

(ABET Binder)
PRAIRIE VIEW A&M UNIVERSITY
COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
MCEG 4306-P02 DYNAMIC SYSTEMS & CONTROLS
Spring Semester 2022

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Course Description in ABET Format

MCEG 4306 Dynamic Systems and Controls

1.	Course number and name:	MCEG 4306 Dynamic Systems and Controls
2.	Credits and contact hours:	3 credit hours; 3 contact hours
3.	Instructor's or course coordinator's name:	Dr. Shield Lin
4.	Text book: Title:	System Dynamics for Engineering Students
	Author:	Nicolae Lobontiu
	Publisher, Year & Edition:	Academic Press, an Imprint of Elsevier, San Diego, CA, 2018, 2 nd Edition
	a. Other Supplemental material	MATLAB/Simulink, Release R2020a, MathWorks Inc., Natick, Massachusetts.
5.	Specific course information	
	a. Brief description of the content of the course (catalog description)	The scope of this course includes mathematical modeling, analysis, and feedback control of dynamic systems. Topics include free and forced vibrations of single and multiple degrees of freedom systems. Transient, steady-state, and stability of linear feedback control systems will be studied in the course.
	b. Prerequisites	MCEG 2053 – Engineering Mechanics II. MATH 2043 – Differential Equations.
	c. *Indicate whether a required, elective, or selected elective (as per definition provided) course in the program	Required.
6.	Specific goals for the course	Students will be able to:
	a. specific outcomes of instruction (e.g. The student will be able to explain the significance of current research about a particular topic.)	Upon completion of this course, the students will be able to solve dynamic system problems and have an overview of feedback control systems. The students will learn to: • derive equations from simple components, proceed to the objects' assembly, and arrive at the system interaction level; • use modeling, procedures, and techniques to solve dynamic system equations; and • use computer tools to solve dynamic and feedback control systems.
	b. Explicitly indicate which of the student outcomes listed in Criterion 3 or any other outcomes are addressed by the course.	This course addresses: Outcome 1 Ability to identify, formulate, and solve complex engineering problems (CEP) by applying principles of engineering, science, and mathematics.
7	Brief list of topics to be covered	1. Mechanical elements and systems (12 hours) 2. Modeling and solution techniques of dynamics systems (12 hours) 3. Electromechanical coupling systems (9 hours) 4. Feedback control systems (12 hours)

Detailed Course Syllabus
(Distributed to Students at the beginning of the Semester)
ABET Outcomes measured using course should be
explicitly listed in the syllabus



**MCEG 4306 Dynamic Systems and Controls
Spring Semester 2022**

Instructor: Dr. Shield Lin
Section # and CRN: Section P02, CRN # 20244
Office Location: S. R. Collins Building, Room 302A
Office Phone: (936) 261-9958
Email Address: shlin@pvamu.edu
Office Hours: 12:30 – 3:30 p.m. Tuesday and Thursday
Mode of Instruction: Face to Face

Course Location: S.R. Collins Building, Room 114
Class Days & Times: 9:30 – 10:50 a.m. Monday and Wednesday
Catalog Description: The scope of this course includes mathematical modeling, analysis, and feedback control of dynamic systems. Topics include free and forced vibrations of single and multiple degrees of freedom systems. Transient, steady-state, and stability of linear feedback control systems will be studied in the course.

Prerequisites: MATH 2320 – Differential Equations and MCEG 2302 – Engineering Mechanics II
Co-requisites: N/A

Required Text(s): “System Dynamics for Engineering Students, Concepts and Applications” by Nicolae Lobontiu, 2nd Edition, Academic Press, Elsevier, 2017, ISBN 978-0-12-804559-6.

Recommended Text(s): “Feedback Control of Dynamic Systems,” by G. F. Franklin, J.D. Powell, and A. Emami-Naeini, 6th Edition, Pearson, New Jersey, 2009.
“Theory of Vibration with Applications,” by William Thomson, 5th Edition, Prentice Hall, 1998.

Course Learning Outcomes:

	Upon successful completion of this course, students will be able to:	Student Learning Outcome # Alignment	Core Curriculum Objective Alignment
1	derive dynamic equations from components, proceed to the objects' assembly, and arrive at the system interaction level;	#1	Develop critical thinking skills.
2	use modeling, procedures, and techniques with a computer tool to solve dynamic system problems;	#1	Develop critical thinking skills.
3	conduct engineering design utilizing a modern computer tool for dynamic systems and write a project report.	#2	Develop communication skills.

The Mechanical Engineering Program Objectives are to produce graduates who have:

1. the techniques and skills necessary for modern mechanical engineering practices, and an ability to function effectively in multi-disciplinary teams.
2. an ability in life-long learning skills, a knowledge of contemporary issues, and an ability to communicate effectively.
3. an understanding of global and societal context in the aspects of professional and ethical responsibility and the impact of engineering solutions on society.
4. the qualifications to be employed by major industries and government agencies in the State of Texas and the nation.
5. the qualifications to pursue graduate degrees.

Fall 2018-Spring 2022 Student Learning Outcomes Assessment Matrix for MCEG Courses ENGINEERING ACCREDITATION COMMISSION OF ABET NEW STUDENT LEARNING OUTCOMES (Please read the important information below the table)										
MCEG Courses	ABET EAC Student Outcomes 1-7								Totals	Yearly Average
	Fall 2018	Spring 19	Fall 2019	Spring 2020	Fall 2020	Spring 2021	Fall 2021	Spring 2022		
1021 Mech. Drawing & Desg.	2	2	2	2	2SACS		2SACS			
2013 Thermodynamics I	8SACS	8SACS	8SACS	8SACS	8SACS		8SACS			
2023 Materials Sci/Engr.	8SACS	8SACS	8SACS	8SACS	8SACS	8SACS		8SACS		
2053 Engineering Mechanics II	8SACS	8SACS	8SACS	8SACS	8SACS		8SACS			
3011 Measurement Lab	7	6	7	6	7	6	7	6		
3013 Heat Transfer	1		1		2	1	2	1		
3021 Thermal Sci. Lab	6	6	6	6		6		6		
3023 Thermodynamics II	1	1	1	1		2		2		
3031 Manufacturing Lab		2		4		2		4		
3033 Manufacturing Proc		2		4		2		4		
3043 Machine Design I	2		2		2		2			
3053 Kinematic Design	1		1		1		1			
3063 Fluid Mechanics	1	6	1	6	1	6	1	6		
4043 Machine Design II	2		2		2		2			
4063 Dynamic Systems		1		1		1		1		
4093 Finite Element	1		1		1		1			
4472 Senior Project I	4, 5		4, 5		4, 5		4, 5			
4482 Senior Project II		2, 3, 7		2, 3, 7		2, 3, 7		2, 3, 7		
Not Assessed										
Frequency										
Outcome Description & Breakdown into Performance criteria										
Outcome									Totals	Yearly Average
1 1.1: Identify & Formulate CEP (i-vi) based on E, S, M										
1 1.2: Solve CEP (i-vi) based on E, S, M	5	2	5	2	3	2	3	2	24	6.0
2 2.1: Ability to Define/Understand the Problem and then Plan										
2 2.2: Ability to Conduct a Review of the Literature, Generate	2	3	2	1	3	4	3	2	20	5.0
2 2.3: Ability to Perform Preliminary and Detailed Design										
3 (Oral) 3.1: Ability to Organize, Plan, Design, Prepare and Use										
3 (Oral) 3.2: Ability to Articulate Subject Knowledge (Technical										
3 (Oral) 3.3: Appearance and Ability to Provide Good Oral Delivery										
3 (Written) 3.4: Ability to prepare an executive summary	0	1	0	1	0	1	0	1	4	1.0
3 (Written) 3.5: Ability to organize, plan and properly format a written										
3 (Written) 3.6: Ability to compose original texts and properly apply the										
3 (Written) 3.7: Ability to provide appropriate discussion/conclusions										
4 4.1: Recognize ethical and professional responsibilities in engine	1		1	2	1		1	2	8	2.0
4 4.2: Make informed judgments on ethical and professional res		0		0		0		0	0	
5 5.1: Function effectively on a team whose members together pr	1	0	1	0	1	0	1	0	4	1.0
5 5.2: Establish goals, plan tasks, and meet objectives										
6 6.1: Ability to develop appropriate experiments									0	
6 6.2: Ability to conduct experiments	0	3	0	3	0	3	0	3	12	3.0
6 6.3: Ability to analyze and Interpret experimental data										
6 6.4: Draw appropriate conclusions and recommendations										
7 7.1: Students are able to acquire new knowledge	0	1	0	1	0	1	0	1	4	1.0
7 7.2: Apply new knowledge to solve problems									0	
8 8.1: Identify & Formulate EP (i-vi) based on E, S, M	3	3	3	3	3	3	3	3	24	6
8 8.2: Solve EP based on E, S, M									0	

IMPORTANT INFORMATION

8 Assessment is based on the the new outcome 1 but for regular problems, NOT CEP type problems. It is for SACS reporting only and will not be reported to ABET. However, the 1 page assessment summary and grade sheet will be needed for SACS assessment report.

MCEG 1021 assessment is based on NX design project. Since ability to design using NX is important, it will be reported to ABET and SACS, and thus an assessment report will be needed.

Major Course Requirements

Method of Determining Final Course Grade

Course Grade Requirement	Value	Total
1. attendance, discussion, homework, quiz, etc.	Various points	20%
2. Test 1	100	15%
3. Test 2	100	15%
4. Test 3	100	15%
5. Test 4	100	15%
6. Design Project	100	20%
Total:		100%

Grading Criteria and Conversion: 100

A = 90-100

B = 80-89

C = 70-79

D = 60-69

F = 0-59

If a student has stopped attending the course (i.e. "stopped out") at any point after the first day of class but did not officially withdraw from the course and has missed assignments and exams, including the final exam, and performed below the grade level of a D, a grade of FN (failed-non attendance) will be assigned for the final course grade to ensure compliance with the federal Title IV financial aid regulations. In contrast, if the student has completed all assignments and exams, including the final exam, but performed below the grade level of a D, a grade of F will be assigned for the final course grade.

Detailed Description of Major Assignments:

Assignment Title or Grade Requirement	Description
1. Test 1	Derive equations of mechanical components and degree of freedom.
2. Test 2	Derive equations of dynamic systems and obtain solutions.
3. Test 3	Modeling and simulation of dynamic systems in MATLAB/Simulink.
4. Test 4	Fundamentals and applications of feedback control systems.
5. Design Project	Model/simulate a dynamic system, design for specified goals, and write a report.

Course Procedures or Additional Instructor Policies

Taskstream

Taskstream is a tool that Prairie View A&M University uses for assessment purposes. One of your assignments may be required to be submitted as an "artifact," an item of coursework that serves as evidence that course objectives are met. If applicable, more information will be provided during the semester by your department, but for general information, you can visit Taskstream via the link in eCourses.

Assignments & Due Dates

All assignments must be submitted to Canvas before the due day and time, **30% of grade deduction** for the late submission between "Due" time and "Available Until" time. After "Available Until" time, the grade for the assignment **will be 0**.

Email Messages

For non-urgent issues, the instructor encourages the students to use emails to communicate with the instructor. The instructor will generally reply email messages in a few hours, at least within 24 hours.

Office Hours

The students, as a group or an individual, are welcome to schedule zoom meetings with the instructor during the office hours. Scheduling can be done by an email request.

Grading Feedback

Other than the final design project, the instructor will grade assignments, discussions, and tests within one week. The results of grading and comments will be posted on Canvas.

Professionalism of On-line Discussions

When responding to discussion assignments, all responders shall be professional during the entire

communication. We shall be considerate to others and use encouraging words even you do not agree with other's opinions. We also must adhere to the privacy of personal data.

Semester Calendar for Spring Semester 2022

Week	Description
Week One: 01/19	Introduction of Dynamic Systems and Controls and Software Access
Week Two: 01/24 01/26	Mechanical Elements - Spring Mechanical Elements - Inertia
Week Three: 01/31 02/02	Mechanical Elements - Damper Mechanical Systems – Degrees of Freedom
Week Four: 02/07 02/09	Equivalence of Mechanical Elements in Dynamic Systems Test 1
Week Five: 02/14 02/16	Single Degree of Freedom System – Free Response Single Degree of Freedom System – Forced Response
Week Six: 02/21 02/23	Multiple Degree of Freedom System – Free Response Multiple Degree of Freedom System – Forced Response
Week Seven: 02/28 03/02	Summary of Equations in Dynamic Systems Fundamentals of Simulink
Week Eight: 03/07 03/09	Applications of Simulink in Dynamic Systems Test 2
Week Nine: 03/14-19	Spring Break Week
Week Ten: 03/21 03/23	Laplace Transform in Dynamic System Founders Day/Honors Convocation (class cancelled)
Week Eleven: 03/28 03/30	Transfer Functions and Time Response Solving Dynamic Systems in Frequency-Domain
Week Twelve: 04/04 04/06	Frequency-Domain Application – Mechanical Vibration Test 3
Week Thirteen: 04/11 04/13	Concepts of Feedback Control of Dynamic Systems Block Diagrams in Control Systems
Week Fourteen: 04/18 04/20	Transfer Functions in Control Systems Active Control vs. Passive Control Systems
Week Fifteen: 04/25 04/27	Test 4 Project Presentation and Discussions
Week Sixteen: 05/02 05/04	Project Presentation and Discussions Guest Speaker/Industrial Applications

Student Support and Success

John B. Coleman Library

The John B. Coleman Library's mission is to enhance the scholarly pursuit of knowledge, to foster intellectual curiosity, and to promote life-long learning and research through our innovative services, resources, and cultural programs, which support the Prairie View A&M University's global mission of teaching, service, and research. It maintains library collections and access both on campus, online, and through local agreements to further the educational goals of students and faculty. Website: <https://www.pvamu.edu/library/>; Phone: 936-261-1500

Academic Advising Services

Academic Advising Services offers students a variety of services that contributes to student success and leads towards graduation. We assist students with understanding university policies and procedures that affect academic progress. We support the early alert program to help students get connected to success early in the semester. We help refer students to the appropriate academic support services when they are unsure of the best resource for their needs. Faculty advisors support some students in their respective colleges. Your faculty advisor can be identified in PantherTracks. Advisors with Academic Advising Services are available to all students. We are located across campus. Find your advisor's location by academic major at www.pvamu.edu/advising. Phone: 936-261-5911

The University Tutoring Center

The University Tutoring Center (UTC) offers free tutoring and academic support to all registered PVAMU students. The mission of the UTC is to help provide a solid academic foundation that enables students to become confident, capable, independent learners. Competent and caring staff and peer tutors guide students in identifying, acquiring, and enhancing the knowledge, skills, and attitudes needed to reach their desired goals. Tutoring and academic support are offered face-to-face in the UTC, in virtual face-to-face sessions (<https://www.pvamu.edu/student-success/sass/university-tutoring-center/>), and through online sessions (<https://www.pvamu.edu/pvplace/>). Other support services available for students include Supplemental Instruction, Study Break, Academic Success Workshops, and Algebra Study Jam. Location: J. B. Coleman Library, Rm. 307; Phone: 936-261-1561; Email: pvtutoring@pvamu.edu; Website: <https://www.pvamu.edu/student-success/sass/university-tutoring-center/>

Writing Center

The Writing Center provides well-trained peer tutors to assist students with writing assignments at any stage of the writing process. Tutors help students with various writing tasks from understanding assignments, brainstorming, drafting, revising, editing, researching, and integrating sources. Students have free access to Grammarly online writing assistance. Grammarly is an automated proofreading and plagiarism detection tool. Students must register for Grammarly by using their student email address. In addition, students have access to face-to-face and virtual tutoring services either asynchronously via email or synchronously via Zoom. Location: J. B. Coleman Library, Rm. 209; Phone: 936-261-3724; Website: <https://www.pvamu.edu/student-success/writing-center/>; Grammarly Registration: <https://www.grammarly.com/enterprise/signup>

Academic Early Alert

Academic Early Alert is a proactive system of communication and collaboration between faculty, academic advisors, and PVAMU students that is designed to support student success by promptly identifying issues and allowing for intervention. Academic Early Alerts help students by providing a central location to schedule advising appointments, view advisor contact information, and request assistance. Students who recognize that they have a problem that is negatively affecting their academic performance or ability to continue school may self-refer an Academic Early Alert. To do so, students will log in to PV Place and click on Academic Early Alert on the left sidebar. Phone: 936-261-5902; Website: <https://www.pvamu.edu/student-success/early-alert/>

Student Counseling Services

The Student Counseling Services unit offers a range of services and programs to assist students in maximizing their potential for success: short-term individual, couples, and group counseling, as well as crisis intervention, outreach, consultation, and referral services. The staff is licensed by the State of Texas and assists students who are dealing with academic skills concerns, situational crises, adjustment problems, and emotional difficulties. Information shared with the staff is treated confidentially and in accordance with Texas State Law. Location: Hobart Taylor, 2nd floor; Phone: 936-261-3564; Website:

Office of Testing Services

Testing Services serves to create opportunities by offering a suite of exams that aid in the students' academic and professional success. Currently, we administer entrance (HESI A2), college readiness (TSI assessment), Prior Learning (CLEP, DSST), and proctored exams. Location: Wilhelmina Delco, 3rd Floor, Rm. 305; Phone: 936-261-3627; Email: aetesting@pvamu.edu; Website: www.pvamu.edu/testing

Office of Diagnostic Testing and Disability Services

The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, contact the Office of Disability Services. As a federally-mandated educational support unit, the Office of Disability Services serves as the repository for confidential disability files for faculty, staff, and students. For persons with a disability, the Office develops individualized ADA letters of request for accommodations. Other services include learning style inventories, awareness workshops, accessibility pathways, webinars, computer laboratory with adapted hard and software, adapted furniture, proctoring non-standardized test administrations, ASL interpreters, ALDs, digital recorders, Livescribe, and a comprehensive referral network across campus and the broader community. Location: Hobart Taylor, Rm. 1D128; Phone: 936-261-3583; Website: <https://www.pvamu.edu/disabilityservices/>

Center for Instructional Innovation and Technology Services (CIITS)

Distance Learning, also referred to as Distance Education, is the employment of alternative instructional delivery methods to extend programs and services to persons unable to attend college in the traditional manner. The Center for Instructional Innovation and Technology Services (CIITS) supports student learning through online, hybrid, web-assist, and 2-way video course delivery. For more details and contact information, visit:

<https://www.pvamu.edu/dlearning/distance-learning-2-2/students-2/>; Phone: 936-261-3283

Veteran Affairs

Veterans Services works with student veterans, current military and military dependents to support their transition to the college environment and continued persistence to graduation. The Office coordinates and certifies benefits for both the G.I. Bill and the Texas Hazlewood Act. Location: Evans Hall, Rm. 102; Phone: 936-261-3563; Website: <https://www.pvamu.edu/sa/departments/veteranaffairs/>

Office for Student Engagement

The Office for Student Engagement delivers comprehensive programs and services designed to meet the co-curricular needs of students. The Office implements inclusive and accessible programs and services that enhance student development through exposure to and participation in diverse and relevant social, cultural, intellectual, recreational, community service, leadership development, and campus governance. Location: Memorial Student Center, Rm. 221; Phone: 936-261-1340; Website: <https://www.pvamu.edu/studentengagement/>

Career Services

Career Services supports students through professional development, career readiness, and placement and employment assistance. The Office provides one-on-one career coaching, interview preparation, resume and letter writing, and career exploration workshops and seminars. Services are provided for students at the Northwest Houston Center and College of Nursing in the Medical Center twice a month or on a requested basis. Distance Learning students are encouraged to visit the Career Services website for information regarding services provided. Location: Anderson Hall, 2nd floor; Phone: 936-261-3570; Website: <https://www.pvamu.edu/careerservices/>

University Rules and Procedures

Academic Misconduct

Academic dishonesty is defined as any form of cheating or dishonesty that has the effect or intent of interfering with any academic exercise or fair evaluation of a student's performance. The college faculty can provide additional information, particularly related to a specific course, laboratory, or assignment.

You are expected to practice academic honesty in every aspect of this course and all other courses. Make sure you are familiar with the *University Administrative Guidelines on Academic Integrity*, which can be found on the [Academic Integrity webpage](#). Students who engage in academic misconduct are subject to university disciplinary procedures. As listed in the *University Administrative Guidelines on Academic Integrity*, the University Online Catalog, and the Student Code of Conduct, the following are examples of prohibited conduct. This list is not designed to be all-inclusive or exhaustive. In addition to academic sanctions, any student found to have committed academic misconduct that is also a violation of criminal law may also be subject to disciplinary review and action by the Office of Student Conduct (as outlined in the Student Code of Conduct).

Forms of Academic Dishonesty:

1. **Cheating:** Deception in which a student misrepresents that he/she has mastered information on an academic exercise that he/she has not learned, giving or receiving aid unauthorized by the instructor on assignments or examinations. Examples: unauthorized use of notes for a test; using a "cheat sheet" on a quiz or exam; any alteration made on a graded test or exam which is then resubmitted to the teacher;
2. **Plagiarism:** Careless or deliberate use of the work or the ideas of another; representation of another's work, words, ideas, or data as your own without permission or appropriate acknowledgment. Examples: copying another's paper or answers, failure to identify information or essays from the internet and submitting or representing it as your own; submitting an assignment which has been partially or wholly done by another and claiming it as yours; not properly acknowledging a source which has been summarized or paraphrased in your work; failure to acknowledge the use of another's words with quotation marks;
3. **Collusion:** When more than one student or person contributes to a piece of work that is submitted as the work of an individual;
4. **Conspiracy:** Agreeing with one or more persons to commit an act of academic/scholastic dishonesty; and
5. **Multiple Submission:** Submission of work from one course to satisfy a requirement in another course without explicit permission. Example: using a paper prepared and graded for credit in one course to fulfill a requirement and receive credit in a different course.

Nonacademic Misconduct

The university respects the rights of instructors to teach and students to learn. Maintenance of these rights requires campus conditions that do not impede their exercise. Campus behavior that interferes with either (1) the instructor's ability to conduct the class, (2) the inability of other students to profit from the instructional program, or (3) campus behavior that interferes with the rights of others will not be tolerated. An individual engaging in such disruptive behavior may be subject to disciplinary action. The Office of Student Conduct will adjudicate such incidents under nonacademic procedures.

Sexual Misconduct

Sexual harassment of students and employees at Prairie View A&M University is unacceptable and will not be tolerated. Any member of the university community violating the university's sexual harassment policy will be subject to disciplinary action. In accordance with the Texas A&M University System guidelines, your instructor is obligated to report to the Office of Title IX Compliance (titleixteam@pvamu.edu) any instance of sexual misconduct involving a student, which includes sexual assault, stalking, dating violence, domestic violence, and sexual harassment, about which the instructor becomes aware during this course through writing, discussion, or personal disclosure. The faculty and staff of PVAMU actively strive to provide a learning, working, and living environment that promotes respect that is free from sexual misconduct, discrimination, and all forms of violence. If students, faculty, or staff would like assistance or have questions,

they may contact the Title IX Coordinator at 936-261-2144 or titleixteam@pvamu.edu. More information can be found at www.pvamu.edu/titleix, including confidential resources available on campus.

Pregnancy, Pregnancy-related, and Parenting Accommodations

Title IX of the Education Amendments of 1972 prohibits sex discrimination, which includes discrimination based on pregnancy, marital status, or parental status. Students seeking accommodations related to pregnancy, pregnancy-related conditions, or parenting (reasonably immediate postpartum period) are encouraged to contact Student Disability Services or the Dean of Students' Office for additional information and to request accommodations.

Non-Discrimination Statement

Prairie View A&M University does not discriminate on the basis of race, color, sex, religion, national origin, age, disability, genetic information, veteran status, sexual orientation, or gender identity in its programs and activities. The University is committed to supporting students and complying with The Texas A&M University System non-discrimination policy. It seeks to establish an environment that is free of bias, discrimination, and harassment. If you experience an incident of discrimination or harassment, we encourage you to report it. If you would like to speak with someone who may be able to afford you privacy or confidentiality, there are individuals who can meet with you. The Director of Equal Opportunity & Diversity has been designated to handle inquiries regarding the non-discrimination policies and can be reached at Harrington Science Building, Suite 109 or by phone at 936-261-1744 or 1792.

Class Attendance Policy (See the University Online Catalog for Full Attendance Policy)

Prairie View A&M University requires regular class attendance. Attending all classes supports the full academic development of each learner, whether classes are taught with the instructor physically present or via distance learning technologies such as interactive video and/or the internet. Excessive absenteeism, whether excused or unexcused, may result in a student's course grade being reduced or in the assignment of a grade of "F." Absences are accumulated beginning with the first day of class during regular semesters and summer terms. Each faculty member will include the University's attendance policy in each course syllabus.

Student Academic Appeals Process

Authority and responsibility for assigning grades to students rest with the faculty. However, in those instances where students believe that miscommunication, errors, or unfairness of any kind may have adversely affected the instructor's assessment of their academic performance, the student has a right to appeal by the procedure listed in the University Online Catalog and by doing so within thirty days of receiving the grade or experiencing any other problematic academic event that prompted the complaint.

Technical Considerations

Minimum Recommended Hardware and Software:

- Intel PC or Laptop with Windows 10 or later version; Mac with OS High Sierra*
- Smartphone or iPad/Tablet with Wi-Fi*
- High-speed Internet access
- 8 GB Memory
- Hard drive with 320 GB storage space
- 15" monitor, 800x600, color or 16 bit
- Sound card w/speakers
- Microphone and recording software
- Keyboard & mouse
- Most current version of Google Chrome, Safari, or Firefox

Note: Be sure to enable Java & pop-ups in the Web browser preferences

* Smartphones, Google Chrome books, and Android tablets may not be supported. iPads are the only tablets supported.

Participants should have a basic proficiency of the following computer skills:

- Sending and receiving email
- A working knowledge of the Internet

- Microsoft Word (or a program convertible to Word)
- Acrobat PDF Reader
- Windows or Mac OS
- Video conferencing software

Netiquette (online etiquette)

Students are expected to participate in all discussions and virtual classroom chats as directed. Students are to be respectful and courteous to others on discussion boards. Foul or abusive language will not be tolerated. Do not use ALL CAPS for communicating to others AS IT CAN BE INTERPRETED AS YELLING. Avoid slang terms such as "wassup?" and texting abbreviations such as "u" instead of "you." Limit and possibly avoid the use of emoticons. Be cautious when using humor or sarcasm as tone is sometimes lost in an email or discussion post, and the message might be taken seriously or sound offensive.

Video Conferencing Etiquette

When using Zoom, WebEx, or other video conferencing tools, confirm the visible area is tidy, clear of background clutter, inappropriate or offensive posters, and other distractions. Ensure you dress appropriately and avoid using high traffic or noisy areas. Stay muted when you are not speaking and avoid eating/drinking during the session. Before the class session begins, test audio, video, and lighting to alleviate technology issues.

Technical Support

Students should go to <https://mypassword.pvamu.edu/> if they have password issues. The page will provide instructions for resetting passwords and contact information if login issues persist. For other technical questions regarding eCourses, call the Center for Instructional Innovation and Technology Services at 936-261-3283 or email ciits@pvamu.edu.

Communication Expectations and Standards

Emails or discussion postings will receive a response from the instructor, usually in less than 48 hours. Urgent emails should be marked as such. Check regularly for responses.

Discussion Requirement

Online courses often require minimal to no face-to-face meetings. However, conversations about the readings, lectures, materials, and other aspects of the course can occur in a seminar fashion. The use of the discussion board will accomplish this. The instructor will determine the exact use of discussion boards.

It is strongly suggested that students type their discussion postings in a word processing application such as Word and save it to their PC or a removable drive before posting to the discussion board. This is important for two reasons: 1) If for some reason your discussion responses are lost in your online course, you will have another copy; 2) Grammatical errors can be greatly minimized by the use of the spell-and-grammar check functions in word processing applications. Once the post(s) have been typed and corrected in the word processing application, copy and paste to the discussion board.

COVID-19 Campus Safety Measures

To promote public safety and protect students, faculty, and staff during the coronavirus pandemic, PVAMU has adopted policies and practices to limit virus transmission.

- **Self-monitoring** - Students should follow CDC recommendations for self-monitoring. Students who have a fever or exhibit symptoms of COVID-19 should participate in class remotely and should not participate in face-to-face instruction.
- **Face Coverings** - Face coverings (cloth face covering, surgical mask, etc.) are recommended in classrooms, teaching laboratories, common spaces such as lobbies and hallways, public study spaces, libraries, academic resource, and support offices, and outdoor spaces where 6 feet of physical distancing is challenging to maintain reliably.

- **Physical Distancing** - Physical distancing should be maintained between students, instructors, and others in course and course-related activities where possible.
- **Personal Illness and Quarantine** - Students required to quarantine are to participate in courses and course-related activities remotely and must not attend face-to-face course activities. Students should notify their instructors of the quarantine requirement. Students under quarantine are expected to participate in courses and complete graded work unless they have symptoms that are too severe to participate in course activities. Students experiencing personal injury or illness that is too severe for the student to attend class qualify for an excused absence. To receive an excused absence, students must provide appropriate documentation to the Office for Student Conduct, studentconduct@pvamu.edu.

End of Semester Course Assessment Report

This summary contains the following Information

1. Performance Statistics

- Total number of students in class
- The class average performance (stated as %) in each course outcome assessed in this class and the two previous times course was assessed.
- The acceptable class average for the class in each outcome area as set by your Department.
- Percentage of students who scored below the expected average in each outcome.

2. Implementation Summary

- Brief summary of plans implemented during the semester based on last assessment report

3. Perceived Problems

- Instructor's critical evaluation of perceived problems that affected students' performance. Realistic problems should be identified throughout the semester and summarized in this section of the report.

4. New Plans for Addressing Problems

- Instructor's plans for addressing the perceived problems the next time the class is taught.

5. Overall Trend over last three assessment periods

- Instructors summary of overall performance trend for each outcome measured over the last three periods.

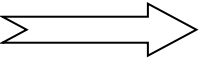
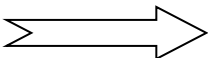
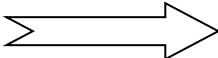
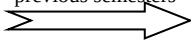
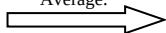
5. Were Expectations Met?

- Answer YES if the class average was above the expected average and the stated percentage of students scored at or above the expected average, else answer NO.

