another; representation of another's work, words, ideas, or data as your own without permission or appropriate acknowledgment. Examples: copying another's paper or answers, failure to identify information or essays from the internet and submitting or representing it as your own; submitting an assignment which has been partially or wholly done by another and claiming it as yours; not properly acknowledging a source which has been summarized or paraphrased in your work; failure to acknowledge the use of another's words with quotation marks;
3. Collusion: When more than one student or person contributes to a piece of work that is submitted as the work of an individual;
4. Conspiracy: Agreeing with one or more persons to commit an act of academic/scholastic dishonesty; and
5. Multiple Submission: Submission of work from one course to satisfy a requirement in another course without explicit permission. Example: using a paper prepared and graded for credit in one course to fulfill a requirement and receive credit in a different course.

Nonacademic Misconduct

The university respects the rights of instructors to teach and students to learn. Maintenance of these rights requires campus conditions that do not impede their exercise. Campus behavior that interferes with either (1) the instructor's ability to conduct the class, (2) the inability of other students to profit from the instructional program, or (3) campus behavior that interferes with the rights of others will not be tolerated. An individual engaging in such disruptive behavior may be subject to disciplinary action. The Office of Student Conduct will adjudicate such incidents under nonacademic procedures.

Sexual Misconduct

Sexual harassment of students and employees at Prairie View A&M University is unacceptable and will not be tolerated. Any member of the university community violating the university's sexual harassment policy will be subject to disciplinary action. In accordance with the Texas A&M University System guidelines, your instructor is obligated to report to the Office of Title IX Compliance (titleixteam@pvamu.edu) any instance of sexual misconduct involving a student, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which the instructor becomes aware during this course through

writing, discussion, or personal disclosure. The faculty and staff of PVAMU actively strive to provide a learning, working, and living environment that promotes respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like assistance or have questions, they may contact the Title IX Coordinator at 936-261-2144 or titleixteam@pvamu.edu. More information can be found at www.pvamu.edu/titleix, including confidential resources available on campus.

Pregnancy, Pregnancy-related, and Parenting Accommodations

Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related conditions, or parenting (reasonably immediate postpartum period) are encouraged to contact Student Disability Services or the Dean of Students' Office for additional information and to request accommodations.

Non-Discrimination Statement

Prairie View A&M University does not discriminate on the basis of race, color, sex, religion, national origin, age, disability, genetic information, veteran status, sexual orientation, or gender identity in its programs and activities. The University is committed to supporting students and complying with The Texas A&M University System non-discrimination policy. It seeks to establish an environment that is free of bias, discrimination, and harassment. If you experience an incident of discrimination or harassment, we encourage you to report it. If you would like to speak with someone who may be able to afford you privacy or confidentiality, there are individuals who can meet with you. The Director of Equal Opportunity & Diversity has been designated to handle inquiries regarding the non-discrimination policies and can be reached at Harrington Science Building, Suite 109 or by phone at 936-261-1744 or 1792.

Class Attendance Policy (See the University Online Catalog for Full Attendance Policy)

Prairie View A&M University requires regular class attendance. Attending all classes supports the full academic development of each learner, whether classes are taught with the instructor physically present or via distance learning technologies such as interactive video and/or the internet. Excessive absenteeism, whether excused or unexcused, may result in a student's course grade being reduced or in the assignment of a grade of "F." Absences are accumulated beginning with the first day of class during regular semesters and summer terms. Each faculty member will include the University's attendance policy in each course syllabus.

Student Academic Appeals Process

Authority and responsibility for assigning grades to students rest with the faculty. However, in those instances where students believe that miscommunication, errors, or unfairness of any kind may have adversely affected the instructor's assessment of their academic performance, the student has a right to appeal by the procedure listed in the University Online Catalog and by doing so within thirty days of receiving the grade or experiencing any other problematic academic event that prompted the complaint.

Technical Considerations

Minimum Recommended Hardware and Software:

- Intel PC or Laptop with Windows 10 or later version; Mac with OS High Sierra*
- Smartphone or iPad/Tablet with Wi-Fi*
- High-speed Internet access
- 8 GB Memory
- Hard drive with 320 GB storage space
- 15" monitor, 800x600, color or 16 bit
- Sound card w/speakers
- Microphone and recording software
- Keyboard & mouse
- Most current version of Google Chrome, Safari, or Firefox

Note: Be sure to enable Java & pop-ups in the Web browser preferences

* Smartphones, Google Chrome books, and Android tablets may not be supported. iPads are the only tablets supported.

Participants should have a basic proficiency of the following computer skills:

- Sending and receiving email
- A working knowledge of the Internet
- Microsoft Word (or a program convertible to Word)
- Acrobat PDF Reader
- Windows or Mac OS
- Video conferencing software

Netiquette (online etiquette)

Students are expected to participate in all discussions and virtual classroom chats as directed. Students are to be respectful and courteous to others on discussion boards. Foul or abusive language will not be tolerated. Do not use ALL CAPS for communicating to others AS IT CAN BE INTERPRETED AS YELLING. Avoid slang terms such as "wassup?" and texting abbreviations such as "u" instead of "you." Limit and possibly avoid the use of emoticons. Be cautious when using humor or sarcasm as tone is sometimes lost in an email or discussion post, and the message might be taken seriously or sound offensive.

Video Conferencing Etiquette

When using Zoom, WebEx, or other video conferencing tools, confirm the visible area is tidy, clear of background clutter, inappropriate or offensive posters, and other distractions. Ensure you dress appropriately and avoid using high traffic or noisy areas. Stay muted when you are not speaking and avoid eating/drinking during the session. Before the class session begins, test audio, video, and lighting to alleviate technology issues.

Technical Support

Students should go to <https://mypassword.pvamu.edu/> if they have password issues. The page will provide instructions for resetting passwords and contact information if login issues persist. For other technical questions regarding eCourses, call the Center for Instructional Innovation and Technology Services at 936-261-3283 or email ciits@pvamu.edu.

Communication Expectations and Standards

Emails or discussion postings will receive a response from the instructor, usually in less than 48 hours. Urgent emails should be marked as such. Check regularly for responses.

Discussion Requirement

Online courses often require minimal to no face-to-face meetings. However, conversations about the readings, lectures, materials, and other aspects of the course can occur in a seminar fashion. The use of the discussion board will accomplish this. The instructor will determine the exact use of discussion boards.

It is strongly suggested that students type their discussion postings in a word processing application such as Word and save it to their PC or a removable drive before posting to the discussion board. This is important for two reasons: 1) If for some reason your discussion responses are lost in your online course, you will have another copy; 2) Grammatical errors can be greatly minimized by the use of the spell-and-grammar check functions in word processing applications. Once the post(s) have been typed and corrected in the word processing application, copy and paste to the discussion board.

COVID-19 Campus Safety Measures

To promote public safety and protect students, faculty, and staff during the coronavirus pandemic, PVAMU has adopted policies and practices to limit virus transmission.

- **Self-monitoring** - Students should follow CDC recommendations for self-monitoring. Students who have a fever or exhibit symptoms of COVID-19 should participate in class remotely and should not participate in face-to-face instruction.
- **Face Coverings** - Face coverings (cloth face covering, surgical mask, etc.) are recommended in classrooms, teaching laboratories, common spaces such as lobbies and hallways, public study

spaces, libraries, academic resource, and support offices, and outdoor spaces where 6 feet of physical distancing is challenging to maintain reliably.

- **Physical Distancing** - Physical distancing should be maintained between students, instructors, and others in course and course-related activities where possible.
- **Personal Illness and Quarantine** - Students required to quarantine are to participate in courses and course-related activities remotely and must not attend face-to-face course activities. Students should notify their instructors of the quarantine requirement. Students under quarantine are expected to participate in courses and complete graded work unless they have symptoms that are too severe to participate in course activities. Students experiencing personal injury or illness that is too severe for the student to attend class qualify for an excused absence. To receive an excused absence, students must provide appropriate documentation to the Office for Student Conduct, studentconduct@pvamu.edu.

End of Semester Course Assessment Report (assessed every fall semester)

This summary contains the following Information

1. Performance Statistics

- **Total number of students in class**
- **The class average performance (stated as %) in each course outcome assessed in this class and the two previous times course was assessed.**
- **The acceptable class average for the class in each outcome area as set by your Department.**
- **Percentage of students who scored below the expected average in each outcome.**

2. Implementation Summary

- **Brief summary of plans implemented during the semester based on last assessment report**

3. Perceived Problems

- **Instructor's critical evaluation of perceived problems that affected students' performance. Realistic problems should be identified throughout the semester and summarized in this section of the report.**

4. New Plans for Addressing Problems

- **Instructor's plans for addressing the perceived problems the next time the class is taught.**

5. Overall Trend over last three assessment periods

- **Instructors summary of overall performance trend for each outcome measured over the last three periods.**

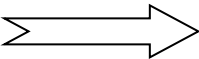
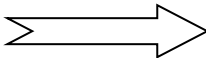
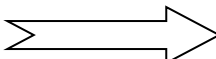
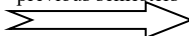
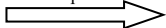
5. Were Expectations Met?

- **Answer YES if the class average was above the expected average and the stated percentage of students scored at or above the expected average, else answer NO.**

END OF SEMESTER COURSE OUTCOME ASSESSMENT REPORT
MCEG 4309-P01 and P02 FINITE ELEMENT ANALYSIS & DESIGN
FALL 2021 SEMESTER

Report Prepared by: Dr. Shield Lin

Report Date: December 2021

Semester	Analysis Type	Number of Students	Outcome 1.			
			Expected Average	Expected Percent of students at or above average	Class Average	% Students meeting Expected Average
Fall 2021	Direct	44	70	70	72.6	73%
Fall 2020	Direct	54	70	70	74.1	74%
Fall 2019	Direct	40	70	70	78.4	77%
Implementation Summary	Items implemented from previous report or after meeting previous instructor 	The students may login the university’s server to use the finite element program, ANSYS. The students can also download, free of charge, from the ANSYS website. Some students downloaded software to their personal laptops, others were using the software in the university’s labs.		Performance Criteria for Outcome 1 Ability to identify, formulate, and solve complex engineering problems (CEP) by applying principles of engineering, science, and mathematics. The instructor used the average of two “in-class” test scores and a design project score to evaluate students’ ability. 1. Test 2 for finite element equation formulation. Although the individual test problems are not full “complex engineering problems,” they represent important equation derivations and solving processes that can be used to solve a complex problem. The “in-class” test is the best means to evaluate students’ individual ability in equation derivation. 2. Test 4 using finite element program ANSYS to solve an engineering problem. Although the test problem itself is not a full “complex engineering problem,” it represents an important step of using a modern computer tool, ANSYS, to solve a complex problem. The “in-class” test is the best means to evaluate students’ individual ability in software skills. 3. A design project of “Heat Sink for Electronic Device” requires the students to investigate the design of an existing heat sink and its operating environment on the market. The product serves as the benchmark design. The students are to modify the design to fit the same operating environment, and prove by finite element simulation that their modified design performs better than the one found on the market. In other words, the students’ modified design can keep the electronic device cooler than the benchmark product under the same operational conditions.		
Perceived Problems	Perceived problems are directly related to the PC and the sub items under them 	Many students did not do well in Test 2 for finite element equation formulation and solving process. However, they did much better in Test 4 using finite element program ANSYS to solve an engineering problem. Most students completed reasonably good design projects of a modified heat sink with better performance than the benchmark.				
Plans for Addressing Problems	Plans for address specific measures to address PC and their sub areas 	In the next year, the instructor plans to add more notes to the equation derivation process, and add more short questions in the PowerPoint presentations for class. The goal is to improve the students’ ability of formulating and solving finite element problems by more interaction with the students in class.				
Overall Trend over Periods	Ascertain if there were improvements over previous semesters 	73% students met expectation in the outcome (1) for this semester.				
Were Expectations Met?	Class Average $\geq 70\%$ and $\geq 70\%$ of Students Performed at or Above Expected Average. 	YES				

Grade Sheet Showing Student and Class Performance in Outcome (1)

Outcome (1) was evaluated in this semester by

1. Test 2 for finite element equation formulation. Although the individual test problems are not full “complex engineering problems,” they represent important equation derivations and solving processes that can be used to solve a complex problem. The “in-class” test is the best means to evaluate students’ individual ability in equation derivation.
2. Test 4 using finite element program ANSYS to solve an engineering problem. Although the test problem itself is not a full “complex engineering problem,” it represents an important step of using a modern computer tool, ANSYS, to solve a complex problem. The “in-class” test is the best means to evaluate students’ individual ability in software skills.
3. A design project of “Heat Sink for Electronic Device” requires the students to investigate the design of an existing heat sink and its operating environment on the market. The product serves as the benchmark design. The students are to modify the design to fit the same operating environment, and prove by finite element simulation that their modified design performs better than the one found on the market. In other words, the students’ modified design can keep the electronic device cooler than the benchmark product under the same operational conditions.

**Grades for Test 2, Test 4, Design Project, and Their Average
(Section P01 has 20 students; Section P02 has 24 students)
Fall 2021 semester**

name	Test 2	Test 4	Project	Average	Average >= 70
name 1	47	80	88	71.50	*
name 2	78	100	90	89.33	*
name 3	38	70	75	61.00	
name 4	47	100	75	74.00	*
name 5	10	100	75	61.67	
name 6	68	100	75	81.00	*
name 7	59	80	75	71.33	*
name 8	53	100	81	78.08	*
name 9	59	100	65	74.67	*
name 10	89	100	88	92.17	*
name 11	67	100	0	55.67	
name 12	75	70	69	71.25	*
name 13	56	100	56	70.75	*
name 14	42	100	0	47.33	
name 15	26	80	69	58.25	
name 16	45	80	69	64.58	
name 17	30	100	0	43.33	
name 18	62	80	69	70.25	*
name 19	72	100	81	84.42	*
name 20	41	90	63	64.50	
name 21	73	100	88	86.83	*
name 22	59	100	69	75.92	*
name 23	58	80	75	71.00	*
name 24	35	100	75	70.00	*
name 25	70	100	69	79.58	*
name 26	92	100	75	89.00	*
name 27	70	100	90	86.67	*
name 28	63	80	88	76.83	*
name 29	62	100	75	79.00	*
name 30	71	100	88	86.17	*
name 31	57	90	75	74.00	*
name 32	35	80	75	63.33	
name 33	48	80	69	65.58	
name 34	40	100	83	74.17	*
name 35	44	90	63	65.50	
name 36	54	80	81	71.75	*
name 37	78	100	81	86.42	*
name 38	31	100	81	70.75	*
name 39	40	100	88	75.83	*
name 40	52	100	81	77.75	*
name 41	53	90	81	74.75	*
name 42	61	80	69	69.92	*
name 43	46	90	69	68.25	
name 44	40	100	75	71.67	*
Class Overall Average:				72.63	
Percentage above 70:				73%	

FINITE ELEMENT ANALYSIS AND DESIGN FALL 2021

Title of Assignment: TEST 1

TOPICS COVERED IN TEST 1

1. Matrix multiplications
2. Solving simultaneous equations by matrix method
3. Solving spring systems via finite element method
4. Solving bar assembly via finite element method

The 1st Test(1)
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
September 9, 2021

1. Matrix A and matrix B are defined as below.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ -1 & -2 \\ 0 & 5 \end{bmatrix}$$

(5%)

- (a) Can we perform the matrix multiplication for A*B? If we can, what's the result?

(5%)

- (b) Can we perform the matrix multiplication for B*A? If we can, what's the result?

(10%)

2. Convert the following 4 algebraic equations to one matrix equation, and write **MATLAB Command Statements** to solve the variables of X_1 , X_2 , X_3 , and X_4 .

(Solutions for X_1 , X_2 , X_3 , and X_4 are not required)

$$X_1 + 2X_2 + 3X_3 + 4X_4 = 70$$

$$6X_1 - 7X_2 - 8X_3 = -30$$

$$12X_3 + 11X_4 = 150$$

$$16X_1 + 14X_3 + 13X_4 = 300$$

MATLAB Statements:

>>

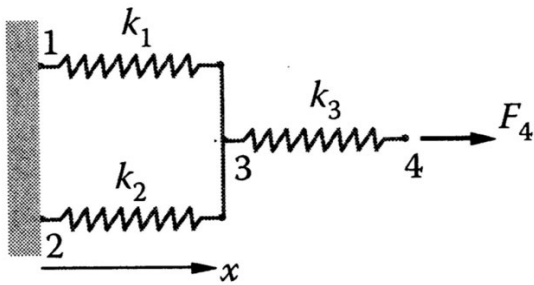
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3. A spring system shown below is restricted in moving in one dimension (horizontal only) as shown in the x coordinate. Given: $k_1 = 5 \text{ N/mm}$, $k_2 = 15 \text{ N/mm}$, $k_3 = 10 \text{ N/mm}$, one external force $F_4 = 40 \text{ N}$ is applied to node 4. Nodes 1 and 2 are attached on a fixed structure.



(2%)

How many nodes in the system? _____

How many elements in the system? _____

(2%)

What are the boundary conditions (write in terms of equation forms)?

(2%)

What are the external forces?

Find:

(10%)

(a) Global stiffness matrix of the spring system

(8%)

(b) Displacements of Nodes 3 and 4

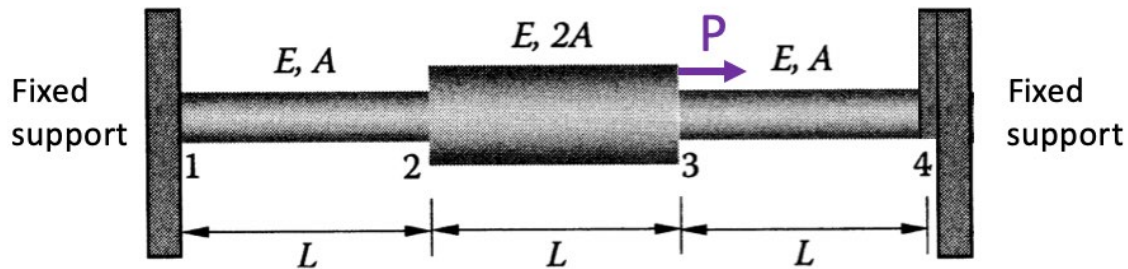
(8%)

(c) Reaction forces at nodes 1 and 2

(8%)

(d) Spring force 1 and Spring force 3; are they tension or compression?

4. A bar assembly is loaded with force $P = 500$ lb at node 3 as shown in the figure. Assume the bar can only move in one dimension in horizontal direction. Node 1 and node 4 are fixed on the supporting structure. The bar assembly is made of a homogeneous material with Young's modulus $E = 10^4$ psi, the length of each element $L = 2$ in, and the cross-sectional area $A = 1$ in², cross-sectional area $2A = 2$ in².



(3%)

(a) What are the variables attached to the 4 nodes?

(3%)

(b) What are the boundary conditions?

(4%)

(c) What are the external loads?

(10%)

(d) Use finite element method to set the global stiffness matrix for the bar assembly.

(10%)

(e) Solve for displacements at node 2 and node 3.

(10%)

(f) Obtain stress in each of the elements.

ABOVE AVERAGE

The 1st Test(1)
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
September 9, 2021

1. Matrix A and matrix B are defined as below.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ -1 & -2 \\ 0 & 5 \end{bmatrix}$$

(5%)

- (a) Can we perform the matrix multiplication for $A \cdot B$? If we can, what's the result?

Yes

$$\begin{bmatrix} 3 \cdot 1 - 2 \cdot 1 + 1 \cdot 0 & 3 \cdot 1 - 2(2) + 1(5) \\ -1(1) + 0 \cdot (-1) + 4(0) & -1(1) + 0(-2) + 4(5) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ -1 & 19 \end{bmatrix}$$



(5%)

- (b) Can we perform the matrix multiplication for $B \cdot A$? If we can, what's the result?

Yes

$$= \begin{bmatrix} 1(3) + 1(-1) & 1(2) + 1(0) & 1(1) + 1(4) \\ -1(3) + (-2)(-1) & -1(2) - 2(0) & (-1)(1) - 2(4) \\ 0(3) + 5(-1) & 0(2) + 5(0) & 0(1) + 5(4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 5 \\ -1 & -2 & -9 \\ -5 & 0 & 20 \end{bmatrix}$$



(10%)

2. Convert the following 4 algebraic equations to one matrix equation, and write

MATLAB Command Statements to solve the variables of X_1 , X_2 , X_3 , and X_4 .

(Solutions for X_1 , X_2 , X_3 , and X_4 are not required)

$$X_1 + 2X_2 + 3X_3 + 4X_4 = 70$$

$$6X_1 - 7X_2 - 8X_3 = -30$$

$$12X_3 + 11X_4 = 150$$

$$16X_1 + 14X_3 + 13X_4 = 300$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -7 & -8 & 0 \\ 0 & 0 & 12 & 11 \\ 16 & 0 & 14 & 13 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 70 \\ -30 \\ 150 \\ 300 \end{bmatrix}$$

MATLAB Statements:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -7 & -8 & 0 \\ 0 & 0 & 12 & 11 \\ 16 & 0 & 14 & 13 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 70 \\ -30 \\ 150 \\ 300 \end{bmatrix}$$

-1

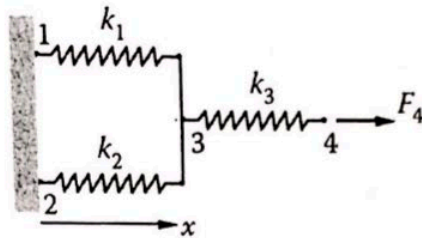
$$\gg A = [1 \ 2 \ 3 \ 4; 6 \ -7 \ -8 \ 0; 0 \ 0 \ 12 \ 11; 16 \ 0 \ 14 \ 13]$$

$$\gg B = [70; -30; 150; 300]$$

$$\gg T = [A]^{-1} \cdot [B]$$

$$\gg T = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \text{ — "this would be solution"}$$

3. A spring system shown below is restricted in moving in one dimension (horizontal only) as shown in the x coordinate. Given: $k_1 = 5 \text{ N/mm}$, $k_2 = 15 \text{ N/mm}$, $k_3 = 10 \text{ N/mm}$, one external force $F_4 = 40 \text{ N}$ is applied to node 4. Nodes 1 and 2 are attached on a fixed structure.



(2%)
How many nodes in the system? 4
How many elements in the system? 3

(2%)
What are the boundary conditions (write in terms of equation forms)?

$$u_1 = 0 \quad u_2 = 0$$

(2%)
What are the external forces?

$$F_3 = 0 \quad F_4 = 40 \text{ N}$$

Find:
(10%)

(a) Global stiffness matrix of the spring system

$$K = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & -k_2 & 0 \\ -k_1 & -k_2 & k_1 + k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 15 & -15 & 0 \\ -5 & -15 & 30 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix}$$

$$K = \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 15 & -15 & 0 \\ -5 & -15 & 30 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix}$$

$$k_1 = 5 \text{ N/mm}$$

$$k_2 = 15 \text{ N/mm}$$

$$k_3 = 10 \text{ N/mm}$$

$$K_1 = \begin{bmatrix} u_1 & u_3 \\ +k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} u_2 & u_3 \\ k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} u_3 & u_4 \\ k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

(8%) $[N/mm]$ $u_1 = u_2 = 0$ $[mm]$ $[N]$

(b) Displacements of Nodes 3 and 4

$$\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 15 & -15 & 0 \\ -5 & -15 & 30 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 0 \\ 40 \end{bmatrix}$$

$30u_3 - 10u_4 = 0$
 $(-10u_3 + 10u_4 = 40)$
 $20u_3 = 40$
 Node 3 $u_3 = 2mm$

$30(2) - 10(u_4) = 0$
 $60 = 10u_4$
 Node 4 $u_4 = 6mm$

✓

(8%)

(c) Reaction forces at nodes 1 and 2

Node 1: $-5(u_3) = F_1 = -5(2) = -10N$ or $\leftarrow$ $10N$ (Left)

Node 2: $F_2 = -15(u_3) = -15(2mm) = -30N$ ($\leftarrow$)

✓

(8%)

(d) Spring force 1 and Spring force 3; are they tension or compression?

Spring 1 $u_2 - u_1$

$F_1 = k_1 \Delta_{1-2} = 5N/mm(2mm - 0mm) = 10N$ (Tension)

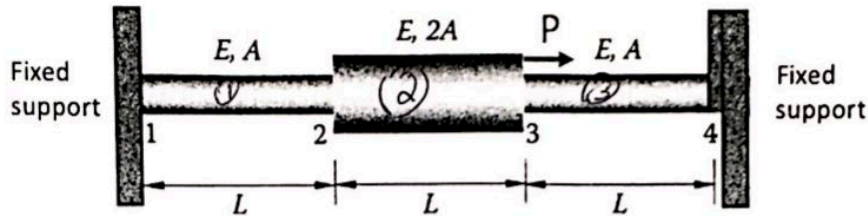
Spring 3 $u_4 - u_3$

$F_3 = k_3 \Delta_{3-4} = 10N/mm(6mm - 2mm) = 40N$ (Tension)

✓

4. A bar assembly is loaded with force $P = 500$ lb at node 3 as shown in the figure. Assume the bar can only move in one dimension in horizontal direction. Node 1 and node 4 are fixed on the supporting structure. The bar assembly is made of a homogeneous material with Young's modulus $E = 10^4$ psi, the length of each element $L = 2$ in, and the cross-sectional area $A = 1$ in², cross-sectional area $2A = 2$ in².

$$k = \frac{EA}{L}$$



(3%)

(a) What are the variables attached to the 4 nodes?

$$u_1, u_2, u_3, u_4$$

(3%)

(b) What are the boundary conditions?

$$u_1 = 0 \quad u_4 = 0$$

(4%)

(c) What are the external loads?

$$F_3 = P = 500 \text{ lb} \quad F_2 = 0 \text{ lb}$$

(10%)

(d) Use finite element method to set the global stiffness matrix for the bar assembly.

$$K = \frac{EA}{L} \begin{bmatrix} & u_1 & u_2 & u_3 & u_4 \\ 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 2 & -2 & 0 \\ 0 & -2 & 2 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix}$$

$$= \frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} = \frac{10^4 \text{ lb/in}^2 (1 \text{ in}^2)}{2 \text{ in}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\frac{5000 \text{ lb/in}}{1 \text{ in}} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$u_1 = u_4 = 0$$

(10%)

(e) Solve for displacements at node 2 and node 3.

$$5000 \text{ lb/in} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \\ 500 \\ F_4 \end{bmatrix}$$

$$\text{Node 2} \quad 5000(3u_2 - 2u_3) = 0$$

$$15000u_2 = 10000u_3 \quad u_3 = \frac{15000u_2}{10000} = 1.5u_2$$

$$5000(-2u_2 + 3u_3) = 500$$

$$\frac{5000}{5000}(-2u_2 + 3(1.5u_2)) = \frac{500}{5000} \quad \frac{1}{10}$$

$$-2u_2 + 4.5u_2 = 0.1$$

$$2.5u_2 = 0.1$$

$$u_2 = 0.04 \text{ in}$$

Node 3

$$u_3 = 1.5(0.04 \text{ in})$$

$$= 0.06 \text{ in}$$

$$F_1 = -200 \text{ lb}$$

$$F_4 = -300 \text{ lb}$$

(10%)

(f) Obtain stress in each of the elements.

Element 1

$$\sigma_1 = E \epsilon = E \begin{bmatrix} 1/L & -1/L \end{bmatrix} \begin{bmatrix} u_2 \\ u_1 \end{bmatrix}$$

$$= \frac{E}{L} [1 \quad -1] \begin{bmatrix} u_2 \\ u_1 \end{bmatrix} = \frac{10^4 \text{ lb/in}^2}{2 \text{ in}} [0.04 \text{ in} - 0 \text{ in}] = 200 \text{ psi}$$

(Tension)

Element 2

$$\sigma_2 = E \epsilon = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

$$= \frac{E}{L} [u_3 - u_2] = \frac{10^4 \text{ lb/in}^2}{2 \text{ in}} [0.06 \text{ in} - 0.04 \text{ in}] = 100 \text{ psi}$$

(Tension)

Element 3

$$\sigma_3 = E \epsilon = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

$$= \frac{E}{L} (u_4 - u_3) = \frac{10^4 \text{ lb/in}^2}{2 \text{ in}} (0 - 0.06 \text{ in}) = -300 \text{ psi (compression)}$$

AVERAGE

The 1st Test(1)
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
September 9, 2021

1. Matrix A and matrix B are defined as below.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ -1 & -2 \\ 0 & 5 \end{bmatrix}$$

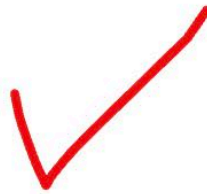
(5%)

(a) Can we perform the matrix multiplication for $A \cdot B$? If we can, what's the result?

Proof: $A(2 \times 3) \cdot B(3 \times 2) : \text{yes we can!}$

By use of calculator:

$$= \begin{bmatrix} 1 & 4 \\ -1 & 19 \end{bmatrix}$$



(5%)

(b) Can we perform the matrix multiplication for $B \cdot A$? If we can, what's the result?

Proof: $B(3 \times 2) \cdot A(2 \times 3) : \text{yes we can!}$

By use of calculator:

$$= \begin{bmatrix} 2 & 2 & 5 \\ -1 & -2 & -9 \\ -5 & 0 & 20 \end{bmatrix}$$



(10%)

2. Convert the following 4 algebraic equations to one matrix equation, and write **MATLAB Command Statements** to solve the variables of X_1 , X_2 , X_3 , and X_4 .

-4 (Solutions for X_1 , X_2 , X_3 , and X_4 are not required)

$$X_1 + 2X_2 + 3X_3 + 4X_4 = 70$$

$$6X_1 - 7X_2 - 8X_3 = -30$$

$$12X_3 + 11X_4 = 150$$

$$16X_1 + 14X_3 + 13X_4 = 300$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -7 & -8 & 0 \\ 0 & 0 & 12 & 11 \\ 16 & 0 & 14 & 13 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 70 \\ -30 \\ 150 \\ 300 \end{bmatrix}$$

MATLAB Statements:

$$>> A = [1 \ 2 \ 3 \ 4; 6 \ -7 \ -8 \ 0; 0 \ 0 \ 12 \ 11; 16 \ 0 \ 14 \ 13]$$

$$>> \text{~~B = [X_1; X_2; X_3; X_4];~~}$$

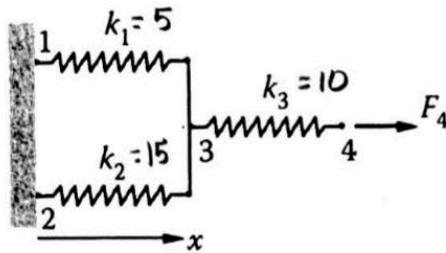
$$>> C = [70; -30; 150; 300]$$

$$>> B = C * A \text{ INS } \rightarrow$$

>>

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 70 \\ -30 \\ 150 \\ 300 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -7 & -8 & 0 \\ 0 & 0 & 12 & 11 \\ 16 & 0 & 14 & 13 \end{bmatrix}^{-1}$$

3. A spring system shown below is restricted in moving in one dimension (horizontal only) as shown in the x coordinate. Given: $k_1 = 5 \text{ N/mm}$, $k_2 = 15 \text{ N/mm}$, $k_3 = 10 \text{ N/mm}$, one external force $F_4 = 40 \text{ N}$ is applied to node 4. Nodes 1 and 2 are attached on a fixed structure.



(2%)

How many nodes in the system? 4

How many elements in the system? 3

(2%)

What are the boundary conditions (write in terms of equation forms)?

$$u_1 = u_2 = 0, F_3 = 0$$

(2%)

What are the external forces?

-1

$$F_4 = 40 \text{ N}$$

Find:

(10%)

(a) Global stiffness matrix of the spring system

$$\begin{aligned}
 k_1 &= \begin{bmatrix} u_1 & u_3 \\ 5 & -5 \\ -5 & 5 \end{bmatrix} \\
 k_2 &= \begin{bmatrix} u_2 & u_3 \\ 15 & -15 \\ -15 & 15 \end{bmatrix} \\
 k_3 &= \begin{bmatrix} u_3 & u_4 \\ 10 & -10 \\ -10 & 10 \end{bmatrix}
 \end{aligned}$$

$$K = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ 5 & 0 & -5 & 0 \\ 0 & 15 & -15 & 0 \\ -5 & -15 & 5+15+10 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

✓

(8%)

(b) Displacements of Nodes 3 and 4

$$\begin{bmatrix} 30 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 40 \end{Bmatrix}$$

$$30(u_3) - 10(u_4) = 0$$

$$30(u_3) - 10(u_4) = 0$$

$$-10(u_3) + 10(u_4) = 40$$

$$30(u_3) - 10(u_3) = 40$$

$$20(u_3) = 40$$

$$\frac{20}{20}$$

$$\boxed{u_3 = 2}$$

$$\frac{60}{10} = u_4$$

$$\boxed{u_4 = 6}$$

(8%)

(c) Reaction forces at nodes 1 and 2

$$F_1 = -5(u_3) = -5(2) = \boxed{-10 \text{ N}}$$

Based off Global stiffness matrix

$$F_2 = -15(u_3 - u_2^0) = -15(2) = \boxed{-30 \text{ N}}$$

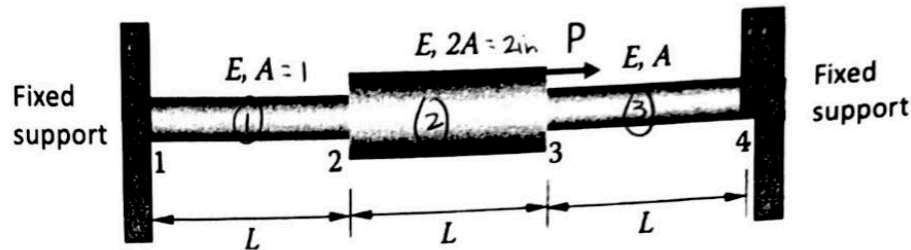
(8%)

(d) Spring force 1 and Spring force 3; are they tension or compression?

$$f_1 = k_1 \Delta u = 5(u_3 - u_1^0) = 5(u_3) = 5(2) = \boxed{10 \text{ N}} \rightarrow \text{tension}$$

$$f_3 = k_3 \Delta u = 10(u_4 - u_3) = 10(6 - 2) = 10(4) = \boxed{40 \text{ N}} \rightarrow \text{tension}$$

4. A bar assembly is loaded with force $P = 500$ lb at node 3 as shown in the figure. Assume the bar can only move in one dimension in horizontal direction. Node 1 and node 4 are fixed on the supporting structure. The bar assembly is made of a homogeneous material with Young's modulus $E = 10^4$ psi, the length of each element $L = 2$ in, and the cross-sectional area $A = 1$ in², cross-sectional area $2A = 2$ in².



(3%)

(a) What are the variables attached to the 4 nodes?

$$u_1, u_2, u_3, u_4$$

(3%)

(b) What are the boundary conditions?

$$u_1 = u_4 = 0, \quad F_2 = 0$$

(4%)

(c) What are the external loads?

-1

$$F_3 = P = 500 \text{ lb}$$

(10%)

(d) Use finite element method to set the global stiffness matrix for the bar assembly.

$$\text{FE: } K_1 = \frac{EA}{L} \begin{bmatrix} u_1 & u_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_2 = \frac{EA}{L} \begin{bmatrix} u_2 & u_3 \\ 2 & -2 \\ -2 & 2 \end{bmatrix} \quad K = \frac{EA}{L}$$

$$K_3 = \frac{EA}{L} \begin{bmatrix} u_3 & u_4 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \\ 500 \\ F_4 \end{Bmatrix}$$

-5

(10%)

(e) Solve for displacements at node 2 and node 3.

$$\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 500 \end{Bmatrix}$$

Applying u_2 :

$$3(3u_2) - 2u_3 = 0$$

$$2(-2u_2) + 3u_3 = 500$$

$$9u_2 - 4u_3 = 1000$$

$$\frac{5u_2 = 1000}{5}$$

$$\boxed{u_2 = 200}$$

$$3(200) - 2u_3 = 0$$

$$\frac{2u_3 = 600}{2}$$

$$\boxed{u_3 = 300}$$

X

(10%)

(f) Obtain stress in each of the elements.

$$\sigma_1 = E\epsilon = EB_1u_1 = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= E \frac{u_2 - u_1}{L} = \frac{E}{L} [200 - 0]$$

$$\sigma_1 = \frac{200(10^4) \text{ psi}}{1 \text{ in}^2} = \boxed{20(10^5) \text{ psi}}$$

$$\sigma_2 = EB_2u_2 = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= E \frac{u_3 - u_2}{L} = \frac{E}{L} [300 - 200]$$

$$\sigma_2 = \frac{100(10^4) \text{ psi}}{2 \text{ in}^2} = \boxed{5(10^5) \text{ psi}}$$

$$\sigma_3 = EB_3u_3 = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$\sigma_3 = E \frac{u_4 - u_3}{L} = \frac{E}{L} [0 - 300]$$

$$\sigma_3 = \frac{-300(10^4)}{1 \text{ in}^2} = \boxed{-30(10^5) \text{ psi}}$$

BELOW AVERAGE

The 1st Test(1)
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
September 9, 2021

1. Matrix A and matrix B are defined as below.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

(5%)

(a) Can we perform the matrix multiplication for $A \cdot B$? If we can, what's the result?

$$\begin{bmatrix} (3)(1) + (2)(-1) + (1)(0) & (3)(1) + (2)(-2) + (1)(5) \\ (-1)(1) + (0)(-1) + (4)(0) & (-1)(1) + (0)(-2) + (4)(5) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ -1 & 19 \end{bmatrix}$$



(5%)

(b) Can we perform the matrix multiplication for $B \cdot A$? If we can, what's the result?

$$\begin{bmatrix} (1)(3) + (-1)(-1) + (0)(0) & (1)(2) + (-1)(0) + (0)(4) & (1)(1) + (-1)(5) + (0)(4) \\ (-1)(3) + (2)(-1) & (-1)(2) + (0)(0) & (-1)(1) + (2)(5) \\ 0 + -5 & 0 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & -4 \\ -1 & -2 & 9 \\ -5 & 0 & 20 \end{bmatrix}$$



(10%)

2. Convert the following 4 algebraic equations to one matrix equation, and write **MATLAB Command Statements** to solve the variables of X_1, X_2, X_3 , and X_4 .
(Solutions for X_1, X_2, X_3 , and X_4 are not required)

$$X_1 + 2X_2 + 3X_3 + 4X_4 = 70$$

$$6X_1 - 7X_2 - 8X_3 = -30$$

$$12X_3 + 11X_4 = 150$$

$$16X_1 + 14X_3 + 13X_4 = 300$$

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & -7 & -8 & 0 \\ 0 & 0 & 12 & 11 \\ 16 & 0 & 14 & -13 \end{bmatrix} & \begin{bmatrix} 70 \\ -30 \\ 150 \\ 300 \end{bmatrix} \end{matrix}$$

MATLAB Statements:

```
>> A = [1 2 3 4; 6 7 8 0; 0 0 12 11; 16 0 14 13]
```

```
>> B = [70; -30; 150; 300]
```

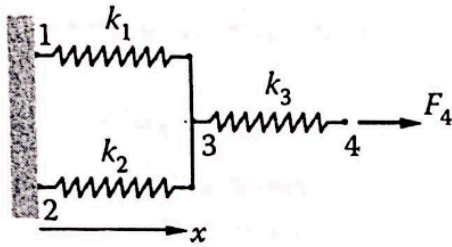
```
>> inv(A)*(B)
```

```
>> ans =
```

```
>>
```



3. A spring system shown below is restricted in moving in one dimension (horizontal only) as shown in the x coordinate. Given: $k_1 = 5 \text{ N/mm}$, $k_2 = 15 \text{ N/mm}$, $k_3 = 10 \text{ N/mm}$, one external force $F_4 = 40 \text{ N}$ is applied to node 4. Nodes 1 and 2 are attached on a fixed structure.



(2%)

How many nodes in the system? 4

How many elements in the system? 3

(2%)

What are the boundary conditions (write in terms of equation forms)?

$$u_1 = u_2 = 0 \quad u_3, u_4$$

(2%)

What are the external forces?

-1

$$F_4 = F_7 = 40 \text{ N}$$

Find:

(10%)

(a) Global stiffness matrix of the spring system

-5

$$k_1 = \begin{bmatrix} u_1 & u_2 \\ 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad k_2 = \begin{bmatrix} u_2 & u_3 \\ 15 & -15 \\ -15 & 15 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \quad k_3 = \begin{bmatrix} u_3 & u_4 \\ 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$\begin{bmatrix} 5 & -5 & 0 & 0 \\ -5 & 5+15 & -15 & 0 \\ 0 & -15 & 15+10 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 0 & 0 \\ -5 & 20 & -15 & 0 \\ 0 & -15 & 25 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ 0 \\ 40 \end{Bmatrix}$$

3

(8%)

(b) Displacements of Nodes 3 and 4

-6

$$\begin{aligned} 15u_3 - 10u_4 &= 0 \\ -25u_3 + 25u_4 &= 50 \end{aligned}$$



$$10u_4 = 50$$

$$u_4 = 5 \text{ mm}$$

$$u_3 = 2 \text{ mm}$$

(8%)

(c) Reaction forces at nodes 1 and 2

-8

$$r_1 = -10 \text{ N}$$

How?

$$r_2 = -25 \text{ N}$$

(8%)

(d) Spring force 1 and Spring force 3; are they tension or compression?

-4

$$\text{Spring 1} = 5$$

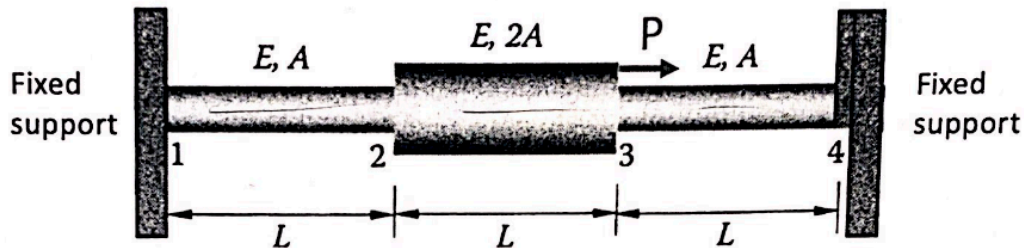
$$\text{Spring 3} = 10$$

$$\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$= \begin{bmatrix} -5 & 5 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = 0 \text{ N} \Rightarrow \text{Spring 1} \\ \text{Compression}$$

$$\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} \Rightarrow = \begin{bmatrix} -10 & 10 \end{bmatrix} \begin{Bmatrix} 2 \\ 5 \end{Bmatrix} = 30 \text{ N} \\ \text{Tension}$$

4. A bar assembly is loaded with force $P = 500$ lb at node 3 as shown in the figure. Assume the bar can only move in one dimension in horizontal direction. Node 1 and node 4 are fixed on the supporting structure. The bar assembly is made of a homogeneous material with Young's modulus $E = 10^4$ psi, the length of each element $L = 2$ in, and the cross-sectional area $A = 1$ in², cross-sectional area $2A = 2$ in².



(3%)

(a) What are the variables attached to the 4 nodes?

$$u_1, u_2, u_3, u_4$$

(3%)

(b) What are the boundary conditions?

$$u_1 = u_4 = 0$$

(4%)

(c) What are the external loads?

-1

$$F_3 = P$$

(10%)

(d) Use finite element method to set the global stiffness matrix for the bar assembly.

$$k_1 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \end{matrix}$$

$$k_2 = \frac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_2 \\ u_3 \end{matrix}$$

$$k_3 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_3 \\ u_4 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$u_1 = 0$

$$\frac{EA}{L} \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P \\ 0 \end{bmatrix}$$

$$= \frac{PL}{EA} \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$$

(10%)

-3

(e) Solve for displacements at node 2 and node 3.

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ 5 \\ R_3 \end{Bmatrix}$$

$$\frac{1 \times 10^4}{2} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 500 \end{Bmatrix}$$

$$5000(-2u_2 + u_3) = 500$$

$$15k u_2 - 10k u_3 = 0$$

$$-10k u_2 + 15k u_3 = 500$$

$$u_2 = \frac{10k}{15k} u_3 = 500$$

$$\frac{2u_3}{3} = u_2 = 0.04$$

$$-20k \frac{u_3}{3} + 15k u_3 = 500$$

(10%)

(f) Obtain stress in each of the elements.

-5

$$\sigma_1 = E \epsilon = E \left[-\frac{1}{L} \quad \frac{1}{L} \right] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{E}{L} (u_2 - u_1) = \frac{E}{L} \left(\frac{PL}{EA} - 0 \right) = \frac{P}{A}$$

$$\sigma_2 = E \frac{(u_3 - u_2)}{L}$$

$$= E \frac{\frac{3PL}{2AE} - \frac{PL}{AE}}{L}$$

$$= \frac{P}{2A}$$

$$\sigma_3 = E \frac{u_1 - u_3}{L}$$

$$= E \frac{\frac{5PL}{EA} - \frac{3PL}{AE}}{L}$$

$$= \frac{P}{A}$$

**FINITE ELEMENT ANALYSIS AND DESIGN
FALL 2021**

Title of Assignment: TEST 2

TOPICS COVERED IN TEST 2

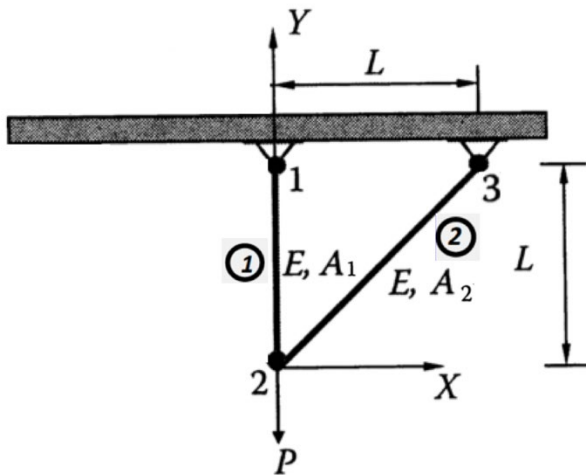
1. Derive equations and solve plane truss problem using finite element method.
2. Derive equations and solve beam structure problem using finite element method.

Name (print) _____ Signature: _____
(last name, first name)

The 2nd Test

For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
October 7, 2021

4. The plane truss is loaded with a downward force $P = 100$ lb as shown below. Truss element 1 is between nodes 2 and 1, and has the cross-sectional area $A_1 = 1$ in². Truss element 2 is between nodes 2 and 3, and has a cross-sectional area $A_2 = \sqrt{2}$ in². Both truss elements have the same Young's modulus $E = 20,000$ lb/in². Length $L = 20$ inches. Use finite element method to solve this problem.



(3%)

- (a) To solve this problem, what are the variables you would define at the three nodes?

Node 1:

Node 2:

Node 3:

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

(3%)

- (c) What are the external forces (write in terms of equations)?

(30%)

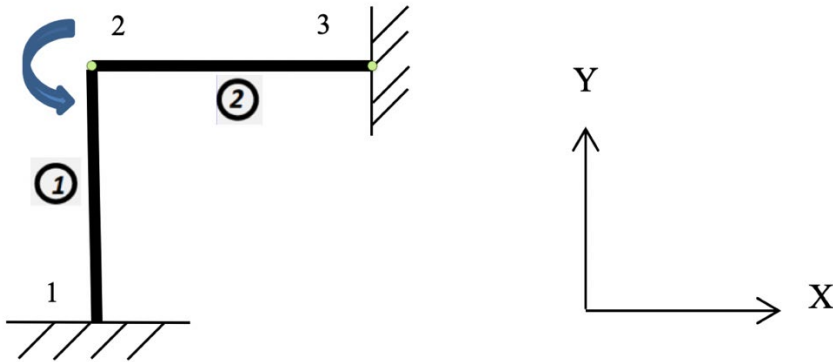
- (d) Use finite element method to find the displacements at node 2 assuming there is no buckling effect for the truss system.

(10%)

(e) Find stresses in the two elements.

5. An L-shape beam shown below has a length $L = 10$ in, modulus of elasticity $E = 10^5$ psi, the area moment of inertia of the beam's cross-section $I = 1 \text{ in}^4$, and the cross-sectional area $A = 1 \text{ in}^2$.

The beam is fixed on its two ends at node 1 and node 3. A positive moment of 100 in-lbs is applied in the z-direction to the beam at node 2. Use finite element method to find linear displacements and angular rotation at point 2.



(3%)

- (a) There will be three variables at each node. What are the variables would you define at the 3 nodes?

Node 1:

Node 2:

Node 3:

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

(3%)

- (c) What are the external loads?

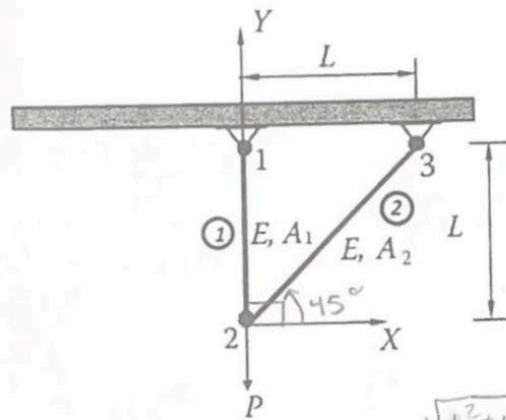
(40%)

(d) Use finite element method to solve for the variables that you have defined at node 2.

ABOVE AVERAGE

The 2nd Test
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
October 7, 2021

1. The plane truss is loaded with a downward force $P = 100$ lb as shown below. Truss element 1 is between nodes 2 and 1, and has the cross-sectional area $A_1 = 1$ in². Truss element 2 is between nodes 2 and 3, and has a cross-sectional area $A_2 = \sqrt{2}$ in². Both truss elements have the same Young's modulus $E = 20,000$ lb/in². Length $L = 20$ inches. Use finite element method to solve this problem.



$$\begin{aligned} P &= 100 \text{ lb} \\ A_1 &= 1 \text{ in}^2 \\ A_2 &= \sqrt{2} \text{ in}^2 \\ E &= 20,000 \text{ lb/in}^2 \\ L &= 20 \text{ in} \\ L_1 &= 20 \text{ in} \end{aligned}$$

$$\sqrt{L^2 + L^2} = L_2 = 28.28 \text{ in}$$

(3%)

- (a) To solve this problem, what are the variables you would define at the three nodes?

Neglect horizontal (v)

Node 1: U_1, V_1

Node 2: u_2, v_2

Node 3: U_3, V_3

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

$$u_1 = v_1 = 0$$

$$U_3 = V_3 = 0$$

(3%)

- (c) What are the external forces (write in terms of equations)?

$$F_{2y} = P = 100 \text{ lb}$$

$$F_{2x} = 0$$

-1

(30%)

(d) Use finite element method to find the displacements at node 2 assuming there is no buckling effect for the truss system.

Element 1 $\theta = 90^\circ$

$L = 0$ $m = 1$

$$K = \frac{EA_1}{L_1} \begin{bmatrix} L^2 & Lm & -L^2 & -Lm \\ Lm & m^2 & -Lm & -m^2 \\ -L^2 & -Lm & L^2 & Lm \\ -Lm & -m^2 & Lm & m^2 \end{bmatrix}$$

$u_i \quad v_i \quad u_j \quad v_j$

$$K = \frac{EA_1}{L_1} \begin{bmatrix} u_2 & v_2 & u_1 & v_1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$K = \frac{20,000 \frac{lb}{in^2} (12 in)^2}{20 in} = 1000 \therefore K_1 =$$

$$K_1 = \begin{bmatrix} u_2 & v_2 & u_1 & v_1 \\ 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & -1000 & 0 & 1000 \end{bmatrix}$$

$u_2 \quad v_2 \quad u_1 \quad v_1$

Element 2 $\theta = 45^\circ$

$L = \frac{\sqrt{2}}{2}$ $m = \frac{\sqrt{2}}{2}$

$$K = \frac{EA_2}{L_2} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$K = 20,000 \frac{lb}{in^2} \sqrt{2} in^2$$

$$= 1000.151 \frac{lb}{in} \therefore K_2 =$$

$$K_2 = \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 500.075 & 500.075 & -500.075 & -500.075 \\ 500.075 & 500.075 & -500.075 & -500.075 \\ -500.075 & -500.075 & 500.075 & 500.075 \\ -500.075 & -500.075 & 500.075 & 500.075 \end{bmatrix}$$

$u_2 \quad v_2 \quad u_3 \quad v_3$

$$K = \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 & 0 & 0 \\ 0 & 0 & 500.075 & 500.075 & -500.075 & -500.075 \\ 0 & -1000 & 500.075 & 500.075 & -500.075 & -500.075 \\ 0 & 0 & -500.075 & -500.075 & 500.075 & 500.075 \\ 0 & 0 & -500.075 & -500.075 & 500.075 & 500.075 \end{bmatrix}$$

$u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3$

$$= \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix}$$

$$u_1 v_1 = 0$$

$$u_3 v_3 = 0$$

$$F_2 x = 0$$

$$F_2 y = 100 \text{ lb}$$

$$\begin{bmatrix} 500.075 & 500.075 \\ 500.075 & 1500.075 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \text{ lb} \end{bmatrix}$$

$$500.075 u_2 + 500.075 v_2 = 0$$

$$500.075 u_2 = -\frac{500.075 v_2}{500.075}$$

$$u_2 = -v_2$$

$$\sigma = \frac{E}{L} [-l -m] \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix}$$

$$500.075 u_2 + 1500.075 v_2 = 100 \text{ lb}$$

$$-500.075 (-v_2) + 1500.075 v_2 = 100 \text{ lb}$$

$$1000 v_2 = 100 \text{ lb}$$

(10%)

(e) Find stresses in the two elements.

$$v_2 = 0.1 \text{ in} \therefore u_2 = -0.1 \text{ in}$$

$$\sigma_1 = \frac{E}{L_1} \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_1 \\ v_1 \end{bmatrix}$$

$$\frac{E}{L_1} (-v_2) = \frac{20,000 \text{ lb}}{20 \text{ in}} (-0.1) = +100 \text{ lb/in}^2$$

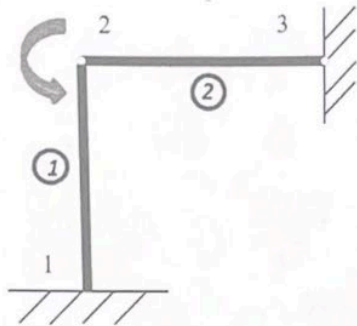
$$\sigma_2 = \frac{E}{L_2} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

$$\frac{E}{L_2} \left[-\frac{\sqrt{2}}{2} u_2 - \frac{\sqrt{2}}{2} v_2 \right] = \frac{20,000 \text{ lb}}{28.28 \text{ in}} \left[-\frac{\sqrt{2}}{2} (-0.1) - \frac{\sqrt{2}}{2} (0.1) \right]$$

$$\sigma_2 = 0$$

2. An L-shape beam shown below has a length $L = 10$ in, modulus of elasticity $E = 10^5$ psi, the area moment of inertia of the beam's cross-section $I = 1$ in⁴, and the cross-sectional area $A = 1$ in².

The beam is fixed on its two ends at node 1 and node 3. A positive moment of 100 in-lbs is applied in the z-direction to the beam at node 2. Use finite element method to find linear displacements and angular rotation at point 2.



$$\begin{aligned} L &= 10 \text{ in} \\ E &= 10^5 \frac{\text{lb}}{\text{in}^2} \\ I &= 1 \text{ in}^4 \\ A &= 1 \text{ in}^2 \\ M_2 &= 100 \text{ in-lb} \end{aligned}$$

(3%)

- (a) There will be three variables at each node. What are the variables would you define at the 3 nodes?

Node 1: u_1, v_1, θ_1

Node 2: u_2, v_2, θ_2

Node 3: u_3, v_3, θ_3

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

$$u_1 = v_1 = \theta_1 = 0$$

$$u_3 = v_3 = \theta_3 = 0$$

(3%)

- (c) What are the external loads?

-2

$$M_2 = 100 \text{ in-lb}$$

$$\cancel{F_{2x} = 10 \text{ lb}}$$

$$M_2 = F_{2x} \cdot L \therefore F_{2x} = \frac{M_2}{L} = 10 \text{ lb}$$

-5

$$E = 10^5 \text{ psi} \quad I = 1 \text{ in}^4 \quad A = 1 \text{ in}^2 \quad L = 10 \text{ in}$$

(40%)

(d) Use finite element method to solve for the variables that you have defined at node 2.

$$K = T^T K^e T$$

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$K_2 = K_1$$

$$K = \begin{bmatrix} 10000 & 0 & 0 & -10000 & 0 & 0 \\ 0 & 1200 & 6000 & 0 & -1200 & 6000 \\ 0 & 6000 & 40000 & 0 & -6000 & 20000 \\ -10000 & 0 & 0 & 10000 & 0 & 0 \\ 0 & -1200 & -6000 & 0 & 1200 & -6000 \\ 0 & 6000 & 20000 & 0 & -6000 & 40000 \end{bmatrix}$$

Element 1 $\theta = 90^\circ$: $L = 0$ $m = 1$

$$T = \begin{bmatrix} L & m & 0 & 0 & 0 & 0 \\ -m & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & m & 0 \\ 0 & 0 & 0 & -m & L & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 10000 & 0 & 0 & -10000 & 0 & 0 \\ 0 & 1200 & 6000 & 0 & -1200 & 6000 \\ 0 & 6000 & 40000 & 0 & -6000 & 20000 \\ -10000 & 0 & 0 & 10000 & 0 & 0 \\ 0 & -1200 & -6000 & 0 & 1200 & -6000 \\ 0 & 6000 & 20000 & 0 & -6000 & 40000 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} \text{Calculator} & 1200 & 0 & 6000 & -1200 & 0 & 6000 \\ 0 & 10000 & 0 & 0 & -10000 & 0 & 0 \\ 6000 & 0 & 40000 & -6000 & 0 & 20000 & 0 \\ -1200 & 0 & -6000 & 1200 & 0 & +6000 & 0 \\ 0 & -10000 & 0 & 0 & 10000 & 0 & 0 \\ 6000 & 0 & 20000 & +6000 & 0 & 40000 & 0 \end{bmatrix} \begin{matrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{matrix}$$

$$K_2 = \begin{bmatrix} 10,000 & 0 & 0 & -10,000 & 0 & 0 \\ 0 & 1200 & 6000 & 0 & -1200 & 6000 \\ 0 & 6000 & 40,000 & 0 & -6000 & 20,000 \\ -10,000 & 0 & 0 & 10,000 & 0 & 0 \\ 0 & -1200 & -6000 & 0 & 1200 & -6000 \\ 0 & 600 & 2000 & 0 & -600 & 40,000 \end{bmatrix} \begin{matrix} u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \end{matrix}$$

$$K = \begin{matrix} & \begin{matrix} u_1 & v_1 & \theta_1 \end{matrix} & \begin{matrix} u_2 & v_2 & \theta_2 \end{matrix} & \begin{matrix} u_3 & v_3 & \theta_3 \end{matrix} \\ \begin{matrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{matrix} & \begin{bmatrix} 1200 & 0 & 6000 \\ 0 & 10000 & 0 \\ 6000 & 0 & 40000 \\ -1200 & 0 & -6000 \\ 0 & -1000 & 0 \\ 6000 & 0 & 20000 \end{bmatrix} & \begin{bmatrix} -1200 & 0 & 6000 \\ 0 & -10000 & 0 \\ -6000 & 0 & 20000 \\ 1200 & 0 & -6000 \\ 0 & 10000 & 0 \\ 4000 & 0 & 40000 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -10000 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

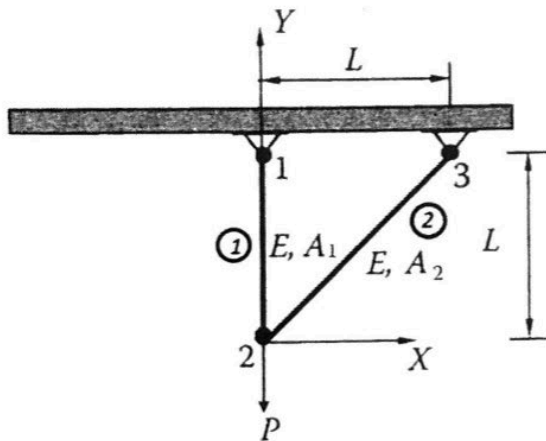
$u_1 = F_x$
 $v_1 = F_y$
 $\theta_1 = M_1$

$$\begin{bmatrix} 1200 & 0 & -600 \\ 0 & 10,000 & 0 \\ 4000 & 0 & 40,000 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 100 \end{bmatrix}$$

AVERAGE

The 2nd Test
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
October 7, 2021

1. The plane truss is loaded with a downward force $P = 100$ lb as shown below. Truss element 1 is between nodes 2 and 1, and has the cross-sectional area $A_1 = 1$ in². Truss element 2 is between nodes 2 and 3, and has a cross-sectional area $A_2 = \sqrt{2}$ in². Both truss elements have the same Young's modulus $E = 20,000$ lb/in². Length $L = 20$ inches. Use finite element method to solve this problem.



(3%)

- (a) To solve this problem, what are the variables you would define at the three nodes?

Node 1: u_1, v_1

Node 2: u_2, v_2

Node 3: u_3, v_3

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

$$u_1 = v_1 = u_3 = v_3 = 0$$

(3%)

- (c) What are the external forces (write in terms of equations)?

$$F_{2y} = -100 \text{ lb}, \quad F_{2x} = 0$$

(30%)

- (d) Use finite element method to find the displacements at node 2 assuming there is no buckling effect for the truss system.

$$K = \frac{EA}{L} \begin{bmatrix} u_i & v_i & u_j & v_j \\ 1^2 & 1m & -1 & -1m \\ 1m & m^2 & -1m & -m^2 \\ -1^2 & -1m & 1^2 & 1m \\ -1m & -m^2 & 1m & m^2 \end{bmatrix}$$

$$K_{1\ 2-1}: \alpha = 90^\circ, \quad l = 0, \quad m = 1$$

$$K_1 = \frac{EA}{L} \begin{bmatrix} u_2 & v_2 & u_1 & v_1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad \text{where, } \frac{EA}{L} = \frac{(20,000)(1)}{20} = \underline{1000}$$

$$K_1 = \begin{bmatrix} u_2 & v_2 & u_1 & v_1 \\ 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & -1000 & 0 & 1000 \end{bmatrix}$$

$$K_{2\ 2-3}: \alpha = 45^\circ, \quad l = \frac{\sqrt{2}}{2}, \quad m = \frac{\sqrt{2}}{2}$$

$$K_2 = \frac{EA}{\sqrt{2}L} \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix} \quad \Rightarrow$$

$$\text{where, } \frac{EA}{\sqrt{2}L} = \frac{(20,000)(\sqrt{2})}{(\sqrt{2})(20)} = \underline{1000}$$

2

$$K_2 = \begin{bmatrix} \begin{matrix} u_2 & v_2 \\ 500 & 500 \\ 500 & 500 \end{matrix} & \begin{matrix} u_3 & v_3 \\ -500 & -500 \\ -500 & -500 \end{matrix} \\ \begin{matrix} -500 & -500 \\ -500 & -500 \end{matrix} & \begin{matrix} 500 & 500 \\ 500 & 500 \end{matrix} \end{bmatrix}$$

Since the boundary conditions: $u_1 = v_1 = u_3 = v_3 = 0$, we reduce the global equation to

$$\begin{bmatrix} 500 & 500 \\ 500 & 1500 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

$$- 500 u_2 + 500 v_2 = 0 \quad \text{--- (1)}$$

$$500 u_2 + 1500 v_2 = -100 \quad \text{--- (2)}$$

$$1000 v_2 = -100$$

$$v_2 = -0.1 \quad \text{plug in to eq. (1)}$$

$$500 u_2 + 500(-0.1) = 0$$

$$500 u_2 = 50$$

$$u_2 = 0.1$$

(10%)

(e) Find stresses in the two elements.

Stress: Element 1, $\theta = 90^\circ$, $l = 0$, $m = 1$

$$\sigma_1 = \frac{E}{L} \begin{bmatrix} -1 & -m & 1 & m \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix}$$

$$\sigma_1 = \frac{20,000}{20} \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_1 = 1000 * (-1) * (-0.1) = 100 \text{ psi}$$

Stress: Element 2, $\theta = 45^\circ$, $l = \frac{\sqrt{2}}{2}$, $m = \frac{\sqrt{2}}{2}$

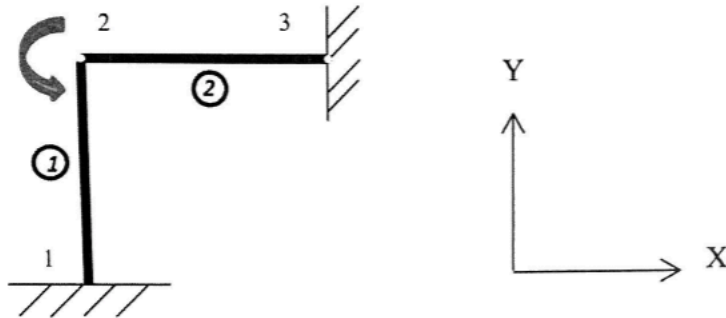
$$\sigma_2 = \frac{E}{L} \begin{bmatrix} -1 & -m & 1 & m \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix}$$

$$\sigma_2 = \frac{20,000}{20} \begin{bmatrix} -0.7071 & -0.7071 & 0.7071 & 0.7071 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_2 = 0$$

2. An L-shape beam shown below has a length $L = 10$ in, modulus of elasticity $E = 10^5$ psi, the area moment of inertia of the beam's cross-section $I = 1$ in⁴, and the cross-sectional area $A = 1$ in².

M_2 The beam is fixed on its two ends at node 1 and node 3. A positive moment of 100 in-lbs is applied in the z-direction to the beam at node 2. Use finite element method to find linear displacements and angular rotation at point 2.



(3%)

- (a) There will be three variables at each node. What are the variables would you define at the 3 nodes?

-1

Node 1: V_1, θ_1 —

Node 2: V_2, θ_2 —

Node 3: V_3, θ_3 —

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

-1

$$V_1 = \theta_1 = V_3 = \theta_3 = 0$$

(3%)

- (c) What are the external loads?



$M_2 = 100$ in-lb, Assume $F_{2y} = 0$



(40%)

(d) Use finite element method to solve for the variables that you have defined at node 2.

-20 Stiffness matrix for element 1, $L = 10 \text{ in}$, $\frac{EI}{L^3} = \frac{(100000)(1)}{10^3} = 100$

$$K_1 = \frac{EI}{L^3} \begin{bmatrix} V_1 & Q_1 & V_2 & Q_2 \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

should use 6x6 matrix

$$K_1 = 100 \begin{bmatrix} 12 & 60 & -12 & 60 \\ 60 & 400 & -60 & 200 \\ -12 & -60 & 12 & -60 \\ 60 & 200 & -60 & 400 \end{bmatrix}$$

$\Rightarrow K_1 =$

$$\begin{bmatrix} V_1 & Q_1 & V_2 & Q_2 \\ 1200 & 6000 & -1200 & 6000 \\ 6000 & 40000 & -6000 & 20000 \\ -1200 & -6000 & 1200 & -6000 \\ 6000 & 20000 & -6000 & 40000 \end{bmatrix}$$

Stiffness matrix for element 2, $L = 10 \text{ in}$, $\frac{EI}{L^3} = \frac{(100000)(1)}{10^3} = 100$

$$K_2 = \frac{EI}{L^3} \begin{bmatrix} V_2 & Q_2 & V_3 & Q_3 \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K_2 = 100 \begin{bmatrix} V_2 & Q_2 & V_3 & Q_3 \\ 12 & 60 & -12 & 60 \\ 60 & 400 & -60 & 200 \\ -12 & -60 & 12 & -60 \\ 60 & 200 & -60 & 400 \end{bmatrix}$$

$\Rightarrow K_2 =$

$$\begin{bmatrix} V_2 & Q_2 & V_3 & Q_3 \\ 1200 & 6000 & -1200 & 6000 \\ 6000 & 40000 & -6000 & 20000 \\ -1200 & -6000 & 1200 & -6000 \\ 6000 & 20000 & -6000 & 40000 \end{bmatrix}$$

Applying the boundary conditions: $V_1 = Q_1 = V_3 = Q_3$, we can reduce the global matrix to find V_2, Q_2

$\Rightarrow$

$$\begin{bmatrix} 2400 & 0 \\ 0 & 80000 \end{bmatrix} \begin{bmatrix} V_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \end{bmatrix}$$

Using Matlab:

$$A = \begin{bmatrix} 2400 & 0 & ; & 0 & 80000 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 & ; & 100 \end{bmatrix}$$

$$ans = A \backslash b$$

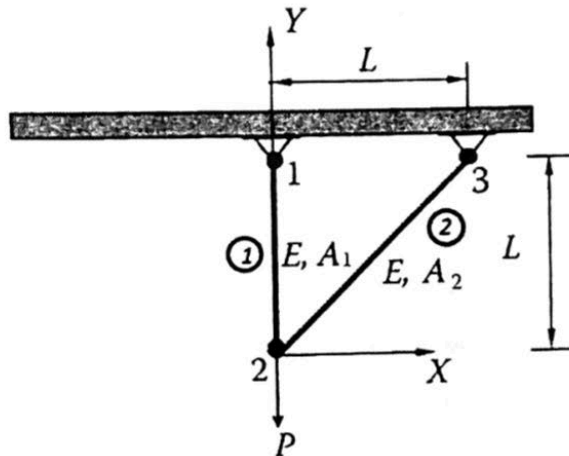
$$V_2 = 0$$

$$\alpha_2 = 0.001 \text{ radian}$$

BELOW AVERAGE

The 2nd Test
For MCEG 4309 - Finite Element Analysis & Design, Fall 2021
October 7, 2021

1. The plane truss is loaded with a downward force $P = 100$ lb as shown below. Truss element 1 is between nodes 2 and 1, and has the cross-sectional area $A_1 = 1$ in². Truss element 2 is between nodes 2 and 3, and has a cross-sectional area $A_2 = \sqrt{2}$ in². Both truss elements have the same Young's modulus $E = 20,000$ lb/in². Length $L = 20$ inches. Use finite element method to solve this problem.



- (3%) (a) To solve this problem, what are the variables you would define at the three nodes?

Node 1: u_1, v_1

Node 2: u_2, v_2

Node 3: u_3, v_3

- (4%) (b) What are the boundary conditions (write in terms of equations)?

$$v_1 = u_1 = 0 \quad u_3 = 0 = v_3$$

- (3%) (c) What are the external forces (write in terms of equations)?

$$F_{2y} = -P \quad F_{2x} = 0$$

-1

(30%)

-10

(d) Use finite element method to find the displacements at node 2 assuming there is no buckling effect for the truss system.

$$k = \frac{AE}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$k' = \frac{1 \times 20,000}{20} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & -1000 & 0 & 1000 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 & 0 & 0 \\ 0 & 0 & 1414.2 & 1414.2 & -1414.2 & -1414.2 \\ 0 & -1000 & 1414.2 & 2414.2 & -1414.2 & -1414.2 \\ 0 & 0 & -1414.2 & -1414.2 & 1414.2 & 1414.2 \\ 0 & 0 & -1414.2 & -1414.2 & 1414.2 & 1414.2 \end{bmatrix}$$

$$k^2 = \frac{\sqrt{2} \times 20,000}{20} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1414.2 & 1414.2 & -1414.2 & -1414.2 \\ 1414.2 & 1414.2 & -1414.2 & -1414.2 \\ -1414.2 & -1414.2 & 1414.2 & 1414.2 \\ -1414.2 & -1414.2 & 1414.2 & 1414.2 \end{bmatrix}$$

$$u_1 = v_1 = u_3 = v_3 = 0$$

$$Q = [u_2 \ v_2]^T$$

$$P = [0 \ 0 \ 0 \ -100 \ 0 \ 0]^T$$

$$[K][Q] = [P]$$

$$\begin{bmatrix} 1414.2 & 1414.2 \\ 1414.2 & 2414.2 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

$$1414.2 u_2 + 1414.2 v_2 = 0$$

$$1414 u_2 + 2414.2 v_2 = -100$$

$$v_2 = -u_2$$

$$1414.2(u_2) + 2414.2(-u_2) = -100$$

$$u_2 = 0.1 \text{ in}$$

$$v_2 = -0.1 \text{ in}$$

(10%)

(e) Find stresses in the two elements.

-3

$$\sigma = \frac{E}{L} \begin{bmatrix} -c & -s & c & s \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \frac{20,000}{20} \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ -0.1 \end{bmatrix}$$

$$\sigma^1 = 499.9 \text{ lb/in}^2$$

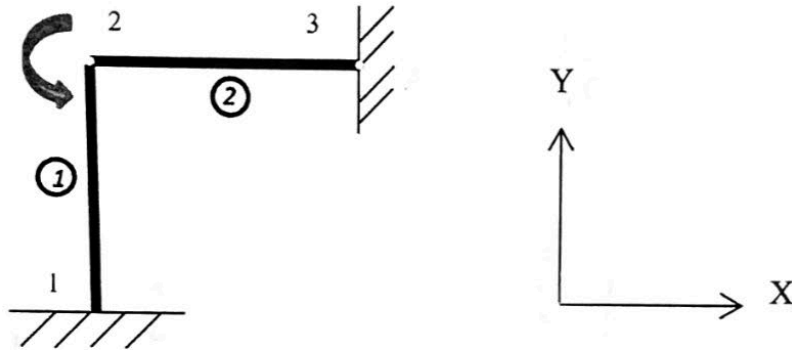
$$\sigma = \frac{20,000}{20} \begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma^2 = 1000 \begin{bmatrix} -0.1 + 0.1 \end{bmatrix}$$

$$\sigma^2 = 0 \text{ lb/in}^2$$

2. An L-shape beam shown below has a length $L = 10$ in, modulus of elasticity $E = 10^5$ psi, the area moment of inertia of the beam's cross-section $I = 1$ in⁴, and the cross-sectional area $A = 1$ in².

The beam is fixed on its two ends at node 1 and node 3. A positive moment of 100 in-lbs is applied in the z-direction to the beam at node 2. Use finite element method to find linear displacements and angular rotation at point 2.



(3%)

- (a) There will be three variables at each node. What are the variables would you define at the 3 nodes?

Node 1: v_1, u_1, θ_1

-1 Node 2:

Node 3: v_3, u_3, θ_3

(4%)

- (b) What are the boundary conditions (write in terms of equations)?

$$v_1 = 0, u_1 = 0, \theta_1 = 0$$

$$v_3 = 0, u_3 = 0, \theta_3 = 0$$

(3%)

- (c) What are the external loads?

-1

$$m_2 = 100 \text{ in-lbs}$$

$$F_{2x}, F_{2y} = ?$$

(40%)

(d) Use finite element method to solve for the variables that you have defined at node 2.

-25

$$[K] \begin{bmatrix} u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2 \\ \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix}$$

$$\frac{AE}{L} = \frac{1 \times 10^7}{10} = 1 \times 10^6$$

$$\frac{12EI}{L^3} = \frac{12 \times 10^6}{10^3} = 12000$$

$$\frac{AEI}{L} = \frac{4 \times 10^6}{10} = 4 \times 10^5$$

$$\frac{6EI}{L^2} = \frac{6 \times 10^6}{10^2} = 60,000$$

$$\frac{2EI}{L} = \frac{2 \times 10^6}{10} = 2 \times 10^5$$

$$[K]^{ie} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0.12 & 0.6 & 0 & -0.12 & 0.6 \\ 0 & 0.6 & 4 & 0 & -0.6 & 2 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.12 & -0.6 & 0 & 0.12 & -0.6 \\ 0 & 0.6 & 2 & 0 & -0.6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \theta \sin \theta & 0 & 0 \\ 0 & 0 & 0 & -\sin \theta \cos \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta = 270^\circ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K]^e = [T]^T [K]^{ie} [T]$$

$$[F]^e = \begin{bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \\ 5 \\ 10 \end{bmatrix}$$

$$[F]^e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[F]^e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -5 \\ 5 \\ 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

not complete.

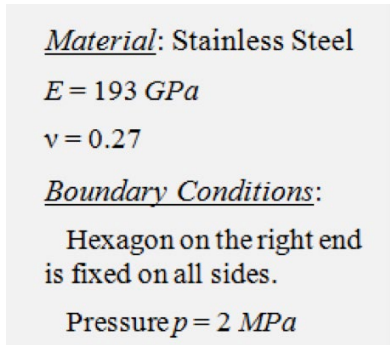
**FINITE ELEMENT ANALYSIS AND DESIGN
FALL 2021**

Title of Assignment: TEST 3

TOPICS COVERED IN TEST 3

1. Create a personalized geometry for a two-dimensional wrench with the number of characters in the student's first name and last name.
2. Use finite element program, ANSYS, to perform analysis to obtain total deformation and equivalent stress.

MCEG 4309 - Finite Element Analysis & Design
November 16, 2021



After you complete the finite element analysis, upload the following two pages to eCourse:

- (The screenshots should include the file trees, graphical results, and ANSYS version)

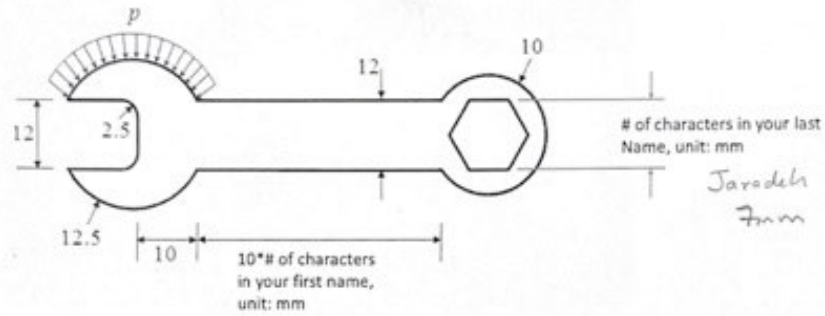
ABOVE AVERAGE

Bashir Jaradeh

Test 3

MCEG 4309 - Finite Element Analysis & Design

November 16, 2021



Material: Stainless Steel

$E = 193 \text{ GPa}$

$\nu = 0.27$

Boundary Conditions:

Hexagon on the right end is fixed on all sides.

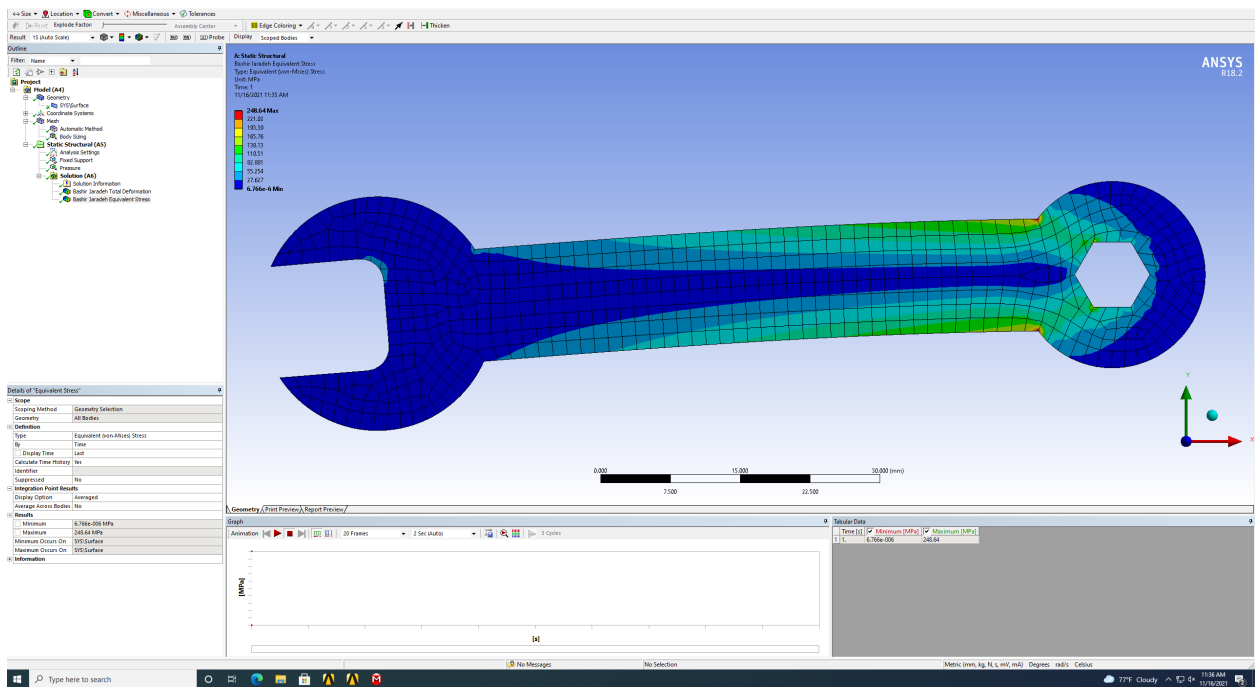
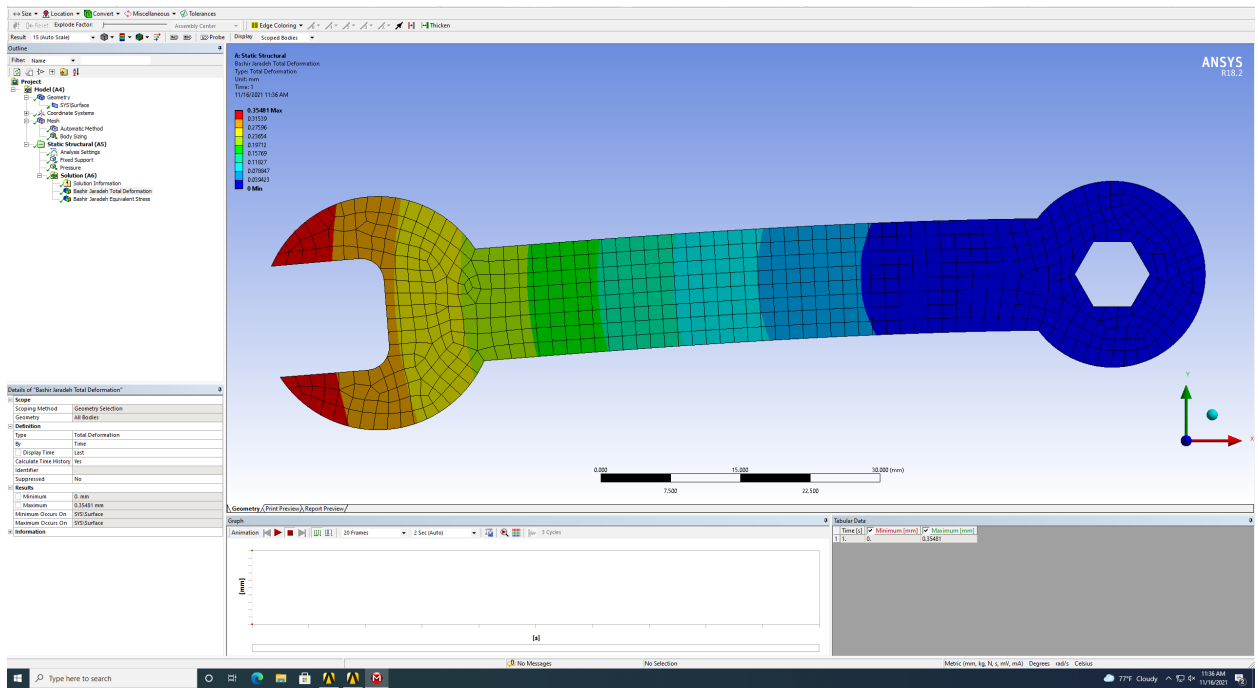
Pressure $p = 2 \text{ MPa}$

A wrench shown in the figure is made of stainless steel and has a uniform thickness of 3 mm. You are requested to generate a personalized geometry in terms of your first name and last name. Determine the total deformation and the equivalent stresses under the given distributed load and boundary conditions.

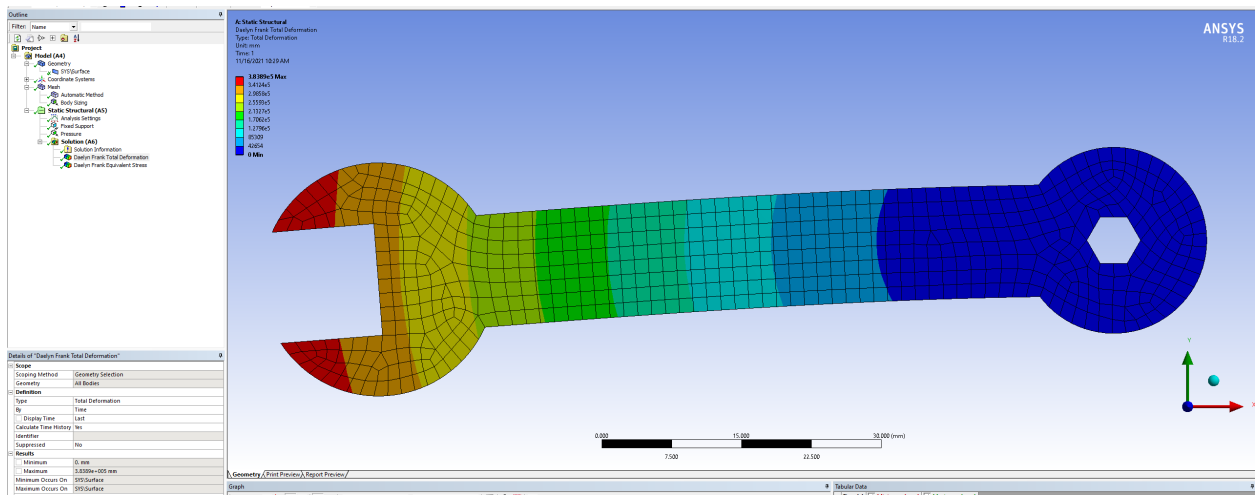
After you complete the finite element analysis, upload the following two pages to eCourse:

- (1) Total Deformation (use rename to add your name in front of Total Deformation)
- (2) Equivalent Stress (use rename to add your name in front of Equivalent Stress)

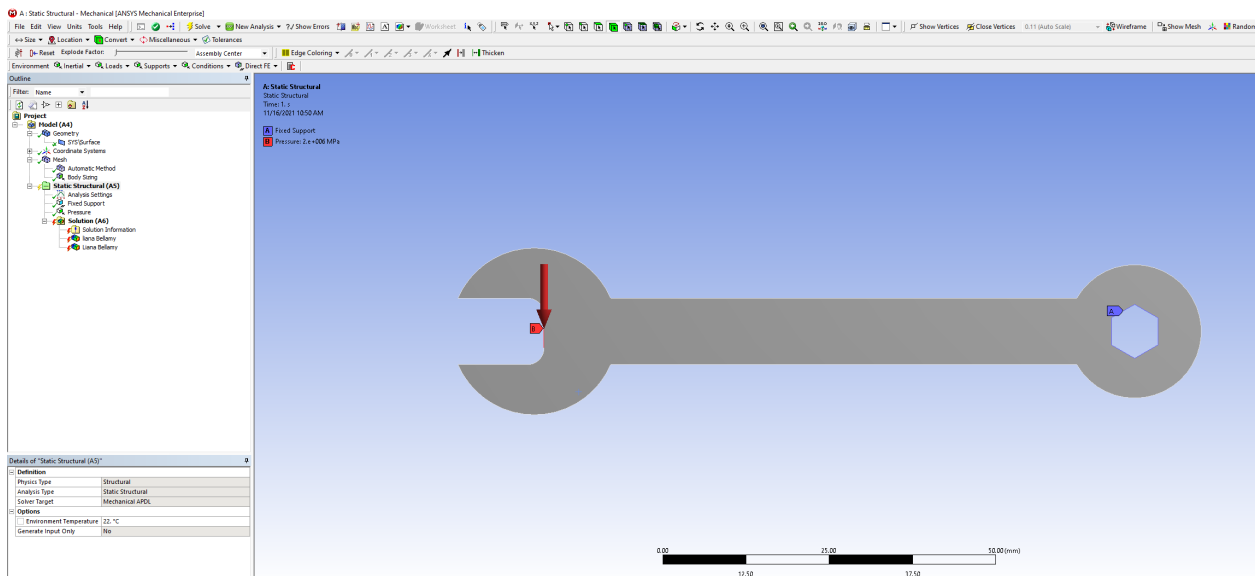
(The screenshots should include the file trees, graphical results, and ANSYS version)



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Assignment Comments



Did not complete analysis.



Shield Lin, Nov 19 at 10:52am

FINITE ELEMENT ANALYSIS AND DESIGN FALL 2021

Title of Assignment: TEST 4

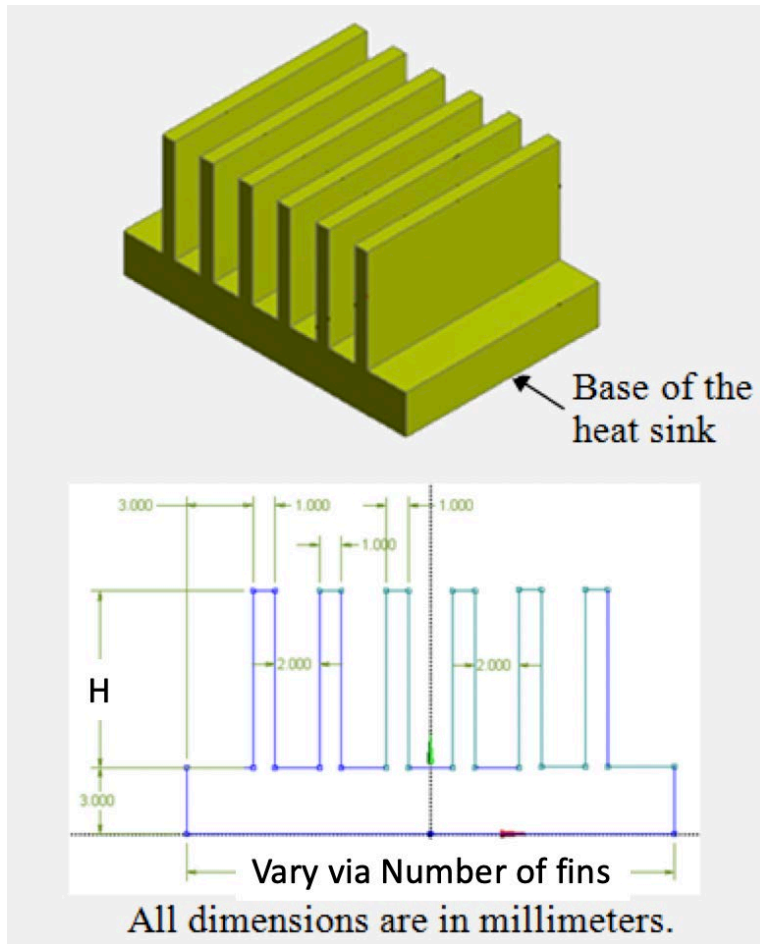
TOPICS COVERED IN TEST 4

1. Design a heat sink geometry using personal attributes.
2. Using ANSYS to solve for steady-state temperature.
3. Using ANSYS to solve for thermal stress under the temperature change.

Test 4

MCEG 4309 - Finite Element Analysis & Design

November 30, 2021



a heat sink made of aluminum with thermal conductivity $k = 170 \text{ W/(m}\cdot\text{K)}$, density $\rho = 2800 \text{ kg/m}^3$, specific heat $c = 870 \text{ J/(kg}\cdot\text{K)}$, Young's modulus $E = 70 \text{ GPa}$, Poisson's ratio $\nu = 0.3$ and thermal expansion coefficient $\alpha = 22 \times 10^{-6}/^\circ\text{C}$. A fan forces air over all surfaces of the heat sink except for the base, where a heat flux q' is prescribed. The surrounding air is 28°C with a heat transfer coefficient of $h = 30 \text{ W/(m}^2\cdot^\circ\text{C)}$. Study the steady-state thermal response of the heat sink with an initial temperature of 28°C and a constant heat flux input of $q' = 1000 \text{ W/m}^2$.

The base of the heat sink is fixed.

Draw a "personalized" geometry of a heat sink with the unit of millimeters.

The Height of Fins of the heat sink: $H = \text{Number of characters in your last name}$.

Number of fins = Number of characters in your first name.

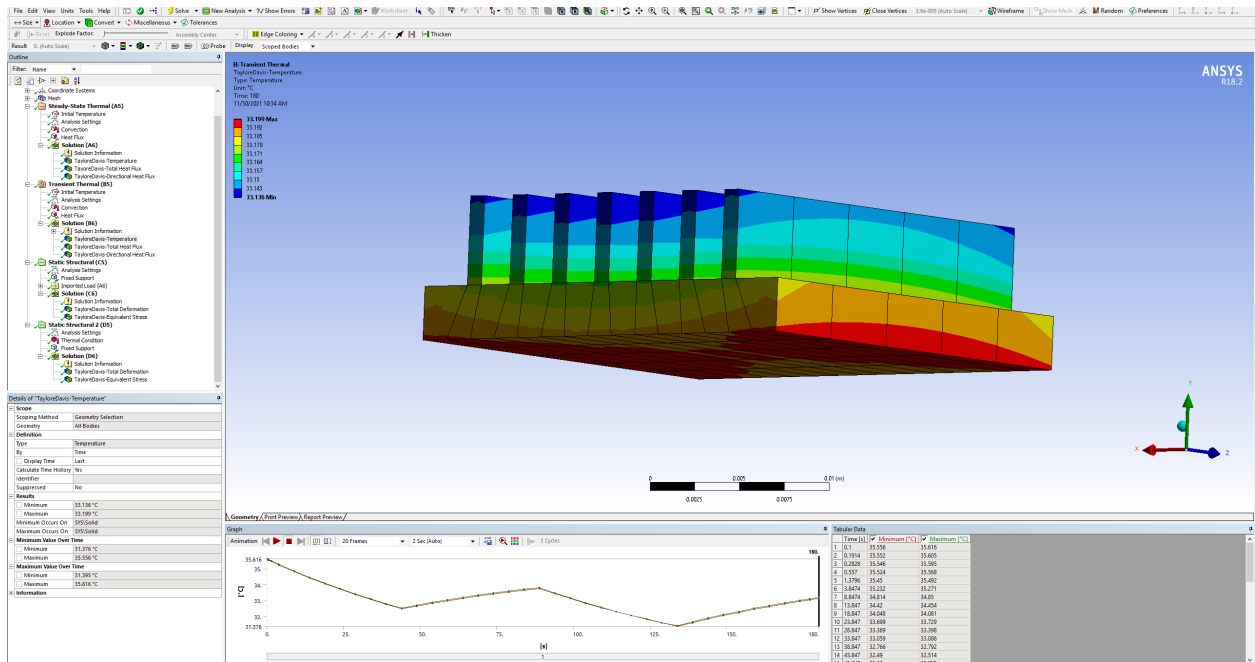
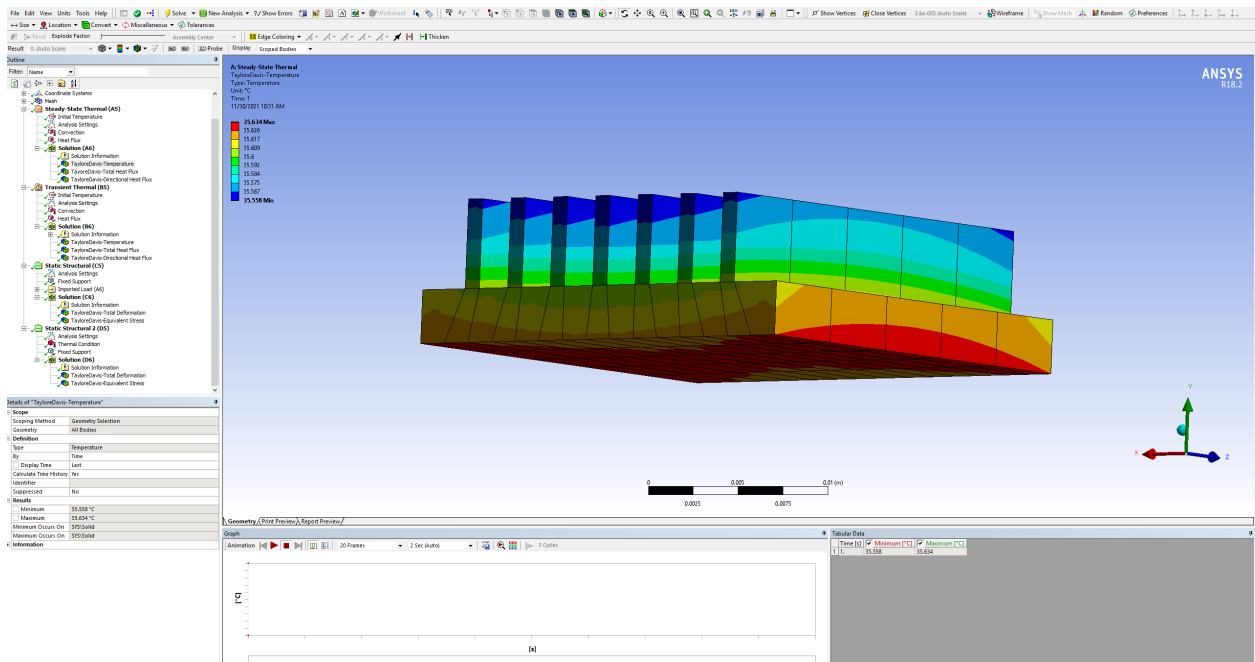
The depth of heat sink is 25 mm (in "pull" direction). Each fin has a width of 1 mm. The gaps between fins are 2 mm. 3 mm between side fin and the edge of the base.

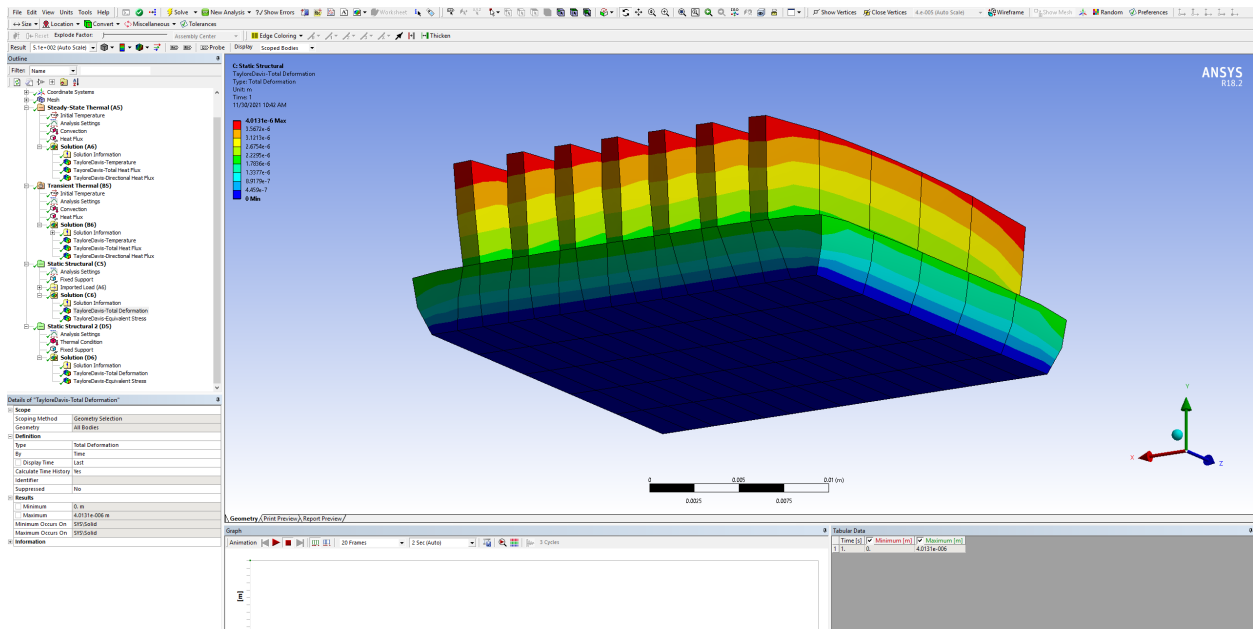
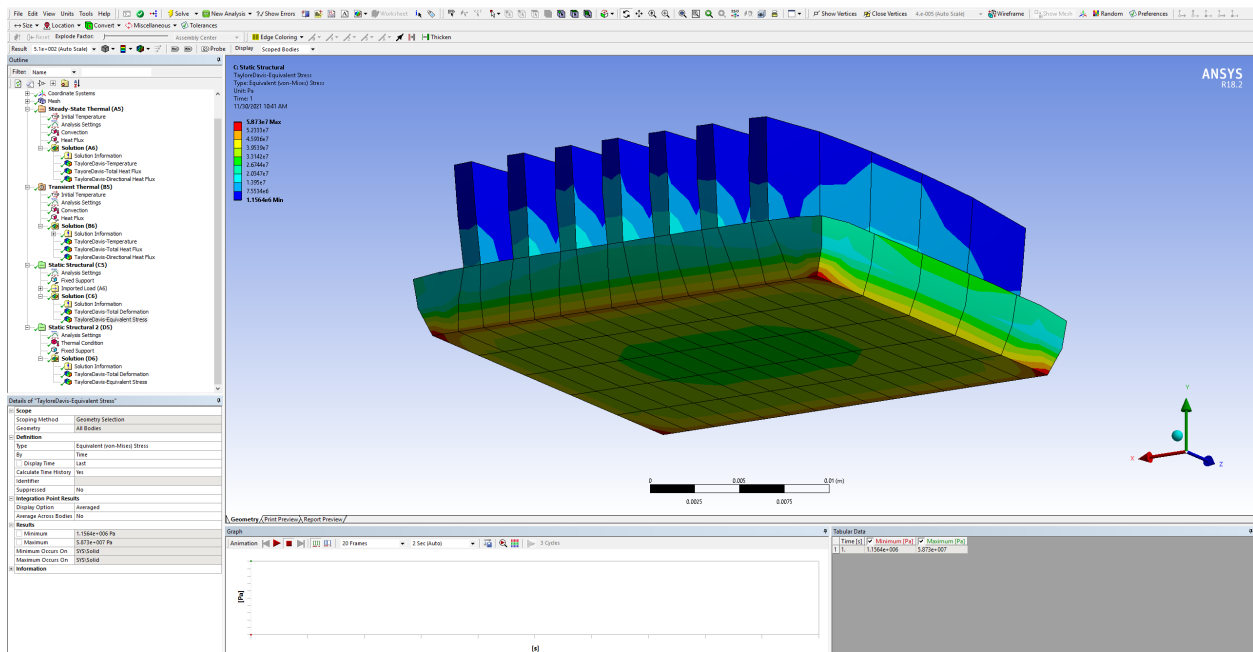
Perform ANSYS finite element analysis and obtain the following results:

- (1) "Temperature" plot of Steady-State Thermal Analysis; add **your name** in front of Temperature.
- (2) "Equivalent Stress" plot of Thermal Stress Analysis under the given heat flux with the fixed support to the bottom face of the heat sink. Add **your name** in front of Equivalent Stress.

(The two screenshots should include the file trees, graphical results, and ANSYS version)

ABOVE AVERAGE

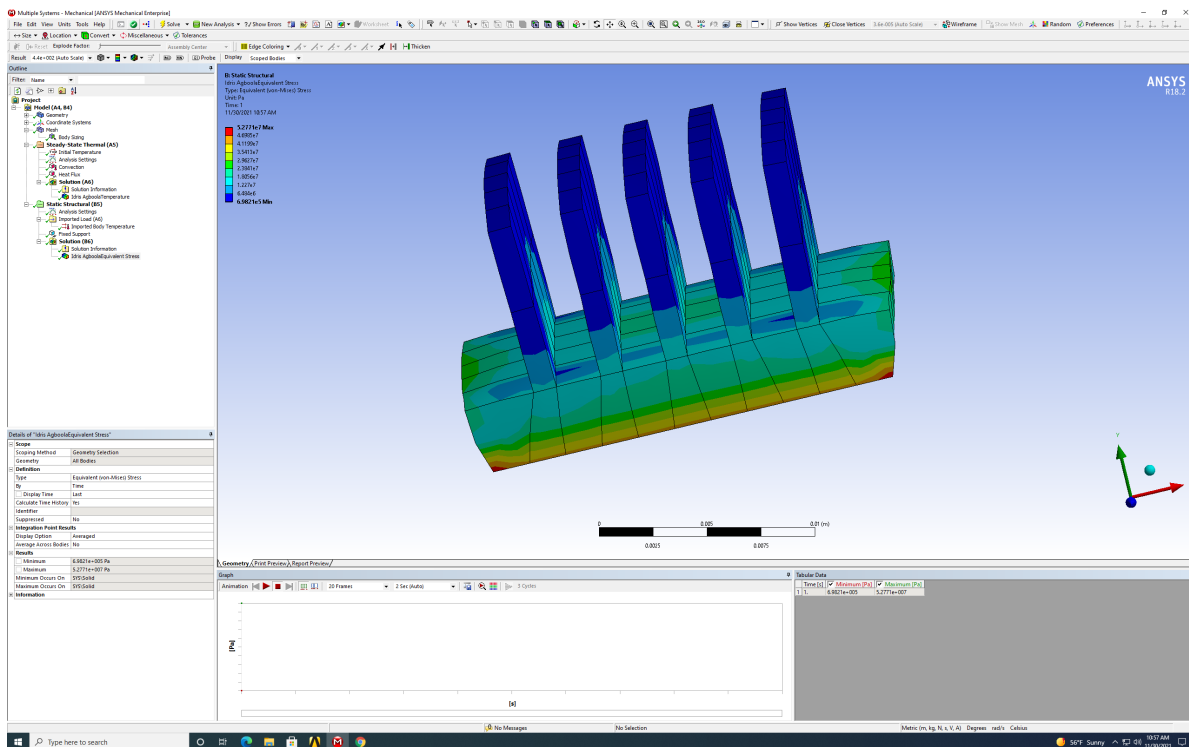
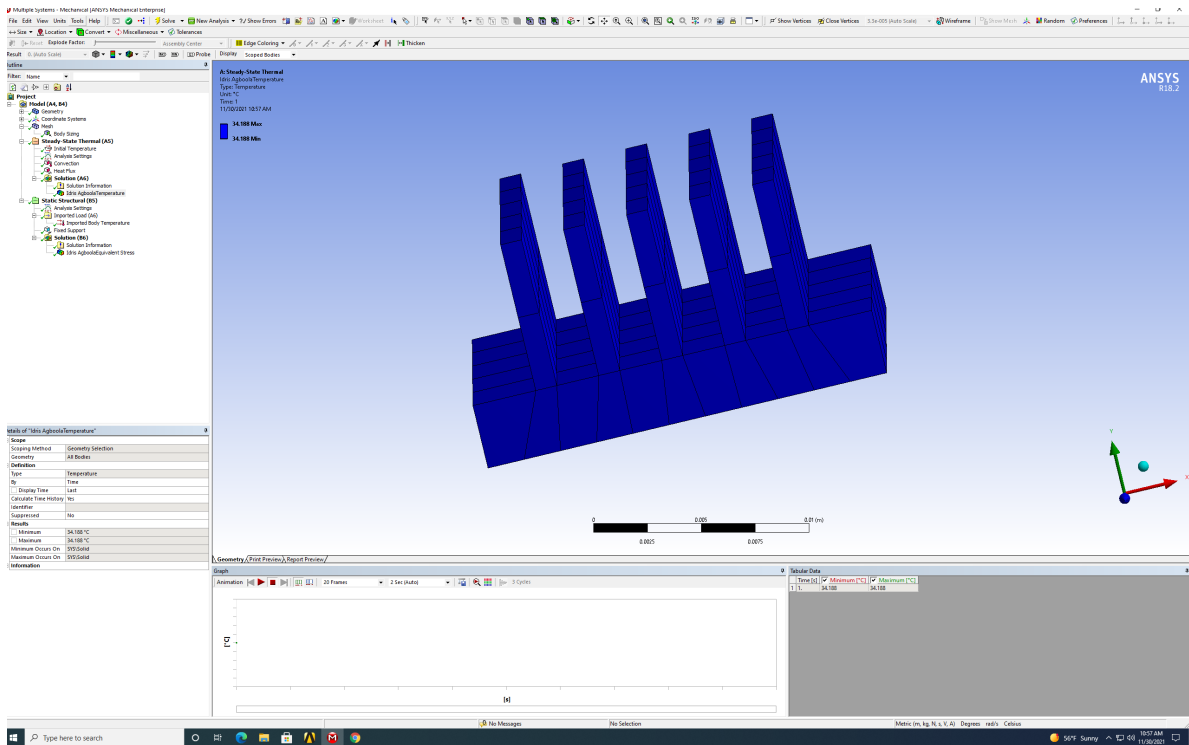




(Submitted more results than required.)

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Assignment Comments



Wrong temperature.



Shield Lin, Dec 4 at 5:31pm

FINITE ELEMENT ANALYSIS AND DESIGN
Fall 2021

Title of Assignment: Homework 1

TOPICS COVERED IN HOMEWORK 1

1. Matrix Algebra
2. Using MATLAB to solve Matrix Equation

Homework:

1. Given matrices

$$[A] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}, [B] = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}, \text{ and } [C] = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

perform the following operations: (Use hand calculation)

- a. $[A] + [B] = ?$
- b. $[A] - [B] = ?$
- c. $3[A] = ?$
- d. $[A][B] = ?$
- e. $[A][C] = ?$

2. Determine the inverse of the following matrices:

$$[A] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad [B] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$$

3. Solve the following set of equations, resulting from a model of a fin, using MATLAB.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.0408 & 0.0888 & -0.0408 & 0 & 0 \\ 0 & -0.0408 & 0.0888 & -0.0408 & 0 \\ 0 & 0 & -0.0408 & 0.0888 & -0.0408 \\ 0 & 0 & 0 & -0.0408 & 0.04455 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0.144 \\ 0.144 \\ 0.144 \\ 0.075 \end{bmatrix}$$

ABOVE AVERAGE

Q.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

(a) $[A] + [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$

Note:-

Each element of matrices will be added to each other in order to obtain Addition of matrices

Example:-

$(a_{ij})_A$ will be added to $(a_{ij})_B$ and so on.
 $i = \text{Row Number}$
 $j = \text{Column Number}$

$$[A] + [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 2+2 & 1-1 \\ 7+5 & 0+3 & -7+3 \\ 1+4 & -5+5 & 3-7 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 5 & 0 & -4 \end{bmatrix}$$

(b) $[A] - [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$

$$= \begin{bmatrix} 4-1 & 2-2 & 1-1 \\ 7-5 & 0-3 & -7-3 \\ 1-4 & -5-5 & 3-7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix}$$

$$3[A] = 3 \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$$

Note:- Each and every elements will be multiplied by 3.

$$= \begin{bmatrix} 3 \times 4 & 3 \times 2 & 3 \times 1 \\ 3 \times 7 & 3 \times 0 & 3 \times (-7) \\ 3 \times 1 & 3 \times (-5) & 3 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix}$$

$$(4) [A][B] = ?$$

$$[A][B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$(m \times n) \cdot (n \times p) = m \times p$$



Multiplication
will result
matrix of

$$\Rightarrow (3 \times 3)$$

for the first element of multiplication of matrix will be obtained by multiplying first row to the first column of matrix.

for example

$$c_{11} = (4 \times 1) + (2 \times 5) + (1 \times 4) = 4 + 10 + 4 = \underline{\underline{18}}$$

$$c_{12} = (4 \times 2) + (2 \times 3) + (1 \times 5) = 8 + 6 + 5 = \underline{\underline{19}}$$

$$c_{13} = (4 \times -1) + (2 \times 3) + (1 \times -7) = -4 + 6 - 7 = \underline{\underline{-5}}$$



$$C_{21} = (7 \times 1) + (0 \times 5) + (-7 \times 4) = 7 - 28 = -21$$

$$C_{22} = (7 \times 2) + (0 \times 3) + (-7 \times 5) = 14 + (-35) = -21$$

$$C_{23} = (7 \times -1) + (0 \times 8) + (-7 \times -7) = (-7) + 49 = 42$$

$$C_{31} = (1 \times 1) + (-5 \times 5) + (3 \times 4) = 1 - 25 + 12 = -12$$

$$C_{32} = (1 \times 2) + (-5 \times 3) + (3 \times 5) = 2 - 15 + 15 = 2$$

$$C_{33} = (1 \times -1) + (-5 \times 3) + (3 \times -7) = -1 - 15 - 21 = -37$$

$$A \cdot B = \begin{bmatrix} 18 & 19 & -5 \\ -21 & -21 & 42 \\ -12 & 2 & -37 \end{bmatrix}$$

$$\textcircled{5} \quad [A][C] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$(m \times n)(n \times p) = mp$$

Multiplication
will result matrix
of

$$\frac{3 \times 3}{m \times n}$$

$$\frac{3 \times 1}{n \times p}$$

$$= 3 \times 1$$

$$\begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix}$$

$$C_{11} = (4 \times 1) + (2 \times -2) + (1 \times 4) = 4 - 4 + 4 = 4$$

$$C_{21} = (7 \times 1) + (0 \times -2) + (-7 \times 4) = 7 + 0 - 28 = -21$$

$$C_{31} = (1 \times 1) + (-5 \times -2) + (3 \times 4) = 1 + 10 + 12 = 23$$

$$[A][C] = \begin{bmatrix} 4 \\ -21 \\ 23 \end{bmatrix}$$

2. Determine the inverse of the following matrices:

$$[A] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad [B] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$$

1 $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

We know $A = IA$

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$$

Now we will convert I.H.S. to identity matrix I,

Applying $R_1 \rightarrow \frac{1}{5} R_1$, $R_2 \rightarrow \frac{1}{2} R_2$, $R_3 \rightarrow \frac{1}{8} R_3$

and $R_4 \rightarrow \frac{1}{4} R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix} A$$

i.e. $I = A^{-1}A$

Hence

$$A^{-1} = \begin{bmatrix} 1/5 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$$

12 $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$

Since $B = IB$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B$$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow 3R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} B$$

Applying $R_2 \rightarrow \frac{1}{3} R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ 3 & 0 & 1 \end{bmatrix} B$$

Applying $R_3 \rightarrow R_3 - 4R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 28/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ 17/3 & -4/3 & 1 \end{bmatrix} B$$

Applying $R_3 \rightarrow \frac{3}{28} R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ 17/28 & -1/7 & 3/28 \end{bmatrix} B$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 & 0 \\ -2/3 & 1/3 & 0 \\ 17/28 & -1/7 & 3/28 \end{bmatrix} B$$

Applying $R_1 \rightarrow R_1 - \frac{4}{3}R_3$, $R_2 \rightarrow R_2 + \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6/7 & -1/7 & -1/7 \\ -13/28 & 2/7 & 1/28 \\ 17/28 & -1/7 & 3/28 \end{bmatrix} B$$

$$I = B^{-1}B$$

Thus

$$B^{-1} = \begin{bmatrix} 6/7 & -1/7 & -1/7 \\ -13/28 & 2/7 & 1/28 \\ 17/28 & -1/7 & 3/28 \end{bmatrix}$$



Solve the following set of equations, resulting from a model of a fin, using MATLAB.

3.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.0408 & 0.0888 & -0.0408 & 0 & 0 \\ 0 & -0.0408 & 0.0888 & -0.0408 & 0 \\ 0 & 0 & -0.0408 & 0.0888 & -0.0408 \\ 0 & 0 & 0 & -0.0408 & 0.04455 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 100 \\ 0.144 \\ 0.144 \\ 0.144 \\ 0.075 \end{Bmatrix}$$

```
>> A=[1 0 0 0 0;-0.0408 0.0888 -0.0408 0 0;-0.0408 0.0888 -0.0408 0 0;-0.0408 0.0888 -0.0408 0 0;-0.0408 0.0888 -0.0408 0 0;-0.0408 0.04455]
```

```
A =
    1.0000    0    0    0    0
   -0.0408    0.0888   -0.0408    0    0
    0   -0.0408    0.0888   -0.0408    0
    0    0   -0.0408    0.0888   -0.0408
    0    0    0   -0.0408    0.04455
```

```
>> B=[100;0.144;0.144;0.144;0.075]
```

```
B =
```

```
100.0000
    0.1440
    0.1440
    0.1440
    0.0750
```

```
>> C=inv(A)*B
```

```
C =
```

```
100.0000
  75.0387
  59.7901
  51.5633
  48.9064
```

*C=inverse of (A)*B.

We inserted the formula to compute.

*After entering the formula.

We get the solution: each element of A matrix is Multiplied with B matrix and we get the solution.

*First write down the given array.

A=[1 0 0 0 0;-0.0408 0.0888 -0.408 0 0; & so on]

As you seen above every element in coloumn has given single space between each element (;) semi colon for next line(column.).

*After entering value we get our A matrix.

* As we know the formula is AX=B,

So, we had already written A value. Now we are inserting B values.

B=[100;0.144;0.144;0.144;0.075]

All values in column so use (;) semicolon for every column.

*After entering B value we get B matrix.

- First insert the A matrix.
- Then insert B matrix and
- Then the formula. (C=inv(A)*B or C=A\B)
- we will get the solution

Excellent Work!

AVERAGE

Aug. 29, 2021

Homework Ch. 0

Garrett Warren

1. $[A] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$, $[B] = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$, $[C] = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

a. $[A] + [B]$

$$\begin{bmatrix} (4+1) & (2+2) & (1+(-1)) \\ (7+5) & (0+3) & (-7+3) \\ (1+4) & (-5+5) & (3-7) \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 5 & 0 & -4 \end{bmatrix} \quad \text{Ans.}$$

b. $[A] - [B]$

$$\begin{bmatrix} (4-1) & (2-2) & (1-(-1)) \\ (7-5) & (0-3) & (-7-3) \\ (1-4) & (-5-5) & (3+7) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix} \quad \text{Ans.}$$

c. $3[A]$

$$\begin{bmatrix} 3(4) & 3(2) & 3(1) \\ 3(7) & 3(0) & 3(-7) \\ 3(1) & 3(-5) & 3(3) \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix} \quad \text{Ans.}$$

d. $[A][B]$

$$\begin{bmatrix} (4)(1) & (2)(2) & (1)(-1) \\ (7)(5) & (0)(3) & (-7)(3) \\ (1)(4) & (-5)(5) & (3)(-7) \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ 35 & 0 & -21 \\ 4 & -25 & -21 \end{bmatrix} \quad \text{Ans.}$$

e. $[A][C]$

$$\begin{bmatrix} 4(1) + 2(-2) + 1(4) \\ 7(1) + 0(-2) + (-7)(4) \\ 1(1) + (-5)(-2) + 3(4) \end{bmatrix} = \begin{bmatrix} 4 \\ -21 \\ 23 \end{bmatrix} \quad \text{Ans.}$$

2.

$$[A] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad [B] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$$

Inverse:

$$[A] = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$[B] = \begin{bmatrix} \frac{6}{7} & -\frac{1}{7} & -\frac{1}{7} \\ -\frac{13}{28} & \frac{2}{7} & \frac{1}{28} \\ \frac{17}{28} & -\frac{1}{7} & \frac{3}{28} \end{bmatrix}$$

```

Command Window

>> A = [100; 0.144; 0.144; 0.144; 0.075;]

A =

100.0000
  0.1440
  0.1440
  0.1440
  0.0750

>> B = [1 0 0 0 0; -0.0408 0.0888 -0.0408 0 0; 0 -0.0408 0.0888 -0.0408 0; 0 0 -0.0408 0.0888 -0.0408; 0 0 0 -0.0408 0.0888]

B =

1.0000    0    0    0    0
-0.0408    0.0888   -0.0408    0    0
  0   -0.0408    0.0888   -0.0408    0
  0    0   -0.0408    0.0888   -0.0408
  0    0    0   -0.0408    0.0445

>> Y = inv(B)

Y =

1.0000    0    0    0    0
 0.6880 16.8623 12.1906  9.6701  8.8561
 0.4974 12.1906 26.5325 21.0467 19.2751
 0.3945  9.6701 21.0467 36.1375 33.0956
 0.3613  8.8561 19.2751 33.0956 52.7564

>> X = Y*A

```

```
>> X = Y*A
```

```
X =
```

```
100.0000
 75.0387
 59.7901
 51.5633
 48.9064
```



BELOW AVERAGE

1.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

a. $[A] + [B] =$

$$= \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$$
$$= \begin{bmatrix} (4+1) & (2+2) & (1+(-1)) \\ (7+5) & (0+3) & ((-7)+3) \\ (1+4) & ((-5)+5) & (3+(-7)) \end{bmatrix}$$

Answer $= \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 6 & 0 & -4 \end{bmatrix}$

b. $[A] - [B] =$

$$= \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$$
$$= \begin{bmatrix} (4-1) & (2-2) & (1-(-1)) \\ (7-5) & (0-3) & ((-7)-3) \\ (1-4) & ((-5)-5) & (3-(-7)) \end{bmatrix}$$

Answer $= \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix}$



$$c. 3[A] =$$

$$= 3 \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 3(4) & 3(2) & 3(1) \\ 3(7) & 3(0) & 3(-7) \\ 3(1) & 3(-5) & 3(3) \end{bmatrix}$$

$$\text{Answer} = \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix}$$

$$d. [A][B] =$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1) + 2(5) + 1(4) & 4(2) + 2(3) + 1(5) & 4(-1) + 2(3) + 1(-7) \\ 7(1) + 0(5) + -7(4) & 7(2) + 0(3) + -7(5) & 7(-1) + 0(3) + -7(-7) \\ 1(1) + -5(5) + 3(4) & 1(2) + -5(3) + 3(5) & 1(-1) + -5(3) + 3(-7) \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 19 & -5 \\ -21 & -21 & 42 \\ -12 & 2 & -37 \end{bmatrix}$$

$$e. [A][C] =$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1) + 2(-2) + 1(4) \\ 7(1) + 0(-2) + -7(4) \\ 1(1) + -5(-2) + 3(4) \end{bmatrix} = \begin{bmatrix} 4 \\ -21 \\ 23 \end{bmatrix}$$

2. Inverse Matrices

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}$$

Answer

$$= \begin{bmatrix} 1/5 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/8 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$$

Answer

$$= \begin{bmatrix} 6/7 & -1/7 & -1/7 \\ -13/28 & 2/7 & 1/28 \\ 17/28 & -1/7 & 3/28 \end{bmatrix}$$



3.

-10

Missing Problem 3

FINITE ELEMENT ANALYSIS AND DESIGN FALL 2021

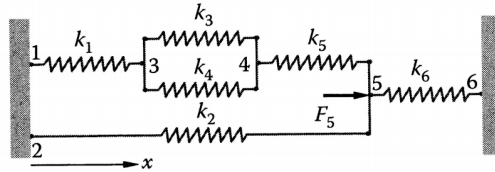
Title of Assignment: Homework 2

TOPICS COVERED IN HOMEWORK 2

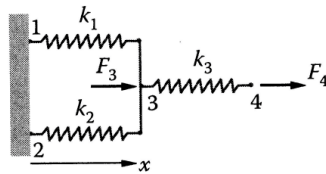
1. Finite element formulation for spring systems
2. Calculate nodal displacements
3. Calculate forces at boundaries and in the springs.

Homework:

- 1.4 Consider the spring system shown below. Find the global stiffness matrix for the entire system.



- 1.6 A spring system is shown below

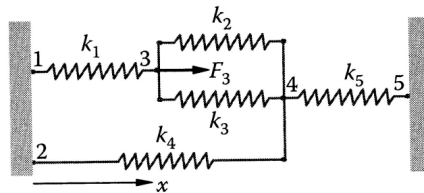


Given: $k_1 = 80 \text{ N/mm}$, $k_2 = 100 \text{ N/mm}$, $k_3 = 160 \text{ N/mm}$, $F_3 = 200 \text{ N}$, $F_4 = 100 \text{ N}$, and nodes 1 and 2 are fixed;

Find:

- Global stiffness matrix
- Displacements of nodes 3 and 4
- Reaction forces at nodes 1 and 2
- Forces in springs 1 and 2

- 1.7 A spring system is shown below



Given: $k_1 = k_2 = k_3 = k_4 = k_5 = k$, $F_3 = P$, and nodes 1, 2, and 5 are fixed;

Find:

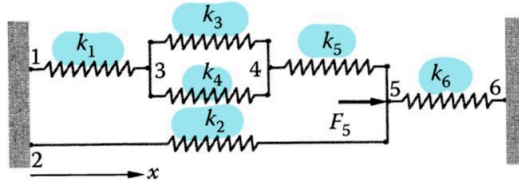
- Global stiffness matrix
- Displacements of nodes 3 and 4
- Reaction forces at nodes 1, 2, and 5
- Forces in springs 2, 4, and 5

ABOVE AVERAGE

Homework:

1.4 Consider the spring system shown below. Find the global stiffness matrix for the entire system.

6 elements



element 1

$$K_1 = \begin{bmatrix} k_{11} & -k_{13} \\ -k_{13} & k_{33} \end{bmatrix}$$

element 2

$$K_2 = \begin{bmatrix} k_{22} & -k_{25} \\ -k_{25} & k_{55} \end{bmatrix}$$

element 3

$$K_3 = \begin{bmatrix} k_{33} & -k_{34} \\ -k_{34} & k_{44} \end{bmatrix}$$

$$K_4 = \begin{bmatrix} k_{33} & -k_{34} \\ -k_{34} & k_{44} \end{bmatrix}$$

element 5

$$K_5 = \begin{bmatrix} k_{44} & -k_{45} \\ -k_{45} & k_{55} \end{bmatrix}$$

element 6

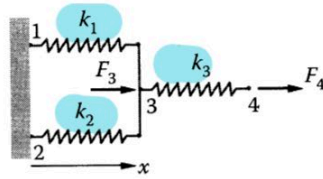
$$K_6 = \begin{bmatrix} k_{55} & -k_{56} \\ -k_{56} & k_{66} \end{bmatrix}$$

Global Stiffness Matrix:

$$\begin{bmatrix} k_1 & 0 & -k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & -k_2 & 0 \\ -k_1 & 0 & k_1 + k_3 + k_4 & -k_3 - k_4 & 0 & 0 \\ 0 & 0 & -k_3 + (-k_4) & k_3 + k_4 + k_5 & -k_5 & 0 \\ 0 & -k_2 & 0 & -k_5 & k_2 + k_5 + k_6 & -k_6 \\ 0 & 0 & 0 & 0 & -k_6 & k_6 \end{bmatrix}$$



1.6 A spring system is shown below



3 elements

Given: $k_1 = 80 \text{ N/mm}$, $k_2 = 100 \text{ N/mm}$, $k_3 = 160 \text{ N/mm}$, $F_3 = 200 \text{ N}$, $F_4 = 100 \text{ N}$, and nodes 1 and 2 are fixed;

Find:

- Global stiffness matrix
- Displacements of nodes 3 and 4
- Reaction forces at nodes 1 and 2
- Forces in springs 1 and 2

Given:

$$k_1 = 80 \text{ N/mm}$$

$$k_2 = 100 \text{ N/mm}$$

$$k_3 = 160 \text{ N/mm}$$

$$F_3 = 200 \text{ N}$$

$$F_4 = 100 \text{ N}$$

node 1 & 2 fixed = 0

$$\text{element 1} \quad \begin{matrix} u_1 & u_3 \\ K_1 & -K_3 \\ -K_3 & K_3 \end{matrix}$$

$$\text{element 2} \quad \begin{matrix} u_2 & u_3 \\ K_2 & -K_3 \\ -K_3 & K_3 \end{matrix}$$

$$\text{element 3} \quad \begin{matrix} u_3 & u_4 \\ K_3 & -K_4 \\ -K_4 & K_4 \end{matrix}$$

a) Global Stiffness

$$\begin{bmatrix} K_1 & 0 & -K_1 & 0 \\ 0 & K_2 & -K_2 & 0 \\ -K_1 & -K_2 & K_1 + K_2 + K_3 & -K_3 \\ 0 & 0 & -K_3 & K_3 \end{bmatrix} =$$

$$\begin{bmatrix} 80 & 0 & -80 & 0 \\ 0 & 100 & -100 & 0 \\ -80 & -100 & 340 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ 200 \\ 100 \end{Bmatrix}$$

b)

$$\begin{bmatrix} 340 & -100 \\ -100 & 100 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 200 \\ 100 \end{bmatrix}$$

$$\begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 340 & -100 \\ -100 & 100 \end{bmatrix}^{-1} \begin{bmatrix} 200 \\ 100 \end{bmatrix}$$

$$\begin{aligned} u_3 &= 1.6667 \\ u_4 &= 2.2917 \end{aligned}$$

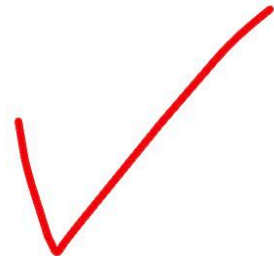
c)

$$\begin{bmatrix} 80 & 0 & -80 & 0 \\ 0 & 100 & -100 & 0 \\ -80 & -100 & 340 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1.6667 \\ 2.2917 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ 200 \\ 100 \end{Bmatrix}$$

$$\begin{aligned} R_1 &= -133.336 \\ R_2 &= -166.67 \end{aligned}$$

d)

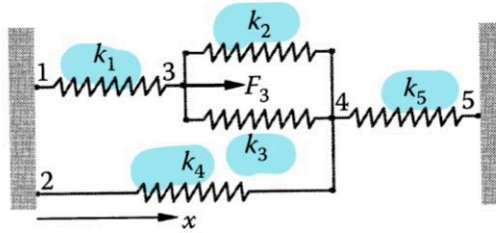
$$\begin{aligned} F_1 &= K_1(u_3 - u_1) = 80(1.6667 - 0) = 133.34 \text{ N} \\ F_2 &= K_2(u_3 - u_2) = 100(1.6667 - 0) = 166.67 \text{ N} \end{aligned}$$



1.7 A spring system is shown below

elements 5

nodes 5



Given: $k_1 = k_2 = k_3 = k_4 = k_5 = k$, $F_3 = P$, and nodes 1, 2, and 5 are fixed;

Find:

- Global stiffness matrix
- Displacements of nodes 3 and 4
- Reaction forces at nodes 1, 2, and 5
- Forces in springs 2, 4, and 5

$$K_1 = \begin{bmatrix} K_{11} & -K_{31} \\ -K_{13} & K_{33} \end{bmatrix} \quad K_2 = \begin{bmatrix} K_{33} & -K_{43} \\ -K_{34} & K_{44} \end{bmatrix}$$

$$K_3 = \begin{bmatrix} K_{33} & -K_{43} \\ -K_{34} & K_{44} \end{bmatrix} \quad K_4 = \begin{bmatrix} K_{22} & -K_{42} \\ -K_{24} & K_{44} \end{bmatrix}$$

$$K_5 = \begin{bmatrix} K_{44} & -K_{54} \\ -K_{45} & K_{55} \end{bmatrix}$$

gegeben
 $K_1 = K_2 = K_3 = K_4 = K_5 = K$
 $F_3 = P$

$u_1, u_2, u_5 = 0$

a)

$$\begin{bmatrix} K_1 & 0 & -K_1 & 0 & 0 \\ 0 & K_4 & 0 & -K_4 & 0 \\ -K_1 & 0 & K_1 + K_2 + K_3 & -K_2 - K_3 & 0 \\ 0 & -K_4 & -K_2 - K_3 & K_2 + K_3 + K_4 + K_5 & -K_5 \\ 0 & 0 & 0 & -K_5 & K_5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ P \\ 0 \\ R_5 \end{bmatrix}$$

b)

$$\begin{bmatrix} K_1 + K_2 + K_3 & -K_2 - K_3 \\ -K_2 - K_3 & K_2 + K_3 + K_4 + K_5 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix}$$

$$K \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix} = \begin{aligned} 2(3Ku_3 - 2Ku_4) &= P \Rightarrow 6Ku_3 - 4Ku_4 = P \\ 3(-2Ku_3 + 4Ku_4) &= 0 \Rightarrow -6Ku_3 + 12Ku_4 = 0 \end{aligned}$$

$8Ku_4 = 2P$

$u_4 = P/4K$

$u_3 = P/2K$



c)

$$K \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 3 & -2 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ 0 \\ 0 \\ R_5 \end{bmatrix}$$

$$\begin{aligned} R_1 &= -k u_3 = -k (P/2k) = -P/2 \\ R_2 &= -k u_4 = -k (P/4k) = -P/4 \\ R_5 &= -k u_4 = -k (P/4k) = -P/4 \end{aligned}$$



D) $k_1 = k_2 = k_3 = k_4 = k_5 = k$

$$F_2 = k_2 (u_1 - u_3) = k_2 (P/4k - P/2k) = \frac{-k_2 P}{4k} = \boxed{-P/4}$$

$$F_4 = k_4 (u_4 - u_2) = k_2 (P/4k - 0) = \frac{k_4 P}{4k} = \boxed{P/4}$$

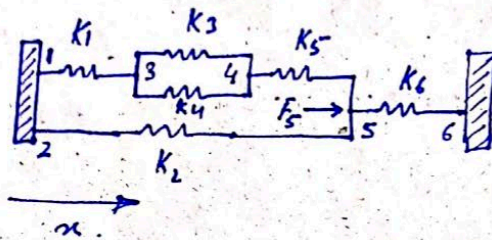
$$F_5 = k_5 (u_5 - u_4) = k_5 (0 - P/4k) = \boxed{-P/4}$$



AVERAGE

1.4
=

Given



Required

Global stiffness matrix.

Solution

As from global stiffness matrix.

$$[K] = [K_1] + [K_2] + [K_3] + [K_4] + [K_5] + [K_6] \rightarrow \text{①}$$

$$\text{So } [K_1] = k_1 \begin{bmatrix} \text{①} & \text{③} \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} \text{①} \\ \text{③} \end{matrix}, [K_2] = k_2 \begin{bmatrix} \text{②} & \text{⑤} \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} \text{②} \\ \text{⑤} \end{matrix}$$

$$[K_1] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}, [K_2] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

$$[K_3] = k_3 \begin{bmatrix} \text{③} & \text{④} \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} \text{③} \\ \text{④} \end{matrix} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

$$[K_4] = k_4 \begin{bmatrix} \text{③} & \text{④} \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} \text{③} \\ \text{④} \end{matrix} = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix}$$

$$[K_5] = k_5 \begin{bmatrix} \text{④} & \text{⑤} \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} \text{④} \\ \text{⑤} \end{matrix} = \begin{bmatrix} k_5 & -k_5 \\ -k_5 & k_5 \end{bmatrix}$$

$$[K_6] = K_6 \begin{matrix} \textcircled{1} & \textcircled{6} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} K_6 & -K_6 \\ -K_6 & K_6 \end{bmatrix}$$

So by putting values in eqn (A) we get.

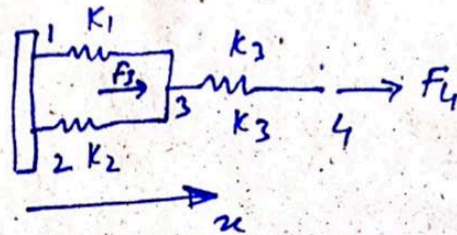
$$[K] = \begin{bmatrix} K_1 & 0 & -K_1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 & -K_2 & 0 \\ -K_1 & 0 & (K_1+K_2+K_4) & -K_3-K_4 & 0 & 0 \\ 0 & 0 & -(K_3+K_4) & K_5+K_4+K_3 & -K_5 & 0 \\ 0 & -K_2 & 0 & -K_5 & K_2+K_5+K_6 & -K_6 \\ 0 & 0 & 0 & 0 & -K_6 & K_6 \end{bmatrix}$$

$$[K] = \begin{bmatrix} K_1 & 0 & -K_1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 & -K_2 & 0 \\ -K_1 & 0 & (K_1+K_2+K_4) & -K_3-K_4 & 0 & 0 \\ 0 & 0 & -(K_3+K_4) & (K_5+K_4+K_3) & -K_5 & 0 \\ 0 & -K_2 & 0 & -K_5 & (K_2+K_5+K_6) & -K_6 \\ 0 & 0 & 0 & 0 & -K_6 & K_6 \end{bmatrix}$$

Ans

1.6

Given.



$$K_1 = 80 \text{ N/mm}, \quad K_2 = 100 \text{ N/mm}$$

$$K_3 = 160 \text{ N/mm}, \quad F_3 = 200 \text{ N}$$

$$F_4 = 100 \text{ N}$$

node 1 and 2 are fixed.

Required.

- ① Global stiffness matrix.
- ② Displacement of node 3 & 4.
- ③ Reaction forces at node 1 & 2.
- ④ Forces in spring 1 & 2.

Solution.

① Taking node 1 & 3

$$K_{13} = \begin{bmatrix} \textcircled{1} & \textcircled{3} \\ K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{matrix} 1 \\ 3 \end{matrix}$$

Now node 2 & 3

$$K_{23} = \begin{bmatrix} \textcircled{2} & \textcircled{3} \\ K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{matrix} 2 \\ 3 \end{matrix}$$



and node ③ & ④

$$K_{34} = \begin{bmatrix} \textcircled{3} & \textcircled{4} \\ K_3 & -K_3 \\ -K_3 & K_3 \end{bmatrix} \begin{matrix} \textcircled{3} \\ \textcircled{4} \end{matrix}$$

So

$$\{K\} = \begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ K_1 & 0 & -K_1 & 0 \\ 0 & K_2 & -K_2 & 0 \\ -K_1 & -K_2 & (K_1+K_2+K_3) & -K_3 \\ 0 & 0 & -K_3 & K_3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

putting K_1, K_2, K_3 values.

$$K = \begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ 80 & 0 & -80 & 0 \\ 0 & 100 & -100 & 0 \\ -80 & -100 & 340 & -160 \\ 0 & 0 & -160 & 160 \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix}$$


⑥

As node 1 & 2 are fixed, so

$$u_1 = 0 \text{ \& } u_2 = 0$$

therefore

$$\begin{bmatrix} -80 & -100 & 340 & -160 \\ 0 & 0 & -160 & 160 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 200 \\ 100 \end{bmatrix}$$

So

$$340 u_3 - 160 u_4 = 200 \rightarrow \textcircled{A}$$

$$-160 u_3 + 160 u_4 = 100 \rightarrow \textcircled{B}$$

Solving Eqn $\textcircled{A}$ & $\textcircled{B}$

$$340 u_3 - 160 u_4 = 200$$

$$-160 u_3 + 160 u_4 = 100$$

$$180 u_3 = 300$$

$$u_3 = 300/180$$

$$\boxed{u_3 = 1.666 \text{ mm}}$$

and

$$340 (1.666) - 160 u_4 = 200$$

$$(566.66 - 200)/160 = u_4$$

$$\boxed{u_4 = 2.29 \text{ mm}}$$



⑥

© Now reaction forces at node 1 & 2

$$\begin{bmatrix} 80 & 0 & -80 & 0 \\ 0 & 100 & -100 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1.66 \\ 2.29 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 200 \\ 100 \end{bmatrix}$$

$$-80 \times 1.66 = F_1$$

$$F_1 = -132.8 \text{ N.}$$

$$\Rightarrow -100 \times 1.66 = F_2$$

$$F_2 = -166 \text{ N.}$$

② Now forces in spring 1 & 2.

As $F = kx$, so

$$F_{k1} = k_1 (u_3 - u_1)$$

$$F_{k1} = 80 (1.66 - 0) = 132.8 \text{ N.}$$

↗

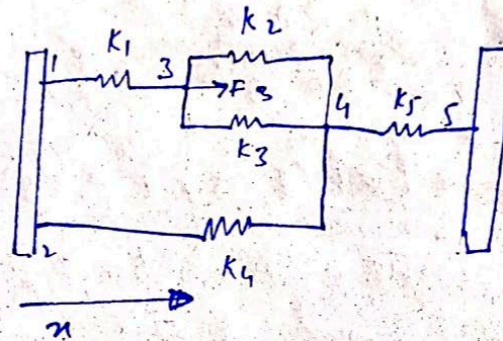
$$F_{k2} = k_2 (u_3 - u_2) = 100 (1.66)$$

$$F_{k2} = 166 \text{ N.}$$

⑦

1.7

Given



$$k_1 = k_2 = k_3 = k_4 = k_5 = k$$

$F_3 = P$, nodes 1, 2, 5 are fixed.

Solution:

① Global stiffness matrix = ?

$$K_1 = \begin{matrix} & \textcircled{1} & \textcircled{3} \\ \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{pmatrix} & \begin{matrix} 1 \\ 3 \end{matrix} \end{matrix}$$

$$K_2 = \begin{matrix} & \textcircled{3} & \textcircled{4} \\ \begin{pmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} & \begin{matrix} 3 \\ 4 \end{matrix} \end{matrix}$$

$$K_3 = \begin{matrix} & \textcircled{3} & \textcircled{4} \\ \begin{pmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{pmatrix} & \begin{matrix} \textcircled{3} \\ \textcircled{4} \end{matrix} \end{matrix}$$

$$K_4 = \begin{matrix} & \textcircled{2} & \textcircled{4} \\ \begin{pmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{pmatrix} & \begin{matrix} \textcircled{2} \\ \textcircled{4} \end{matrix} \end{matrix}$$



$$K_5 = \begin{matrix} & \textcircled{4} & \textcircled{5} \\ \textcircled{4} & k_5 & -k_5 \\ \textcircled{5} & -k_5 & k_5 \end{matrix}$$

So stiffness matrix becomes.

$$K = \begin{bmatrix} k & 0 & -k & 0 & 0 \\ 0 & k & 0 & -k & 0 \\ -k & 0 & 3k & -2k & 0 \\ 0 & -k & -2k & 4k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ P_3 \\ 0 \\ R_5 \end{bmatrix}$$

3) Displacement of nodes 3 & 4. ✓

As node 1, 2, 5 are fixed, so from matrix,

$$\begin{bmatrix} 3k & -2k \\ -2k & 4k \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix}$$

$$3k u_3 - 2k u_4 = P \rightarrow \textcircled{A}$$

$$-2k u_3 + 4k u_4 = 0 \rightarrow \textcircled{B}$$

From Eqn $\textcircled{A}$ & $\textcircled{B}$.

⑨

$$2(3kU_3) - 2(2kU_4) = 2P$$

$$-2kU_3 + 4kU_4 = 0$$

$$4kU_3 = 2P$$

$$U_3 = \frac{2P}{4k}$$

$$U_3 = P/2k$$

and

$$-2k[P/2k] + 4kU_4 = 0$$

$$4kU_4 = P$$

$$U_4 = P/4k$$

⑩ Reaction forces at node 1, 2, 5
from matrix

$$K \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 3 & -2 & 0 \\ 0 & -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ U_3 \\ U_5 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ 0 \\ 0 \\ R_5 \end{bmatrix}$$

So $R_1 = KU_1 + 0 - KU_3 + 0 + 0$

(10)

$$R_1 = \frac{-PK}{2k}$$

$$\boxed{R_1 = -P/2}$$

Now

$$R_2 = 0 + Ku_2 + 0 - u_4 k + 0$$

$$R_2 = \frac{-Pk}{4k}$$

$$\boxed{R_2 = -P/4}$$

and

$$R_5 = 0 + 0 + 0 - ku_4 + u_5 k.$$

$$R_5 = \frac{-Pk}{4k}$$

$$\boxed{R_5 = -P/4}$$

④ Force in spring 2, 4, 5

As force in spring is $F = kx$,
So here

$$F_{K2} = \frac{(u_3^{u4} - u_4)}{k} = (P/2k - P/4k)k$$

$$\boxed{F_{K2} = P/4}$$

①

$$F_{k4} = (u_4 - u_L) k = (P/4k - 0)$$

$$F_{k4} = P/4k$$

And

$$F_{k5} = (\underbrace{u_4 - u_5}_{u_5 - u_4}) k$$

$$F_{k5} = (P/4k - 0)$$

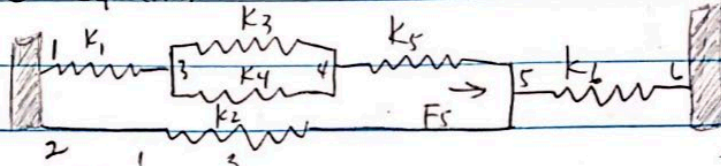
$$F_{k5} = P/4k$$

Ans.

BELOW AVERAGE

MCEG 4093 HW Chap. 1

1.4 Find the global stiffness matrix for the entire system.



$$K_1 = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$K_4 = k_4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 3 \\ 4 \end{matrix} \begin{matrix} 3 \\ 4 \end{matrix}$$

$$K_2 = k_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 2 \\ 3 \end{matrix} \begin{matrix} 2 \\ 3 \end{matrix}$$

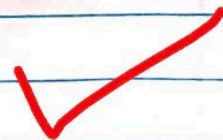
$$K_5 = k_5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 4 \\ 5 \end{matrix} \begin{matrix} 4 \\ 5 \end{matrix}$$

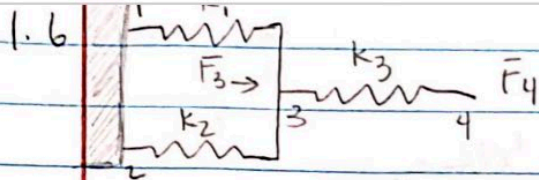
$$K_6 = k_6 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 5 \\ 6 \end{matrix} \begin{matrix} 5 \\ 6 \end{matrix}$$

$$K_3 = k_3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 3 \\ 4 \end{matrix} \begin{matrix} 3 \\ 4 \end{matrix}$$

Variables = 6 nodes

$$K = \begin{matrix} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \begin{matrix} k \end{matrix} & \begin{bmatrix} k_1 & k_2 & -k_1 & 0 & -k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -k_1 & 0 & k_1+k_3+k_4 & -k_3-k_4 & 0 & 0 \\ 0 & 0 & -k_3-k_4 & k_3+k_4+k_5 & -k_5 & 0 \\ 0 & -k_2 & 0 & -k_5 & k_2+k_5 & -k_6 \\ 0 & 0 & 0 & 0 & -k_6 & k_6 \end{bmatrix} \end{matrix}$$





$$k_1 = 80 \text{ N/mm}, k_2 = 100 \text{ N/mm}, k_3 = 160 \text{ N/mm}, F_3 = 200 \text{ N}, F_4 = 100 \text{ N}$$

a.) Global Stiffness Matrix

$$K_1 = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \quad K_3 = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

$$K = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & -k_2 & 0 \\ -k_1 & -k_2 & k_1 + k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix}$$

$$= \begin{bmatrix} 80 & 0 & -80 & 0 \\ 0 & 100 & -100 & 0 \\ -80 & -100 & 340 & -160 \\ 0 & 0 & -160 & 160 \end{bmatrix}$$



b.) Displacement of Nodes 3 & 4

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$$u_1 = u_2 = 0$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$F_3 = 200 \\ F_4 = 100$$

Wrong matrix multiplication.

$$b) (cont.) \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ -80 & -100 & 340 & -160 \\ 0 & 0 & -160 & 160 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

$$-80(0) - 100(0) + 340u_3 - 160u_4 = 200$$

$$-110u_3 + 160u_4 = 100$$

$$170u_3 = 300$$

$$u_3 = \frac{5}{3}$$

$$-110\left(\frac{5}{3}\right) + 160u_4 = 100$$

$$u_4 = \frac{55}{26} \text{ mm}$$

c.) Reaction forces at nodes 1, 2

$$\begin{bmatrix} 80 & 0 & -8 & 0 \\ 0 & 100 & -100 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{3} \\ \frac{55}{26} \end{bmatrix} = \begin{bmatrix} 200 \\ 100 \end{bmatrix}$$

$$F_1 = 80(0) + 0\left(-80\frac{5}{3}\right) + 0\left(\frac{55}{26}\right) = 133.3 \text{ N}$$

$$F_2 = 0 + 100(0) - 100\left(\frac{5}{3}\right) + 0 = -166.67 \text{ N}$$

d.) Forces in springs 1 & 2

$$(u_3 - u_1)k_1 = F_{k1}$$

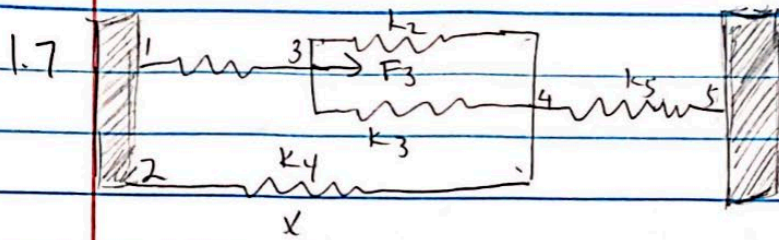
$$= \left(\frac{5}{3} - 0\right)(80)$$

$$(F_{k1} = 133.3 \text{ N})$$

$$(u_3 - u_2)k_2 = F_{k2}$$

$$= 100\left(\frac{5}{3}\right) - (0)$$

$$= 166.67 \text{ N}$$



$k_1 = k_2 = k_3 = k_4 = k_5 = k$, $F_3 = P$, nodes 1, 2 & 5 are fixed

a.) Global Stiffness Matrix

$$k_1 = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad k_4 = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad k_5 = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_3 = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 3 & -2 & 0 \\ 0 & -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \\ 0 \\ 0 \end{bmatrix}$$

b.) Displacements Nodes 3 & 4

$$[K][u] = [F]$$

$$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix}$$

$$\begin{aligned} 3ku_3 - 2ku_4 &= P \\ -2ku_3 + 4ku_4 &= 0 \end{aligned} \Rightarrow \begin{aligned} u_3 &= \frac{P}{2k} \\ u_4 &= \frac{P}{4k} \end{aligned}$$

$$c.) \begin{bmatrix} R_1 \\ R_2 \\ 0 \\ 0 \\ R_5 \end{bmatrix} = k \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 3 & -2 & 0 \\ 0 & -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \\ 0 \end{bmatrix}$$

$$R_1 = -k \frac{P}{2k} = \left[-\frac{P}{2} \right]$$

$$R_2 = -k \frac{P}{4k} = \left[-\frac{P}{4} \right]$$

$$R_5 = \left[-\frac{P}{4} \right]$$



d.) Forces in 2, 4, 2 & 5

$$2.) \quad k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} \quad F_2 = \begin{bmatrix} P/4 \\ P/4 \end{bmatrix}$$

$$= k \begin{bmatrix} u_3 & -u_4 \\ -u_3 & u_4 \end{bmatrix}$$

$$4.) = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_4 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{P}{4k} \end{bmatrix}$$

$$F_4 = \begin{bmatrix} -P/4 \\ P/4 \end{bmatrix}$$

$$5.) = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_4 \\ u_5 \end{bmatrix}$$

$$= k \begin{bmatrix} u_4 \\ u_5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} \frac{P}{4k} \\ 0 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} P/4 \\ -P/4 \end{bmatrix}$$

FINITE ELEMENT ANALYSIS AND DESIGN
Fall 2021

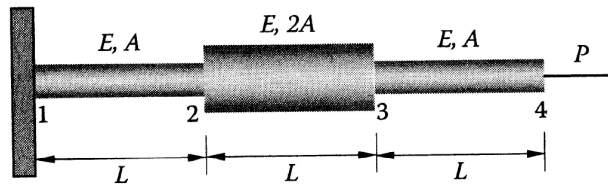
Title of Assignment: Homework 3

TOPICS COVERED IN HOMEWORK 3

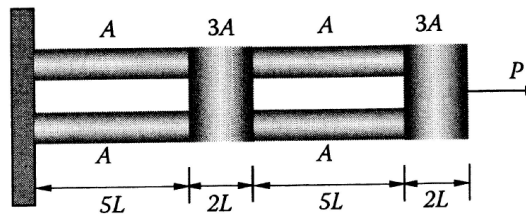
Use finite element method to solve bar assembly

PROBLEMS

- 2.1 A bar assembly is loaded with force P at one end and constrained at the other end, as shown in the figure below. Determine:
- The displacement at node 2 and node 3
 - The stress in the bar assembly



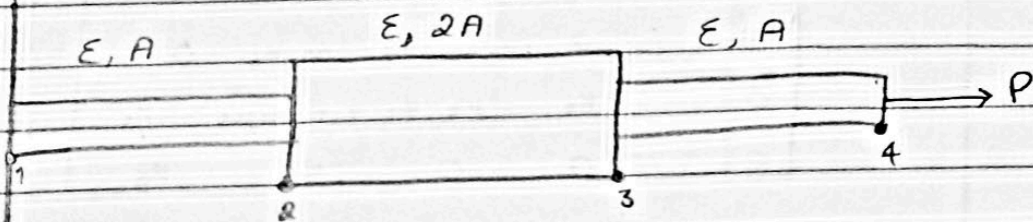
- 2.2 A simple structure with the profile shown below is loaded with force P , and constrained at one end. Young's modulus E is the same for all the bars. The cross-sectional areas are shown in the figure. Use 1-D bar elements to approximate the deformation and the stresses in the structure.