END OF SEMESTER COURSE OUTCOME ASSESSMENT REPORT
MCEG 4306-P02 Dynamic Systems and Controls
 SPRING 2022 SEMESTER

Report Prepared by: Dr. Shield Lin

Report Date: May 9, 2022

Semester	Analysis Type	Number of Students	Outcome 1 (the course was evaluated under Outcome a in 2018 and before)			
			Expected Average	Expected Percent of students at or above average	Class Average	% Students meeting Expected Average
Spring 2022	Direct	35	70	70	74.8	71%
Spring 2021	Direct	35	70	70	73.5	69%
Spring 2020	Direct	21	70	70	76.9	71%
Implementation Summary	Items implemented from previous report or after meeting previous instructor 	This class covers two main areas: mechanical vibrations and automatic controls; a great deal of materials for the students to learn in one semester. As stated in the last ABET assessment report, the instructor balanced the course materials between equation derivation and computer simulation. Namely, required the students to derive dynamic equations from given physical systems, and to solve the equations with a computer tool, MATLAB/Simulink.				
Perceived Problems	Perceived problems are directly related to the PC and the sub items under them 	On average, the students did well for the balanced approach. Most of students did well with the computer tool and obtained output from the simulation; however, some of them could not interpret the output results well.				
Plans for Addressing Problems	Plans for address specific measures to address PC and their sub areas 	The instructor plans to ask the students to write a summary statement to explain the result for every output they obtain from running the MATLAB/Simulink.				
Overall Trend over Periods	Ascertain if there were improvements over previous semesters 	The overall trend of this class assessment is about 70% students meeting expected average.				
Were Expectations Met?	Class Average $\geq 70\%$ and $\geq 70\%$ of Students Performed at or Above Expected Average. 	Yes				

Performance Criteria for Outcome 1 - An ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics:

1. Students are able to use techniques and skills acquired in their studies necessary for engineering practice.

(i) Students are able to utilize problem solving methods in their discipline, such as formulating differential equations of motion from mechanical systems by Newton's method and Lagrange's method. Students are also able to solve equations via eigen value and eigen vector approach, Laplace/Inverse Laplace transform, transfer function approach, and bode plot. Feedback loop is introduced to control a dynamic system.

2. Students are able to use modern engineering tools necessary for engineering practice.

Students are able to solve and simulate dynamic systems using current software used in the discipline, MATLAB/Simulink.

**Grade Sheet Showing
Student and Class Performance in Outcome (1)
Outcome (1) was evaluated in this semester by**

Assessing students' ability to solve complex engineering problems and to use modern engineering tools necessary for engineering practice (outcome 1).

1. Test 1: Calculation of elements in a dynamic System: mass moment of inertia, spring constant, damping coefficient, etc. Determine Degree of freedom of system.
2. Test 2: Derive equations of motion of dynamic systems and solve for dynamic equations using MATLAB and Simulink.
3. Test 3: Use MATLAB to convert equations between time domain and s domain via Laplace Transform; use step function in MATLAB to solve a dynamic motion of a MEMS problem; and use bode plot to illustrate transmissibility and phase change of a dynamic system under a wide range of frequency of harmonic input.
4. Test 4: Use Simulink to simulate a given dynamic system and find out the force transmitted to the wall (supporting structure). Add a PID controller to command an actuator to the dynamic system in the purpose of reducing the force transmitted to the wall.

(Although the individual test problems in the four tests are not full “complex engineering problems,” they represent important equation derivations and solving processes that can be used to solve a complex engineering problem. The “in-class” tests are the best means to evaluate students' ability.)

5. Project – Simulation of an Active Suspension System for a Vehicle using self-collected research information. It is a complex engineering problem, has no obvious solution, includes many component parts, and involves mechanical, control, and possible other disciplines, such as electrical, hydraulic, pneumatic, etc.

MCEG 4306-P02 Dynamic Systems and Controls						
Grades Contributed to ABET Outcome (1)						
Solving Complex Engineering Problems						
	Test 1	Test 2	Test 3	Test 4	Project	Average
Student Name	85	75	95	100	80	87.0
Student Name	50	45	83	90	80	69.6
Student Name	80	65	80	90	80	79.0
Student Name	70	75	70	95	80	78.0
Student Name	65	65	60	85	70	69.0
Student Name	70	50	50	95	70	67.0
Student Name	65	75	65	90	70	73.0
Student Name	35	60	50	80	70	59.0
Student Name	80	75	70	100	90	83.0
Student Name	70	75	65	100	80	78.0
Student Name	95	95	95	100	90	95.0
Student Name	85	70	60	60	70	69.0
Student Name	85	85	80	90	80	84.0
Student Name	70	80	65	100	70	77.0
Student Name	70	75	60	90	80	75.0
Student Name	75	80	77	100	80	82.4
Student Name	70	60	70	90	70	72.0
Student Name	92	55	60	95	70	74.4
Student Name	55	55	65	100	70	69.0
Student Name	70	55	50	90	80	69.0
Student Name	93	85	65	90	80	82.6
Student Name	75	50	60	100	80	73.0
Student Name	55	75	65	90	80	73.0
Student Name	65	55	55	90	80	69.0
Student Name	65	60	65	90	80	72.0
Student Name	80	85	90	90	90	87.0
Student Name	72	65	80	90	80	77.4
Student Name	70	55	40	90	70	65.0
Student Name	73	70	65	85	80	74.6
Student Name	70	50	70	100	60	70.0
Student Name	83	80	80	95	80	83.6
Student Name	85	45	70	80	65	69.0
Student Name	75	60	55	90	60	68.0
Student Name	85	75	55	90	70	75.0
Student Name	65	65	60	95	70	71.0
Expected Average: 70%						
Class Average:						74.8
Number of Students meeting Expected Average:						25
% of Students meeting Expected Average:						71.4%

DEPARTMENT
OUTCOMES SPECIFIC ASSIGNMENT COVER SHEET
Dynamic Systems and Controls

Spring 2022

Instructor: Dr. Shield Lin

Title of Assignment

TEST 1: calculation of elements in a dynamic system: mass moment of inertia, spring constant, and damping coefficient; determine degree of freedom of a system.

Outcome Measured Using this Assignment

Outcome 1

An ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics.

Check the complex engineering problem characteristics related to this assignment

Assignment problem	
☐	involves wide-ranging or conflicting technical issues,
☐	has no obvious solution
☐	Addresses problems not encompassed by current standards and codes
☐	involves diverse groups of stakeholders
☐	includes many component parts or sub-problems
☐	involves multiple disciplines
☒	has significant consequences in a range of contexts
Assignment problem involves the application of the principles of	
☒	engineering
☐	science
☒	mathematics

Brief justification of why this assignment is suitable for this outcome

This test evaluates the students' ability to identify and calculate important elements in a dynamic system, such as equivalent values of mass moment of inertia, spring constant, and damping coefficient. Although the individual test problems are not full "complex engineering problems," they represent important identification and solving processes that can be used to solve a complex engineering problem. The "in-class" tests are the best means to evaluate students' ability.

DEPARTMENT
OUTCOMES SPECIFIC ASSIGNMENT COVER SHEET
Dynamic Systems and Controls

Spring 2022

Instructor: Dr. Shield Lin

Title of Assignment

TEST 2 Formulation of Equations for Dynamic Systems and Solution Procedures

Outcome Measured Using this Assignment

Outcome 1

An ability to identify, formulate, and solve **complex engineering problems** by applying principles of engineering, science, and mathematics.

Check the complex engineering problem characteristics related to this assignment

Assignment problem	
	involves wide-ranging or conflicting technical issues,
	has no obvious solution
	Addresses problems not encompassed by current standards and codes
	involves diverse groups of stakeholders
	includes many component parts or sub-problems
	involves multiple disciplines
X	has significant consequences in a range of contexts
Assignment problem involves the application of the principles of	
X	engineering
	science
X	mathematics

Brief justification of why this assignment is suitable for this outcome

This test evaluates the students' ability to formulate differential equations of motion from dynamic systems with mass moment of inertia, springs, and dampers. Students are also requested to solve the dynamic equations and display the variable results via MATLAB/Simulink. Although the individual test problems are not full "complex engineering problems," they represent important equation derivations and solving processes that can be used to solve a complex engineering problem. The "in-class" tests are the best means to evaluate students' ability.

DEPARTMENT

OUTCOMES SPECIFIC ASSIGNMENT COVER SHEET

Dynamic Systems and Controls

Spring 2022

Instructor: Dr. Shield Lin

Title of Assignment

TEST 3 Use s-domain (Laplace Transform) and frequency domain (bode plot) to solve dynamic equations and frequency input functions.

Outcome Measured Using this Assignment

Outcome 1

An ability to identify, formulate, and solve **complex engineering problems** by applying principles of engineering, science, and mathematics.

Check the complex engineering problem characteristics related to this assignment

Assignment problem	
	involves wide-ranging or conflicting technical issues,
	has no obvious solution
	Addresses problems not encompassed by current standards and codes
	involves diverse groups of stakeholders
	includes many component parts or sub-problems
	involves multiple disciplines
X	has significant consequences in a range of contexts
Assignment problem involves the application of the principles of	
X	engineering
	science
X	mathematics

Brief justification of why this assignment is suitable for this outcome

Covert differential equations from time domain to s-domain via Laplace Transform; solve equation variables, and display results and output in graphical forms via MATLAB. Bode plot is used to investigate system responses for wide ranges of frequencies in input harmonic functions to a dynamic system. Although the individual test problems are not full “complex engineering problems,” they represent important solving techniques that can be used to solve a complex engineering problem. The “in-class” tests are the best means to evaluate students’ ability.

DEPARTMENT

OUTCOMES SPECIFIC ASSIGNMENT COVER SHEET

Dynamic Systems and Controls

Spring 2022

Instructor: Dr. Shield Lin

Title of Assignment

TEST 4 Add a PID controller to a given dynamic system in the purpose of reducing the output force transmitted to a supporting structure.

Outcome Measured Using this Assignment

Outcome 1

An ability to identify, formulate, and solve **complex engineering problems** by applying principles of engineering, science, and mathematics.

Check the complex engineering problem characteristics related to this assignment

Assignment problem	
	involves wide-ranging or conflicting technical issues,
	has no obvious solution
	Addresses problems not encompassed by current standards and codes
	involves diverse groups of stakeholders
	includes many component parts or sub-problems
	involves multiple disciplines
X	has significant consequences in a range of contexts
Assignment problem involves the application of the principles of	
X	engineering
	science
X	mathematics

Brief justification of why this assignment is suitable for this outcome

Use Simulink to simulate a given dynamic system and find out the force transmitted to the wall (supporting structure). Add a PID controller to command an actuator to the dynamic system in the purpose of reducing the force transmitted to the wall. Although the individual test problem is not a full “complex engineering problem,” it represents important knowledge and skill that can be used to solve a complex engineering problem. The “in-class” tests are the best means to evaluate students’ ability.

DEPARTMENT
OUTCOMES SPECIFIC ASSIGNMENT COVER SHEET
Dynamic Systems and Controls

Spring 2022

Instructor: Dr. Shield Lin

Title of Assignment

Project: Simulation of an Active Suspension System for a Vehicle using self-collected research information.

Outcome Measured Using this Assignment

Outcome 1

An ability to identify, formulate, and solve **complex engineering problems** by applying principles of engineering, science, and mathematics.

Check the complex engineering problem characteristics related to this assignment

Assignment problem	
	involves wide-ranging or conflicting technical issues,
X	has no obvious solution
	Addresses problems not encompassed by current standards and codes
	involves diverse groups of stakeholders
X	includes many component parts or sub-problems
X	involves multiple disciplines
X	has significant consequences in a range of contexts
Assignment problem involves the application of the principles of	
X	engineering
	science
X	mathematics

Brief justification of why this assignment is suitable for this outcome

The students are to research an active suspension system used on a vehicle, describe the functionality of the active suspension system, define the main variables, derive equations, and use the collected data and parameters to simulate the suspension system via Simulink. Explain the simulation results and justify the purpose of having active suspension systems in the modern vehicles. The assignment has no obvious solution, includes many component parts, and involves mechanical, control, and at least one other discipline, such as electrical, hydraulic, pneumatic, etc.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: TEST 1

TOPICS COVERED IN TEST 1

1. Spring Constant Equivalent of a System
2. Calculation of Damping Coefficient
3. Equivalent of Mass Moment of Inertia
4. Equivalent of Stiffness
5. Degree of Freedom of Dynamic Systems

(Last name, First name)

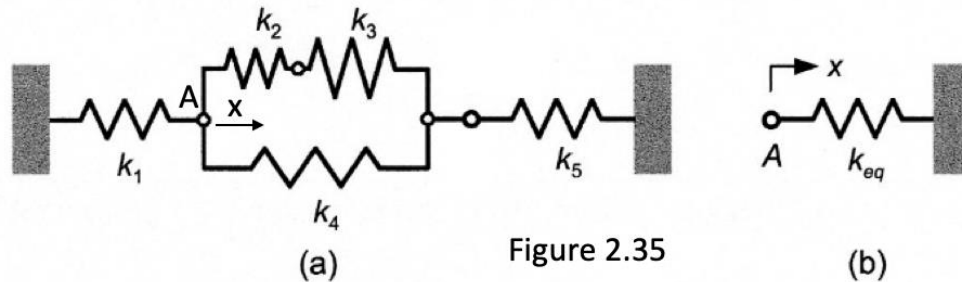
(Signature)

The 1st Test

MCEG 4306 – Dynamic Systems and Controls, Spring 2022
February 9, 2022

(20%) problem 1.

Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20$ N/m, $k_2 = 30$ N/m, $k_3 = 15$ N/m, $k_4 = 25$ N/m, and $k_5 = 35$ N/m, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.



(20%)

(2) A translatory-motion bearing with a cylindrical rod as shown in the figure.

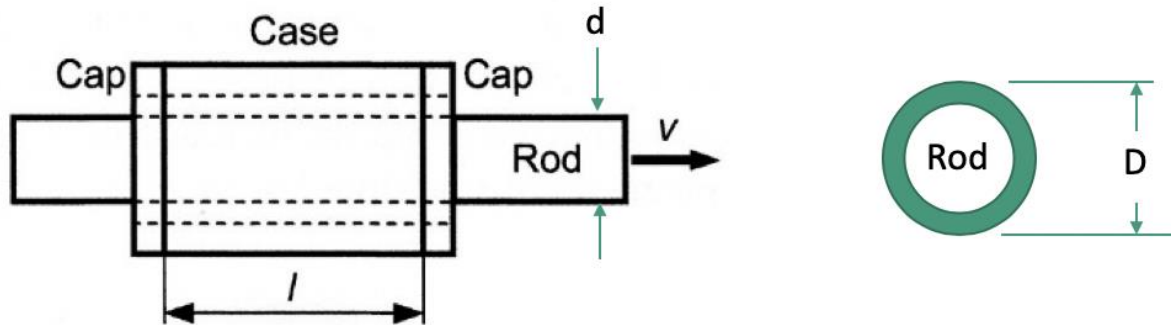
Calculate the damping coefficient based on the geometric parameters and the fluid dynamic viscosity μ .

length of bearing: $l = 0.04$ m;

diameter of bearing: $D = 0.012$ m;

diameter of rod: $d = 0.01$ m;

bearing fluid dynamic viscosity $\mu = 0.003$ N s/m².



(25%)

(3) Three shafts mesh through their respective gears as illustrated in the figure. Calculate the total mass moment of inertia of the system transferred to the middle shaft. The shafts have the same material. Their material property and geometric parameters are as follows:

Density $\rho = 7800 \text{ kg/m}^3$,

$d_1 = 0.012 \text{ m}$,

$l_1 = 0.15 \text{ m}$,

$d_2 = 0.01 \text{ m}$,

$l_2 = 0.1 \text{ m}$,

$d_3 = 0.009 \text{ m}$,

$l_3 = 0.13 \text{ m}$,

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.

Mass moment of inertia for wheels and gears are:

$$J_1 = 6 \cdot 10^{-4} \text{ kg m}^2$$

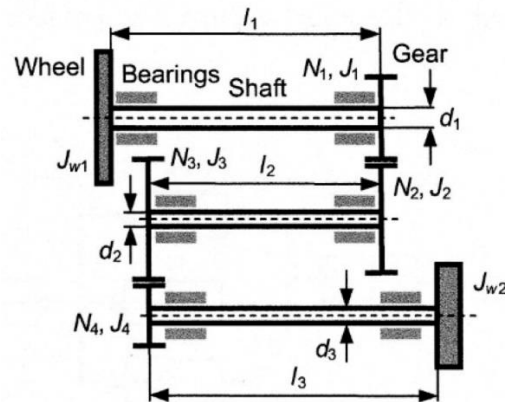
$$J_2 = 8 \cdot 10^{-4} \text{ kg m}^2$$

$$J_3 = 9 \cdot 10^{-4} \text{ kg m}^2$$

$$J_4 = 4 \cdot 10^{-4} \text{ kg m}^2$$

$$J_{w1} = 1 \cdot 10^{-3} \text{ kg m}^2$$

$$J_{w2} = 9 \cdot 10^{-4} \text{ kg m}^2$$



(25%)

(4) Three flexible shafts mesh through their respective gears as illustrated in the figure. Under the condition the two wheels (J_{w1} and J_{w2}) are locked (which functioning as fixed supports), determine the equivalent torsional stiffness of the system transferred to the middle shaft (the students can choose at either location of N_2 or N_3). The material property and geometric parameters are as follows:

The shear modulus $G = 100(10)^9 \text{ (N/m}^2\text{)}$

$d_1 = 0.012 \text{ m,}$

$l_1 = 0.15 \text{ m,}$

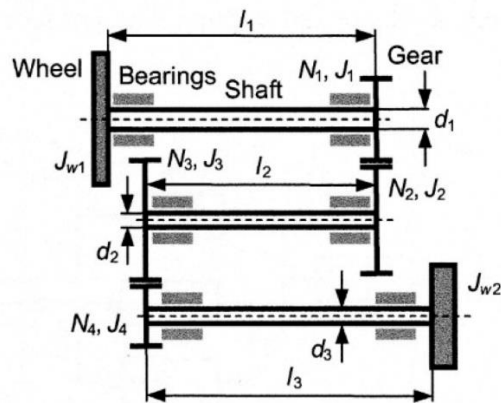
$d_2 = 0.01 \text{ m,}$

$l_2 = 0.1 \text{ m,}$

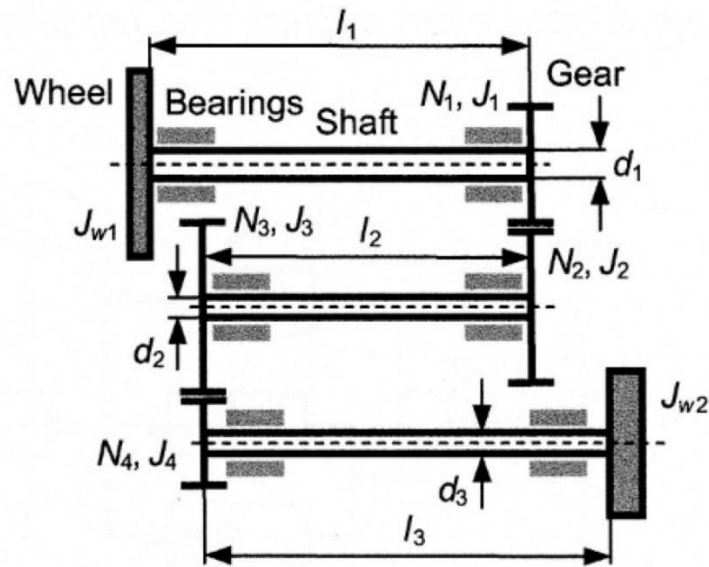
$d_3 = 0.009 \text{ m,}$

$l_3 = 0.2 \text{ m,}$

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.



- (5) Determine the degree of freedom (DOF) of the mechanical systems shown in the figure.
(The wheels are not locked.)



(5%)

- (a) Assume all shafts are rigid, the system's DOF = _____.

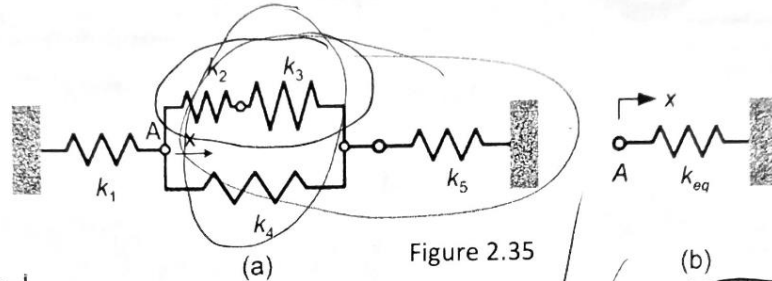
(5%)

- (b) Assume all shafts are flexible (like springs), the system's DOF = _____.

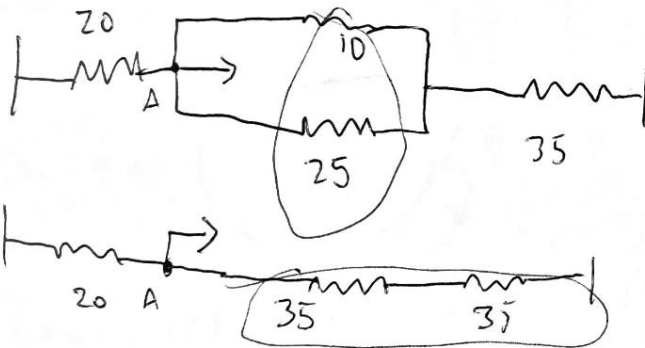
ABOVE AVERAGE

(20%) problem 1.

Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$, $k_4 = 25 \text{ N/m}$, and $k_5 = 35 \text{ N/m}$, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.



$$\left(\frac{1}{k_2} + \frac{1}{k_3} \right)^{-1} = \left(\frac{1}{30} + \frac{1}{15} \right)^{-1} = 10$$



$$\left(\frac{1}{35} + \frac{1}{35} \right)^{-1} = 17.5$$

$$k_{eq} = 20 + 17.5 = 37.5 \text{ N/m}$$

$$\begin{aligned} & \frac{1}{\left(\left(\frac{1}{k_2} + \frac{1}{k_3} \right)^{-1} + k_4 \right)^{-1} + \frac{1}{k_5}} \\ & + k_1 \\ & \left(\frac{1}{\left(\frac{1}{k_2} + \frac{1}{k_3} \right)^{-1} + k_4} + \frac{1}{k_5} \right)^{-1} + k_1 \end{aligned}$$

(20%)

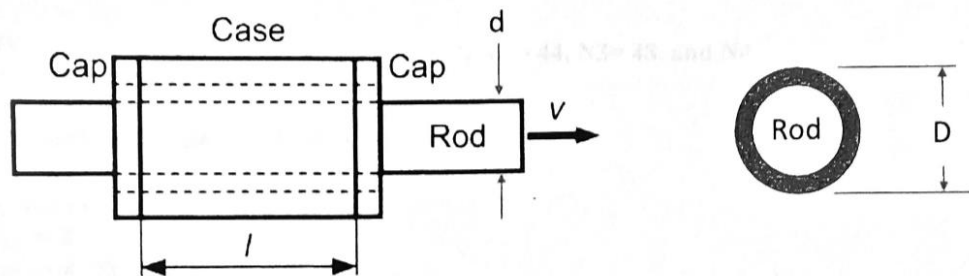
(2) A translatory-motion bearing with a cylindrical rod as shown in the figure. Calculate the damping coefficient based on the geometric parameters and the fluid dynamic viscosity μ .

length of bearing: $l = 0.04$ m;

diameter of bearing: $D = 0.012$ m;

diameter of rod: $d = 0.01$ m;

bearing fluid dynamic viscosity $\mu = 0.003$ N s/m².



$$f_d = C_t V = \tau A \quad \tau = \mu \frac{dv(z)}{dz}$$

$$v(z) = \frac{z}{g} \cdot v$$

$$dv(z)/dz = v/g$$

$$\tau = \mu \cdot v/g \quad g = (D-d)/2$$

$$A = \pi d \cdot l$$

$$f_d = \tau \cdot A = \frac{2\pi \cdot \mu \cdot d \cdot l}{D-d} (v)$$

$C_t \cdot v$

$$C_t = \frac{2\pi \cdot \mu \cdot d \cdot l}{D-d} = \frac{2\pi \cdot 0.003 \text{ N s/m}^2 \cdot 0.01 \text{ m} \cdot 0.04 \text{ m}}{0.012 - 0.01 \text{ m}}$$

$$C_t = 0.00377 \text{ N s/m}$$



(25%)

(3) Three shafts mesh through their respective gears as illustrated in the figure. Calculate the total mass moment of inertia of the system transferred to the middle shaft. The shafts have the same material. Their material property and geometric parameters are as follows:

Density $\rho = 7800 \text{ kg/m}^3$,

$d_1 = 0.012 \text{ m}$,

$l_1 = 0.15 \text{ m}$,

$d_2 = 0.01 \text{ m}$,

$l_2 = 0.1 \text{ m}$,

$d_3 = 0.009 \text{ m}$,

$l_3 = 0.13 \text{ m}$,

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.

Mass moment of inertia for wheels and gears are:

$J_1 = 6 \cdot 10^{-4} \text{ kg m}^2$

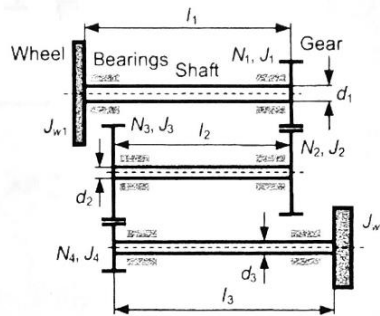
$J_2 = 8 \cdot 10^{-4} \text{ kg m}^2$

$J_3 = 9 \cdot 10^{-4} \text{ kg m}^2$

$J_4 = 4 \cdot 10^{-4} \text{ kg m}^2$

$J_{w1} = 1 \cdot 10^{-3} \text{ kg m}^2$

$J_{w2} = 9 \cdot 10^{-4} \text{ kg m}^2$



$$J_{eqm} = J_3 + J_2 + \left(\frac{N_2}{N_1}\right)^2 (J_{w1} + J_1) + \left(\frac{N_3}{N_4}\right)^2 (J_{w2} + J_4)$$

$$J_{eqm} = 9 \times 10^{-4} + 8 \times 10^{-4} + \left(\frac{44}{22}\right)^2 (1 \times 10^{-3} + 6 \times 10^{-4}) + \left(\frac{48}{16}\right)^2 (4 \times 10^{-4} + 9 \times 10^{-4})$$

$$J_{eqm} = 0.0198 \text{ kg m}^2$$



(25%)

(4) Three flexible shafts mesh through their respective gears as illustrated in the figure. Under the condition the two wheels (J_{w1} and J_{w2}) are locked (which functioning as fixed supports), determine the equivalent torsional stiffness of the system transferred to the middle shaft. The material property and geometric parameters are as follows:

$$G = 100(10)^9 \text{ (N/m}^2\text{)}$$

$$d_1 = 0.012 \text{ m,}$$

$$l_1 = 0.15 \text{ m,}$$

$$d_2 = 0.01 \text{ m,}$$

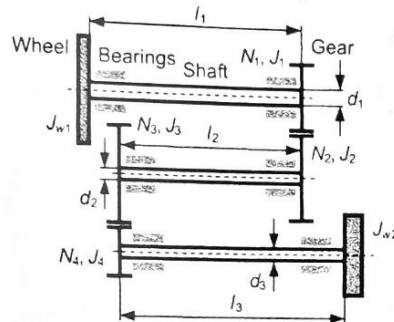
$$l_2 = 0.1 \text{ m,}$$

$$d_3 = 0.009 \text{ m,}$$

$$l_3 = 0.2 \text{ m.}$$

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.

-5



$$k_r = \frac{G I_p}{l} = \frac{G \cdot \pi d^4 / 32}{l}$$

$$k_{eqm} = k_{1eq} + k_{2eq} + k_{3eq} \quad \text{X}$$

$$k_{1eq} = \left(\frac{N_2}{N_1} \right)^2 \cdot k_1$$

$$k_{3eq} = \left(\frac{N_3}{N_4} \right)^2 \cdot k_3 \quad \checkmark$$

$$\frac{G \pi d_2^4}{32 l_2} + \left(\frac{N_2}{N_4} \right)^2 \cdot \frac{G \pi d_1^4}{32 l_1} + \left(\frac{N_3}{N_1} \right)^2 \left(\frac{G \pi d_3^4}{32 l_3} \right)$$

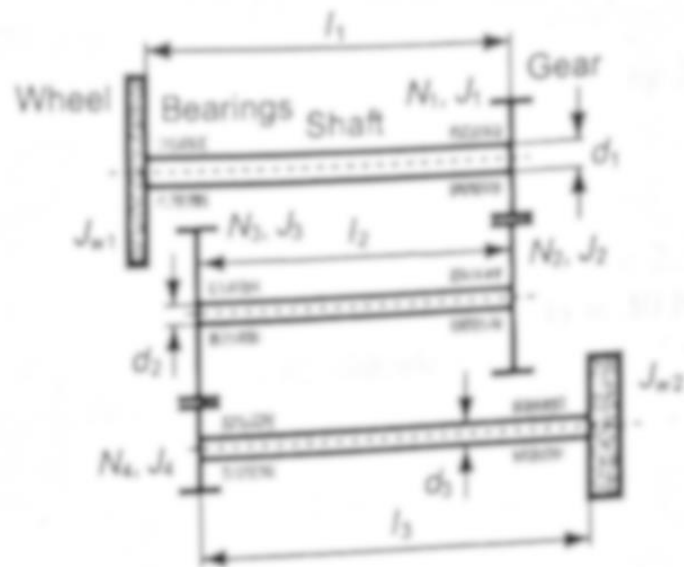
$k_2 \quad + \quad k_{eq1} \quad \quad k_{eq3}$

$$\frac{G \pi}{32} \left(\frac{d_2^4}{l_2} + \left(\frac{N_2}{N_1} \right)^2 \frac{d_1^4}{l_1} + \left(\frac{N_3}{N_4} \right)^2 \frac{d_3^4}{l_3} \right)$$

$$\frac{100 \times 10^9 \pi}{32} \left(\frac{0.01^4}{0.1} + \left(\frac{44}{22} \right)^2 \cdot \frac{0.012^4}{0.15} + \left(\frac{48}{16} \right)^2 \cdot \frac{0.009^4}{0.2} \right) = \boxed{9308.98 \text{ Nm}}$$

4

- (5) Determine the degree of freedom (DOF) of the mechanical systems shown in the figure.
(The wheels are not locked.)



- (3%) (a) Assume all shafts are rigid, the system's DOF = 1
- (3%) (b) Assume all shafts are flexible (like springs), the system's DOF = 4

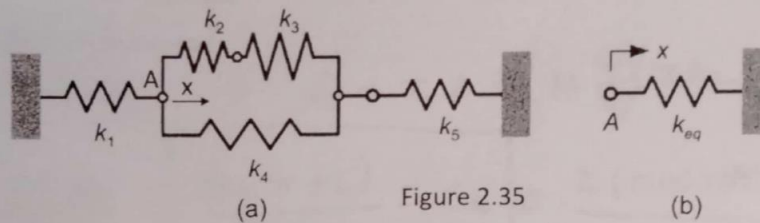


AVERAGE

(20%) problem 1.

Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$, $k_4 = 25 \text{ N/m}$, and $k_5 = 35 \text{ N/m}$, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.

-5



$$\frac{1}{k_{s1}} = \frac{1}{k_2} + \frac{1}{k_3} \rightarrow \frac{1}{k_{s1}} = \frac{k_3}{k_2 k_3} + \frac{k_2}{k_2 k_3} = \frac{k_2 + k_3}{k_2 k_3} \rightarrow k_{s1} = \frac{k_2 k_3}{k_2 + k_3}$$

$$k_{s1} = \frac{(30 \text{ N/m})(15 \text{ N/m})}{30 + 15} = 10 \text{ N/m}$$

$$k_p = k_{s1} + k_4 = 10 \text{ N/m} + 25 \text{ N/m} = 35 \text{ N/m}$$

$$\frac{1}{k_{s2}} = \frac{1}{k_1} + \frac{1}{k_p} \rightarrow \frac{k_p k_1}{k_p + k_1} = k_{s2} = \frac{(35 \text{ N/m})(20 \text{ N/m})}{35 \text{ N/m} + 20 \text{ N/m}}$$

$$k_{s2} = 12.727 \text{ N/m}$$

$$k_{s2} + k_5 = k_{eq} = (12.727 \text{ N/m}) + (35 \text{ N/m}) = 47.727 \text{ N/m} = k_{eq}$$

X

(20%)

(2) A translatory-motion bearing with a cylindrical rod as shown in the figure.

Calculate the damping coefficient based on the geometric parameters and the fluid dynamic viscosity μ .

length of bearing: $l = 0.04$ m; ✓

diameter of bearing: $D = 0.012$ m; ✓

diameter of rod: $d = 0.01$ m; ✓

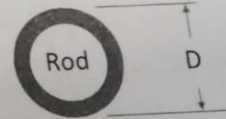
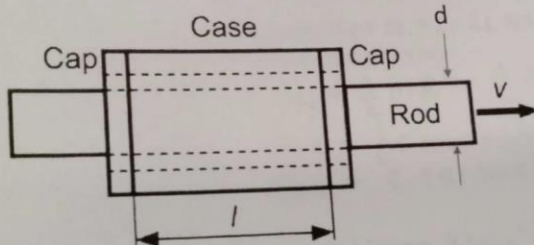
bearing fluid dynamic viscosity $\mu = 0.003$ N s/m². ✓

$$\eta = \frac{D-d}{2}$$

$$\frac{z}{\eta} = \frac{V(z)}{V}$$

$$\frac{dz}{\eta} = \frac{dV(z)}{V}$$

$$\frac{dV(z)}{dz} = \frac{V}{\eta}$$



$$\tau = \mu \frac{dv(z)}{dz} = \mu \frac{V}{\eta} \rightarrow f_d = \tau A = \left(\mu \frac{V}{\eta} \right) (\pi d L)$$

$$\frac{f_d}{V} = C_t = \frac{\mu \pi d L}{\eta} = \boxed{\frac{2(\mu \pi d L)}{D-d}} = C_t = \frac{2(0.003 \text{ N s/m}^2)(\pi)(0.01 \text{ m})(0.04 \text{ m})}{0.012 \text{ m} - 0.01 \text{ m}}$$

$$\boxed{C_t = 0.003769 \text{ N s/m}}$$



(25%)

(3) Three shafts mesh through their respective gears as illustrated in the figure. Calculate the total mass moment of inertia of the system transferred to the middle shaft. The shafts have the same material. Their material property and geometric parameters are as follows:

Density $\rho = 7800 \text{ kg/m}^3$,

$d_1 = 0.012 \text{ m}$,

$l_1 = 0.15 \text{ m}$,

$d_2 = 0.01 \text{ m}$,

$l_2 = 0.1 \text{ m}$,

$d_3 = 0.009 \text{ m}$,

$l_3 = 0.13 \text{ m}$.

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.

Mass moment of inertia for wheels and gears are:

$J_1 = 6 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_2 = 8 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_3 = 9 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_4 = 4 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_{w1} = 1 \cdot 10^{-3} \text{ kg m}^2$ ✓

$J_{w2} = 9 \cdot 10^{-4} \text{ kg m}^2$

$$J_t = J_1 + J_{w1} + J_{x_t} = (6 \cdot 10^{-4}) + (1 \cdot 10^{-3}) + (2.38 \cdot 10^{-6})$$
$$J_t = 0.0016 \text{ kg m}^2$$

$$J_m = J_2 + J_3 + J_{x_m} = (8 \cdot 10^{-4}) + (9 \cdot 10^{-4}) + (7.658 \cdot 10^{-7}) = 0.0017 \text{ kg m}^2$$

$$J_b = J_4 + J_{w2} + J_{x_b} = (4 \cdot 10^{-4}) + (9 \cdot 10^{-4}) + (6.5314 \cdot 10^{-7})$$
$$J_b = 0.0013 \text{ kg m}^2$$

Mass moment of inertia for wheels and gears are:

$J_1 = 6 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_2 = 8 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_3 = 9 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_4 = 4 \cdot 10^{-4} \text{ kg m}^2$ ✓

$J_{w1} = 1 \cdot 10^{-3} \text{ kg m}^2$ ✓

$J_{w2} = 9 \cdot 10^{-4} \text{ kg m}^2$

$$J_{x_t} = \frac{1}{2} m R^2 = \frac{1}{2} \rho \frac{\pi}{4} d_1^2 l_1 \left(\frac{d_1}{2}\right)^2 =$$

$$J_{x_t} = \frac{1}{2} (7800 \text{ kg/m}^3) \left(\frac{\pi}{4} (0.012)^2\right) (0.15 \text{ m}) \left(\frac{0.012}{2}\right)^2$$

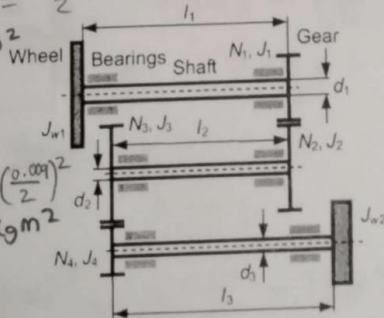
$$J_{x_t} = 2.381829 \times 10^{-6} \text{ kg m}^2$$

$$J_{x_m} = \frac{1}{2} \rho \frac{\pi}{4} d_2^2 l_2 \left(\frac{d_2}{2}\right)^2 = \frac{1}{2} (7800 \text{ kg/m}^3) \left(\frac{\pi}{4} (0.01)^2\right) (0.1 \text{ m}) \left(\frac{0.01}{2}\right)^2$$
$$J_{x_m} = 7.65763 \times 10^{-7} \text{ kg m}^2$$

$$J_{x_b} = \frac{1}{2} \rho \frac{\pi}{4} d_3^2 l_3 \left(\frac{d_3}{2}\right)^2$$

$$J_{x_b} = \frac{1}{2} (7800 \text{ kg/m}^3) \left(\frac{\pi}{4} (0.009)^2\right) (0.13 \text{ m}) \left(\frac{0.009}{2}\right)^2$$

$$J_{x_b} = 6.5314 \times 10^{-7} \text{ kg m}^2$$



$$J_{meq} = J_m + J_t \left(\frac{N_2}{N_1}\right)^2 + \left(\frac{N_3}{N_4}\right)^2 J_b$$

$$J_{meq} = (0.0017 \text{ kg m}^2) + (0.0016 \text{ kg m}^2) \left(\frac{44}{22}\right)^2 + \left(\frac{48}{16}\right)^2 (0.0013 \text{ kg m}^2)$$

$$J_{meq} = 0.0017 + 0.0064 + 0.0117$$

$$J_{meq} = 0.0198 \text{ kg m}^2$$

(25%)

(4) Three flexible shafts mesh through their respective gears as illustrated in the figure. Under the condition the two wheels (J_{w1} and J_{w2}) are locked (which functioning as fixed supports), determine the equivalent torsional stiffness of the system transferred to the middle shaft. The material property and geometric parameters are as follows:

$$G = 100(10^9) \text{ (N/m}^2\text{)}$$

$$d_1 = 0.012 \text{ m,}$$

$$l_1 = 0.15 \text{ m,}$$

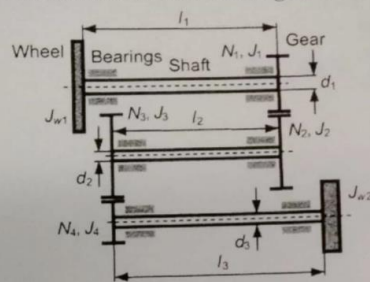
$$d_2 = 0.01 \text{ m,}$$

$$l_2 = 0.1 \text{ m,}$$

$$d_3 = 0.009 \text{ m,}$$

$$l_3 = 0.2 \text{ m,}$$

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.



$$k_1 = \frac{G \pi d_1^4}{32 l_1} \quad k_2 = \frac{G \pi d_2^4}{32 l_2}$$

$$k_3 = \frac{G \pi d_3^4}{32 l_3}$$

$$k_{eq,1} = \left(\frac{N_2}{N_1}\right)^2 k_1 = \left(\frac{44}{22}\right)^2 1357.168 \text{ Nm}$$

$$k_{eq,1} = 5428.672 \text{ Nm}$$

$$k_{eq,3} = \left(\frac{N_3}{N_4}\right)^2 k_3 = \left(\frac{48}{16}\right)^2 322.06 \text{ Nm}$$

$$k_{eq,3} = 2898.54 \text{ N}\cdot\text{m}$$

Calculations:

$$k_1 = \frac{(100 \times 10^9 \text{ N/m}^2) \pi (0.012 \text{ m})^4}{32 (0.15 \text{ m})} = \frac{G \pi d_1^4}{32 l_1} = 1357.168 \text{ Nm}$$

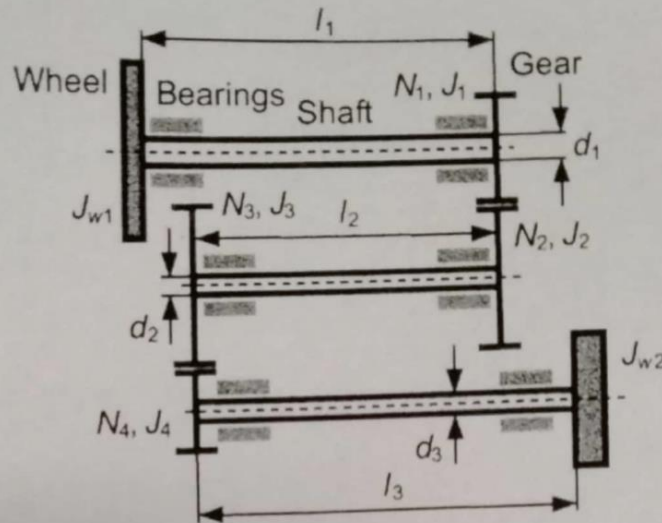
$$k_2 = \frac{G \pi d_2^4}{32 l_2} = \frac{(100 \times 10^9 \text{ N/m}^2) \pi \times (0.01 \text{ m})^4}{32 (0.1 \text{ m})} = 981.747 \text{ Nm}$$

$$k_3 = \frac{G \pi d_3^4}{32 l_3} = \frac{(100 \times 10^9 \text{ N/m}^2) \pi \times (0.009 \text{ m})^4}{32 (0.2 \text{ m})} = 322.06 \text{ Nm}$$

$$k_{eq} = k_2 + k_{eq,1} + k_{eq,3} = (981.747 \text{ Nm}) + 5428.672 \text{ Nm} + \dots$$

$$2898.54 \text{ N}\cdot\text{m} = \boxed{9308.959 \text{ Nm} = k_{eq}}$$

- (5) Determine the degree of freedom (DOF) of the mechanical systems shown in the figure. (The wheels are not locked.)



-10

(5%)

(a) Assume all shafts are rigid, the system's DOF = 2.

(5%)

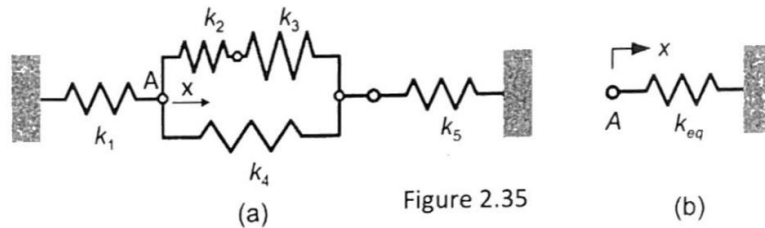
(b) Assume all shafts are flexible (like springs), the system's DOF = 5.

BELOW AVERAGE

(20%) problem 1.

Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$, $k_4 = 25 \text{ N/m}$, and $k_5 = 35 \text{ N/m}$, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.

-5



Given: $k_1 = 20 \text{ N/m}$ $k_4 = 25 \text{ N/m}$
 $k_2 = 30 \text{ N/m}$ $k_5 = 35 \text{ N/m}$
 $k_3 = 15 \text{ N/m}$

Find: (a) calculate the equivalent stiffness k_{eq}
(b) which corresponds to point A of original chain.

Analysis:

$$K = \frac{\left(\frac{k_2 k_3}{k_3 + k_2} + k_4 \right) \times k_1}{\left(\frac{k_2 k_3}{k_3 + k_2} + k_4 \right) + k_1}$$

$$= \frac{\left(\frac{(30)(15)}{30+15} + 25 \right) \times 20}{\left(\frac{(30)(15)}{30+15} + 25 \right) + 20}$$

$$= \frac{\left(\frac{450}{45} + 25 \right) \times 20}{\left(\frac{450}{45} + 25 \right) + 20}$$

$$= \frac{700}{55} = 12.72 \text{ N/m}$$

$$K_{eq} = K_{LHS} + k_5$$

$$= 12.72 \text{ N/m} + 35 \text{ N/m}$$

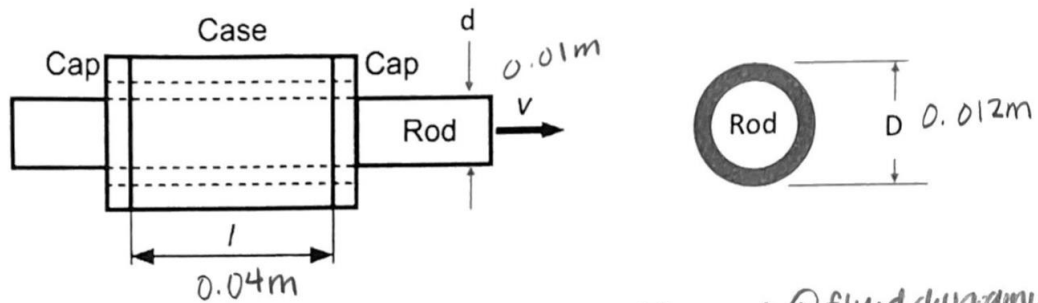
$$K_{eq} = 47.72 \text{ N/m}$$

(20%)

-5

(2) A translatory-motion bearing with a cylindrical rod as shown in the figure. Calculate the damping coefficient based on the geometric parameters and the fluid dynamic viscosity μ .

length of bearing: $l = 0.04 \text{ m}$;
diameter of bearing: $D = 0.012 \text{ m}$;
diameter of rod: $d = 0.01 \text{ m}$;
bearing fluid dynamic viscosity $\mu = 0.003 \text{ N s/m}^2$.



Given: $l = 0.04 \text{ m}$
 $D = 0.012 \text{ m}$
 $d = 0.01 \text{ m}$
 $\mu = 0.003 \text{ N/s m}^2$

Find: Damping coefficient @ fluid dynamic viscosity μ .

Analysis:

$$\begin{aligned} C_v &= \frac{\pi \mu d \cdot l}{2(D-d)} \\ &= \frac{\pi (0.003 \text{ N s/m}^2) (0.01 \text{ m}) (0.04 \text{ m})}{2(0.012 \text{ m} - 0.01 \text{ m})} \\ &= \frac{3.768 \times 10^{-6}}{4 \times 10^{-3}} \\ &= 9.42 \times 10^{-4} \text{ N m s} \end{aligned}$$

X

(25%)

-5

(3) Three shafts mesh through their respective gears as illustrated in the figure. Calculate the total mass moment of inertia of the system transferred to the middle shaft. The shafts have the same material. Their material property and geometric parameters are as follows:

Density $\rho = 7800 \text{ kg/m}^3$,

$d_1 = 0.012 \text{ m}$,

$l_1 = 0.15 \text{ m}$,

$d_2 = 0.01 \text{ m}$,

$l_2 = 0.1 \text{ m}$,

$d_3 = 0.009 \text{ m}$,

$l_3 = 0.13 \text{ m}$,

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.

Mass moment of inertia for wheels and gears are:

$J_1 = 6 \cdot 10^{-4} \text{ kg m}^2$

$J_2 = 8 \cdot 10^{-4} \text{ kg m}^2$

$J_3 = 9 \cdot 10^{-4} \text{ kg m}^2$

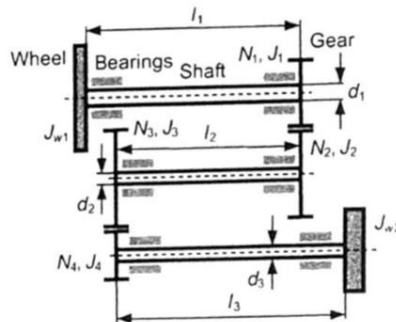
$J_4 = 4 \cdot 10^{-4} \text{ kg m}^2$

$J_{w1} = 1 \cdot 10^{-3} \text{ kg m}^2$

$J_{w2} = 9 \cdot 10^{-4} \text{ kg m}^2$

Find J_{eq} ?

$$J_{eq} = J_1 + J_2 \left(\frac{N_2}{N_1} \right)^2$$



$$\begin{aligned}
 J_{eq} &= J_{w1} + J_1 + J_{s1} \\
 &= (1 \times 10^{-3} \text{ kg m}^2) + (6 \times 10^{-4} \text{ kg m}^2) + \frac{\rho V \times R^2}{2} \\
 &= 1.6 \times 10^{-3} \text{ kg m}^2 + 7800 \text{ kg/m}^3 \times \frac{R^2 l_1 \times R}{2} \\
 &= 1.6 \times 10^{-3} + 2.1 \times 10^{-6} \\
 &= 1.6024 \times 10^{-3}
 \end{aligned}$$

$$\begin{aligned}
 J_{eq}^2 &= J_2 + J_3 + J_{s2} \\
 &= (8 \times 10^{-4} \text{ kg m}^2 + 9 \times 10^{-4} \text{ kg m}^2) + \frac{7800 \times R^4 \times l_2}{2} \\
 &= 1.7 \times 10^{-3} + 7.6 \times 10^{-7} \\
 &= 1.70076 \times 10^{-3} \text{ kg m}^2
 \end{aligned}$$

$$\begin{aligned}
 J_{eq}^3 &= J_4 + J_3 + \frac{7800 \pi R^4 \times 0.13}{2} \\
 &= 4 \times 10^{-4} + 9 \times 10^{-4} + \frac{7800 (\pi) (0.009/2)^4 \times 0.13}{2} \\
 &= 1.3 \times 10^{-3} + 0.0065 \times 10^{-4} \\
 &= 1.30065 \times 10^{-3} \text{ kg m}^2
 \end{aligned}$$

$$\frac{J_2 \times \omega_2^2}{2} = \frac{J_{2eq} \times \omega_2^2}{2} + \frac{J_{1eq} \times \omega_1^2}{2} + \frac{J_{3eq} \times \omega_3^2}{2}$$

$$J_2' = J_{2eq} + J_{eq} \left(\frac{M}{N_2} \right)^2 + J_{3eq} \left(\frac{N_3}{N_2} \right)^2$$

$$J_2' = (1.70076 \times 10^{-3}) + (1.6024 \times 10^{-3}) \times \left(\frac{44}{22} \right)^2 + 1.30065 \times 10^{-3} \times \left(\frac{48}{16} \right)^2$$

$$= 3.303 \times 10^{-3} \times (4) + 1.30065 \times 10^{-3} (9)$$

$$J_2' = 2.38 \times 10^{-2} \text{ Kg m}^2$$



(25%)

-10

(4) Three flexible shafts mesh through their respective gears as illustrated in the figure. Under the condition the two wheels (J_{w1} and J_{w2}) are locked (which functioning as fixed supports), determine the equivalent torsional stiffness of the system transferred to the middle shaft. The material property and geometric parameters are as follows:

$$G = 100(10)^9 \text{ (N/m}^2\text{)}$$

$$d_1 = 0.012 \text{ m.}$$

$$l_1 = 0.15 \text{ m.}$$

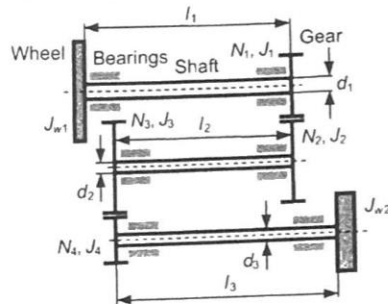
$$d_2 = 0.01 \text{ m.}$$

$$l_2 = 0.1 \text{ m.}$$

$$d_3 = 0.009 \text{ m.}$$

$$l_3 = 0.2 \text{ m.}$$

The number of teeth of the gears: $N_1 = 22$, $N_2 = 44$, $N_3 = 48$, and $N_4 = 16$.



$$K_{eq} = \left(\frac{N_1}{N_2} \right)^2 K_2$$

Analysis

Gear Ratio

$$\frac{N_1}{N_4} = \frac{T_4}{T_1}$$

$$K = \frac{GJ}{L}$$

$$K_{eq} = K_1 + K_2 + K_3$$

$$= \frac{GJ_1}{L_1} + \frac{GJ_2}{L_2} + \frac{GJ_3}{L_3}$$

$$K_e = \frac{G\pi d_1^4}{32 \times l_1} + \frac{G\pi d_2^4}{32 \times l_2} + \frac{G\pi d_3^4}{32 \times l_3} + \dots$$

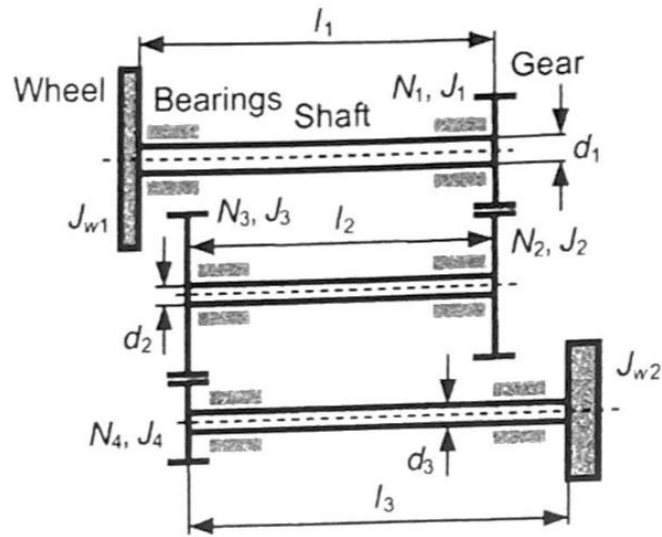
$$= \frac{100 \times 10^9 \pi (0.012 \text{ m})^4}{32 (0.15)} + \frac{100 (10)^9 \pi (0.01 \text{ m})^4}{32 (0.1)} + \frac{100 (10)^9 \pi (0.009 \text{ m})^4}{32 (0.2 \text{ m})}$$

$$= 1356.48 + 981.25 + 321.90 = 2659.63$$

$$K_e = \frac{G\pi}{32} = \frac{100 \times 10^9 \pi}{32} [1356.48 + 981.25 + 321.90]$$

$$= \boxed{2.609 \times 10^{13} \text{ Nm}}$$

- (5) Determine the degree of freedom (DOF) of the mechanical systems shown in the figure.
(The wheels are not locked.)



-10

(5%)

- (a) Assume all shafts are rigid, the system's DOF = 2.

(5%)

- (b) Assume all shafts are flexible (like springs), the system's DOF = 3.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: TEST 2

TOPICS COVERED IN TEST 2

1. Derive Equations of Motion of Dynamic Systems
2. Solve for Dynamic Equations using MATLAB and Simulink

Test 2

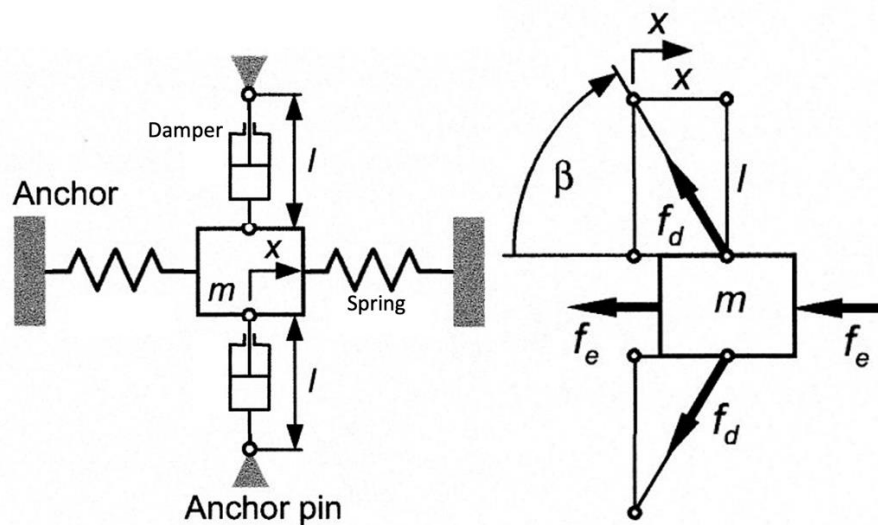
MCEG 4306 – Dynamic Systems and Controls, Spring 2022

March 9, 2022

1. A body mass $m = 0.25$ kg can move along x -direction being supported by two identical springs of stiffness $k = 10$ N/m and two identical dampers with damping coefficient $c = 0.2$ N s/m as shown in the figure. For an initial displacement of the body mass $x(0) = 0.2$ m and zero initial velocity $v(0) = 0$ m/s. $l = 0.4$ m

- Derive the mathematical model of the body motion.
- Calculate the time history $x(t)$ and plot it by using MATLAB's ode45 operator for a duration of 8 sec.

In the MATLAB statement, **define the function $dx = (\text{your name})(t, x)$ and title as 'your name'. Your MATLAB screenshot should include "user's path."**

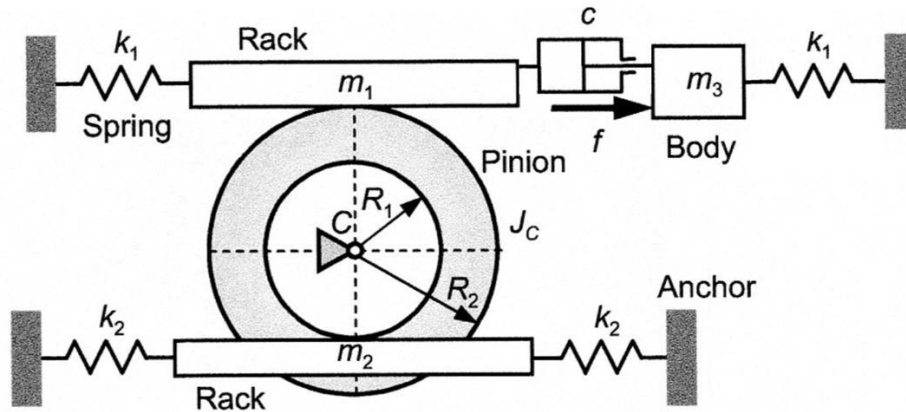


- (2) A mechanical system is shown in the figure, which is formed of a pinion, two racks, a mass body, four springs, and one damper. The pinion comprises two concentric disks forming a single solid part that rotates around a fixed axis passing its center C. The pinion meshes (without slippage) with two racks that consequently undergo translatory and parallel motions.

Given system parameters: $m_1 = 1\text{kg}$, $m_2 = 2.1\text{kg}$, $J_c = 0.05\text{ kg m}^2$, $m_3 = 0.6\text{kg}$, $k_1 = 120\frac{\text{N}}{\text{m}}$, $k_2 = 100\frac{\text{N}}{\text{m}}$, $c = 32\text{ N}\frac{\text{s}}{\text{m}}$,

$$R_1 = 0.015\text{m}, R_2 = 0.03\text{m}$$

A harmonic force $f = 5 \sin(t)$ N is applied at body mass m_3 .



- (i) Derive the mathematical equations of the system.
- (ii) Use Simulink to solve the equations and plot output for 10 sec. **Output must include your name, such as x_your_first_name, thada_your_last_name, etc.**

ABOVE AVERAGE

Test 2
MCEG 4306 – Dynamic Systems and Controls, Spring 2022
March 9, 2022

1. A body mass $m = 0.25$ kg can move along x -direction being supported by two identical springs of stiffness $k = 10$ N/m and two identical dampers with damping coefficient $c = 0.2$ N s/m as shown in the figure. For an initial displacement of the body mass $x(0) = 0.2$ m and zero initial velocity $v(0) = 0$ m/s. $l = 0.4$

- (i) Derive the mathematical model of the body motion.
(ii) Calculate the time history $x(t)$ and plot it by using MATLAB's ode45 operator for a duration of 8 sec.
In the MATLAB statement, define the function $dx = (\text{your name}) (t, x)$ and title as 'your name'. Your MATLAB screenshot should include "user's path."