ABOVE AVERAGE

MCEG4093 - Finite Element Analysis and Design

Chapter 2 Homework Part 1



Determine:

- the displacement at node 2 and node 3
- the stress in the bar assembly

Element	Node (1)	Node (2)
K_1	1	2
K_2	2	3
K_3	3	4

$$\Rightarrow K_1 = EA/L = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix} \begin{matrix} u_1 \\ u_2 \end{matrix}, K_2 = 2EA/L = \begin{bmatrix} 2EA/L & -2EA/L \\ -2EA/L & 2EA/L \end{bmatrix} \begin{matrix} u_2 \\ u_3 \end{matrix}$$

$$K_3 = EA/L = \begin{bmatrix} EA/L & -EA/L \\ -EA/L & EA/L \end{bmatrix} \begin{matrix} u_3 \\ u_4 \end{matrix}$$

$$\Rightarrow K = EA/L \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$\Rightarrow$ load and boundary conditions are

$$u_1 = 0, F_4 = P$$

$$a) \Rightarrow EA/L \begin{bmatrix} & & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ P \end{Bmatrix}$$

$$\Rightarrow \begin{matrix} & & 3 & -2 & 0 \\ & EA/L & -2 & 3 & -1 \\ & & 0 & -1 & 1 \end{matrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \end{Bmatrix}$$

$$\Rightarrow 3(AE/L)u_2 - 2(AE/L)u_3 = 0 \quad \text{--- (1)}$$

$$-2(AE/L)u_2 + 3(AE/L)u_3 - u_4 = 0 \quad \text{--- (2)}$$

$$-(AE/L)u_3 + (AE/L)u_4 = 0 \quad \text{--- (3)}$$

$\Rightarrow$ From (1)

$$\Rightarrow 3u_2 - 2u_3 = 0 \Rightarrow 3u_2 = 2u_3 \Rightarrow u_2 = \frac{2}{3}u_3$$

$\Rightarrow$ From (2)

$$\Rightarrow 2u_2 + 3u_3 - u_4 = 0 \Rightarrow 2\left(\frac{2}{3}u_3\right) + 3u_3 - u_4 = 0$$

$$\Rightarrow \frac{4}{3}u_3 + 3u_3 - u_4 = 0 \Rightarrow \frac{4}{3}u_3 + \frac{9}{3}u_3 - u_4 = 0$$

$$\Rightarrow u_4 = \frac{5}{3}u_3$$

$\Rightarrow$ From (3)

$$\Rightarrow -u_3 + u_4 = 0 \Rightarrow -u_3 + \left(\frac{5}{3}\right)u_3 = PL/AE$$

$$\Rightarrow -\frac{3}{3}u_3 + \frac{5}{3}u_3 = PL/AE \Rightarrow \frac{2}{3}u_3 = PL/AE$$

$$\Rightarrow u_3 = \frac{3PL}{2AE}$$

$$\Rightarrow u_4 = \frac{5}{3}\left(\frac{3PL}{2AE}\right) \Rightarrow u_4 = \frac{5PL}{2AE}$$

$$\Rightarrow u_2 = \frac{2}{3}\left(\frac{3PL}{2AE}\right) \Rightarrow u_2 = \frac{PL}{AE}$$



b) Stress in element ①:

$$\sigma_1 = E/L(u_2 - u_1) = E/L \left(\frac{Px}{AE} - 0 \right) \Rightarrow \sigma_1 = \frac{P}{A}$$

Stress in element ②:

$$\sigma_2 = E/L(u_3 - u_2) = E/L \left(\frac{3Px}{2AE} - \frac{Px}{AE} \right) \Rightarrow \sigma_2 = \frac{P}{2A}$$

Stress in element ③:

$$\sigma_3 = E/L(u_4 - u_3) = E/L \left(\frac{5Px}{2AE} - \frac{3Px}{2AE} \right) \Rightarrow \sigma_3 = \frac{P}{A}$$

Problem 2-2

Element	Node (1)	Node (2)
①	1	2
②	1	2
③	2	3
④	3	4
⑤	3	4
⑥	4	5

$$K_1 = EA/5L \begin{bmatrix} u_1 & u_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} u_1, K_2 = EA/5L \begin{bmatrix} u_1 & u_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} u_1$$

$$K_3 = 3AE/2L \begin{bmatrix} u_2 & u_3 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} u_2, K_4 = EA/5L \begin{bmatrix} u_3 & u_4 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} u_3$$

$$a) K_5 = \frac{EA}{5L} \begin{bmatrix} & u_3 & u_4 \\ & 1 & -1 \\ & -1 & 1 \end{bmatrix} \begin{matrix} u_3 \\ u_4 \end{matrix}, K_6 = \frac{3AE}{2L} \begin{bmatrix} & u_4 & u_5 \\ & 1 & -1 \\ & -1 & 1 \end{bmatrix} \begin{matrix} u_4 \\ u_5 \end{matrix}$$

$$\Rightarrow K = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 \\ 2EA/5L & -2AE/5L & 0 & 0 & 0 \\ -2AE/5L & 19AE/10L & -3AE/2L & 0 & 0 \\ 0 & -3AE/2L & 19AE/10L & -2AE/5L & 0 \\ 0 & 0 & -2AE/5L & 19AE/10L & -3AE/2L \\ 0 & 0 & 0 & -3AE/2L & 3AE/2L \end{bmatrix}$$

From boundary conditions and load conditions:

$$u_1 = 0, F_5 = P$$

$$\Rightarrow \frac{EA}{L} \begin{bmatrix} 19/10 & -3/2 & 0 & 0 \\ -3/2 & 19/10 & -2/5 & 0 \\ 0 & -2/5 & 19/10 & -3/2 \\ 0 & 0 & -3/2 & 3/2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P \end{bmatrix}$$

Use MATLAB to solve would be easier.

$$\Rightarrow \frac{19}{10} \left(\frac{AE}{L} \right) u_2 - \frac{3}{2} \left(\frac{AE}{L} \right) u_3 = 0 \quad \text{--- (1)}$$

$$- \frac{3}{2} \left(\frac{AE}{L} \right) u_2 + \frac{19}{10} \left(\frac{AE}{L} \right) u_3 - \frac{2}{5} \left(\frac{AE}{L} \right) u_4 = 0 \quad \text{--- (2)}$$

$$- \frac{2}{5} \left(\frac{AE}{L} \right) u_3 + \frac{19}{10} \left(\frac{AE}{L} \right) u_4 - \frac{3}{2} \left(\frac{AE}{L} \right) u_5 = 0 \quad \text{--- (3)}$$

$$- \frac{3}{2} \left(\frac{AE}{L} \right) u_4 + \frac{3}{2} \left(\frac{AE}{L} \right) u_5 = P \quad \text{--- (4)}$$

$\Rightarrow$ From (1):

$$\frac{19}{10} u_2 - \frac{3}{2} u_3 = 0 \Rightarrow \frac{19}{10} u_2 = \frac{3}{2} u_3 \Rightarrow u_2 = \frac{15}{19} u_3$$

$\Rightarrow$ From (2):

$$- \frac{3}{2} u_2 + \frac{19}{10} u_3 - \frac{2}{5} u_4 = 0 \Rightarrow - \frac{3}{2} \left(\frac{15}{19} u_3 \right) + \frac{19}{10} u_3 - \frac{2}{5} u_4 = 0$$

$$\Rightarrow - \frac{45}{38} u_3 + \frac{19}{10} u_3 - \frac{2}{5} u_4 = 0 \Rightarrow \frac{293}{95} u_3 - \frac{2}{5} u_4 = 0$$

$$\Rightarrow \frac{293}{95} u_3 = \frac{2}{5} u_4 \Rightarrow u_3 = \frac{38}{293} u_4$$

From (3):

$$\Rightarrow -\frac{2}{5} u_3 + \frac{19}{10} u_4 - \frac{3}{2} u_5 = 0 \Rightarrow -\frac{2}{5} \left(\frac{38}{293} u_4 \right) + \frac{19}{10} u_4 - \frac{3}{2} u_5 = 0$$

$$\Rightarrow -\frac{76}{1465} u_4 + \frac{19}{10} u_4 = \frac{3}{2} u_5 \Rightarrow \frac{1083}{586} u_4 = \frac{3}{2} u_5$$

$$\Rightarrow u_4 = \frac{293}{361} u_5$$

From (4):

$$\Rightarrow -\frac{3}{2} u_4 + \frac{3}{2} u_5 = \frac{PL}{AE} \Rightarrow -\frac{3}{2} \left(\frac{293}{361} u_5 \right) + \frac{3}{2} u_5 = \frac{PL}{AE}$$

$$\Rightarrow \frac{102}{361} u_5 = \frac{PL}{AE} \Rightarrow u_5 = \frac{361 PL}{102 AE}$$

$$\Rightarrow u_4 = \frac{293}{361} \left(\frac{361 PL}{102 AE} \right) \Rightarrow u_4 = \frac{293 PL}{102 AE}$$

$$\Rightarrow u_3 = \frac{38}{293} \left(\frac{293 PL}{102 AE} \right) \Rightarrow u_3 = \frac{38 PL}{102 AE}$$

$$\Rightarrow u_2 = \frac{15}{19} \left(\frac{38 PL}{102 AE} \right) \Rightarrow u_2 = \frac{5 PL}{17 AE}$$

b) $\sigma_1 = \frac{E}{L} (u_2 - u_1) = \frac{E}{L} \left(\frac{5 PL}{17 AE} - 0 \right) \Rightarrow \sigma_1 = \frac{5P}{17A}$

$$\sigma_2 = \frac{E}{L} (u_2 - u_1) = \frac{E}{L} \left(\frac{5 PL}{17 AE} - 0 \right) \Rightarrow \sigma_2 = \frac{5P}{17A}$$

$$\sigma_3 = \frac{E}{L} (u_3 - u_2) = \frac{E}{L} \left(\frac{38 PL}{102 AE} - \frac{5 PL}{17 AE} \right) \Rightarrow \sigma_3 = \frac{4P}{51A}$$

$$\sigma_4 = \frac{E}{L} (u_4 - u_3) = \frac{E}{L} \left(\frac{293 PL}{102 AE} - \frac{38 PL}{102 AE} \right) \Rightarrow \sigma_4 = \frac{5P}{2A}$$

$$\sigma_5 = \frac{E}{L} (u_4 - u_3) \Rightarrow \sigma_4 = \frac{5P}{2A}$$

$$\sigma_6 = \frac{E}{L} (u_5 - u_4) = \frac{E}{L} \left(\frac{361 PL}{102 AE} - \frac{293 PL}{102 AE} \right) \Rightarrow \sigma_6 = \frac{2P}{3A}$$



Good efforts, however, answers are not correct.

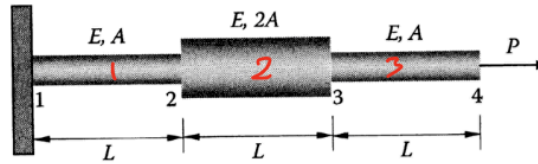
AVERAGE

2.1 A bar assembly is loaded with force P at one end and constrained at the other end, as shown in the figure below. Determine:

- The displacement at node 2 and node 3
- The stress in the bar assembly

$$F_4 = P$$

$$u_1 = 0$$



$$K_1 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad K_2 = \frac{EA}{L} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad K_3 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+2 & -2 & 0 \\ 0 & -2 & 2+1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{matrix}$$

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ F_2 \\ F_3 \\ P \end{bmatrix}$$

external forces of F_2 and F_3 are both equal to 0

$$\begin{array}{l} u_2 \quad u_3 \\ \begin{array}{l} \times 2 \\ \times 3 \end{array} \end{array} \quad \begin{array}{l} 3u_2 - 2u_3 = F_2 \\ -2u_2 + 3u_3 = F_3 \end{array}$$

$$\begin{array}{r} 6u_2 - 4u_3 = 2F_2 \\ + \quad -6u_2 + 9u_3 = 3F_3 \\ \hline \end{array}$$

$$5u_3 = 5F$$

$$u_3 = \frac{FL}{EA}$$

$$3u_2 - 2\frac{FL}{EA} = F$$

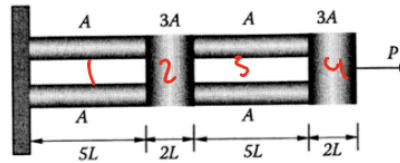
$$u_2 = \frac{F}{3} - \frac{2FL}{3EA}$$

$$\frac{EA}{L}(-u_3 + u_4) = P$$

$$\frac{EA}{L}\left(-\frac{FL}{EA} + u_4\right) = P$$

$$\frac{PL}{EA} + \frac{FL}{EA} = u_4$$

- 2.2 A simple structure with the profile shown below is loaded with force P , and constrained at one end. Young's modulus E is the same for all the bars. The cross-sectional areas are shown in the figure. Use 1-D bar elements to approximate the deformation and the stresses in the structure.



$$K_1 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad K_2 = \frac{EA}{2L} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad K_3 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_4 = \frac{EA}{2L} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \text{cross-sectional area are different.}$$

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 4 & -3 & 0 & 0 \\ 0 & -3 & 4 & -1 & 0 \\ 0 & 0 & -1 & 4 & -3 \\ 0 & 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} 0 \\ F_2 \\ F_3 \\ F_4 \\ P \end{bmatrix}$$

- ① $u_1 - u_2 = 0 \quad \therefore u_1 = u_2$
- ② $-u_1 - 4u_2 - 3u_3 = F_2 \quad \therefore -5u_2 - 3u_3 = F_2$
- ③ $-3u_2 + 4u_3 - u_4 = F_3 \quad \therefore -3u_2 - 3u_3 = F_3$
- ④ $-u_3 + 4u_4 - 3u_5 = F_4 \quad \therefore -u_3 + 4u_4 - 3u_5 = F_4$
- ⑤ $-3u_4 + 3u_5 = P$

$$u_2, u_3 \quad \begin{cases} -5u_2 - 3u_3 = F \\ x \rightarrow (-3u_2 - 3u_3 = F \end{cases}$$

$$\begin{array}{r} -5u_2 - 3u_3 = F \\ + \quad 3u_2 + 3u_3 = -F \\ \hline -2u_2 = 0 \end{array}$$

$$u_2 = 0$$

$$u_3 = \frac{FL}{3EA}$$

$$\therefore \frac{EA}{L} [-5(0) - 3u_3] = F$$

$$u_4, u_5$$

$$\begin{array}{r} -u_3 + 4u_4 - 3u_5 = F \\ + \quad -3u_4 + 3u_5 = F \end{array}$$

$$\frac{EA}{L} [-u_3 + u_4] = 2F$$

$$u_4 = \frac{2FL}{EA} - \frac{FL}{3EA}$$

$$u_5 = \left(\frac{2FL}{EA} - \frac{FL}{3EA} \right) + \frac{FL}{EA}$$

X

Missing stress calculation.

BELOW AVERAGE

2.1

Finite Ch 2 HW

Leonardo Herrera

P21621306

a)

$$K_1 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \end{matrix} \quad u_1 = 0$$

$$K_2 = \frac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_2 \\ u_3 \end{matrix} \quad F_2 = P$$

$$K_3 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_3 \\ u_4 \end{matrix}$$

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1+2 & -2 & 0 \\ 0 & -2 & 2+1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{matrix} = \begin{matrix} F_1 \\ F_2 \\ F_3 \\ P \end{matrix}$$

$$\frac{EA}{L} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{matrix} = \begin{matrix} F_1 \\ F_2 \\ F_3 \\ P \end{matrix}$$

$$\frac{EA}{L} (3u_2 - 2u_3) = F_2$$

$$\frac{EA}{L} (-u_2) = F_1$$

$$u_2 = -\frac{F_1 L}{EA}$$

$$\frac{EA}{L} (-u_3 + u_4) = P$$

$$u_3 = -\frac{PL}{EA} + u_4$$

$$\frac{EA}{L} (-2u_2 + 3u_3 - u_4) = F_3$$

$$\frac{EA}{L} (-u_3 + u_4) = P$$

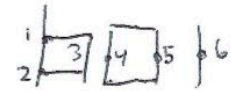
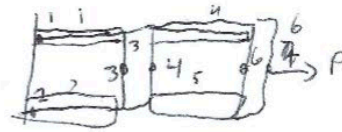
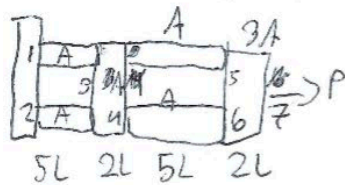
$$\frac{EA}{L} (-2u_2 + 2u_3) = F_3 + P$$

$$\begin{aligned}\sigma_1 &= E \epsilon_1 = E B_1 u_1 = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &= E \left(\frac{u_2 - u_1}{L} \right) = \frac{E}{L} \left(-\frac{F_1 L}{EA} \right) = -\frac{F_1}{A}\end{aligned}$$

$$\begin{aligned}\sigma_2 &= E \epsilon_2 = E B_2 u_2 = (E) \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \\ &= E \left(\frac{u_3 - u_2}{L} \right) = \frac{E}{L} \left(-\frac{PL}{EA} + u_4 + \frac{F_1 L}{EA} \right) \\ &= -\frac{P}{A} + \frac{E u_4}{L} + \frac{F_1}{A}\end{aligned}$$

$$\begin{aligned}\sigma_3 &= E B_3 u_3 = E \begin{bmatrix} -1/L & 1/L \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} \\ &= E \left(\frac{u_4 - u_3}{L} \right) = \frac{E}{L} \left(u_4 + \frac{PL}{EA} - u_4 \right) \\ &= \frac{P}{A}\end{aligned}$$

2.2



$$k_1 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_1 \\ u_3 \end{matrix}$$

$$k_4 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_4 \\ u_5 \end{matrix}$$

$$k_2 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_2 \\ u_3 \end{matrix}$$

$$k_5 = \frac{EA}{5L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_4 \\ u_5 \end{matrix}$$

$$k_3 = \frac{3EA}{2L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_3 \\ u_4 \end{matrix}$$

$$k_6 = \frac{3EA}{2L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} u_5 \\ u_6 \end{matrix} \quad u_1 = u_2 = 0$$

$$\frac{EA}{L} \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ 1/5 & 0 & -1/5 & 0 & 0 & 0 \\ 0 & 1/5 & -1/5 & 0 & 0 & 0 \\ -1/5 & -1/5 & 1/5+1/5 & -3/2 & 3/2 & 0 \\ 0 & 0 & -3/2 & 1/5+3/2+1/5 & -2/5 & 0 \\ 0 & 0 & 0 & -2/5 & 2/5+3/2 & -3/2 \\ 0 & 0 & 0 & 0 & -3/2 & 3/2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ P \end{bmatrix}$$

$$\sigma_1 = E \epsilon_1 = E \frac{u_3 - u_1}{L} = \frac{E}{L} (u_3) = \frac{E}{L} \left(-5 \frac{F_1 L}{EA} \right) = -\frac{5F_1}{A}$$

$$\sigma_2 = \sigma_1 = -\frac{5F_1}{A}$$

$$\sigma_3 = E \epsilon_3 = E \frac{u_4 - u_2}{L}$$

$$\sigma_4 = E \epsilon_4 = E \frac{u_5 - u_4}{L}$$

$$\sigma_5 = \sigma_4 = E \frac{u_5 - u_4}{L}$$

$$\sigma_6 = E \epsilon_6 = E \left(\frac{u_6 - u_5}{L} \right)$$

F1 is an unknown reactional force, cannot be used in the solution.

FINITE ELEMENT ANALYSIS AND DESIGN
Fall 2021

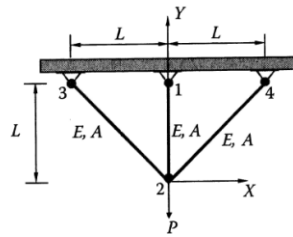
Title of Assignment: Homework 4

TOPICS COVERED IN HOMEWORK 4

1. Use finite element method to solve a plane truss problem.

(Homework)

- 2.3 The plane truss is loaded with force P as shown below. Constants E and A for each bar are as shown in the diagram. Determine:
- The nodal displacements
 - The reaction forces
 - The stresses in bar elements



ABOVE AVERAGE

(2.3)

Given Data

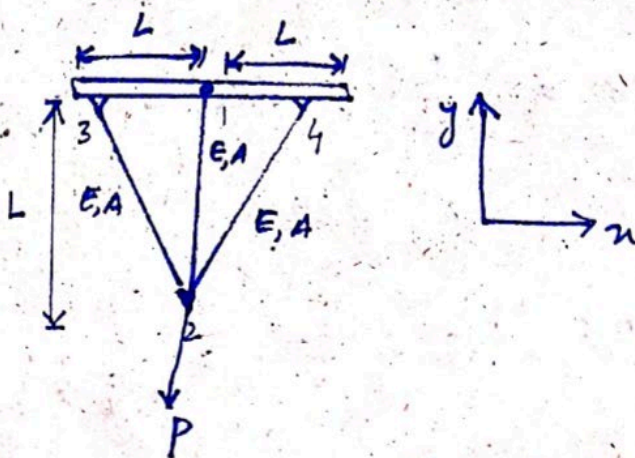
Force P

E, A for each bar

Required

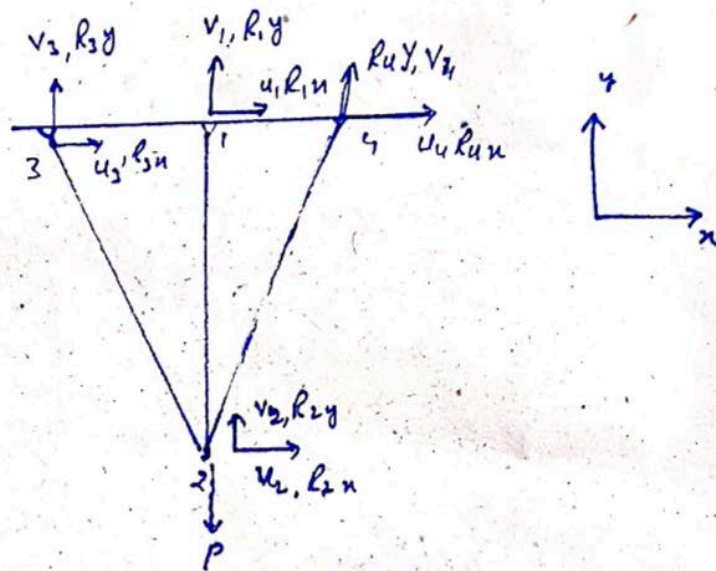
- (a) Nodal displacement
- (b) The reaction forces
- (c) Stresses in each bar

Solution



②

Now form free body diagram



②

Now to find nodal displacement, As there are three elements, so for element 1.

Define angle before writing cos and sin values.

$$\cos \theta = 0$$

$$\sin \theta = -1, \quad L = L$$

For Element 2

$$\cos \theta = \frac{1}{\sqrt{2}}, \quad \sin \theta = -\frac{1}{\sqrt{2}}, \quad L = \sqrt{2}L$$

For Element 3

$$\cos \theta = -\frac{1}{\sqrt{2}}, \quad \sin \theta = \frac{1}{\sqrt{2}}, \quad L = \sqrt{2}L$$

③

So

$$K_1 = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{matrix}$$

$$K_2 = \frac{AE}{\sqrt{2}L} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{matrix} u_3 \\ v_3 \\ u_2 \\ v_2 \end{matrix}$$

$$K_3 = \frac{AE}{\sqrt{2}L} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{matrix} u_4 \\ v_4 \\ u_2 \\ v_2 \end{matrix}$$



So Global matrix will become

$$[K] = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.707 & 0 & -0.35 & 0.35 & -0.35 & -0.35 \\ 0 & -1 & 0 & 1.707 & 0.35 & -0.35 & -0.35 & -0.35 \\ 0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 & 0 & 0 \\ 0 & 0 & 0.35 & -0.35 & -0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \end{bmatrix}$$

(4)

From boundary conditions,

1, 3, 4 are fixed, so
force will become.

$$F = Kx$$

$$\begin{bmatrix} R_{1x} \\ R_{1y} \\ 0 \\ -P \\ R_{3x} \\ R_{3y} \\ R_{4x} \\ R_{4y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & -0.35 & 0.35 & -0.35 & -0.35 \\ 0 & -1 & 0 & 1.7 & 0.35 & -0.35 & -0.35 & -0.35 \\ 0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 & 0 & 0 \\ 0 & 0 & 0.35 & -0.35 & -0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \end{bmatrix}$$

So

$$\left[\frac{AE}{L} \right] \left[\frac{\sqrt{2}+1}{\sqrt{2}} \right] (v_2) = -P.$$

$$v_2 = \frac{-PL}{AE(1.707)}$$

$$v_2 = \frac{-PL}{AE} (0.585230814)$$

And

$$\left(\frac{AE}{\sqrt{2}L} \right) (u_2) = 0$$

$$u_2 = 0$$

⑤

⑥ Now to find reaction forces

⇒ At node - 1.

$$\begin{bmatrix} R_{1x} \\ R_{1y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.58p \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So

$$R_{1x} = 0$$

$$R_{1y} = 0.58p$$

⇒ At node - 3

$$\begin{bmatrix} R_{3x} \\ R_{3y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.35 & 0.35 & 0.35 & -0.35 & 0 & 0 \\ 0 & 0 & 0.35 & -0.35 & -0.35 & 0.35 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.58p \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So

$$R_{3x} = (-0.58p)(0.35)$$

$$R_{3x} = -0.203p$$

and

$$R_{3y} = 0.203p$$



⇒ Reaction forces at node - 4.

$$\begin{bmatrix} R_{ux} \\ R_{uy} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.58P \\ 0 \end{bmatrix}$$

$$R_{ux} = -(0.35)(-0.58P)$$

$$\boxed{R_{ux} = 0.203 P}$$

and

$$R_{uy} = (-0.35)(-0.58P)$$

$$\boxed{R_{uy} = 0.203 P}$$

② Now finding stresses in each element.

⇒ Element - 01.

$$\sigma_1 = \frac{K}{AE} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.58P \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \frac{E}{L}$$

$$\sigma_1 = -(0.58P)(-1)/A$$

$$\boxed{\sigma_1 = \frac{0.58P}{A}}$$

⑦

⇒ Element - 02

$$\delta_2 = \frac{L}{EA} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.58P \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \frac{E}{\sqrt{2}L}$$

$$\delta_2 = \frac{(-0.58P)(-\frac{1}{\sqrt{2}})}{\sqrt{2}A} = \frac{(-0.58P)(-1)}{(\sqrt{2} \times \sqrt{2})A}$$

$$\delta_2 = \frac{0.58P}{2A}$$

⇒ Element - 03

$$\delta_3 = \frac{L}{EA} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.58P \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \frac{E}{\sqrt{2}L}$$

$$\delta_3 = \frac{(-0.58P)(-1)}{(\sqrt{2} \times \sqrt{2})A}$$

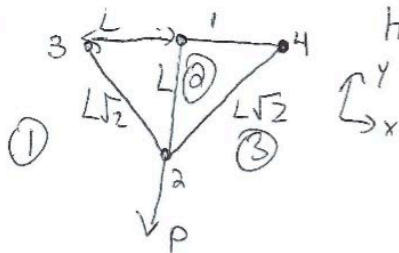
$$\delta_3 = \frac{0.58P}{2A}$$

Good work except the matrix in the wrong order.

Ans.

AVERAGE

2.3



HW Chapter 2 part 2

Leonardo Herrera
P2162801

$$K = \frac{EA}{L} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element 1

$$\theta_1 = 135^\circ \quad l = -\frac{\sqrt{2}}{2}, \quad m = \frac{\sqrt{2}}{2}$$

$$K_1 = \frac{EA}{2L\sqrt{2}} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

Element 2

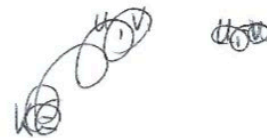
$$\theta_2 = 90^\circ \quad l = 0, \quad m = 1$$

$$K_2 = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Element 3

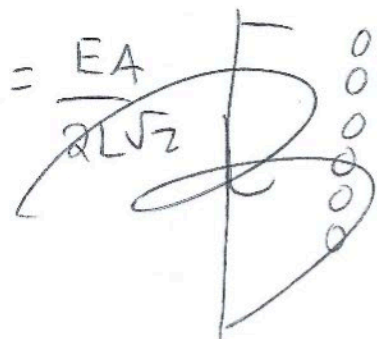
$$\theta = 45^\circ \quad l = \frac{\sqrt{2}}{2}, \quad m = \frac{\sqrt{2}}{2}$$

$$K_3 = \frac{EA}{2\sqrt{2}L} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$



$$u_1 = v_1 = u_3 = v_3 = u_4 = v_4 = 0 = u_2 = 0$$

from node 1 to 2, the angle is 270 degrees.

$$k = \frac{EA}{2L\sqrt{2}}$$


$$u_1 = v_1 = u_2 = u_3 = v_3 = u_4 = v_4 = 0$$

$$K = \frac{EA}{2L\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 & 2\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -1 & -1 \\ 0 & 2\sqrt{2} & 0 & 2+2\sqrt{2} & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\frac{EA}{2L\sqrt{2}} \begin{bmatrix} 0 \\ 2\sqrt{2} \\ 0 \\ 2+2\sqrt{2} \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ 0 \\ P \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix}$$

$$\frac{EA}{2L\sqrt{2}} (2+2\sqrt{2}) V_2 = P$$

$$V_2 = \frac{2\sqrt{2}}{EA(2+2\sqrt{2})} PL$$

$$F_{1x} = 0$$

$$F_{1y} = \frac{2\sqrt{2} \cdot \cancel{V_2} \cdot EA}{\cancel{2L\sqrt{2}}} = \frac{2\sqrt{2}}{EA(2+2\sqrt{2})} \cdot \frac{\cancel{EA}}{L} = \frac{2\sqrt{2}}{L(2+2\sqrt{2})}$$

$$F_{2x} = 0$$

$$F_{2y} = P$$

$$F_{3x} = V_2 \frac{EA}{2L\sqrt{2}} = \frac{2\sqrt{2}}{EA(2+2\sqrt{2})} \cdot \left(\frac{\cancel{EA}}{2\sqrt{2}} \right) = \frac{1}{L(2+2\sqrt{2})}$$

$$F_{3y} = -\frac{1}{L(2+2\sqrt{2})}$$

Need show more calculation process.

$$F_{4x} = -\frac{1}{L(2+2\sqrt{2})}$$

$$F_{4y} = -\frac{1}{L(2+2\sqrt{2})}$$

$$\text{Element 1 } \theta = 135^\circ$$

$$\sigma_1 = \frac{E}{L} \left[\frac{\sqrt{2}}{2} \frac{V_2}{2} + \frac{\sqrt{2}}{2} \frac{V_2}{2} \right] \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\sigma_1 = \frac{E}{L} \left[\frac{\sqrt{2}}{2} \right] (V_2) = \frac{E}{L} \cdot \frac{\sqrt{2}}{2} \cdot \frac{2\sqrt{2}}{EA(2+2\sqrt{2})} = \frac{2}{L(2+2\sqrt{2})} \quad \text{P/A}$$

$$\text{Element 2}$$

$$\sigma_2 = \frac{E}{L} (V_2 - V_1) = \frac{E}{L} \left(\frac{2\sqrt{2}}{EA(2+2\sqrt{2})} \right) = \frac{2\sqrt{2}}{L(2+2\sqrt{2})}$$

$$\text{Element 3}$$

$$\sigma_3 = \frac{E}{L} \left(\frac{\sqrt{2}}{2} \right) (V_2) = \frac{E}{L} \left(\frac{\sqrt{2}}{2} \right) \left(\frac{2\sqrt{2}}{EA(2+2\sqrt{2})} \right) = \frac{2}{L(2+2\sqrt{2})}$$

BELOW AVERAGE

2.3

$$\sin 45^\circ = \frac{L}{L_2 - 4}$$

$$L_2 - 4 = \sqrt{2}L$$

$$L_2 - 3 = \sqrt{2}L$$

$$[K^{(2-4)}] = \frac{AE}{L_2 - 4} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$$[K^{(2-4)}] = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

$$[K^{(2-4)}] = \frac{AE}{L_2 - 4} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

$$[K^{(2-4)}] = \frac{AE}{\sqrt{2}L} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

$$[K^{(2-1)}] = \frac{AE}{L_2-4} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$[K^{(2-4)}] = \frac{AE}{\sqrt{2}L} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$$

$$[K] = [K^{(2-1)}] + [K^{(2-4)}] + [K^{(2-4)}]$$

Global Stiff matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.7 & 0.35 & -0.35 & -0.35 & -0.35 \\ 0 & -1 & 0.7 & 0.7 & -0.35 & -0.35 & -0.35 & -0.35 \\ 0 & 0 & -0.35 & -0.35 & 0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix}$$

Disp. $[F] = [K]d$

$$\begin{bmatrix} F_{24} \\ F_{23} \\ 0 \\ P \\ F_{24} \\ F_{23} \\ F_{24} \\ F_{24} \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.70 & 0.35 & -0.35 & -0.35 & -0.35 \\ 0 & -1 & 0.7 & 0.70 & -0.35 & -0.35 & -0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0.35 & 0.35 & 0 & 0 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \\ 0 & 0 & -0.35 & -0.35 & 0 & 0 & 0.35 & 0.35 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ P \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 0.70 & 0.70 \\ 0.70 & 0.70 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix}$$

$$\frac{AE}{L} (0.70u_2 + 0.70v_2) = 0$$

$$\frac{AE}{L} (0.70u_2 + 0.70v_2) = P \quad \text{X}$$

Missing reaction forces and stresses.

**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

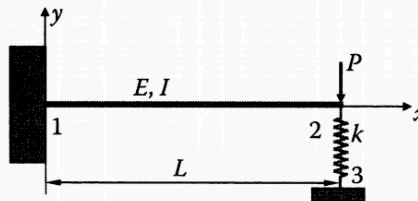
**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

Title of Assignment: Homework 5

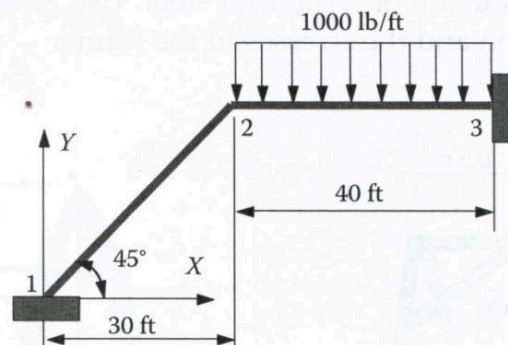
TOPICS COVERED IN HOMEWORK 5

2. Use finite element method to solve a structure with beam and spring.
3. Use finite element method to solve beam structure.

- 3.2 The cantilever beam is supported by a spring at the end as shown in the figure. Using FEM, determine the deflection and rotation at the node 2.

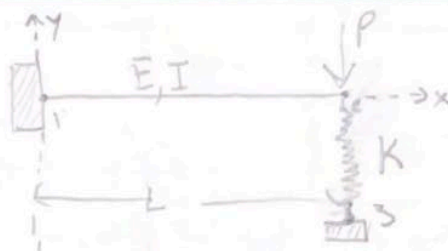


- 3.5 The plane frame is subjected to the uniformly distributed load and is fixed at the ends as shown in the figure. Assume $E = 30 \times 10^6$ psi, $A = 100$ in.², and $I = 1000$ in.⁴ for both elements of the frame. Using FEM, find the displacement and rotation of node 2.



ABOVE AVERAGE

3.2



3 Nodes
2 elements

horizontal = neglect
 $\therefore u = 0$

$$V_1 = 0$$

$$\theta_1 = 0$$

$$F_{2y} = -P$$

$$M_2 = 0$$

$$V_2$$

$$\theta_2$$

$$V_3 = 0$$

$$K_1 = \frac{EI}{L^3} \begin{bmatrix} V_1 & \theta_1 & V_2 & \theta_2 \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} V_3 \\ K_{33} \\ -K_{23} \end{bmatrix} \begin{bmatrix} V_3 \\ -K_{33} \\ K_{23} \end{bmatrix}$$

$$\frac{EI}{L^3} \begin{bmatrix} V_1 & \theta_1 & V_2 & \theta_2 & V_3 \\ 12 & 6L & -12 & 6L & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 \\ -12 & -6L & 12+K & -6L & -K \\ 6L & 2L^2 & -6L & 4L^2 & 0 \\ 0 & 0 & -K & 0 & K \end{bmatrix} \begin{bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \end{bmatrix}$$

$$\frac{EI}{L^3} \begin{bmatrix} 12+K & -6L \\ -6L & 4L^2 \end{bmatrix} \begin{bmatrix} V_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -P \\ 0 \end{bmatrix}$$

$$\begin{cases} (12+K)V_2 - (6L)\theta_2 = \frac{-PL^3}{EI} \\ -6L V_2 + 4L^2 \theta_2 = 0 \end{cases}$$

$$4L^2 \theta_2 = \frac{6L V_2}{4L^2} = \frac{3}{2L} V_2 \Rightarrow \theta_2 = \frac{3}{2L} V_2$$

$$(12+K)V_2 - 6L \left(\frac{3}{2L} V_2 \right) = \frac{-PL^3}{EI}$$

$$(12+K)V_2 - 9V_2 = -$$

3.2 cont.

$$(12+k)v_2 - 9v_2 = \frac{-PL^3}{EI}$$

$$12v_2 + kv_2 - 9v_2$$

$$3v_2 + kv_2 = \frac{-PL^3}{EI} \quad \therefore v_2 = \frac{-PL^3}{EI(3+k)}$$

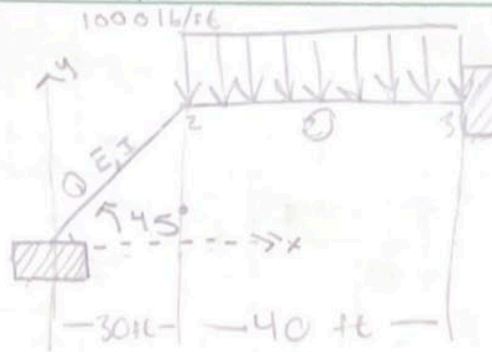
$$\theta_2 = \frac{3}{2L} v_2 \quad \therefore \frac{3}{2L} \frac{-PL^3}{EI(3+k)}$$

$$\theta_2 = \frac{-3PL^2}{2EI(3+k)}$$

ANS.

$$\begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \underline{-\frac{PL^2}{EI(3+k)}} \begin{bmatrix} L \\ 3/2 \end{bmatrix}$$

3.5



Given:

$$E = 30 \times 10^6 \text{ psi} \quad A = 100 \text{ in}^2 \quad I = 1000 \text{ in}^4$$

$$L_1 = \sqrt{L^2} = 30\sqrt{2} \text{ ft}$$

$$L_2 = 40 \text{ ft}$$

$$\begin{aligned} U_1 &= 0 \\ V_1 &= 0 \\ \theta_1 &= 0 \end{aligned}$$

$$\begin{aligned} U_2 &= 0 \\ V_2 &= 0 \\ \theta_2 &= 0 \end{aligned}$$

$$\begin{aligned} U_3 &= 0 \\ V_3 &= 0 \\ \theta_3 &= 0 \end{aligned}$$

$$F_{2y} = 1000 \frac{L_2}{2}$$

$$F_{3y} = 1000 \frac{L_2}{2} \quad F_{3x} = 0$$

$$M_2 = -\frac{1000 L_2^2}{12}$$

$$K_1 = \begin{bmatrix} \frac{EA}{L_1} & 0 & 0 & -\frac{EA}{L_1} & 0 & 0 \\ 0 & \frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} & 0 & -\frac{12EI}{L_1^3} & \frac{6EI}{L_1^2} \\ 0 & \frac{6EI}{L_1^2} & \frac{4EI}{L_1} & 0 & -\frac{6EI}{L_1^2} & \frac{2EI}{L_1} \\ -\frac{EA}{L_1} & 0 & 0 & \frac{EA}{L_1} & 0 & 0 \\ 0 & -\frac{12EI}{L_1^3} & -\frac{6EI}{L_1^2} & 0 & \frac{12EI}{L_1^3} & -\frac{6EI}{L_1^2} \\ 0 & \frac{6EI}{L_1^2} & \frac{2EI}{L_1} & 0 & -\frac{6EI}{L_1^2} & \frac{4EI}{L_1} \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ \theta_1 \\ U_2 \\ V_2 \\ \theta_2 \end{Bmatrix}$$



3.5 cond

$$K_i' = 10^6 \times$$

$$\begin{bmatrix} u_1 & v_1 & \theta_1 & u_2 & v_2 & \theta_2 \\ 70.71 & 0 & 0 & -70.71 & 0 & 0 \\ 0 & 4.71 & 100 & 0 & -4.71 & 100 \\ 0 & 100 & 2828.43 & 0 & -100 & 1414.21 \\ -70.71 & 0 & 0 & 70.71 & 0 & 0 \\ 0 & -4.71 & -100 & 0 & 4.71 & -100 \\ 0 & 100 & 1414.21 & 0 & -100 & 2828.43 \end{bmatrix}$$

Element 1 $\theta = 45^\circ \therefore L = \frac{\sqrt{2}}{2} \quad m = \frac{\sqrt{2}}{2}$

$$K = T^T K' T$$

$$T = \begin{bmatrix} L & m & 0 & 0 & 0 & 0 \\ -m & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & m & 0 \\ 0 & 0 & 0 & -m & L & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^T = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3.5 cont

$$K_1 = T^T K_1' T$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 70.71 & 0 & 0 & -70.71 & 0 & 0 \\ 0 & 4.71 & 100 & 0 & -4.71 & 100 \\ 0 & 100 & 2828.43 & 0 & -100 & 1414.21 \\ 70.71 & 0 & 0 & 70.71 & 0 & 0 \\ 0 & -4.71 & -100 & 0 & 4.71 & -100 \\ 0 & 100 & 1414.21 & 0 & -100 & 2828.43 \end{bmatrix} \times 10^6$$

$$\cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = K_1$$

$$K_1 = 10^6 \times \begin{bmatrix} U_1 & V_1 & \Theta_1 & U_2 & V_2 & \Theta_2 \\ u_1 & -457.26 & 33 & -70.711 & -37.71 & -70.711 \\ v_1 & 527.97 & 37.71 & 70.711 & -33 & -37.71 \\ \theta_1 & 19728.3 & 70.711 & 2828.43 & 70.711 & -70.71 \\ u_2 & 457.26 & -33 & 70.711 & 37.71 & 33 \\ v_2 & -528 & -37.71 & -70.711 & 33 & 37.71 \\ \theta_2 & 9828.76 & 70.71 & 1414.21 & 70.711 & -70.71 \end{bmatrix}$$

$$K_2 = 10^6 \begin{bmatrix} u_2 & v_2 & \Theta_2 & u_3 & v_3 & \Theta_3 \\ 100 & 0 & 0 & -100 & 0 & 0 \\ 0 & 13.33 & 200 & 0 & -13.33 & 200 \\ 0 & 200 & 4000 & 0 & -200 & 2000 \\ -100 & 0 & 0 & -100 & 0 & 0 \\ 0 & -13.33 & -200 & 0 & 13.33 & -200 \\ 0 & 200 & 2000 & 0 & -200 & 4000 \end{bmatrix}$$

Accidentally
Made
L=30

3.5 cont Global Stiffness

	U_1	V_1	θ_1	U_2	V_2	θ_2	U_3	V_3	θ_3		
	457.26	33	-70.711	-37.71	-33	-70.711	0	0	0	U_1	F_x
	527.97	37.71	70.711	-33	-37.71	70.711	0	0	0	V_1	F_{1y}
	1928.3	70.711	2928.43	70.711	-70.711	1414.21	0	0	0	θ_1	M_1
10^6	457.26	-33	70.711	137.71	33	70.711	100	0	0	U_2	F_{2x}
	-528	-37.71	-70.711	33	51.04	129.289	0	-13.33	200	V_2	F_{2y}
	9828.76	70.711	1414.21	70.711	129.289	6828.43	0	-200	2000	θ_2	M_2
	0	0	0	-100	0	0	100	0	0	U_3	F_{3x}
	0	0	0	0	-13.33	-200	0	13.33	-200	V_3	F_{3y}
	0	0	0	0	200	2000	0	-200	4000	θ_3	M_3

$$\begin{bmatrix} 137.71 & 33 & 70.711 \\ 33 & 51.04 & 129.289 \\ 70 & 129.289 & 6828.43 \end{bmatrix} \begin{bmatrix} U_2 \\ V_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ M_2 \end{bmatrix}$$

$$A \cdot X = b$$

$$F_x = 0$$

$$F_y = \frac{1000 \cdot 40}{2} = 20000 \text{ lb}$$

$$M_2 = \frac{-1000 \cdot 40^2}{12} = -\frac{400,000}{3} \text{ lb-ft}$$

$$X = A^{-1}b$$

3.5 cont

$$137.71 u_2 + 33 v_2 + 70.711 \theta_2 = 0$$

$$33 u_2 + 51.04 v_2 + 129.289 \theta_2 = 2000$$

$$70.711 u_2 + 129.289 v_2 + 6828.43 \theta_2 = \frac{400000}{3}$$

$$X = \begin{bmatrix} u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A^{-1} \\ 137.71 & 33 & 70.711 \\ 33 & 51.04 & 129.289 \\ 70.711 & 129.289 & 6828.43 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2000 \\ -\frac{400000}{3} \end{bmatrix}$$

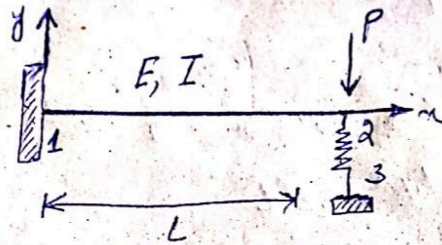
$$\therefore \begin{bmatrix} u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -13.47 \\ 101.889 \\ -21.31 \end{bmatrix}$$

process is correct, but the answers are incorrect.

AVERAGE

3.2

Given



Required

Deflection and Rotation at node 02.

Solution

We know the stiffness matrix for beam element as

$$K = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^3 & -6L & 2L^2 \\ -12 & -6 & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \rightarrow \textcircled{A}$$

And for spring element

$$K' = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \rightarrow \textcircled{B}$$

Combining eqn (A) and (B) to form global stiffness matrix.

$$\begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \end{bmatrix} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 \\ -12L & -6L & (12+k) & -6L & -k \\ 6L & 2L^2 & -6L & 4L^2 & 0 \\ 0 & 0 & -k & 0 & k \end{bmatrix} \frac{EI}{L^3} = \begin{bmatrix} f_1(1) \\ m_1(1) \\ f_2(1) + f_2(2) \\ m_2(1) \\ f_3(2) \end{bmatrix}$$

Now apply BCs which are

$$v_1 = 0, \theta_1 = 0, v_2 = 0$$

$$v_3 = 0$$

So

$$\begin{bmatrix} 0 \\ 0 \\ v_2 \\ \theta_2 \\ 0 \end{bmatrix} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 \\ -12L & -6L & (12+k) & -6L & -k \\ 6L & 2L^2 & -6L & 4L^2 & 0 \\ 0 & 0 & -k & 0 & k \end{bmatrix} \frac{EI}{L^3} = \begin{bmatrix} f_1(1) \\ m_1(1) \\ f_2(1) + f_2(2) \\ m_2(1) \\ f_3(2) \end{bmatrix}$$

Therefore

$$\begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} \begin{bmatrix} (12+k) & -6L \\ -6L & 4L^2 \end{bmatrix} \frac{EI}{L^3} = \begin{bmatrix} f_2(1) + f_2(2) \\ m_2(1) \end{bmatrix}$$

$$\text{Hence } f_2(1) + f_2(2) = -P$$

$$m_2(1) = 0$$

③

$$v_2 \frac{EI}{L^3} (12+k) - 6L \left(\frac{EI}{L^3} \right) \theta_2 = -P \rightarrow \textcircled{A}$$

$$-6L \left(\frac{EI}{L^3} \right) v_2 + 4L^2 \left(\frac{EI}{L^3} \right) \theta_2 = 0 \rightarrow \textcircled{B}$$

From eqn $\textcircled{B}$.

$$-6L \left(\frac{EI}{L^3} \right) v_2 + 4L^2 \left(\frac{EI}{L^3} \right) \theta_2 = 0$$

$$-6L(v_2) + 4L^2 \theta_2 = 0$$

$$v_2 = \frac{4L^2 \theta_2}{6L}$$

$$v_2 = \frac{2}{3} (L \theta_2)$$

From eqn $\textcircled{A}$

$$\frac{EI}{L^3} \left[v_2 (12+k) - 6L \theta_2 \right] = -P$$

$$\text{put } \frac{EI}{L^3} = \kappa$$

$$v_2 (12+k) - 6L \theta_2 = -P/\kappa$$

$$(\theta_2) \left(\frac{2}{3} \right) (12+k) - 6L \theta_2 = -P/\kappa$$

$$L \theta_2 \left[\left(\frac{2}{3} (12+k) - 6 \right) \right] = \frac{-P}{\kappa}$$

$$L \theta_2 = \frac{-P/\kappa \left(\frac{2}{3} (12+k) - 6 \right)}{\left(\frac{2}{3} (12+k) - 6 \right)}$$

$$\theta_2 = \frac{-P}{L \kappa \left[\frac{2}{3} (12+k) - 6 \right]}$$

incorrect.

④

and

$$V_2 = \frac{2}{3} E \left(\frac{-P}{K \pi (2/3(12+K) - 6)} \right)$$

$$V_2 = \frac{2}{3} (-P) / \frac{2\pi}{3} (12+K) - 6\pi$$



Ans.

③.5

Given Data

Fixed loads at ends
uniformly distributed load

$$E = 30 \times 10^6 \text{ psi}$$

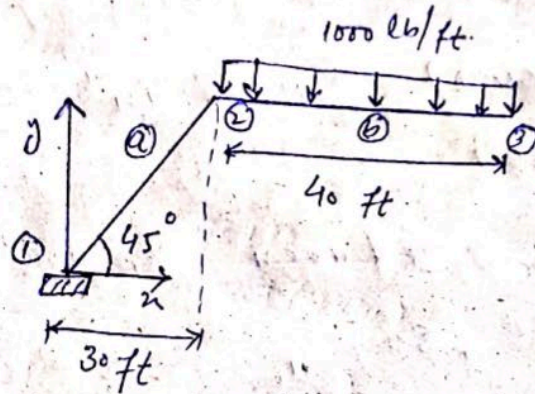
$$I = 1000 \text{ in}^4$$

Required

Displacement and rotation
at node - 2

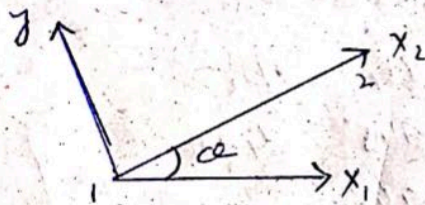
⑤

Solution.



Now from FBD for ② & ③

For ②



$$\therefore \alpha = 360^\circ$$

$$L = \sqrt{(360)^2 + (360)^2}$$

$$L = 509 \text{ in}$$

$$As \quad S = C$$

So

$$360 / 509 = 0.70$$

$$\frac{E}{L} = \frac{30 \times 10^6}{509} = 58.94 \text{ k/in}^2$$



⑥

missing coordinate transformation.

$$\frac{12I}{L^3} = \frac{12(1000)}{(12 \times 30\sqrt{2})^2} = 0.046 \text{ in}^2$$

$$\frac{6I}{L} = \frac{6(1000)}{509} = 11.78 \text{ in}^3$$

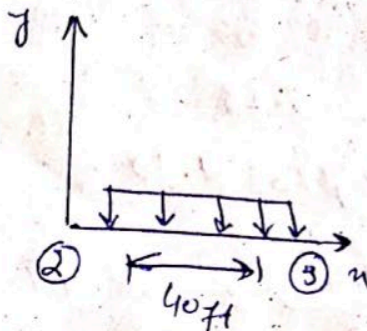
As angle between X_1 and X_2 is 45° , so

$$K^{(1)} = 58.93 \begin{bmatrix} u_2 & v_2 & \phi_2 \\ 50.02 & 49.98 & 8.33 \\ 49.98 & 50.02 & -8.33 \\ 8.33 & -8.33 & 4000 \end{bmatrix}$$

should be 6x6 matrix

$$K^{(1)} = \begin{bmatrix} u_2 & v_2 & \phi_2 \\ 2948 & 2945 & 491 \\ 2945 & 2948 & -491 \\ 491 & -491 & 235700 \end{bmatrix}$$

Now FBD for element ⑥



⑦

As in element ⑥ angle between x_1 and x_2 is zero, so

$$s = 0, \quad c = 1$$

$$\frac{E}{L} = \frac{30 \times 10^6}{(12 \times 40)} = 52.5 \text{ k/in}$$

$$\frac{12I}{L^2} = \frac{12(1000)}{(12 \times 40)^2} = 0.052 \text{ in}^2$$

$$\frac{6I}{L} = \frac{6(1000)}{12 \times 40} = 12.52 \text{ in}^3$$

So

$$K = 62.50 \begin{bmatrix} u_2 & v_2 & \phi_2 \\ 100 & 0 & 0 \\ 0 & 0.052 & 12.52 \\ 0 & 12.52 & 4,000 \end{bmatrix}$$

should be 6x6 matrix

$$K = \begin{bmatrix} u_2 & v_2 & \phi_2 \\ 6250 & 0 & 0 \\ 0 & 3.26 & 781.24 \\ 0 & 781.23 & 250000 \end{bmatrix}$$

⑧

So Global beam matrix is

$$\begin{bmatrix} u_L \\ v_L \\ \phi_L \end{bmatrix} \begin{bmatrix} 9198 & 2945 & 491 \\ 2945 & 2951 & 290 \\ 491 & 290 & 485,700 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \text{ k} \\ -1600 \text{ K.in} \end{bmatrix}$$

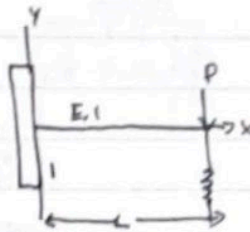
upon solving system of equation

$$\begin{aligned} u_L &= 0.003 \text{ in} \\ v_L &= -0.0098 \text{ in} \\ \phi_L &= -0.003345 \text{ rad} \end{aligned}$$

Ans.

BELOW AVERAGE

Problem 1: 3.2.)



Determine deflection + rotation @ the node 2

$$[K]^{(1)} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad [K]^{(2)} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix} k' & -k' \\ -k' & k' \end{bmatrix}$$

where $k' = \frac{L^3}{EI} \cdot k$

$$[K_{eq}] = \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+k' & -6L & 0 & 0 \\ 6L & 2L^2 & -6L & 4L^2 & -k & 0 \\ 0 & 0 & 0 & 0 & k' & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

~~A3~~ $[F] = [K][x]$

$$\begin{bmatrix} F_y \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{bmatrix} = [K_{eq}] \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix} \quad \text{--- (1)}$$

$$v_1 = \theta_1 = v_3 = \theta_3 = 0 \quad ; \quad F_y = M_1 = M_2 = F_{3y} = M_3 = 0$$

$$F_{2y} = -P$$

$$\begin{Bmatrix} -P \\ 0 \end{Bmatrix} = \begin{bmatrix} 12+k' & -6L \\ -6L & 4L^2 \end{bmatrix} \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix}$$

$$-P = (12+k')v_2 - 6L\theta_2$$

$$0 = -6Lv_2 + 4L^2\theta_2$$

$$\boxed{v_2 = \frac{2L}{3}\theta_2}$$

$$-P = (12+k') \frac{2L}{3} \theta_1 - 6L\theta_2$$

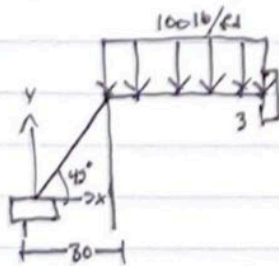
$$\theta_2 = \frac{-3P}{2L(3+k')}$$

The answers can not contain k' .

$$V_2 = \frac{-P}{(3+k')}$$

missing most calculations.

Problem 2: 3.5.)



Find displacement and rotation of node 2

$$E = 30 \times 10^6 \text{ kpsi} \quad A = 100 \text{ in}^2 \quad I = 1000 \text{ in}^4$$

$$= 2.068 \times 10^{11} \text{ N/m}^2 \quad = 0.645 \text{ m}^2 \quad = 0.00041628 \text{ m}^4$$

Deflection @ Node 2

$$= \frac{W_1 b_1^3}{12 L_1} (4L_1 - 3b_1) + \frac{W_2 b_2^3}{12 L_2} (4L_2 - 3b_2)$$

$$W_1 = 1000 \text{ lb/ft} \quad L_1 = 40 \times 30.48 \quad L_2 = 30 \times \cos 45 \times 30.48$$

$$= 1488 \text{ kg/m} \quad = 1219.2 \text{ cm} \quad = 64.6 \text{ m}$$

$$= 12.19 \text{ m}$$

$$b_1 = A_1 / I_1 = 0.645 / 12.19 \quad b_2 = A / I_2 = 0.645 / 6.46$$

$$= 0.0529 \text{ m} \quad = 0.099 \text{ m}$$

$$f = \frac{1488 \times 0.0529^3}{12 \times 12.19^2} (4 \times 12.19 - 3 \times 0.0529) + \frac{1488 \times 0.099^3}{12 \times 6.46^2} (4 \times 6.46 - 3 \times 0.099)$$

$$= 0.079 \text{ m}$$

Bending Moment Equation

$$\frac{H}{I} = \frac{C}{Y} = \frac{F}{R} \Rightarrow M = \frac{E I}{L} = \frac{2.068 \times 10^{11} \times 0.0004163}{6.46} = 13317152$$

$$5x M_{12} = \frac{2EI}{L} \left[\theta_B - \frac{3\delta}{1} \right]$$

$$1331715L = \frac{2 \times 2.06 \times 10^4 \times 0.00041623}{12.19}$$

$$\left[\theta_B = - \frac{3 \times 0.079}{12.19} \right]$$

$$0.947 - \theta_B = \frac{3 \times 0.079}{12.19}$$

$$\boxed{\theta_B = 0.946}$$

X

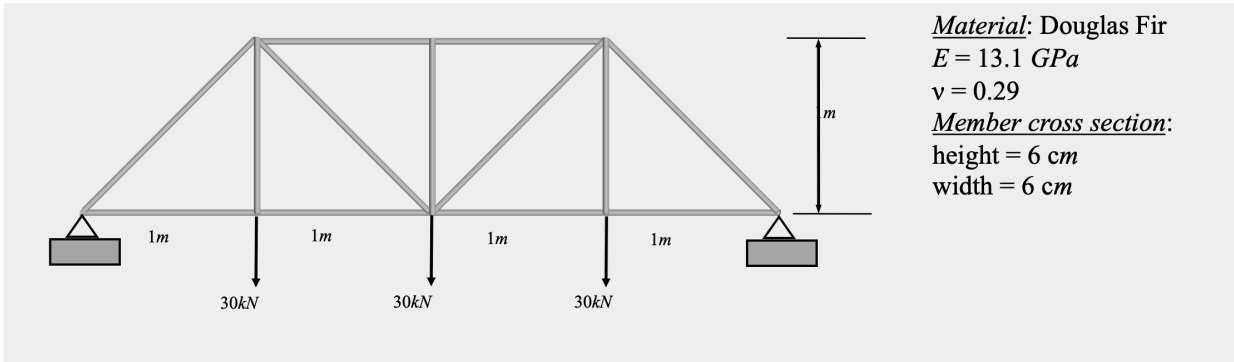
**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

Title of Assignment: Homework 6

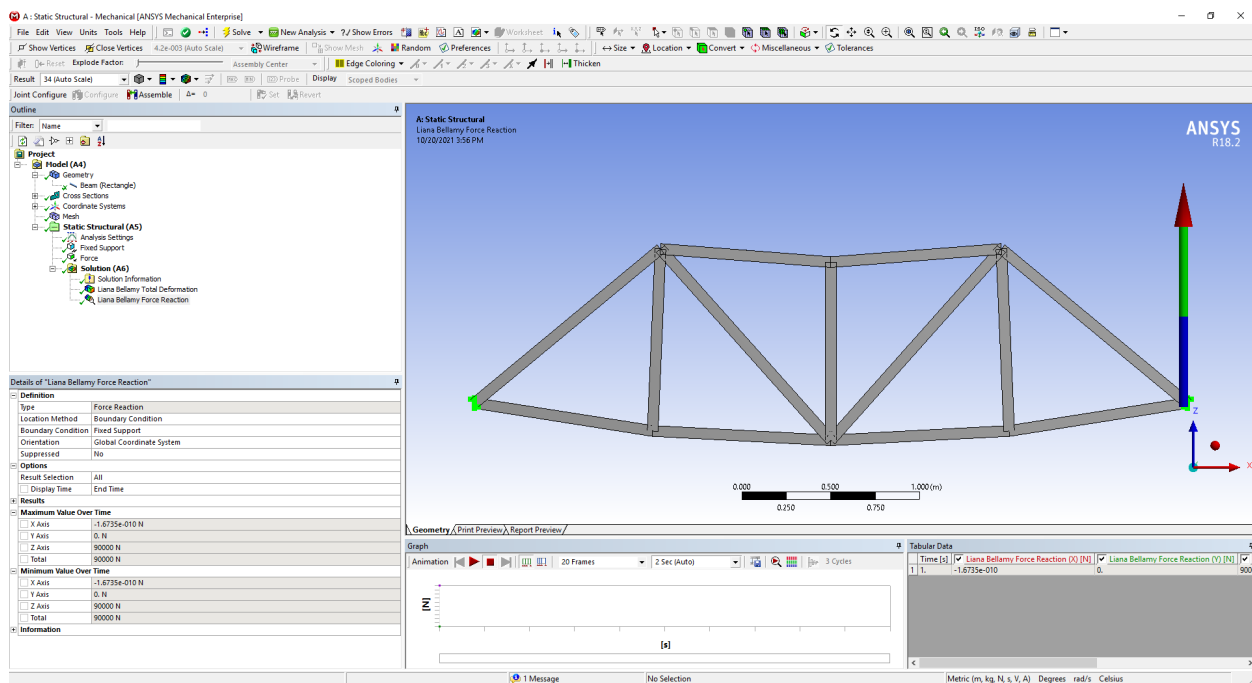
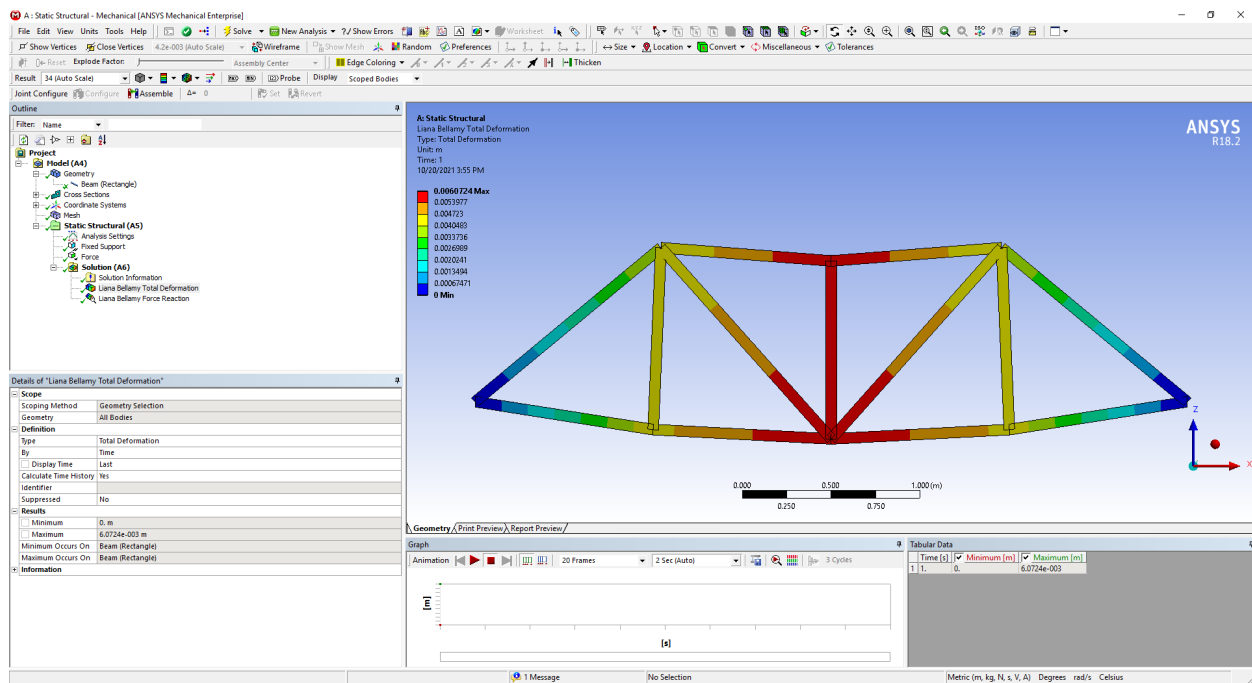
TOPICS COVERED IN HOMEWORK 6

Use static structural program in ANSYS to analyze deformation and reactional force in a truss bridge.

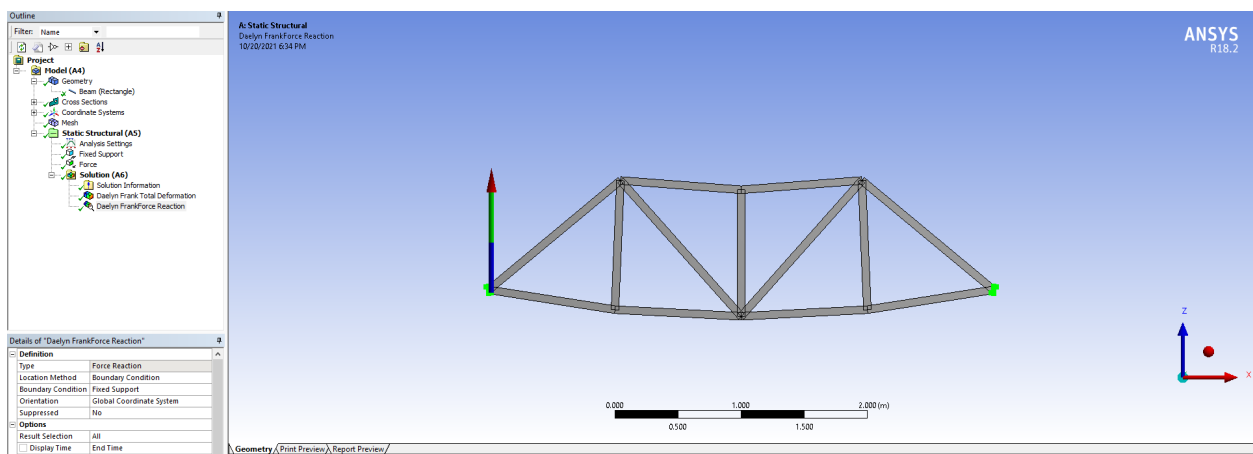
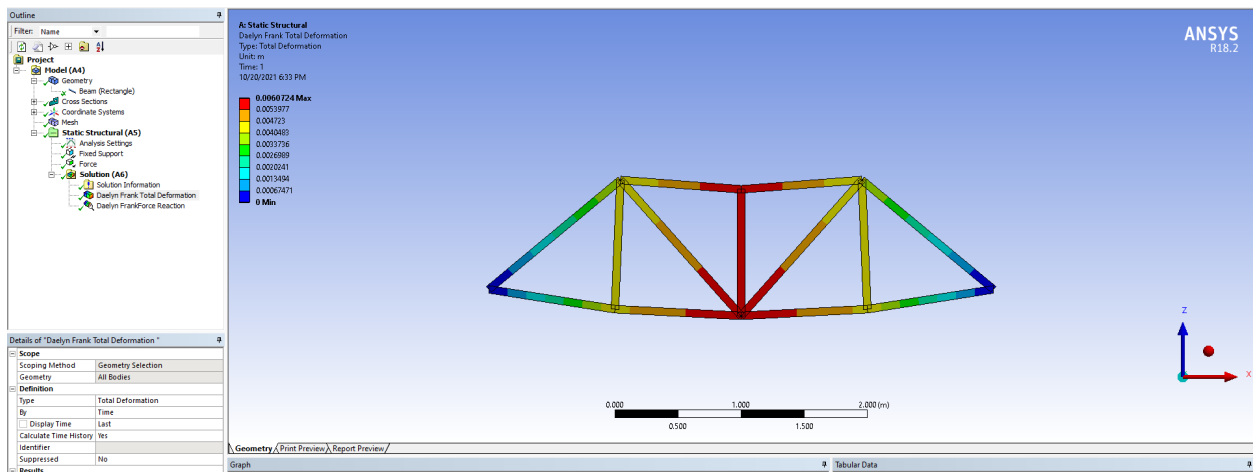
<Problem Description> Truss bridges can span long distances and support heavy weights without intermediate supports. They are economical to construct and are available in a wide variety of styles. Consider the following planar truss, constructed of wooden timbers, which can be used in parallel to form bridges. Determine the deflections at each joint of the truss under the given loading conditions.



ABOVE AVERAGE

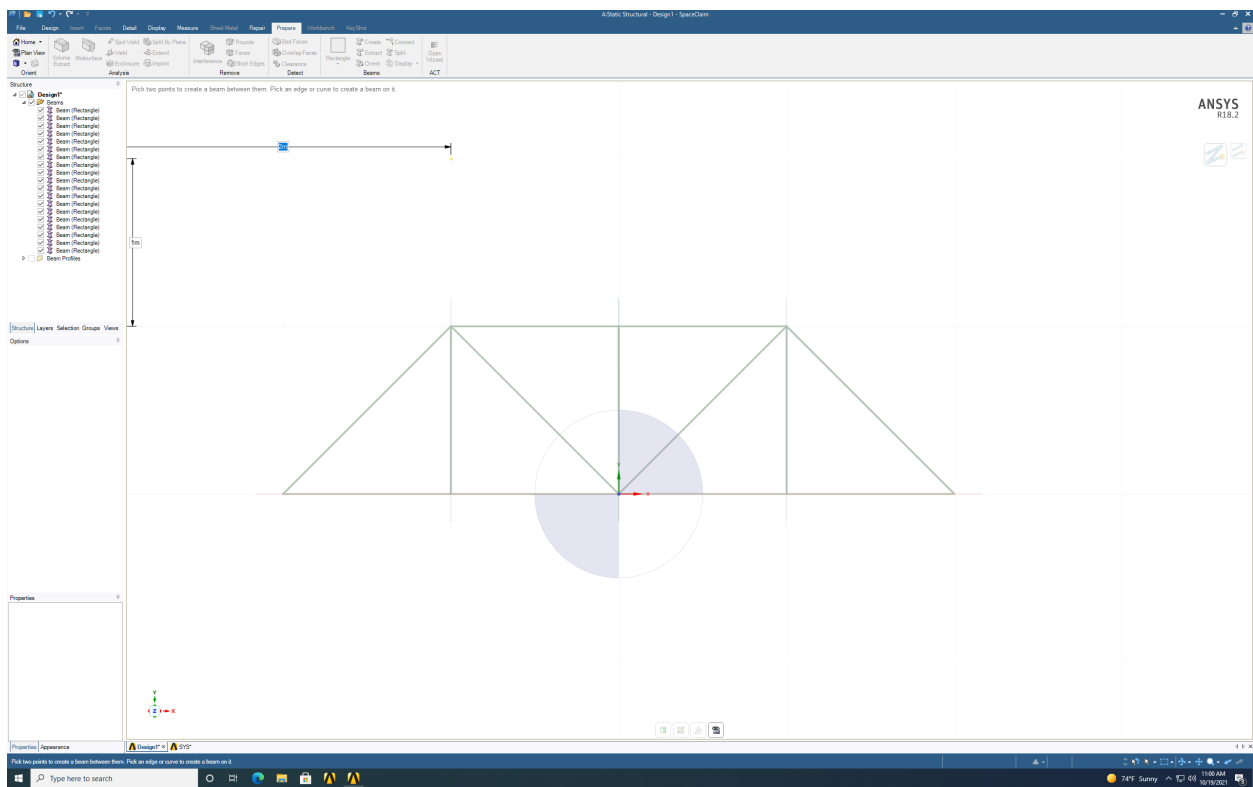


AVERAGE



The magnitude of force reaction is not shown in the solution.

BELOW AVERAGE



Built the structure; however, failed to define boundary conditions, external loads, and complete the analysis.

**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

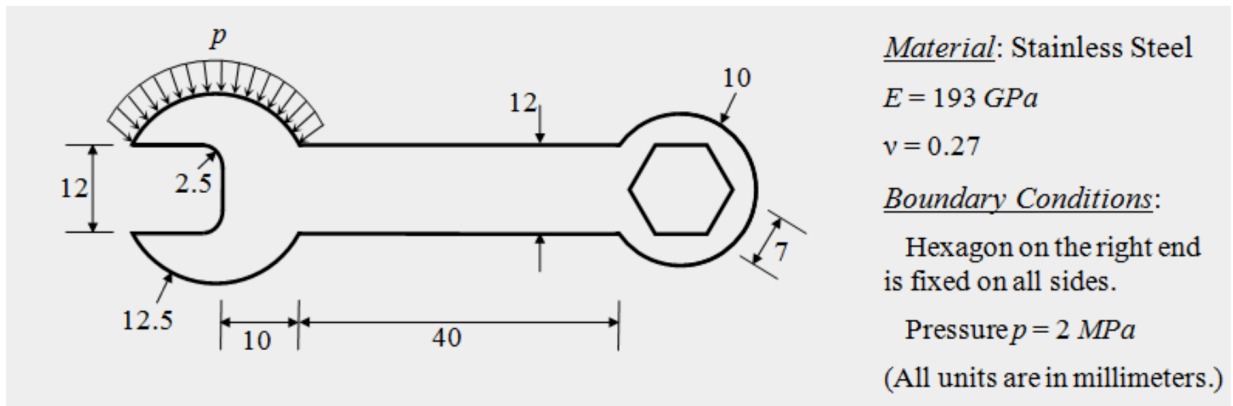
Title of Assignment: Homework 7

TOPICS COVERED IN HOMEWORK 7

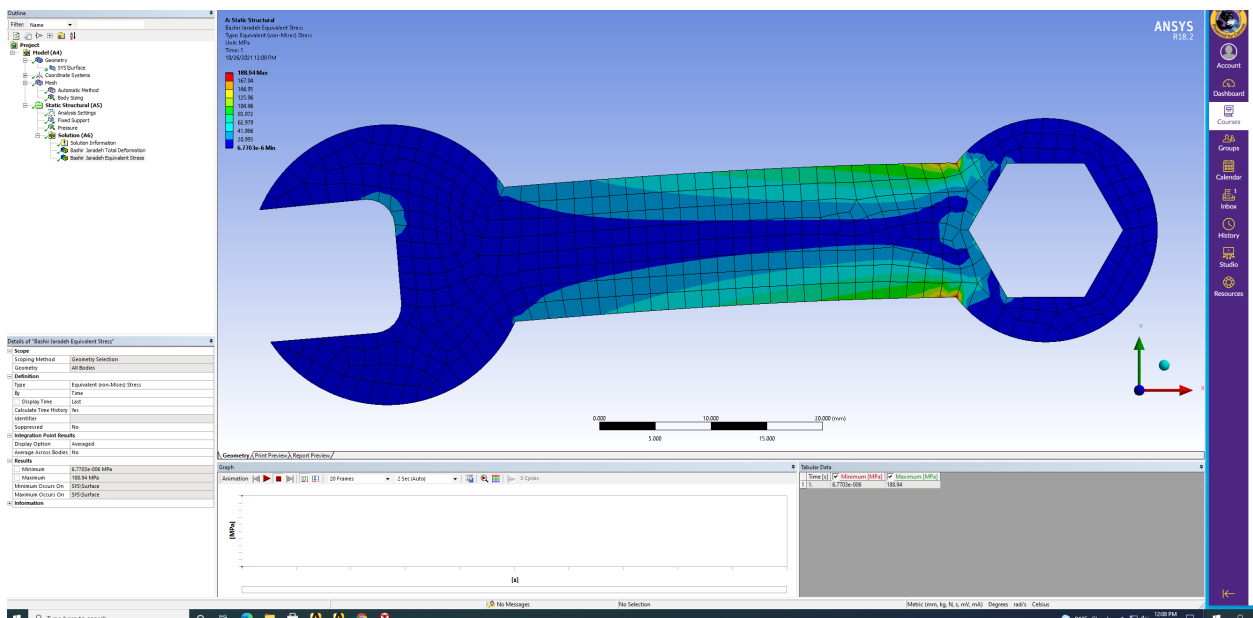
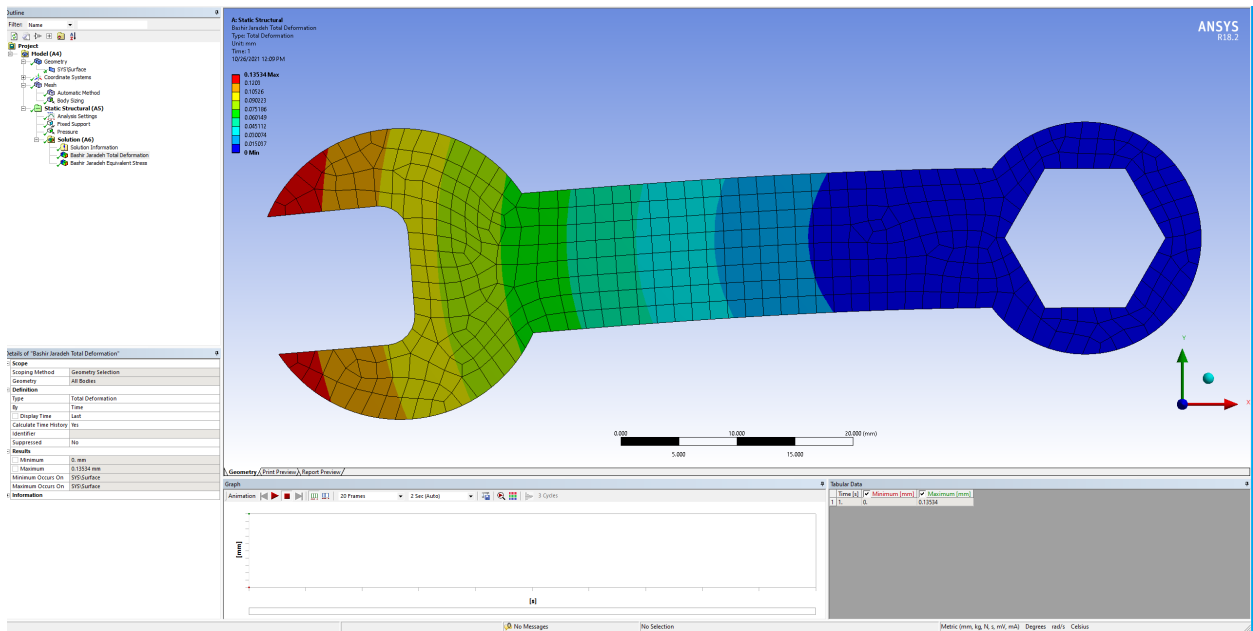
Use static structural program in ANSYS to analyze the deformation and stress in a wrench.

Case Study 4.5

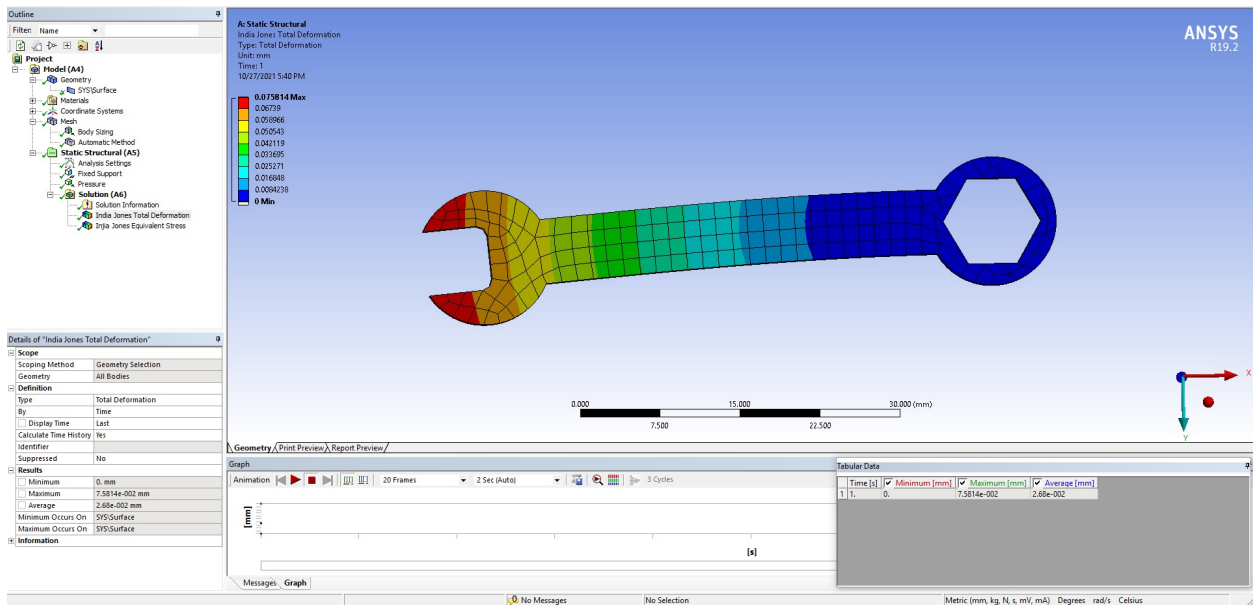
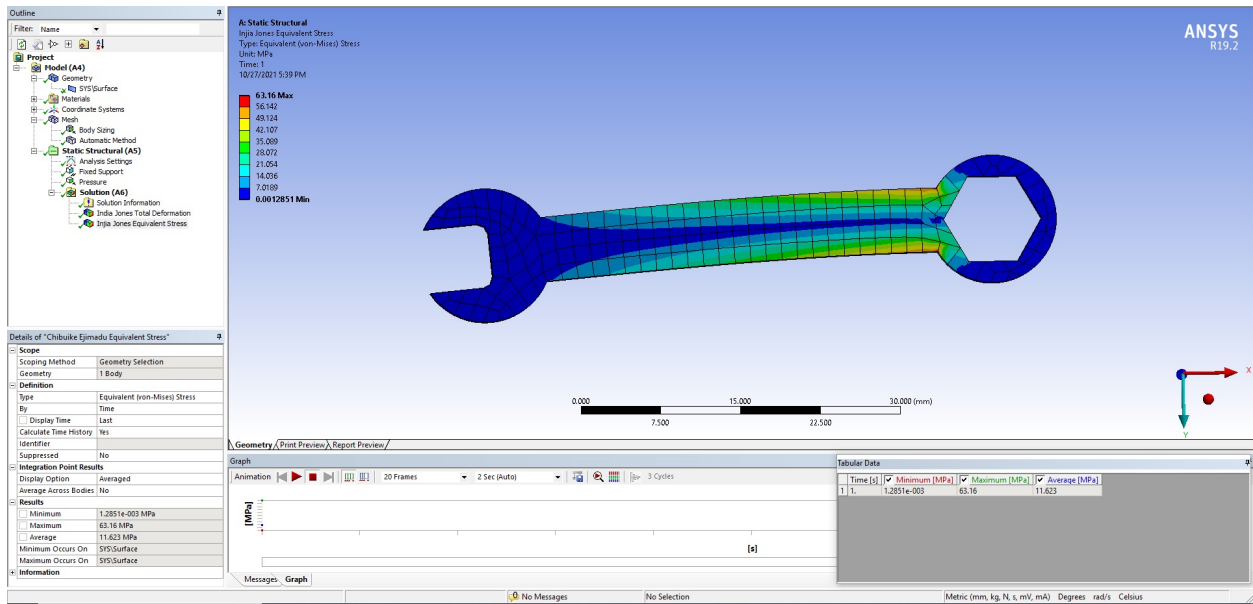
<Problem Description> A combination wrench is a convenient tool that is used to apply torque to loosen or tighten a fastener. The wrench shown below is made of stainless steel and has a thickness of 3 mm . Determine the maximum deformation and the distribution of von Mises stresses under the given distributed load and boundary conditions.