$\Delta l = \sqrt{l^2 + x^2} - l$
 $(l^2 + x^2)^{1/2} - l$
time deriv
 $\frac{1}{2} (0.4^2 + x^2)^{-1/2} \cdot 2x \cdot \dot{x}$

funcn to tell displacement

$f_e = k \cdot \Delta x$
 $f_d = c \cdot \dot{x}$

$m \ddot{x} = -2f_e - 2f_d \cos \beta$
 $\cos \beta = \frac{x}{\sqrt{x^2 + l^2}}$

$m \ddot{x} + 2c \left(1 - \frac{l}{\sqrt{l^2 + x^2}}\right) \dot{x} = -2k \left(\sqrt{l^2 + x^2} - l\right)$

$f_e = k \cdot (x - x(0))$
 $f_d = c \cdot (0.4^2 + x^2)^{-1/2} \cdot x \cdot \dot{x}$

clean work shown on back of this page

$$\sqrt{l^2 + x^2} - l = \text{displacement } f(x)$$

$$f'(x) = \frac{1}{2} (l^2 + x^2)^{-1/2} \cdot 2x \cdot \dot{x}$$

$$= (l^2 + x^2)^{-1/2} x \cdot \dot{x}$$

$$f_d = c f'(x) = c (l^2 + x^2)^{-1/2} \cdot x \cdot \dot{x} \quad \checkmark$$

$$\cos \beta = \frac{x}{\sqrt{l^2 + x^2}}$$

$$f_e = k(x - x_0)$$

$$m \ddot{x} = -2f_e - 2f_d \cos \beta$$

$$m \ddot{x} = -2k(x - 0.2) - 2 \cdot c \cdot \frac{x}{\sqrt{l^2 + x^2}} \cdot \frac{x}{\sqrt{l^2 + x^2}} \cdot \dot{x}$$

$$m \ddot{x} = -2k(x - 0.2) - 2 \cdot c \cdot \frac{x^2}{l^2 + x^2} \cdot \dot{x}$$

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{2k}{m}(x_1 - 0.2) - \frac{2c}{m} \cdot \frac{x_1^2}{0.4^2 + x_1^2} \cdot x_2$$

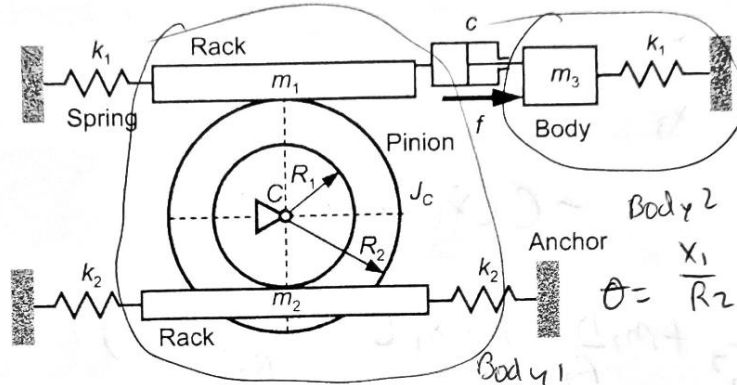
$$\ddot{x}_2 = -80(x_1 - 0.2) - 1.6 \frac{x_1^2}{0.4^2 + x_1^2} \cdot x_2$$

$x(0)$ is the initial condition of x when $t=0$, it should not be part of the equation.
 mat lab simulation takes too long

- (2) A mechanical system is shown in the figure, which is formed of a pinion, two racks, a mass body, four springs, and one damper. The pinion comprises two concentric disks forming a single solid part that rotates around a fixed axis passing through its center C. The pinion meshes (without slippage) with two racks that consequently undergo translatory and parallel motions.

Given system parameters: $m_1 = 1 \text{ kg}$, $m_2 = 2.1 \text{ kg}$, $J_c = 0.05 \text{ kg m}^2$, $m_3 = 0.6 \text{ kg}$, $k_1 = 120 \frac{\text{N}}{\text{m}}$, $k_2 = 100 \frac{\text{N}}{\text{m}}$, $c = 32 \frac{\text{N s}}{\text{m}}$, $R_1 = 0.015 \text{ m}$, $R_2 = 0.03 \text{ m}$

A harmonic force $f = 5 \sin(t) \text{ N}$ is applied at body mass m_3 .



- Derive the mathematical equations of the system.
- Use Simulink to solve the equations and plot output for 10 sec. **Output must include your name, such as x_your_first_name, thada_your_last_name, etc.**

Body 1

$$T_1 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_2 \dot{x}_2^2 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J_c \frac{\dot{x}_1^2}{R_1^2} + \frac{1}{2} m_2 \dot{x}_1^2 \cdot \frac{R_1^2}{R_2^2}$$

$$U_1 = 2\left(\frac{1}{2} k_2 x_2^2\right) + \frac{1}{2} k_1 x_1^2 = k_2 x_1^2 \frac{R_1^2}{R_2^2} + \frac{1}{2} k_1 x_1^2$$

$$F_1 = \frac{1}{2} c (\dot{x}_1 - \dot{x}_3)^2$$

$$Q_1 = 0$$

Body 2

$$T_2 = \frac{1}{2} m_3 \dot{x}_3^2; U_2 = \frac{1}{2} k_1 x_3^2; F_2 = \frac{1}{2} c (\dot{x}_1 - \dot{x}_3)^2$$

$$Q_2 = f$$

$$\frac{d}{dt} \frac{\partial T_1}{\partial \dot{x}_1} + \frac{\partial U_1}{\partial x_1} + \frac{\partial F_1}{\partial \dot{x}_1} = 0 \Rightarrow m_1 \ddot{x}_1 + \frac{J_c}{R_2^2} \ddot{x}_1 + m_2 \ddot{x}_1 \frac{R_1^2}{R_2^2}$$

$$\frac{d}{dt} \frac{\partial T_2}{\partial \dot{x}_3} + \frac{\partial U_2}{\partial x_3} + \frac{\partial F_2}{\partial \dot{x}_3} = f = m_3 \ddot{x}_3 + k_1 x_3 +$$

$$m_1 \ddot{x}_1 + \frac{J_c}{R_2^2} \ddot{x}_1 + m_2 \ddot{x}_1 \frac{R_1^2}{R_2^2} + 2k_2 x_1 \frac{R_1^2}{R_2^2} + k_1 x_1 + c(\dot{x}_1 - \dot{x}_3) = 0$$

$$\frac{d}{dt} \frac{\partial T_2}{\partial \dot{x}_3} + \frac{\partial U_2}{\partial x_3} + \frac{\partial F_2}{\partial \dot{x}_3} = f$$

$$m_3 \ddot{x}_3 + k_1 x_3 - c(\dot{x}_1 - \dot{x}_3) = f$$

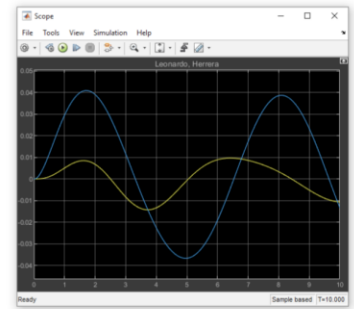
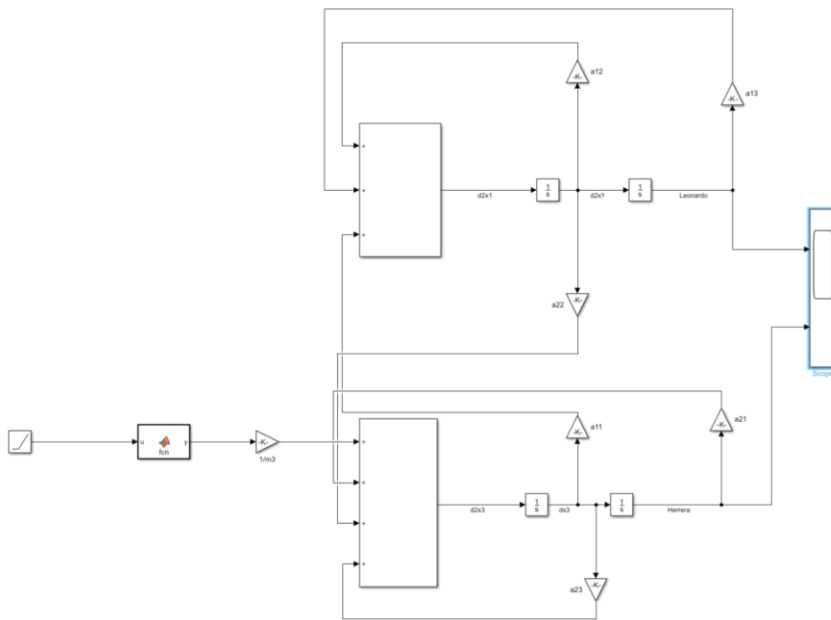
$$\ddot{x}_1 \left(m_1 + \frac{J_c}{R_2^2} + m_2 \frac{R_1^2}{R_2^2} \right) + x_1 \left(2k_2 \frac{R_1^2}{R_2^2} + k_1 \right) + c\dot{x}_1 - c\dot{x}_3 = 0$$

$$\ddot{x}_1 (57.08) = \frac{170}{32} \dot{x}_3 - 32\dot{x}_1 - x_1 (2.97824)$$

$$\ddot{x}_1 = 0.5606 \dot{x}_3 - 0.5606 \dot{x}_1 - 0.8289 x_1$$

$$\dot{x}_3 = \frac{f - k_1 x_3 + c\dot{x}_1 - c\dot{x}_3}{m_3}$$

$$\ddot{x}_3 = 1.6667 f - 200 x_3 + 53.333 \dot{x}_1 - 53.333 \dot{x}_3$$

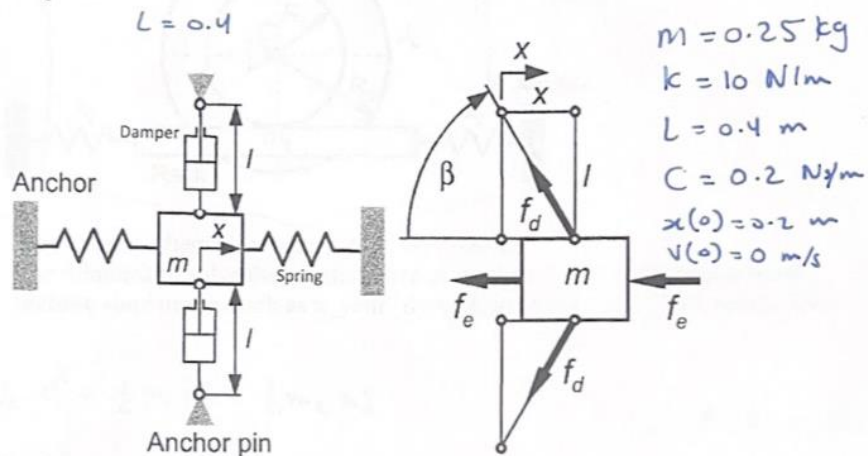


AVERAGE

Test 2
MCEG 4306 – Dynamic Systems and Controls, Spring 2022
March 9, 2022

-10

1. A body mass $m = 0.25$ kg can move along x -direction being supported by two identical springs of stiffness $k = 10$ N/m and two identical dampers with damping coefficient $c = 0.2$ N s/m as shown in the figure. For an initial displacement of the body mass $x(0) = 0.2$ m and zero initial velocity $v(0) = 0$ m/s.
- Derive the mathematical model of the body motion.
 - Calculate the time history $x(t)$ and plot it by using MATLAB's ode45 operator for a duration of 8 sec.
In the MATLAB statement, define the function $dx = (\text{your name})(t, x)$ and title as 'your name'. Your MATLAB screenshot should include "user's path."



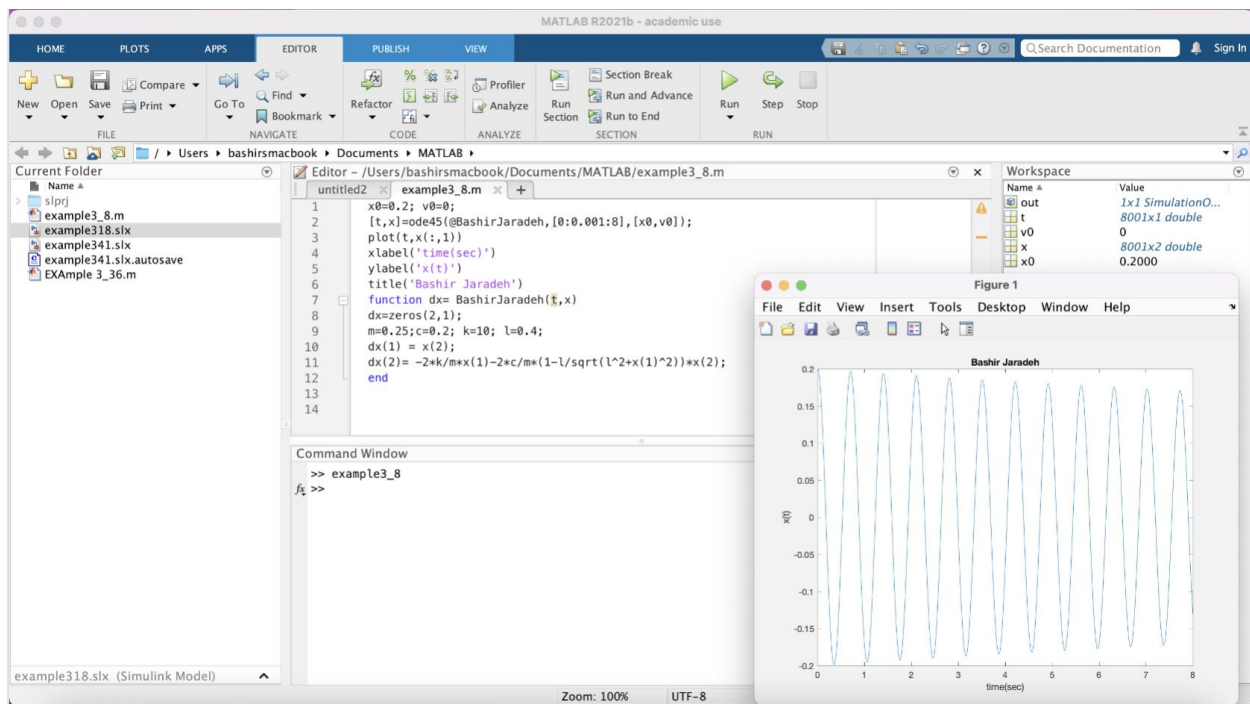
$$m \cdot \ddot{x} = -2f_e - 2f_d \cdot \cos \beta$$

$$m \cdot \ddot{x} + 2k \cdot x + 2C \cdot \left(1 - \frac{l}{\sqrt{l^2 + x^2}}\right) \dot{x} = 0$$

$$\ddot{x} = -\frac{2k}{m} \cdot x - \frac{2C}{m} \left(1 - \frac{l}{\sqrt{l^2 + x^2}}\right) \dot{x} = 0$$

$$\begin{cases} \cos \beta = \frac{x}{\sqrt{l^2 + x^2}} \\ f_e = k \cdot x \\ f_d = C \cdot (\sqrt{l^2 + x^2} - l) \end{cases}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{2k}{m} \cdot x_1 - \frac{2C}{m} \left(1 - \frac{l}{\sqrt{l^2 + x_1^2}}\right) x_2 \end{cases}$$



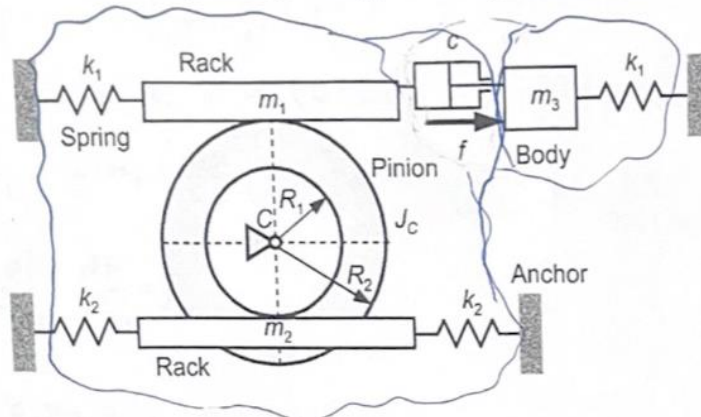
-5

- (2) A mechanical system is shown in the figure, which is formed of a pinion, two racks, a mass body, four springs, and one damper. The pinion comprises two concentric disks forming a single solid part that rotates around a fixed axis passing through its center C. The pinion meshes (without slippage) with two racks that consequently undergo transitory and parallel motions.

Given system parameters: $m_1 = 1 \text{ kg}$, $m_2 = 2.1 \text{ kg}$, $J_c = 0.05 \text{ kg m}^2$, $m_3 = 0.6 \text{ kg}$, $k_1 = 120 \frac{\text{N}}{\text{m}}$, $k_2 = 100 \frac{\text{N}}{\text{m}}$, $c = 32 \frac{\text{N s}}{\text{m}}$,

$$R_1 = 0.015 \text{ m}, R_2 = 0.03 \text{ m}$$

A harmonic force $f = 5 \sin(t) \text{ N}$ is applied at body mass m_3 .



- Derive the mathematical equations of the system.
- Use Simulink to solve the equations and plot output for 10 sec. **Output must include your name, such as x_your_first_name, thada_your_last_name, etc.**

$$T_1 = \frac{1}{2} \cdot J_c \cdot \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$T_2 = \frac{1}{2} m_3 \dot{x}_3^2$$

$$U_1 = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 + \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_1 x_1^2 + k_2 x_2^2$$

$$U_2 = \frac{1}{2} k_1 x_3^2$$

$$F = \frac{1}{2} \cdot c \cdot (\dot{x}_1 - \dot{x}_3)^2$$

$$F = 0$$

$$\begin{aligned} x_2 &= \theta \cdot R_1 \Rightarrow \theta = \frac{\dot{x}_2}{R_1} = \frac{\dot{x}_1}{R_2} \\ \dot{x}_2 &= \frac{\dot{x}_1 R_1}{R_2} \end{aligned}$$

$$\frac{d}{dt} \frac{dT}{dx_1} + \frac{dU}{dx_1} + \frac{dF}{dx_1} = 0$$

missing k1x1

$$J_c \left(\frac{\ddot{x}_1}{R_2} \right) \frac{1}{R_2} + m_1 \ddot{x}_1 + m_2 \left(\frac{\ddot{x}_1 R_1}{R_2} \right) \left(\frac{R_1}{R_2} \right) + 2k_2 \left(\frac{x_1 R_1}{R_2} \right) \left(\frac{R_1}{R_2} \right) + C(\dot{x}_1 - \dot{x}_3) = 0$$

$$\ddot{x}_1 (55.555) + (1) \ddot{x}_1 + \ddot{x}_1 (0.25) + x_1 (60) + 32 \dot{x}_1 - 32 \dot{x}_3 = 0$$

$$(56.8055) \ddot{x}_1 + x_1 (60) + 32 \dot{x}_1 - 32 \dot{x}_3 = 0$$

$$\ddot{x}_1 = -x_1 \left(\frac{60}{56.8055} \right) - \frac{32}{56.8055} \dot{x}_1 + \frac{32}{56.8055} \dot{x}_3$$

$$\frac{d}{dt} \frac{dT}{dx_3} + \frac{dU}{dx_3} + \frac{dF}{d\dot{x}_3} = f$$

missing.

$$m_3 \ddot{x}_3 + k_1 x_3 = f$$

$$0.6 \ddot{x}_3 + 120 x_3 = f$$

$$\ddot{x}_3 = \frac{-120}{0.6} x_3 + \frac{1}{0.6} f$$

$$\begin{cases} \ddot{x}_1 = -1.0562357 x_1 - 0.563325 \dot{x}_1 + 0.563325 \dot{x}_3 \\ \ddot{x}_3 = -200 x_3 + \frac{1}{0.6} f \end{cases}$$

BELOW AVERAGE

Test 2
MCEG 4306 – Dynamic Systems and Controls, Spring 2022
March 9, 2022

-15

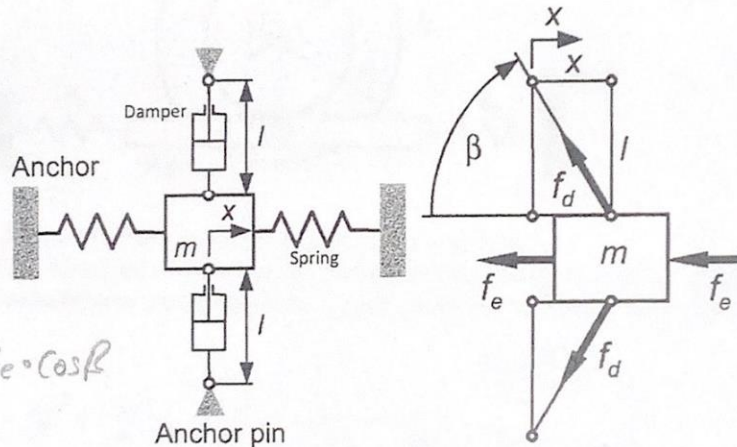
1. A body mass $m = 0.25 \text{ kg}$ can move along x -direction being supported by two identical springs of stiffness $k = 10 \text{ N/m}$ and two identical dampers with damping coefficient $c = 0.2 \text{ N s/m}$ as shown in the figure. For an initial displacement of the body mass $x(0) = 0.2 \text{ m}$ and zero initial velocity $v(0) = 0 \text{ m/s}$. $L = 0.4 \text{ m}$
- Derive the mathematical model of the body motion.
 - Calculate the time history $x(t)$ and plot it by using MATLAB's ode45 operator for a duration of 8 sec.
- In the MATLAB statement, define the function $dx = (\text{your name}) (t, x)$ and title as 'your name'. Your MATLAB screenshot should include "user's path."

$m = 0.25 \text{ kg}$
 $k = 10 \text{ N/m}$
 $c = 0.2 \text{ N s/m}$
 $x(0) = 0.2 \text{ m}$
 $v(0) = 0 \text{ m/s}$
 $L = 0.4 \text{ m}$

$m \cdot \ddot{x} = -2F_d - 2F_e \cdot \cos \beta$

$m \cdot \ddot{x} + 2c \cdot \dot{x} + 2k \cdot \left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right) \cdot x = 0$

$\ddot{x} = \frac{2k}{m} \left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right) \cdot x - \frac{2c}{m} \cdot \dot{x}$



HOMEPLOTSAPPSEDITORPUBLISHFILE VERSIONSVIEW

/ > MATLAB Drive >

Harold_2_1.m x

CURRENT FOLDER

1x0=0.2; v0=0;
2[t, x]=ode45(@RodriguezHarold,(0:0.01:8),[x0,v0]);
3plot(t,x(:,1))
4xlabel('time(sec)')
5ylabel('x(t)')
6title('Rodriguez Harold')
7function dx= RodriguezHarold(t,x)
8dx=zeros(2,1);
9m=0.25; c=0.2; k=10; l=0.4;
10dx(2)= -2*k/m*x(1)-2*c/m*(1-1/sqrt(1^2+x(1)^2))*x(2);
11

Output?

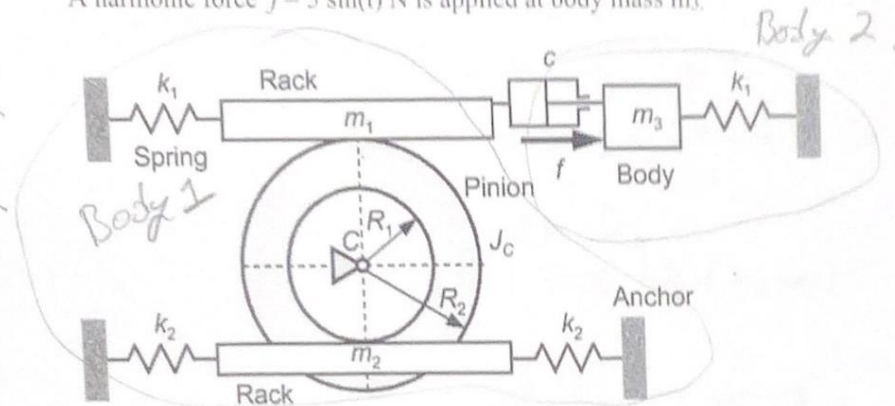
-30

$m_1 = 1 \text{ kg}, m_2 = 2.1 \text{ kg}, J_c = 0.05 \text{ kg m}^2, m_3 = 0.6 \text{ kg}$
 $k_1 = 120 \frac{\text{N}}{\text{m}}, k_2 = 100 \frac{\text{N}}{\text{m}}, c = 32 \frac{\text{N s}}{\text{m}}$
 $R_1 = 0.015 \text{ m}, R_2 = 0.03 \text{ m}$
 $f = 5 \sin(t) \text{ N}$

(2) A mechanical system is shown in the figure, which is formed of a pinion, two racks, a mass body, four springs, and one damper. The pinion comprises two concentric disks forming a single solid part that rotates around a fixed axis passing its center C. The pinion meshes (without slippage) with two racks that consequently undergo translatory and parallel motions.

Given system parameters: $m_1 = 1 \text{ kg}, m_2 = 2.1 \text{ kg}, J_c = 0.05 \text{ kg m}^2, m_3 = 0.6 \text{ kg}, k_1 = 120 \frac{\text{N}}{\text{m}}, k_2 = 100 \frac{\text{N}}{\text{m}}, c = 32 \frac{\text{N s}}{\text{m}}, R_1 = 0.015 \text{ m}, R_2 = 0.03 \text{ m}$

A harmonic force $f = 5 \sin(t) \text{ N}$ is applied at body mass m_3 .



- Derive the mathematical equations of the system.
- Use Simulink to solve the equations and plot output for 10 sec. **Output must include your name, such as x_your_first_name, thada_your_last_name, etc.**

Body 1
 $T_1 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_2 \dot{x}_2^2$
 $U_1 = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$
 $F_1 = (\frac{1}{2} c \dot{x}_1^2) \times 2$
 $x = 5 \sin t$

Body 2
 $T_2 = \frac{1}{2} m_3 \dot{x}_3^2$
 $U_2 = \frac{1}{2} k_3 x_1^2 + \frac{1}{2} k_3 (x_1 - x_3)^2$
 $\theta = \frac{x_1}{R_1}, \theta = \frac{x_2}{R_2}$
 $T_2 = \frac{1}{2} m_3 \dot{x}_3^2$
 $U_2 = \frac{1}{2} k_1 (x_1 - x_3)^2$
 $F_2 = \frac{1}{2} c \dot{x}_2^2$
 $Q_2 = F$

$$T_1 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$U_1 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} \left(\frac{1}{2} m_1 R_1^2 \right) \left(\frac{\dot{x}_1}{R_1} \right)^2 + \frac{1}{2} m_2 \left(x_1 - \frac{R_1}{R_2} \right)^2$$

$$F_1 = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_1 (x_1 - x_2)^2$$

$$C_1 = \left(\dot{x}_1 - \frac{R_1}{R_2} \right)^2$$

$\theta = \theta$

Body 1

$$\bullet m_1 \ddot{x}_1 + m_1 \ddot{x}_1 \frac{R_1^2}{R_2^2} + m_2 x_1 \frac{R_1^2}{R_2^2}$$

$$\bullet k_1 x_1 + k_1 (x_1 - x_2)$$

$$\bullet 2 C_1 \ddot{x}_1 \frac{R_1^2}{R_2^2}$$

$\bullet \theta$

$$\frac{d}{dt} \frac{dT_1}{dx_1} = m_1 \ddot{x}_1 + J_c \frac{\ddot{x}_1}{R_2^2} + m_2 x_1 \frac{R_1^2}{R_2^2}$$

$$\frac{dU_1}{dx_1} = k_1 x_1 + k_1 (x_1 - x_2) + 2 k_2 x_1 \frac{R_1^2}{R_2^2}$$

$$\frac{dF_1}{dx_1} = 0$$

$$\bullet \ddot{x}_1 \left(m_1 + \frac{1}{2} m_1 \frac{R_1^2}{R_2^2} + m_2 \frac{R_1^2}{R_2^2} \right)$$

$$\bullet 2 k_1 x_1 - k_1 l$$

$$\bullet x_1 \left(2 c \frac{R_1^2}{R_2^2} \right)$$

Body 2

$$\bullet T_2 = \frac{1}{2} m_3 \dot{x}_2^2$$

$$\bullet U_2 = \frac{1}{2} k_3 x_2^2 + \frac{1}{2} k_1 (x_1 - x_2)^2$$

$$\bullet m_3 \ddot{x}_2$$

$$\bullet k_3 x_2 + k_1 (x_1 - x_2)$$

$$\frac{d}{dt} \frac{dT_2}{dx_2} = m_3 \ddot{x}_2$$

$$\frac{dU_2}{dx_2} = -k_1 (x_1 - x_2)$$

$$\frac{dF_2}{dx_2} = (x_1 - x_2)$$

$$\bullet m_3 \ddot{x}_2$$

$$\bullet k_1 x_2 + k_1 x_1 - k_1 l$$

Equations=?

Missing Simulink.

DYNAMIC SYSTEMS AND CONTROLS

SPRING 2022

Title of Assignment: TEST 3

TOPICS COVERED IN TEST 3

1. Use MATLAB to covert equations between time domain and s domain via Laplace Transform.
2. Use step function in MATLAB to solve a dynamic motion of a MEMS problem.
3. Use bode plot to illustrate transmissibility and phase change of a dynamic system under a wide range of frequency of harmonic input.

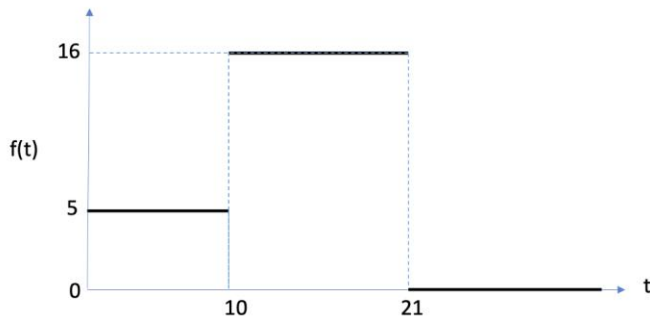
1. Use MATLAB to find Laplace transforms of the following time functions:

10%

(i) $f(t) = 3\sin \omega^2 t - 4 \cos \omega^2 t + 5e^{-\omega t}$

10%

(ii) $f(t)$ is the black ink time function shown in the graph.



When you take screen shots in MATLAB, include file name/path in the image, such as the following:



40%

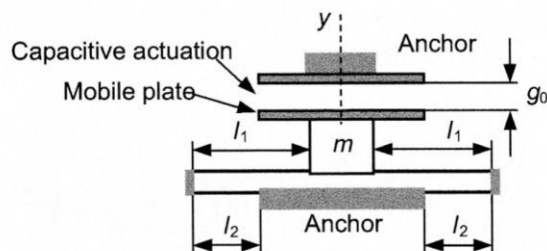
2. The MEMS in the figure is formed of two pairs of beams and a shuttle mass and is subjected to capacitive actuation. Use a lumped-parameter model to formulate the system's mathematical model for the motion about the y direction. All beams have a circular cross section of diameter $d = 3 \mu\text{m}$. Known also are the shuttle mass $m = 12 \times 10^{-6} \text{ kg}$, $l_1 = 120 \mu\text{m}$, $l_2 = 60 \mu\text{m}$, Young's modulus $E = 200 \text{ GPa}$. Plot the system's response using its transfer function in MATLAB when a step function electrostatic force $f(t) = 110 \times 10^{-6} h(t) \text{ N}$ is applied by the capacitive unit. The mobile plate's initial displacement $y(0) = 6 \mu\text{m}$, and initial velocity $\dot{y}(0) = 12 \mu\text{m/s}$. Damping coefficient $c = 0.4 \text{ N s/m}$ for the mobile plate motion, ignore the mass contribution from the beams.