ABOVE AVERAGE

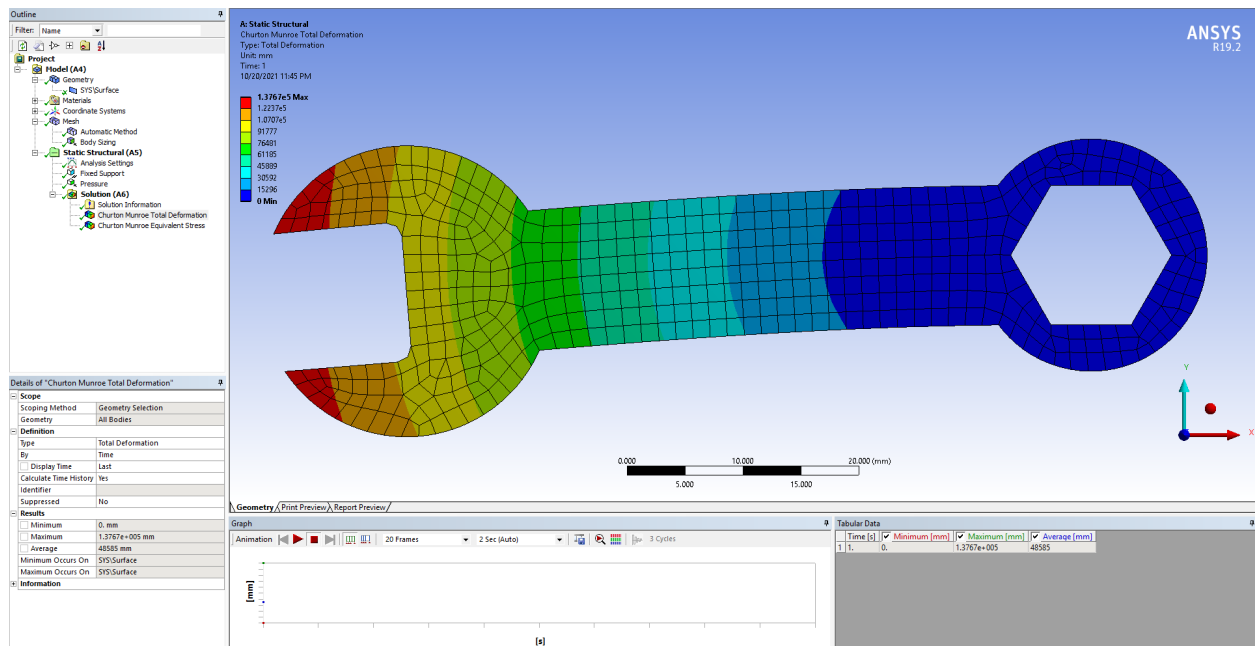


AVERAGE

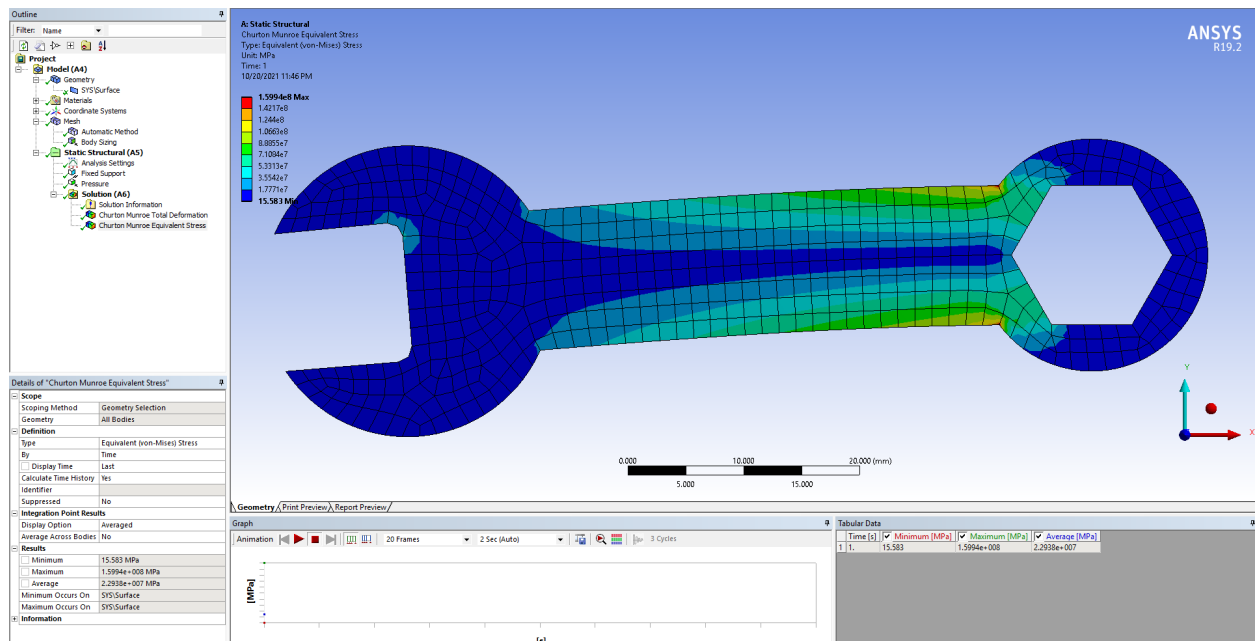


Analysis is correct, but geometry dimension is incorrect.

BELOW AVERAGE



Deformation is greatly exceeded the material elastic range.



Stress value is greatly exceeded the yielding stress of the material.

**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

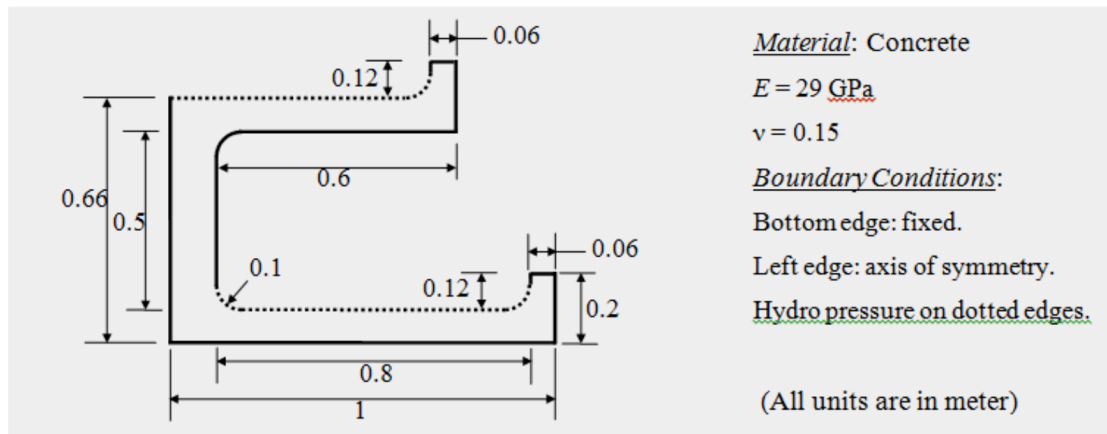
Title of Assignment: Homework 8

TOPICS COVERED IN HOMEWORK 8

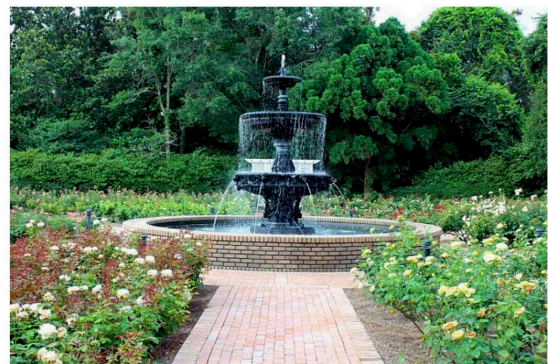
Use static structural program in ANSYS to analyze the deformation and stress in a concrete garden fountain under hydrostatic pressure and rotational force.

Case Study 5.8

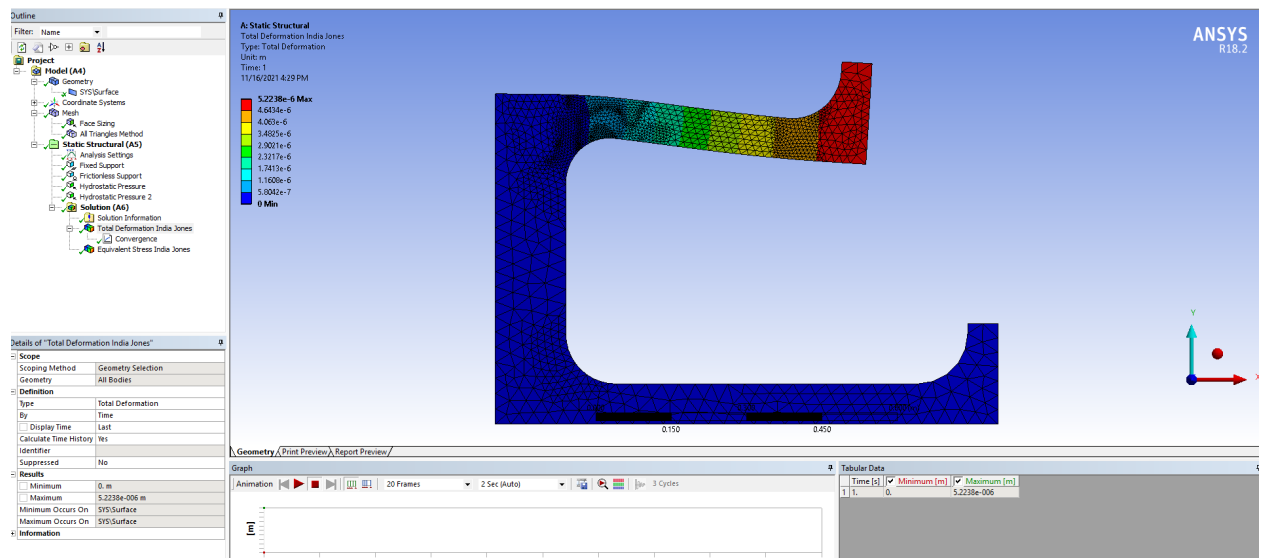
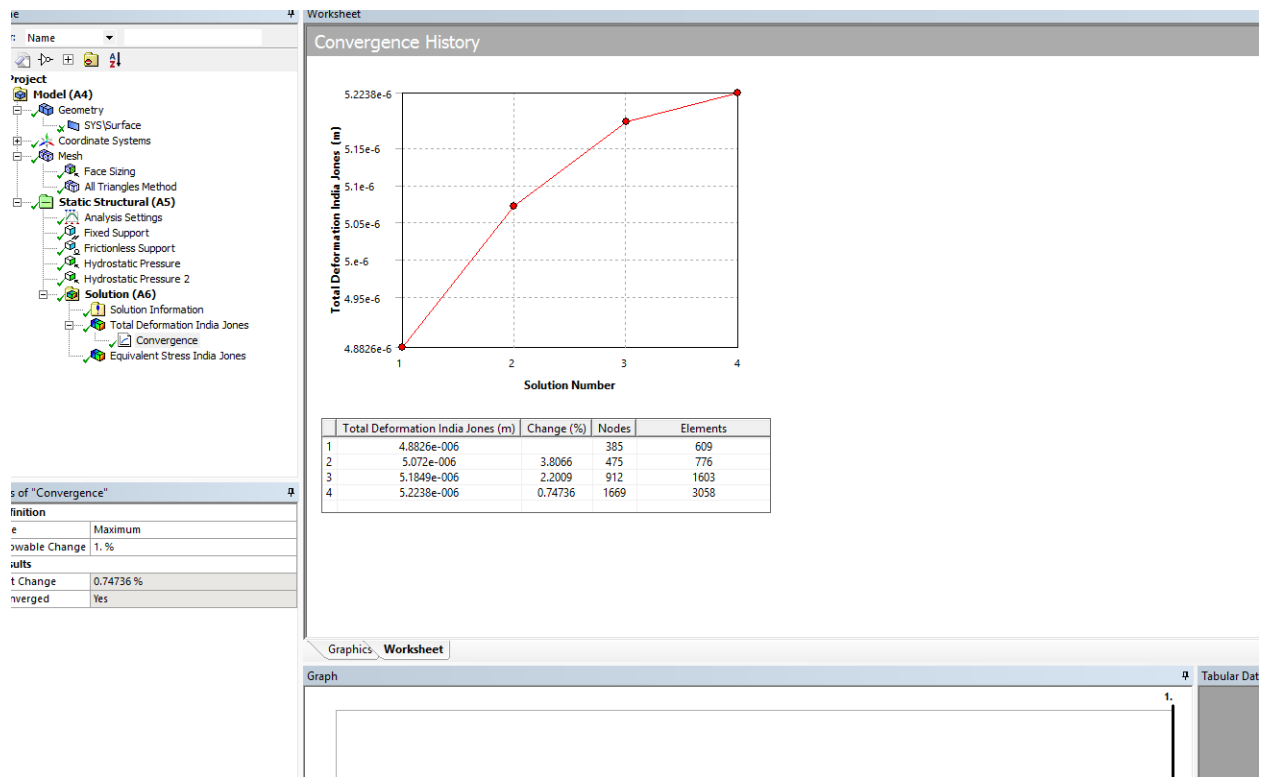
<Problem Description> Garden fountains are popular amenities that are often found at theme parks and hotels. As a fountain structure is usually an axisymmetric geometry with axisymmetric loads and support, only a 2-D model, sliced through the 3-D geometry, is needed to correctly predict the deformation of or stress in the structure. The figure below gives the dimensions of the planar cross section of a two-tier garden fountain made of concrete. Determine the maximum deformation and von Mises stress under the given hydrostatic pressure. Use adaptive meshing to improve solution convergence.

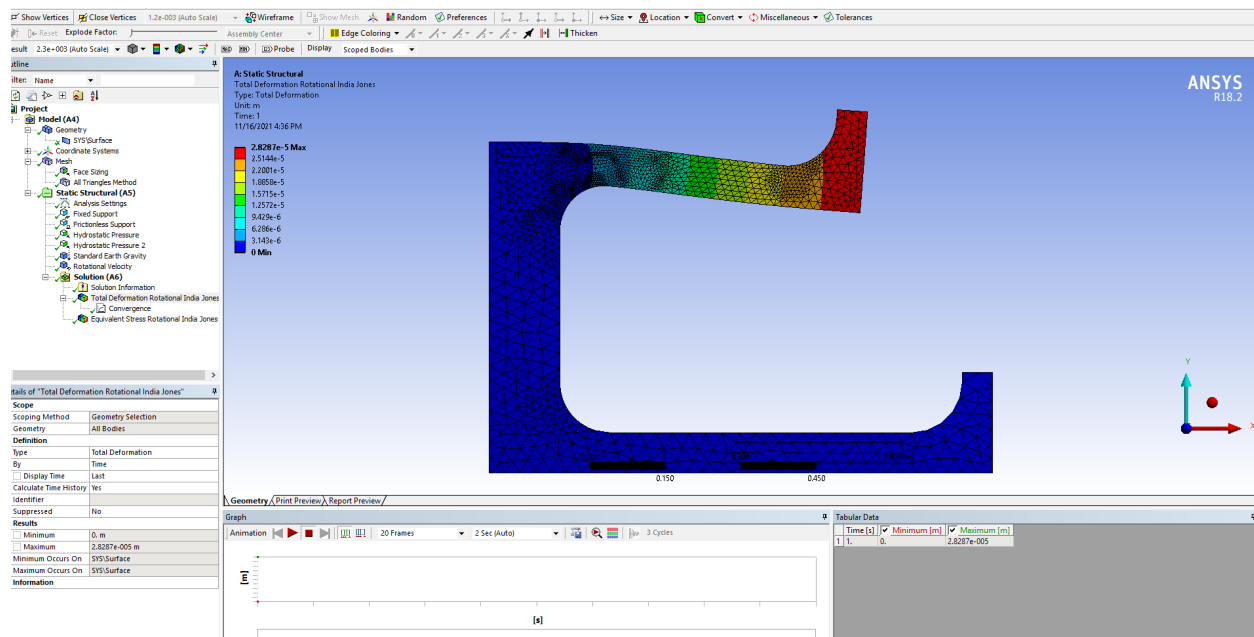
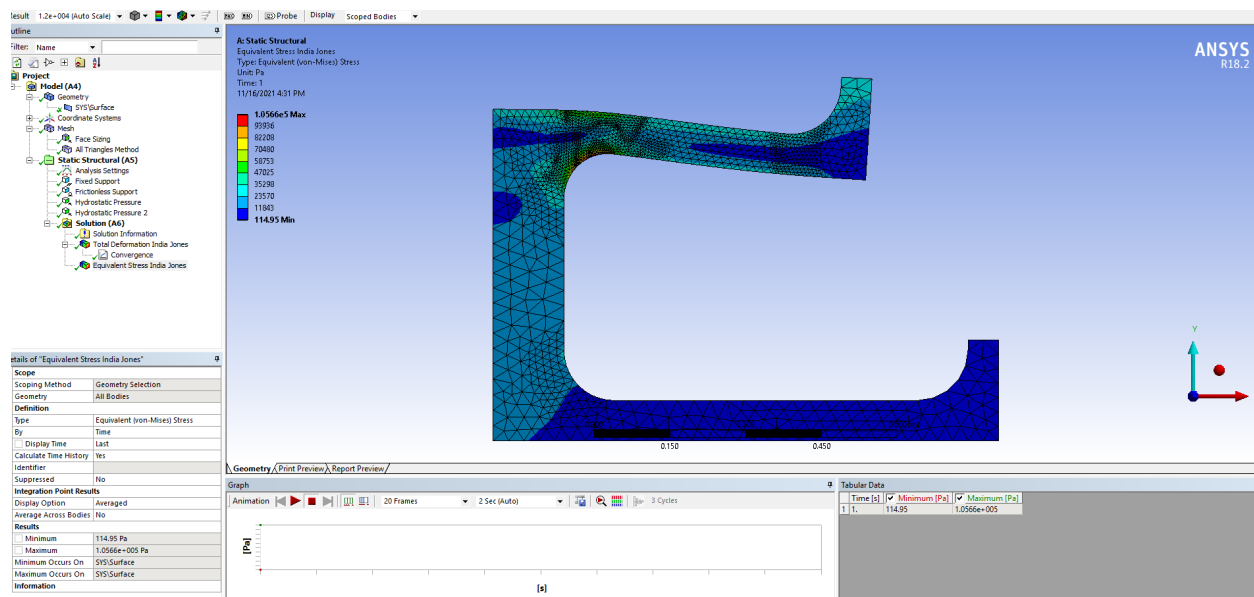


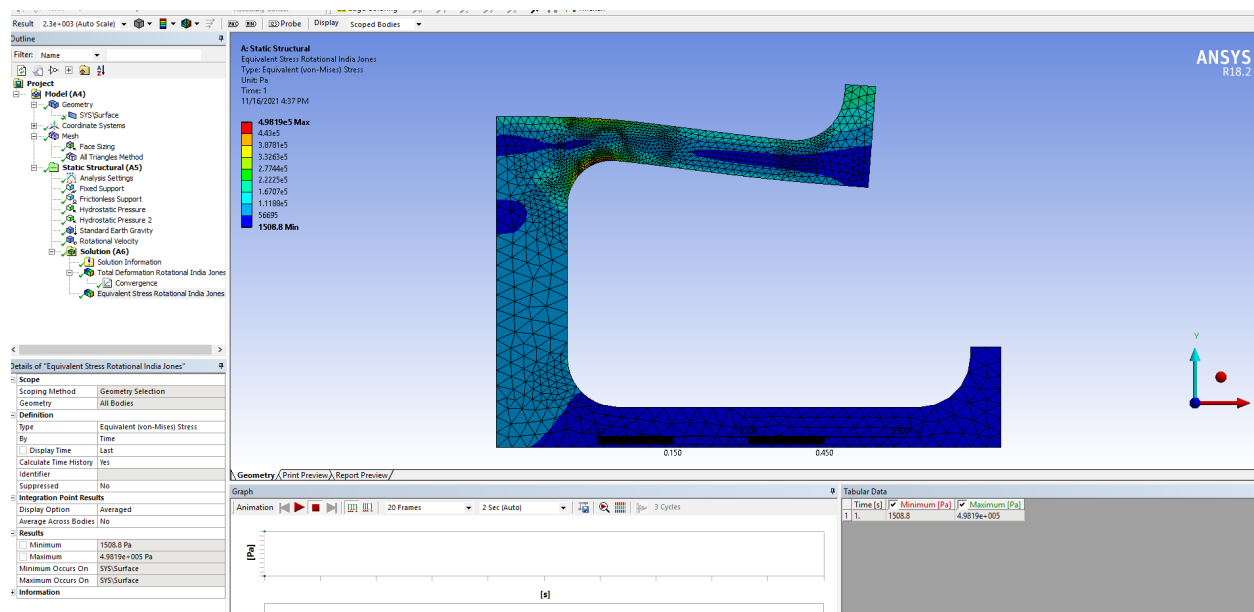
Add body force
(gravitational force = $-\rho g$)
and
radial centrifugal force = $\rho r \omega^2$
to
the analysis



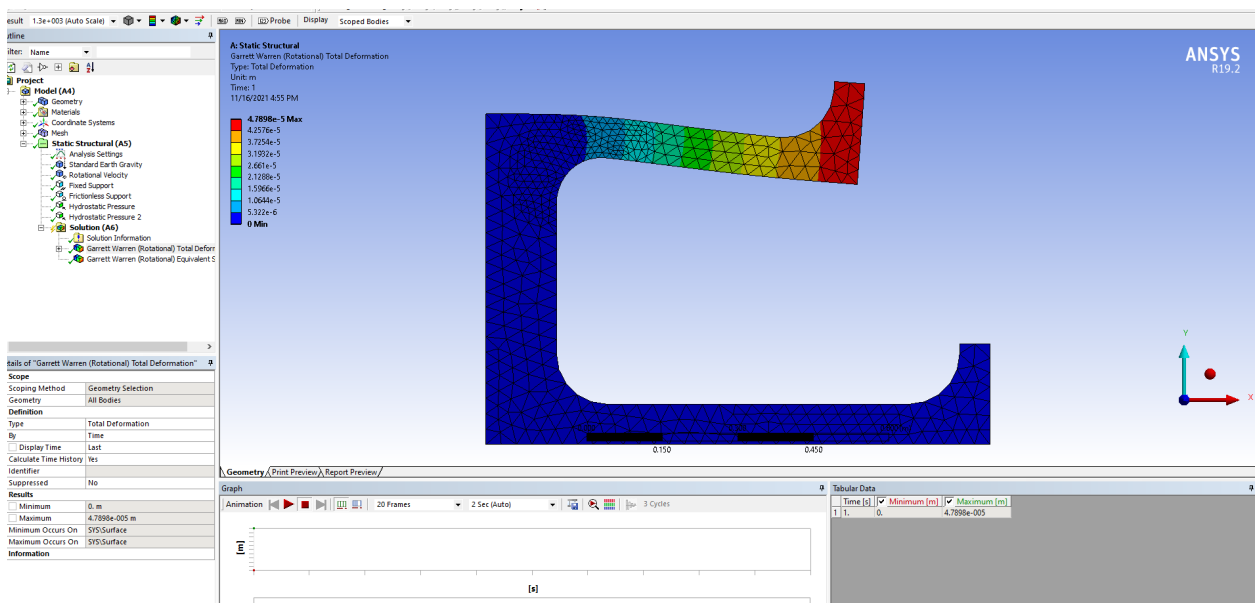
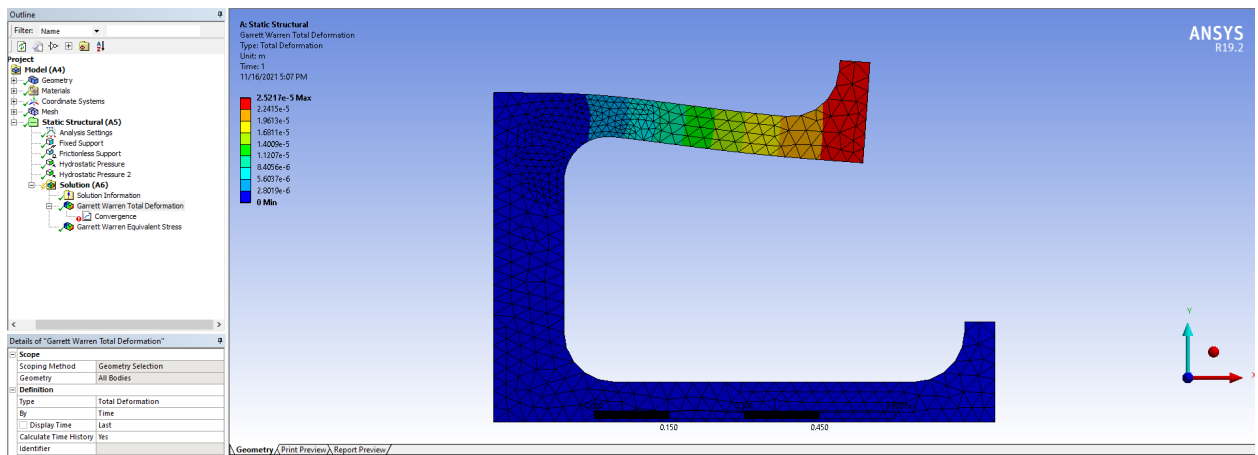
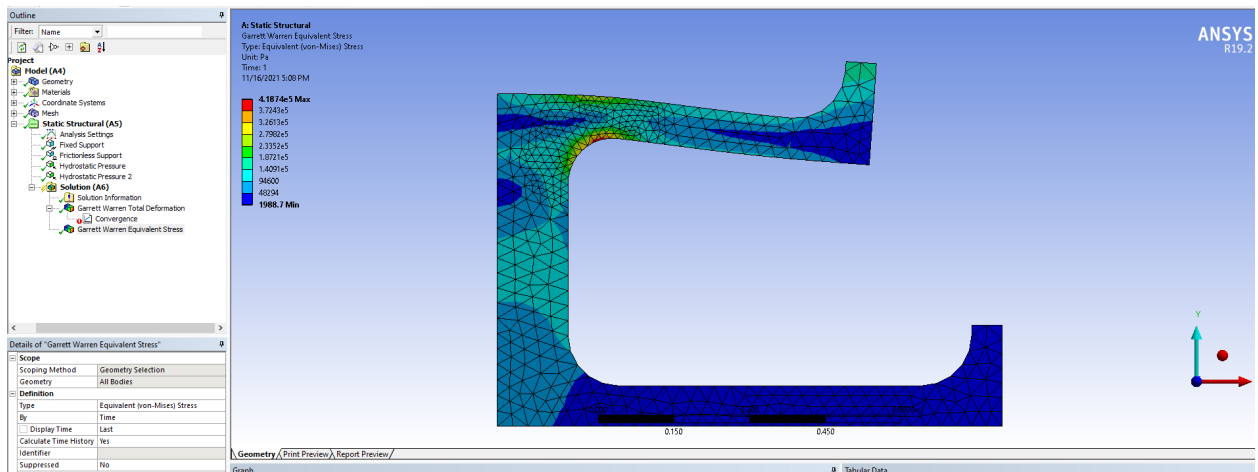
ABOVE AVERAGE

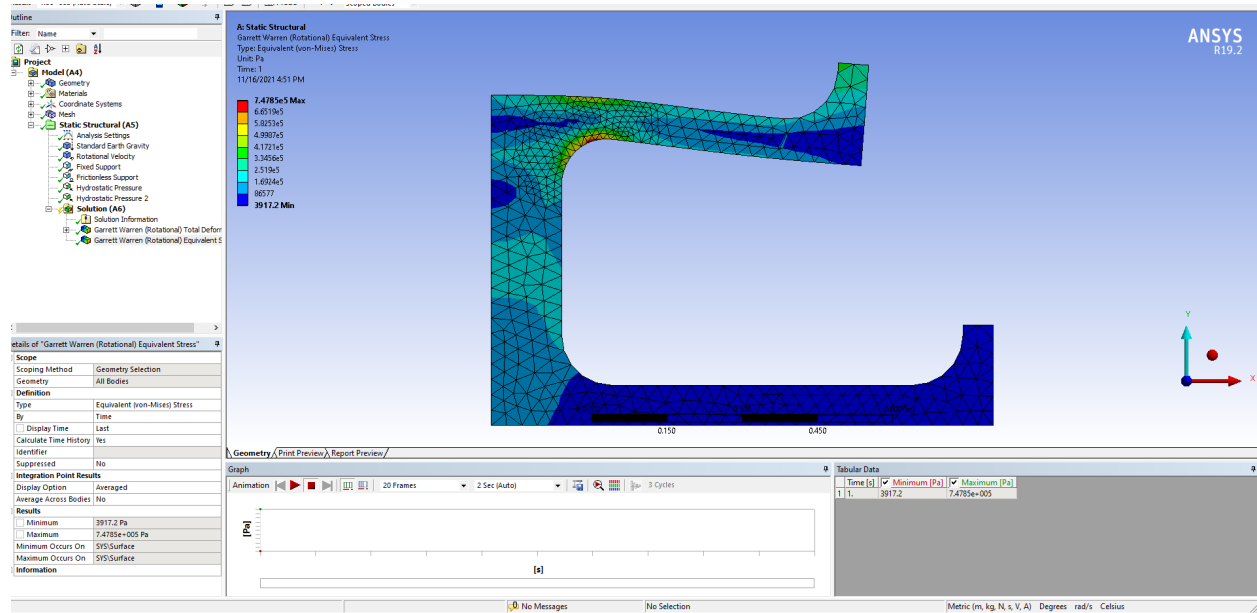




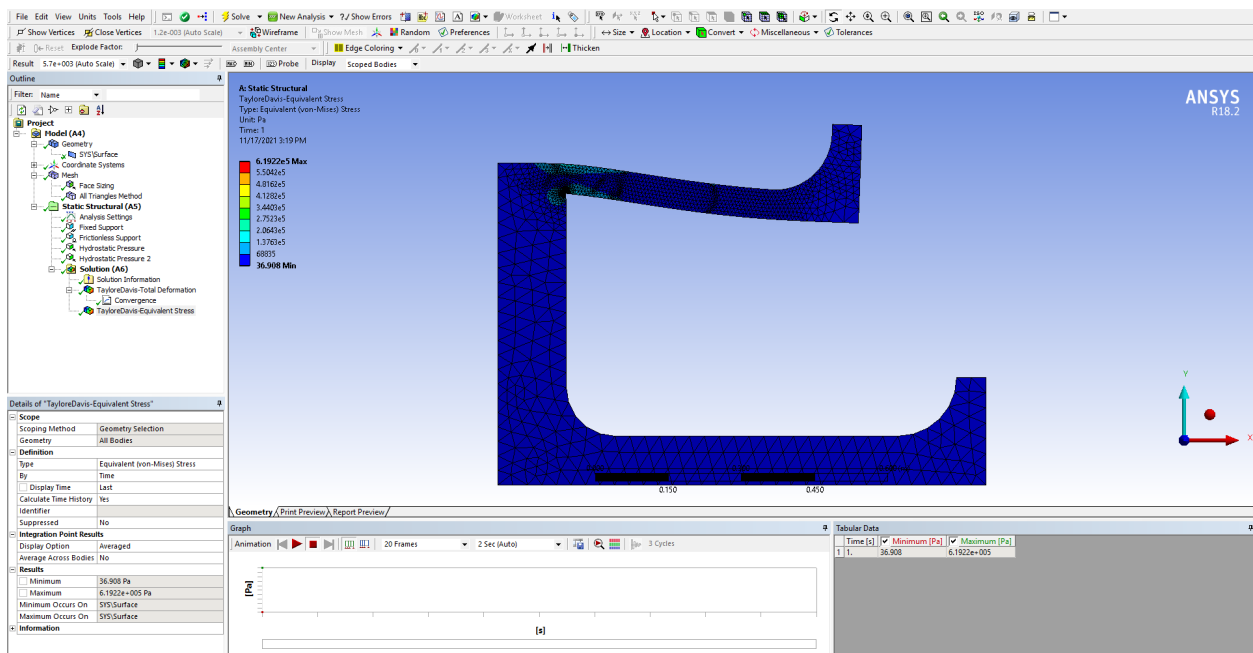
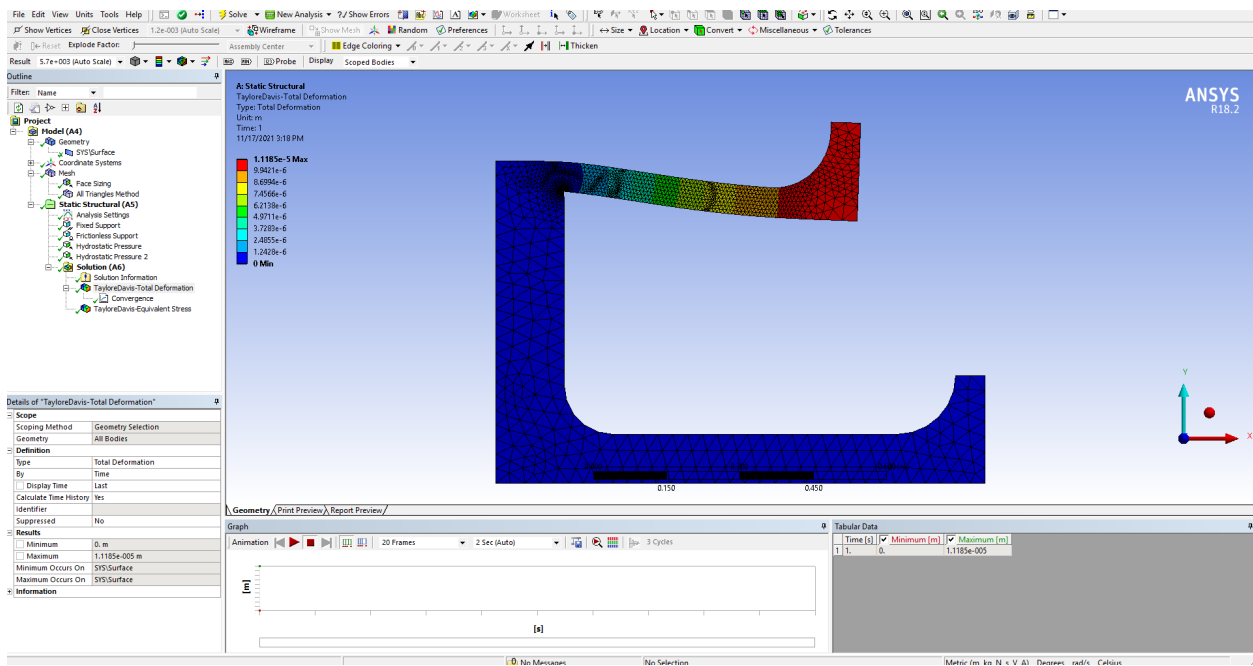


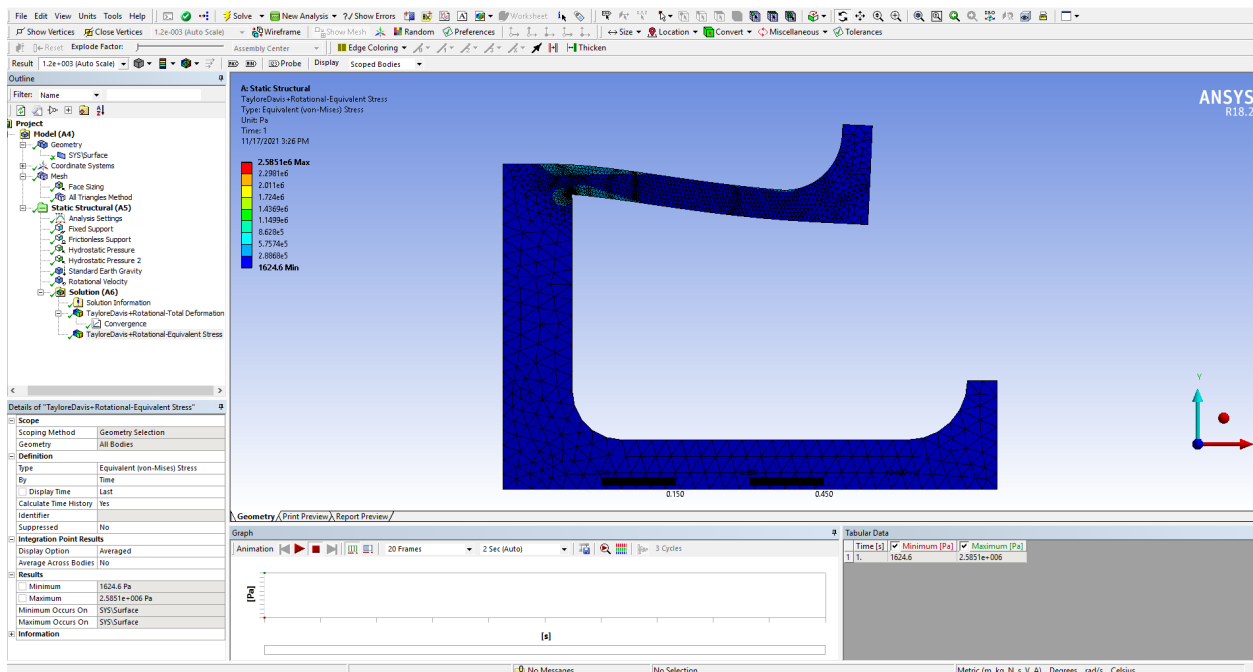
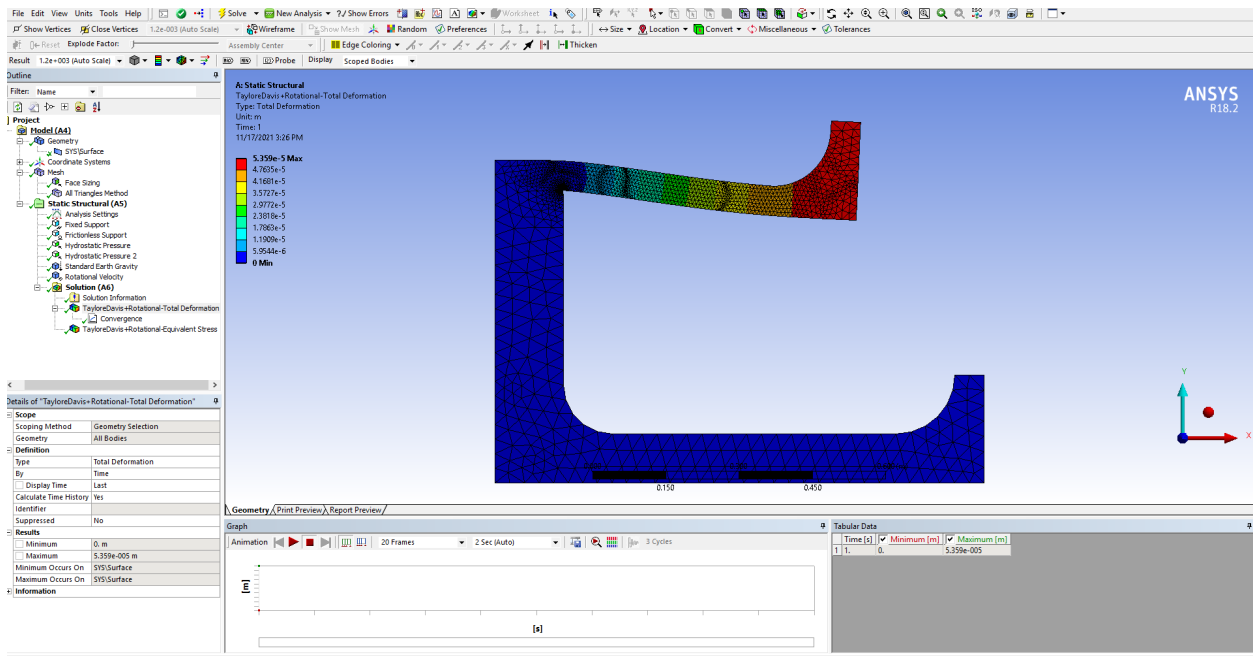
AVERAGE





BELOW AVERAGE





Comments: Geometry was not created correctly, such as missing a round corner at the upper deck.

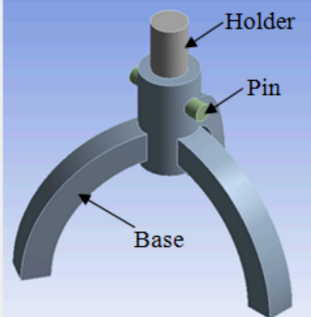
**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

Title of Assignment: Homework 9

TOPICS COVERED IN HOMEWORK 9

Use static structural program in ANSYS to analyze the deformation and stress in a three-part assembly: pin, holder, and base.

<Problem Description> A base stand assembly includes a base, a holder and a pin, as shown in the following figure. The stand assembly is made of structural steel. Assume a no-separation condition for all contact regions. Determine the deformation and von Mises stress distributions of the assembly under the given load and boundary conditions.



Material: Structural Steel ($E = 200 \text{ GPa}$, $\nu = 0.3$)

Boundary Conditions:

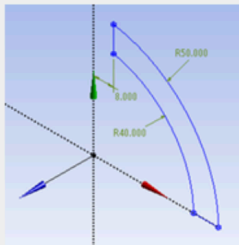
The bottom faces of the leg-base are fixed.

A downward force of 1 kN is applied to the holder's top face.

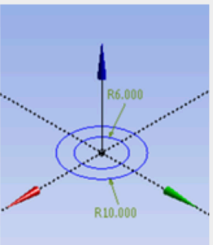
Geometry Construction:

The bottom of the hub-base is 35 mm above the ground level.

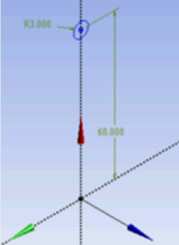
The holder is 36 mm tall, 18 mm of which is in contact with the hub-base.



Sketch of the leg-base
(extrude 5mm on both sides)

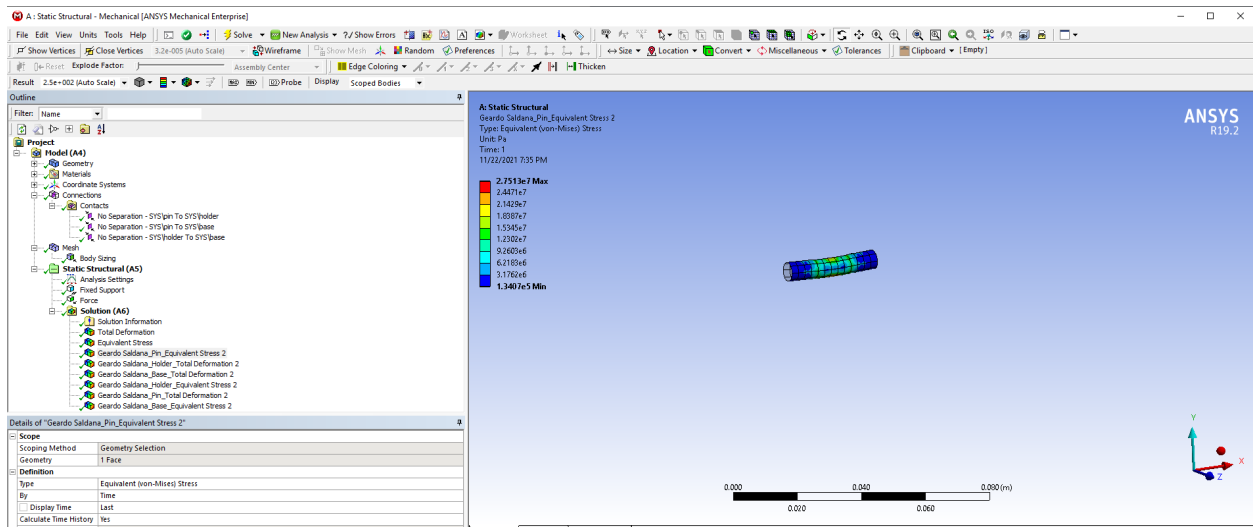
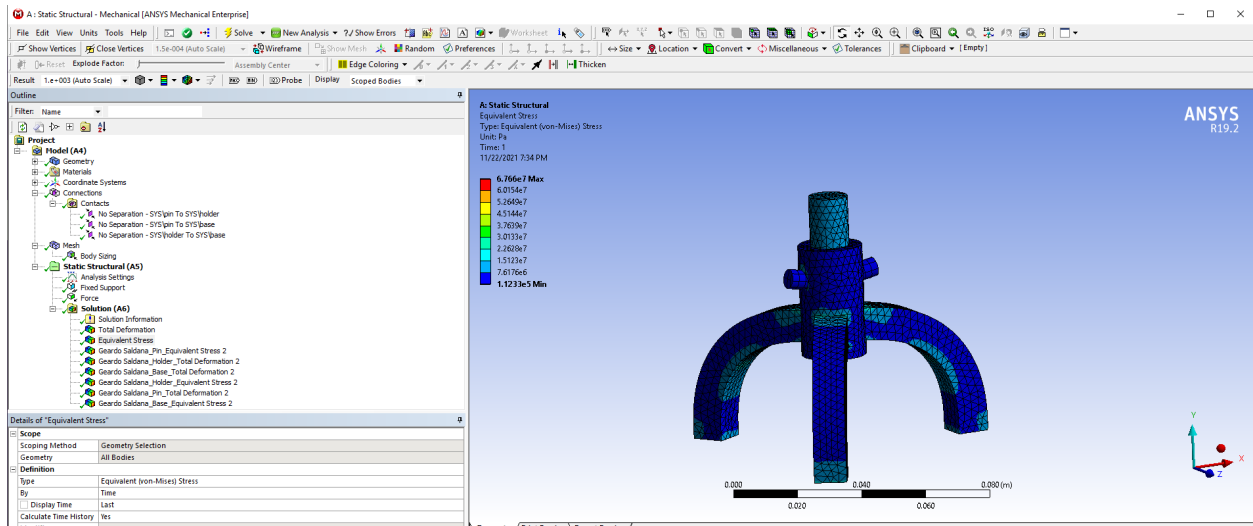
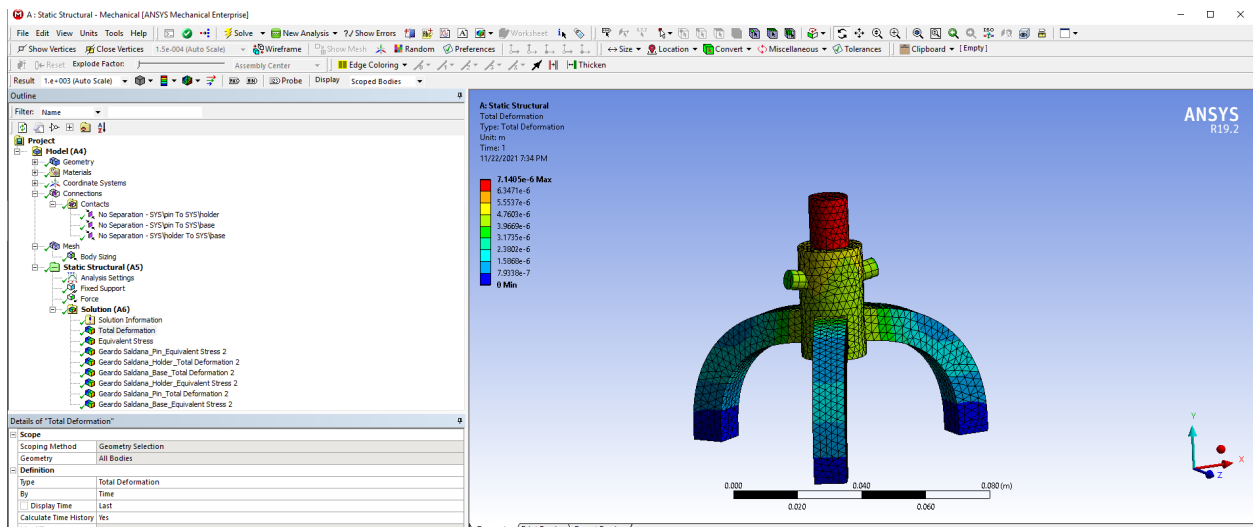


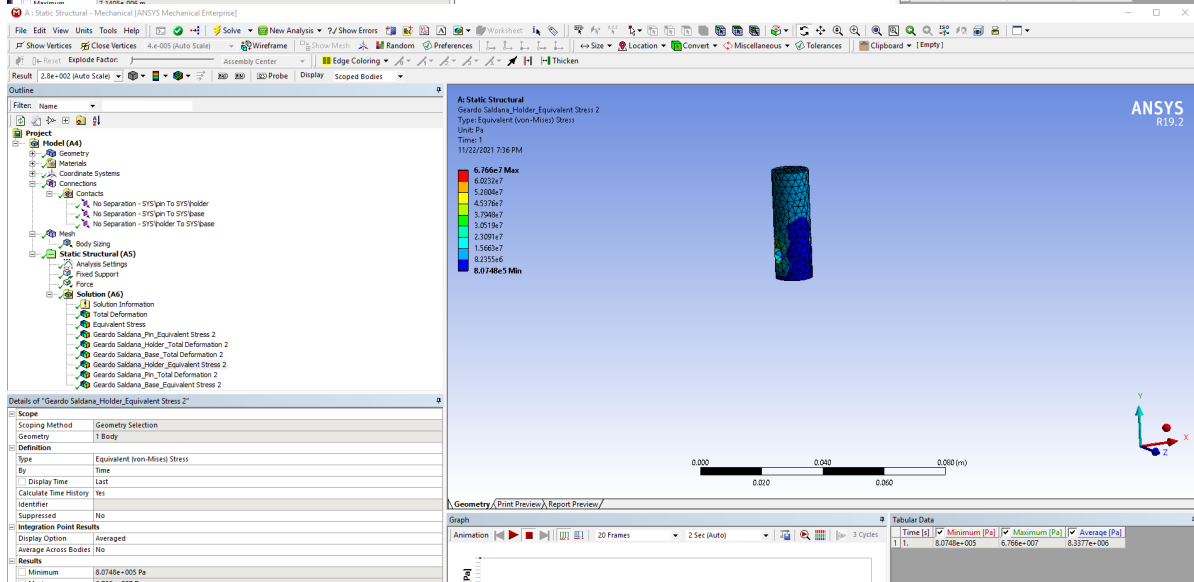
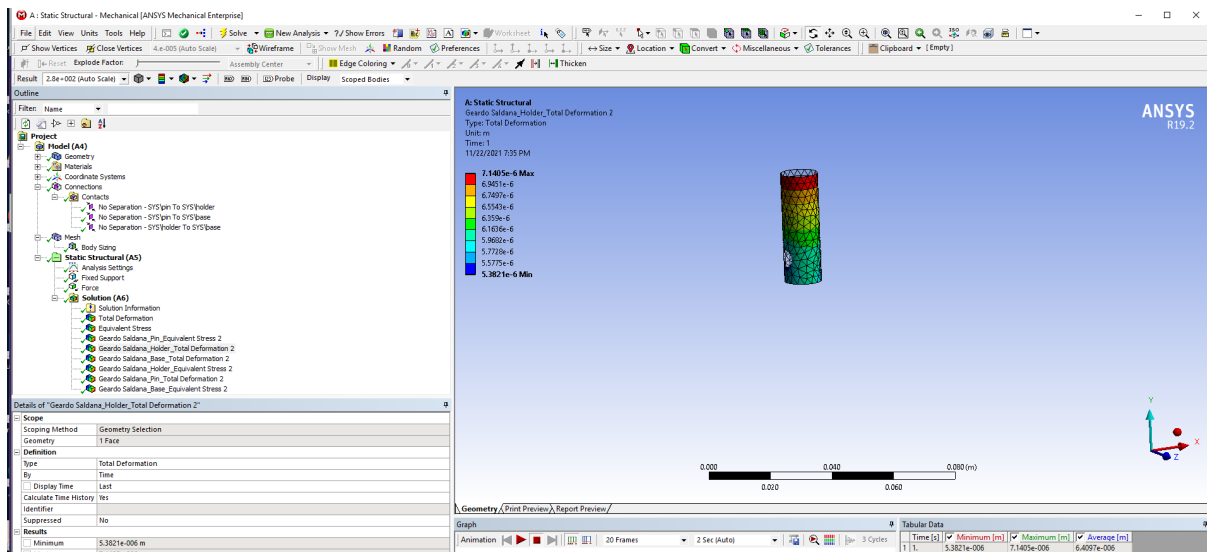
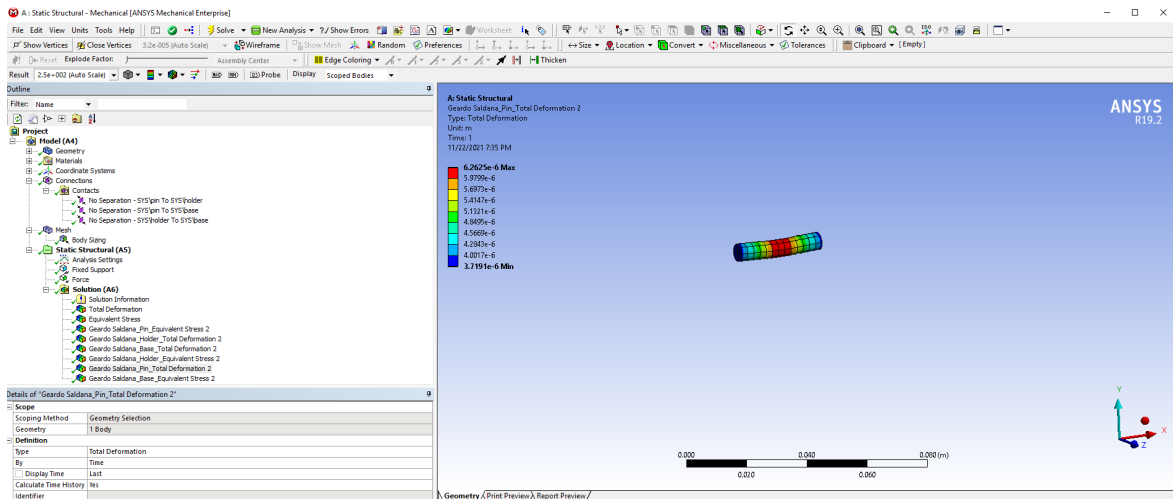
Sketch of the hub-base
(extrude 35mm on one side)

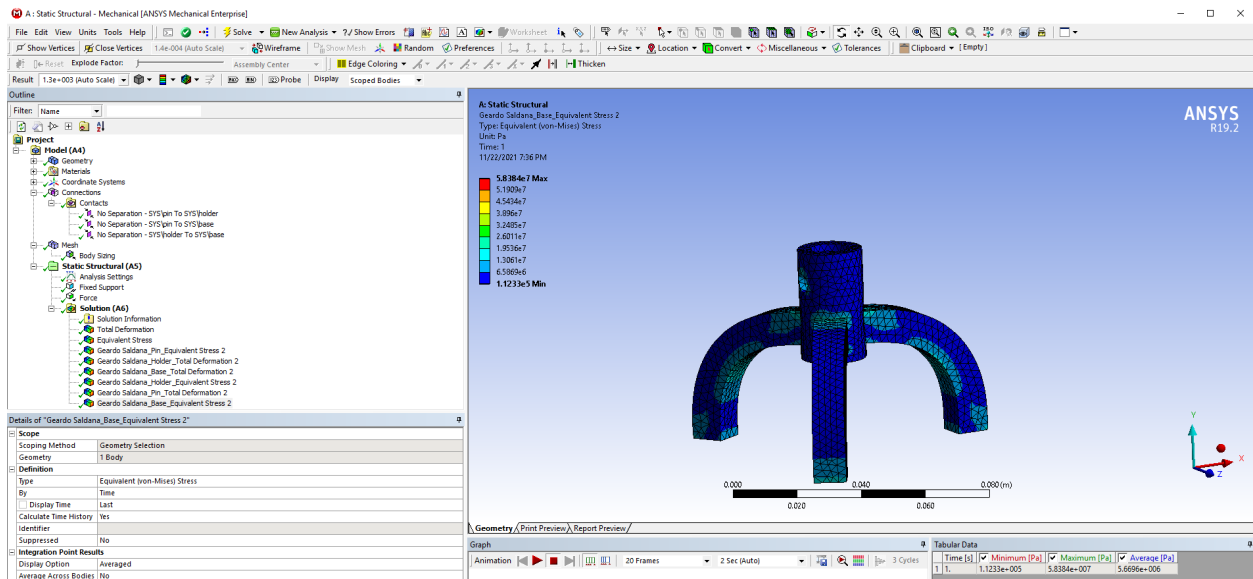
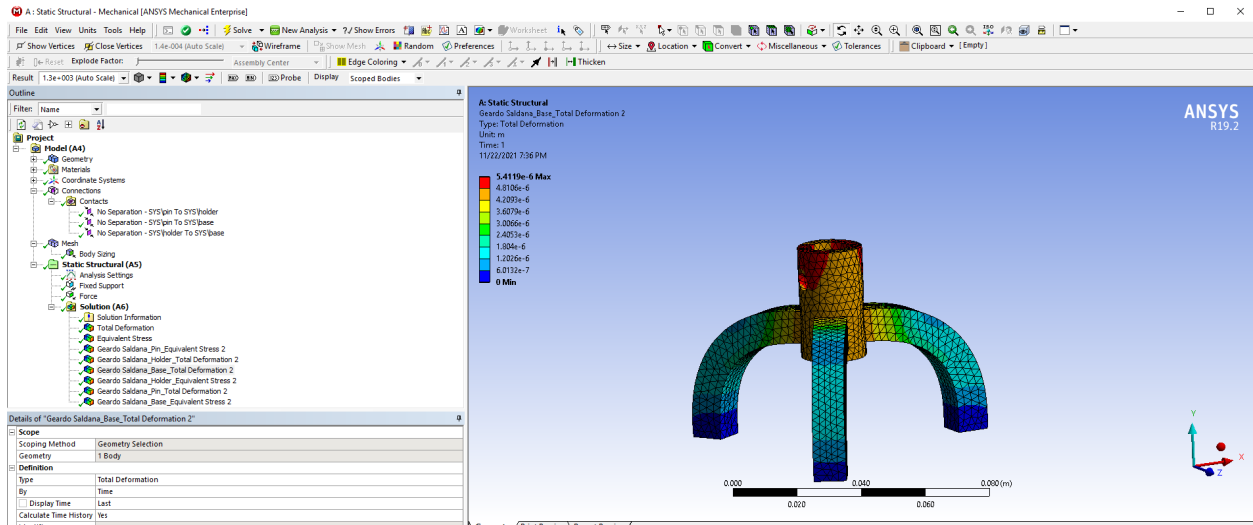


Sketch of the pin (extrude
15mm on both sides)

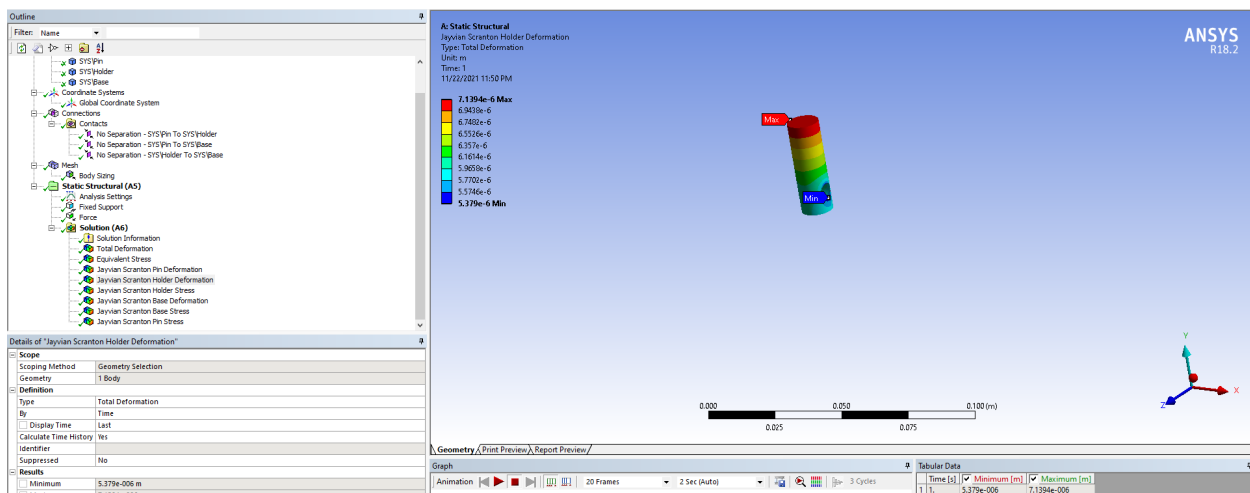
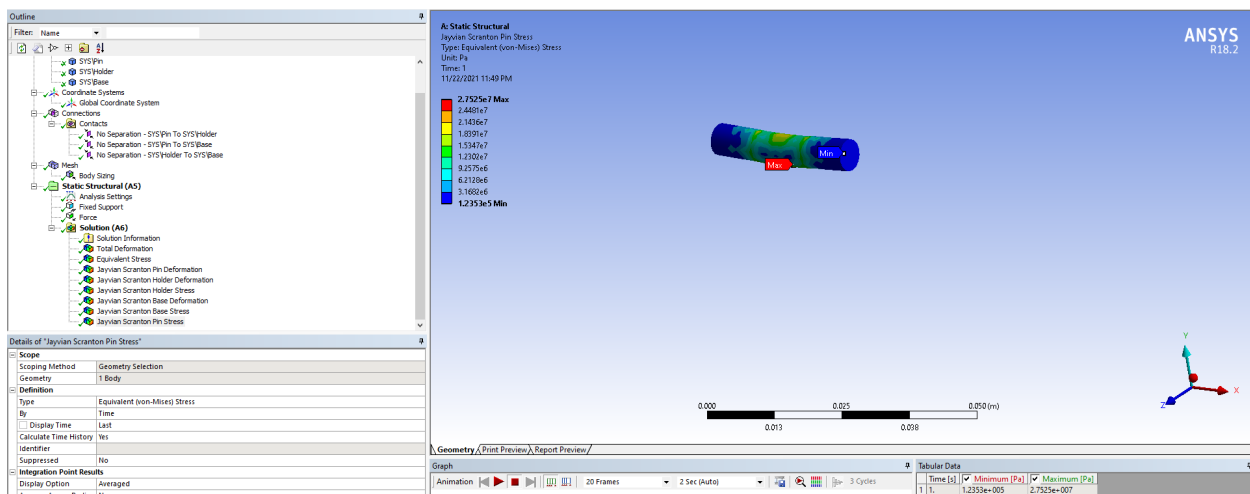
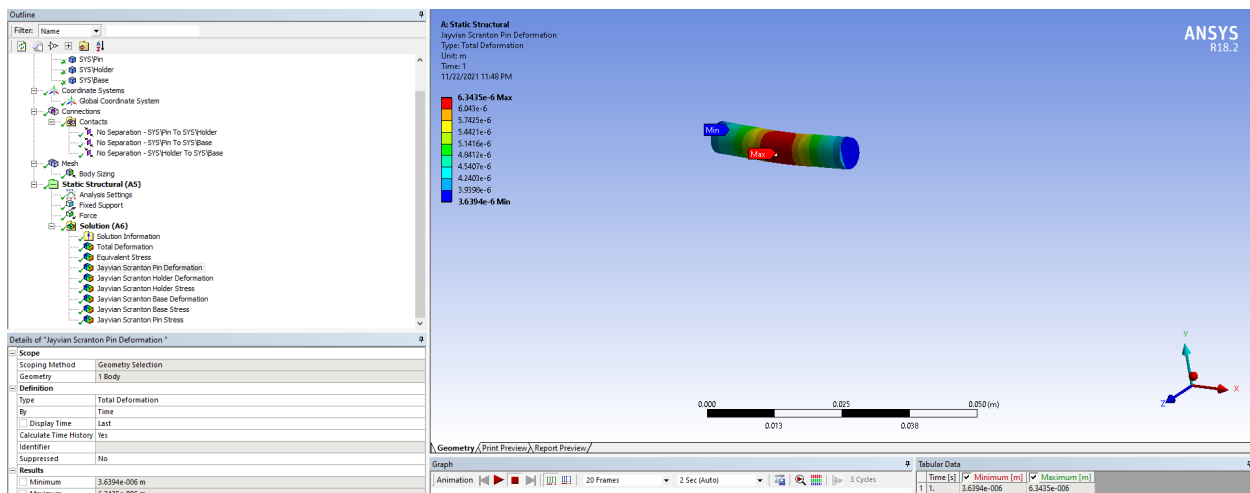
ABOVE AVERAGE

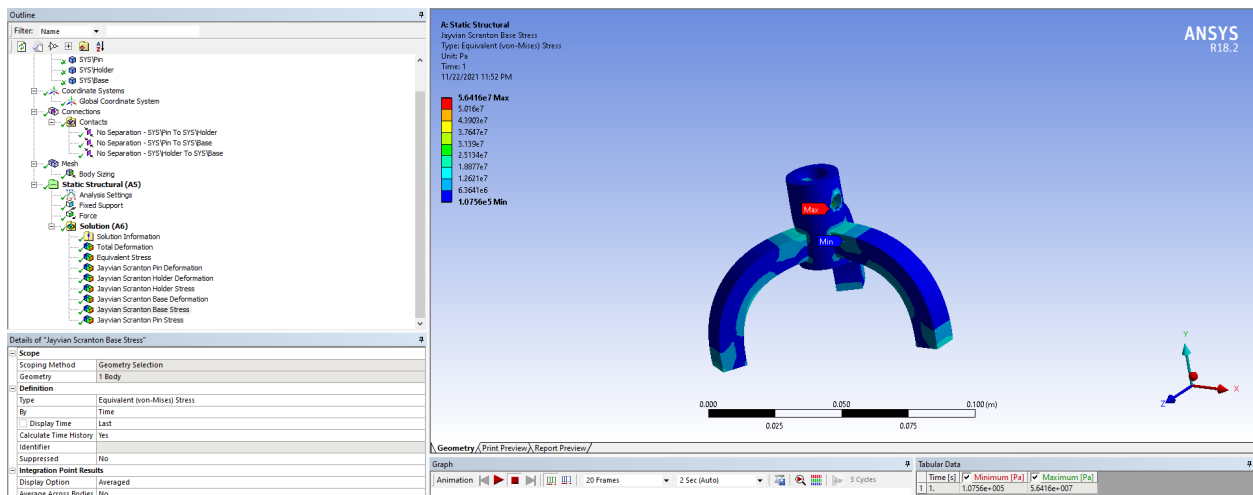
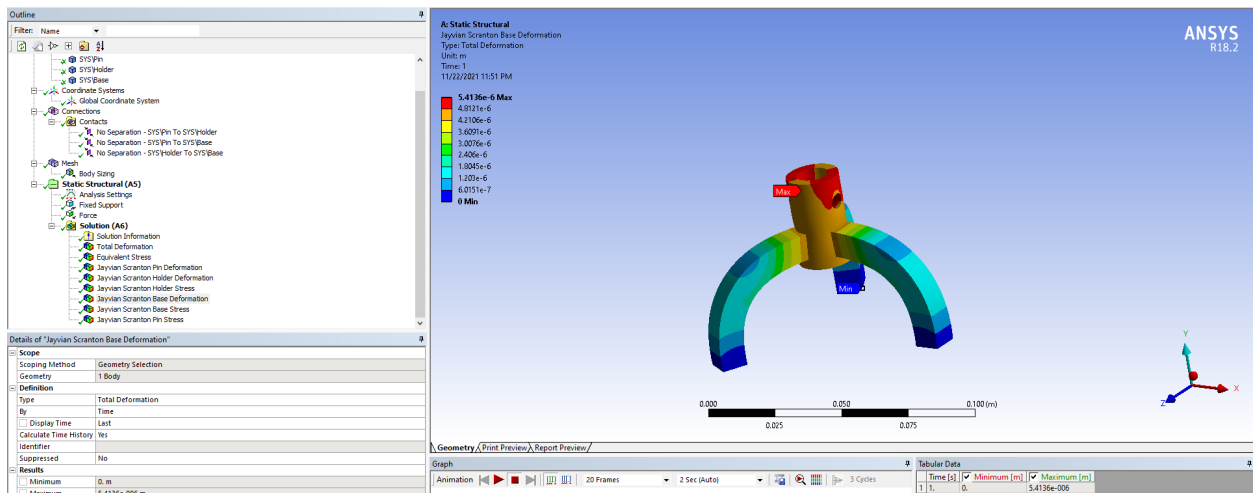
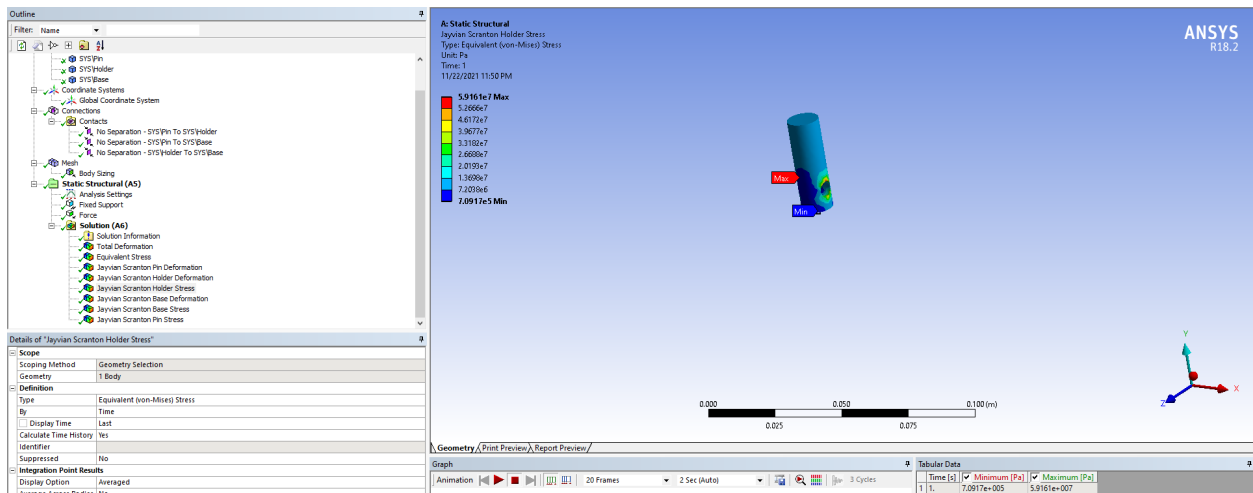




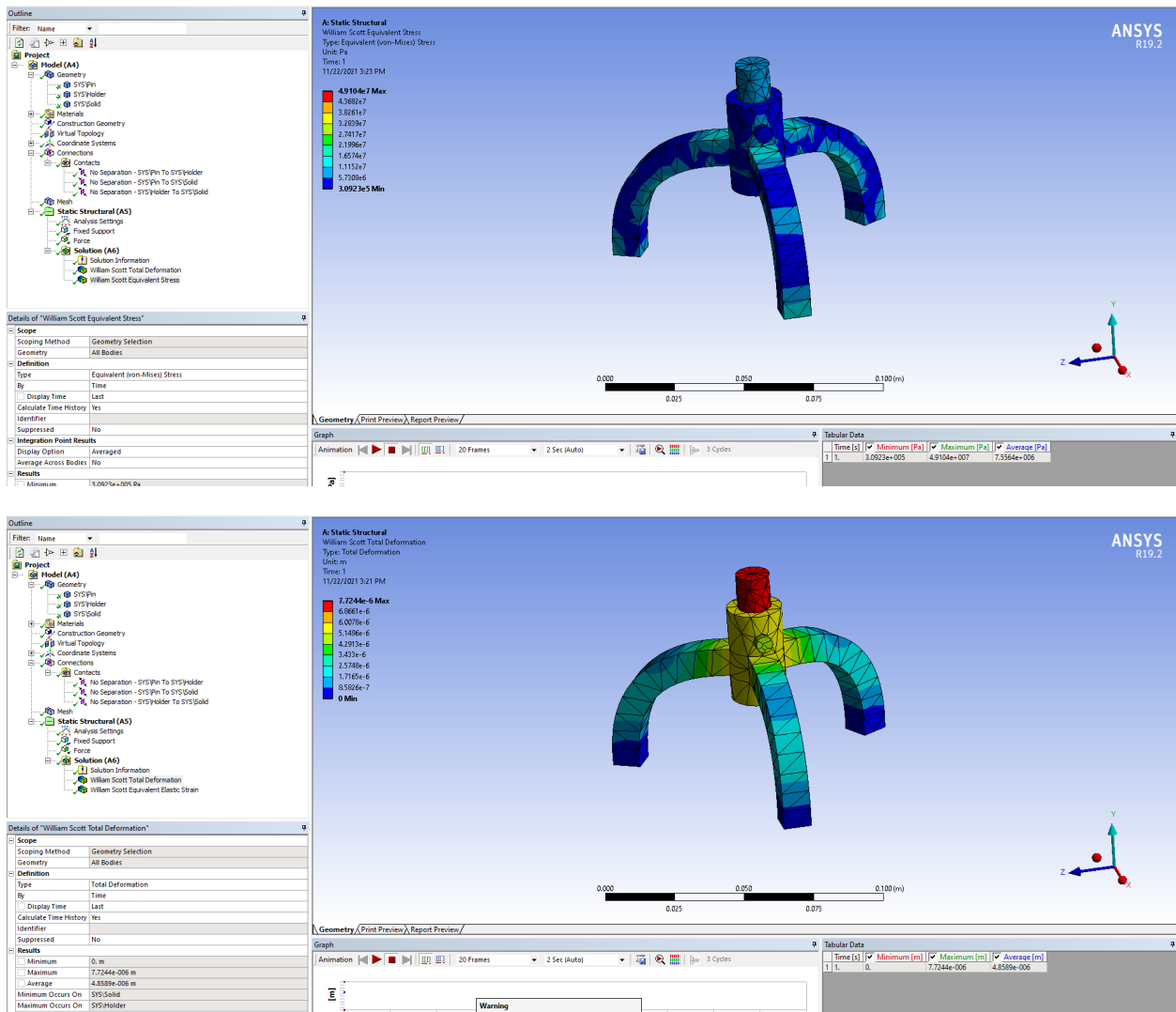


AVERAGE





BELOW AVERAGE



Assignment Comments



Missing component analysis.



Shield Lin, Dec 4 at 6:30pm

**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

Title of Assignment: Homework 10

TOPICS COVERED IN HOMEWORK 10

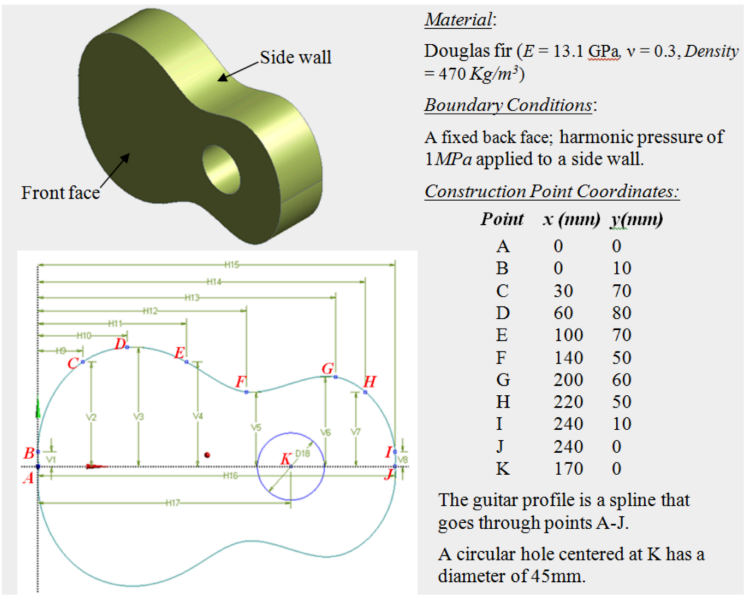
Use modal analysis program in ANSYS to analyze the vibration mode in terms of deformation and natural frequencies of an acoustic guitar.