(i) Use MATLAB to plot the amplitude of the mobile plate

Add a title('your name step response') in your MATLAB statement.

Take screen shot of your MATLAB statements including file name/path and the graphic result.

(ii) Is the initial capacitive gap $g_0 = 10 \mu\text{m}$ sufficient for the mobile plate motion?



40%

3. A sinusoidal force acts on the mechanical system illustrated in the following figure. Considering that $m = 1 \text{ kg}$, $k = 200 \text{ N/m}$, and $c = 25 \text{ Ns/m}$, analyze the transmissibility of the input force to the force on the wall.

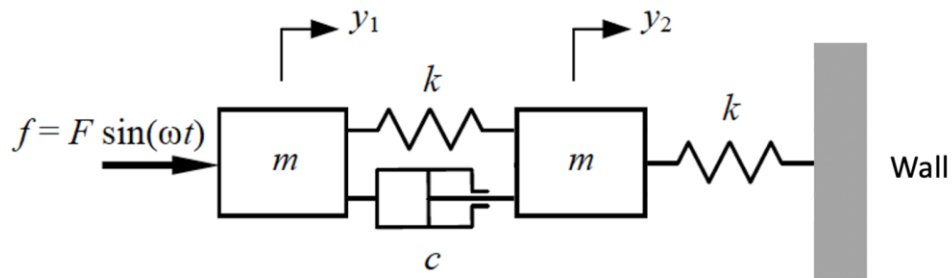
Make bode plot for Magnitude (absolute value) in MATLAB by adding:

```
opts = bodeoptions('cstprefs');
```

```
opts.MagUnits = 'abs';
```

Include your name in the title by adding:

```
opts.Title.String = 'your name bode plot';
```



ABOVE AVERAGE

NAVIGATE EDIT BREAKPOINTS RUN

Users ▸ zalexander4 ▸ Documents ▸ MATLAB

Editor - H:\Dynamic Systems\LP.m

```

1 - syms t w
2 - f(t)=3*sin(w^2*t)-4*cos(w^2*t)+5*exp(-w*t);
3 - laplace(f(t))
4 - pretty(laplace(f(t)))
5

```

Command Window

New to MATLAB? See resources for [Getting Started](#).

```

>> LP

ans =

(3*w^2)/(s^2 + w^4) - (4*s)/(s^2 + w^4) + 5/(s + w)

```

$$\frac{3w^2}{s^2 + w^4} - \frac{4s}{s^2 + w^4} + \frac{5}{s + w}$$

fx >>



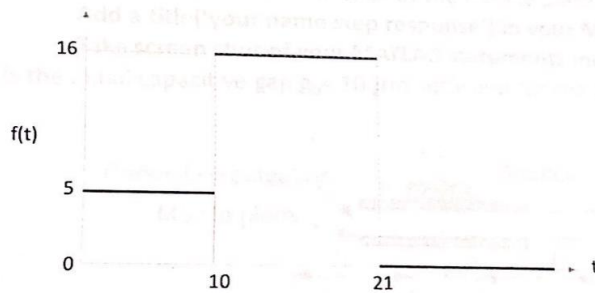
1. Use MATLAB to find Laplace transforms of the following time functions:

10%

(i) $f(t) = 3\sin \omega^2 t - 4 \cos \omega^2 t + 5e^{-\omega t}$

10%

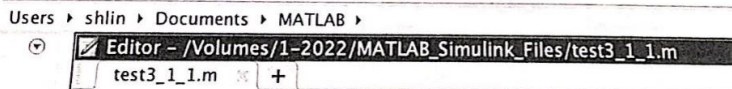
(ii) $f(t)$ is the black ink time function shown in the graph.



$$f(t) = \begin{cases} 5 & 0 \leq t \leq 10 \\ 16 & 10 \leq t \leq 21 \\ 0 & 21 \leq t \end{cases}$$

$f_1 = 5$
 $f_2 = 16$
 $f_3 = 0$

When you take screen shots in MATLAB, include file name/path in the image, such as the following:



)

$$= \mathcal{L}\{f(t)\} = \int_0^{10} 5 \cdot e^{-s \cdot t} dt + \int_{10}^{21} 16 \cdot e^{-s \cdot t} dt + \int_{21}^{\infty} 0 \cdot e^{-s \cdot t} dt$$

✓

zalexander4 ▸ Documents ▸ MATLAB

Editor - H:\Dynamic Systems\LP_heavyside.m

```
1 - syms s t
2 - f1=5; f2=16; f3=0;
3 - Fs=simplify(int(f1*exp(-s*t),t,0,10)+int(f2*exp(-s*t),t,10,21));
4 - pretty(Fs);
```

Command Window

New to MATLAB? See resources for [Getting Started](#).

```
>> LP_heavyside
exp(-10 s) 11 - exp(-21 s) 16 + 5
-----
s
```

f₁ >>

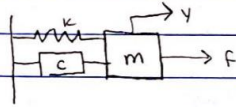


Test 3

Zoe Alexander

#12) $d = 3 \text{e-}6 \text{ m}$, $m = 12 \text{e-}6 \text{ m}$, $\ell_1 = 120 \text{e-}6 \text{ m}$, $\ell_2 = 60 \text{e-}6 \text{ m}$, $E = 200 \text{e}9 \text{ Pa}$
 $f(t) = 110 \text{e-}6 \cdot h(t) \text{ N}$, $y(0) = 60 \text{e-}6 \text{ m}$, $\dot{y}(0) = 12 \text{e-}6 \text{ m/s}$, $c = 0.4 \text{ N}\cdot\text{s/m}$

fixed guided beam i) $k_1 = \frac{12 E \pi d^4}{64 \ell_1^3}$, $k_{\text{half}} = \frac{k_1 \cdot k_2}{k_1 + k_2}$, $k = 2 \left(\frac{k_1 \cdot k_2}{k_1 + k_2} \right)$
 $k_2 = \frac{12 E \pi d^4}{64 \ell_2^3}$



$$m \ddot{y} + f_d + f_s - f = 0 \Rightarrow m \ddot{y} + c \dot{y} + k y = f \cdot h(t)$$

$$\mathcal{L}\{m \ddot{y}\} + \mathcal{L}\{c \dot{y}\} + \mathcal{L}\{k y\} = \mathcal{L}\{f \cdot h(t)\}$$

$$m[s^2 Y - s \cdot y(0) - \dot{y}(0)] + c[s \cdot Y - y(0)] + k[Y] = f$$

$$m \cdot s^2 \cdot Y - m \cdot s \cdot y(0) - m \dot{y}(0) + c \cdot s \cdot Y - c \cdot y(0) + k Y = f$$

$$[m \cdot s^2 + c \cdot s + k] Y - m \cdot s \cdot y(0) - m \dot{y}(0) - c \cdot y(0) = f$$

$$Y[m \cdot s^2 + c \cdot s + k] = f + m \cdot s \cdot y(0) + m \dot{y}(0) + c \cdot y(0)$$

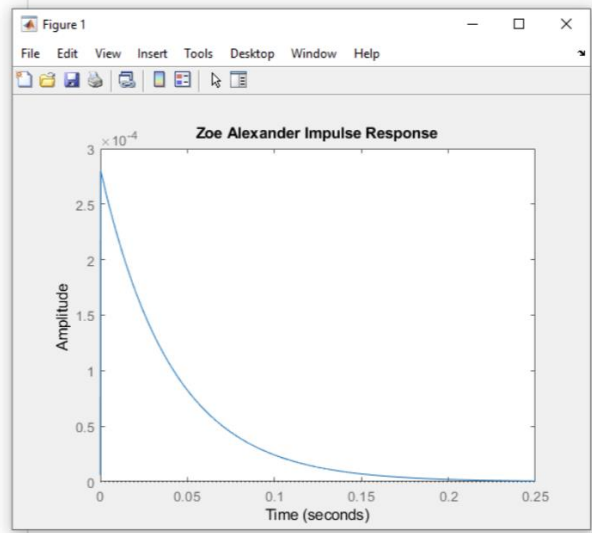
$$Y(s) = \frac{(m \cdot y(0))s + m \dot{y}(0) + c \cdot y(0) + f}{m \cdot s^2 + c \cdot s + k}$$

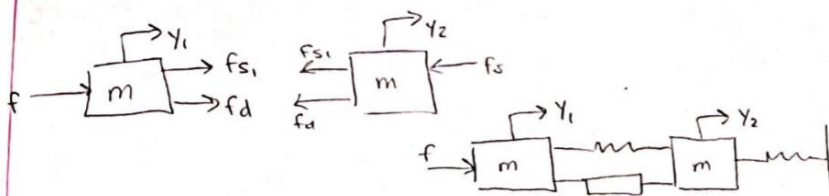
$$Y(s)/(1/s) = G = \frac{[m \cdot y(0)] \cdot s + [m \dot{y}(0) + c \cdot y(0) + f]}{[m]s^2 + [c]s + k}$$

ii) the initial gap is not sufficient since the max amplitude is $2.75 \text{e-}4 \text{ m}$ which is greater than $g_0 = 10 \text{e-}6 \text{ m}$.

$$2.75 \text{e-}4 \text{ m} > 10 \text{e-}6 \text{ m}$$

```
Chapter7.Transfer.m
1 - m=12e-6; L1=120e-6; L2=60e-6; d=3e-6; E=200e9; c=0.4; f=110e-6;
2 - y0=6e-6; y_dot0=12e-6;
3 - k1=(12*E*pi*d^4)/(64*L1^3);
4 - k2=(12*E*pi*d^4)/(64*L2^3);
5 - k=(2*k1*k2)/(k1+k2);
6 - G=tf([m*y0, m*y_dot0+c*y0+f],[m, c, k]);
7 - impulse(G); title('Zoe Alexander Impulse Response');
```





#3) $m = 1 \text{ kg}$, $k = 200 \text{ N/m}$, $c = 25 \text{ N}\cdot\text{s/m}$

① $m\ddot{y}_1 - f + f_{s1} + f_d = 0 \Rightarrow m\ddot{y}_1 + k(y_1 - y_2) + c(\dot{y}_1 - \dot{y}_2) = f$

② $m\ddot{y}_2 + f_{s1} + f_{s2} + f_d = 0 \Rightarrow m\ddot{y}_2 + k(y_2 - y_1) + k(y_2) + c(\dot{y}_2 - \dot{y}_1) = 0$

③ $f_b - f_{s2} = 0 \Rightarrow f_b = k y_2$

no initial conditions

① $m(s^2 Y_1) + k(Y_1 - Y_2) + c(s Y_1 - s Y_2) = F(s)$

② $m(s^2 Y_2) + k(Y_2 - Y_1) + k(Y_2) + c(s Y_2 - s Y_1) = 0$

③ $F_b(s) = k Y_2$

① $[m \cdot s^2 + k + cs] Y_1 = F(s) + [k + cs] Y_2$

② $[m \cdot s^2 + 2k + cs] Y_2 = [k + cs] Y_1 \Rightarrow Y_2 = \frac{cs + k}{ms^2 + cs + 2k} \cdot Y_1$

③ $F_b(s) = k \cdot Y_2 = \frac{c \cdot k \cdot s + k^2}{ms^2 + cs + 2k} Y_1$

① $[m \cdot s^2 + cs + k] Y_1 = F(s) + [cs + k] \left[\frac{cs + k}{ms^2 + cs + 2k} \right] Y_1$

$\Rightarrow [m \cdot s^2 + cs + k - \frac{(cs + k)^2}{ms^2 + cs + 2k}] Y_1 = F(s)$

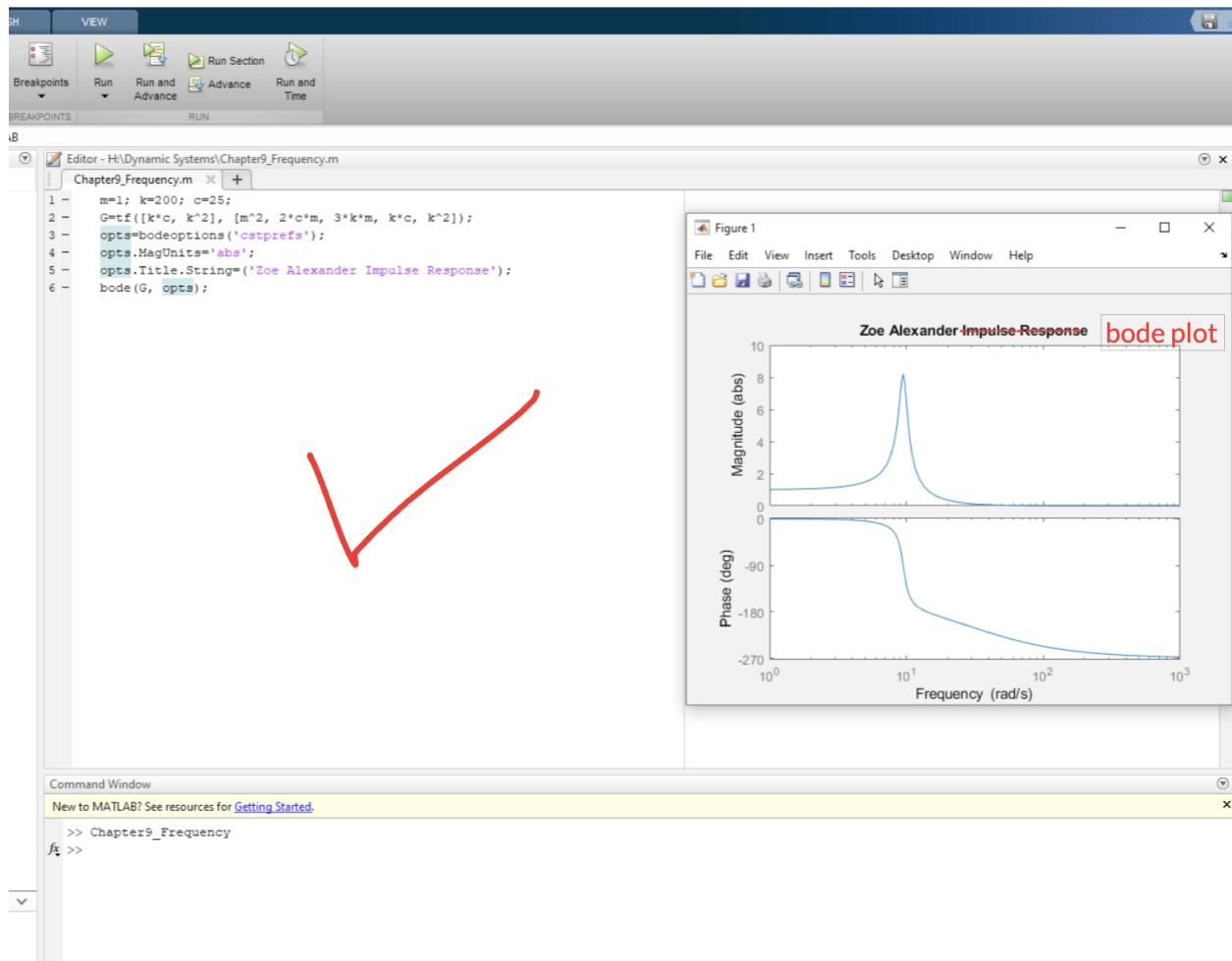
$\therefore F(s) = \frac{(m \cdot s^2 + cs + k)(m \cdot s^2 + cs + 2k) - (cs + k)^2}{m \cdot s^2 + cs + 2k} \cdot Y_1$

$G = \frac{F_b(s)}{F(s)} = \frac{(c \cdot k \cdot s + k^2)}{(m \cdot s^2 + cs + 2k)} \cdot \frac{(m \cdot s^2 + cs + 2k)}{(m \cdot s^2 + cs + k)(m \cdot s^2 + cs + 2k) - (cs + k)^2}$

$G = \frac{c \cdot k \cdot s + k^2}{(m \cdot s^2 + cs + k)(m \cdot s^2 + cs + 2k) - (cs + k)^2} = \frac{ck s + k^2}{m^2 s^4 + 2cm s^3 + 3km s^2 + cks + k^2}$

Numerator: $k \cdot c$, k^2

Denominator: m^2 , $2cm$, $3km$, kc , k^2



AVERAGE

```

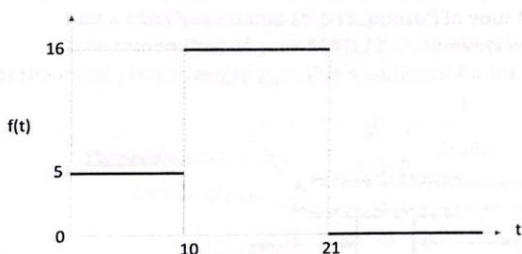
Users > Zion > Documents >
Editor - C:\Users\Zion\Documents\laplacefortest3.m
laplacefunctions.m x Untitled.m x laplacefortest3.m x +
1- syms w t phi
2-
3- f1 = 3 * sin(phi^2 * t) - 4*cos(phi^2*t) + 5 * exp(-phi*t);
4-
5- disp 'Function'
6- L1 = laplace(f1);
7- pretty(L1)

Command Window
>> laplacefortest3
Function
      2
  3 phi      4 s      5
----- + -----
  4      2      4      2      phi + s
phi + s      phi + s

fx>>

```

(ii) $f(t)$ is the black ink time function shown in the graph.

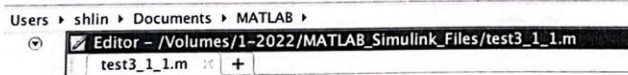


$$\begin{aligned}
 F(t) &= 5[u_0(t) - u_{10}(t)] + 16[u_{10}(t) - u_{21}(t)] \\
 F(t) &= 5u_0(t) - 5u_{10}(t) + 16u_{10}(t) - 16u_{21}(t) \\
 F(t) &= 5u_0(t) + 11u_{10}(t) - 16u_{21}(t)
 \end{aligned}$$

where $u_c(t) = \begin{cases} 1 & \text{if } t \geq c \\ 0 & \text{if otherwise} \end{cases}$

$$L\{u_c(t)\} = \frac{e^{-cs}}{s}$$

When you take screen shots in MATLAB, include file name/path in the image, such as the following:

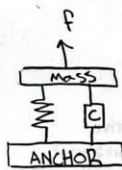


Laplace transform of $F(t)$

$$\begin{aligned}
 L\{F(t)\} &= 5L\{u_0(t)\} + 11L\{u_{10}(t)\} - 16L\{u_{21}(t)\} \\
 F(s) &= \frac{5}{s} + 11 \cdot \frac{e^{-10s}}{s} - 16 \cdot \frac{e^{-21s}}{s} \\
 F(s) &= \frac{5 + 11e^{-10s} - 16e^{-21s}}{s} \quad \text{ANSWER}
 \end{aligned}$$

Q2:

Schematic



Summation of Forces

$$\sum F = Ma$$

$$\rightarrow F - K_{eq} \cdot y - C \cdot \dot{y} = M \cdot \ddot{y}$$

-5

Spring Constants

$$K_{k1} = 12 \cdot \frac{E}{L_1^3} \cdot \frac{\pi d^4}{64} = 12 \cdot \frac{(200)}{(120)^3} \cdot \frac{\pi (3)^4}{64} = 0.00552$$

$$K_{k2} = 12 \cdot \frac{E}{L_2^3} \cdot \frac{\pi d^4}{64} = 12 \cdot \frac{(200)}{(60)^3} \cdot \frac{\pi (3)^4}{64} = 0.04417$$

Combined spring constant

$$\frac{1}{K_{se}} = \frac{1}{K_{k1}} + \frac{1}{K_{k2}} \rightarrow \frac{1}{0.00552} + \frac{1}{0.04417} = \frac{1}{K_{se}} = 203.8 \rightarrow K_{se} = \frac{1}{203.8} \rightarrow 2 \rightarrow K_{eq} = \frac{2}{203.8}$$

Substitute values

$$(1) F - K_{eq} \cdot y - C \cdot \dot{y} = M \cdot \ddot{y}$$

$$(2) m \cdot \ddot{y} + c \cdot \dot{y} + K_{eq} \cdot y = f$$

$$\downarrow \quad \downarrow \quad \downarrow$$

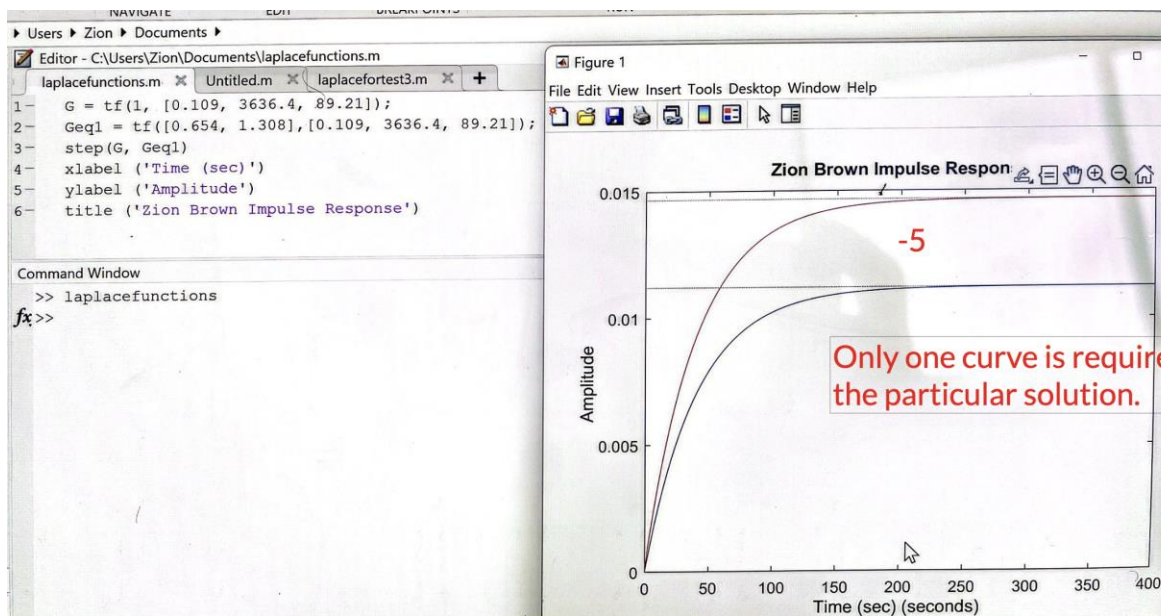
$$a_2 \cdot \ddot{y} + a_1 \cdot \dot{y} + a_0 \cdot y = \delta$$

$$\rightarrow \frac{(12 \times 10^{-6}) \ddot{y} + (0.4) \dot{y} + (\frac{2}{203.8}) y = \delta}{110 \cdot 10^{-6}}$$

input force is a step function not an impulse function.

$$\text{Step input } f = \frac{a_2 \cdot y_0 \cdot s^2 + [a_2 \cdot \dot{y}(0) + a_1 \cdot y(0)] \cdot s + 1}{a_2 \cdot s^2 + a_1 \cdot s + a_0} = \frac{(0.109)(6) \cdot s^2 + [0.109(12) + (3636.4)(0)] \cdot s + 1}{0.109 \cdot s^2 + 3636.4 \cdot s + 89.21}$$

Matlab



Q₃) Split bodies, write equations

Convert time domain to s domain

-10

$$\Sigma F = Ma$$

$$\rightarrow (1) m \ddot{y}_1 = f - K(y_1 - y_2) - C \dot{y}_1 \quad \times$$

$$(2) m \ddot{y}_2 = -K(y_1 - y_2) - K y_2 \quad \times$$

$$(3) f_t = K y_2 \quad \checkmark$$

$$\mathcal{L}[\dot{y}(t)] = s \cdot Y(s) - y(0)$$

$$\mathcal{L}[\ddot{y}(t)] = s^2 \cdot Y(s) - s y(0) - \dot{y}(0)$$

$$(1) m \cdot s^2 \cdot Y_1(s) = F(s) - K(Y_1(s) - Y_2(s)) - C \cdot s Y_1(s)$$

$$(2) m \cdot s^2 \cdot Y_2(s) = K(Y_1(s) - Y_2(s)) - K Y_2(s)$$

$$(3) F_t(s) = K Y_2(s)$$

System of Equations, solve for F(s)

$$(2) m \cdot s^2 \cdot Y_2(s) = K Y_1(s) - K Y_2(s) - K Y_2(s)$$

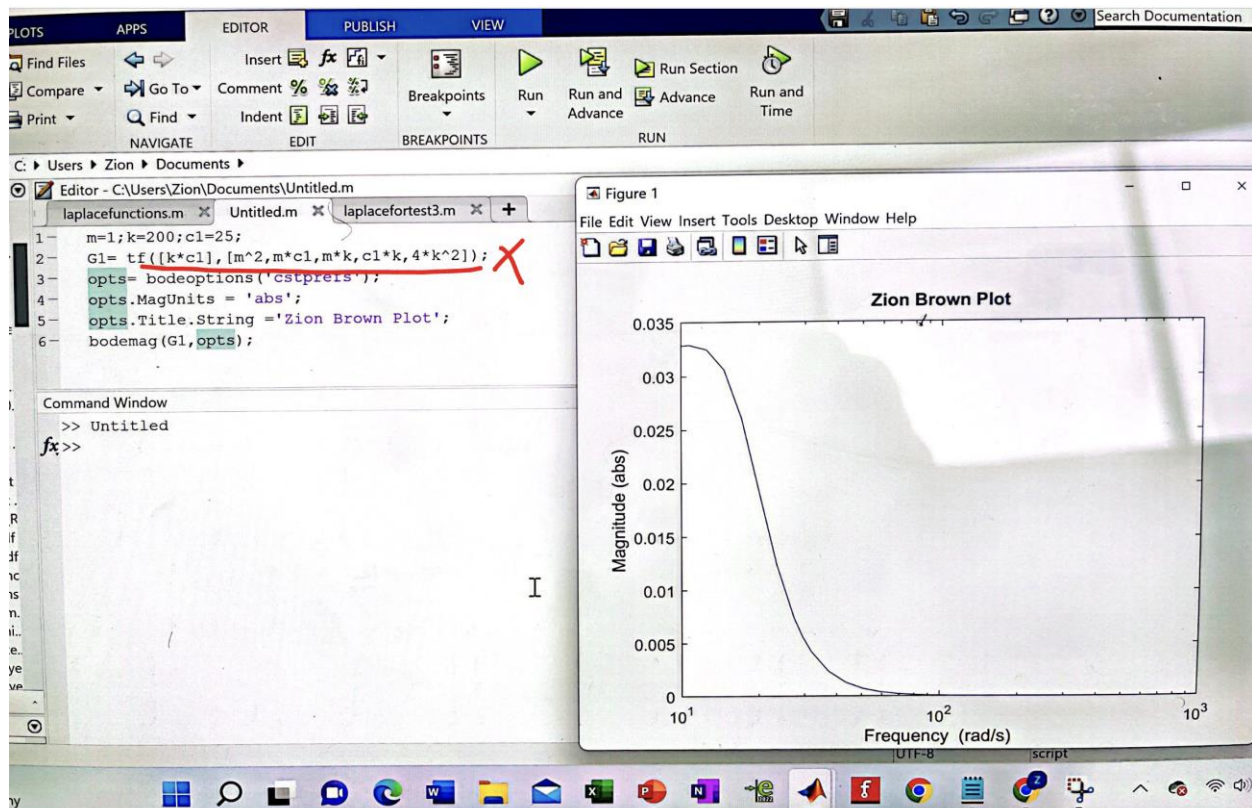
$$\rightarrow \therefore K Y_1(s) = m \cdot s^2 Y_2(s) - 2K Y_2(s)$$

$$(1) m \cdot s^2 \cdot Y_1(s) = F(s) - (m \cdot s^2 Y_2(s) - 2K Y_2(s)) - K Y_2(s) - K Y_2(s)$$

$$\rightarrow \therefore F(s) = m \cdot s^2 Y_1(s) + (m \cdot s^2 Y_2(s) - 2K Y_2(s)) + 2K Y_2(s) \rightarrow$$

$$\rightarrow \frac{F_t(s)}{F(s)} = \frac{\text{output}}{\text{Input}} \rightarrow \text{matlab}$$

$$F(s) \xrightarrow{b} m \cdot s^2 Y_1(s) + m \cdot s^2 Y_2(s) + 4K Y_2(s)$$



BELOW AVERAGE

Editor - FE\ESLQ1.mlx

test3_1.m test3_1.m Untitled2.mlx +

```
%Prob (Test3_1)
%define variables
syms w t
f1 = 3*sin(w^2)*t-4*cos(w^2)*t+5*exp(-w*t);
%calculate laplace
fprintf('Laplace of f1\n\n')
L1 = laplace(f1);
pretty(L1)
|
```

Laplace of f1

$$\frac{3 \sin^2(w)}{s} - \frac{4 \cos^2(w)}{s} + \frac{5}{s + w}$$


Editor - Untitled4.mlx

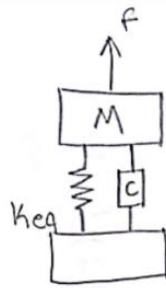
Untitled2.mlx Untitled3.mlx Untitled4.mlx +

```
%define variables
syms b t
f = 5*b*heaviside(t)+16*heaviside(t-1)-b*heaviside(t-3);
%determine laplace
L = laplace(f);
pretty(L)
fprintf(['laplace of multistep function\n\n'])
```

laplace of multistep function

$$\frac{16 \exp(-s)}{s} + \frac{5 b}{s} - \frac{b \exp(-3 s)}{s}$$


2)



$$\sum F = Ma$$

$$F - k_{eq} \cdot y - c \cdot \dot{y} = M \cdot \ddot{y} \quad (1)$$

→ Spring constant

$$k_1 = \frac{12 \cdot E \cdot I}{l_1^3} \rightarrow \frac{12 \cdot 200}{(120)^3} \cdot \frac{\pi (3)^4}{64} = 0.005522$$

$$k_2 = \frac{12 \cdot E \cdot I}{l_2^3} \rightarrow \frac{12 \cdot 200}{(60)^3} \cdot \frac{\pi (3)^4}{64} = 0.44179$$

→ Combine Spring constant

$$\frac{1}{k_{se}} = \frac{1}{k_1} + \frac{1}{k_2} \rightarrow \frac{1}{0.005522} + \frac{1}{0.44179} \rightarrow \frac{1}{k_{se}} = 183.357$$

$$k_{se}(183.357) = 1$$

$$k_{se} = \left(\frac{1}{183.357} \right) \times 2 = \frac{2}{183.357}$$

→ substitute values

$$(1) \quad F - k_{eq} \cdot y - c \cdot \dot{y} = M \cdot \ddot{y}$$

$$F = m \cdot \ddot{y} + c \dot{y} + k_{eq} \cdot y$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$a_2 \cdot \ddot{y} + a_1 \cdot \dot{y} + a_0 \cdot y = \delta F$$

$$\frac{a_2 \cdot \ddot{y} + a_1 \cdot \dot{y} + a_0 \cdot y}{F} = \delta$$

It's a step function, not an impulse function.

$$\frac{(12 \times 10^{-6}) \cdot \ddot{y} + (0.4) \cdot \dot{y} + \left(\frac{2}{183.357} \right) \cdot y}{110 \times 10^{-6}} = \delta$$

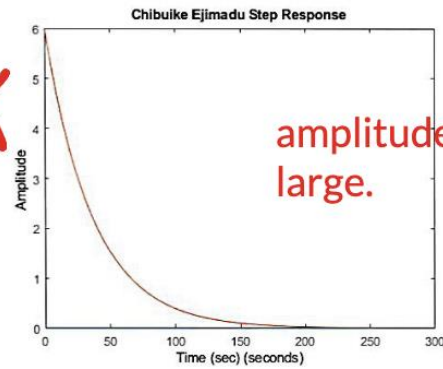
$$0.11 \ddot{y} + 3636.36 \dot{y} + 99.16 y$$

$(a_2) \qquad (a_1) \qquad (a_0)$

→ unit step input

$$\frac{a_2 \cdot y(0) \cdot s^2 + [a_2 \cdot \dot{y}(0) + a_1 \cdot y(0)] + 1}{a_2 \cdot s^2 + a_1 \cdot s + a_0} \rightarrow \text{Next page}$$

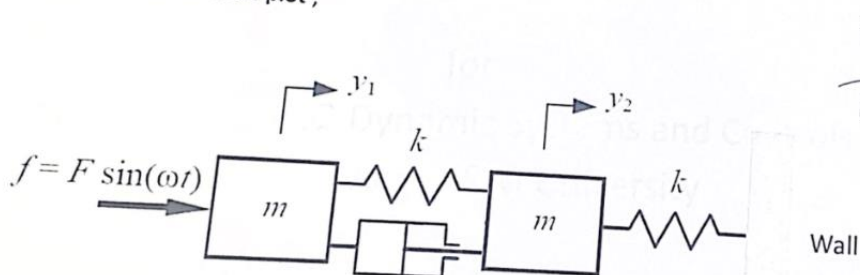
```
G = tf(1, [0.11, 3636.36, 99.16]);
Geq1 = tf([0.66, 21819.48, 1], [0.11, 3636.36, 99.16]);
impz(G, Geq1)
xlabel('Time (sec)')
ylabel('Amplitude')
title('Chibuike Ejimadu Step Response')
```



amplitude is way too large.