<Problem Description>

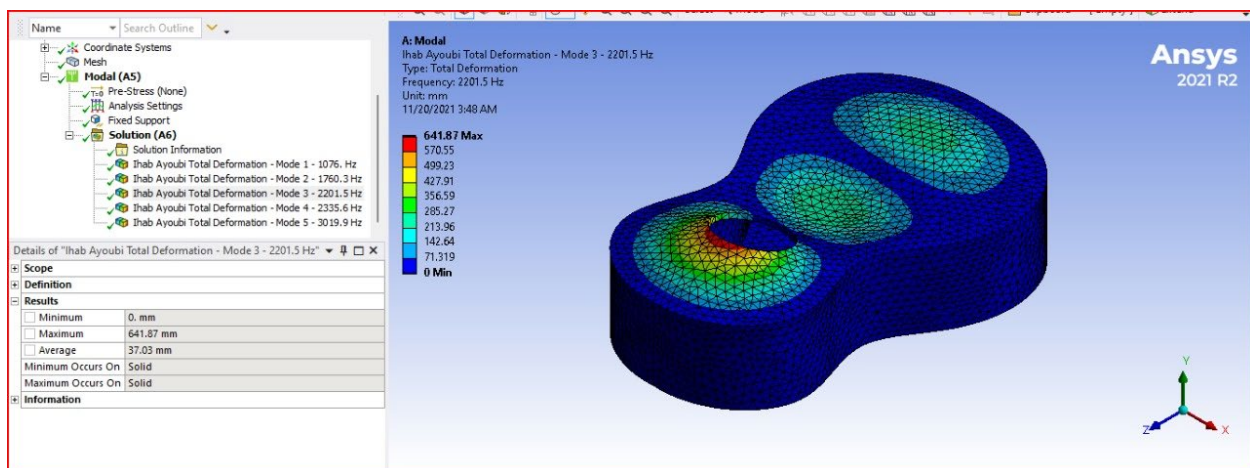
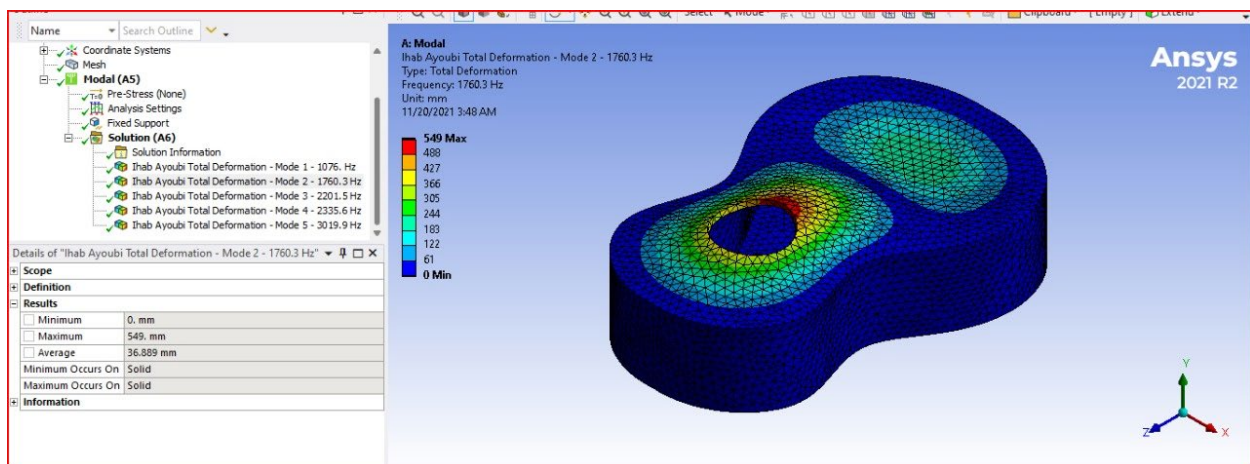
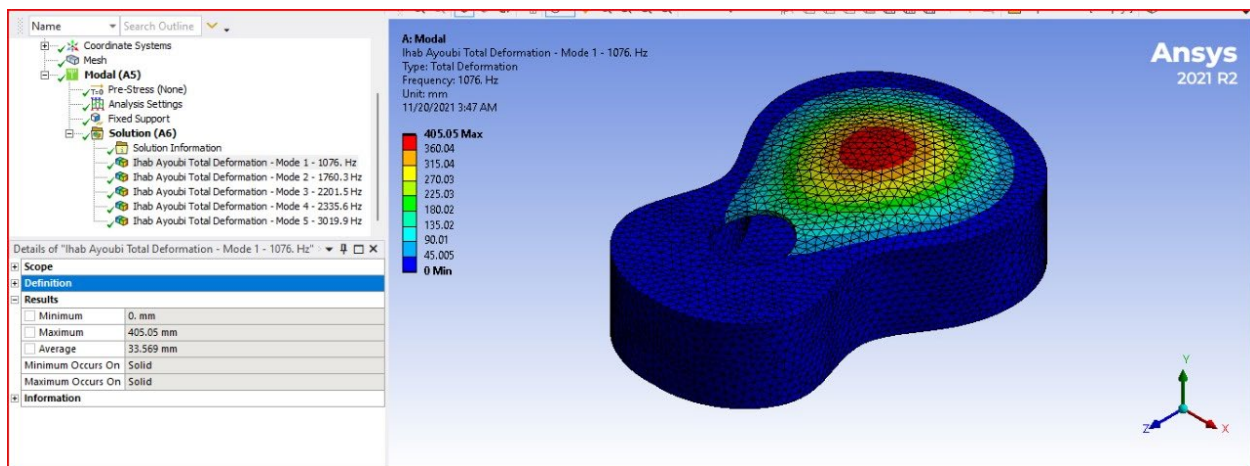
The right figure gives the dimensions of a simplified acoustic guitar model. The guitar has a wall thickness of 3 mm, and is made of Douglas fir wood.

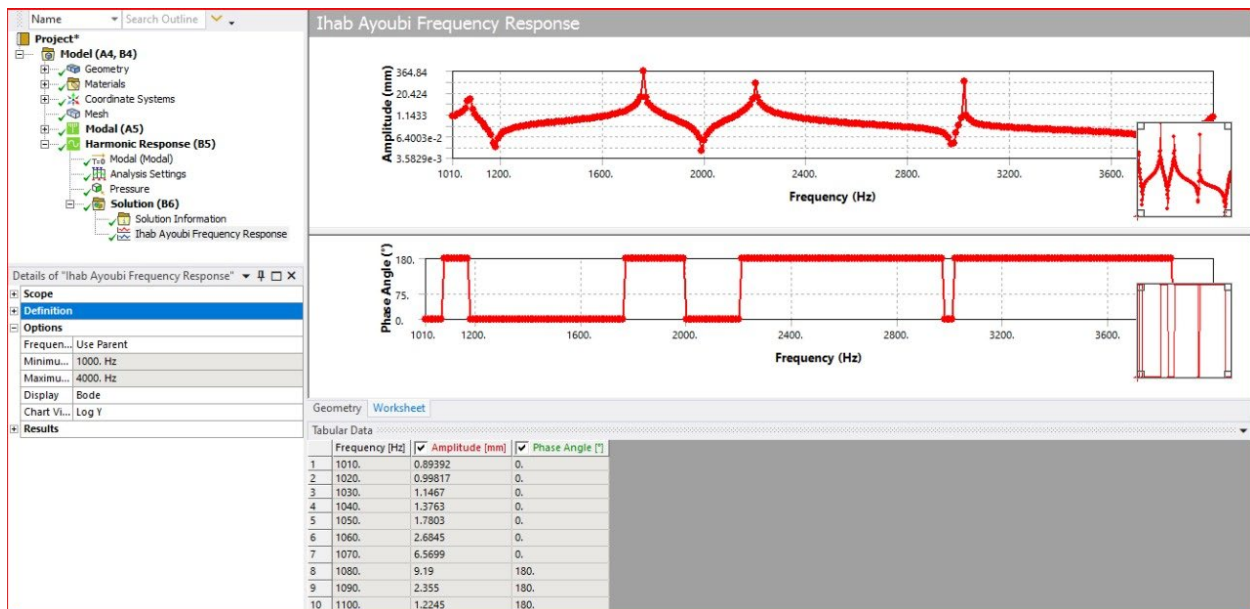
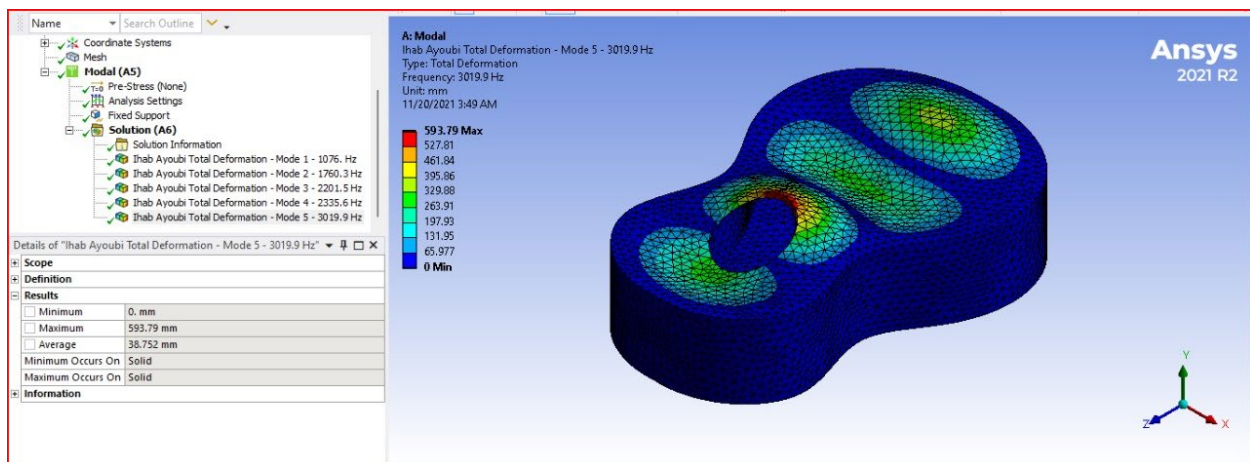
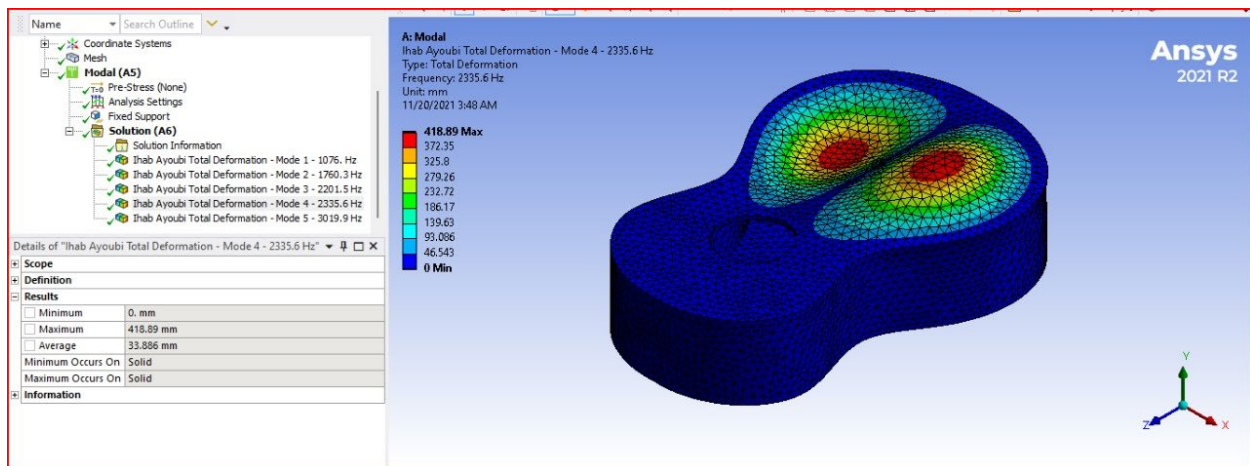
Assuming the back surface of the guitar is fixed, find the first ten natural frequencies and plot the first five vibration modes of the guitar.

Suppose a harmonic pressure loading of magnitude 1MPa is applied to a side wall of the guitar. Plot the frequency response of the z displacement.

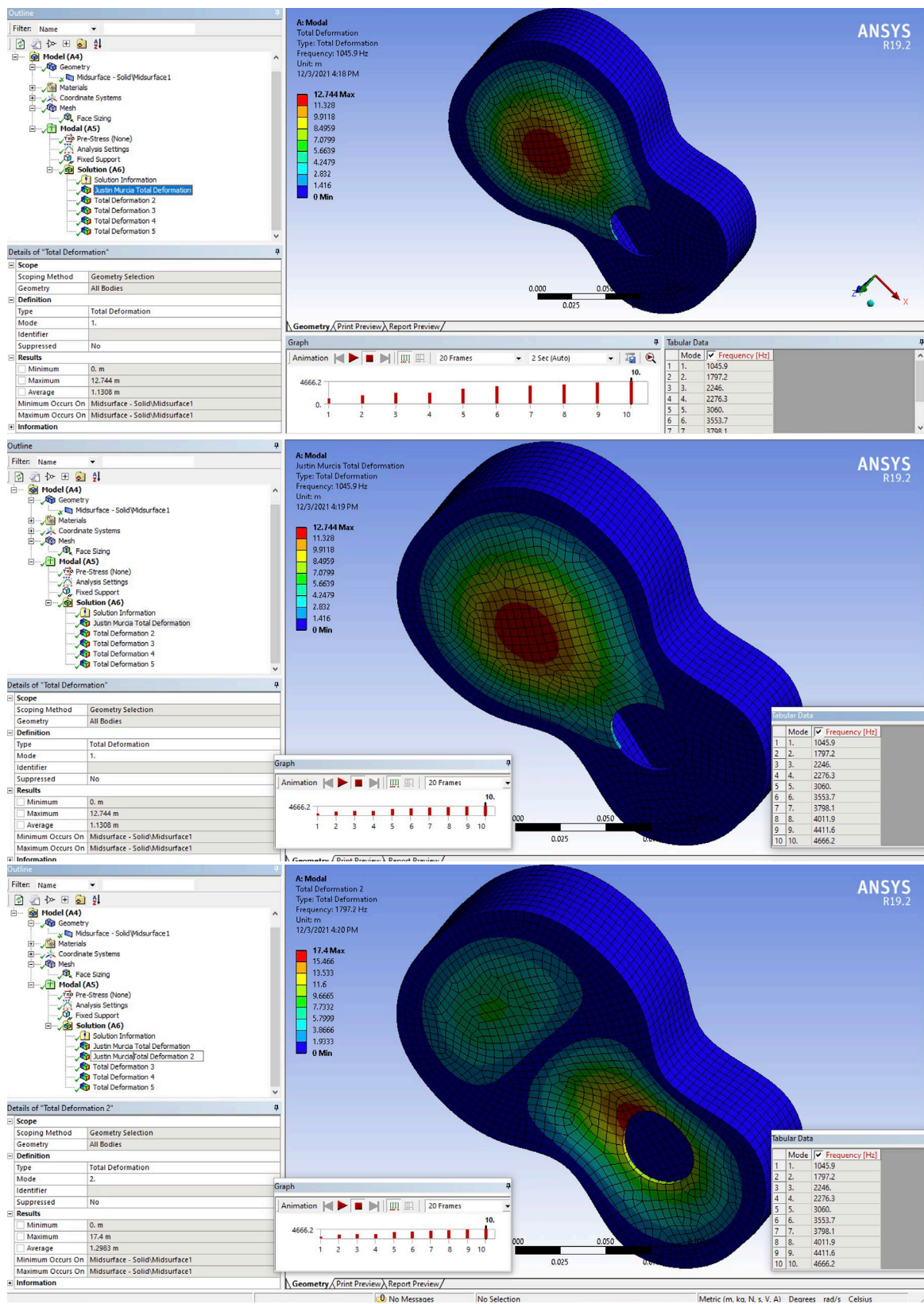


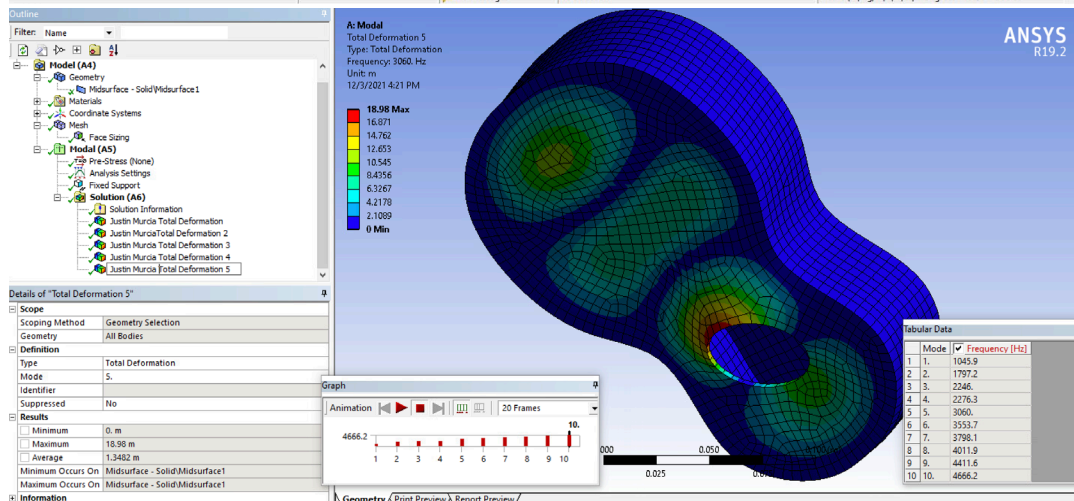
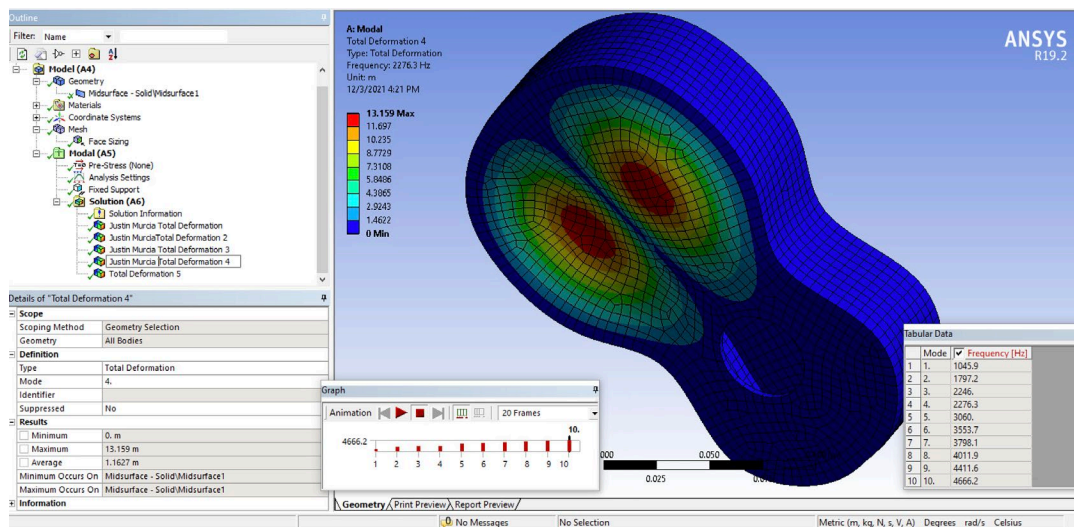
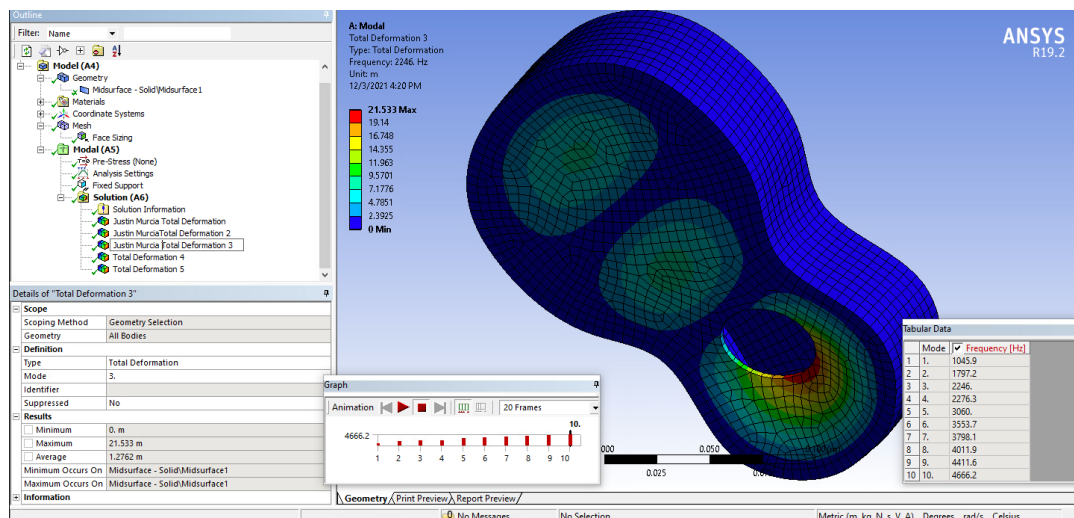
ABOVE AVERAGE





AVERAGE

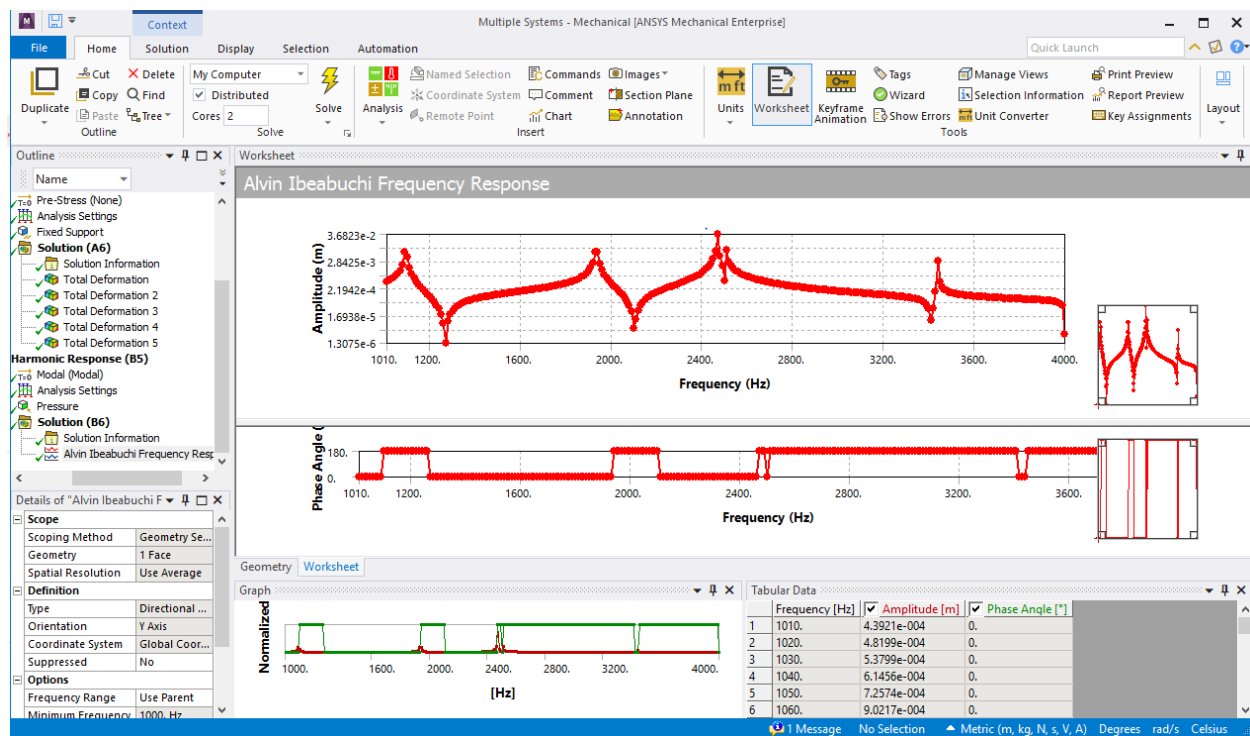




Comments:

Mode 2 and Mode 1 are duplicate; missing frequency analysis.

BELOW AVERAGE



Comments:

Only submitted frequency response; missing the results of five vibration modes.

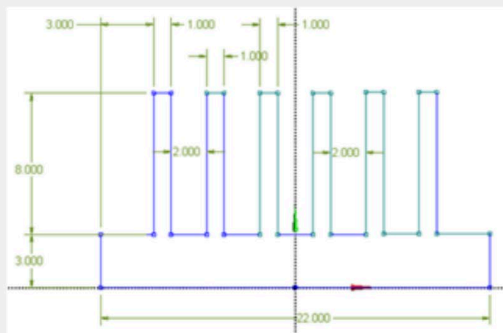
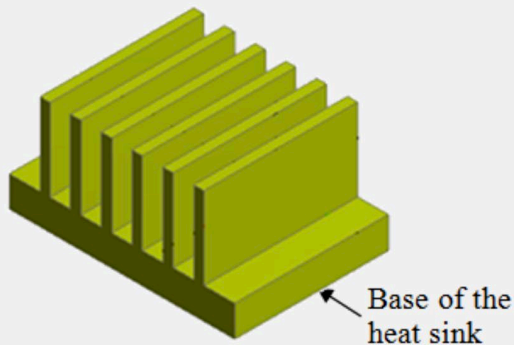
FINITE ELEMENT ANALYSIS AND DESIGN FALL2021

Title of Assignment: Homework 11

TOPICS COVERED IN HOMEWORK 11

Use steady-state thermal, transient thermal, and steady-state structure programs in ANSYS to analyze the temperature distribution in a heat sink under a heat flux for steady state and transient thermal response. The 2nd case is for temperature distribution under thermal conditions with different temperature environments. Also find out thermal stress under temperature load in multi-physics situation.

<Problem Description> Heat sinks are commonly used to enhance heat dissipation from electronic devices. In this Case Study, we conduct thermal analysis of a heat sink made of aluminum with thermal conductivity $k = 170 \text{ W/(m}\cdot\text{K)}$, density $\rho = 2800 \text{ kg/m}^3$, specific heat $c = 870 \text{ J/(kg}\cdot\text{K)}$, Young's modulus $E = 70 \text{ GPa}$, Poisson's ratio $\nu = 0.3$ and thermal expansion coefficient $\alpha = 22 \times 10^{-6} / ^\circ\text{C}$. A fan forces air over all surfaces of the heat sink except for the base, where a heat flux q' is prescribed. The surrounding air is 28°C with a heat transfer coefficient of $h = 30 \text{ W/(m}^2\cdot^\circ\text{C)}$. **(Part A):** Study the steady-state thermal response of the heat sink with an initial temperature of 28°C and a constant heat flux input of $q' = 1000 \text{ W/m}^2$. **(Part B):** Suppose the heat flux is a square wave function with period of 90 seconds and magnitudes transitioning between 0 and 1000 W/m^2 . Study the transient thermal response of the heat sink in 180 seconds by using the steady-state solution as the initial condition. **(Part C):** Suppose the base of the heat sink is fixed. Study the thermal stress response of the heat sink by using the steady-state solution as the temperature load.



All dimensions are in millimeters.

Material: Aluminum

$k = 170 \text{ W/(m}\cdot\text{K)}$

$\rho = 2800 \text{ kg/m}^3$; $c = 870 \text{ J/(kg}\cdot\text{K)}$

$E = 70 \text{ GPa}$; $\nu = 0.3$

$\alpha = 22 \times 10^{-6} / ^\circ\text{C}$

Boundary Conditions:

Air temperature of 28°C ; $h = 30 \text{ W/(m}^2\cdot^\circ\text{C)}$.

Steady-state: $q' = 1000 \text{ W/m}^2$ on the base.

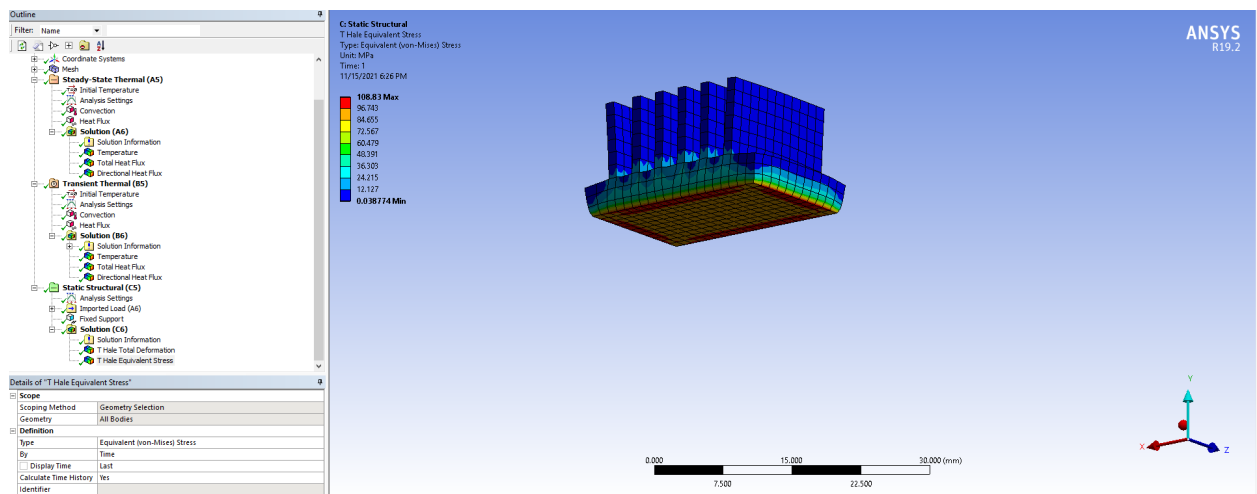
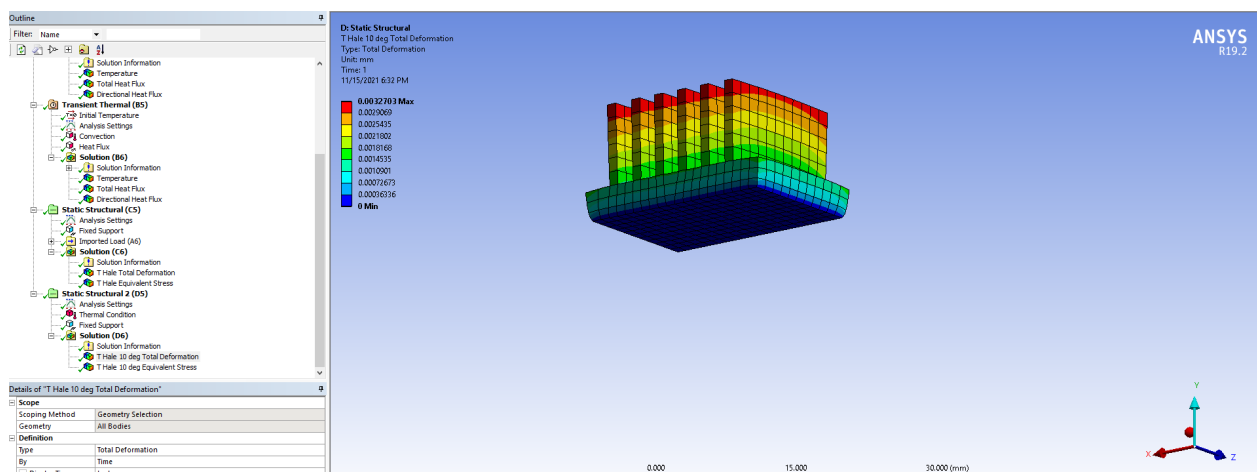
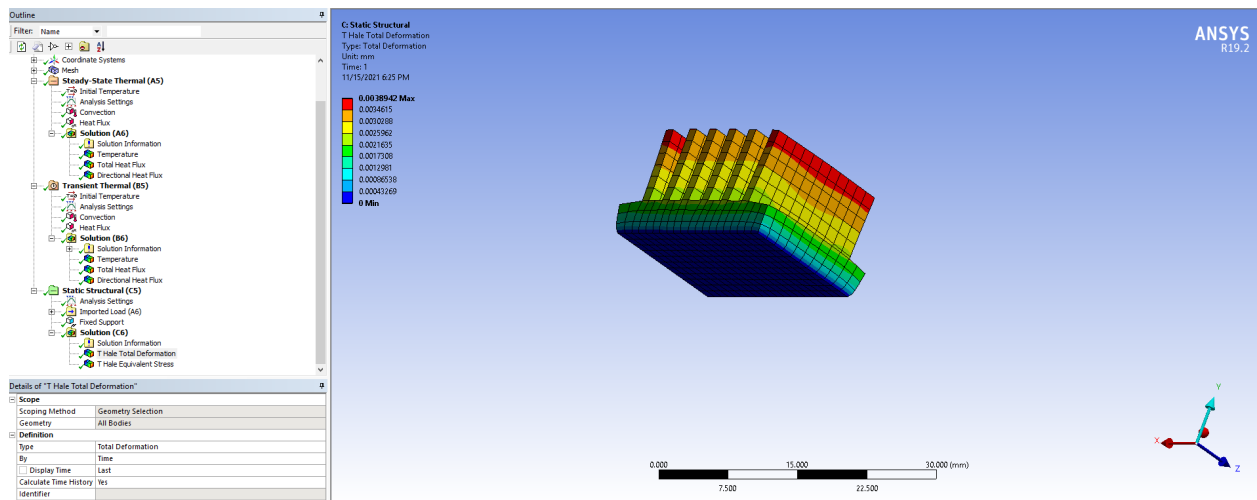
Transient: Square wave heat flux on the base.

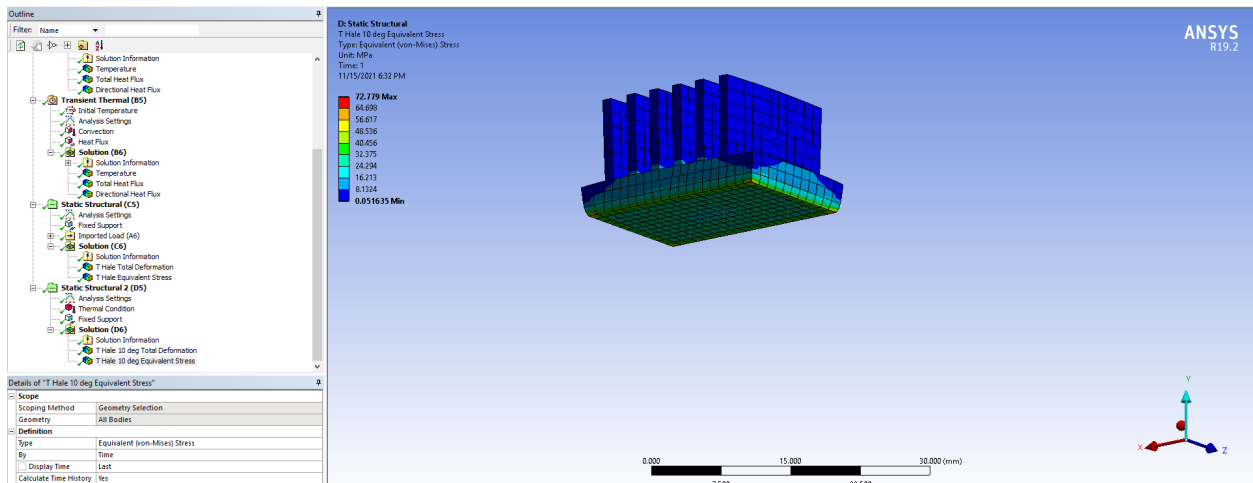
Initial Conditions:

Steady-state: Uniform temperature of 28°C .

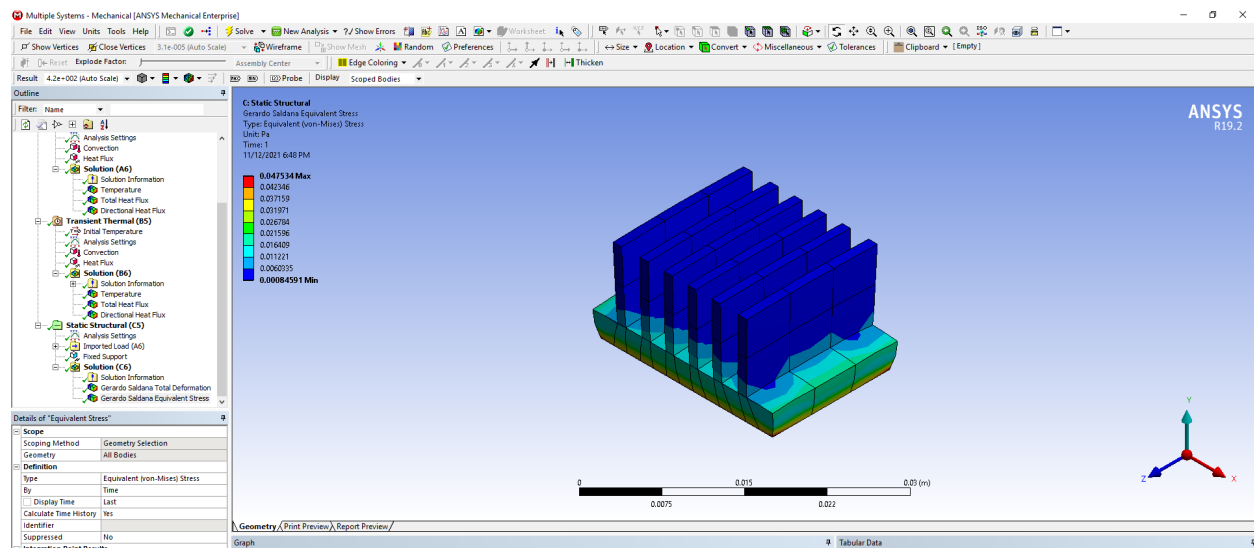
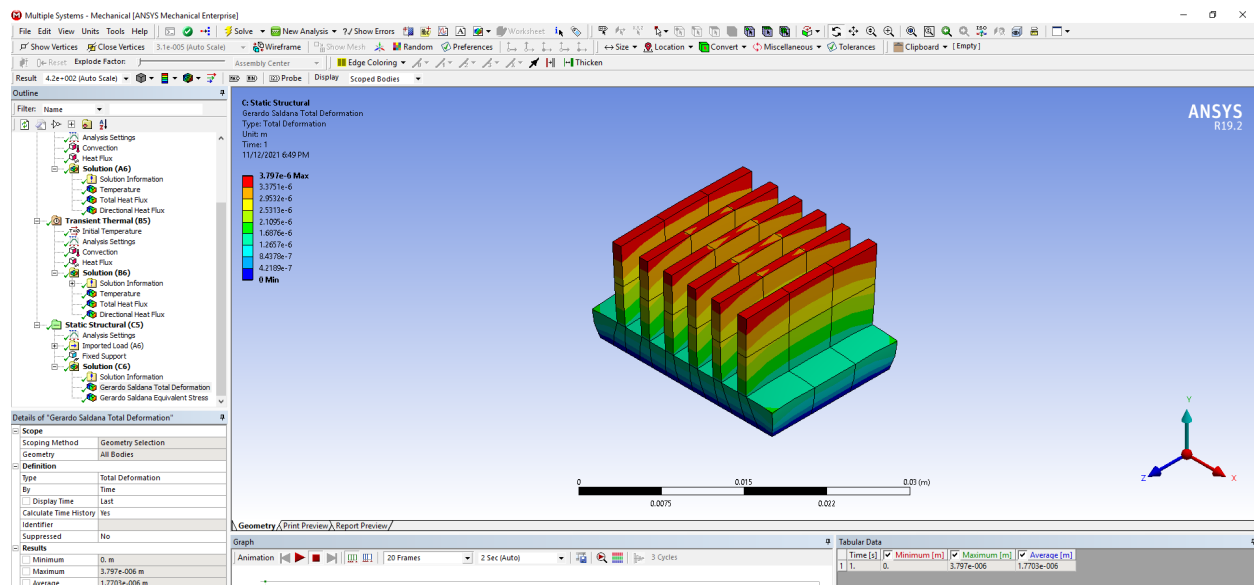
Transient: Steady-state temperature results.

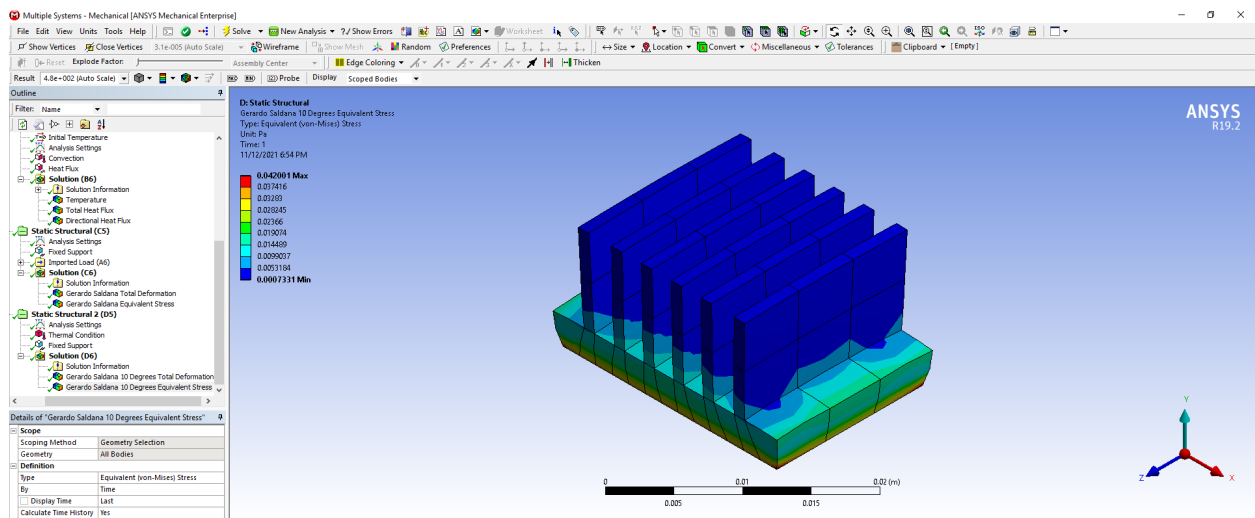
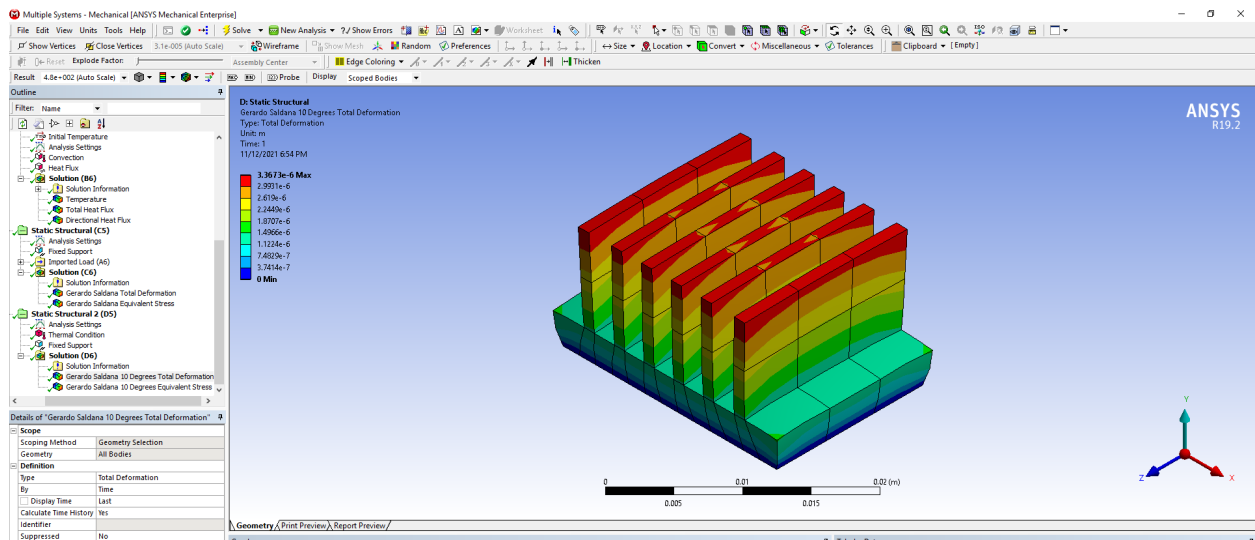
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Assignment Comments

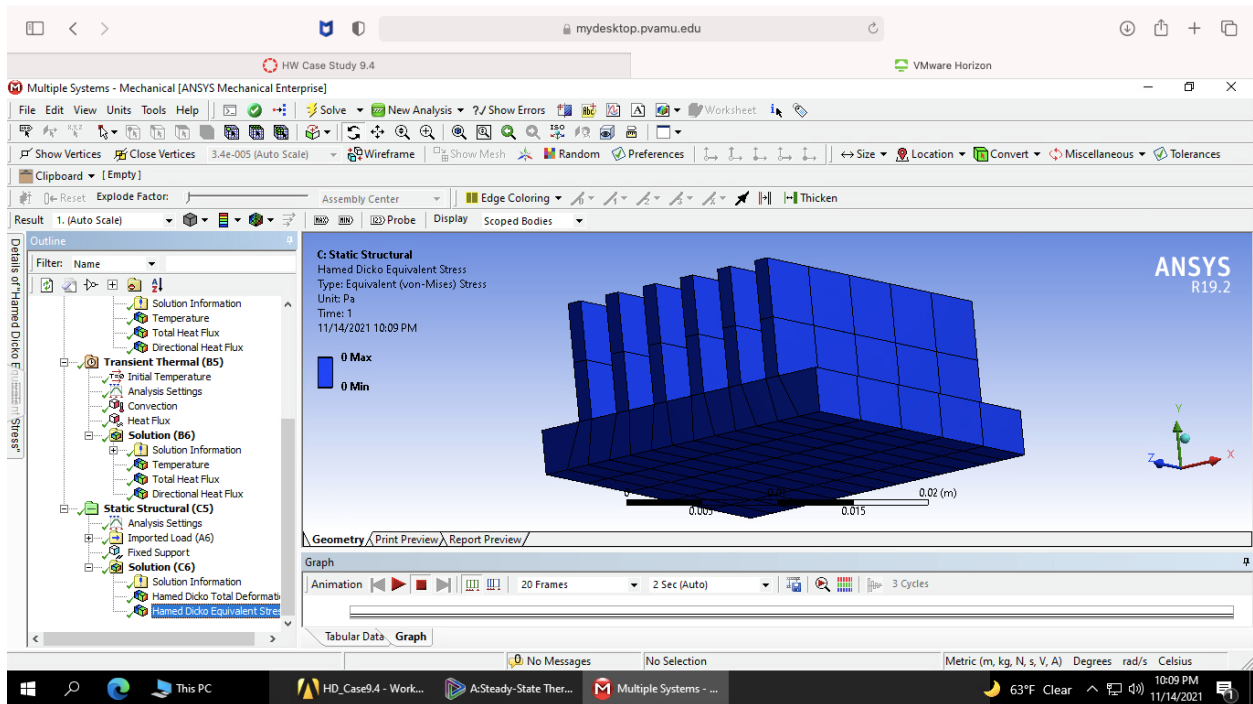
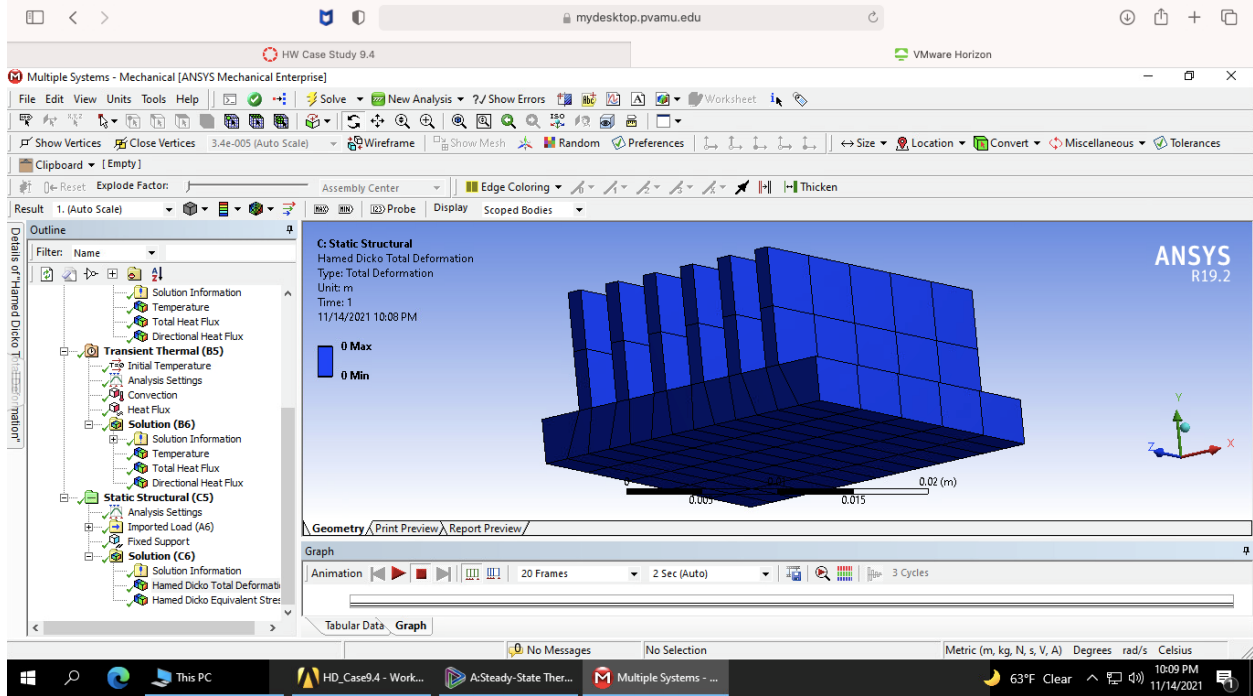


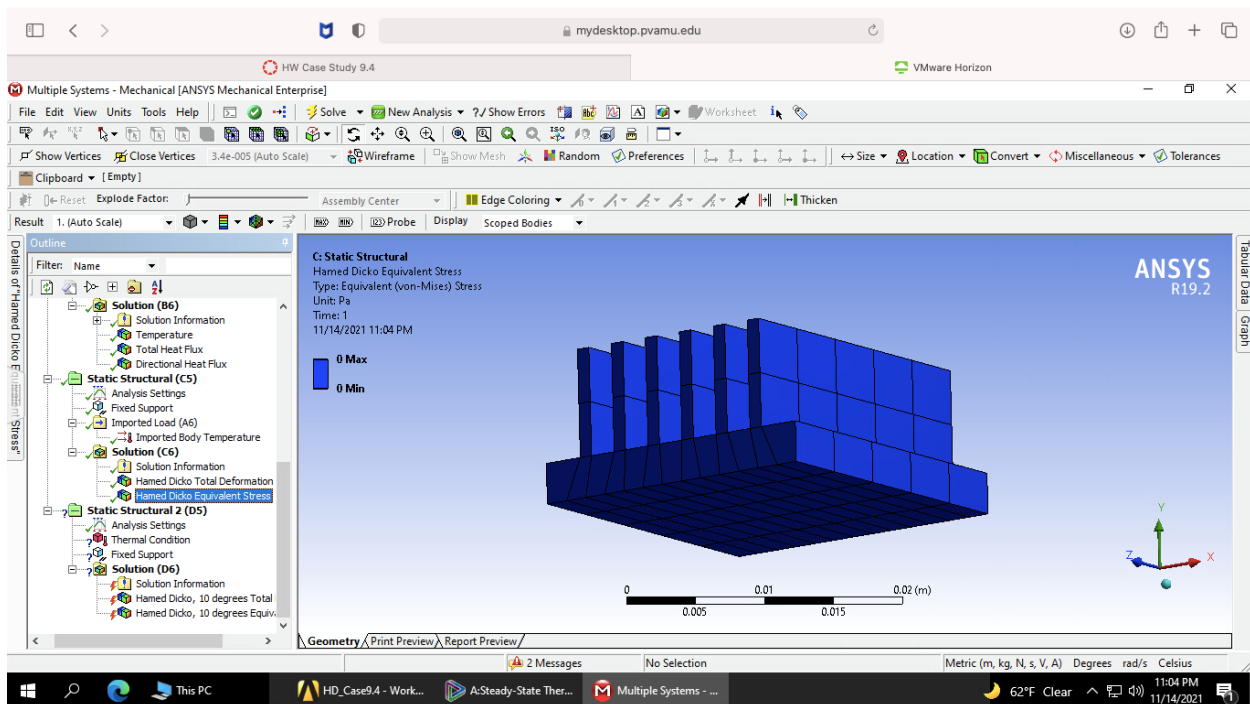
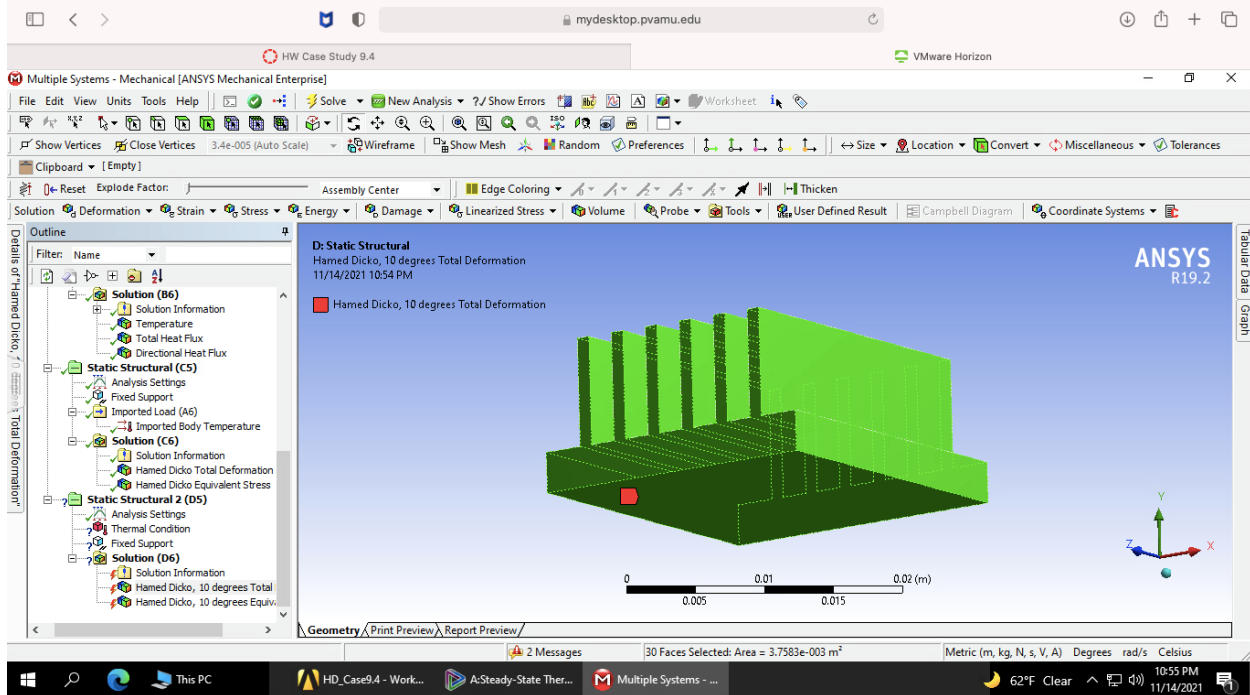
equivalent stresses are too low.



Shield Lin, Nov 19 at 4:38pm

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Comments:

Wrong boundary conditions/thermal loads result the zero values of deformation and thermal stresses.

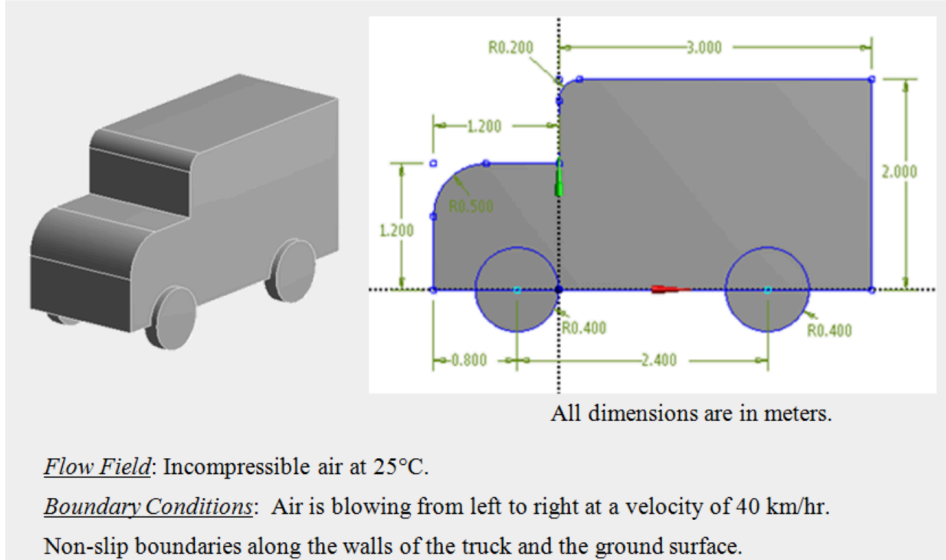
**FINITE ELEMENT ANALYSIS AND DESIGN
FALL2021**

Title of Assignment: Homework 12

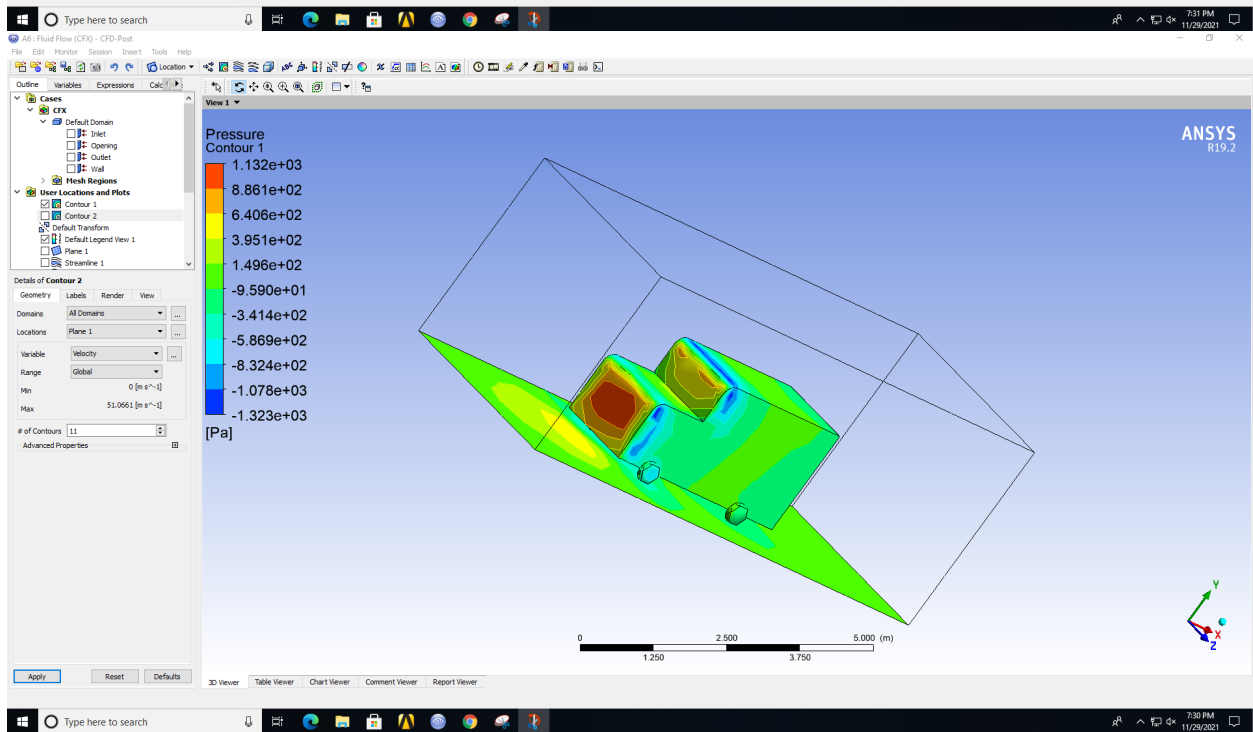
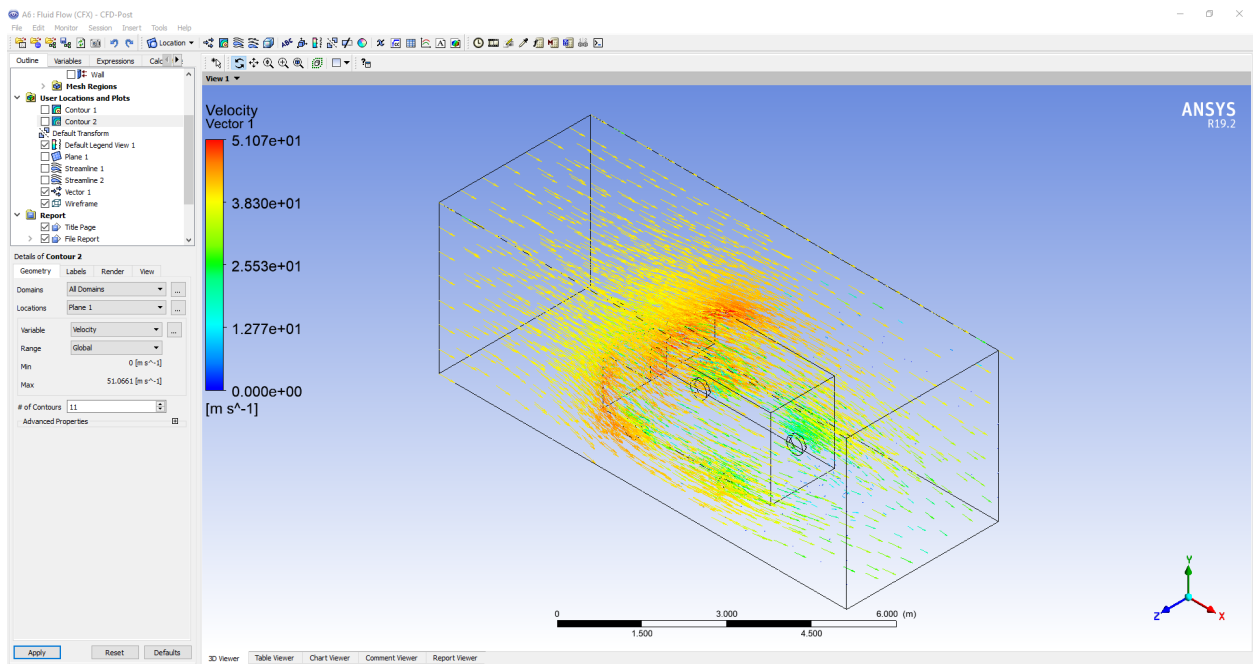
TOPICS COVERED IN HOMEWORK 12

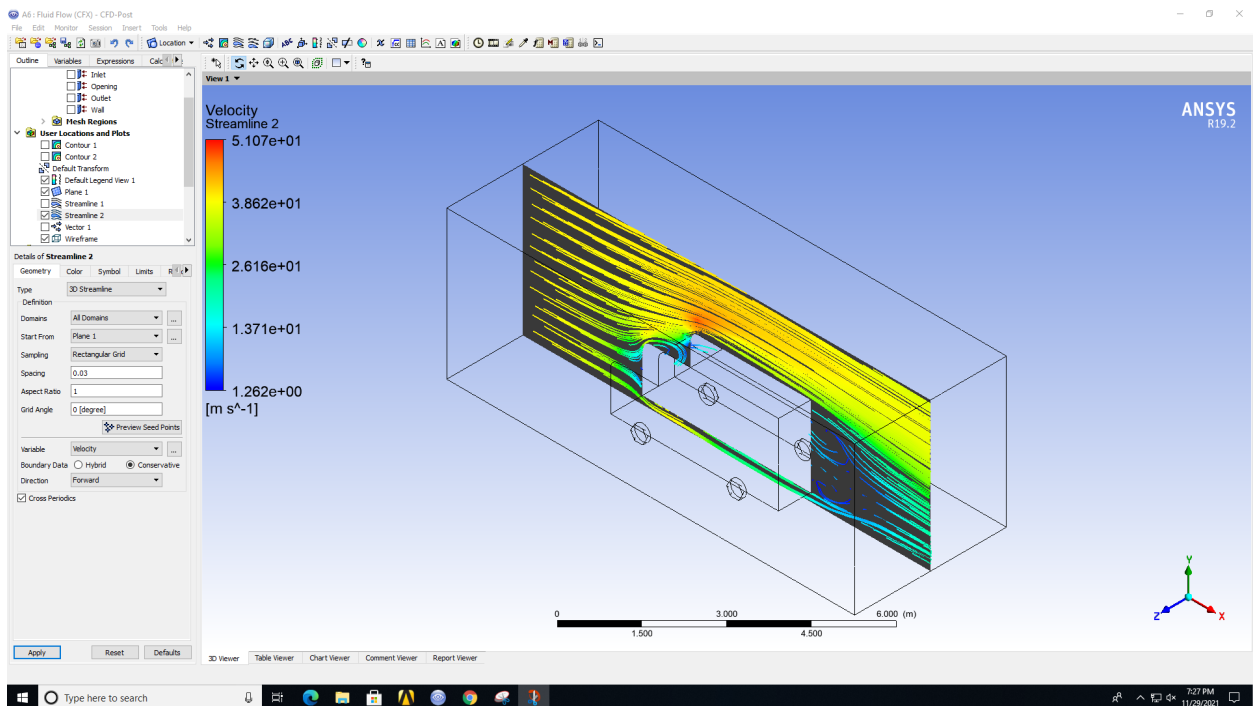
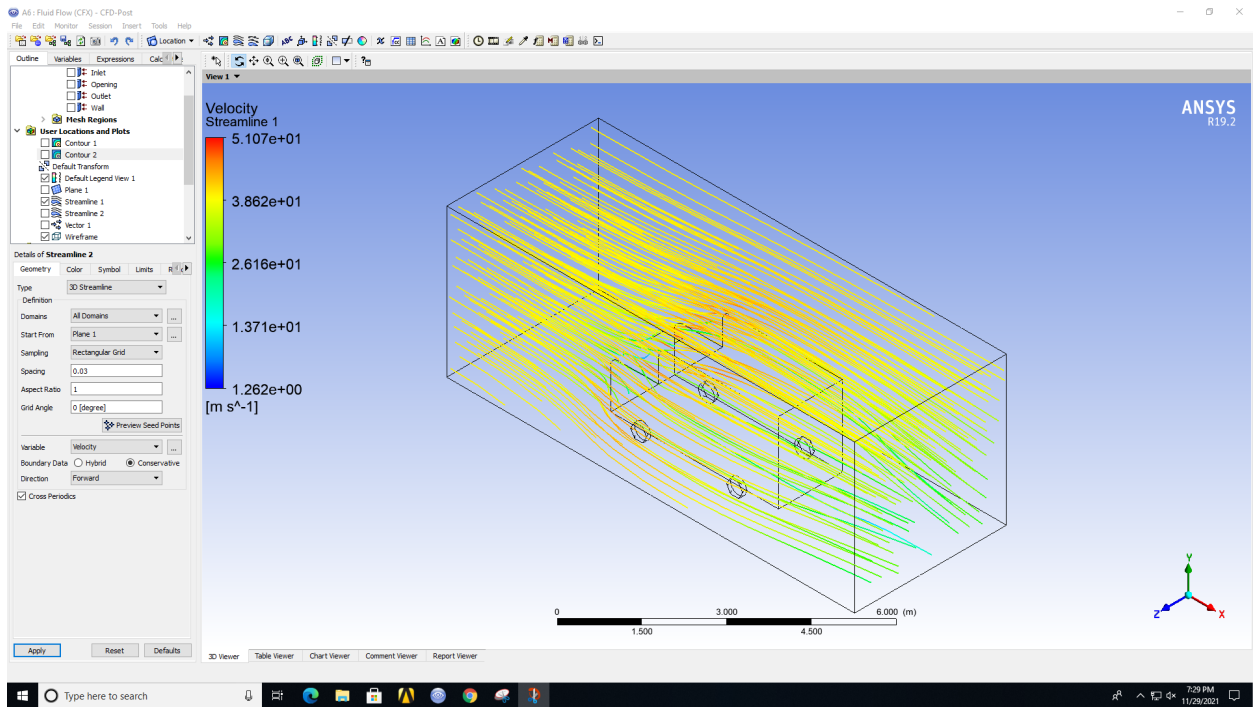
Use computational fluid flow program in ANSYS to analyze the pressure contour and velocity stream of a truck driving in a constant speed.

<Problem Description> The aerodynamic performance of vehicles can be improved by utilizing computational fluid dynamics simulation. Conduct fluid analysis of the air flow passing through a truck. Assume air at room temperature of 25°C for the flow field with an air velocity of 40 km/hr blowing from left to right. Use non-slip boundary conditions along the walls of the truck and the ground surface. Find the airflow pattern as well as the pressure and velocity distributions of the flow field around the truck.

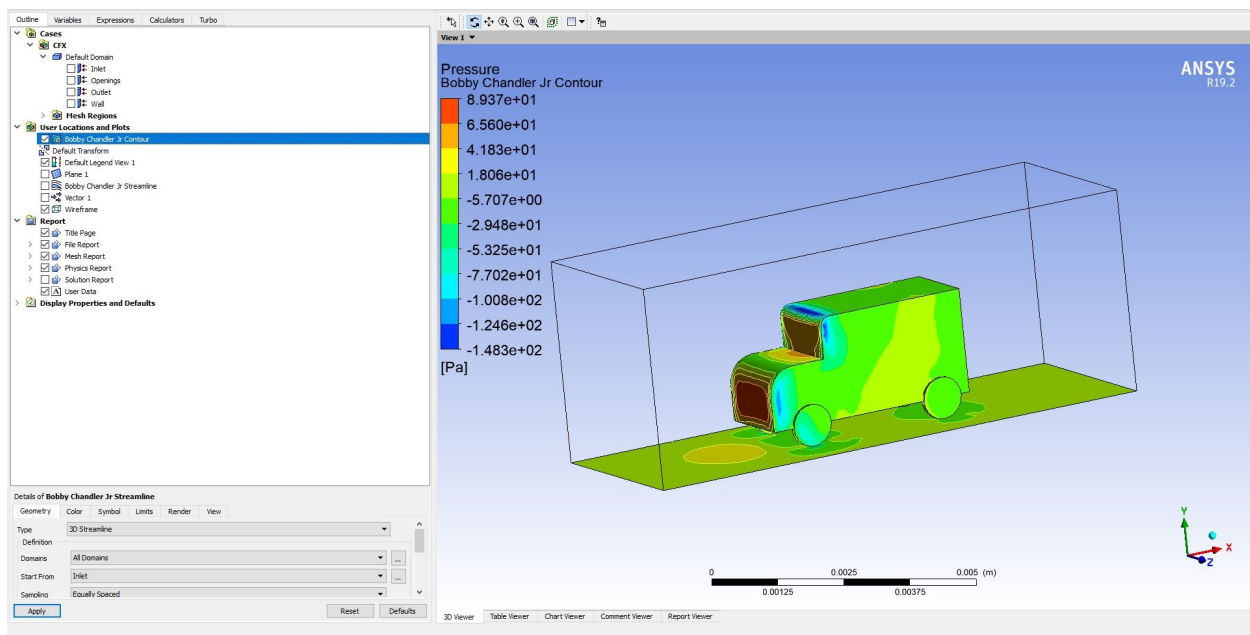
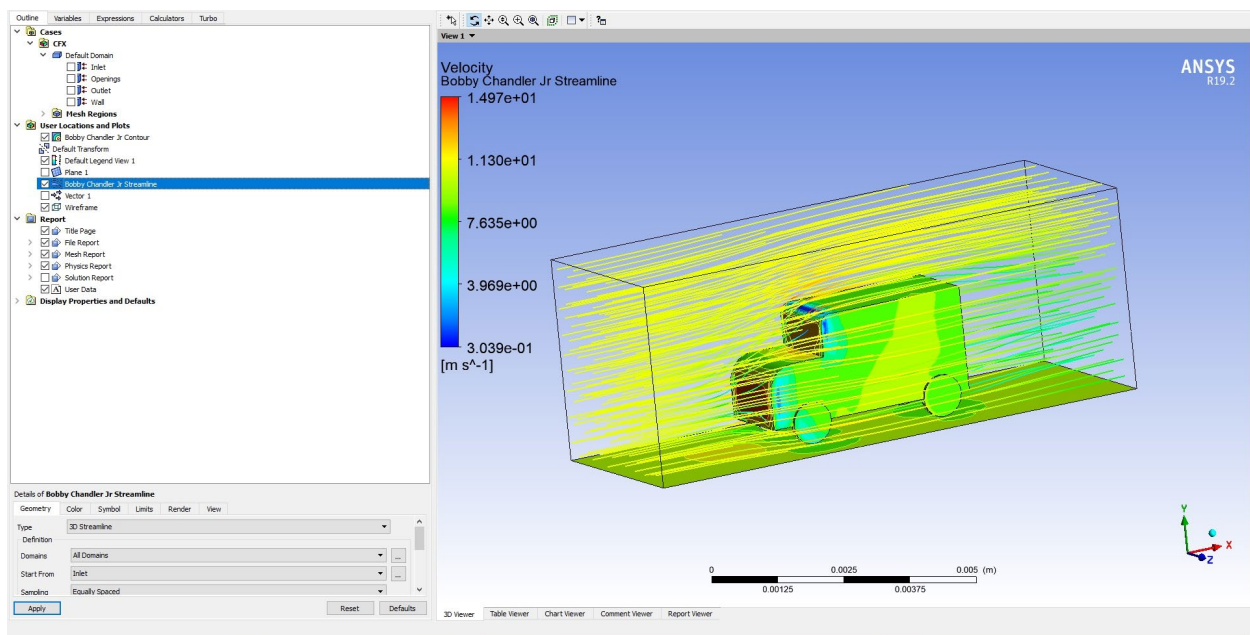


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AVERAGE



BELOW AVERAGE



Comments: Did not complete analysis.

FINITE ELEMENT ANALYSIS AND DESIGN

FALL 2021

Title of Assignment: Design Project

TOPICS COVERED IN DESIGN PROJECT

Title: Heat Sink for Electronic Device

Many electronic devices generate a great deal of heat during operations. Overheating could cause malfunction and reduce the lifespan of the devices. A heat sink with other accessories is commonly used to take the excessive heat away and keep the electronic device under an acceptable temperature.

- (1) You are required to investigate the design of an existing heat sink on the market. The product will serve as the benchmark design. Use a Finite Element Analysis Program (ANSYS or an equivalent) to determine temperature distribution in the benchmark heat sink under a force convective condition under an electrical fan. Calculate the convective coefficient based on air speed and configuration of the heat sink, and apply the coefficient in your ANSYS analysis.
- (2) You shall modify or completely redesign the heat sink using the same material and within the original outer dimensions. Prove via the Finite Element Analysis Program that **“your redesigned heat sink is better,”** i.e., your heat sink can **“keep the electronic device cooler”** than the benchmark heat sink under the same operational conditions.
- (3) If you are allowed to select a different material for the heat sink, what material will you select? Perform Finite Element Analysis of your heat sink using the new material. Discuss the performance of your heat sink with the new material.
- (4) Discuss the estimated cost of your heat sink in comparison to the benchmark.

Design Project
For MCEG 4309 - Finite Element Analysis & Design
Grading Rubric

Title: Design & Finite Element Analysis of a Heat Sink for Electronic Device

Grading Rubric

Item	Content Description	Assigned Points	Points Received	Relevant to ABET assessment
1. Engineering Drawings.	Create engineering drawings with detailed dimensions and materials from a heat sink on the market & design a modified heat sink with new configuration and dimensions.	20		<i>an ability to design a system, component, or process to meet desired needs.</i>
2. Information Research & Calculation of Parameters.	Describe the critical operational conditions of the heat sink. Research and calculate all parameters required for finite element analysis.	20		<i>a recognition of the need for, and an ability to engage in life-long learning.</i>
3. Finite Element Analysis	Create solid models on computer, input parameters to a finite element program- ANSYS or equivalent, solve and obtain results.	20		<i>an ability to use the techniques, skills, and modern engineering tools, (such as ANSYS) necessary for engineering practice.</i>
4. Design Justification, Conclusion, and Discussions.	Interpolate finite element results, justify your design is better based on the analysis results, make a conclusion, and add discussions on materials and manufacturing cost.	20		<i>an ability to design a system, component, or process to meet desired needs.</i>
5. Report Writing	Use of technical writing format which generally includes abstract, introduction, equations, figures, tables, conclusion, references, etc. All parameters used in the analysis must be clearly justified by referring to the origin of technical data or calculated using appropriate methods.	20		<i>an ability to communicate effectively</i>
	Total	100		

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**MECHANICAL ENGINEERING DEPARTMENT
PRAIRIE VIEW A&M UNIVERSITY**



**MCEG 4309-P01
Finite Element Analysis and Design
Fall 2021**

COVER PAGE

**REPORT ON
“Heat Sink for Electronic Device”**

**Submitted by
Mohammed Bahaj**

**On
December 1, 2021**

**To
Dr. Shield Lin**

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Introduction

This report is about the steady state and transient thermal analysis of a heat sink. ANSYS 2019 R3 was used to analyze the temperature and heat flux in the heat sink when a load of 28 degree Celsius and 1000 W/m² heat flux was applied. For part b the thermal response of the heat sink for an applied temperature of 28 degree Celsius and 1000 W/m² heat flux input for a period of 180 seconds. After the two thermal analysis steady state and transient, the heat sink was tested for the static response for the steady state thermal conditions applied and only the bottom face of the heat sink was fixed. The results were obtained for total deformation and equivalent stress. Lastly, the final results for the static analysis were obtained using the initial temperature of 22 and final temperature of 32 degree were applied alongside the steady state thermal loads and total deformation, and stress were determined. This report contains the methodology, 3d model, material selected, meshing, boundary conditions and results for each of the analysis performed on heat sink (Lim, 2005) (Gupta, 2014).