40%

3. A sinusoidal force acts on the mechanical system illustrated in the following figure. Considering that $m = 1$ kg, $k = 200$ N/m, and $c = 25$ Ns/m, analyze the transmissibility of the input force to the force on the wall. Make bode plot for Magnitude (absolute value) in MATLAB by adding:
`opts = bodeoptions('cstprefs');`
`opts.MagUnits = 'abs';`
 Include your name in the title by adding:
`opts.Title.String = 'your name bode plot';`



$$\frac{F_t(s)}{F(s)}$$

$$\begin{aligned} \textcircled{1} \quad m \ddot{y}_1 &= F - K(y_1 - y_2) - c \dot{y}_2 \\ \textcircled{2} \quad m \ddot{y}_2 &= K(y_1 - y_2) - c \dot{y}_2 - K y_2 \\ \textcircled{3} \quad F_t &= c \dot{y}_2 + K y_2 \end{aligned}$$

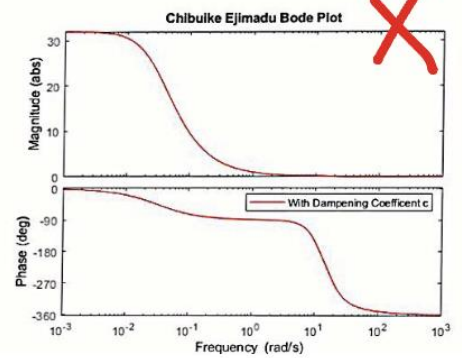
$$\begin{aligned} \mathcal{L}[\dot{y}(t)] &= s \cdot Y(s) - y(0) \\ \mathcal{L}[\ddot{y}(t)] &= s^2 \cdot Y(s) - s y(0) - \dot{y}(0) \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad m \cdot s^2 \cdot Y(s) &= F(s) - K(Y_1(s) - Y_2(s)) - c s Y_2(s) \\ \textcircled{2} \quad m s^2 Y_2(s) &= K(Y_1(s) - Y_2(s)) - c s Y_2(s) - K Y_2(s) \\ \textcircled{3} \quad F_t(s) &= c s Y_2(s) + K Y_2(s) = (c s + K) Y_2(s) \end{aligned}$$

```
opts = bodeoptions('c:\tprefs');
opts.Magunits = 'abs';
opts.Title.String = "Chibuikwe Ejimadu Bode Plot"
m=1; k=200; c=25;
g1= tf([k*c,k^2], [m^2,m*c,3*m*k,c*k,k^2,2*c^2]);
bode(g1, 'n',opts);
legend('With Dampening Coefficient c')
```

opts =

```
FreqUnits: 'auto'
FreqScale: 'log'
MagUnits: 'abs'
MagScale: 'linear'
MagVisible: 'on'
MagLowerLimMode: 'auto'
PhaseUnits: 'deg'
PhaseVisible: 'on'
PhaseWrapping: 'off'
PhaseMatching: 'off'
PhaseMatchingFreq: 0
ConfidenceRegionNumberSD: 1
MagLowerLim: 1
PhaseMatchingValue: 0
PhaseWrappingBranch: -180
IOGrouping: 'none'
InputLabel: [1x1 struct]
OutputLabels: [1x1 struct]
InputVisible: 'on'
OutputVisible: 'on'
Title: [1x1 struct]
XLabel: [1x1 struct]
YLabel: [1x1 struct]
TickLabel: [1x1 struct]
Grid: 'off'
GridColor: [0.1500 0.1500 0.1500]
Xlim: [1 10]
Ylim: [1 10]
XlimMode: 'auto'
YlimMode: 'auto'
```



DYNAMIC SYSTEMS AND CONTROLS SPRING 2022

Title of Assignment: TEST 4

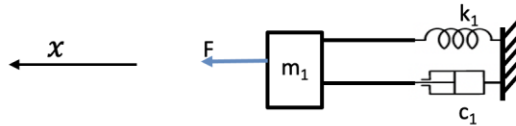
TOPICS COVERED IN TEST 4

1. Use Simulink to simulate a given dynamic system and find out the force transmitted to the wall (supporting structure).
2. Add a PID controller to command an actuator to the dynamic system in the purpose of reducing the force transmitted to the wall.

Test 4
MCEG 4063 Dynamic Systems and Controls
April 25, 2022

A dynamic system can be modeled as a mass, spring, damper system as shown in the schematic diagram. The system vibrates under a sine wave exciter force. The spring and damper are connected to a wall (fixed structure). When the system vibrates, force transmitted to wall = spring force + damping force.

where exciter force $F(t) = 15 \sin(2t)$ N, $M_1 = 5$ kg, $C_1 = 10$ Ns/m, $K_1 = 20$ N/m



(50%)

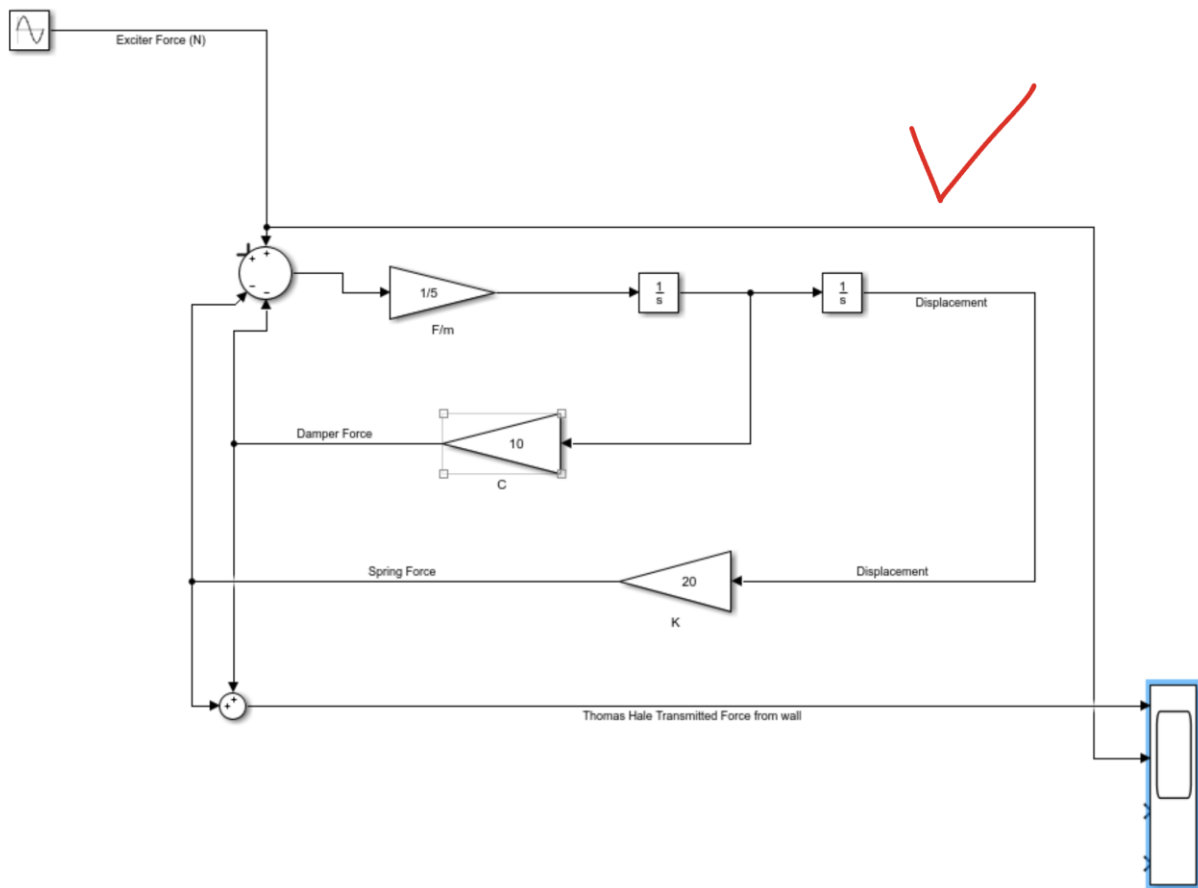
- (a) You are requested to simulate the system using Simulink for 20 sec,
Output to scope: (1) exciter force $F(t)$ and (2) **Your name** force transmitted to wall.

(50%)

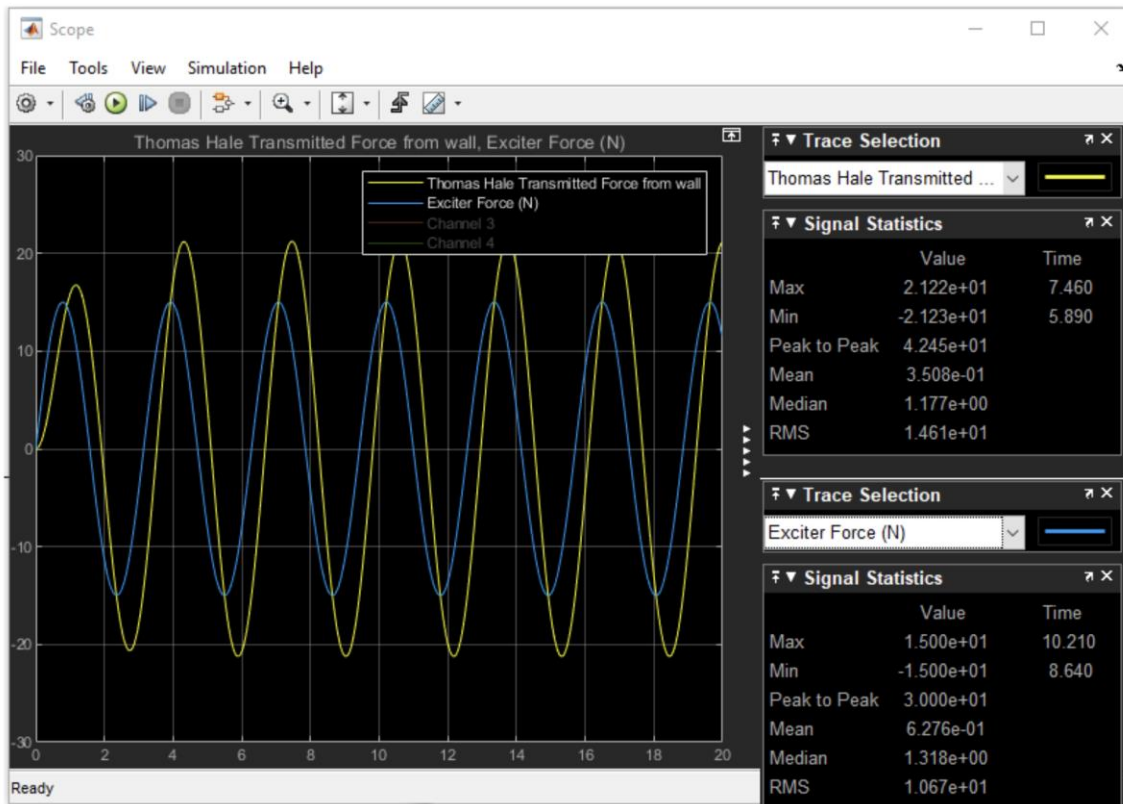
- (b) Add a PID controller to command a linear actuator to provide actuation force to the dynamic system in the purpose of reducing the force transmitted to wall. Simulate the control system which would achieve smaller force transmitted to wall using Simulink for 20 sec, Output to scope: (1) exciter force, (2) **Your name** force transmitted to wall, and (3) **Your name** voltage command from the PID controller.

(Add a summary statement on top of your Simulink output)

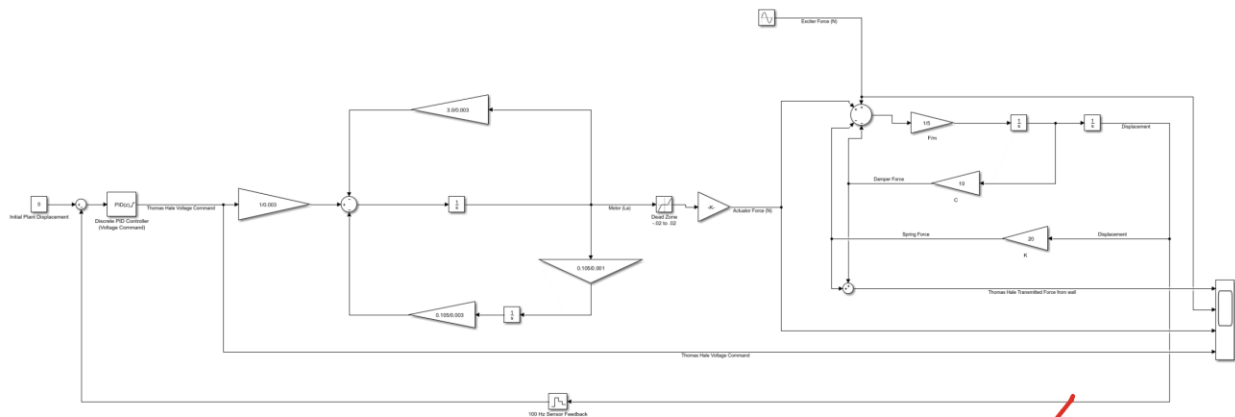
ABOVE AVERAGE



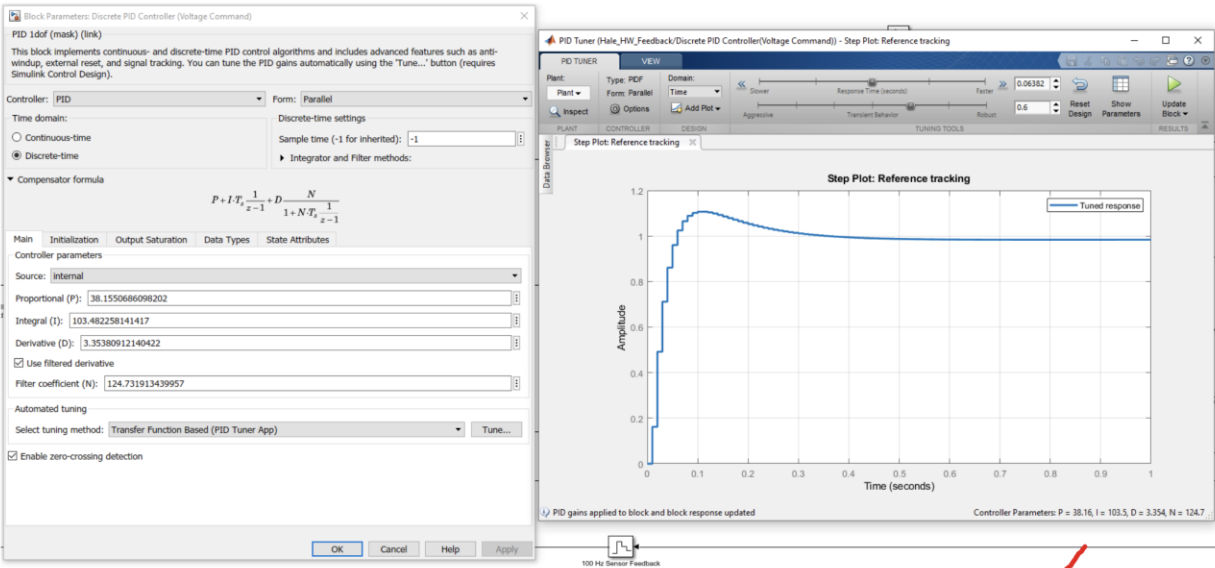
- (a) From the graph, for a time interval of 20 secs, we can see the transmitted force to the wall fluctuates from -21.23N to 21.22N. The exciter force minimum was achieved at 8.640s, while the max was seen at 10.210s.



Full Simulink diagram with PID controller, exciter force, voltage command and transmitted force to wall. Additionally an output from the Actuator was added for the sake of graphical comparison below.

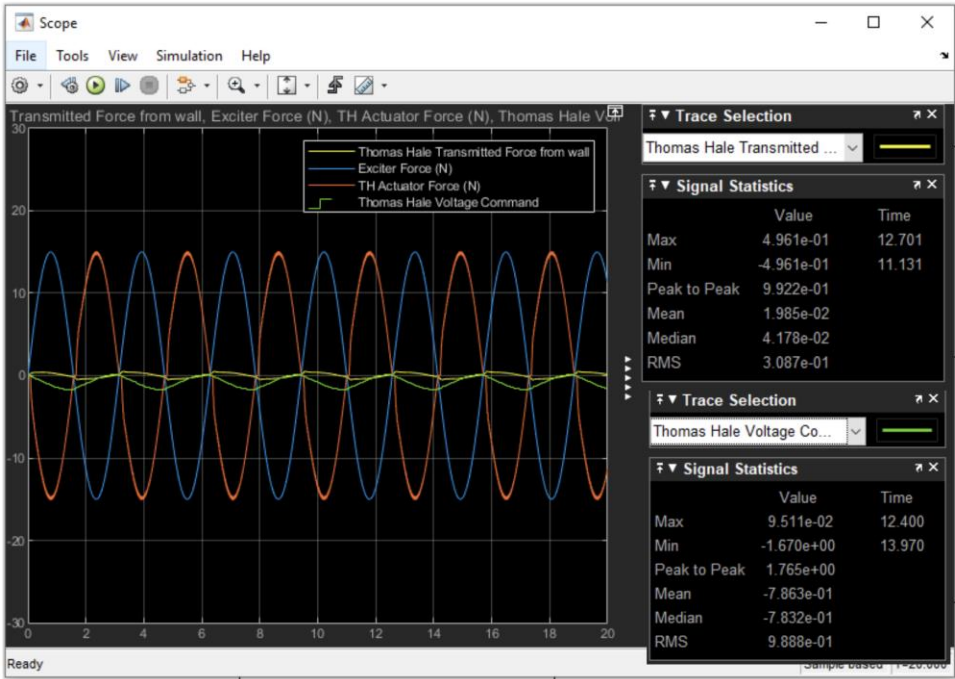


Assuming a 12 V source, the PID is tuned as follows, showing the new values after tuning on left. The step curve on the right, indicating successful tuning.



Analysis:

Adding the PID controller to the system allows of a more regulated vibration through the cycle. As you can see from the output , the force transmitted to the wall has decreased to a range of -.4961 to .4961. This system reduced the overall felt force the wall by 20.73N. This is a huge improvement and could likely save the longevity of equipment it is paired with. Below we can see the graphical representation of the Actuator countering the amplitude of the exciter force.



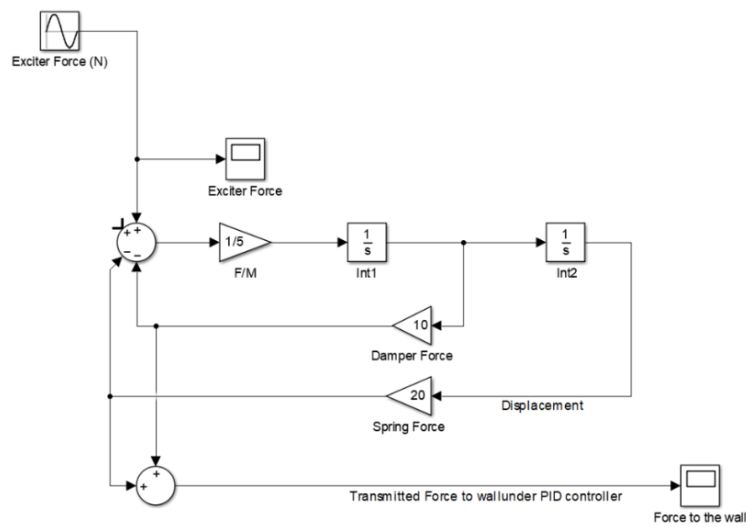
AVERAGE

First of all, let's make a Simulink model with our spring mass damper system were

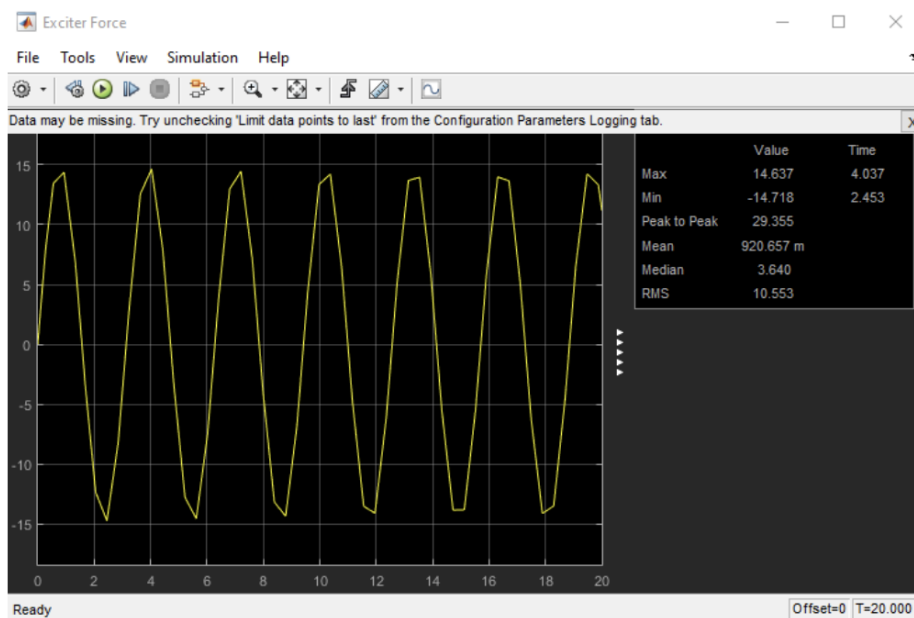
$M=5$, $C=10$, $K=20$

Frequency amplitude = 15

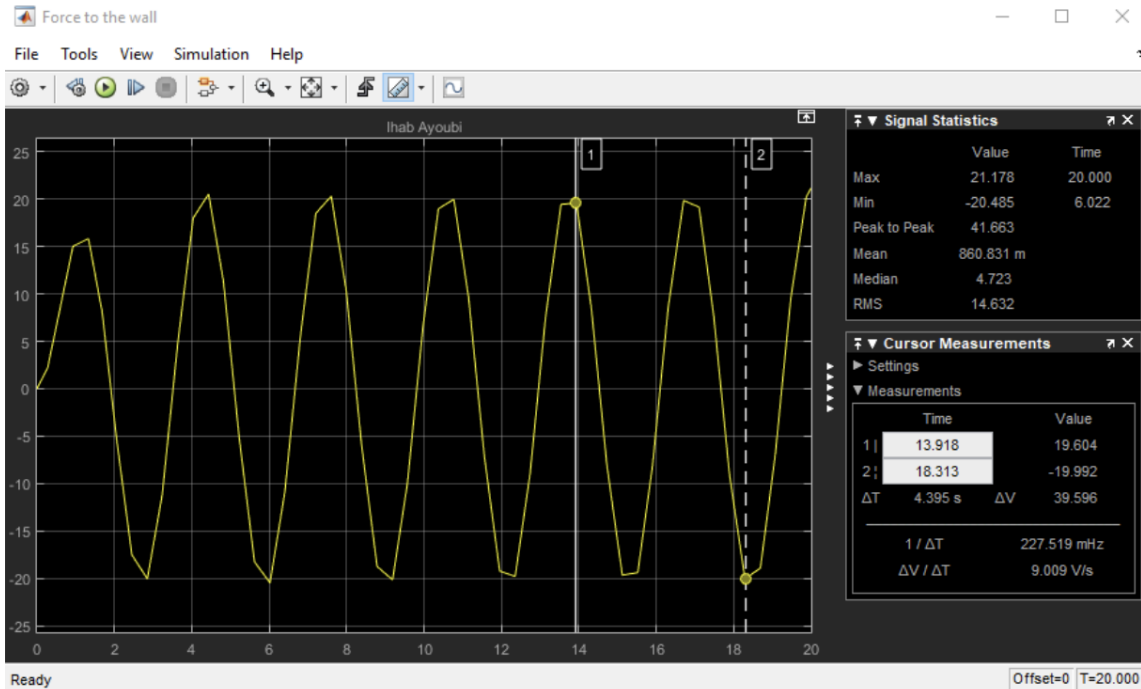
Frequency Time period = 20



After plotting exciter force

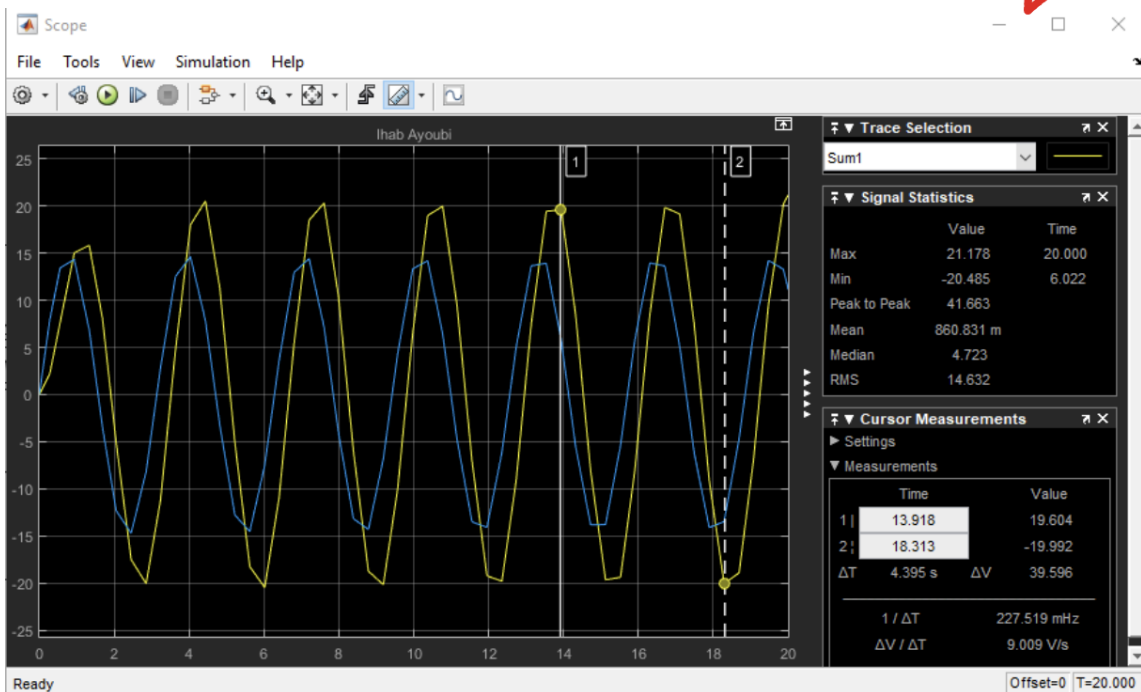


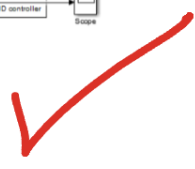
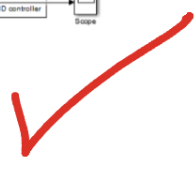
And now force to the wall (With my name as title) we get



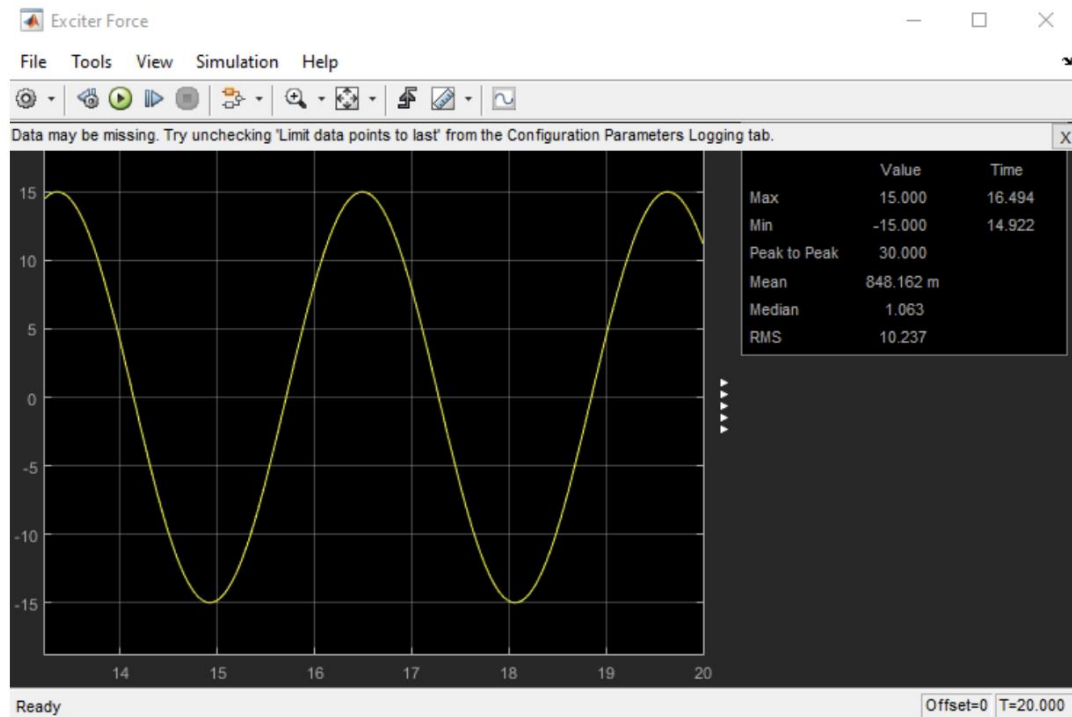
We get max Amplitude of 21.178 as shown in image above.

Combined graph for exciter and force on wall



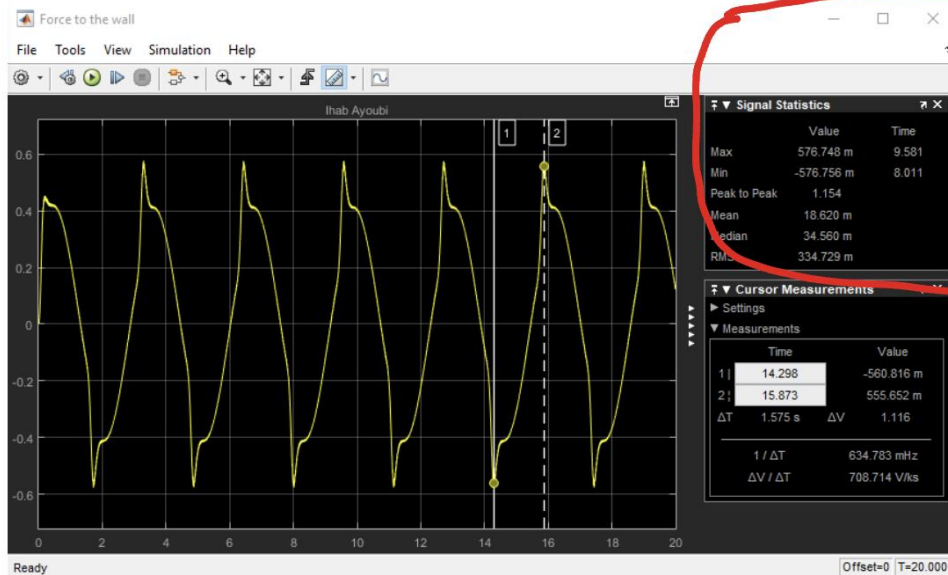


Now plotting exciter force



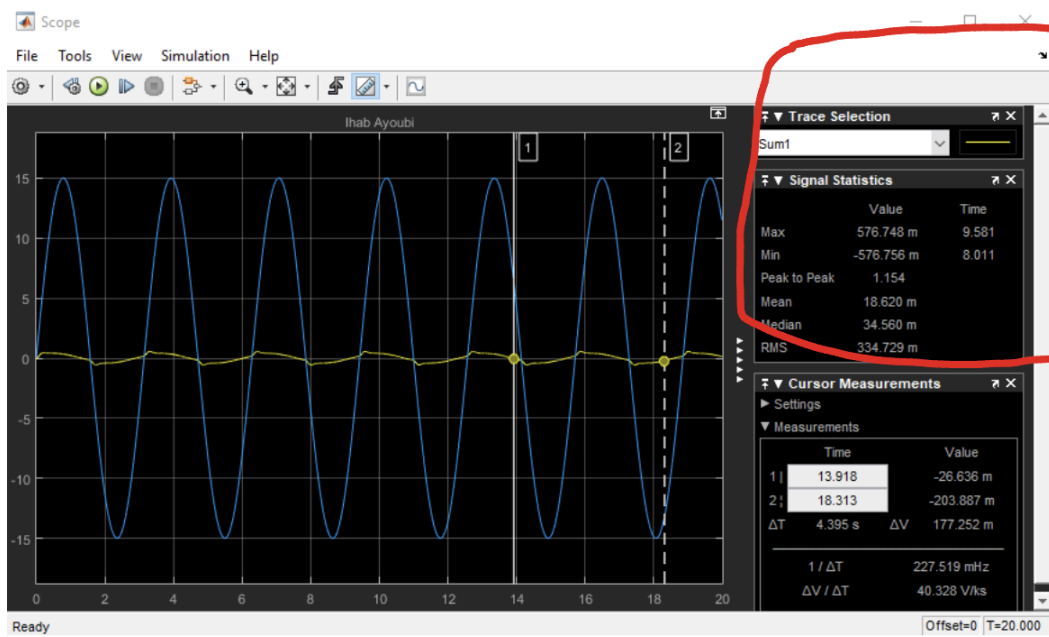
And Force on wall (With my name as title)

the data shows 576 m.



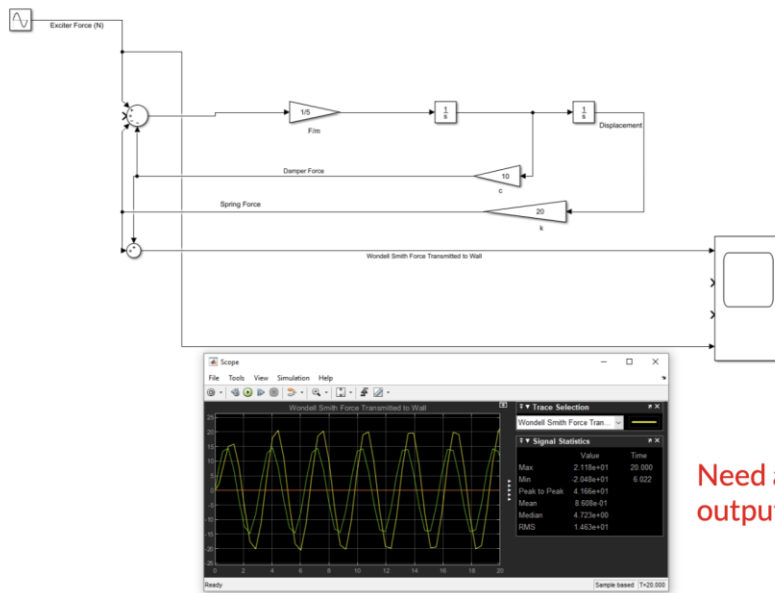
Now Max displacement is 0.576 as compared to 21.178 without controller.

Combined exciter and force on wall graph gives

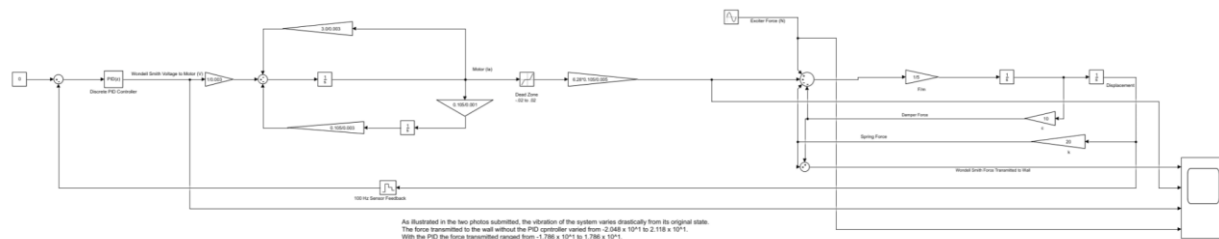


Missing voltage command output.

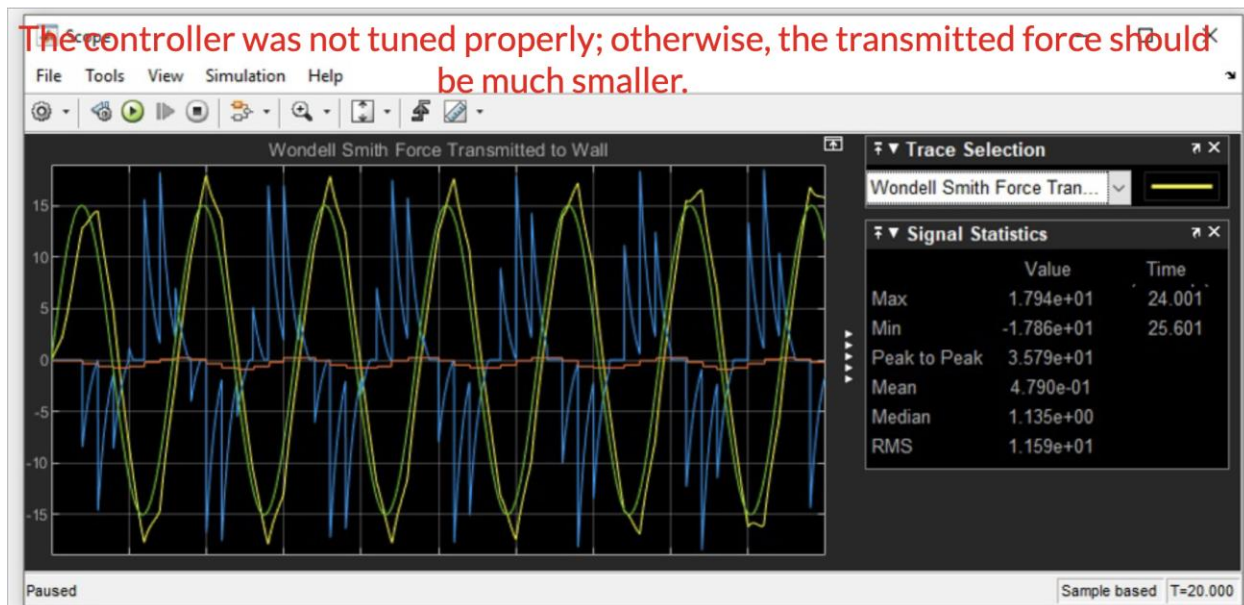
BELOW AVERAGE



Need a summary statement to explain the output parameters.



The controller was not tuned properly; otherwise, the transmitted force should be much smaller.



**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 1

TOPICS COVERED IN HOMEWORK 1

1. Equivalent Stiffness of Mechanical System

PROBLEMS

- 2.1 Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$, $k_4 = 25 \text{ N/m}$, and $k_5 = 35 \text{ N/m}$, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.

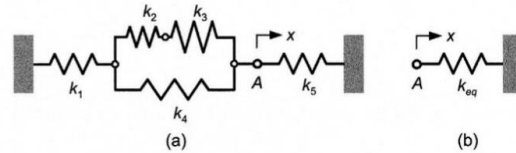


FIGURE 2.35

- 2.4 Two identical elastic shaft segments are connected to a rigid disk acted on by a moment m , as illustrated in Figure 2.38(a). To reduce the rotation angle at O by 40%, the design of Figure 2.38(b) is used instead of the original one. Assuming all shaft segments are built from the same material and have constant circular cross sections, express the diameter d_2 in terms of the diameter d_1 , and the lengths l_1 and l_2 . Also calculate the value of d_2 for $d_1 = 0.012 \text{ m}$, $l_1 = 0.24 \text{ m}$, and $l_2 = 0.16 \text{ m}$.

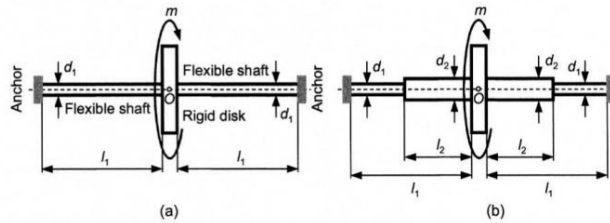


FIGURE 2.38

Fixed-Fixed Shaft With: (a) Two Identical Segments; (b) Two Pairs of Identical Segments.

- 2.7 Determine the equivalent lumped-parameter spring model of the six-beam microdevice sketched in Figure 2.41, and evaluate the stiffness of the single equivalent spring to be attached to the moving rigid link along the y direction. All beams have identical square cross section with side length $h = w = 1 \mu\text{m}$. Known also are $l_1 = 100 \mu\text{m}$, $l_2 = 140 \mu\text{m}$, $l_3 = 200 \mu\text{m}$, and the beams Young's modulus $E = 155 \text{ GPa}$. The allowable displacement of the moving link is $10 \mu\text{m}$. Calculate the corresponding force f_y applied at the moving link center along the y direction.

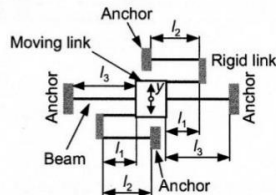


FIGURE 2.41

Microdevice With Six Beams for Translatory Motion.

ABOVE AVERAGE

2.1

MCEG 4306

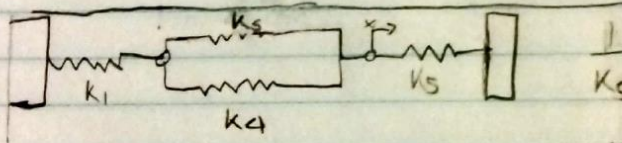
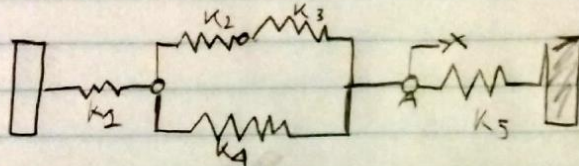
Known: $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$,
 $k_4 = 25 \text{ N/m}$, $k_5 = 35 \text{ N/m}$

Find equivalent stiffness of spring ensemble

Assumptions: The Spring ensemble is overall parallel.

Analysis Laws: $k_s = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \dots}$, $k_p = k_1 + k_2 + k_3$

Schematic / Solution:

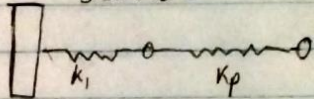


$$\frac{1}{k_s} = \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{30} + \frac{1}{15}$$

$$\frac{1}{k_s} = \frac{1}{10} \rightarrow k_s = 10 \text{ N/m}$$

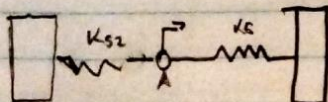
$$k_p = k_s + k_4 = 10 \text{ N/m} + 25 \text{ N/m} = 35 \text{ N/m} = k_p$$

series



$$\frac{1}{k_{s2}} = \frac{1}{k_1} + \frac{1}{k_p} = \frac{1}{20 \text{ N/m}} + \frac{1}{35 \text{ N/m}} = \frac{11}{140}$$

$$\frac{1}{k_{s2}} = \frac{11}{140} \rightarrow k_{s2} = 12.727 \text{ N/m}$$



$$k_{eq} = k_{s2} + k_5$$

$$k_{eq} = 12.727 \text{ N/m} + 35 \text{ N/m}$$

$$k_{eq} = 47.727 \text{ N/m}$$

2.4

Pg 1

Knowns: Figure 2.39(b) the angle of rotation is reduced to 60%, $d_1 = 0.012\text{m}$, $L_1 = 0.24\text{m}$, $L_2 = 0.16\text{m}$

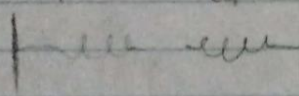
Express d_2 in terms of d_1 , L_1 , L_2 . Find the value of d_2 .

Analysis:

$$k_r = \frac{G \cdot I_p}{L} \quad I_p = \frac{\pi d^4}{32}$$

Solution: $m_e = k_r \cdot \Delta\theta$
 $k_{eq} \theta = k_{eq}^* (60\% \cdot \theta)$

Original configuration



$$k_{eq} = 2k_1 = 2 \frac{G I_p}{L} = \frac{2 G \pi d_1^4}{32 L_1}$$

2nd reinforced configuration

$$\frac{1}{k_s} = \frac{1}{k_s} = \frac{1}{k_1^*} + \frac{1}{k_2}$$

$$\frac{1}{k_s} = \frac{1(k_2)}{(k_1^*)(k_2)} + \frac{1(k_1^*)}{(k_1^*)(k_2)}$$

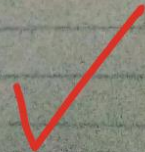
$$\frac{1}{k_s} = \frac{k_2 + k_1^*}{k_1^* k_2} \rightarrow k_s = \frac{(k_1^*)(k_2)}{k_2 + k_1^*}$$

$$k_{eq} = 2k_s = \frac{2 k_1^* k_2}{k_2 + k_1^*}$$

$$k_1^* = \frac{G \cdot I_p}{(L_1 - L_2)} = \frac{G \pi d_1^4}{32(L_1 - L_2)}$$

$$k_2 = \frac{G \cdot I_{p2}}{L_2} = \frac{G \pi d_2^4}{32 L_2}$$

$$k_{eq}^* = 2 \frac{k_1^* k_2}{(k_1^* + k_2)} = 2 \left(\frac{\frac{G \pi d_1^4}{32(L_1 - L_2)}}{\frac{G \pi d_1^4}{32(L_1 - L_2)} + \frac{G \pi d_2^4}{32 L_2}} \right) \left(\frac{G \pi d_2^4}{32 L_2} \right)$$



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2.4 continued

$$k_{eq}^* = \frac{G\pi}{32} \left(\frac{d_1^4}{l_1 - l_2} \right) \left(\frac{G\pi d_2^4}{32 l_2} \right) = (2) \frac{\frac{d_1^4}{(l_1 - l_2)} \left(\frac{G\pi d_2^4}{32 l_2} \right)}{\frac{G\pi}{32} \left(\frac{d_1^4}{(l_1 - l_2)} + \frac{d_2^4}{l_2} \right)}$$

$$= 2 \cdot \frac{(l_1 - l_2)(l_2)}{l_2 d_1^4 + \frac{d_2^4 (l_1 - l_2)}{(l_1 - l_2)(l_2)}} = \frac{2(\pi G/32) (d_1^4 d_2^4)}{d_1^4 l_2 + d_2^4 (l_1 - l_2)}$$

$$k_{eq}^* = \frac{2(\pi G/32) (d_1^4 d_2^4)}{d_1^4 l_2 + (l_1 - l_2) d_2^4}$$

$$k_{eq} = 0.6 k_{eq}^*$$

$$\frac{2\pi G d_1^4}{32 l_1} = 0.6 \left(\frac{2(\pi G/32) d_1^4 d_2^4}{d_1^4 l_2 + (l_1 - l_2) d_2^4} \right)$$

$$\frac{1}{l_1} = \frac{0.6 d_2^4}{d_1^4 l_2 + (l_1 - l_2) d_2^4}$$

$$l_1 = \frac{d_1^4 l_2 + (l_1 - l_2) d_2^4}{0.6 d_2^4}$$

$$0.6 d_2^4 l_1 = d_1^4 l_2 + (l_1 - l_2) d_2^4$$

$$0.6 d_2^4 l_1 - (l_1 - l_2) d_2^4 = d_1^4 l_2$$

$$d_2^4 (0.6 l_1 - l_1 + l_2) = d_1^4 l_2$$

$$d_2^4 (0.4 l_1 + l_2) = d_1^4 l_2$$

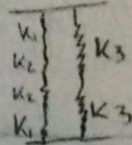
$$d_2^4 = \frac{d_1^4 l_2}{l_2 - 0.4 l_1}$$

$$d_2^4 = \frac{l_2 d_1^4}{l_2 - 0.4 l_1}$$

$$d_2 = \sqrt[4]{\frac{L_2 d_1^4}{L_2 - 0.4 L_1}} = \sqrt[4]{\frac{(0.16\text{m})(0.012\text{m})^4}{(0.16\text{m} - 0.4(0.24\text{m}))}}$$

$$\boxed{d_2 = 0.015089 \text{ m}}$$


2.7 Find the equivalent lumped parameter, k_{eq} , for applied at the moving center



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k_1 and k_2 are in series.
 k_3 is parallel.

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \rightarrow \frac{k_2}{k_1 k_2} + \frac{k_1}{k_1 k_2}$$

$$\frac{1}{k_s} = \frac{k_2 + k_1}{k_1 k_2} \rightarrow k_s = \frac{k_1 k_2}{k_2 + k_1}$$

$$k_p = k_3$$

$$k_{eq} = 2(k_s + k_p) = 2\left(\frac{k_1 k_2}{k_2 + k_1} + k_3\right) = k_{eq}$$

Finding k_1, k_2, k_3

Knowns: $h = w = 1 \mu m$, $L_1 = 100 \mu m$, $L_2 = 140 \mu m$, $L_3 = 200 \mu m$, $E = 155 GPa$, $\Delta y = 10 \mu m$

Analysis:

$$k_t = \frac{12 E I_z}{L^3}$$

$$I_z = \frac{w h^3}{12}$$

$$k_t = \frac{12 E \left(\frac{w h^3}{12}\right)}{L^3}$$

note: $w = h \rightarrow \frac{E h^4}{L^3} = k$

$$k_1 = \frac{E h^4}{L_1^3} = \frac{(155 \times 10^9 Pa) (1 \times 10^{-6} m)^4}{(100 \times 10^{-6} m)^3} = 0.155 Pa \cdot m^4/m^3$$

$$Pa = 1 N/m^2$$

$$k_1 = 0.155 (N/m^2) (m^4)/m^3 = 0.155 N \cdot m^2/m^3$$

$$k_1 = 0.155 N/m$$

$$k_2 = \frac{E h^4}{L_2^3} = \frac{(155 \times 10^9 Pa) (1 \times 10^{-6} m)^4}{(140 \times 10^{-6} m)^3} = 0.0565 N/m$$

$$k_3 = \frac{E h^4}{L_3^3} = \frac{(155 \times 10^9 Pa) (1 \times 10^{-6} m)^4}{(200 \times 10^{-6} m)^3} = 0.0194 N/m$$

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2.7 (page 2 of 2 of 2.7)

$$k_{eq} = 2 \left(\frac{k_1 k_2}{k_2 + k_1} + k_3 \right)$$

$$k_{eq} = \left(\frac{(0.155 \text{ N/m})(0.0565 \text{ N/m})}{(0.0565) + (0.155 \text{ N/m})} + 0.0194 \text{ N/m} \right)$$

$$k_{eq} = 0.0414 \text{ N/m} + 0.0194 \text{ N/m}$$
$$k_{eq} = 0.1216 \text{ N/m}$$

$$f_y = k_{eq} \times \Delta y$$

$$f_y = (0.1216 \text{ N/m})(10 \text{ } \mu\text{m})$$

$$f_y = \boxed{1.216 \text{ } \mu\text{N}}$$

AVERAGE

PROBLEMS

- 2.1 Several springs are connected as illustrated in Figure 2.35(a). Knowing the individual springs stiffnesses $k_1 = 20 \text{ N/m}$, $k_2 = 30 \text{ N/m}$, $k_3 = 15 \text{ N/m}$, $k_4 = 25 \text{ N/m}$, and $k_5 = 35 \text{ N/m}$, calculate the equivalent stiffness k_{eq} of this spring ensemble, see Figure 2.35(b), which corresponds to point A of the original chain.

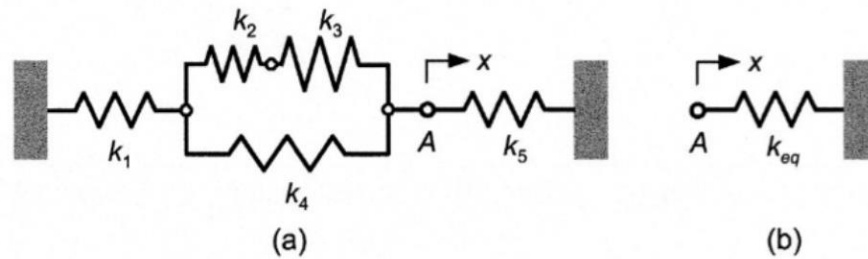
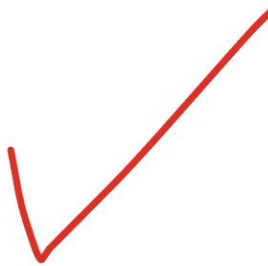


FIGURE 2.35

$$\begin{aligned}
 k_{eq} &= \frac{k_1 \cdot \left(\frac{k_2 \cdot k_3}{k_2 + k_3} + k_4 \right)}{k_1 + \frac{k_2 \cdot k_3}{k_2 + k_3} + k_4} + k_5 \\
 &= \frac{(20) \left(\frac{(30)(15)}{30 + 15} + 25 \right)}{20 + \frac{(30)(15)}{30 + 15} + 25} + 35 \text{ N/m}
 \end{aligned}$$

$$= 47.727 \text{ N/m}$$



- 2.4 Two identical elastic shaft segments are connected to a rigid disk acted on by a moment m , as illustrated in Figure 2.38(a). To reduce the rotation angle at O by 40%, the design of Figure 2.38(b) is used instead of the original one. Assuming all shaft segments are built from the same material and have constant circular cross sections, express the diameter d_2 in terms of the diameter d_1 , and the lengths l_1 and l_2 . Also calculate the value of d_2 for $d_1 = 0.012$ m, $l_1 = 0.24$ m, and $l_2 = 0.16$ m.

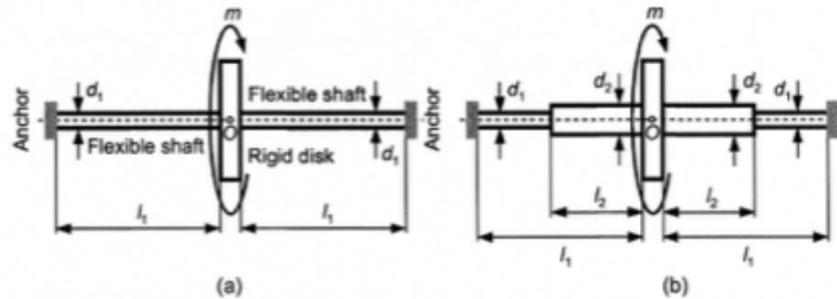


FIGURE 2.38

Fixed-Fixed Shaft With: (a) Two Identical Segments; (b) Two Pairs of Identical Segments.

TORSION EQUATION:

$$\frac{T}{J} = \frac{G\theta}{l}$$

T = torque
 J = polar moment of inertia ($\frac{\pi}{32}d^4$)
 G = modulus of rigidity
 θ = angle of twist
 l = length of shaft

$$\theta_a = \frac{Tl_1}{GJ_1}$$

* REDUCE ROTATION BY 40%.

$$\theta_a(1 - 0.4) = \theta_b$$

$$0.6\theta_a = \theta_b$$

$$\theta_b = \frac{T(l_1 - l_2)}{GJ_1} + \frac{Tl_2}{GJ_2}$$

$$\theta_b = \frac{T}{G} \left[\frac{l_1 - l_2}{J_1} + \frac{l_2}{J_2} \right]$$

$$0.6 \left(\frac{Tl_1}{GJ_1} \right) = \frac{T}{G} \left[\frac{l_1 - l_2}{J_1} + \frac{l_2}{J_2} \right] \quad J = \frac{\pi}{32}d^4$$

$$(0.6) \left(\frac{l_1}{\frac{\pi}{32}d_1^4} \right) = \frac{l_1 - l_2}{\frac{\pi}{32}d_1^4} + \frac{l_2}{\frac{\pi}{32}d_2^4}$$

$$0.6 \frac{l_1}{d_1^4} = \frac{l_1 - l_2}{d_1^4} + \frac{l_2}{d_2^4}$$

$$\frac{-l_2}{d_2^4} = \frac{l_1 - l_2}{d_1^4} - 0.6 \frac{l_1}{d_1^4} = \frac{1}{d_1^4} [l_1 - l_2 - 0.6l_1]$$

$$\frac{l_2}{d_2^4} = \frac{1}{d_1^4} [0.4l_1 - l_2]$$

$$d_2^4 = \frac{l_2 d_1^4}{[0.4l_1 + l_2]} = \frac{l_2 d_1^4}{[l_2 - 0.4l_1]}$$

$$d_2 = d_1 \left[\frac{l_2}{-0.4l_1 + l_2} \right]^{1/4}$$

$$d_2 = 0.012 \left[\frac{0.16}{-0.4(0.24) + 0.16} \right]^{1/4}$$

$$d_2 = 0.015089 \text{ m}$$



- 2.7 Determine the equivalent lumped-parameter spring model of the six-beam microdevice sketched in Figure 2.41, and evaluate the stiffness of the single equivalent spring to be attached to the moving rigid link along the y direction. All beams have identical square cross section with side length $h = w = 1 \mu\text{m}$. Known also are $l_1 = 100 \mu\text{m}$, $l_2 = 140 \mu\text{m}$, $l_3 = 200 \mu\text{m}$, and the beams Young's modulus $E = 155 \text{ GPa}$. The allowable displacement of the moving link is $10 \mu\text{m}$. Calculate the corresponding force f_y applied at the moving link center along the y direction.

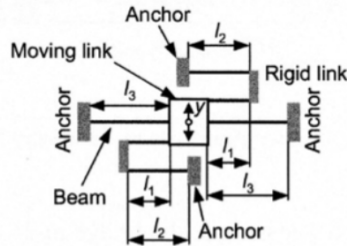


FIGURE 2.41

Microdevice With Six Beams for Translatory Motion.

$$\begin{aligned}
 h = w &= 1 \mu\text{m} = 1 \times 10^{-3} \text{ mm} \\
 l_1 &= 100 \mu\text{m} = 100 \times 10^{-3} \text{ mm} \\
 l_2 &= 140 \mu\text{m} = 140 \times 10^{-3} \text{ mm} \\
 l_3 &= 200 \mu\text{m} = 200 \times 10^{-3} \text{ mm} \\
 E &= 155 \text{ GPa} = 155 \times 10^3 \text{ N/mm}^2 \\
 y &= 10 \mu\text{m} = 10 \times 10^{-3} \text{ mm}
 \end{aligned}$$

$$y = \frac{\omega L^3}{48EI}$$

$$L = l_3 = (200 \times 10^{-3} \text{ mm})^2 = 400 \times 10^{-3} \text{ mm}$$

$$\omega = f_y$$

$$I = \frac{hw^3}{12} = \frac{(1)(1)^3}{12} \times [10^{-3}]^4 \text{ mm} = \frac{1}{12} \times [10^{-3}]^4 \text{ mm}$$

$$E = 155 \times 10^3 \text{ N/mm}^2$$

$$y = 10 \times 10^{-3} \text{ mm}$$

$$10 \times 10^{-3} = \frac{f_y [400 \times 10^{-3}]^3}{48 (155 \times 10^3) (\frac{1}{12} \times 10^{-12})}$$

$$10^{-2} = \frac{f_y (0.4)^3}{6.2 \times 10^{-7}}$$

$$\therefore f_y = 9.78 \times 10^{-8} \text{ N}$$

BELOW AVERAGE

MCEG 4306 - P02

Problem 2.1

$$k_g = k_{1234} + k_5 = \frac{k_1 \left(\frac{l_2 \cdot k_3}{l_2 + k_3} + 1 \right) + k_5}{k_1 + \frac{l_2 \cdot k_3}{l_2 + k_3} + k_5} = 47.707 \text{ N/m}$$

Problem 2.4

$$k_g = 2k_1 = 2 \cdot \frac{\pi G d_1^4}{32 l_1}$$

$$k_1 = \frac{\pi G d_1^4}{32 (1 - l_2)} ; k_2 = \frac{\pi G d_2^4}{32 l_2}$$

$$\beta^* = 0.6 \cdot 0 ; \beta^* = r / k_g^* ; \beta = r / k_g$$
$$\therefore k_g = 0.6 k_g^*$$

$$\beta = \sqrt[4]{\frac{l_2}{I_2 - 0.9 \cdot l_1}} \cdot d_1 = 0.015 \text{ m}$$

Need show more calculation process.