Methodology

ANSYS 2019 R3 workbench was used for thermal and static analysis, four different analysis were performed on the same geometry, material and meshing. The workbench analysis models used for each of the analysis are given below:

Part A: Steady State Thermal Analysis

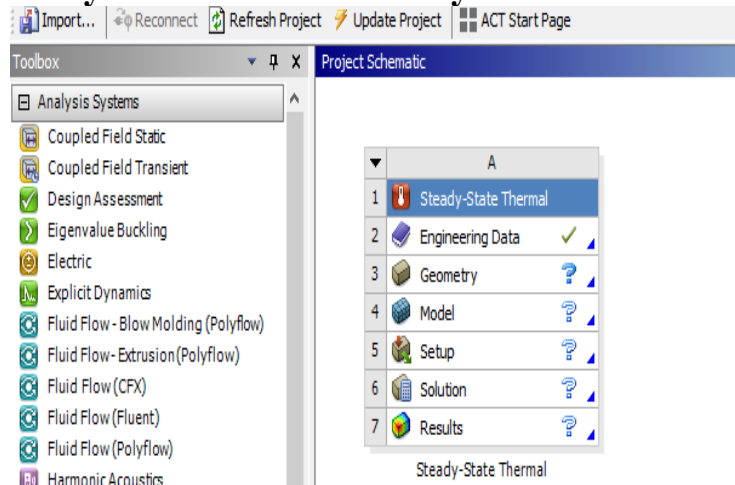


Figure 1. Steady State Thermal Analysis WB

Part B: Transient Thermal Analysis

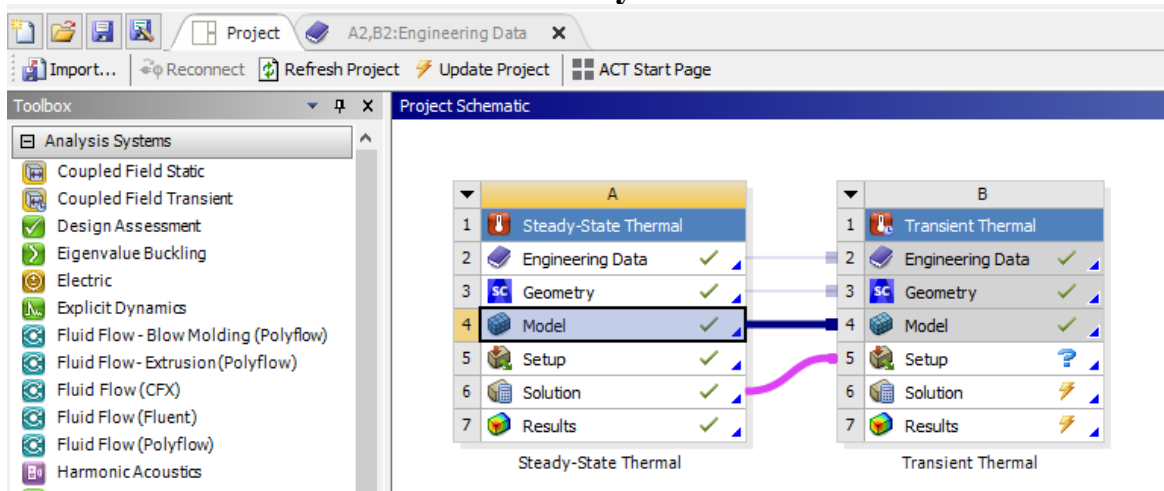


Figure 2. Transient Thermal Analysis WB

Part C: Static Structural Analysis 1

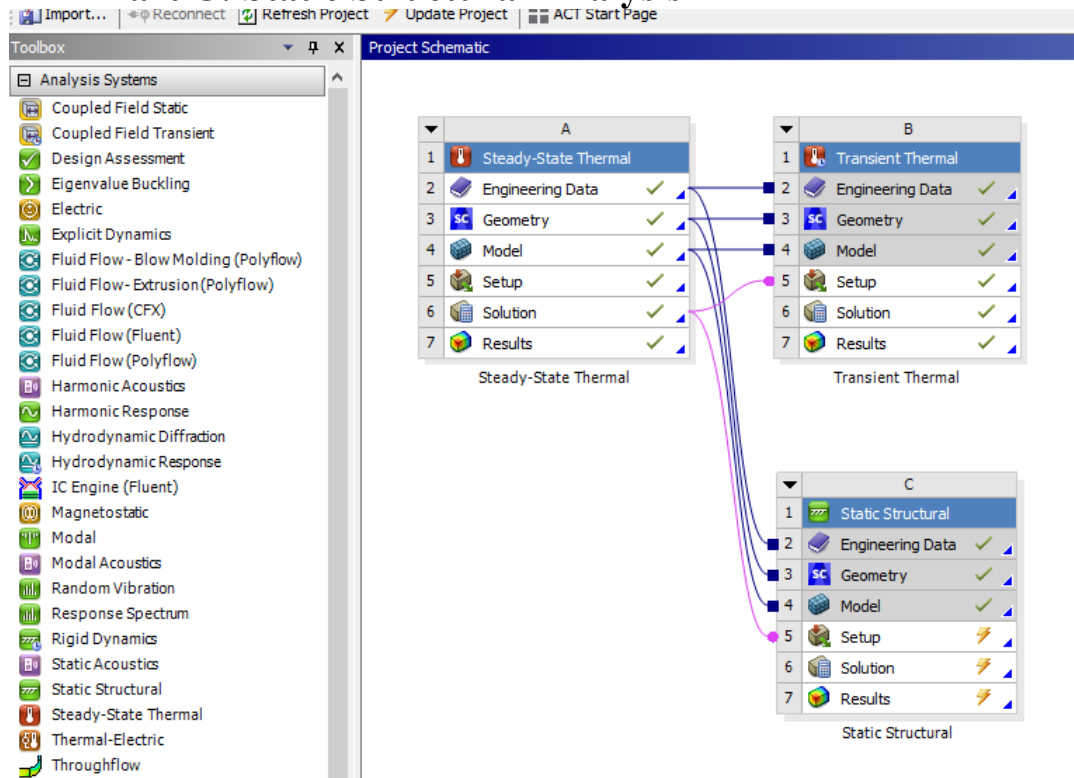


Figure 3. Static Analysis WB

Part D: Static Structural Analysis 2 (10 degrees)

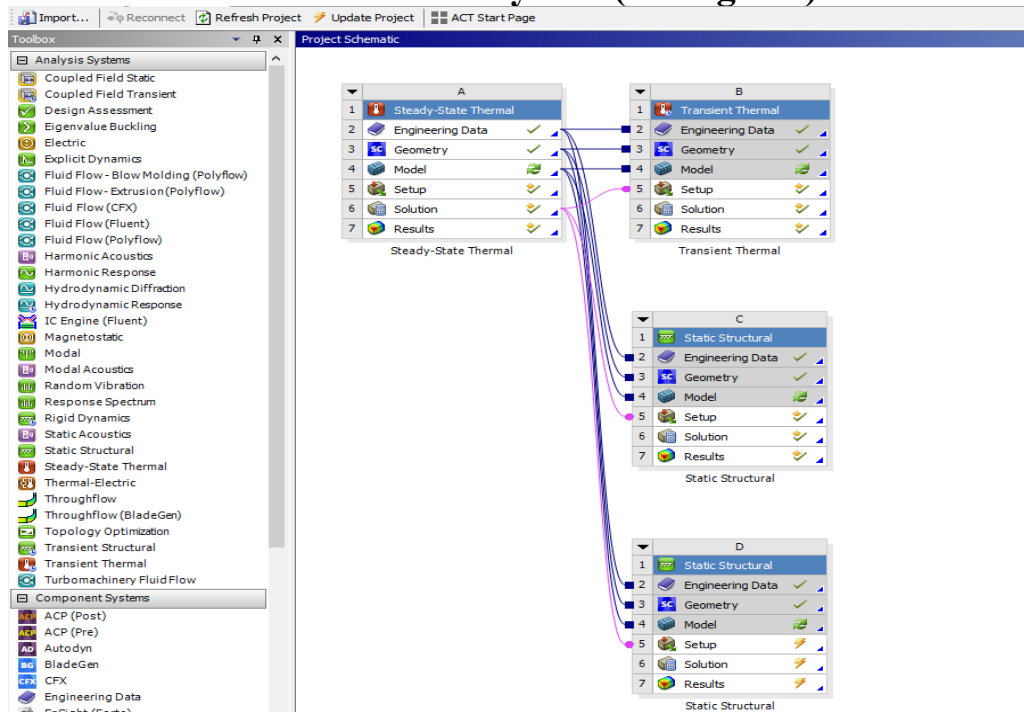
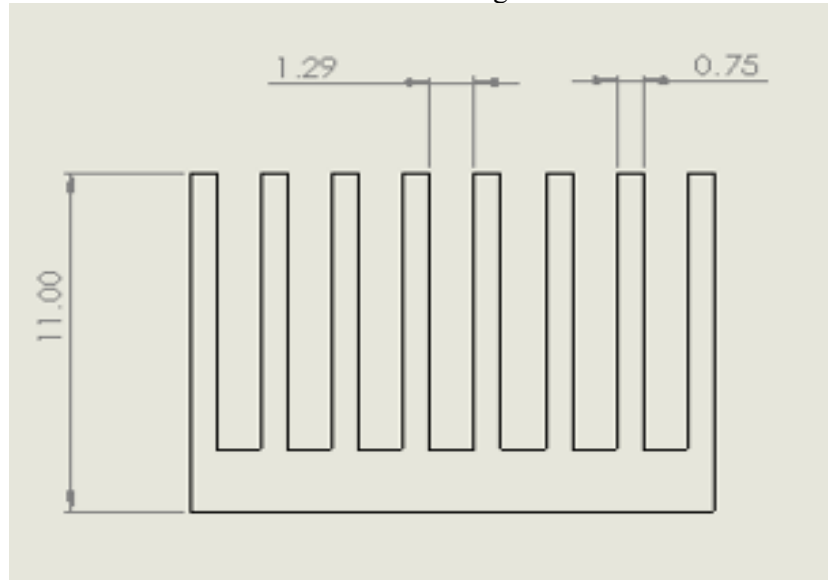


Figure 4. Static Analysis 2 (10 degrees) WB

Convective Heat Transfer Coefficient Calculation

The heat transfer analysis of the heat sinks requires the calculation and determination of the conditions like the heat load, convection heat transfer coefficient and temperatures for the finite element analysis of the model and for the comparisons of the results for different design using the same loading conditions. The external dimensions of the heat sink are shown in the figure below.



$$\text{Heat Transfer Coefficient} = h_c = \frac{(N_u * K)}{L}$$

Thermal conductivity of air = $K = 0.026 \text{ W/(m * C)}$

Room Temperature = 22°C

Temperature of convection = 28°C

Length of the heat sink = $L = 11 \text{ mm} = 0.011\text{m}$

$$Gr = \frac{g * L^3 * \beta * (T_p - T_a)}{\eta^2}$$

$$Pr = \frac{\mu * cp}{k}$$

η = kinematic viscosity of air = $1.86 * 10^{-5}$

$$Gr = \frac{9.81 * 0.011^3 * \frac{28 - 22}{273.16 + 22}}{(1.86 * 10^{-5})^2}$$

$$Gr = 772.44$$

Taking $N_u = 12.89$

$$\text{Heat Transfer Coefficient} = h_c = \frac{(N_u * K)}{L}$$

$$h_c = \frac{(N_u * K)}{L} = \frac{12.89 * 0.026}{0.011}$$

$$h_c = 30.46 \text{ W/m}^2\text{C}$$

Convection loads:

Using the following values for the finite element analysis as the convective loading and heat flux.

Convective heat transfer coefficient = $h = 30.4 \text{ W/m}^2 \text{ } ^\circ\text{C} = 30 \text{ W/m}^2 \text{ } ^\circ\text{C}$

Temperature of the heat sink = 28°C

Heat Flux from the device = 1000 W/m^2

Heat Sink Model 1 Benchmark Model

Heat sinks are used for the heat transfer for the electronic components; a computer processors heat sink was taken for the analysis of the design. The heat sink in use for the processor on the motherboard of the CPU in the figure below, the dimensions of the heat sink include the length, width and height of 15, 11 and 11 mm respectively. The design is shown in the figure below.

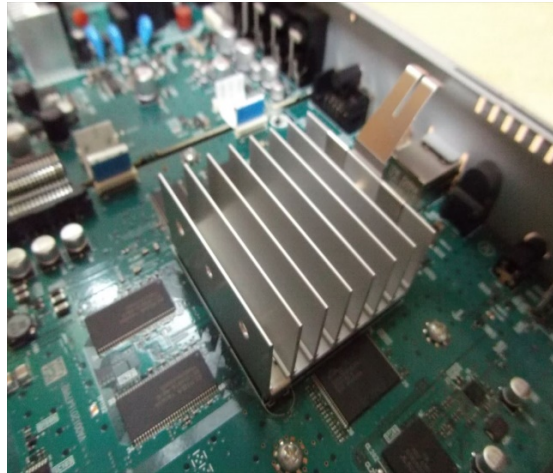


Figure 5. Heat Sink Model

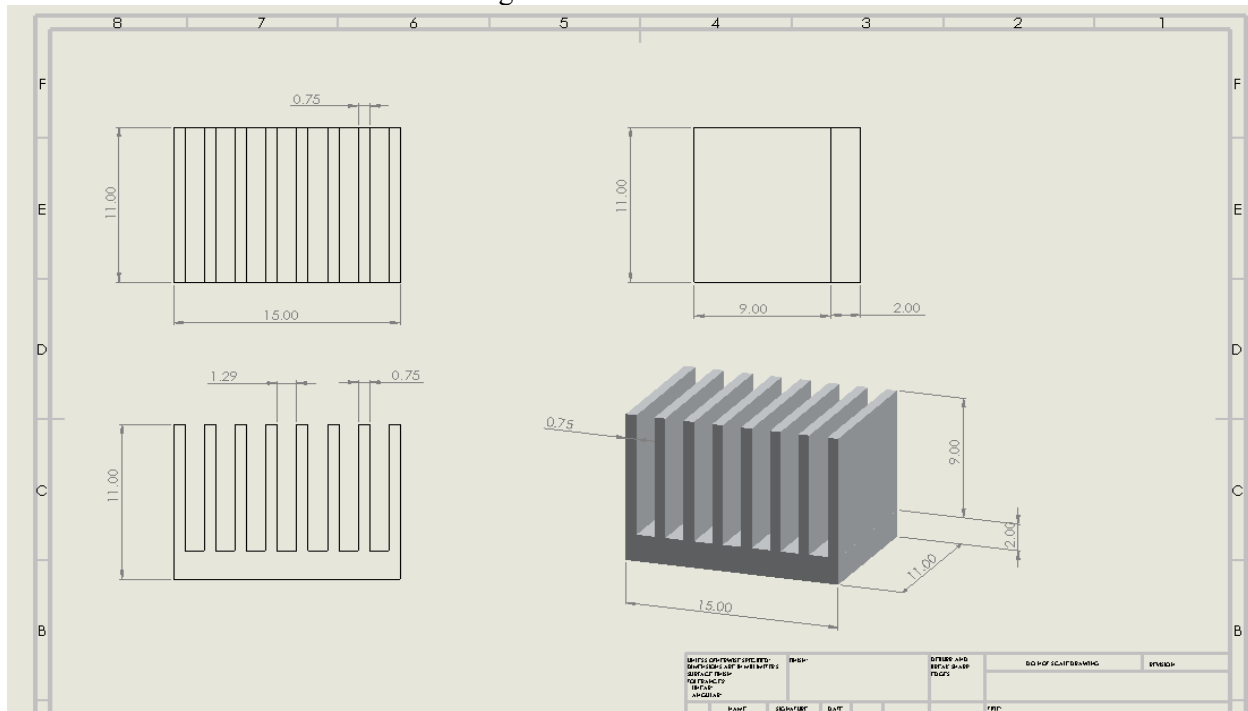
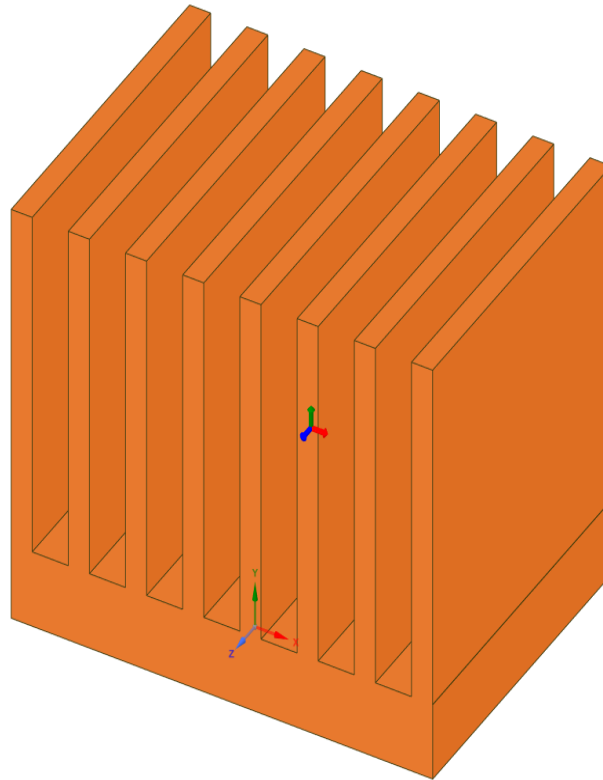


Figure 6. 2d Drawings of Heat Sink



Volume	924mm ³
Center of volume	(0.7898, 4.5357, -2.0883)mm
Total surface area	2126mm ²
Principal moment and axis	19853.8204mm ⁴ (1, -0.00034, 0)
Principal moment and axis	28446.7562mm ⁴ (0.00034, 1, 0)
Principal moment and axis	29666.5767mm ⁴ (0, 0, 1)
Known relative accuracy %	0.001

Figure 7. 3d Model of Heat sink

Volume	924mm ³
Center of volume	(0.7898, 4.5357, -2.0883)mm
Total surface area	2126mm ²
Principal moment and axis	19853.8204mm ⁴ (1, -0.00034, 0)
Principal moment and axis	28446.7562mm ⁴ (0.00034, 1, 0)
Principal moment and axis	29666.5767mm ⁴ (0, 0, 1)
Known relative accuracy %	0.001

Figure 8. Heat Sink Model Properties (Measurements)

Material Selection

The material selected for the analysis was Aluminum; all the materials properties of the material used are given in the figure below:

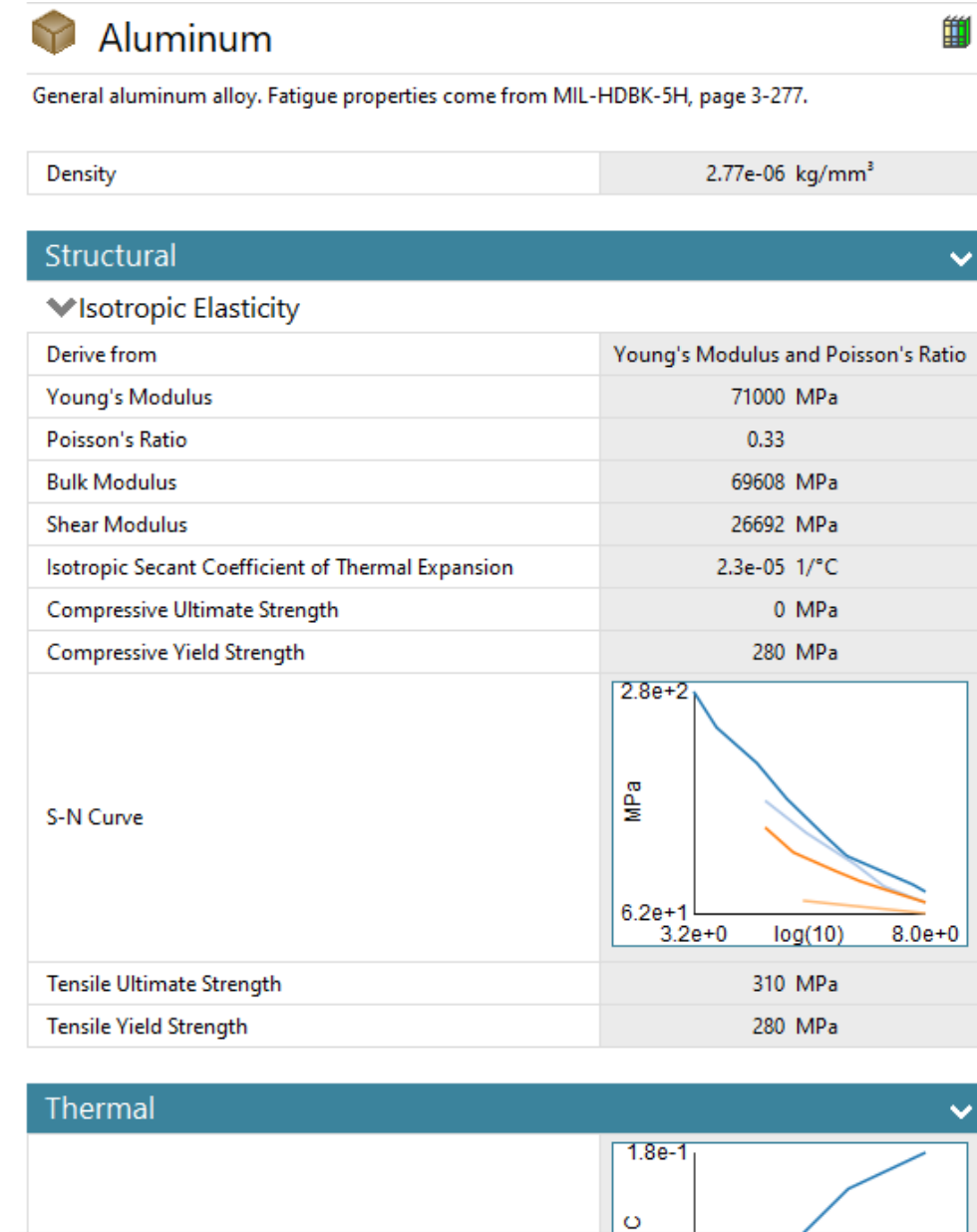


Figure 9. Aluminum Material properties

Meshing

The meshing for the heat sink model 1 was done using the mesh element size of 5×10^{-4} m, the meshed model and mesh statistics are given below:

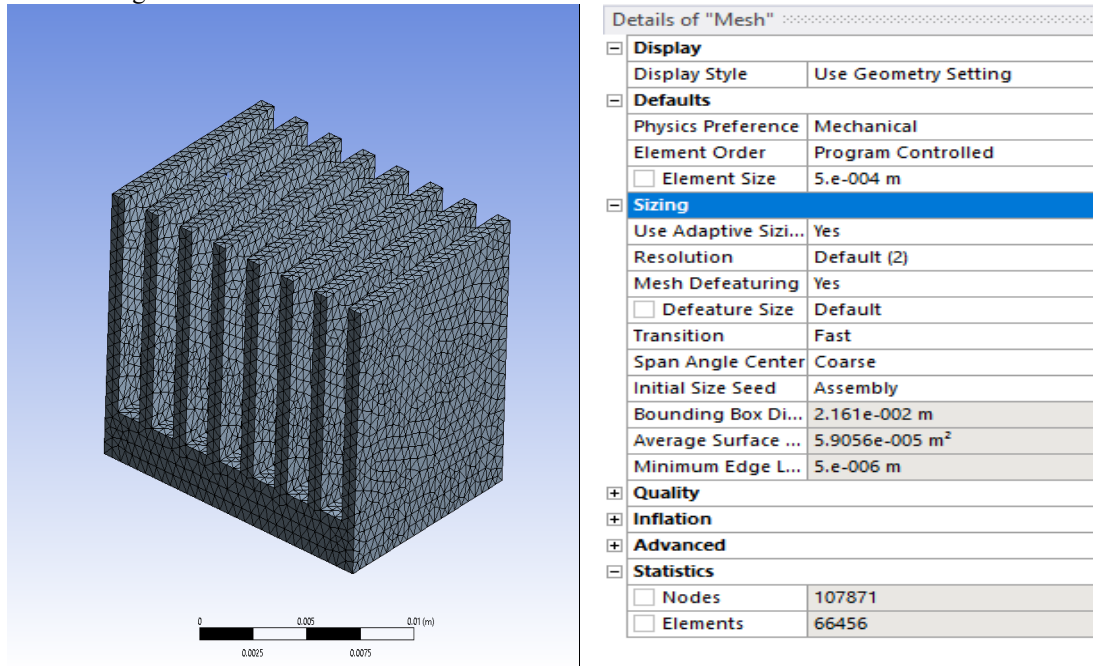


Figure 10. Meshed Model (5×10^{-3} m)

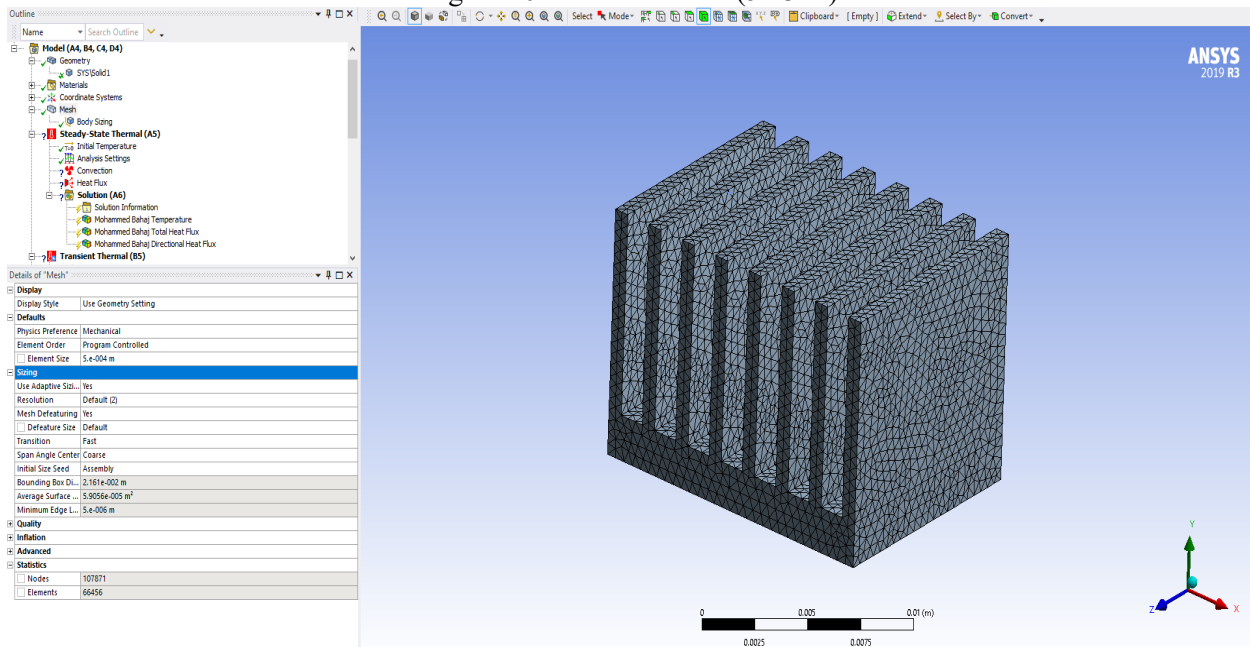


Figure 11. Meshed Model of heat sink 2 (5×10^{-3} m) statics and Model

Boundary Conditions

The boundary conditions section includes the applied thermal loads, such as the temperature, heat flux, and the fixtures for the static analysis.

Part A: Steady State Thermal Analysis

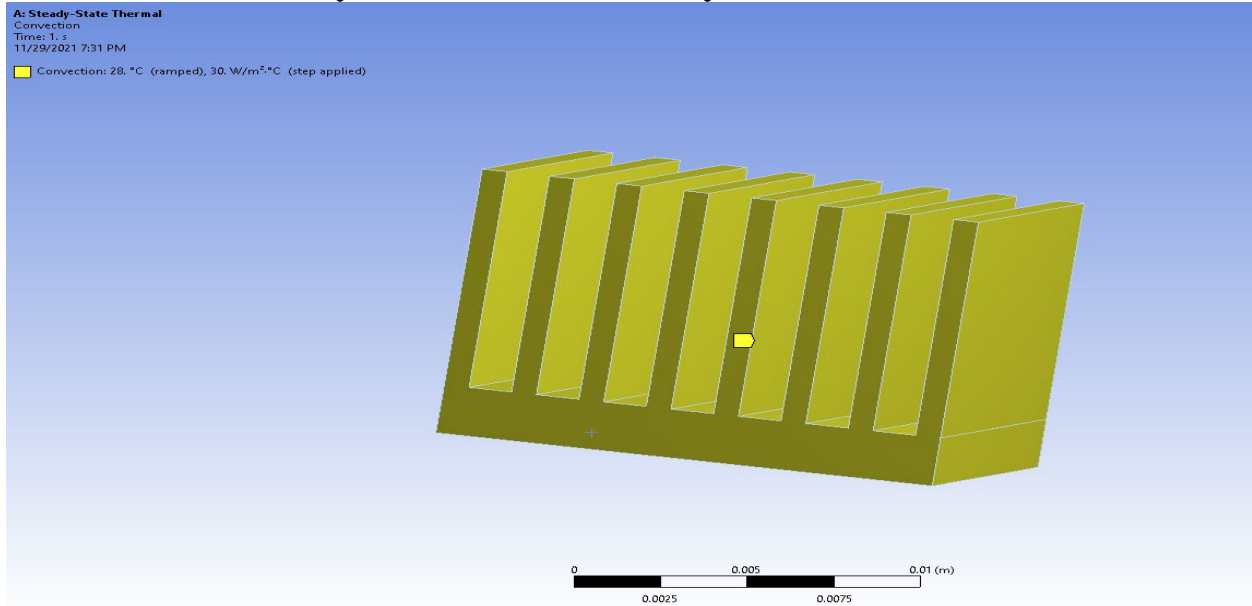


Figure 12. Convection Loads

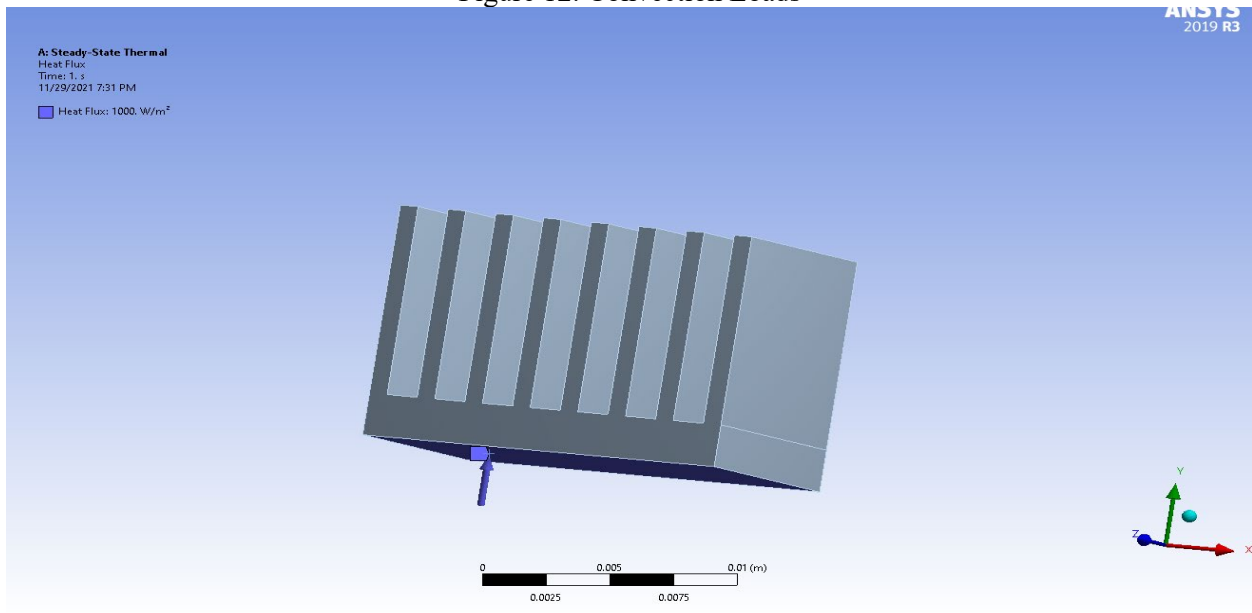


Figure 13. Heat Flux 1000 W/m²

Part B: Transient Thermal Analysis

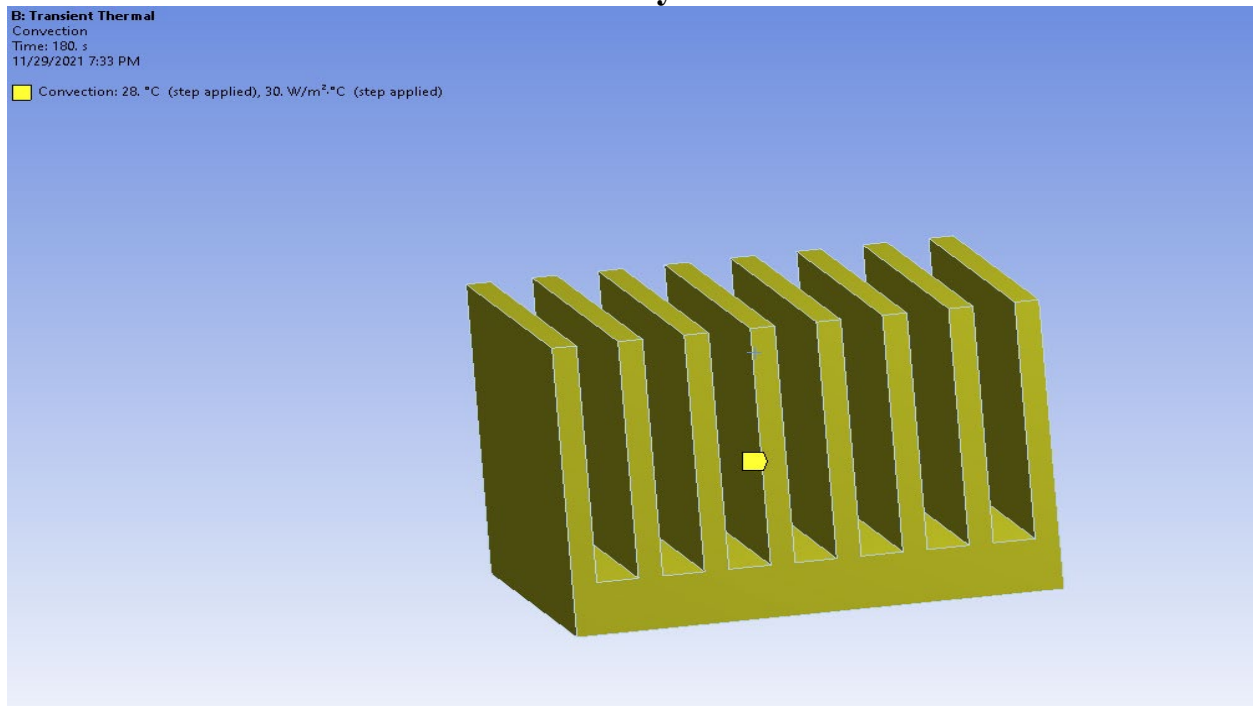


Figure 14. Convection Load 28 degree Celsius and 30 W/m.c

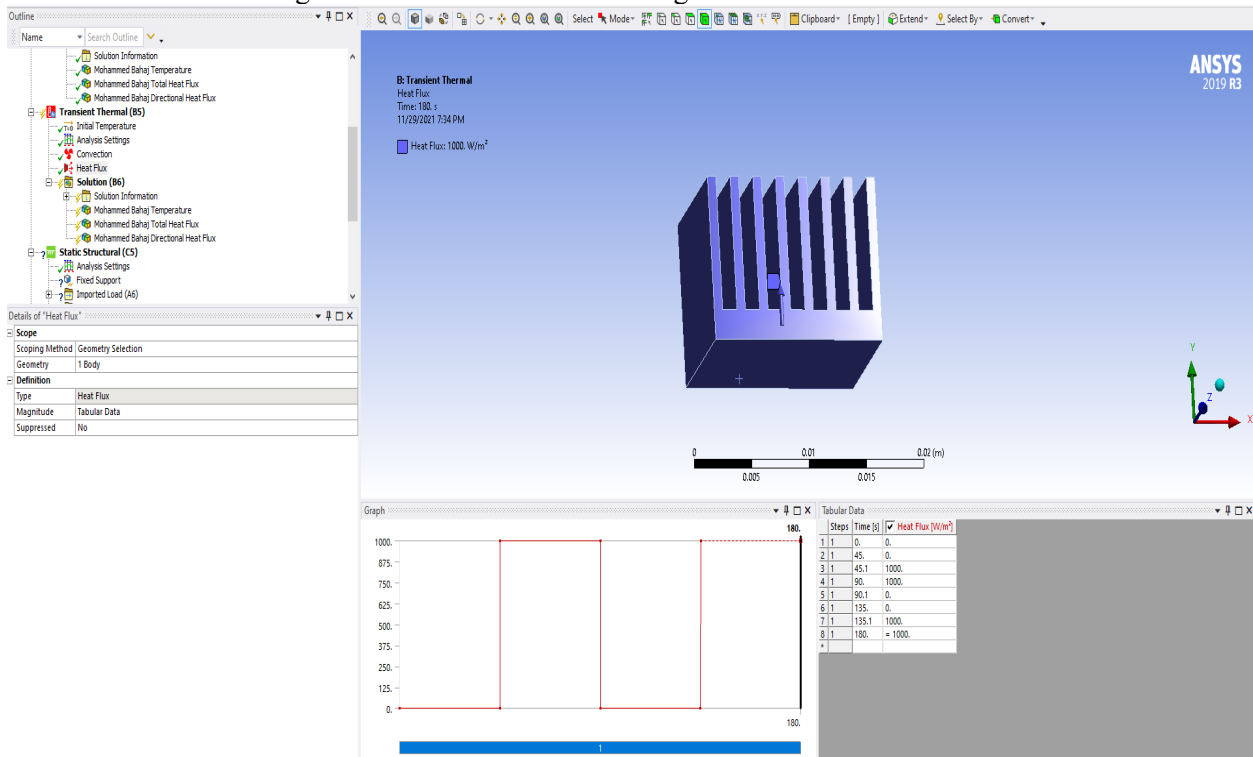


Figure 15. Tabular Heat Flux

Part C: Static Structural Analysis 1

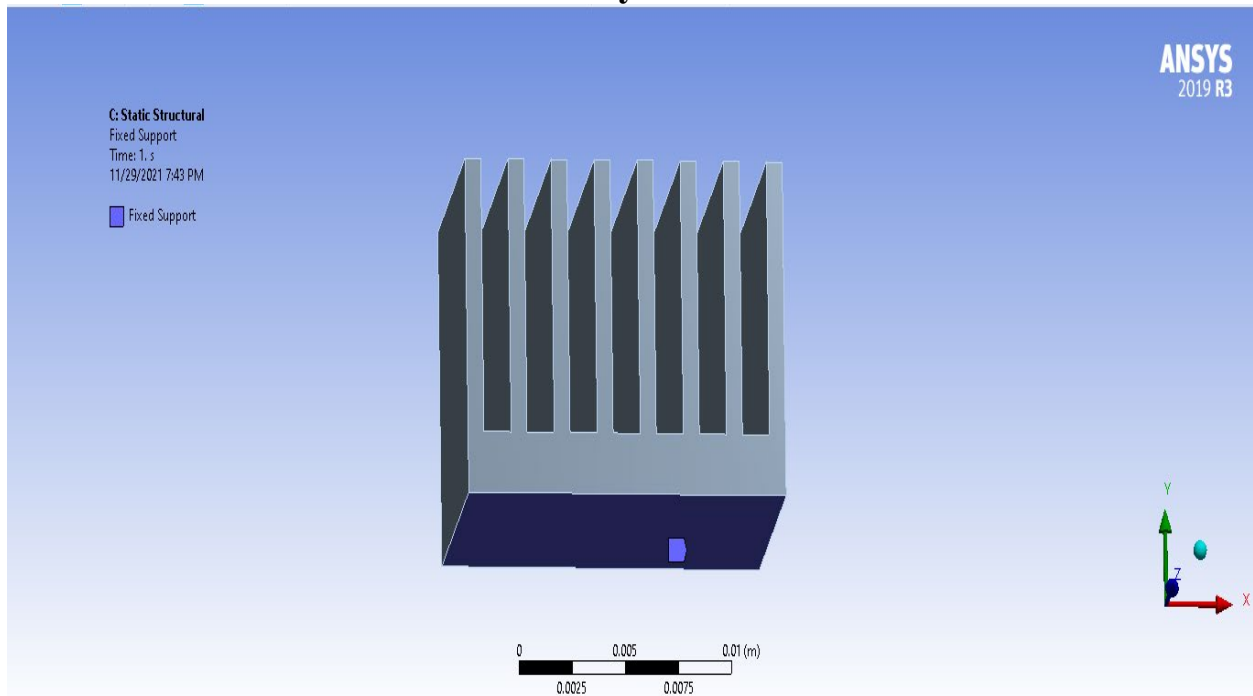


Figure 16 Fixed Support at bottom face

Part D: Static Structural Analysis 2 (10 degrees)

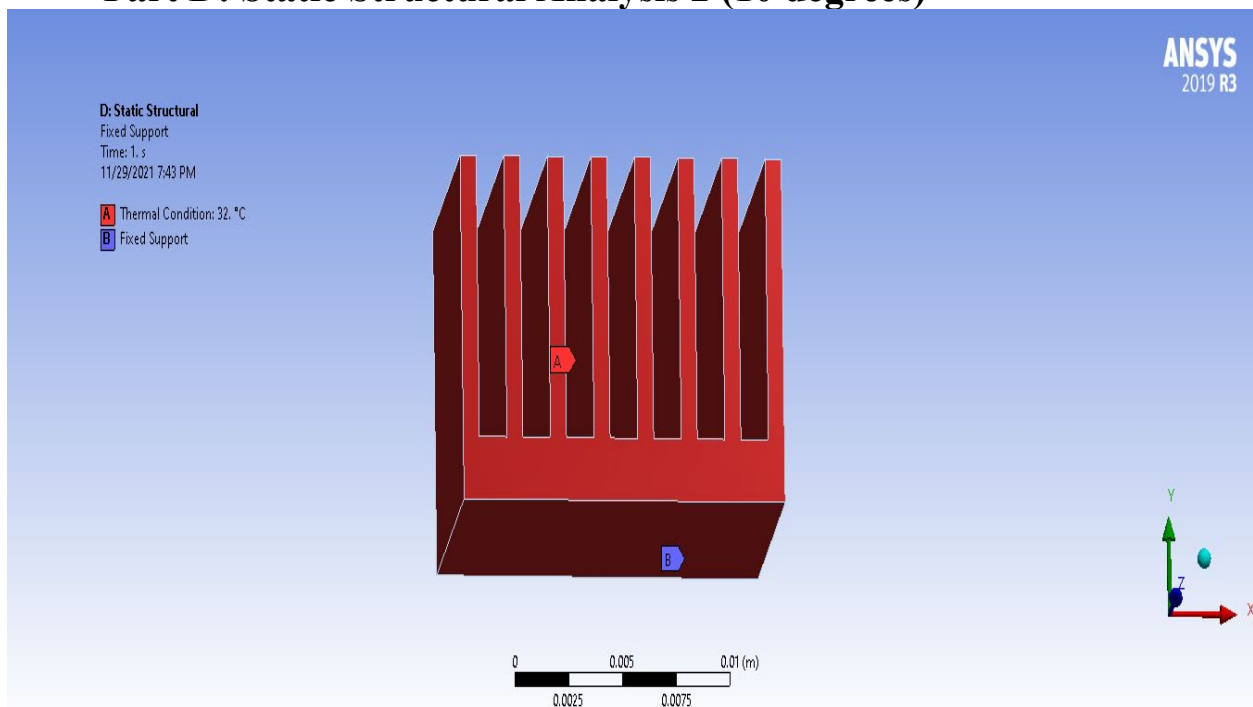


Figure 17. Fixed Support at bottom face
Results for Steady state Thermal

The results of all the four different analysis approaches used are given in this section.

Part A: Steady State Thermal Analysis

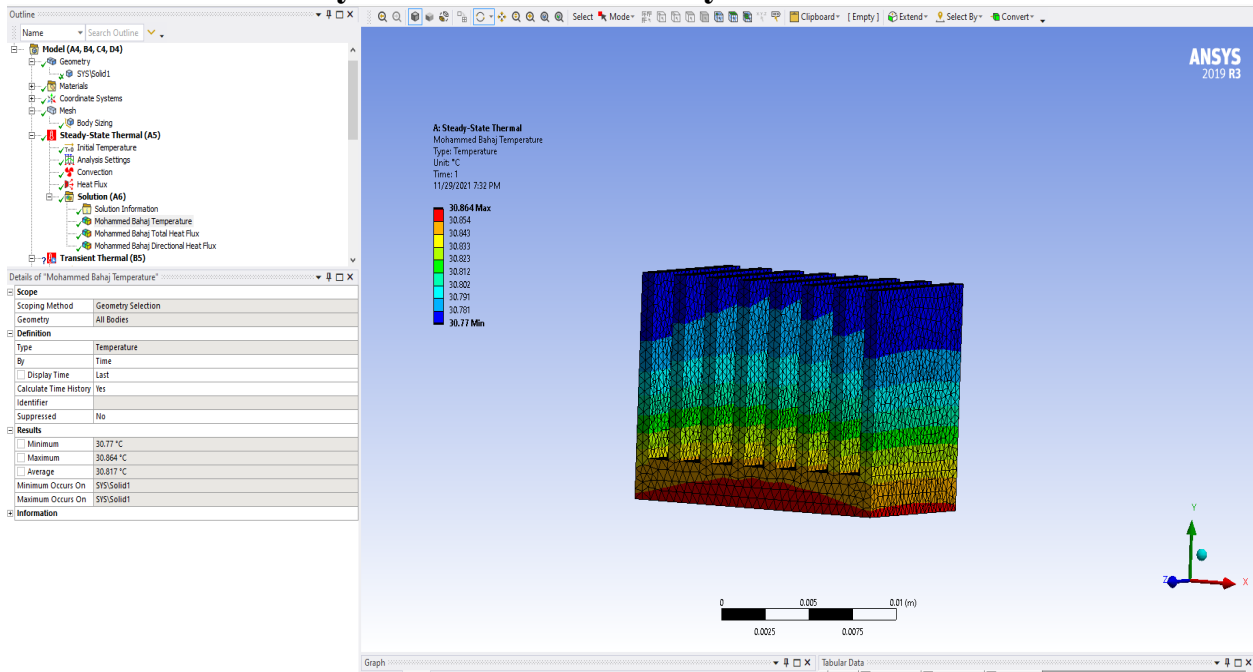


Figure 18. Temperature Steady State Thermal

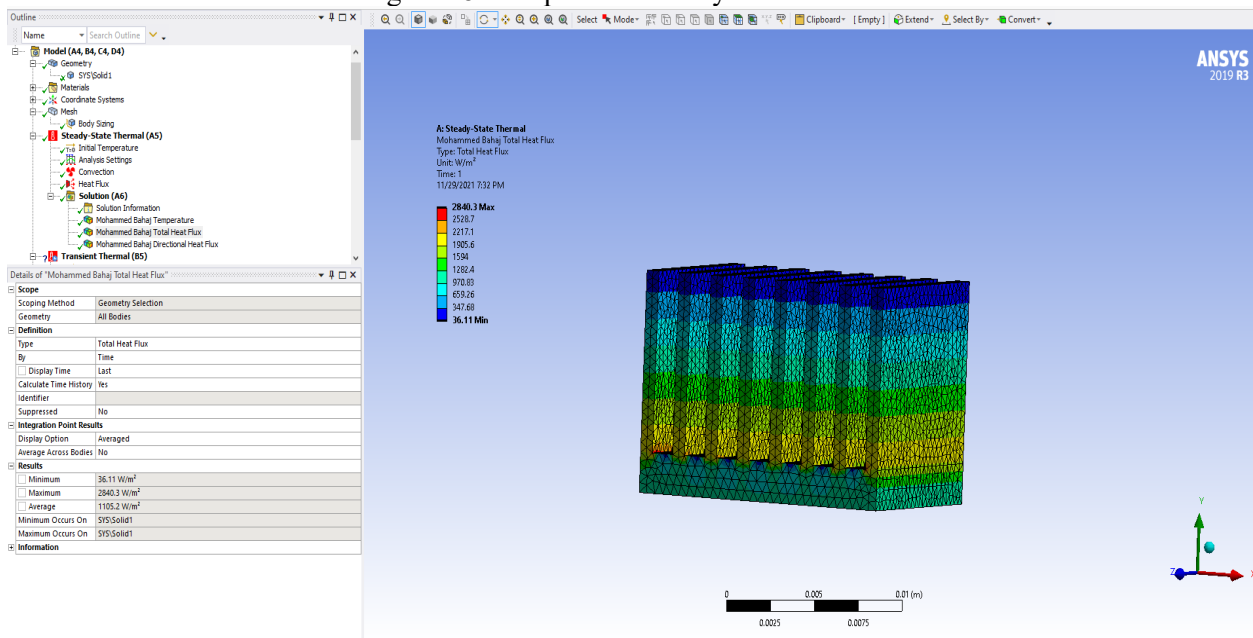


Figure 19. Heat flux Steady State Thermal

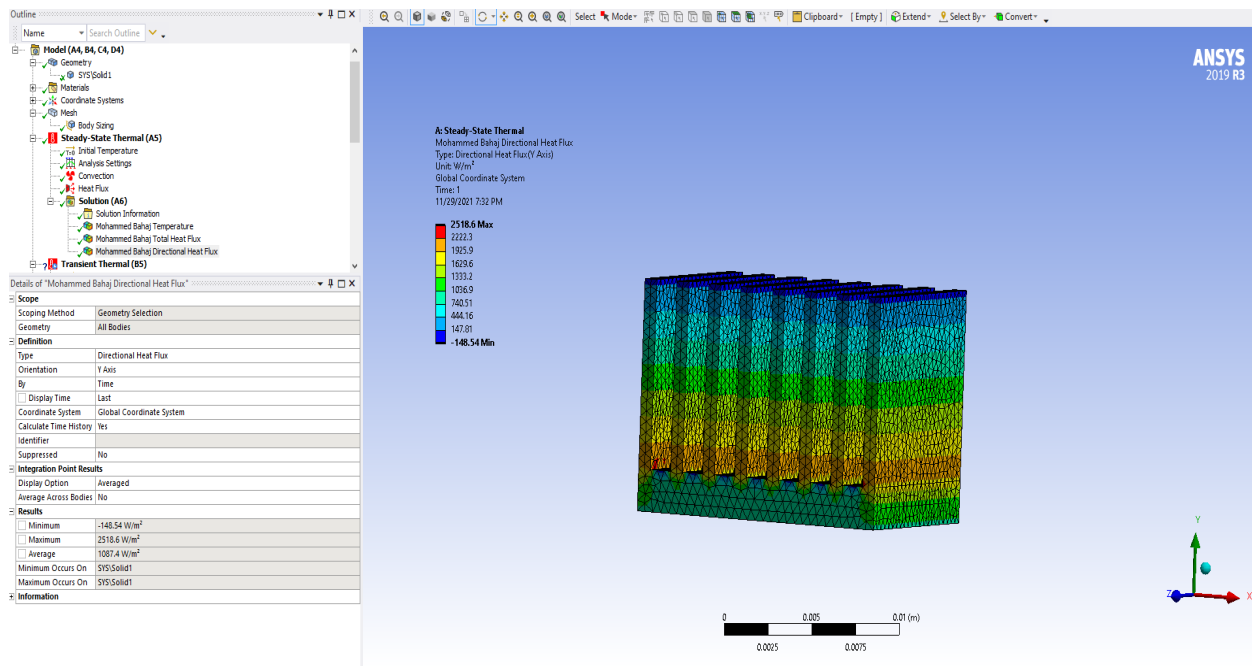


Figure 20. Heat flux Directional (Y-axis) Steady State Thermal

Part B: Transient Thermal Analysis

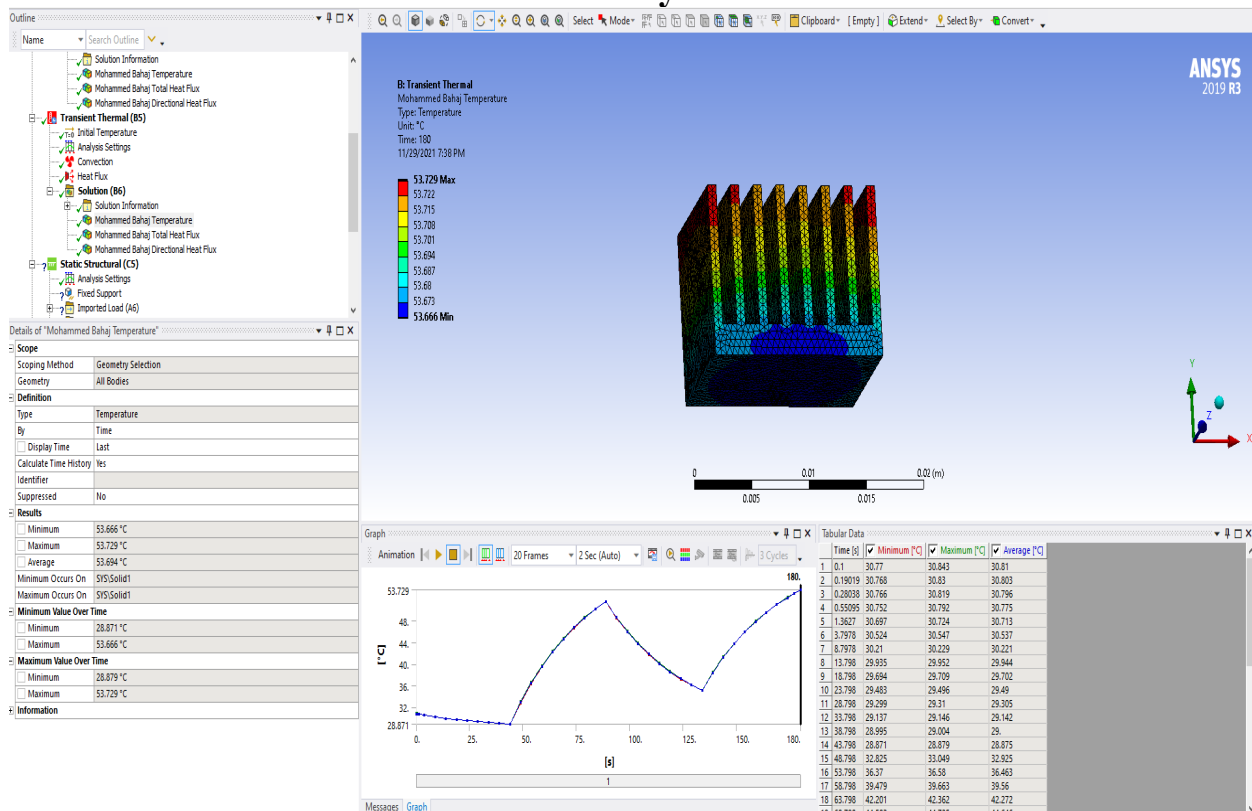


Figure 21. Temperature Transient Thermal

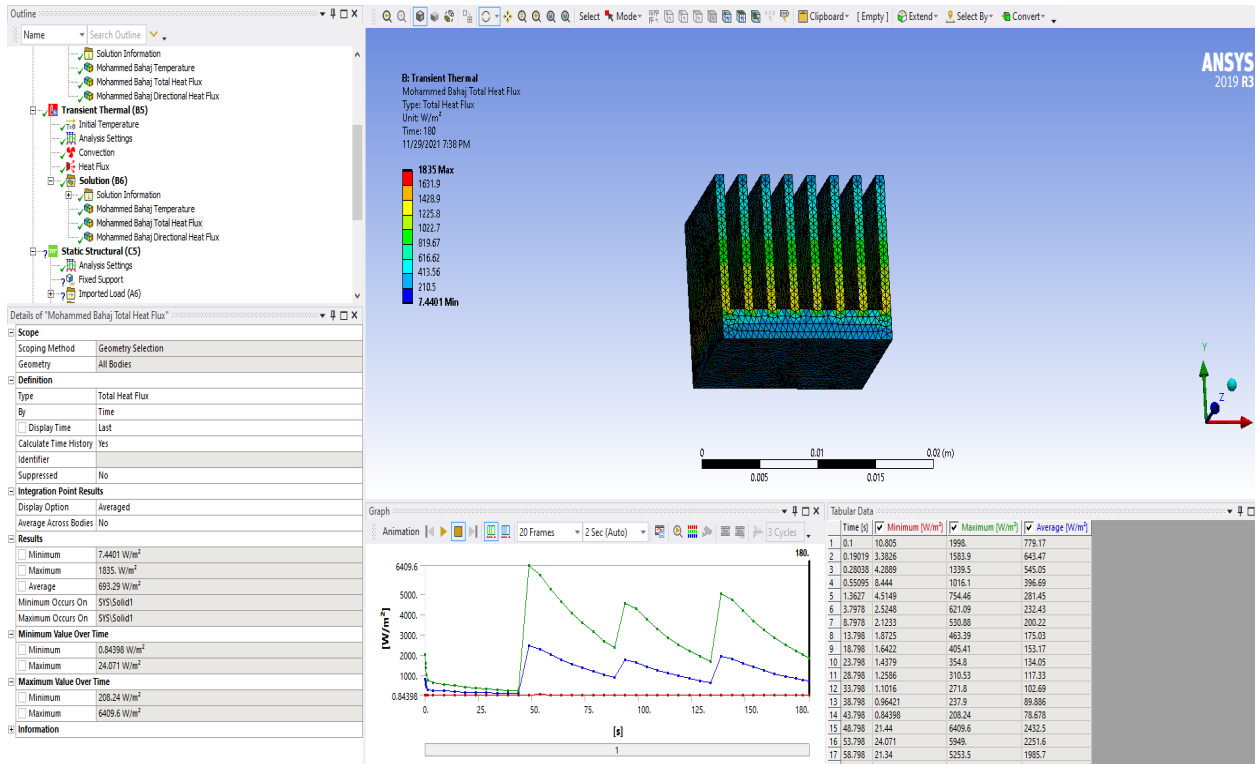


Figure 22. Total Heat Flux Transient Thermal

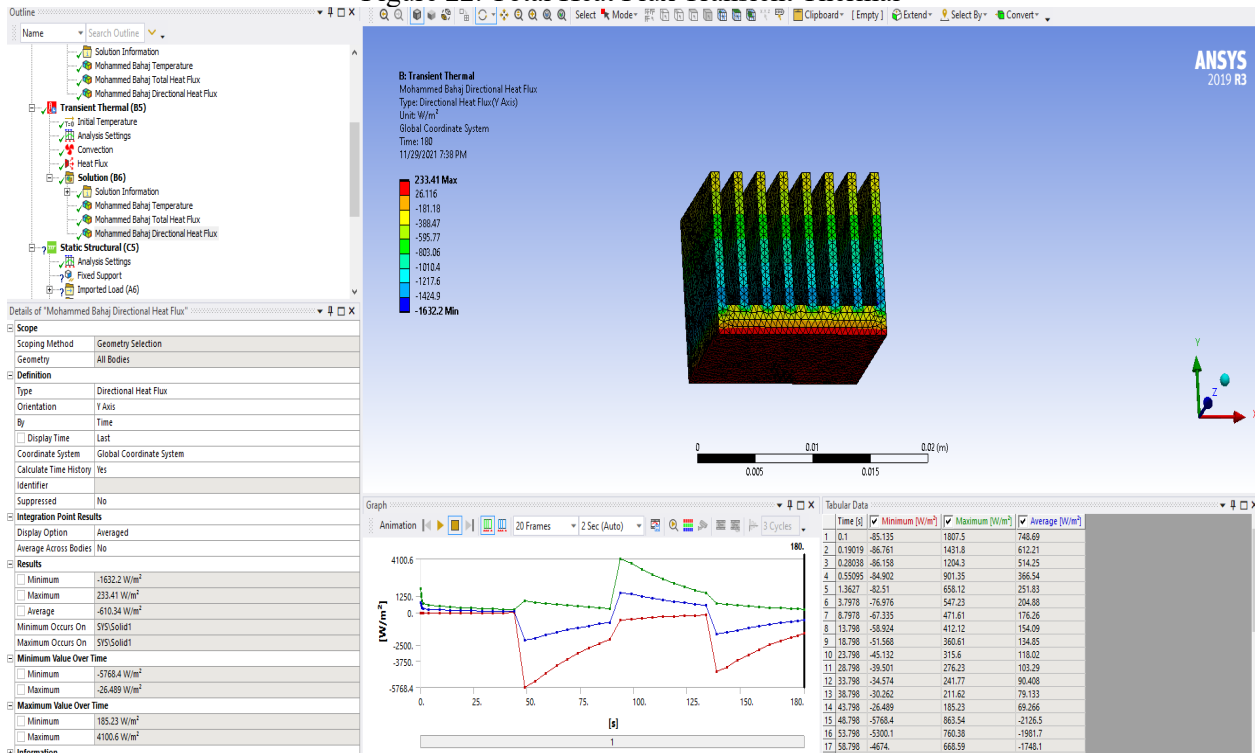


Figure 23. Heat Flux (Y-axis) Transient Thermal

Part C: Static Structural Analysis 1

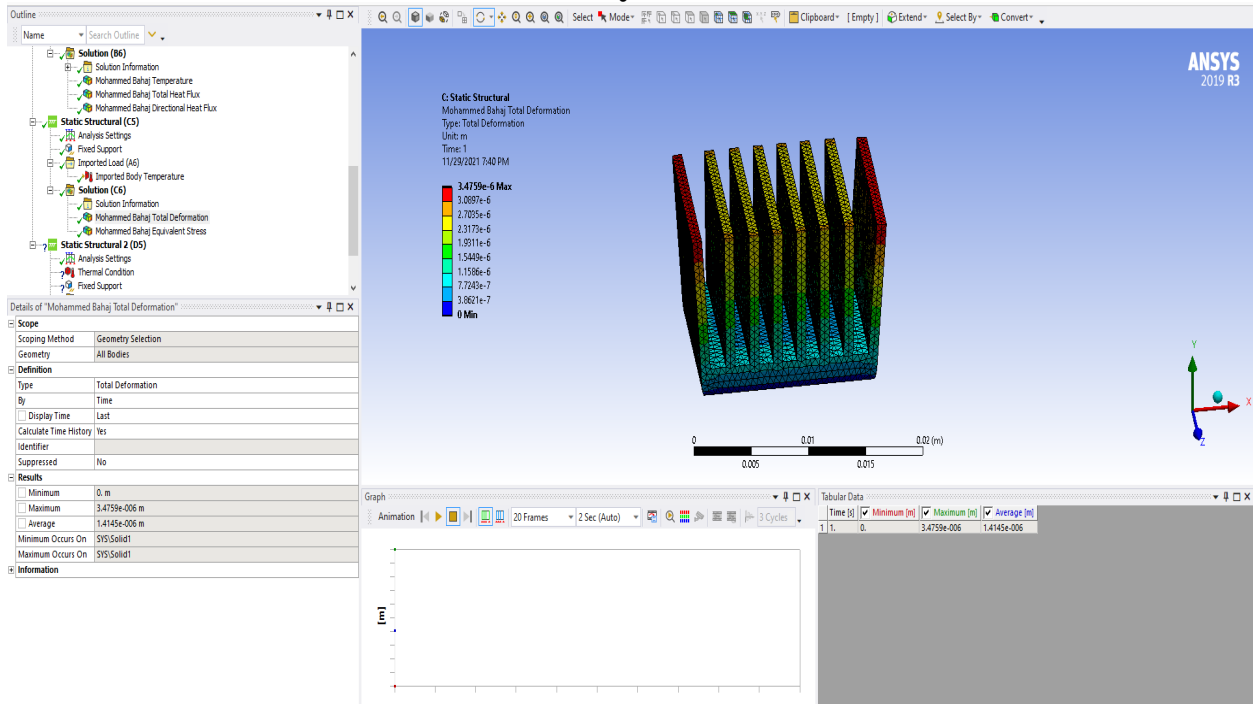


Figure 24. Total Deformation Static Analysis (1)

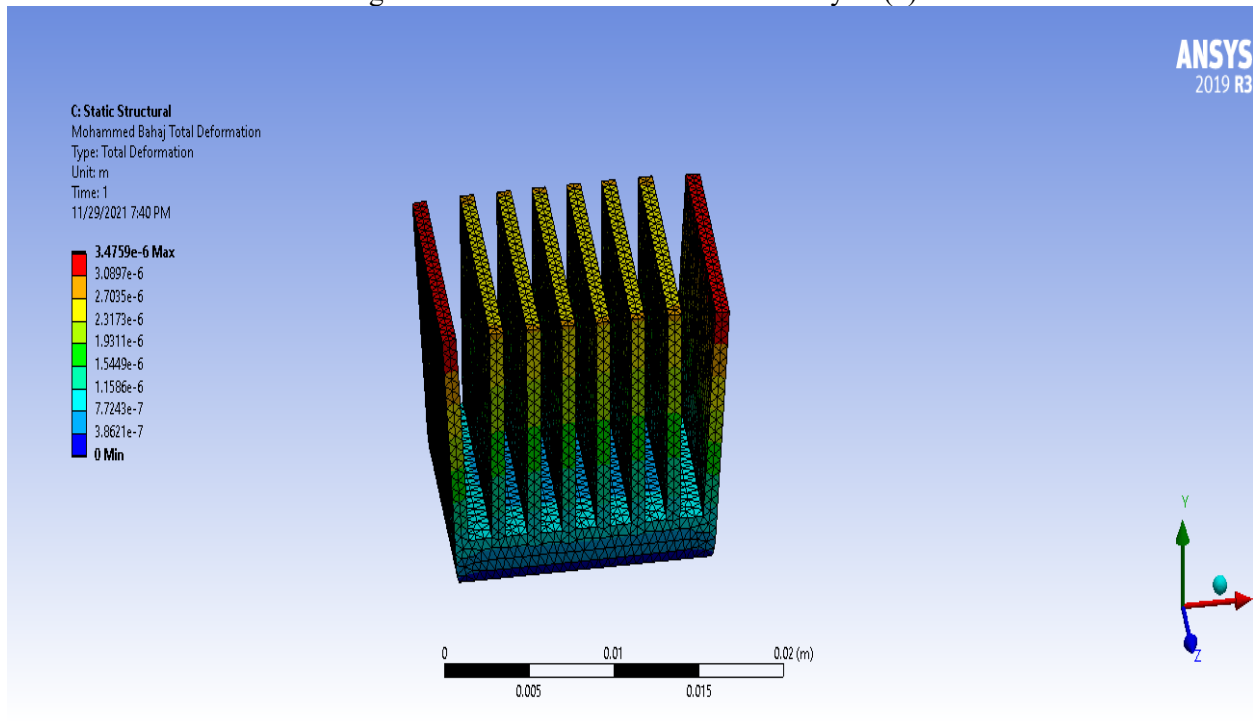


Figure 25. Total Deformation Static Analysis (1)

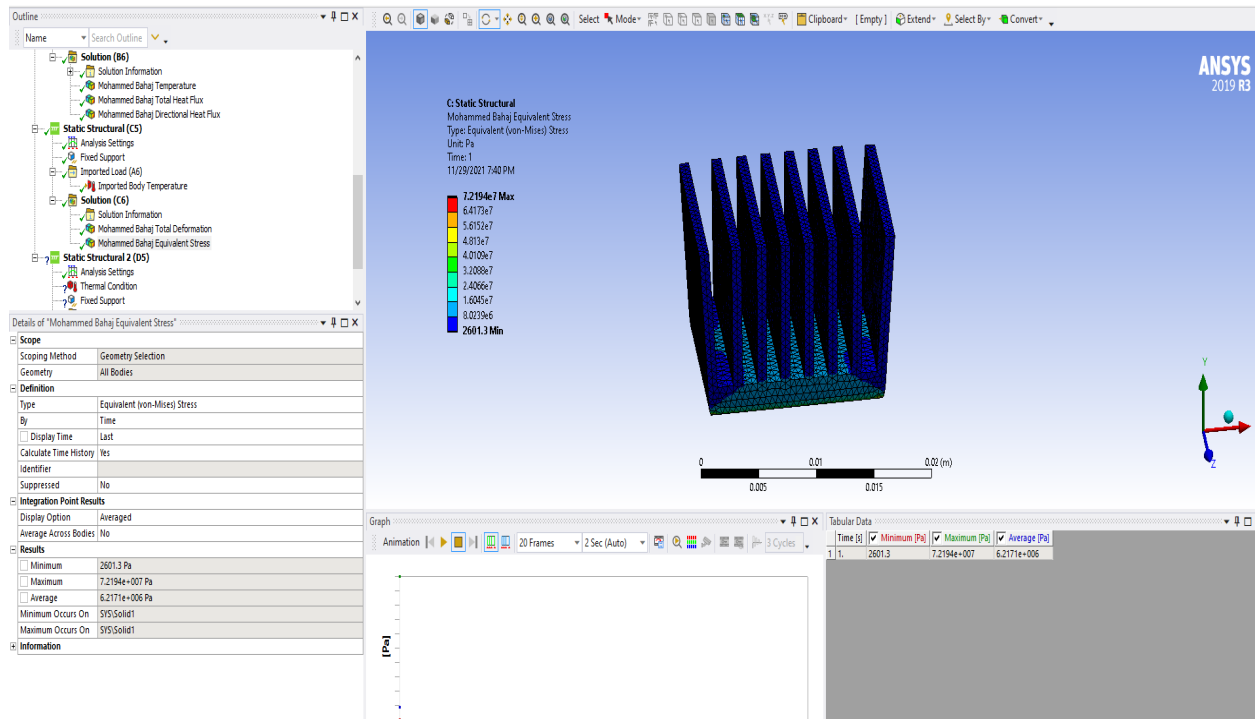


Figure 26. Equivalent Stress (Von Mises) Static Analysis (1)

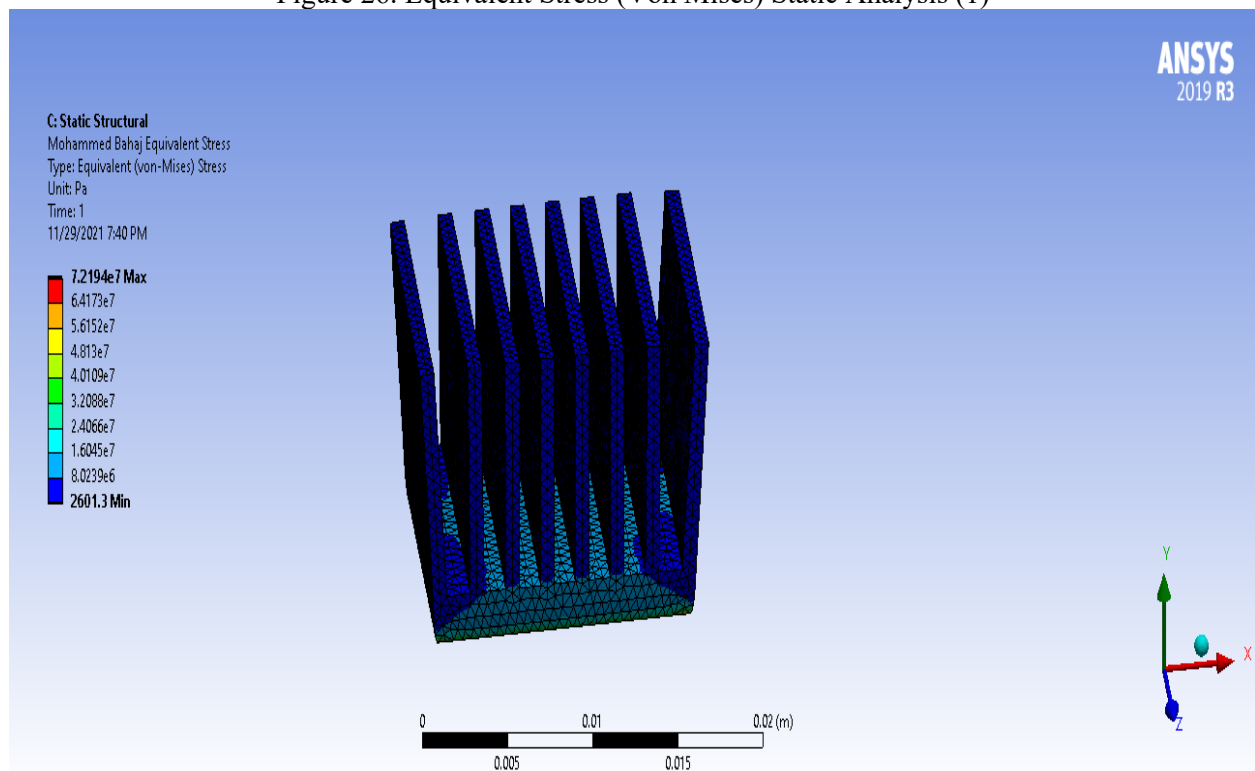


Figure 27. Equivalent Stress (Von Mises) Static Analysis (1)

Part D: Static Structural Analysis 2 (10 degrees)

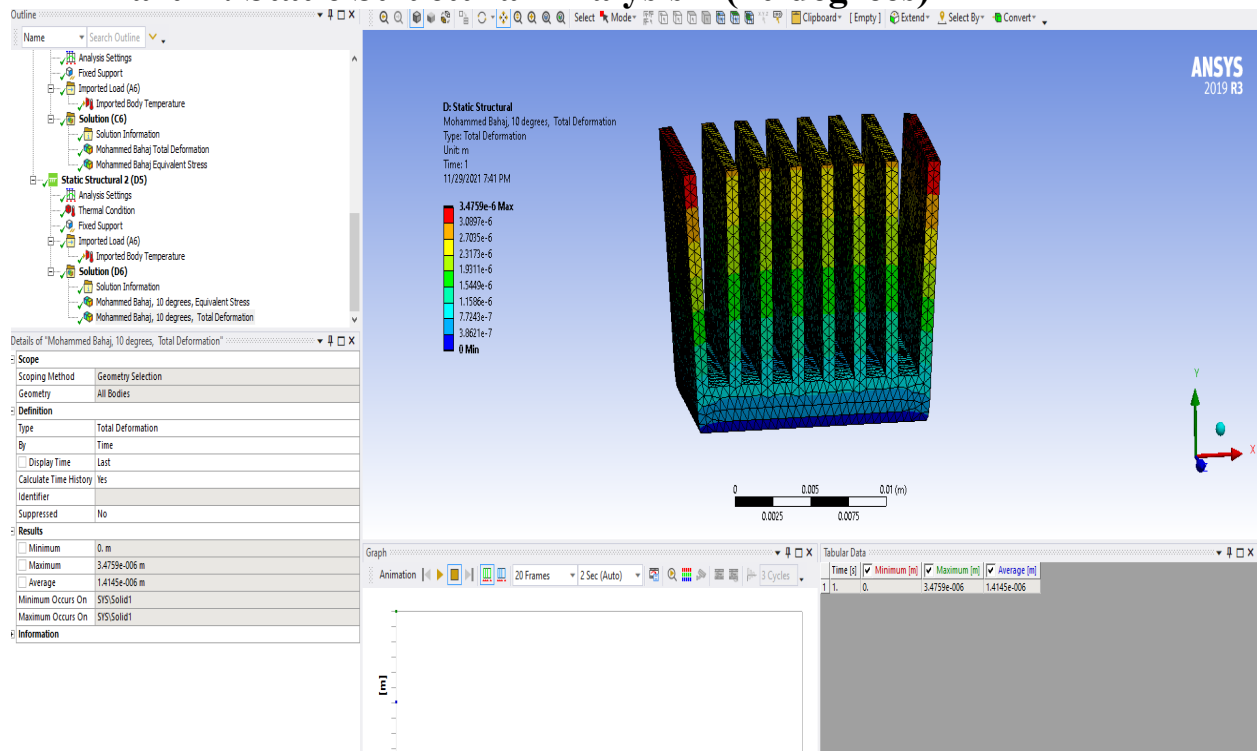


Figure 28. Total Deformation Static Analysis (2) 10 degrees

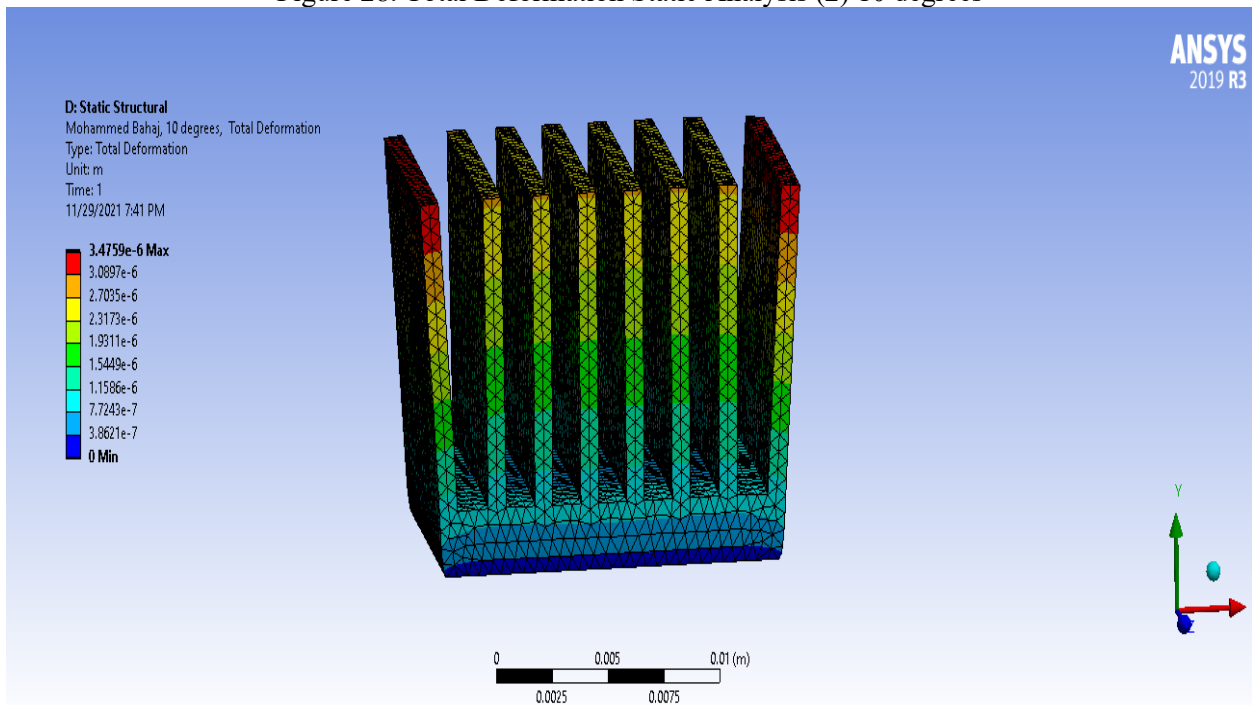


Figure 29. Total Deformation Static Analysis (2) 10 degrees

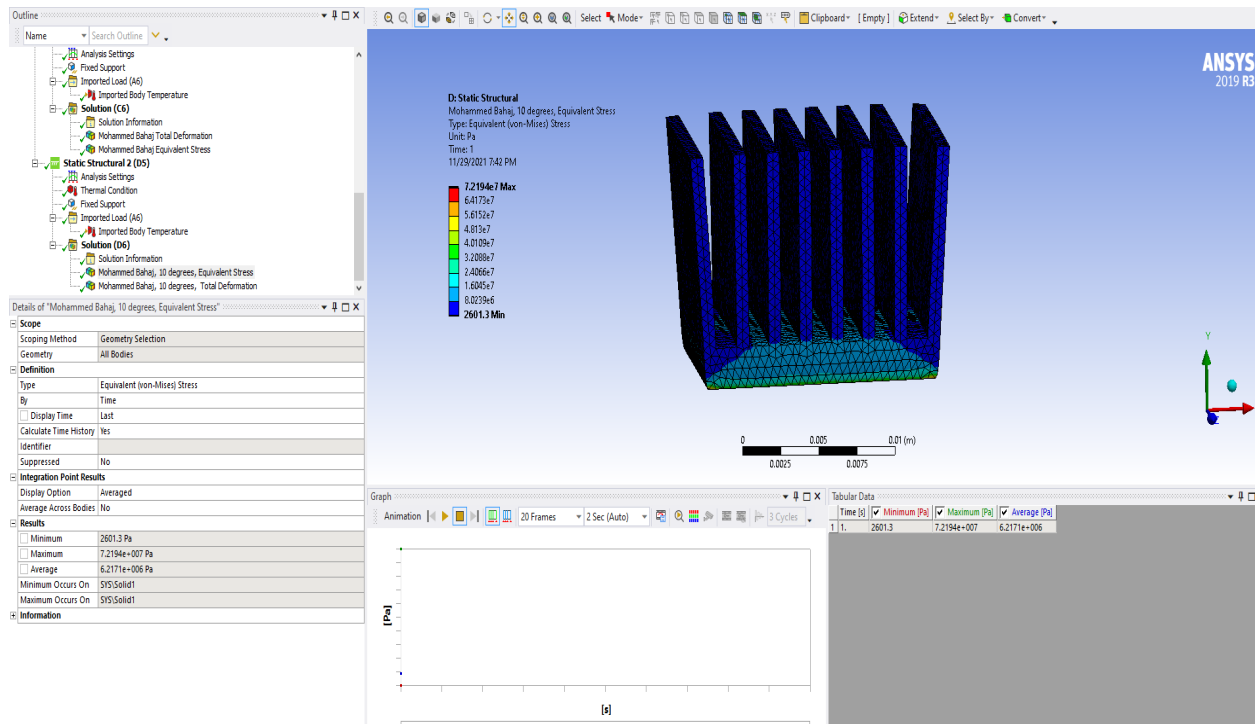


Figure 30. Equivalent Stress (Von Mises) Static Analysis (2) 10 degrees

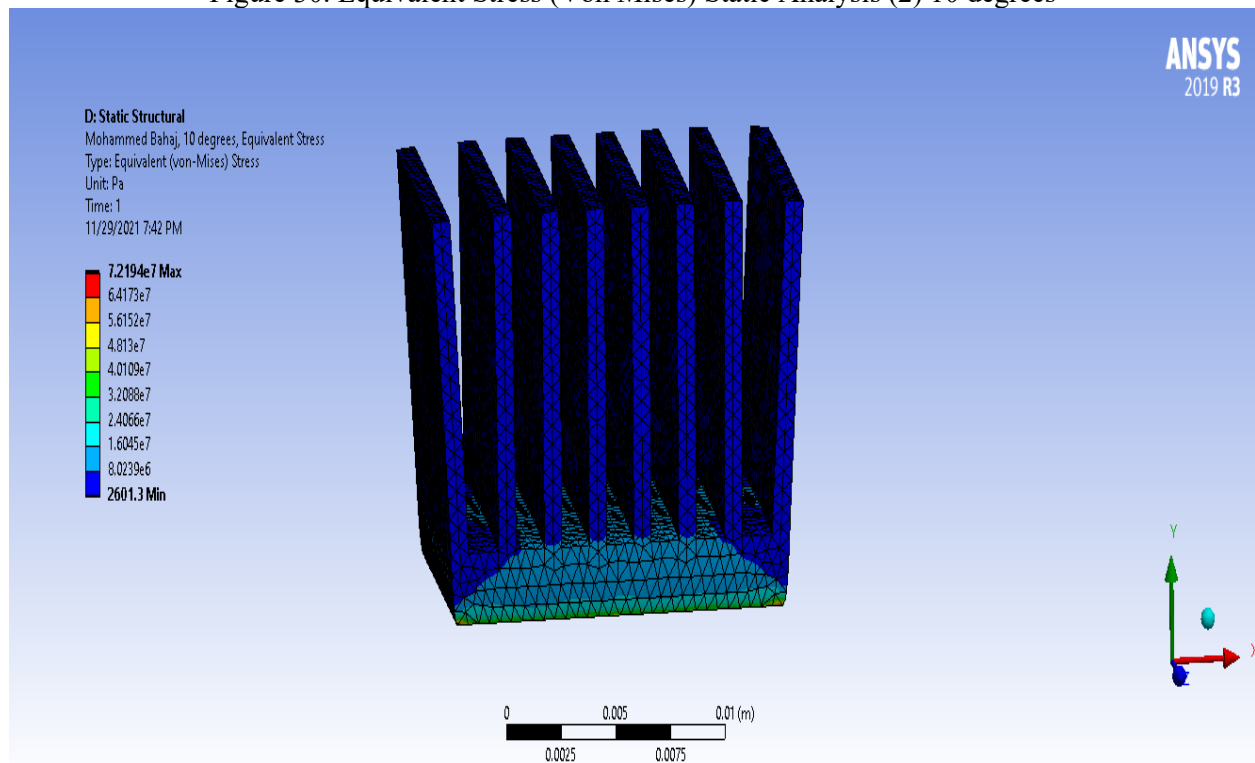


Figure 31. Equivalent Stress (Von Mises) Static Analysis (2) 10 degrees

Heat Sink Model 2

3D Model

The 3d model of heat sink with a different design was created using the following dimensions, the 3d model,

dimension and mass properties are given below:

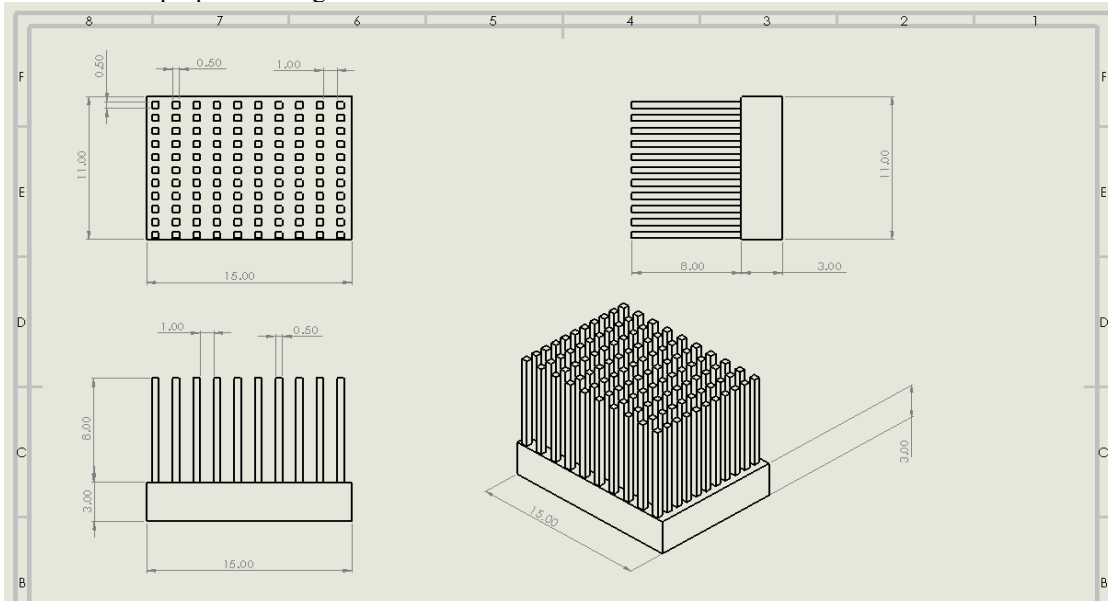


Figure 32. 2D sketch of heat sink 2

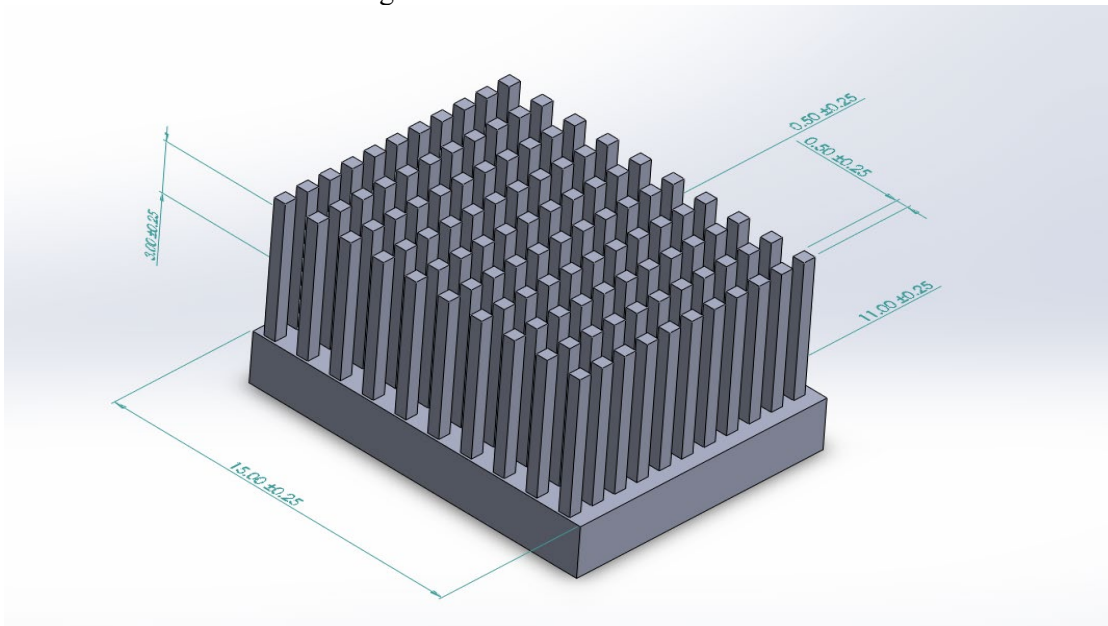


Figure 33. 3d Model Heat Sink 2

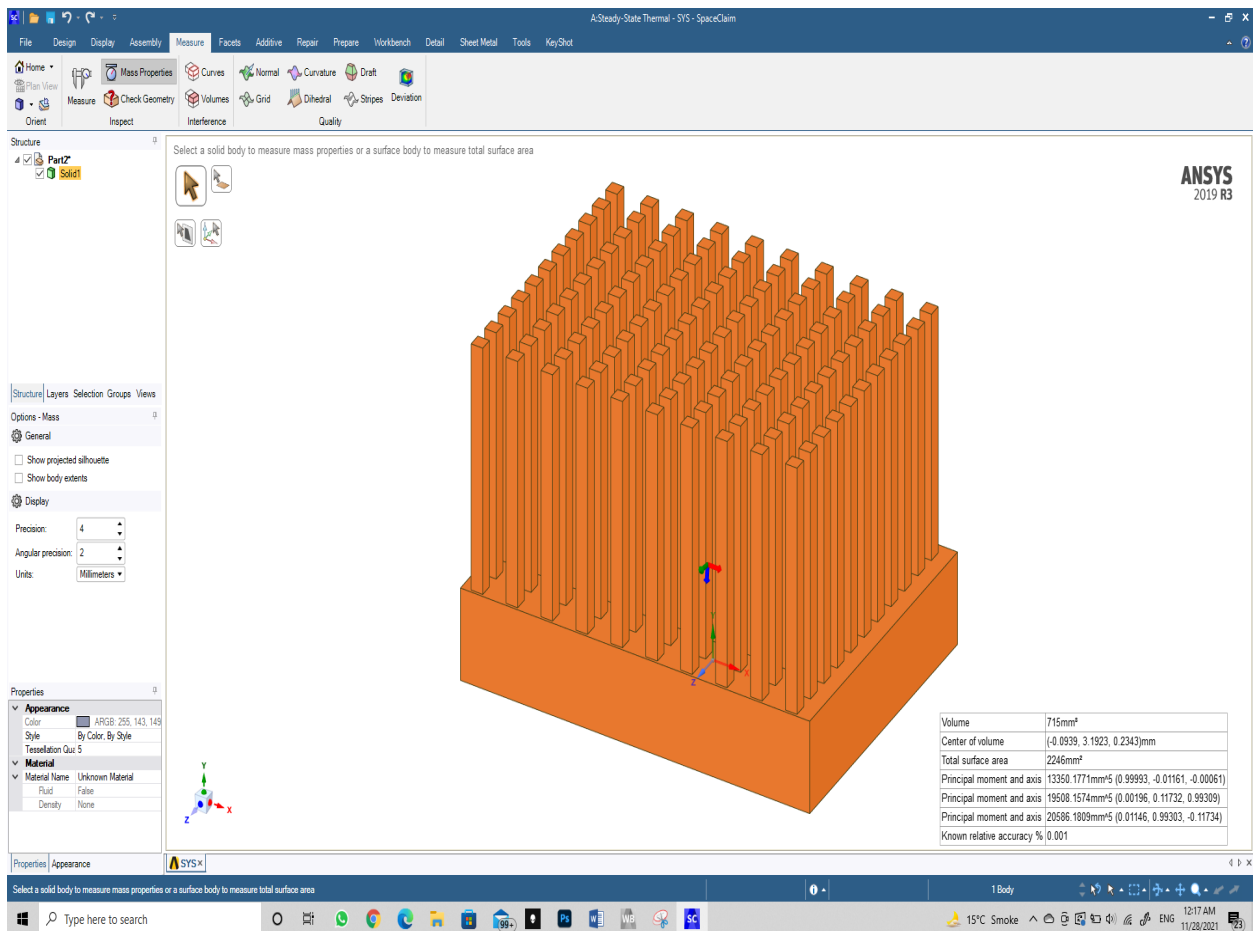


Figure 34. 3d Model Heat Sink 2 Volume and Mass Properties

Volume	715mm ³
Center of volume	(-0.0939, 3.1923, 0.2343)mm
Total surface area	2246mm ²
Principal moment and axis	13350.1771mm ⁴ (0.99993, -0.01161, -0.00061)
Principal moment and axis	19508.1574mm ⁴ (0.00196, 0.11732, 0.99309)
Principal moment and axis	20586.1809mm ⁴ (0.01146, 0.99303, -0.11734)
Known relative accuracy %	0.001

Figure 35. 3d Model Heat Sink 2 Properties

Material Selection

The material selected for the analysis was Aluminum; all the materials properties of the material used are given in the figure below:

Aluminum



General aluminum alloy. Fatigue properties come from MIL-HDBK-5H, page 3-277.

Density	2.77e-06 kg/mm ³
Structural	
▼ Isotropic Elasticity	
Derive from	Young's Modulus and Poisson's Ratio
Young's Modulus	71000 MPa
Poisson's Ratio	0.33
Bulk Modulus	69608 MPa
Shear Modulus	26692 MPa
Isotropic Secant Coefficient of Thermal Expansion	2.3e-05 1/°C
Compressive Ultimate Strength	0 MPa
Compressive Yield Strength	280 MPa
S-N Curve	
Tensile Ultimate Strength	310 MPa
Tensile Yield Strength	280 MPa
Thermal	

Figure 36. Aluminum Material properties

Meshing

The meshing for the heat sink model 2 was done using the mesh element size of 5e-3 m, the meshed model and mesh statistics are given below:

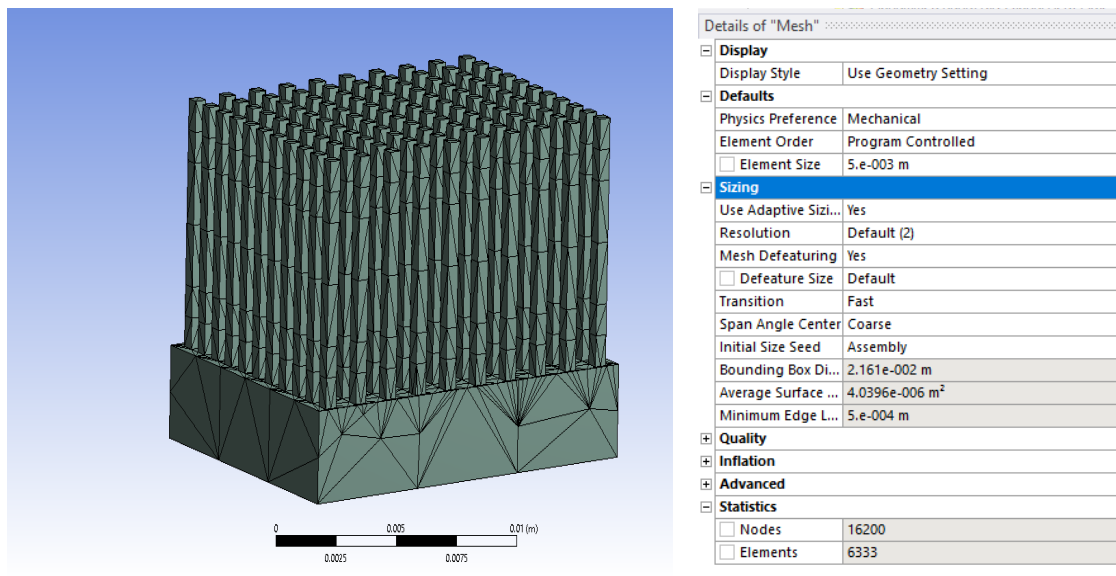


Figure 37. Meshed Model (5e-3 m)

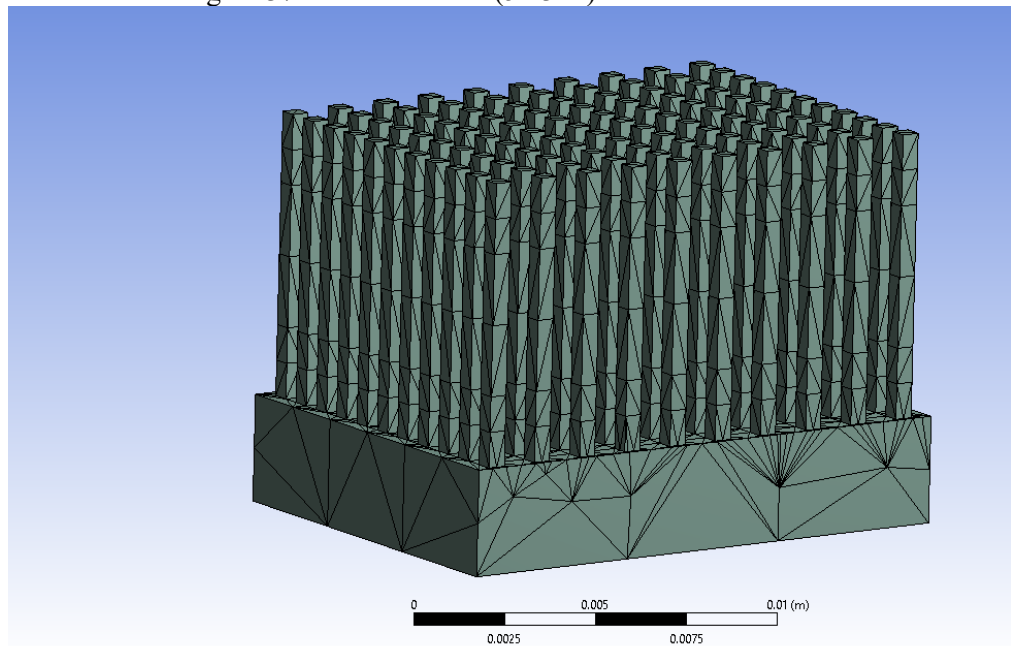


Figure 38. Meshed Model of heat sink 2 (5e-3 m) statics and Model

Boundary Conditions

The boundary conditions section includes the applied thermal loads, such as the temperature, heat flux, and the fixtures for the static analysis.

Part A: Steady State Thermal Analysis

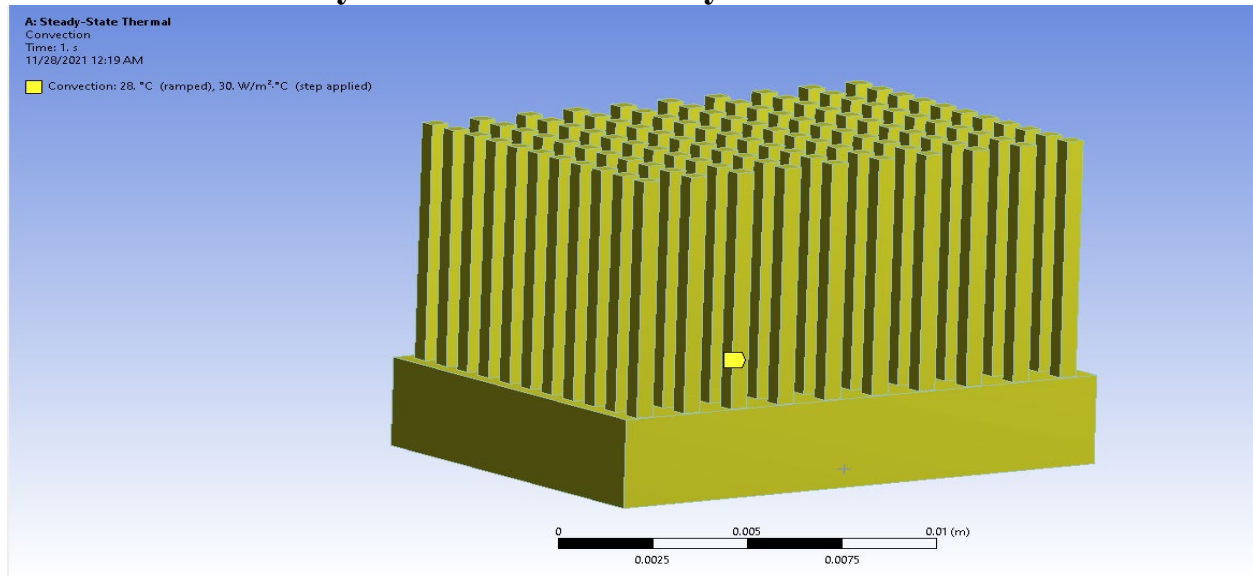


Figure 39. Convection Loads

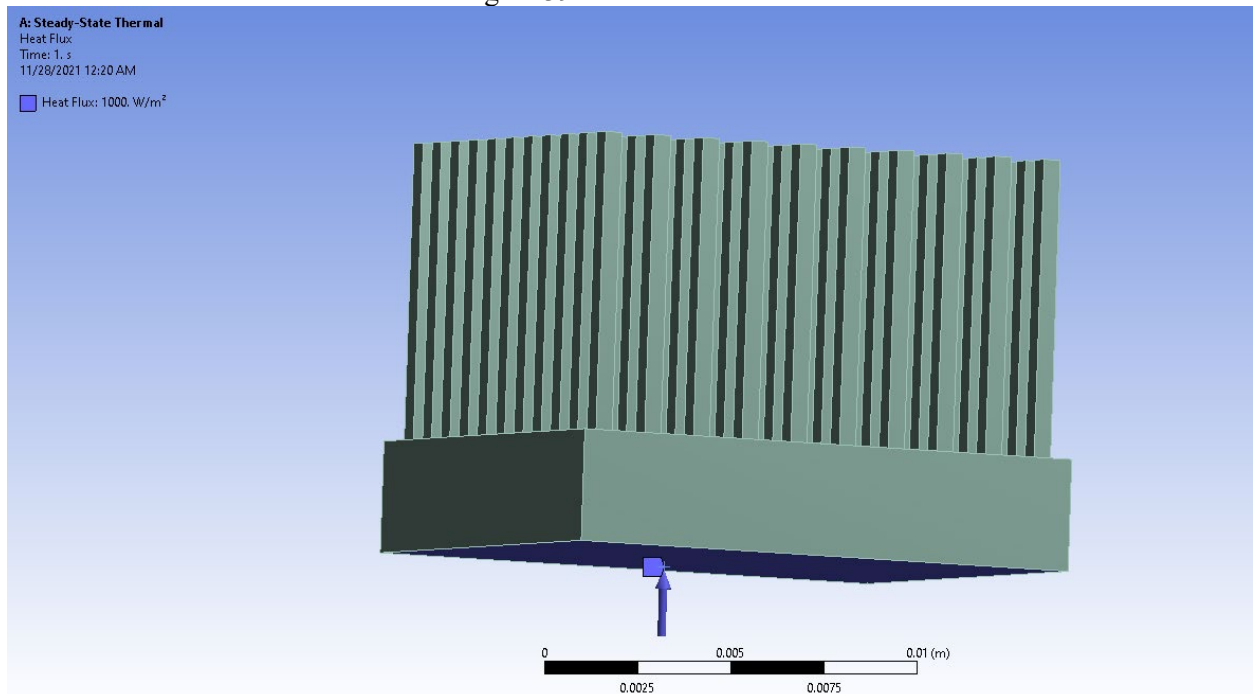
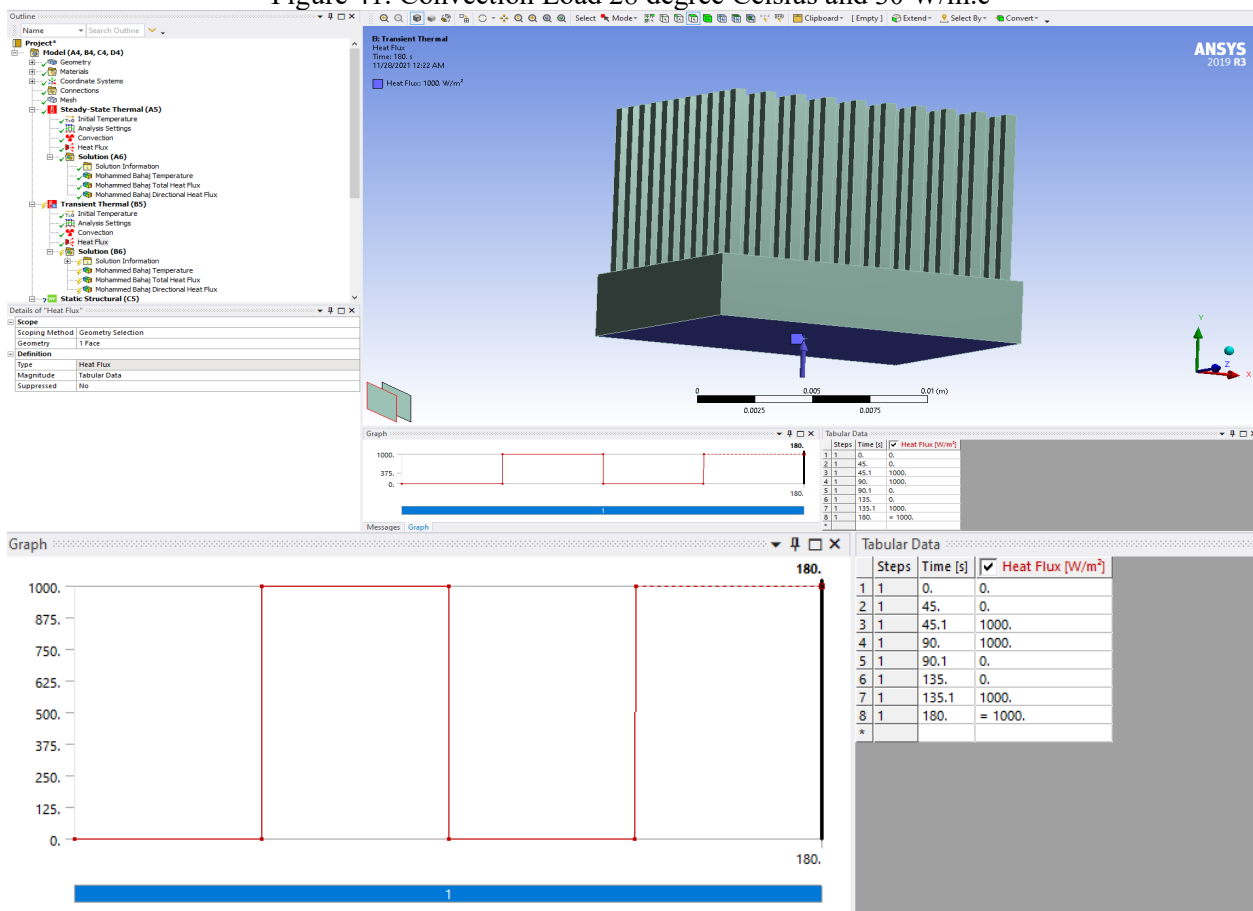
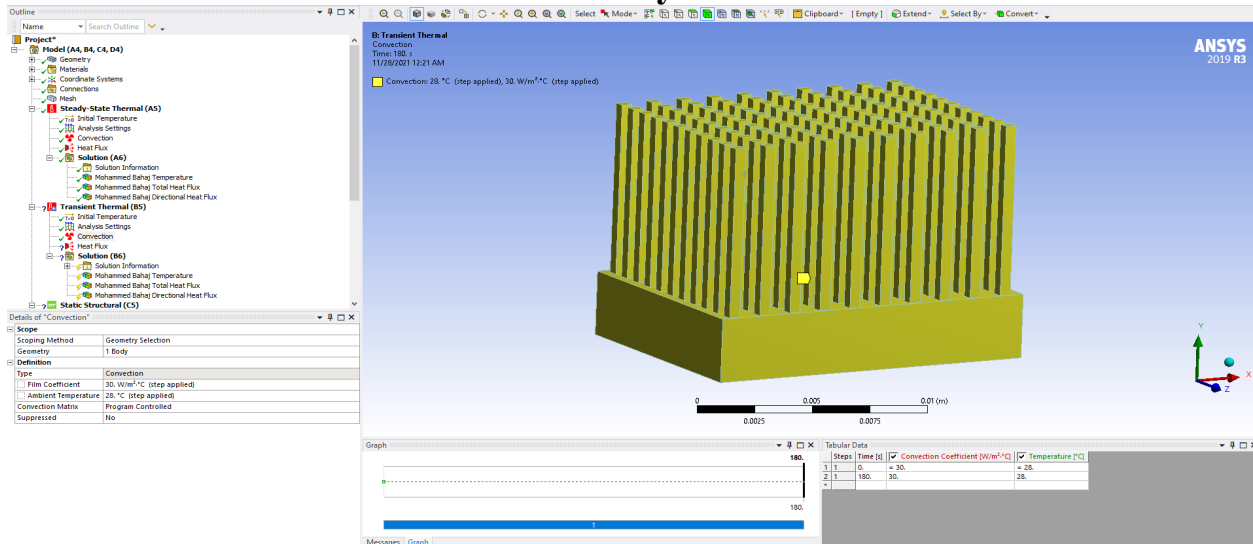


Figure 40. Heat Flux 1000 W/m²

Part B: Transient Thermal Analysis



Part C: Static Structural Analysis 1

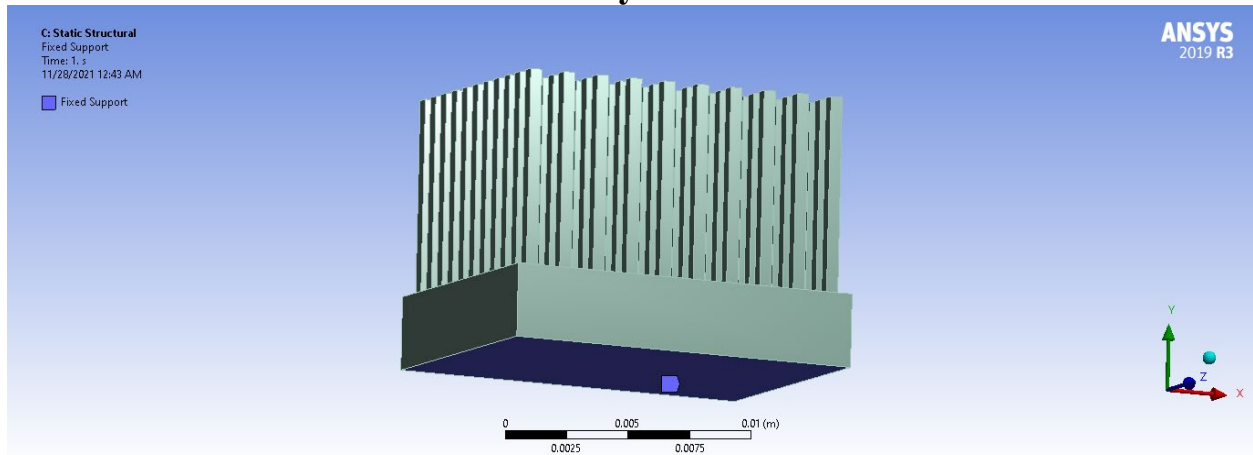


Figure 43. Fixed Support at bottom face

Part D: Static Structural Analysis 2 (10 degrees)

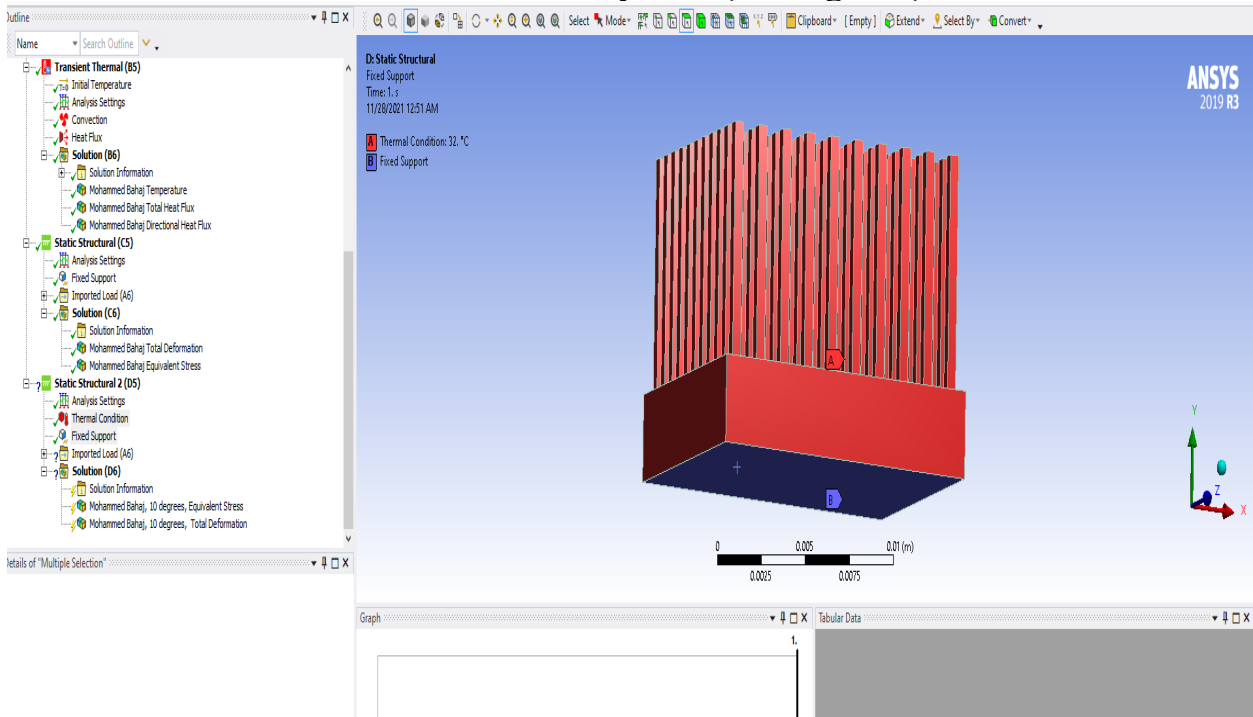


Figure 44. Fixed Support at bottom face
Results for Steady state Thermal

The results of all the four different analysis approaches used are given in this section.

Part A: Steady State Thermal Analysis

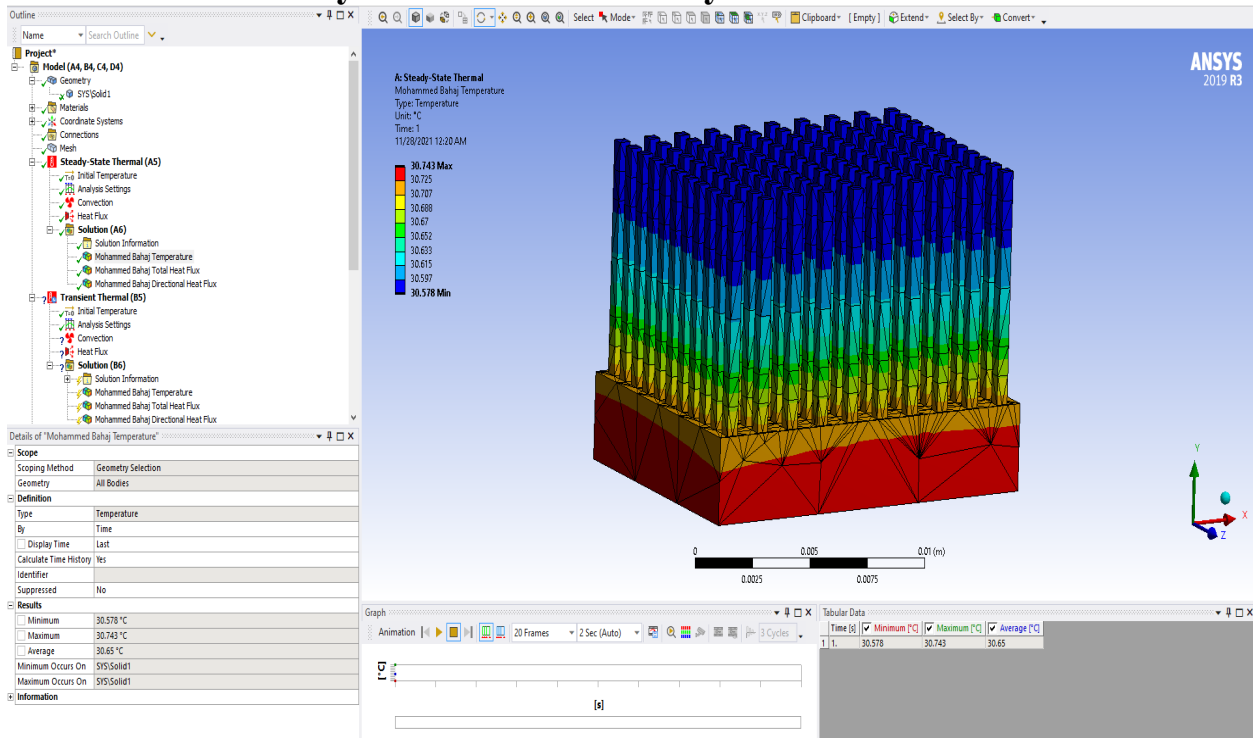


Figure 45. Temperature Steady State Thermal

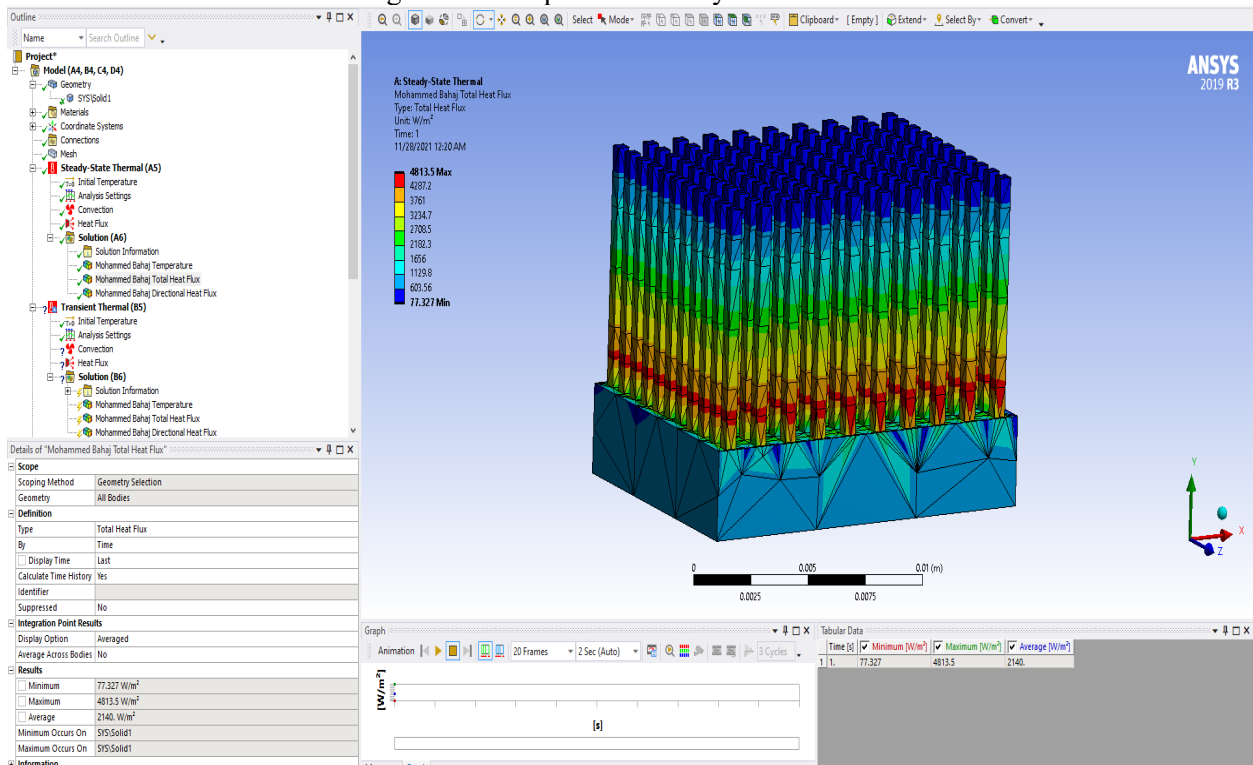


Figure 46. Heat flux Steady State Thermal

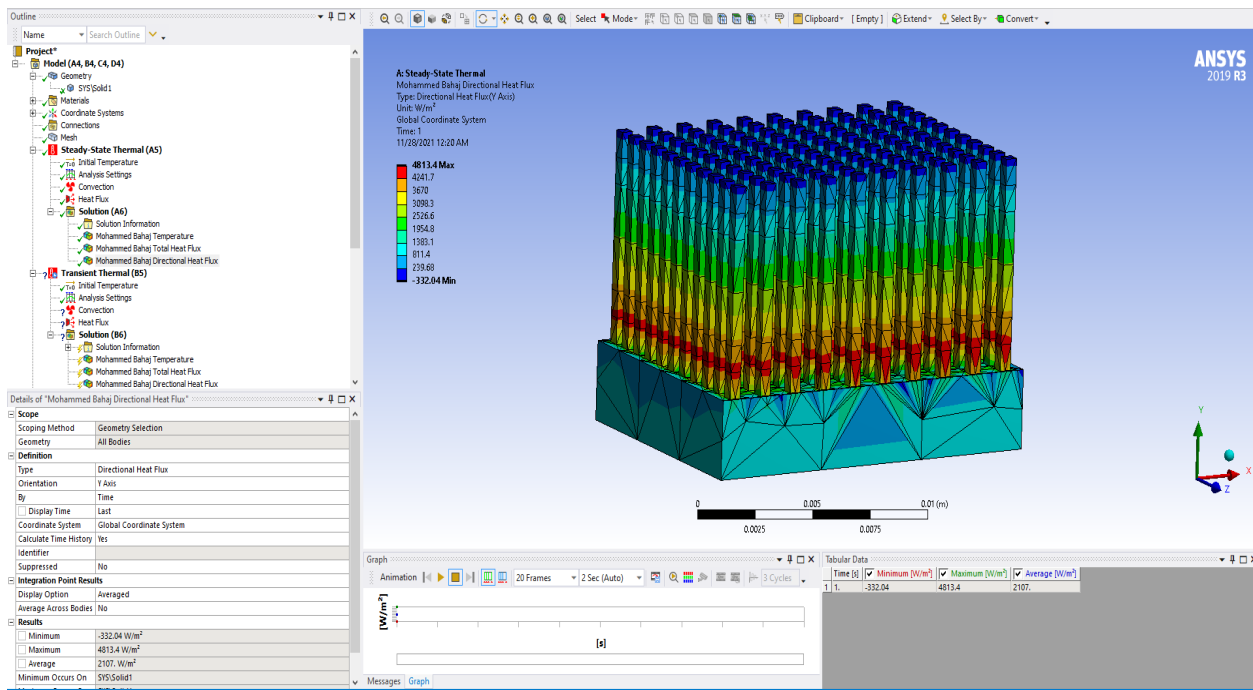


Figure 47. Heat flux Directional (Y-axis) Steady State Thermal

Part B: Transient Thermal Analysis

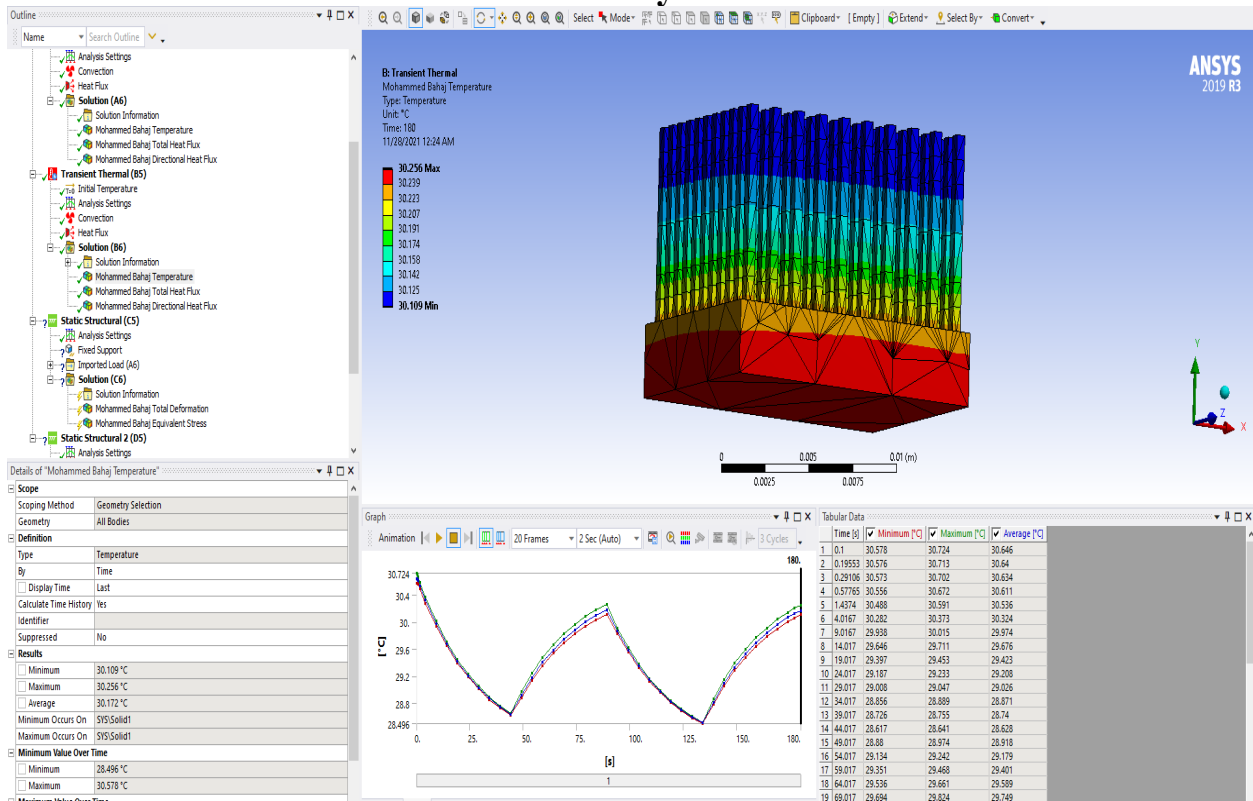


Figure 48. Temperature Transient Thermal

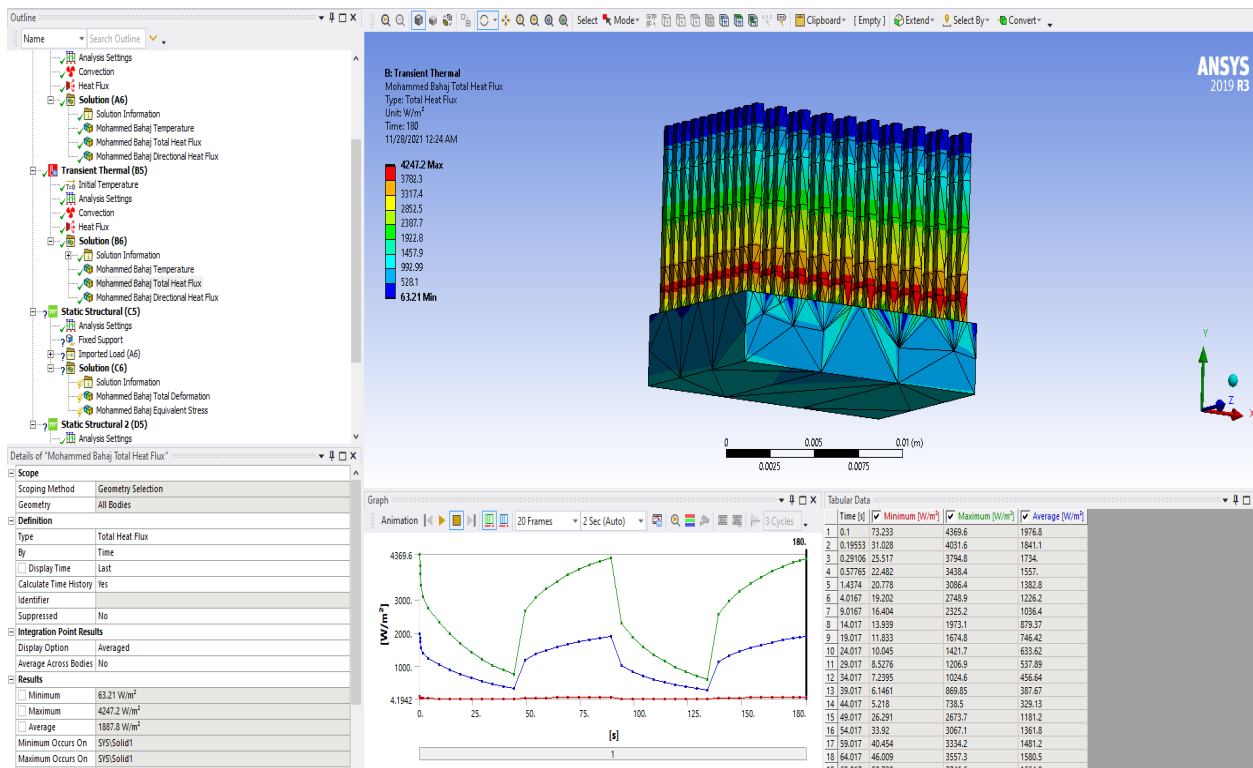


Figure 49. Total Heat Flux Transient Thermal

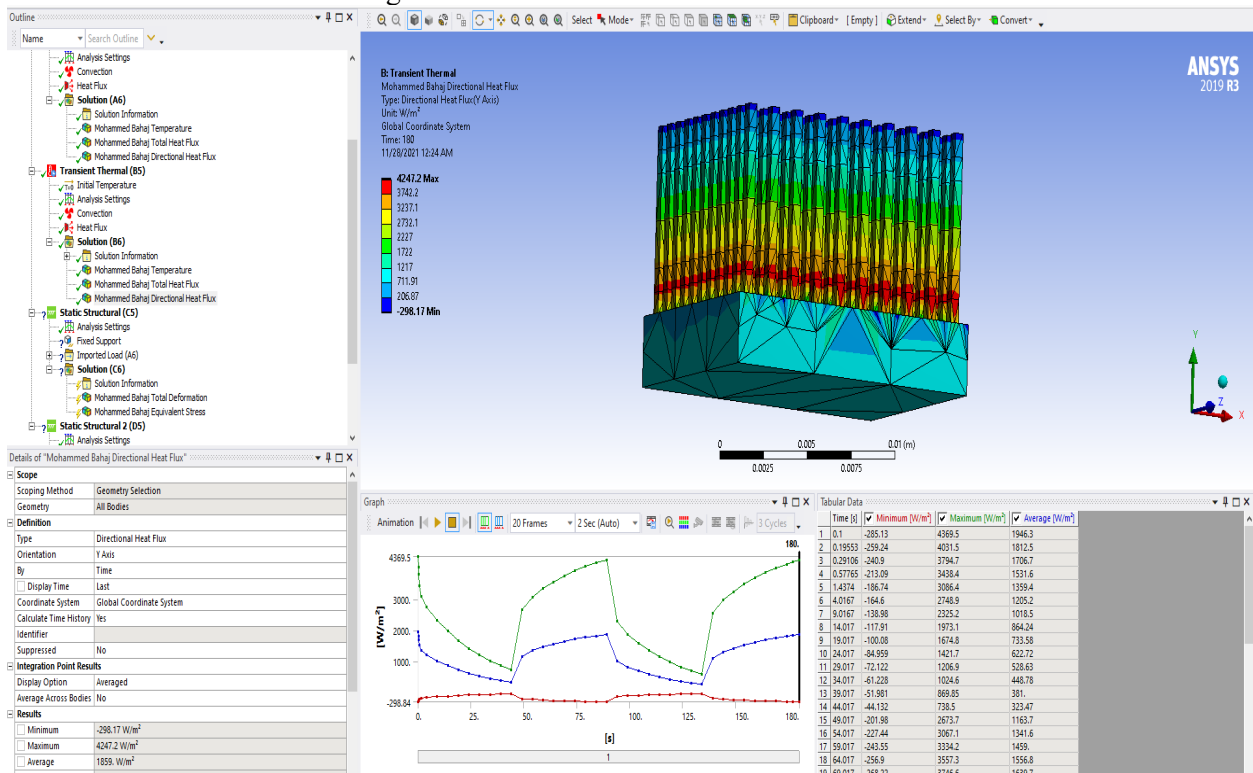


Figure 50. Heat Flux (Y-axis) Transient Thermal

Part C: Static Structural Analysis 1

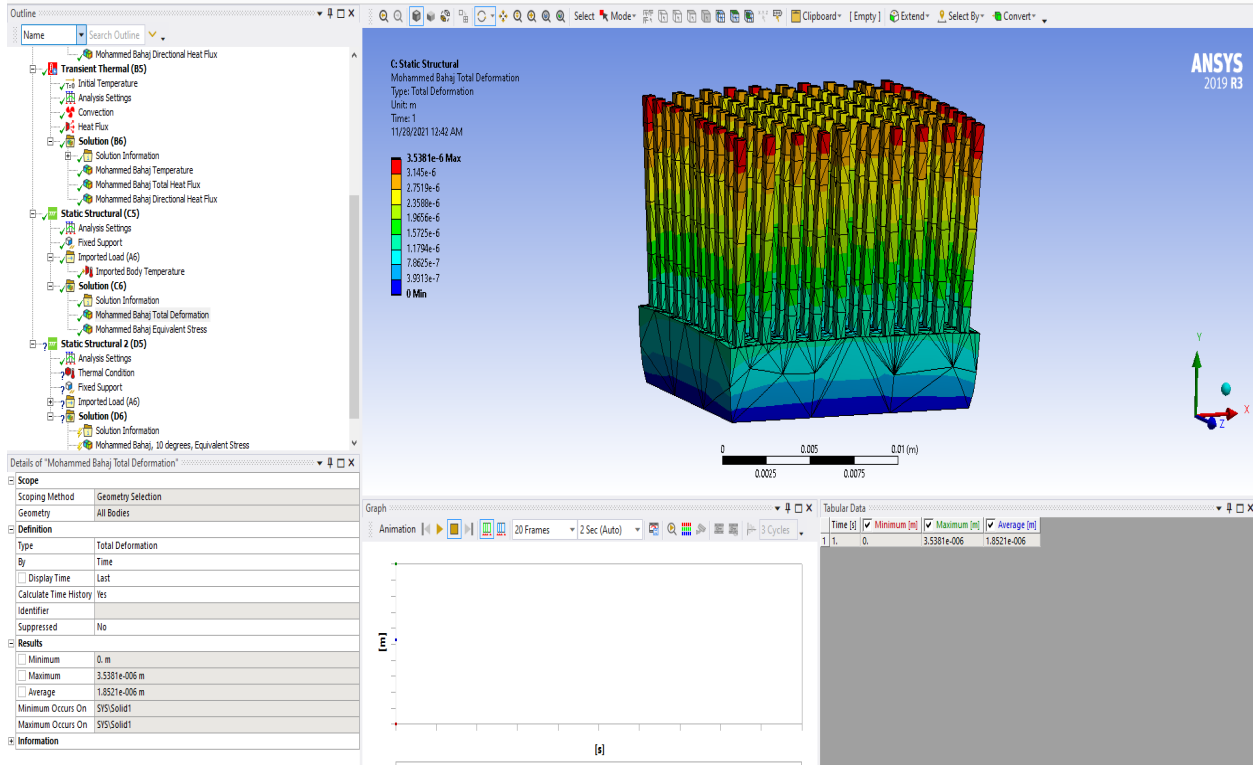


Figure 51. Total Deformation Static Analysis (1)

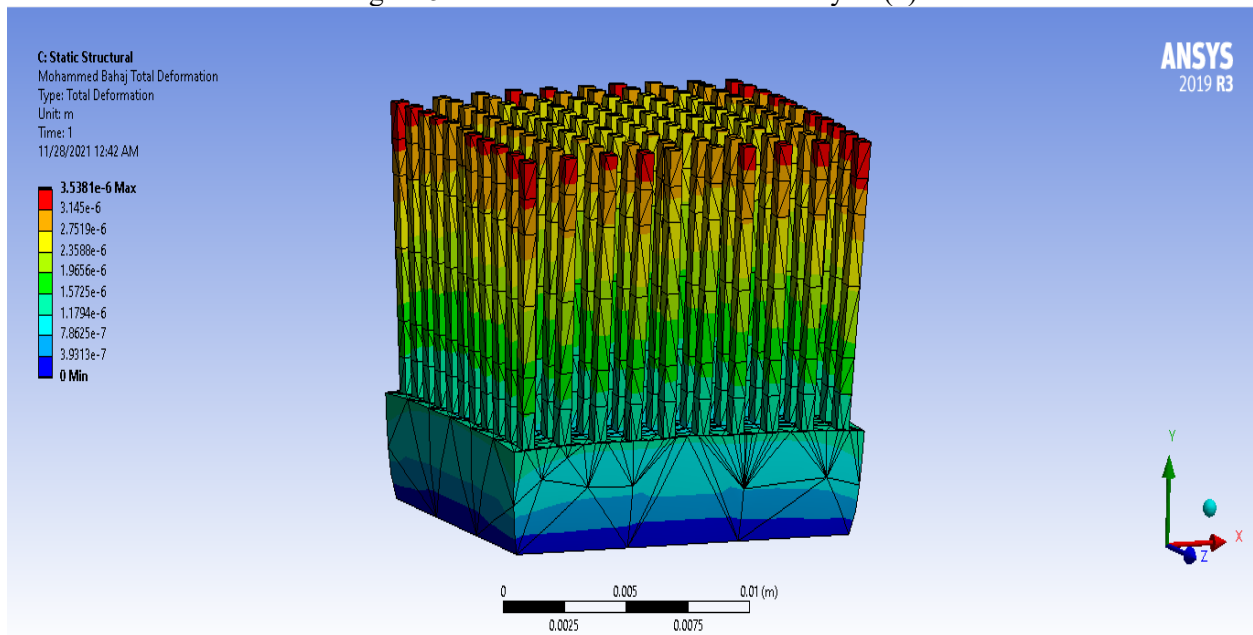


Figure 52. Total Deformation Static Analysis (1)

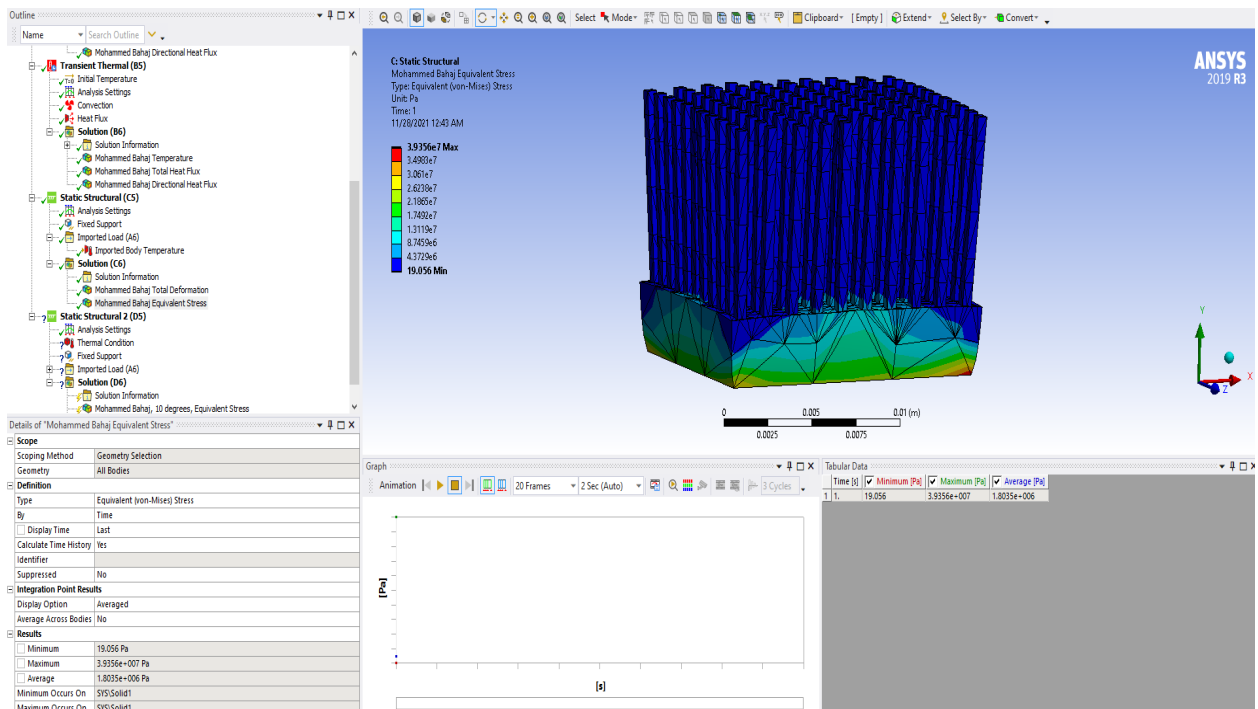


Figure 53. Equivalent Stress (Von Mises) Static Analysis (1)

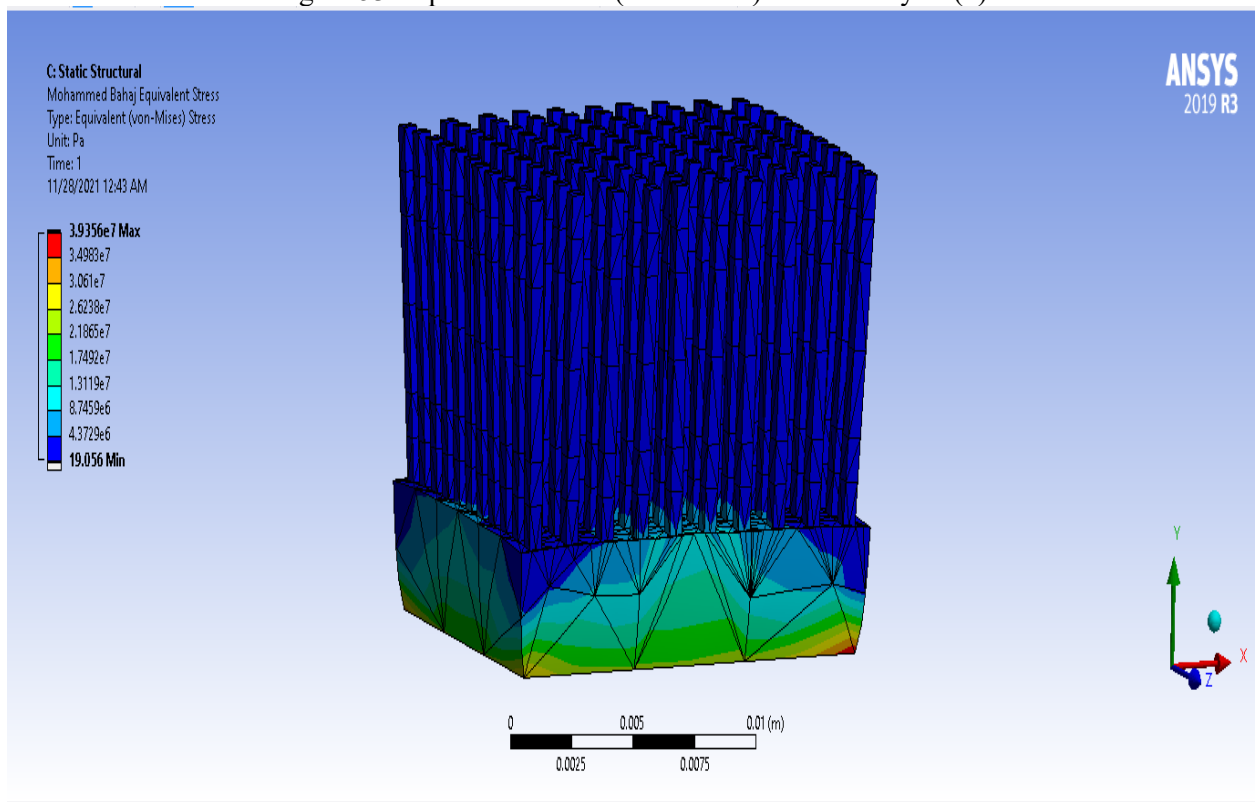


Figure 54. Equivalent Stress (Von Mises) Static Analysis (1)

Part D: Static Structural Analysis 2 (10 degrees)

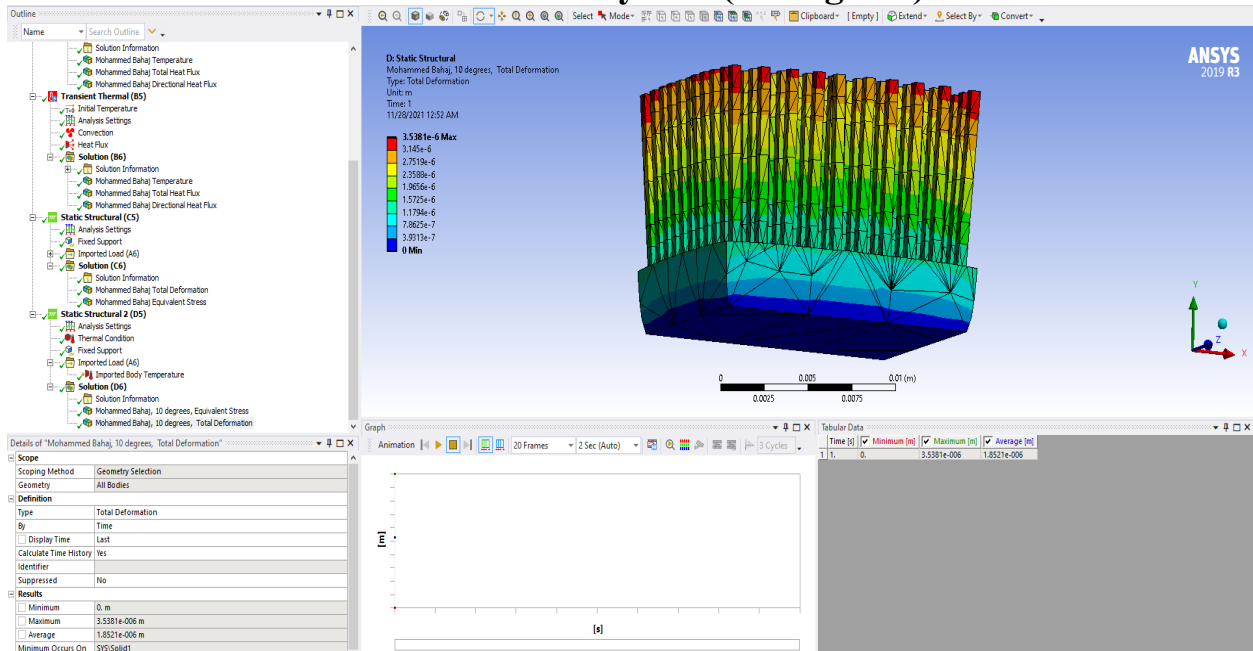


Figure 55. Total Deformation Static Analysis (2) 10 degrees

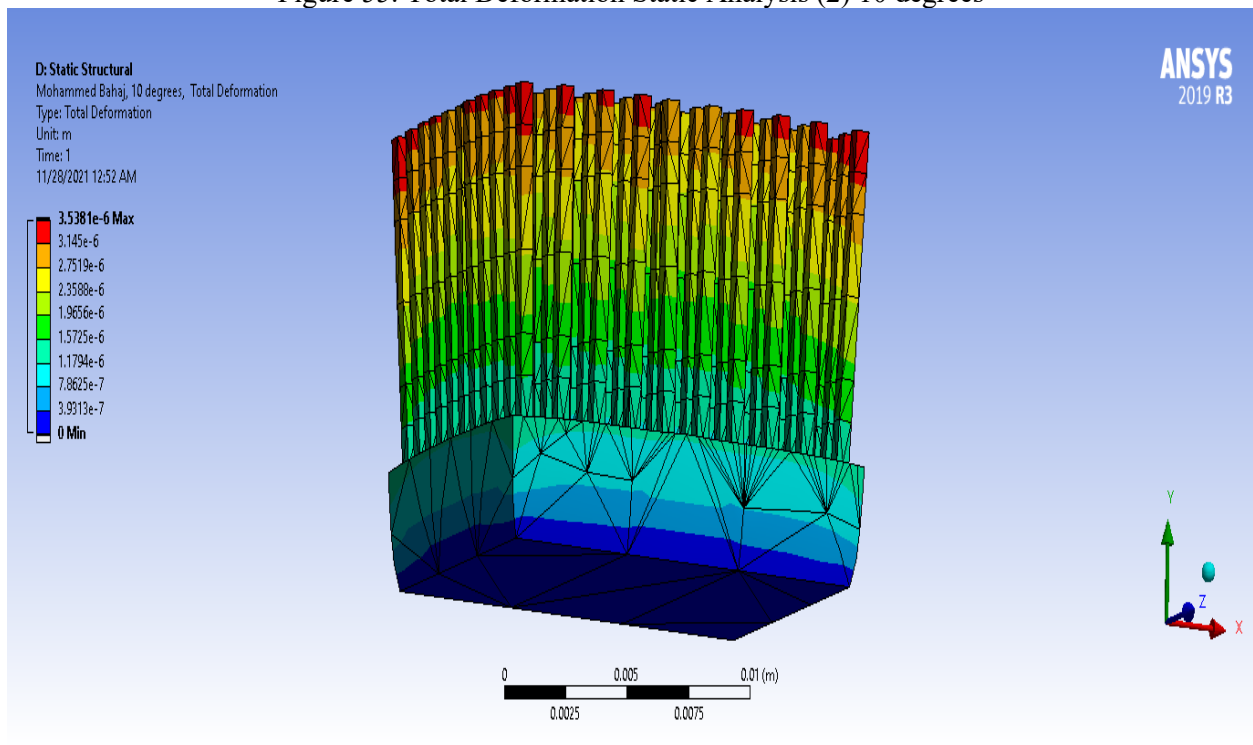


Figure 56. Total Deformation Static Analysis (2) 10 degrees

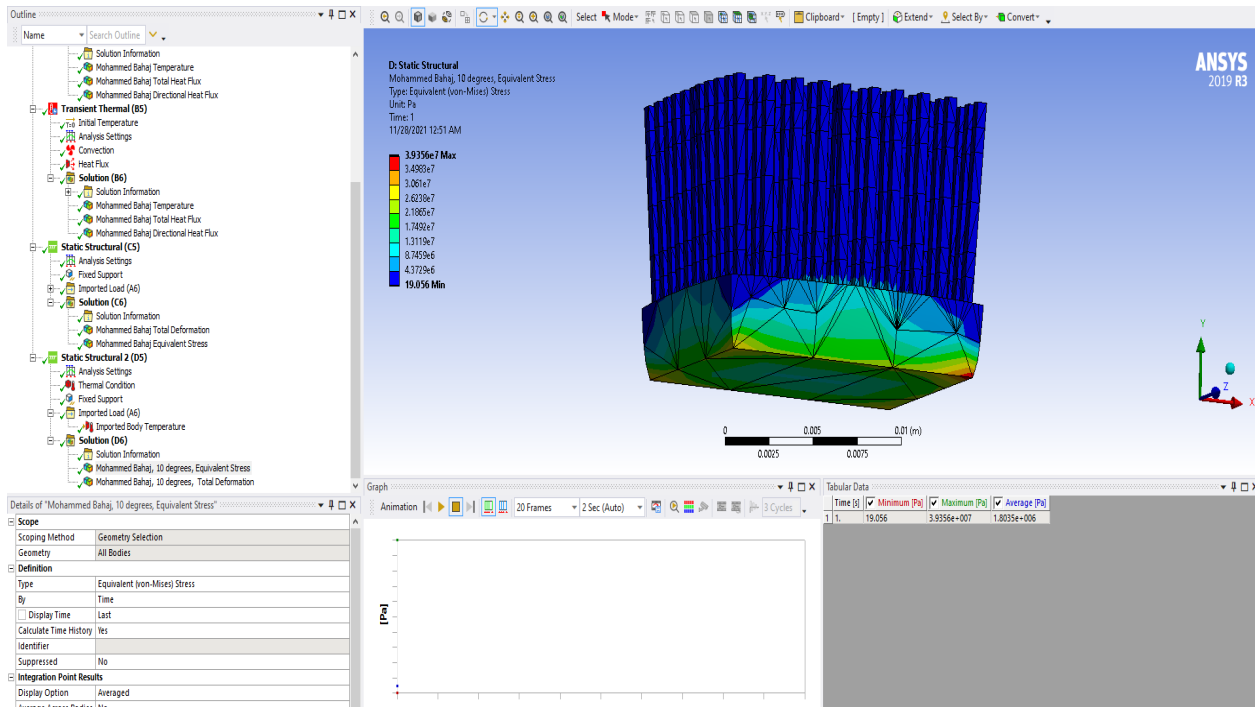


Figure 57. Equivalent Stress (Von Mises) Static Analysis (2) 10 degrees

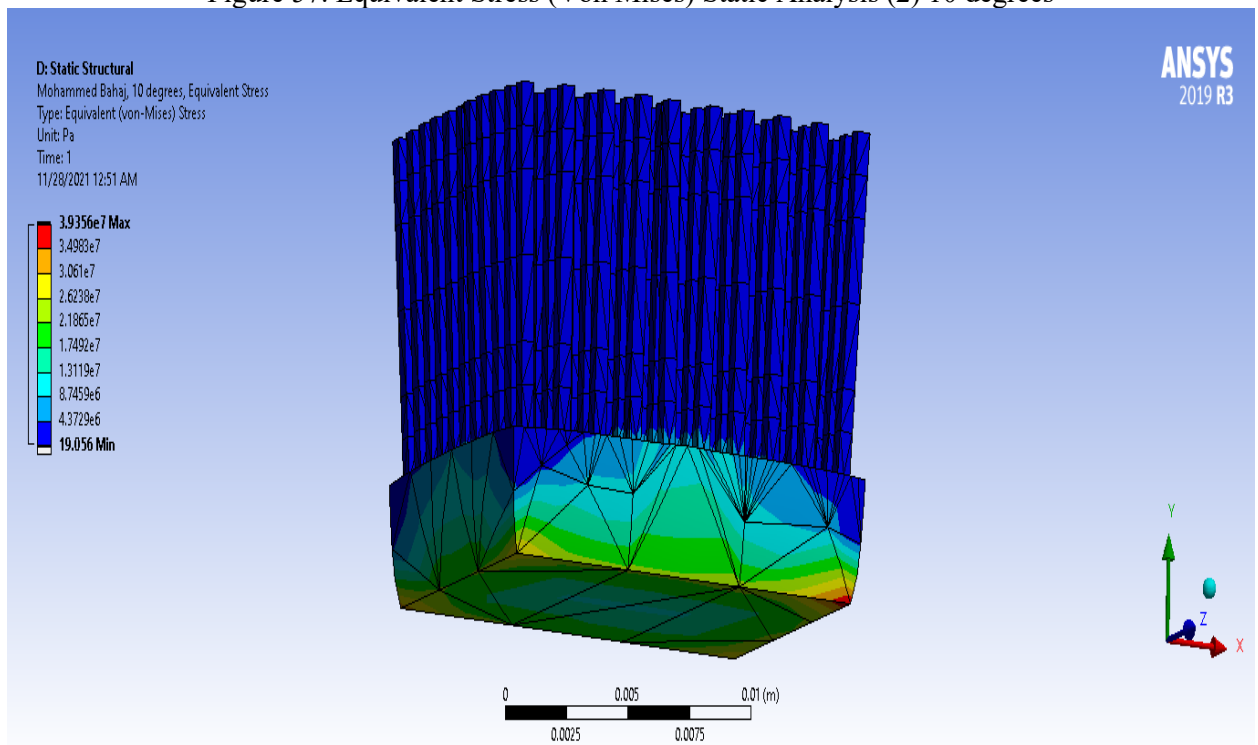


Figure 58. Equivalent Stress (Von Mises) Static Analysis (2) 10 degrees

Discussion and Conclusions

Four different analysis procedures were applied on the heat sink 3d model 1, steady state thermal, transient thermal and two static analysis. One with 22 degree temperature and other with 32 degree temperature. The maximum temperature for steady state thermal was 30.864 degree Celsius, and total heat flux was 2840.3 W/m². Similarly for

the transient thermal with total time of 180 seconds, the maximum temperature in the geometry was 53.729 degree Celsius and total heat flux was observed to be 1835 W/m². For the static analysis with 22-degree temperature and bottom face fixed, the total deformation was 3.47e-6 m and equivalent stress was 7.21e7 Pa. For static analysis with a temperature of 32 degree, the total deformation was 3.47e-6 m and max equivalent stress was 7.21e7 Pascal. So it can be concluded that the maximum temperature that will be generated in the heat sink was 33.33 degree Celsius and max heat flux was 1866.8 W/m².

Similarly for the heat sink model 2, same four analysis procedures and boundary conditions were applied to test the behavior of the heat sink. The maximum temperature for the steady state thermal was 30.743 degree Celsius for model 2, and the total heat flux was 4813.5 W/m². For the transient state, the maximum temperature was 30.256 degree Celsius, and the total heat flux for the heat sink model 2 was 4247.2 W/m². The total deformation for static 1 of the heat sink model 2 was 3.53e-6 m and the total stress was 3.935e7 Pa.

Comparison and Selection of Design

Finally the better design can be selected based on the heat transfer rate. The total flux gives the rate of heat transfer through the body and the higher the heat flux, better the heat sink device as in that case it will transfer more heat and keeps the processor cooler. The second design of heat sink performed better as the heat flux for the second design was 4812 W/m² as compared to 2840 W/m² of the first design. So the second heat sink can be used for better heat transfer as compared to the benchmark design selected for analysis.

References

1. Gupta, D. V. V. a. N. R., 2014. CFD & Thermal Analysis of Heat Sink and its Application in CPU.. *CFD & Thermal Analysis of Heat Sink and its Application in CPU.*, pp. 2-9.
2. Lim, S. L. M. a. K. K., 2005. Thermal Analysis of Heat sink Models using CFD simulation. *Korean Journal of Materials Research*, pp. 829-832.

Comments: It is commendable that a good number of analysis results are included in the report including temperature, heat flux, and thermal stress. However, it should be clearly stated that how these results can be used to justify a better design of a heat sink.

AVERAGE

**MECHANICAL ENGINEERING DEPARTMENT
PRAIRIE VIEW A&M UNIVERSITY**



**MCEG 4309
Finite Element Analysis and Design Project
Fall 2021**

COVER PAGE

**REPORT ON
“Redesign and Perform Finite Element Analysis of a Heat Sink for
Electronic Device”**

**Submitted by
Ihab Ayoubi**

**On
November 25, 2021**

**To
Prof. Shield Lin**

Project Description

Project Title: Redesign and Perform Finite Element Analysis of a Heat Sink for Electronic Device

Many electronic devices generate a great deal of heat during operations. Overheating could cause malfunction and reduce the lifespan of the devices. A heat sink with other accessories is commonly used to take the excessive heat away and keep the electronic device under an acceptable temperature.

- (1) You are required to investigate the design of an existing heat sink on the market. The product will serve as the benchmark design. Use a Finite Element Analysis Program (ANSYS or an equivalent) to determine temperature distribution in the benchmark heat sink under a force convective condition under an electrical fan. Calculate the convective coefficient based on air speed and configuration of the heat sink, and apply the coefficient for your ANSYS analysis.
- (2) You shall modify or completely redesign the heat sink using the same material and within the original outer dimensions. Prove via the Finite Element Analysis Program that “your redesigned heat sink is better,” i.e., your heat sink can “keep the electronic device cooler” than the benchmark heat sink under the same operational conditions.
- (3) If you are allowed to select a different material for the heat sink, what material will you select? Perform Finite Element Analysis of your redesigned heat sink using the new material. Discuss the performance of your heat sink with the new material.
- (4) Discuss the estimated cost of your redesigned heat sink in comparison to the benchmark heat sink.

Grading Rubric:

Item	Content Description	Assigned Points	Points Received	Relevant to ABET assessment
1. Engineering Drawings.	Create engineering drawings with detailed dimensions and materials from a heat sink on the market & design a modified heat sink with new configuration and dimensions.	20		<i>an ability to design a system, component, or process to meet desired needs.</i>
2. Information Research & Calculation of Parameters.	Describe the critical operational conditions of the heat sink. Research and calculate all parameters required for finite element analysis.	20		<i>a recognition of the need for, and an ability to engage in life-long learning.</i>
3. Finite Element Analysis	Create solid models on computer, input parameters to a finite element program- ANSYS or equivalent, solve and obtain results.	20		<i>an ability to use the techniques, skills, and modern engineering tools, (such as ANSYS) necessary for engineering practice.</i>
4. Design Justification, Conclusion, and Discussions.	Interpolate finite element results, justify your design is better based on the analysis results, make a conclusion, and add discussions on materials and manufacturing cost.	20		<i>an ability to design a system, component, or process to meet desired needs.</i>
5. Report Writing	Use of technical writing format which generally includes abstract, introduction, equations, figures, tables, conclusion, references, etc. All parameters used in the analysis must be clearly justified by referring to the origin of technical data or calculated using appropriate methods.	20		<i>an ability to communicate effectively</i>
	Total	100		

Abstract:

A heat sink is a piece of cooling equipment for electrical devices that can provide the necessary cooling performance. This article presents the findings of a research effort to improve and enhance the existing heat sink design used in electrical gadgets. In order to increase the cooling performance of electrical gadgets, the design characteristics of the heat sink and operating parameters, as well as the shape of the heat sink, affect heat convection capacity, and the physical size of the device are some critical factors. There is no doubt that these weapons employ electrical components, which create heat as they operate. As a result of the high temperatures, circuit boards may malfunction or become less efficient in the long run. The issue is exacerbated if the electronic set-up is exposed to extreme temperatures, which shorten the component's lifespan and potentially permanently destroy it [1]. As a result, heat sinks must be designed to disperse the power generated by the device effectively. Designing an effective heat sink for an electrical chip that can handle up to 100 watts of electricity is the goal of this project. Design

formulae were used to address the issue based on the provided requirements and the current heating rate.

Introduction:

The synchronous heat transfer rate phenomena due to convection and radiation are common methods to disperse heat in many industrial systems, such as heat sinks. It is possible to increase the heat transfer rate, reduce the pressure drop, and reduce thermal resistance by modifying heat sink designs. Copper and aluminum are preferred over brass and iron for heat sinks manufacturing. A limited number of pins may be used in a cooling system for maximum efficiency in a small space. The greater heat dissipation occurs when the heat sink's breadth is increased rather than its height, and other design characteristics also impact its thermal performance. Extending the definition of a pin-fin heat sink to include expanded surfaces allows for more in-depth analysis and comparison with traditional heat sink models [2]. The productivity figures of electronic systems and equipment are increasing rapidly but regulating the heat sources has been a concern. When electricity flows through a resistive element, heat is created. These resistive components play different roles depending on the flow of electric current. In order to ensure uninterrupted functioning, heat dissipation is a key consideration.

It is common for Aluminium plates to be used to disperse a tiny amount of energy from heat sinks as part of convection, a process in which heat is transported from fins to the surrounding air. According to the latest research, copper surfaces have been substituted with other conductors to increase heat transmission across the heat sink. New types of conductive heat sinks and increased coolant flow are the keys to reducing the troublesome regions in this manner. Getting too hot in electronic combat devices may lead to a system failure; consequently, proper heat disposal is essential. The cooling system is becoming more crucial in designing electronic items as heat transfer from small components increases and overall shape factors shrink [3]. Digital equipment's part warmth has a negative impact on both its dependability and its lifespan. Managing the thermal performance of a device efficiently may extend the product's life and ensure its dependable operation. As the name suggests, heat sinks help dissipate heat more efficiently from a heated plate to a colder environment. The refrigerating fluid will be considered to be air for the remainder of this study. The connection between solid surfaces and cooling air is the most rapid, and the solid-air contact is the largest hindrance to thermal transfer. This barrier is reduced primarily by dividing the size in physical touch with the coolant that a heat sink may provide [4].

As a result, the working temperature of the gadget may be reduced. The sole objective of a heat sink is to keep the product's temperature from exceeding the maximum permissible value set by the producer. A heat sink model with forced convection heat transfer properties has been investigated using CFD simulation and computations.

Consideration was given to the three geometric characteristics of fin height, thickness, and spacing. A parallel plate heat sink has been used to test the heat sink concept in a series of experiments. This paper's findings and recommendations will be useful to engineers who build cooling systems for electrical devices. Compared to solid fins, porous horizontal rectangular fins are more efficient in transferring heat and reducing weight. Depending on the size of the fins and the material's thermal conductivity, pierced fins facilitate heat transmission [5].

3D modeling:

Initially, an existing heat sink design available in the market was investigated for selecting a product as the benchmark design. It is a commercial product available abundantly in the [market](#) [9], used in CPUs as Raspberry Pi case to keep its temperature low during operations and serve the functions of a cooler. The same dimensions of this product and material were used during the modeling and simulations.



Figure 59. Existing heat sink product design available in market

Two different fins design was modeled on Solidworks. An aluminum plate at the bottom functioned as an electronic chip from which heat will generate during system operations. A copper plate over the electrical chip with expanded surfaces acts as a heat sink. In Figure 2, the heat sink has thin rectangular walls resembling fins that aid in creating cooling phenomena and lowering the operating temperature of the electronic chip. This design idea is taken from a currently available heatsink in the market, and product dimensions are $14 \times 14 \times 8$ mm. Due to thermal contact between the chip and heat sink, facilitating heat transfer between the chip's high-temperature surfaces and the cold heat sink. Additionally, a heat sink (copper) and electronic chip (aluminum) are both made with metals and are in touch, so heat transfer will also occur through the conduction principle.

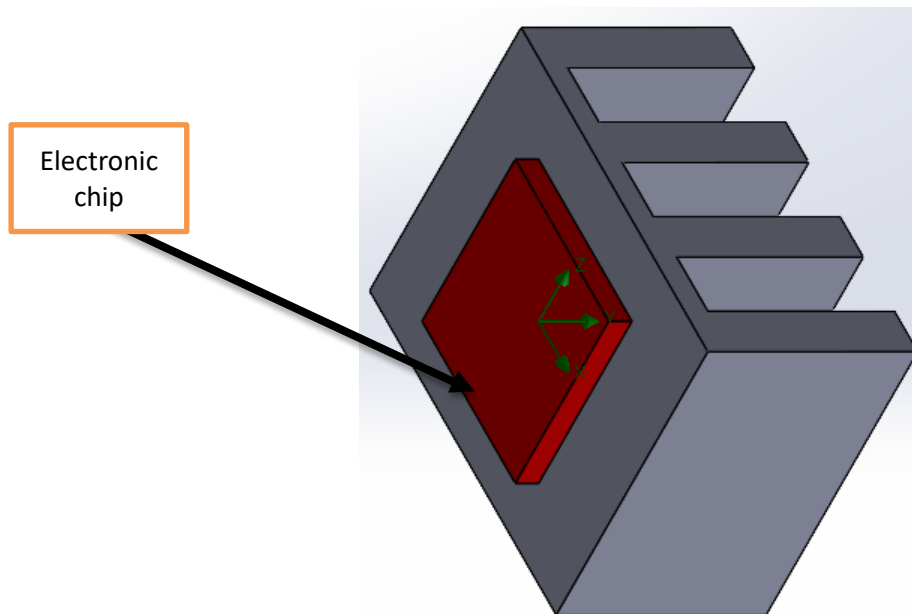


Figure 60. Currently utilizing model in marketplace

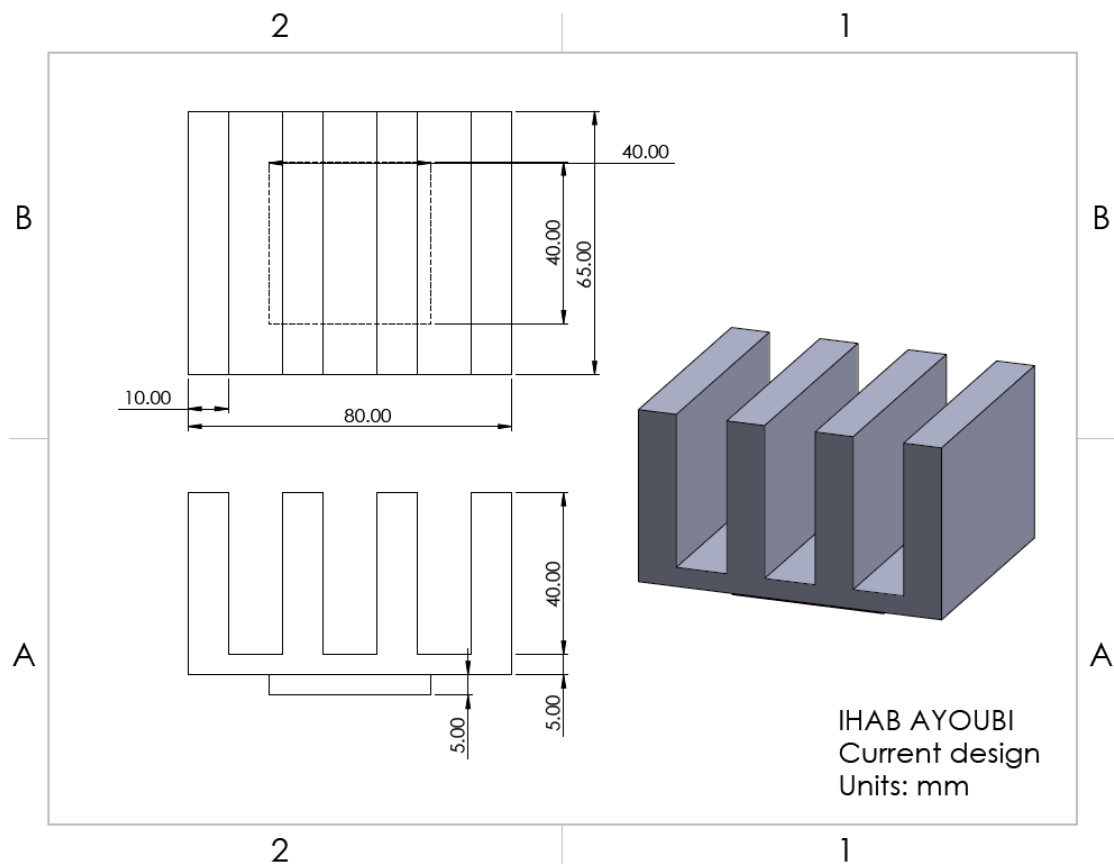


Figure 3. Currently Model Engineering Drawing

The second heat sink design has the same dimensions as the previous model; the aluminum-made electronic chip and copper plate over it were of the same size, just fins design was different. In the 2nd model, long narrow tubes expanded over a copper plate, providing fins function. The modified design of the heat sink has a large number of fins with tight gaps between them, and this model has more surface area covered by prolonged 3d tube-shaped fins.

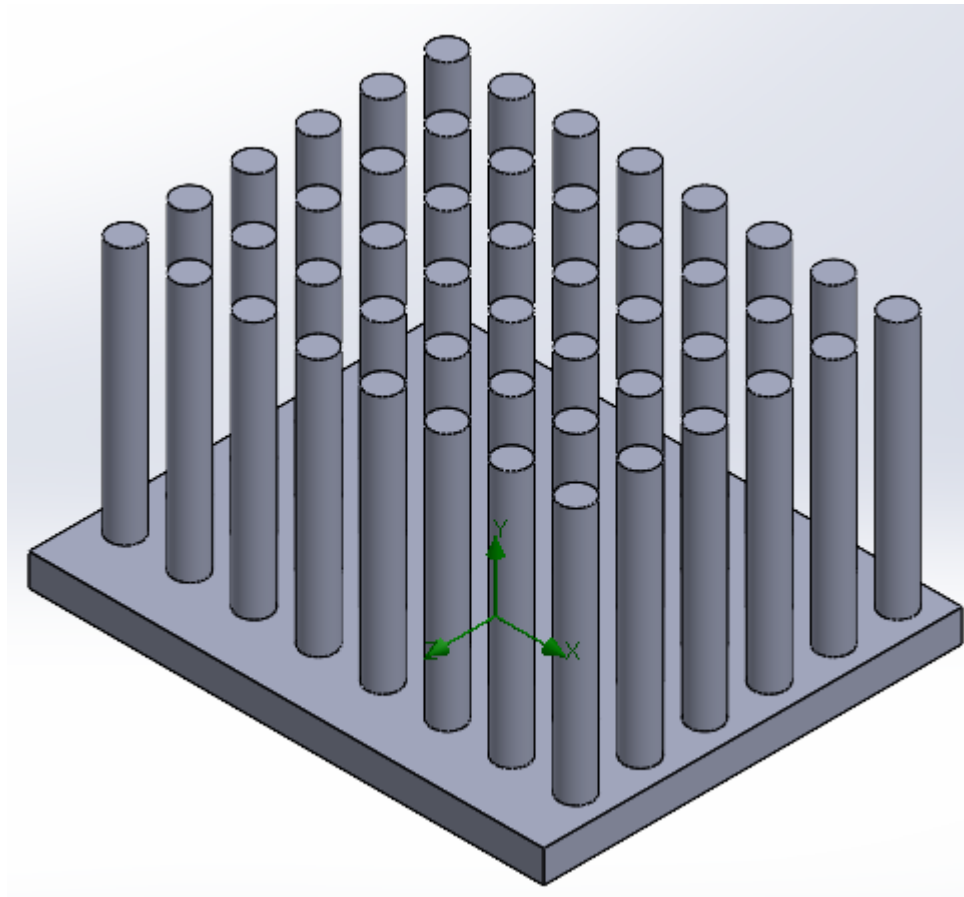


Figure 4. Modified design of heat sink

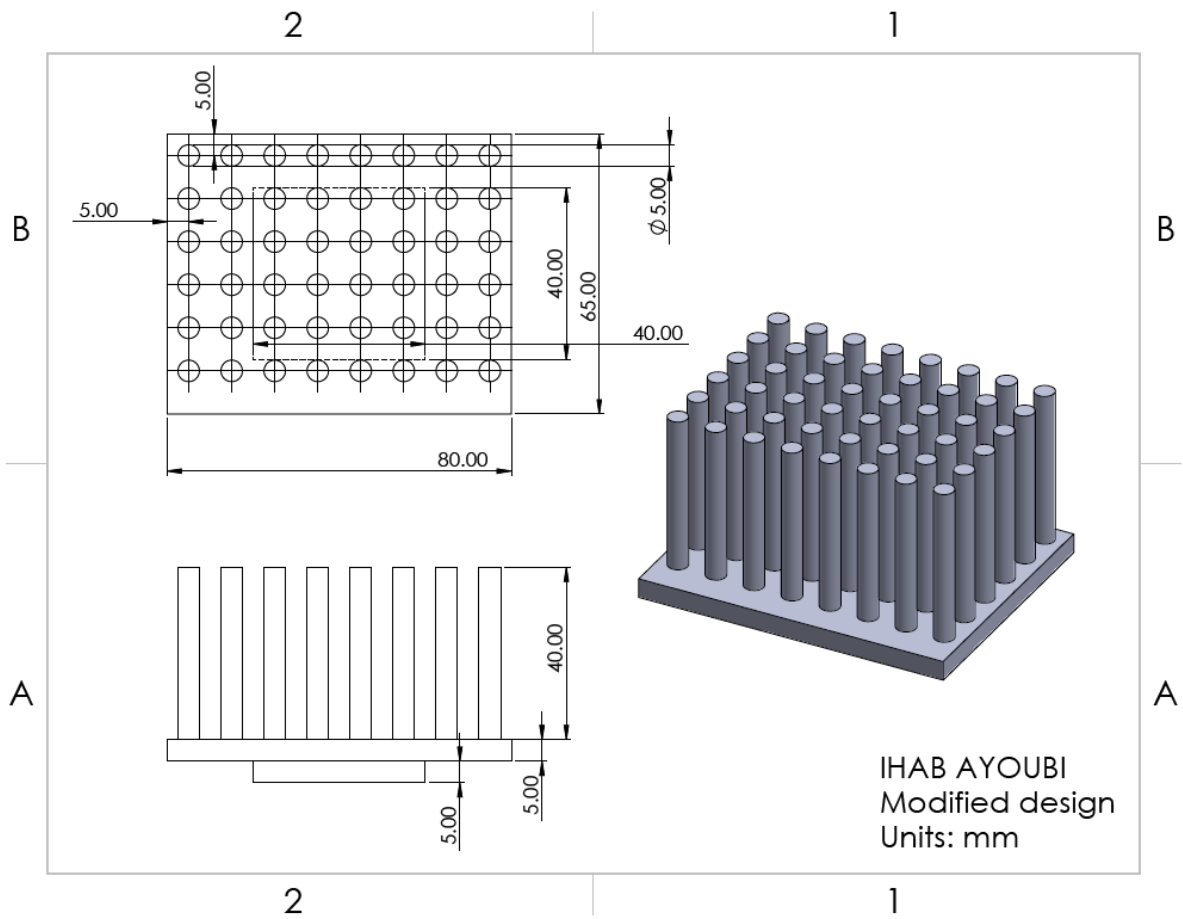
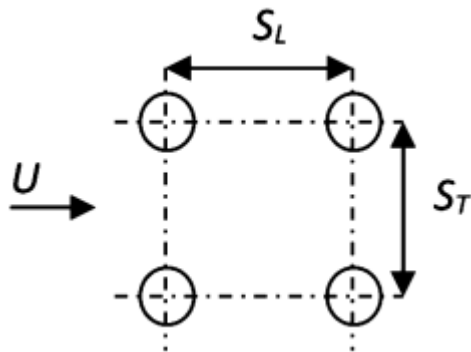


Figure 5. Modified Model Engineering Drawing

Mathematical calculations:

Tube bank type = aligned (in-line)



$$U_T = \frac{S_T U}{S_T - D} = \frac{0.04 \times 5}{0.04 - 0.01} = 6.67 \frac{m}{s}$$

Assumed mean temperature = 25 Celsius

The viscosity of air at 25 Celsius = 1.562×10^{-5}

$$Re = \frac{U_{max} D}{\nu} = \frac{6.67 \times 0.01}{1.562 \times 10^{-5}} = 4270$$

$$Nu = 0.27 \times F \times Re^{0.63} \times Pr^{0.36} \times \left(\frac{Pr}{Pr_s} \right)^{0.25}$$

$$Nu = 0.27 \times 0.945 \times 4270^{0.63} \times 0.7296^{0.36} \left(\frac{0.7296}{0.7073} \right)^{0.25} = 44.47$$

$$h = \frac{k \times Nu}{D} = \frac{0.02545 \times 37}{0.005} = 114.5 \frac{W}{m^2 \cdot ^\circ C}$$

Analysis Methodology:

Multiple simulations were performed in this project, and overall designed heat sinks models' analysis was divided into sorts. Both thermal analyses, as well as computational fluid dynamics (CFD) analysis, were performed well to evaluate the thermal performance of both heat sink designs. The purpose of the thermal analysis was to examine the fins colling efficiency and heat transfer effectiveness between chip and heat sinks via convection. In thermal analysis, a forced convection condition was applied at the heat sink and heat flux condition at the bottom surface of the chip. The heat sink was at cold temperature, and the heat transfer activity was carefully observed to check how much temperature of chip will drop after addition of heat sink. In CFD analysis, the designed model was placed inside a computational domain where the air was flowing, and power condition was applied at the chip lower surface. The heat sink was at ambient temperature, and heat transfer between hot and cold surfaces will occur through conduction plus convection. The air was flowing at room temperature, further speeding up the cooling phenomena as heat transfer with air also occurs. The objective CFD analysis was to evaluate heat sink thermal performance in the case of forced convection and open atmosphere.

Thermal analysis:

Thermal analysis was performed in Ansys, and two conditions were selected for this work. Firstly, aluminum was applied to the electronic chip at the bottom, and copper was assigned to the heat sink laid over it. Both components have a thermal connection between them, and they will treat as one body. [Intel core i7 chip assumed for this project and intel specifies that this chip must operate below 85°C](#) [10]. So, 85°C temperature condition was applied at the lower face of the electronic chip while forced convection coefficient, i.e., $114.5 \frac{W}{m^2 \cdot ^\circ C}$ (calculated) applied at all geometry faces to enable heat transfer inside the model between the hot and cold surfaces from all sides. The ambient temperature was 22 Celsius during simulation. The temperature of the heat sink increased due to heat transfer and distributed irregularly over the entire heat sink surface. The minimum temperature that was observed in this case was 69 Celsius while the temperature at the bottom surface where the heating load was applied was 85 Celsius. The total temperature drop between maximum and minimum was 16 Celsius. The electronic chip temperature also lowered due to heat transfer sensations, and the temperature at the top surface of the electronic chip, which was actually in thermal contact with the heat sink copper plate, was around 75 Celsius. The CAD model dimensions used for analysis in this project has dimensions $14 \times 14 \times 8$ mm exactly similar to the current product available in market.

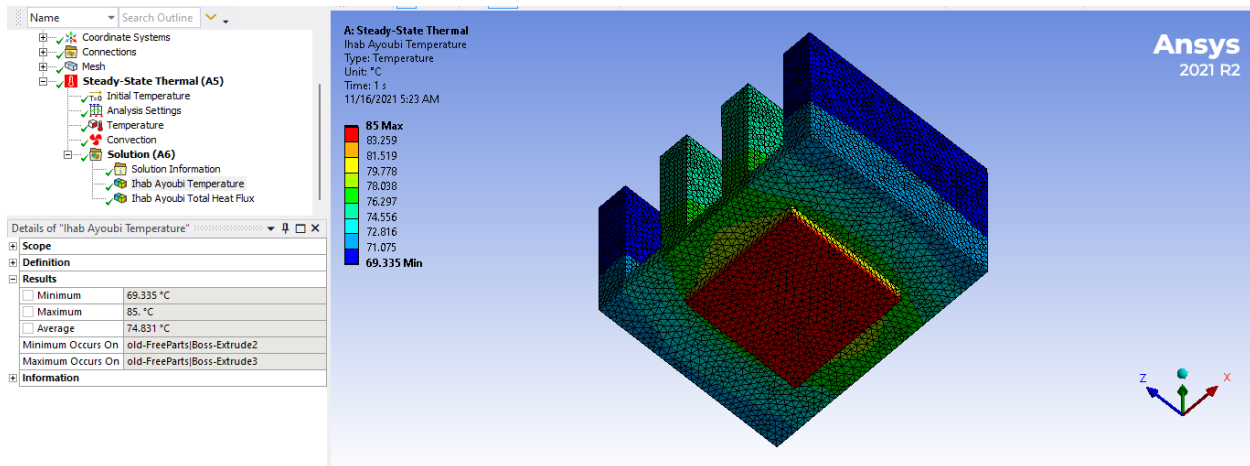


Figure 6. Temperature contours results - V 1

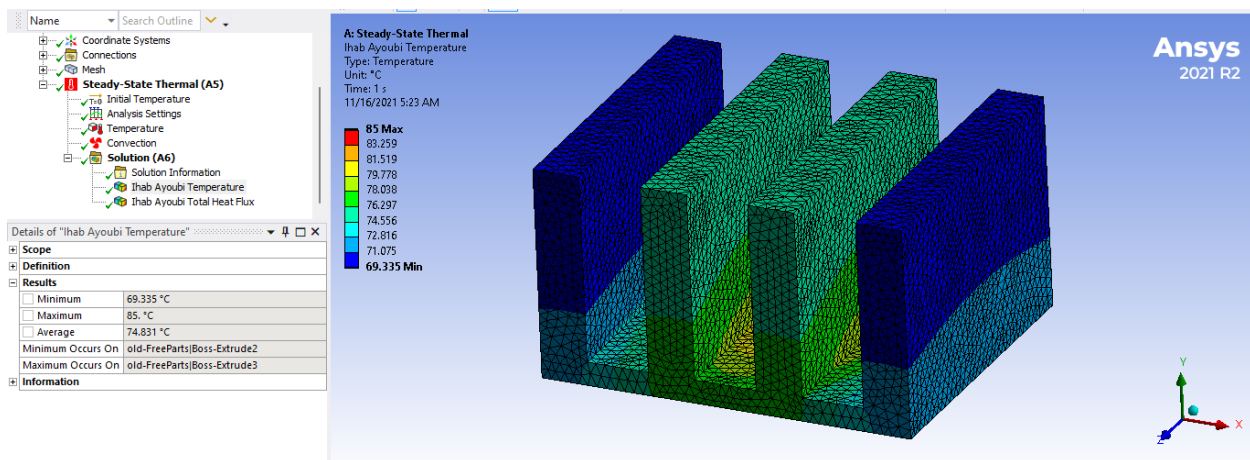


Figure 7. Temperature contours results - V 2

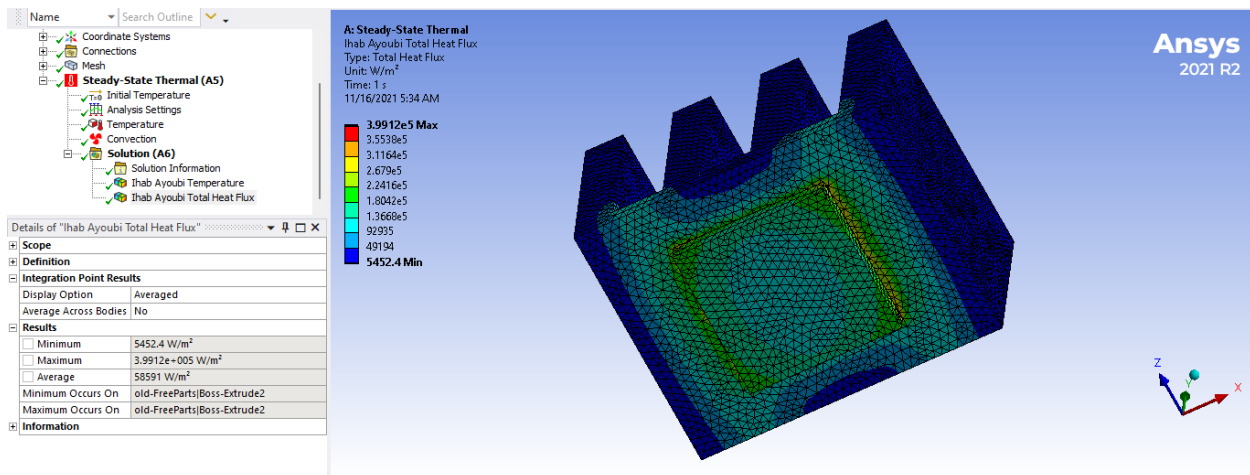


Figure 8. Heat flux distribution

The same thermal analysis was performed for the second heat sink model, and operational conditions remained similar that was used for the old model, i.e., the heat sink was made with copper, electronic chip material was aluminum, temperature of chip surface was 85°C, convection coefficient was $114.5 \frac{W}{m^2 \cdot ^\circ C}$ and bulk ambient temperature was Celsius K. The maximum temperature, in this case, was 85 Celsius, whereas the minimum

temperature was 61.6 celsius. The total temperature drop noticed during this simulation was 24 celsius. The electronic chip temperature dropped by 10 celsius after heat transfer.

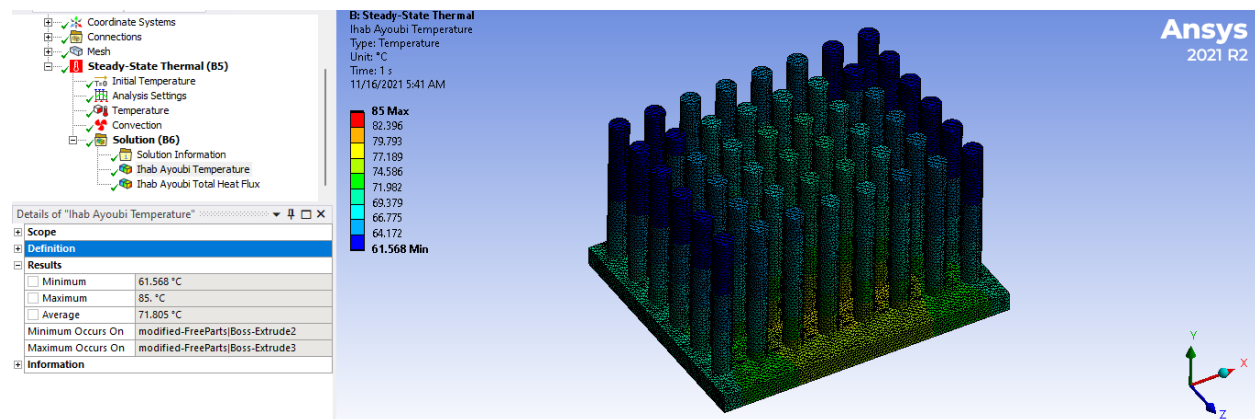


Figure 9. Modified design temperature distribution results – V1

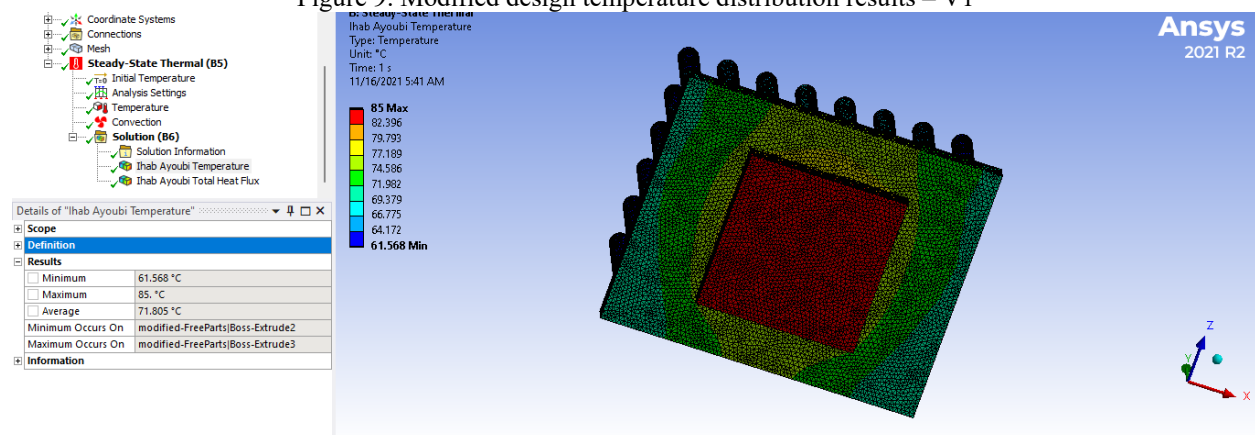


Figure 10. Modified design temperature distribution results – V 2

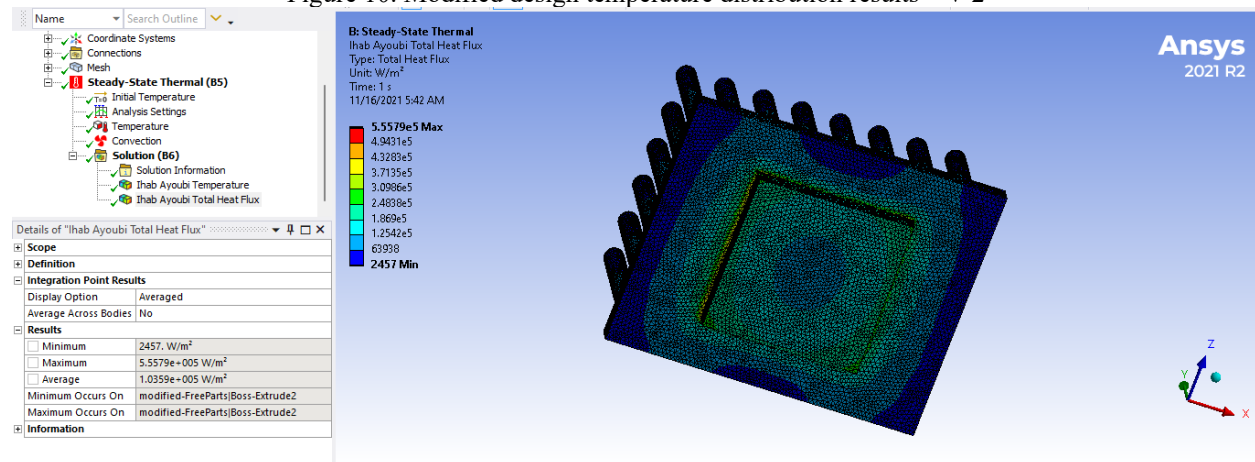


Figure 11. Modified design heat flux computation results

The modified design has a large number of fins than the old design, and circular-shaped tube fins have a large surface area. The circular tubes are aligned very closely, having minimum gaps between them and providing a maximum surface for interacting with the atmosphere or cold air. The whole temperature drop between the current model was 16 Celsius while the newly modified model gave a 24 Celsius temperature drop making this design a better choice for using a heat sink. The absolute difference between heat transfer or temperature drop between old and new models is 8 degrees Celsius, proving that this design can give better thermal efficiency than previously used models in the market.

CFD analysis:

A computational fluid dynamics analysis was also performed on the same designed fins geometries to evaluate their thermal efficiencies. A computational fluid domain was created around the model in which air at ambient temperature was flowing at 5 m/s. A 85 °C temperature condition was put on the bottom surface of the electronic chip and $114.5 \frac{W}{m^2 \cdot ^\circ C}$ Convection heat transfer coefficient was applied at all faces of the heat sink. Initially, the heat sink is at room temperature, but heat transfer between metals occurs due to conduction, and its temperature rises. The chip will transfer some of its heat energy to the heat sink, and due to convection, this energy will transfer from the heat sink to the air. The air temperature will increase that will come in contact with heat sink, and it must transfer a maximum of this power to the air so that heat sink can stay cold temperature. When there is a large gap between the electronic chip and heat sink temperatures, maximum heat transfer will occur, and the chip will continue operating at normal after exchanging power (heat energy) with the heat sink. The selection of heat sink material, fins shape, and size play a crucial role in swapping heat with chilled air. The average heat transfer rate during the simulation was 4 W, and $5937 \frac{W}{m^2}$ surface heat flux spotted (Figure 8) computed by the Solidworks Flow simulation software.

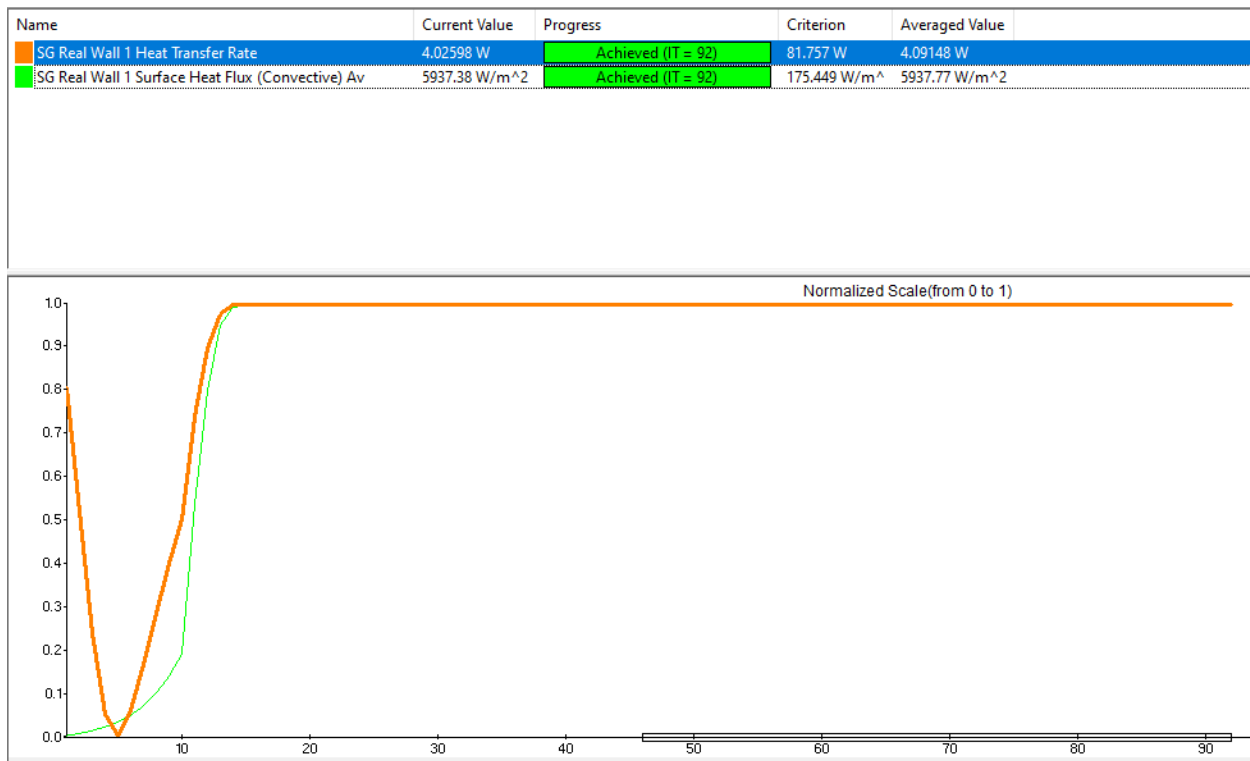


Figure 12. Thermal performance results current model

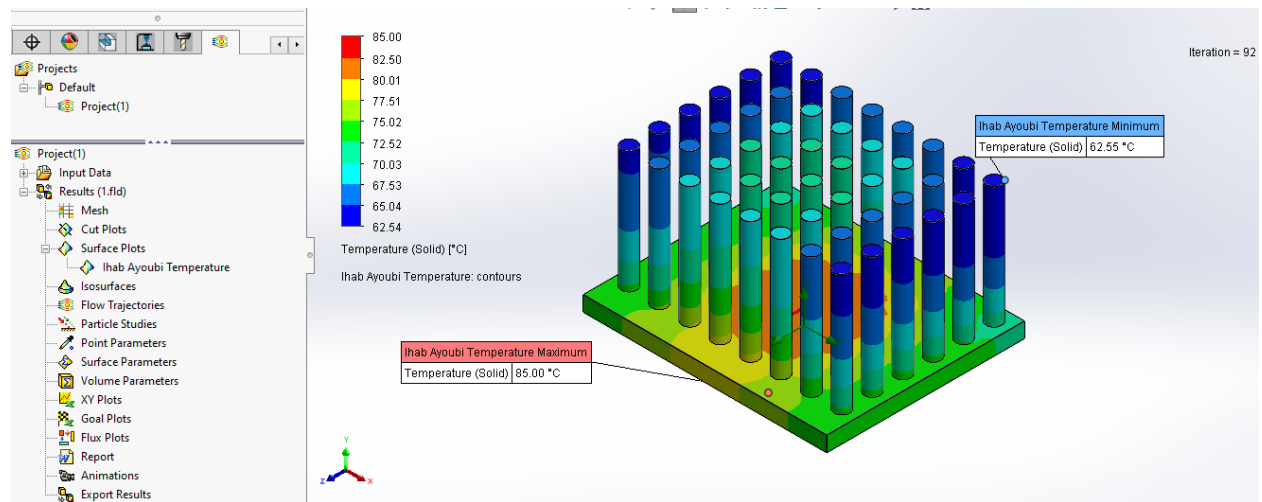


Figure 13. Temperature variation results of proposed model

The maximum and minimum temperature observed during the simulation was 85 Celsius and 62.55 Celsius. The temperature drop of the electronic chip during CFD analysis of the modified model was 23 Celsius. The same CFD analysis with similar boundary conditions was also performed on the old model, where the electronic chip temperature drop was 10 Celsius. The average heat transfer rate during simulation of rectangular fins models was 4.7 W, and surface heat flux noticed was $5172 \frac{W}{m^2}$ (Figure 10). During the large rectangular fins design CFD analysis, a higher heat generation rate was computed with less heat transfer rate and more surface heat flux. On the contrary, it depicts that the modified proposed heat sink is far better to utilize in electronic gadgets because it offers less surface heat flux, more heat transfer rate, and different temperatures of electronic gadgets.

Name	Current Value	Progress	Criterion	Averaged Value
SG Real Wall 1 Heat Transfer Rate	4.67384 W	Achieved (IT = 89)	75.6661 W	4.67384 W
SG Real Wall 1 Surface Heat Flux (Convective) Av	5171.92 W/m ²	Achieved (IT = 89)	167.298 W/m	5171.92 W/m ²

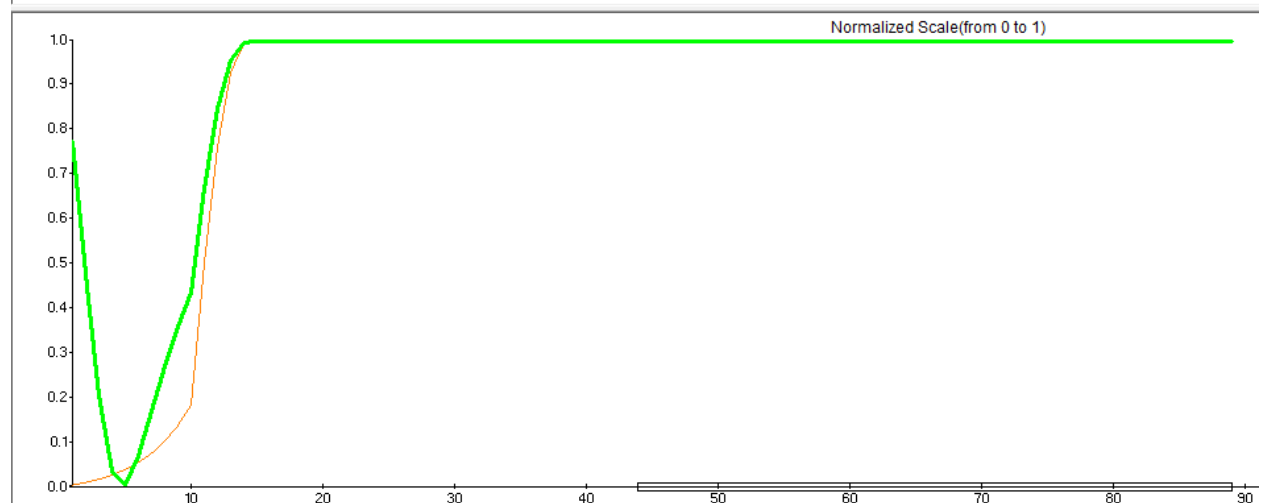


Figure 14. Thermal performance results modified model

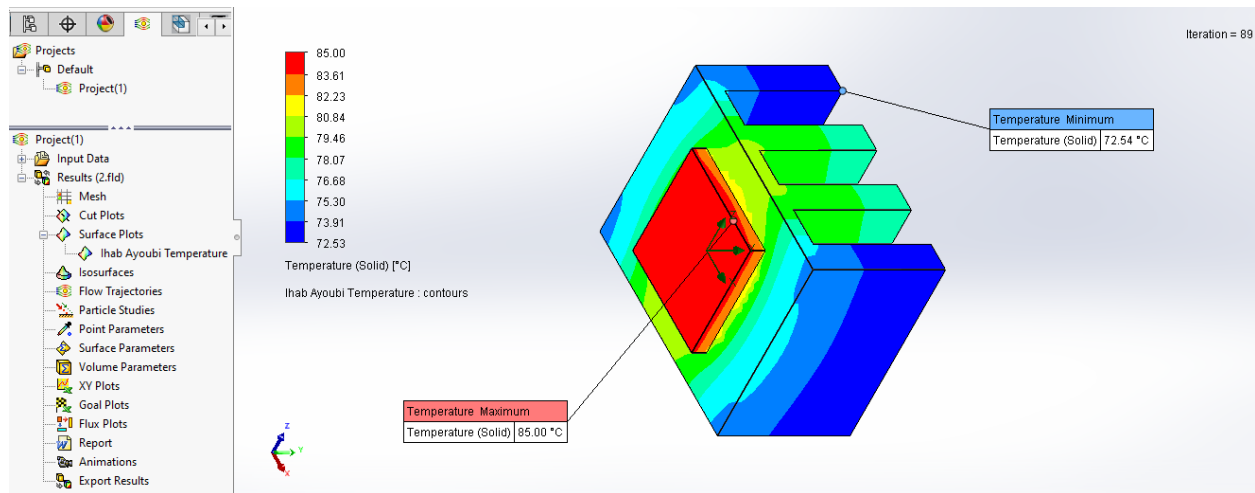


Figure 15. Temperature variation results of old design

Product Cost:

If a manufacturer uses 3d printing to fabricate the designed modified heat sink, it will cost \$24.02. Assume the following parameters that are required for additive manufacturing:

- PLA material is used for 3d printing
- 0.35 mm layer height
- 25% infill
- 30 mm/s print speed
- \$5/hour machining cost
- \$0.03/gram material cost

The total weight of the proposed modified heat sink model is 147.13 grams. Based on the supposed parameters above, the overall time required for additive manufacturing is 3 hours, 55 minutes, and 14 seconds as well as \$24.2 price/product.

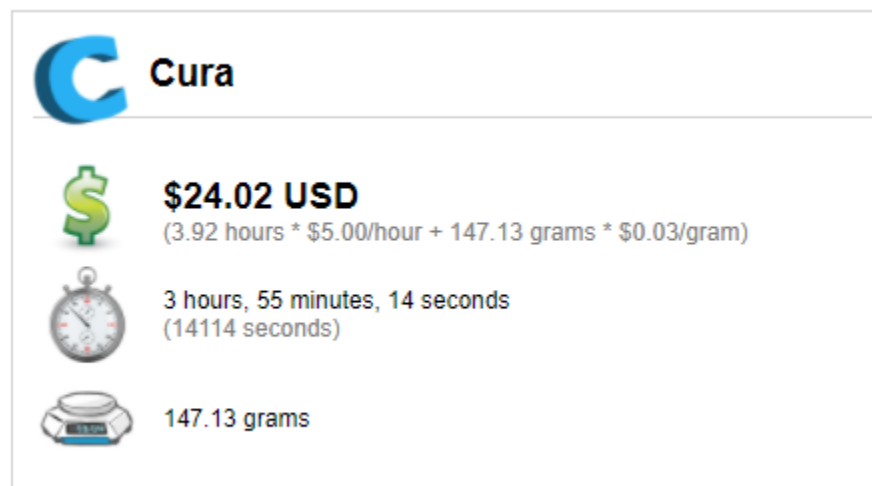


Figure 61. Proposed design cost

Although the 3d printing cost of the old model (Figure 1) is \$21, but the modified model has more number of fins to

increase the cooling performance that will charge a little bit more. Moreover, if a maker prepares molds/dies for bulk production, it will significantly reduce modified design costs.

Discussion:

The outer layer of a heat sink that is in exposure to air is increased by means of a foundation with smooth surfaces and a series of protruding fins. As opposed to fans, which are moving parts, heat sinks are stationary pieces of equipment that benefit from increased airflow. A thermally conductive lubricant is usually used to provide a good thermal interface with a heat sink's bottom, which has to be frictionless. The heat sink is secured to the device using a gripping mechanism, fasteners, or thermal glue, but the pressure applied should not break the device. Thermal resistance, composition, dispersion resistance, fin orientation, and surface color are important considerations when selecting a heat sink or layout. Using thermal models, engineers are able to see what the device's heat and airflow look like from the outside. With the use of thermal simulations, designers can create a chiller, optimize a system to cut down on electricity expenditure, weight, and expense, and validate the thermal design to make sure there are no difficulties with the machinery when it is produced [6]. The most efficient way to cool items within the limits of their design typically necessitates the use of thermal modeling. In the initial stages of electromechanical development, the simulation may be used to check the machine's thermal performance and optimize it. Thermal modeling is a quicker, simpler, and safer alternative to real testing and monitoring for optimizing things like heat sink shape and air velocities.

The steady-state thermal module in Ansys was used to run a series of simulations on the heatsink models that had been developed. The study's goals were to analyze the design for optimal geometric dimensions and compare the thermal efficiency of present and proposed heat sink designs. In developing a heat sink, the choice of material serves an important role since the number of fins boosts heat transfer and the convection also depends on the material's thermal conductivity [7]. Temperature dissipation is the primary factor in determining the dimensions of a heat sink's surface area, as well as the composition and quantity of fins used to cover it.

Conclusion:

Based on the simulation findings, it can be concluded that the heat sink may be employed as a viable approach of cooling electronic equipment. Designers will use the Solidworks flow simulation program to model how dissimilar materials affect the refrigerating performance of the heat sink, intending to improve chip efficiency by increasing thermal efficiency. Material with excellent thermal conductivity helps prolong the maximum temperature, resulting in greater long-term chip reliability. Thermal conductivity and heat capacity are the two most crucial factors determining which material is best suited for a certain application. Increased thermal conductivity and density contribute to a longer time to reach steady-state temperature, as well as a greater amount of energy being absorbed by material throughout its heating [8]. The findings show that copper has a stronger ability to decrease surface temperature than other metals. The purpose of this work is to examine the influence of heat sink design on the chilling capacity and productivity of electronic chips by simulating the thermal transfer across a heat sink.

When subjected to a 85 Celsius thermal load, circular fins cool faster than rectangular pin fins. A circuit board cooled by forced convection was used to test the overall efficacy of various heat sinks designs at a velocity of 5 m/s, and the study presented offers CFD and thermal analysis of varied shape fins heat sinks. There is no doubt that circular pin fins work better, and a heat sink should contain the most fins it can.

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Comments: The analysis is assuming a constant temperature of 85 degree at the contact surface between the chip and the bottom surface of the heat sink. This assumption is usually applied when the heat source is very large that can hold the 85 degree temperature regardless which type heat sink is used. However, a typical chip has a limited heat source. It would be better to use a constant heat flux as the source entering through the contact surface between the chip and heat sink. The 2nd questionable assumption is to use the same convection coefficient to the two different types of heat sink.

Even though the assumption are not perfect. Finite element analysis and report writing are good.

BELOW AVERAGE

**MECHANICAL ENGINEERING DEPARTMENT
PRAIRIE VIEW A&M UNIVERSITY**



**MCEG 4309
Finite Analysis – Fall 2021**

“Heat Sink Project Design”

**Submitted by
Guy Green**

**On
December 1, 2021**

**To
Dr. Shield Lin**

I, Guy Green, declare that this report was entirely written by me. I understand that if any part of this report was copied from previous semester report, or copied from another student, or copied directly from the Internet (or any sources that are not authorized by the instructor), the score for the report will be 0%.

Signature _____ Date _____

Abstract

Being given the outline for the project, many electronic devices generate a great deal of heat during the operations. Anything that has a heat sink has the possibility to overheat and malfunction causing a reduction in the devices lifespan. The purpose of the heat sink is to remove any excessive heat and keep the device under an acceptable temperature. The ideal temperature for the heat sink to perform at its maximum capacity is from 35 to 45 degrees C. In this project report, it will discuss which material is better at cost and efficiency for the device to fulfil its purpose at maximum optimization. For this project, having to find the heat sink's heat transfer coefficient, the film coefficient as seen on ANSYS, to determine the temperature distribution in the device. The temperature distribution can fluctuate different values but will never exceed the certain amount. One way that the temperature can be reduced is the number of fins on the heat sink.

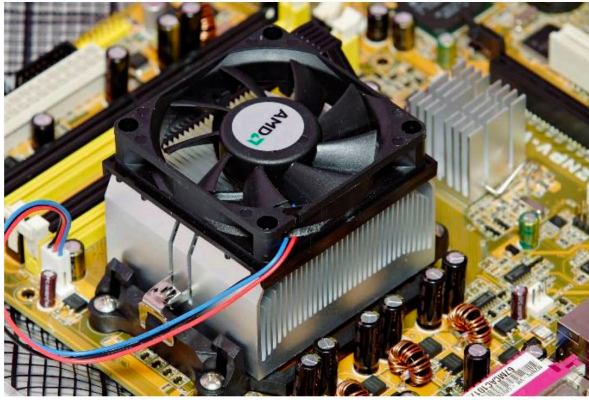


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1. Introduction

1.1 Backgroud

Based on each device's configuration, we find a multitude of heat sink aesthetics, design, and ultimate capabilities. Each heat sink is valuable in applications. Heat sinks are mainly found in CPU's or military devices. For the CPU example, the PC's being used at Prairie View (Dell Precision 3630) and for military purposes, the newest model of the F-35 uses a heat sink but is called a heat exchanger.

1.2 Project Description

Designing and researching a benchmark heat sink that is already on the market. By changing the original material and the original dimensions into a new heat sink using ANSYS, Finite Analysis, and Heat Transfer knowledge to find the right parameters to find the film coefficient, the convection heat transfer coefficient. Comparing the newly designed heat sink, an individual must explain how the newly designed product is more efficient than the ones on the current market. The benchmark is an aluminum heat sink, the dimensions are labeled below with an image of the heat sink.

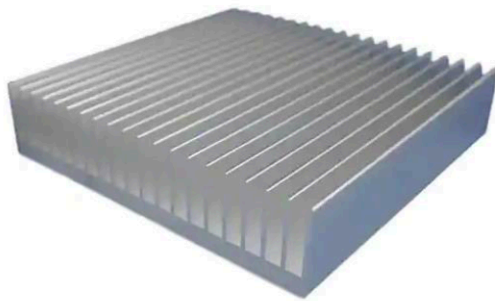


Figure 1.2 - Dimensions of Heat Sink

Length	6.043" (153.50mm)
Width	5.906" (150.00mm)
Diameter	-
Fin Height	1.338" (34.00mm)
Power Dissipation @ Temperature Rise	-
Thermal Resistance @ Forced Air Flow	-
Thermal Resistance @ Natural	-
Material	Aluminum

Figure 2.1 - Benchmark Heat Sink

1.3 Strategy

Over the course of the Fall 2021 semester, the project is similar to Case Study 9.4 that was a homework assignment. In that case study, the class used ANSYS to see the temperature distribution in the heat sink using the heat flux and convection heat transfer coefficient. The project is similar in some ways, using the heat flux, convection, along with ambient air being the temperature of the room. In the project, an assumption was made when it came to the ambient air and the temperature of the fin plate.

1.4 Project Planning

The initial planning of this project is to research and find a heat sink. Once the benchmark heat sink was found, using the dimensions provided, design the heat sink then calculate the the heat transfer coefficient. The heat transfer coefficient is what's used while applying convection onto

the model of ANSYS. The temperature distribution will be compared to the other two designs that will be using different dimensions and a different material.

2. Executive Summary

2.1 Benchmark Heat Sink

The bench mark heat sink is an aluminum heat sink. The thermal conductivity for the aluminum material is $k = 237 \text{ W/m}^2 \cdot \text{K}$. Having to calculate the Reynolds number, Nusselts number, Prandlt number and also Grashoff number to find the heat transfer coefficient. Using the dynamic and kiematic viscosity. Utilizing the resources, using MATLAB to calculate all the unknowns in the equations.

- Prandtl Number - $Pr = \frac{\mu \cdot C_p}{k}$
- Grashof Number - $Gr = \frac{g \beta (T_s - T_a) L^3}{\nu}$
- Nusselt Number - $Nu = 0.59 * Ra^{0.25}$
- Convection Heat Transfer Coefficient - $h = \frac{Nu \cdot k}{L}$

2.2 Benchmark Heat Sink Calculations

In this section, the MATLAB code will show the heat transfer coefficient along with the market model of the heat sink. Showing the temperature distribution within the aluminum heat fin. Aluminum is cheap material and is the reason why many heat sinks are purchased in bulk for manufacturing purposes.

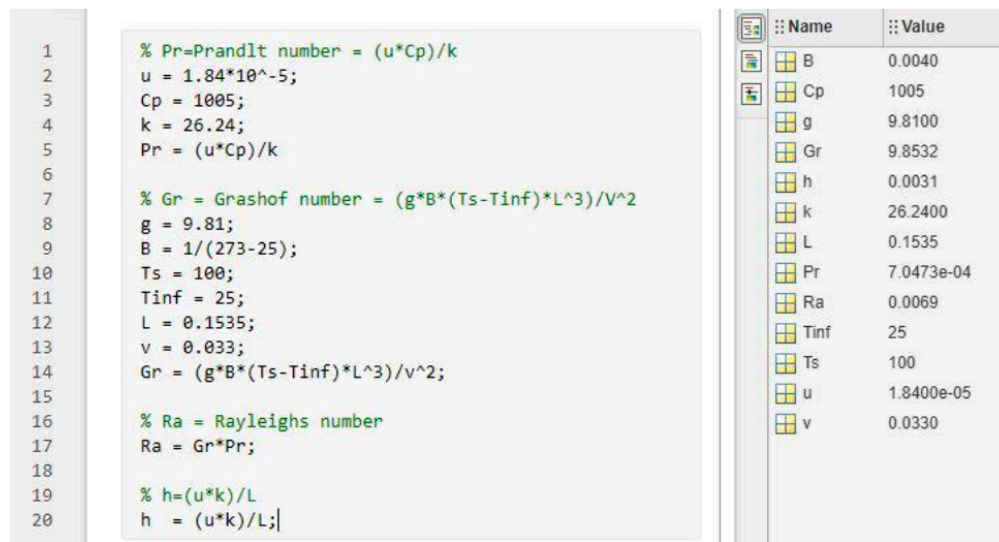


Figure 2.1 – MATLAB Code

2.2.1 Benchmark Model

The three figures below show the temperature distribution in the heat sink and it's fins. 3D Model and drawing sheet of the market product.

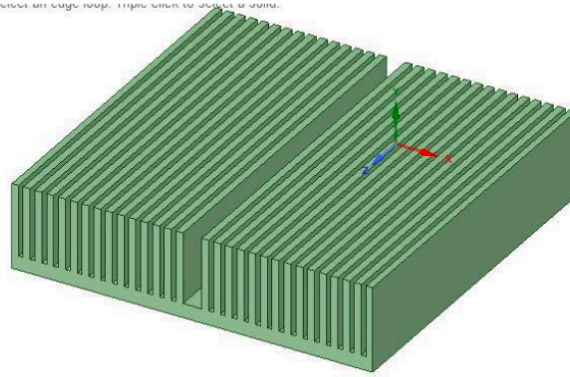


Figure 2.2 – 3D Model

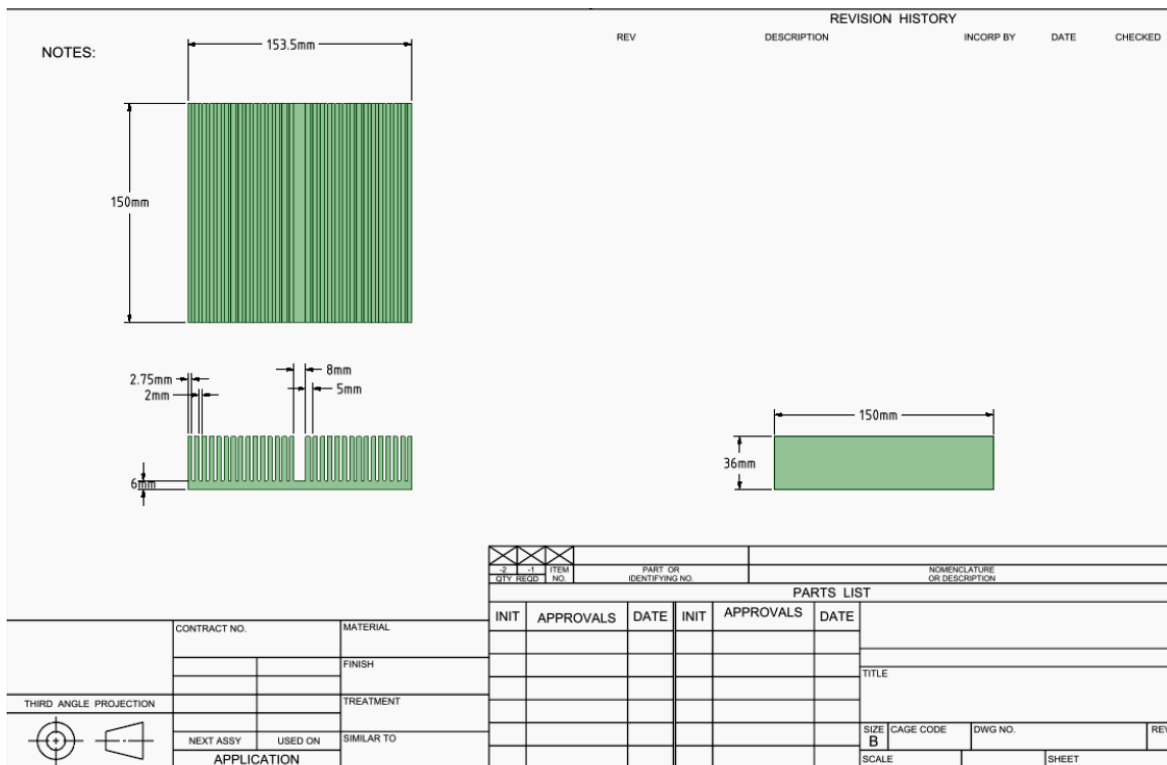


Figure 2.3 – Drawing Sheet of Benchmark Heat Sink

Thermal stresses due to changes of temperatures in structures are also discussed and the effects of constraints of the structures on the thermal stresses are emphasized. Based on the aluminum heat sink, that max temperature is 23.42 while the minimum is 23 flat.

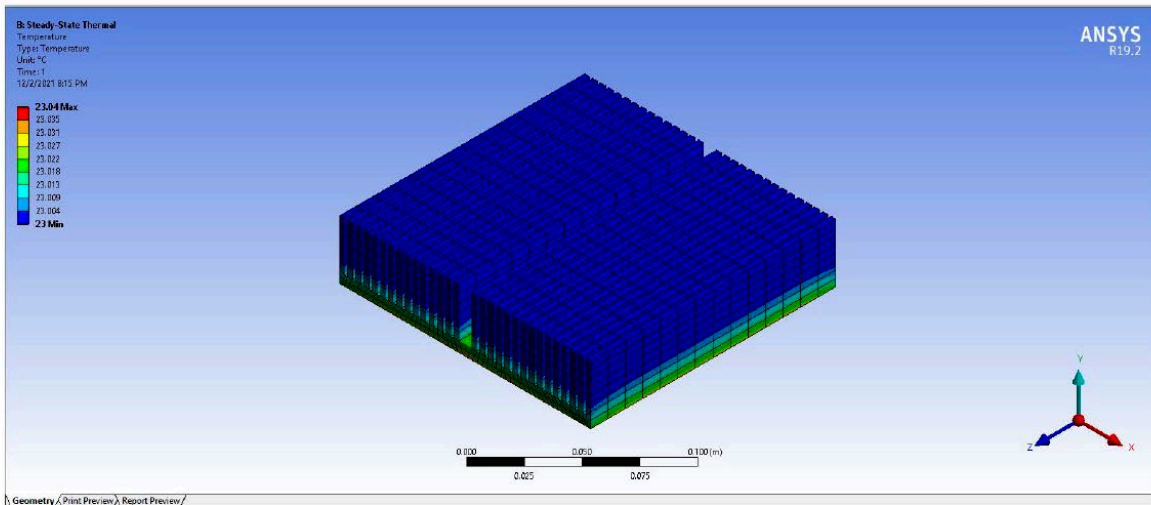


Figure 2.4 – Temperature Distribution

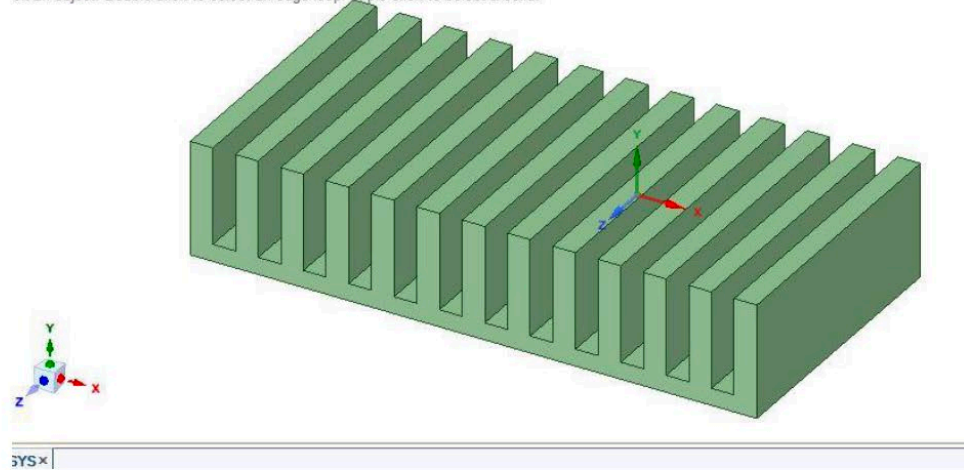
2.3 Aluminum with Different Dimensions

Changes in this heat sink has been the dimensions with the same material. Aluminum no having as high as a conductivity as copper, 232 compared to 402, aluminum is the better conductor in an aspect of ways. The heat convection coefficient has increased by 20 and the temperature distribution between the fins. The amount of fins in the heat sink has tremendously decreased along with all other parameters.

2.3.1 Dimensions Model

Figure 2.5 – 3D Model of Heat Sink

Click an object. Double-click to select an edge loop. Triple-click to select a solid.



2.3.2 Dimensions of 3D Model

Figure 2.6 – Newly Produced Dimensions

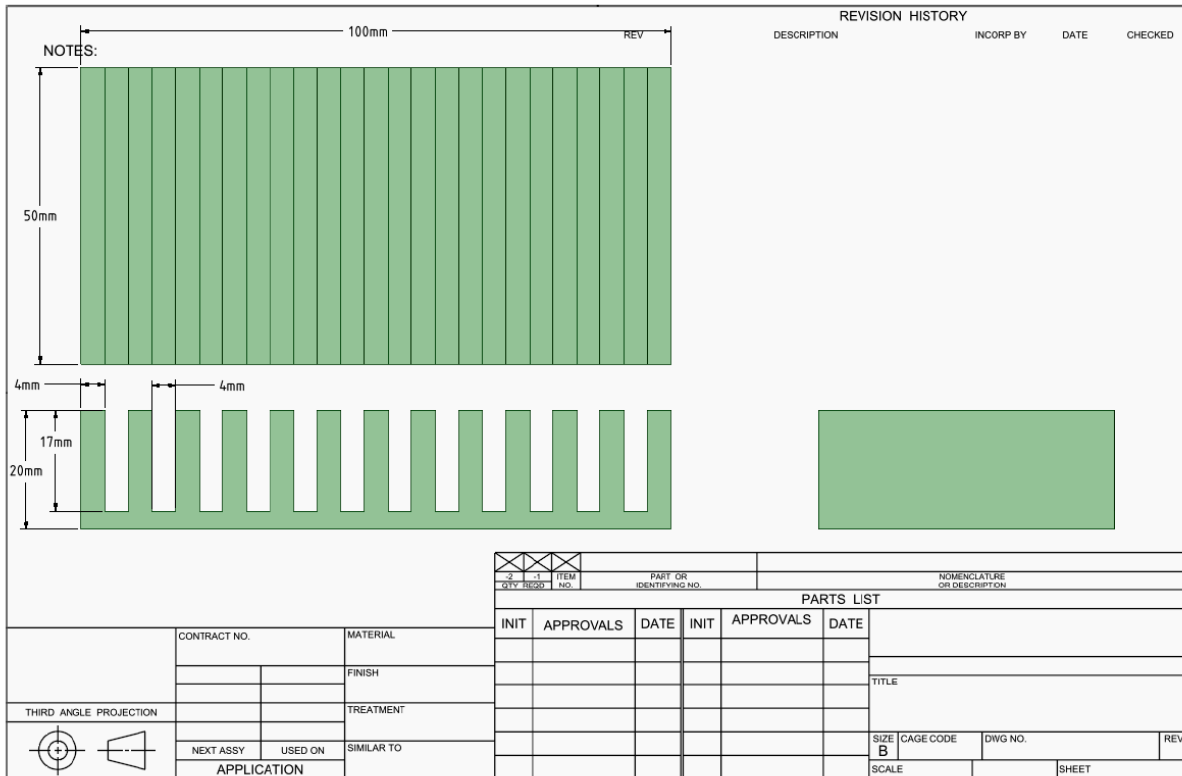
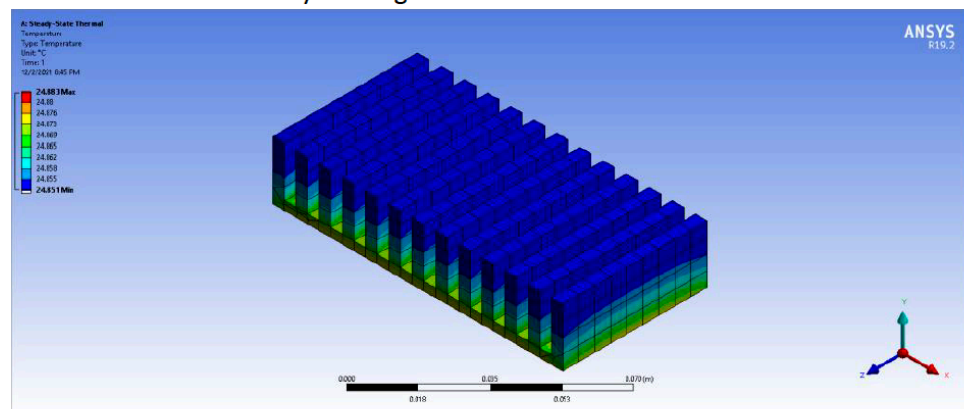


Figure 2.5 – Newly Produced Heat Sink Dimensions

2.3.3 Heat Sink Temperature Distribution

The temperature distribution is shown below in the figure. The temperature between the benchmark and newly profounded dimensions, the temperature has slightly increased. The reason being is because of the new calculated convection coefficient shown above in figure 2.1. The number of fins have decreased, the surface area changed, and the ambient and surface temp stayed the same. If the ambient air were to change, the temperature distribution within the heat sink and fins would've decreased by 2-3 degrees.



2.4 Copper Heat Sink

2.4.1 Copper Heat Sink Equation

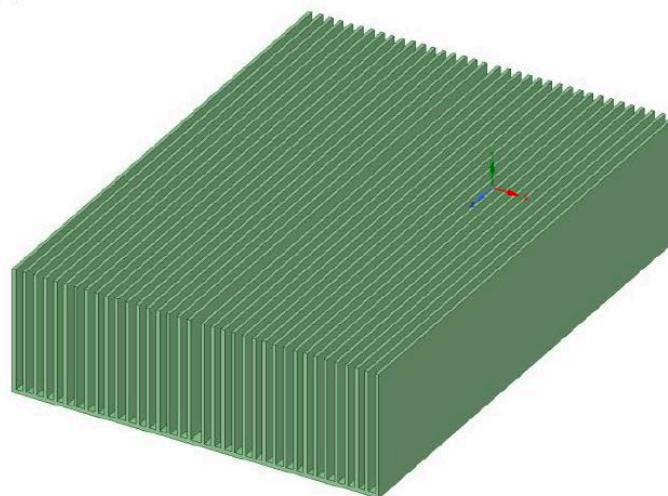
In comparison to the benchmark heat sink, the differences between the two are the number of fins and the material that was used to manufacture the product. Copper is known to have a higher thermal conductivity, copper is the best way to go when it comes to performance and efficiency. When it comes to cost, aluminum is the better option. Using a different method to find the heat transfer convection coefficient, using the formula:

- $Q = hA(T_s - T_\infty)$
Q = Power (W)
h = Heat transfer coefficient (W/m²-K)
Ts = Surface Temp of Fin
Ta = Ambient Air

	Name	Value
1	L = 150;	
2	W = 107;	
3	Bh = 1.2;	
4		
5	As = (2*L)+(2*W)+(2*Bh)	As = 516.4000
6		
7	Ts = 110;	
8	Ta = 25;	
9		
10	Q = 65;	
11		
12	h = (Q/As)*(Ts-Ta))	h = 10.6991

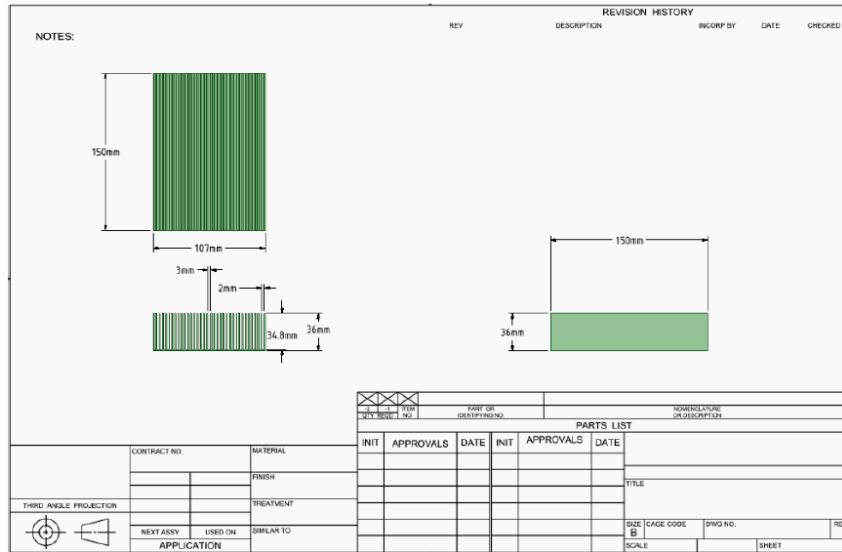
Name	Value
As	516.4000
Bh	1.2000
h	10.6991
L	150
Q	65
Ta	25
Ts	110
W	107

2.4.2 3D Heat Sink



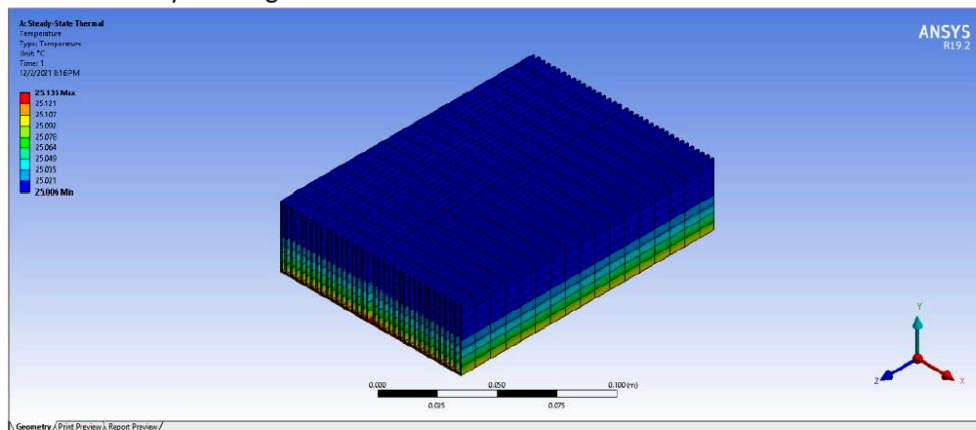
ANSYS
R19.2

2.4.3 Drawing Sheet of Heat Sink



2.4.4 Temperature Distribution in Heat Sink

From the figure below, it shows how there was a temperature increase from the benchmark by two degrees.



3 Aluminum and Copper

3.1 Aluminum Thermal Conductivity

An Aluminum heatsink is the most widely used product for thermal solutions. Aluminium is the second most widely used metal in the world after Iron. After oxygen and silicon, aluminum is the most common element in the Earth's crust. Aluminum is low in weight, low in density at $\rho = 2700 \text{ m}^3/\text{kg}$, easily machined and high in strength ranging from 70 to 700 MPa. Aluminum reacts with oxygen in air to form an extremely thin layer of Aluminum Oxide. Anodizing increases the thickness of the oxide layer and improves the strength of the natural corrosion protection. Compared with other metals, aluminum has a relatively large coefficient of linear expansion. Aluminum's malleability is essential for the extrusion process, and in bending and other forming products. An Aluminum heatsink is an excellent conductor of heat. At bulk sale, 500 pieces, or 500 heat sinks will cost roughly about \$24-25K.

3.2 Copper Heat Sink

A copper heatsink is sometimes used as a heatsink as it is a good conductor of heat. This can be observed if you heat one end of a piece of copper, the other end will quickly reach the same temperature. Copper is used in many heating applications because it doesn't corrode and has a high melting point. The properties of copper is that it has a good thermal and electrical conductivity and a very high tensile strength of 310 MPa, easy to recycle. A Copper heatsink from Radian Thermal Products can be either a standard product, or custom design. For custom copper heatsinks we can offer a design based on thermal requirements or we can manufacture to your design (build to print). We work with a wide range of customers that may need only a few Copper heatsinks, or are a production company that need many thousands Copper heatsinks a month. Copper being such a rare material, its production cost won't be as bad as aluminum but the price for copper is higher than aluminum.

3.3 Aluminum V Copper

Copper is an excellent conductor with a very high thermal and electrical conductivity. Despite the higher efficiency that copper delivers, aluminum heat sinks are typically used due to the fact that aluminum weighs half as much as a copper conductor, with the same conductivity and also less expensive. Many customers add on an anti-oxidation coating, or nickel plate to maintain the look of the copper heat sinks to keep the color and texture over time. Production of the extrusion heat sinks are done in a vapor chamber. Using stamped copper fins that are soldered to the base of the heat sink.

4 Conclusion

4.1 Which Heat Sink?

The copper heat sink seems to be the best decision, even though the cost of copper is much more expensive than aluminum, copper heat sink are more efficient.

Work Cited

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[https://en.wikipedia.org/wiki/Heat_transfer_coefficient#:~:text=The%20overall%20heat%20transfer%20coefficient,-is%20a%20measure&text=R%20%3D%20Resistance\(s\)%20to,transfer%20of%20heat%20takes%20place.](https://en.wikipedia.org/wiki/Heat_transfer_coefficient#:~:text=The%20overall%20heat%20transfer%20coefficient,-is%20a%20measure&text=R%20%3D%20Resistance(s)%20to,transfer%20of%20heat%20takes%20place.)

<https://www.smlease.com/entries/thermal-design/how-to-select-a-heat-sink/>

Comments: use of inconsistent methods to calculate convection coefficient h ; one in Section 2.2 and the other in Section 2.4. The actual parameters including h value, heat flux, ambient temperature, etc. in the analysis are unclear. The conclusion is very brief and does not say much of the modified design.