Problem 2.7

$$k_{eq} = 2 \left(\frac{k_1 k_2}{k_1 + k_2} + k_3 \right)$$

$$k_i = \frac{12EI_z}{L^3} = \frac{EI_z}{L^3}; \quad i=1, 2, 3$$

$$k_{eq} = 0.1215 \text{ N/m}$$

$$k_{eq} \cdot y_a = 1.2155 \mu\text{N}$$

More details please.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 2

TOPICS COVERED IN HOMEWORK 2

1. Calculate Viscous Damping Coefficient
2. Calculate Torsional Stiffness
3. Calculate Equivalent Mass Moment of Inertia

- 2.19** Figure 2.51 shows a translatory-motion bearing. It is formed of a rectangular cross-section rod placed symmetrically in and translating inside a similar box filled with viscous fluid. Calculate the total damping coefficient based on the geometric parameters shown in Figure 2.51 and the fluid dynamic viscosity μ .

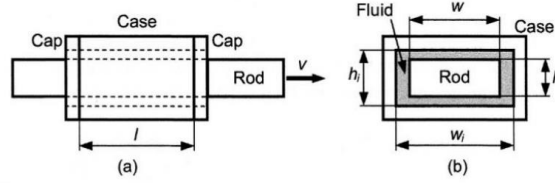


FIGURE 2.51

Translatory Bearing System as a Viscous Damper: (a) Side View; (b) Cross-Section.

- 2.24** Determine the equivalent torsional stiffness transferred to the intermediate rigid shaft axis of the gear transmission sketched in Figure 2.55. All other shafts are flexible; they are constructed from the same material with $G = 90$ GPa N/m² and are defined by $d_1 = 0.005$ m, $l_1 = 0.15$ m, $d_2 = 0.004$ m, $l_2 = 0.2$ m, $d_3 = 0.0045$ m, $l_3 = 0.1$ m, $d_4 = 0.006$ m, and $l_4 = 0.12$ m. Known are also the number of teeth of the gears $N_1 = 36$, $N_2 = 44$, $N_3 = 32$, and $N_4 = 48$.

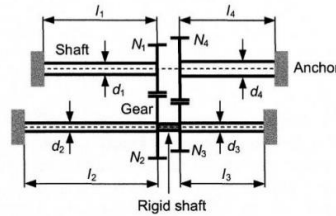


FIGURE 2.55

Rotary-Motion Gear Transmission With Flexible Shafts.

- 2.25** Three rigid shafts mesh through their respective gears as illustrated in Figure 2.56. Calculate the total mass moment of inertia of this system transferred to the middle shaft. The shafts have a mass density $\rho = 7800$ kg/m³ and are defined by $d_1 = 0.012$ m, $l_1 = 0.15$ m, $d_2 = 0.01$ m, $l_2 = 0.1$ m, $d_3 = 0.009$ m, and $l_3 = 0.13$ m. The following parameters define the four gears: the number of teeth $N_1 = 18$, $N_2 = 22$, $N_3 = 28$, $N_4 = 14$, their mass moments of inertia $J_1 = 6 \cdot 10^{-4}$ kg m², $J_2 = 8 \cdot 10^{-4}$ kg m², $J_3 = 9 \cdot 10^{-4}$ kg m², $J_4 = 4 \cdot 10^{-4}$ kg m², as well as the mass moments of inertia of the two end flywheels $J_{w1} = 1 \cdot 10^{-3}$ kg m² and $J_{w2} = 9 \cdot 10^{-4}$ kg m².

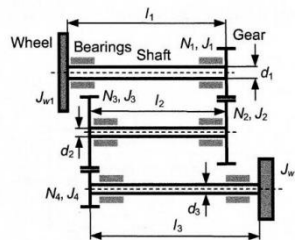


FIGURE 2.56

Rotary-Motion Gear Transmission With Heavy, Rigid Shafts.

ABOVE AVERAGE

2.24

Find the equivalent torsional stiffness transferred to the intermediate rigid shaft axis

Knowns: $d_1, L_1, d_2, L_2, d_3, L_3, d_4, L_4, N_1, N_2, N_3, N_4, G$

$$K_1 = \frac{GJ_1}{L_1} = \frac{G\pi d_1^4}{32L_1} = \frac{(90 \times 10^9 \text{ N/m}^2) \pi (0.005 \text{ m})^4}{32(0.15 \text{ m})}$$

$$K_1 = 36.81 \text{ N.m}$$

$$K_2 = \frac{G\pi d_2^4}{32L_2} = \frac{(90 \times 10^9 \text{ N/m}^2) \pi (0.004 \text{ m})^4}{32(0.2 \text{ m})}$$

$$K_2 = 11.31 \text{ N.m}$$

$$K_3 = \frac{G\pi d_3^4}{32L_3} = \frac{(90 \times 10^9 \text{ N/m}^2) \pi (0.0045 \text{ m})^4}{32(0.1 \text{ m})}$$

$$K_3 = 36.23 \text{ N.m}$$

$$K_4 = \frac{G\pi d_4^4}{32L_4} = \frac{(90 \times 10^9 \text{ N/m}^2) \pi (0.006 \text{ m})^4}{32(0.12 \text{ m})}$$

$$K_4 = 95.426 \text{ N.m}$$

$$K_{eq1} = K_1 \left(\frac{N_2}{N_1} \right)^2 = (36.81 \text{ N.m}) \left(\frac{44}{36} \right)^2 = 55.03 \text{ N.m}$$

$$K_{eq4} = K_4 \left(\frac{N_3}{N_4} \right)^2 = (95.426 \text{ N.m}) \left(\frac{32}{48} \right)^2 = 42.41 \text{ N.m}$$

$$K_{eq} = K_{eq1} + K_{eq4} + K_2 + K_3$$

$$K_{eq} = 144.984 \text{ N.m}$$

2.25

Calculate the total mass moment of inertia of this system transferred to the middle shaft.

knowns: $\rho_{\text{shaft}} = 7800 \text{ kg/m}^3$, $d_1 = 0.012 \text{ m}$, $L_1 = 0.15 \text{ m}$,
 $d_2 = 0.01 \text{ m}$, $L_2 = 0.1 \text{ m}$, $d_3 = 0.009 \text{ m}$, $L_3 = 0.13 \text{ m}$,
 $N_1, N_2, N_3, N_4, J_1, J_2, J_3, J_4, J_{w1}, J_{w2}$

Unknowns: masses and J_x of top shaft, middle shaft, and bottom shafts

Solution:

$$m_{ts} = \rho_s L_1 \frac{\pi d_1^2}{4} = (7800 \text{ kg/m}^3) (0.15 \text{ m}) (\pi) \left(\frac{(0.012 \text{ m})^2}{4} \right)$$

$$m_{ts} = 0.132 \text{ kg}$$

$$m_{ms} = \rho_s L_2 \frac{\pi d_2^2}{4} = (7800 \text{ kg/m}^3) (0.1 \text{ m}) (\pi) \left(\frac{(0.01)^2}{4} \right)$$

$$m_{ms} = 0.06126 \text{ kg}$$

$$m_{bs} = \rho_s L_3 \frac{\pi d_3^2}{4} = (7800 \text{ kg/m}^3) (0.13 \text{ m}) (\pi) \left(\frac{(0.009 \text{ m})^2}{4} \right)$$

$$m_{bs} = 0.0645 \text{ kg}$$

Total mass moment of inertia on Top shaft

$$\bar{J}_t = \bar{J}_{w1} + \bar{J}_1 + \frac{1}{2} m_{ts} \left(\frac{d_1}{2} \right)^2$$

$$\bar{J}_t = (1 \times 10^{-3} \text{ kg m}^2) + (6 \cdot 10^{-4} \text{ kg m}^2) + \frac{1}{2} (0.132 \text{ kg}) \left(\frac{0.012 \text{ m}}{2} \right)^2$$

$$\bar{J}_t = 0.0016023 \text{ kg m}^2$$

Total mass moment of inertia on the middle shaft.

$$\bar{J}_{ms} = \bar{J}_2 + \bar{J}_3 + \frac{1}{2} m_{ms} \left(\frac{d_2}{2} \right)^2$$

$$\bar{J}_{ms} = (8 \cdot 10^{-4} \text{ kg m}^2) + (9 \cdot 10^{-4} \text{ kg m}^2) + \frac{1}{2} (0.06126 \text{ kg}) \left(\frac{0.01 \text{ m}}{2} \right)^2$$

$$\bar{J}_{ms} = 0.0017 \text{ kg m}^2$$

Total mass moment of inertia on the bottom shaft

$$\bar{J}_{bs} = \bar{J}_{w2} + \bar{J}_4 + \frac{1}{2} m_{bs} \left(\frac{d_3}{2} \right)^2$$

$$\bar{J}_{bs} = (9 \cdot 10^{-4} \text{ kg m}^2) + (4 \cdot 10^{-4} \text{ kg m}^2) + \frac{1}{2} (0.0645) \left(\frac{0.009}{2} \right)^2$$

$$\bar{J}_{bs} = 0.0013 \text{ kg m}^2$$

$$\bar{J}_{eq} = \left(\frac{N_2}{N_1} \right)^2 \bar{J}_{bs} + \bar{J}_{ms} + \left(\frac{N_3}{N_4} \right)^2 \bar{J}_{ts}$$

$$\bar{J}_{eq} = \left(\frac{22}{18} \right)^2 (0.0016) + 0.0017 + \left(\frac{28}{14} \right)^2 0.0013 = \boxed{0.00929 = \bar{J}_{eq}}$$

AVERAGE

Homework #2

Given:-

Asked:-

$$\mu = ?$$

and damping coefficient?

Solution:-

Translational :-

$$f_d = C_t \cdot v(t)$$

$$C_t = \frac{M \cdot A}{g} = \frac{(D-d)}{z}$$

$$C_t = \frac{2\pi \mu d l}{D-d}$$

Vertical :-

$$C_1 = \frac{2\pi \mu w \cdot l}{h_i - h} \quad (2.33)$$

Horizontal :-

$$C_2 = \frac{2\pi \mu h l}{w_i - w}$$

$$C_t = C_1 + C_2 = \frac{2\pi \mu w \cdot l}{h_i - h} + \frac{2\pi \mu h l}{w_i - w}$$

The rod has a rectangular cross-section area, not a circular one.

2.24

given:-

$$G = 90 \text{ GPa N/m}^2$$

$$d_1 = 0.005 \text{ m}$$

$$d_3 = 0.0045 \text{ m}$$

$$N_1 = 36 \text{ Teeth}$$

$$l_1 = 0.15 \text{ m}$$

$$l_3 = 0.1 \text{ m}$$

$$N_2 = 44 \text{ Teeth}$$

$$d_2 = 0.004 \text{ m}$$

$$d_4 = 0.006 \text{ m}$$

$$N_3 = 32 \text{ Teeth}$$

$$l_2 = 0.2 \text{ m}$$

$$l_4 = 0.12 \text{ m}$$

$$N_4 = 48 \text{ Teeth}$$

Asked:-

$$k_{eq} = ?$$

Solution:-

$$k_{eq} = k_2 + k_3 + k_1 \left(\frac{N_2}{N_1} \right)^2 + k_4 \left(\frac{N_3}{N_4} \right)^2$$

$$k_1 = \frac{G \cdot I_p}{l_1} = \frac{\pi \cdot G \cdot d_1^4}{32 l_1} = \frac{\pi (90 \text{ GPa}) (0.005)^4}{32 (0.15)} = 36.82$$

$$k_2 = \frac{G \cdot I_p}{l_2} = \frac{\pi \cdot G \cdot d_2^4}{32 l_2} = \frac{\pi (90 \text{ GPa}) (0.004)^4}{32 (0.2)} = 11.31$$

$$k_3 = \frac{G \cdot I_p}{l_3} = \frac{\pi \cdot G \cdot d_3^4}{32 l_3} = \frac{\pi (90 \text{ GPa}) (0.0045)^4}{32 (0.1)} = 36.23$$

$$k_4 = \frac{G \cdot I_p}{l_4} = \frac{\pi \cdot G \cdot d_4^4}{32 l_4} = \frac{\pi (90 \text{ GPa}) (0.006)^4}{32 (0.12)} = 95.43$$

$$k_{eq} = (11.31) + (36.23) + (36.82) \left(\frac{44}{36} \right)^2 + (95.43) \left(\frac{32}{48} \right)^2$$

$$= 144.96 \text{ N.m}$$

2.25

Given:-

$$\rho = 7800 \text{ kg/m}^3$$

$$d_1 = 0.012 \text{ m}$$

$$d_3 = 0.009 \text{ m}$$

$$N_3 = 28 \text{ Teeth}$$

$$l_1 = 0.15 \text{ m}$$

$$l_3 = 0.13 \text{ m}$$

$$N_1 = 14 \text{ Teeth}$$

$$d_2 = 0.01 \text{ m}$$

$$N_1 = 18 \text{ Teeth}$$

$$J_1 = 6 \times 10^{-4} \text{ kg m}^2$$

$$l_2 = 0.1 \text{ m}$$

$$N_2 = 22 \text{ Teeth}$$

$$J_2 = 8 \times 10^{-4} \text{ kg m}^2$$

$$J_{w1} = 1 \times 10^{-3} \text{ kg m}^2$$

$$J_3 = 9 \times 10^{-4} \text{ kg m}^2$$

$$J_{w2} = 9 \times 10^{-4} \text{ kg m}^2$$

$$J_4 = 4 \times 10^{-4} \text{ kg m}^2$$

Asked:-

total mass & moment of inertia?

Solution:-

$$\text{top} \leftarrow J_t = J_{w1} + J_1 + \frac{1}{2} m_t \left(\frac{d_1}{2} \right)^2, \quad m_t = \rho l_1 \frac{\pi d_1^4}{4}$$

$$= (1 \times 10^{-3}) + (6 \times 10^{-4}) + \frac{1}{2} (1.91 \times 10^{-5}) \left(\frac{0.012}{2} \right)^2, \quad = (7800)(0.15) \frac{\pi (0.012)^4}{4}$$

$$= 1.6 \times 10^{-3} \quad = 1.91 \times 10^{-5}$$

$$\text{middle} \leftarrow J_m = J_2 + J_3 + \frac{1}{2} m_m \left(\frac{d_2}{2} \right)^2, \quad m_m = \rho l_2 \frac{\pi d_2^4}{4}$$

$$= (8 \times 10^{-4}) + (9 \times 10^{-4}) + \frac{1}{2} (6.13 \times 10^{-6}) \left(\frac{0.01}{2} \right)^2, \quad = (7800)(0.1) \frac{\pi (0.01)^4}{4}$$

$$= 1.7 \times 10^{-3} \quad = 6.13 \times 10^{-6}$$

$$\text{bottom} \leftarrow J_b = J_{w2} + J_4 + \frac{1}{2} m_b \left(\frac{d_3}{2} \right)^2, \quad m_b = \rho l_3 \frac{\pi d_3^4}{4}$$

$$= (9 \times 10^{-4}) + (4 \times 10^{-4}) + \frac{1}{2} (5.23 \times 10^{-6}) \left(\frac{0.009}{2} \right)^2, \quad = (7800)(0.13) \frac{\pi (0.009)^4}{4}$$

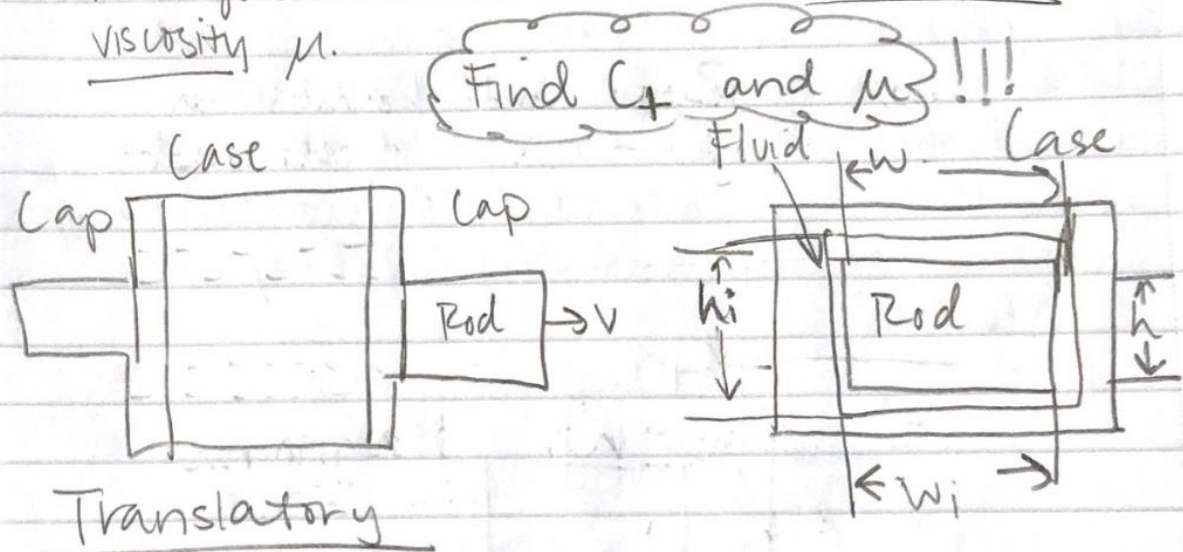
$$= 1.3 \times 10^{-3} \quad = 5.23 \times 10^{-6}$$

$$J_{\text{eq}} = \left(\frac{N_2}{N_1} \right)^2 J_t + J_m + \left(\frac{N_3}{N_4} \right)^2 J_b$$

$$= \left(\frac{22}{18} \right)^2 (1.6 \times 10^{-3}) + (1.7 \times 10^{-3}) + \left(\frac{28}{14} \right)^2 (1.3 \times 10^{-3}) = 0.0093 \text{ kg m}^2$$

BELOW AVERAGE

2.19 Figure 2.51 shows a translatory motion bearing. It is formed of a rectangular cross sectionnd placed symmetrically in and translating inside a similar box filled with viscous fluid. Calculate the total damping coefficient (C_t) based on the geometric parameters shown in Figure 2.51 and the fluid dynamic viscosity μ .



Damping Force/Moment: $f_d = C_t \cdot v(\dot{x}) = C_t \cdot \dot{x}$

Damping (lost Energy): $U_{d,t} = C_t \int \dot{x} \cdot dx = C_t \int \dot{x}^2 dt$

$$A = L \cdot w = h \cdot h_i \quad V(z) = \frac{zV}{g} = \frac{zV}{(h_i - h)/2}$$

$$g = (h_i - h)/2$$

$$V(z) = \frac{2zV}{(h_i - h)}$$

$$\tau = \mu \frac{dv(z)}{dz} \quad \frac{dv(z)}{dz} = \frac{v}{g} = \frac{2v}{(h_i - h)}$$

$$\tau = \mu \cdot \left(\frac{v}{g} \right)' = \mu \frac{2v}{(h_i - h)}$$

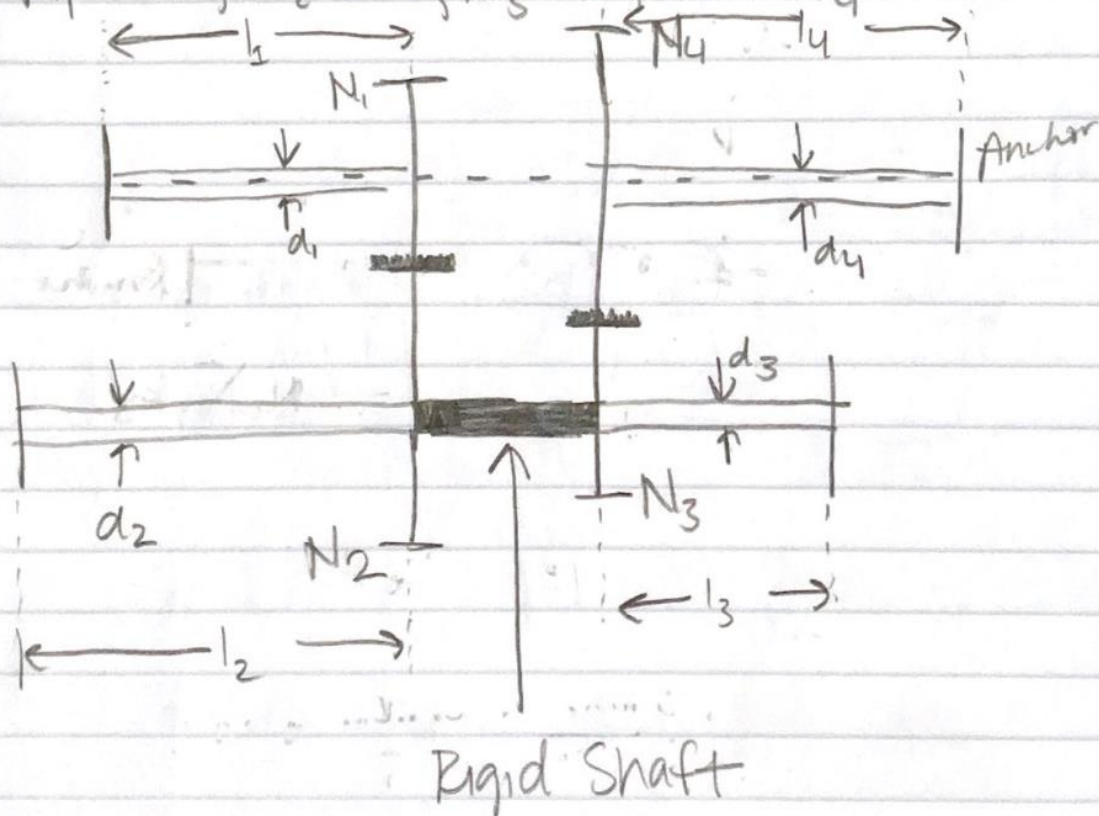
$$F_d = \tau \cdot A = \left(\frac{\mu 2v}{(h_i - h)} \right) \cdot (h_i \cdot h) = C_d \cdot v$$

$$C_d = \frac{2\mu h_i h}{(h_i - h)} \quad \text{damping coefficient}$$

X

$$\mu = \frac{C_d (h_i - h)}{2 h_i \cdot h} \quad \text{dynamic fluid viscosity ?}$$

2.24 Determine the equivalent torsional stiffness transferred to intermediate rigid shaft axis of the gear transmission sketched in figure 2.55. All other shafts are flexible; they are constructed from the same material with $G = 90 \text{ GPa N/m}^2$ and are defined by $d_1 = .005 \text{ m}$, $l_1 = .15 \text{ m}$, $d_2 = .004 \text{ m}$, $l_2 = .2 \text{ m}$, $d_3 = .0045 \text{ m}$, $l_3 = .1 \text{ m}$, $d_4 = .006 \text{ m}$, and $l_4 = .12 \text{ m}$. Known also are the number of teeth of gears $N_1 = 36$, $N_2 = 44$, $N_3 = 32$, and $N_4 = 48$.

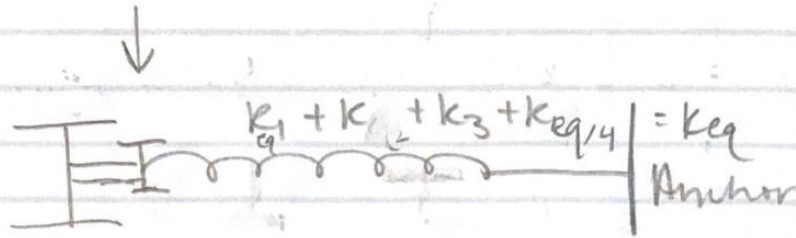
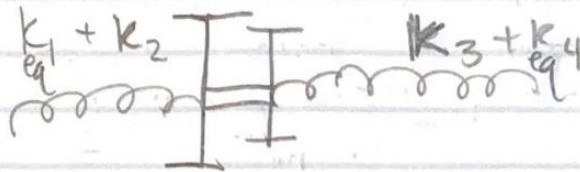


$$k = \frac{G \cdot J_p}{L}$$

$$k_1 = \frac{G \cdot J_{p1}}{L_1} = \frac{G \cdot \pi \cdot d_1^4}{32 L_1}$$

Why?
Refer to
Ex 2.10 on
HW

* Update from figure 2.55



$$k_{eq1} = \left(\frac{N_2}{N_1} \right)^2 \cdot k_1 \quad k_{eq3} = \left(\frac{N_3}{N_4} \right)^2 \cdot k_3$$

$$k_{eq} = k_{eq1} + k_2 + k_3 + k_{eq4}$$

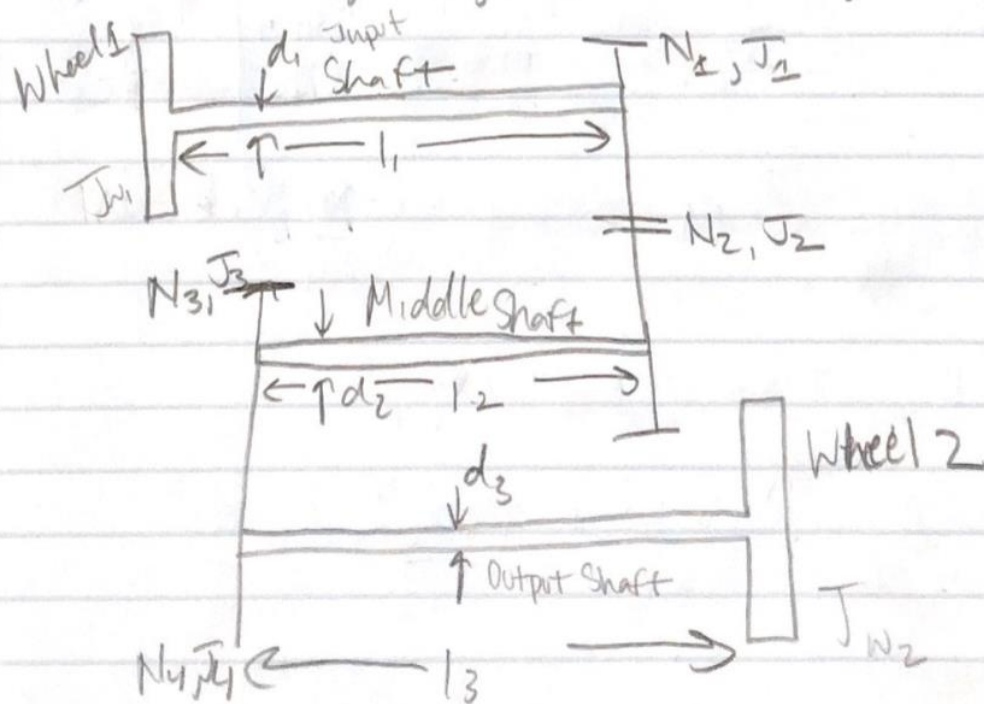
$$= \left(\frac{N_2}{N_1} \right)^2 \cdot \frac{G \pi d_1^4}{32 L_1} + \frac{G \pi d_2^4}{32 L_2} + \frac{G \pi d_3^4}{32 L_3}$$

$$+ \left(\frac{N_3}{N_4} \right)^2 \cdot \frac{G \pi d_4^4}{32 L_4} = \boxed{3.86875 = k_{eq}}$$

Darin McFadden

X

2.25 Three rigid shafts mesh through their respective gears as illustrated in Figure 2.56. Calculate the total mass moment of inertia of this system transferred to the middle shaft. The shafts have a mass density $\rho = 7800 \text{ kg/m}^3$ and are defined by $d_1 = .012 \text{ m}$, $l_1 = .15 \text{ m}$, $d_2 = .01 \text{ m}$, $l_2 = .1 \text{ m}$, $d_3 = .009 \text{ m}$, and $l_3 = .013 \text{ m}$. The following parameters define the four gears: the number of teeth $N_1 = 18$, $N_2 = 22$, $N_3 = 28$, $N_4 = 14$, their mass moments of inertia $J_1 = 6 \times 10^{-4} \text{ kgm}^2$, $J_2 = 8 \times 10^{-4} \text{ kgm}^2$, $J_3 = 9 \times 10^{-4} \text{ kgm}^2$, $J_4 = 4 \times 10^{-4} \text{ kgm}^2$, as well as the mass moments of inertia of the two end flywheels $J_{W1} = 1 \times 10^{-3} \text{ kgm}^2$, $J_{W2} = 9 \times 10^{-4} \text{ kgm}^2$.



Inertia properties are first transferred from wheel 2 to middle shaft

$$J_{eq,m} = (J_2 + J_3) + \frac{N_3^2}{N_4^2} (J_2 + J_4) + (J_{w1} + J_1) + \left(\frac{N_2}{N_3}\right)^2 \cdot J_1$$

$$J_{eq,i} = (J_{w1} + J_1) + \left(\frac{N_2}{N_3}\right)^2 \cdot J_{eq,m}$$

$$J_{eq,o} = (J_{w2} + J_4) + \left(\frac{N_4}{N_3}\right)^2 \cdot J_{eq,m}$$

$$J_{eq,m} = (J_2 + J_3) + \left(\frac{N_2}{N_1}\right)^2 \cdot J_{eq,i} + \left(\frac{N_3}{N_4}\right)^2 \cdot J_{eq,o}$$

$$J_{eq,m} = (J_2 + J_3) + \left(\frac{N_2}{N_1}\right)^2 \cdot (J_1 + J_{w1}) + \left(\frac{N_3}{N_4}\right)^2 \cdot (J_4 + J_{w2})$$

$$J_{eq,m} = 9.29 \times 10^{-3} \text{ kg m}^2$$

Need show more calculation process.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

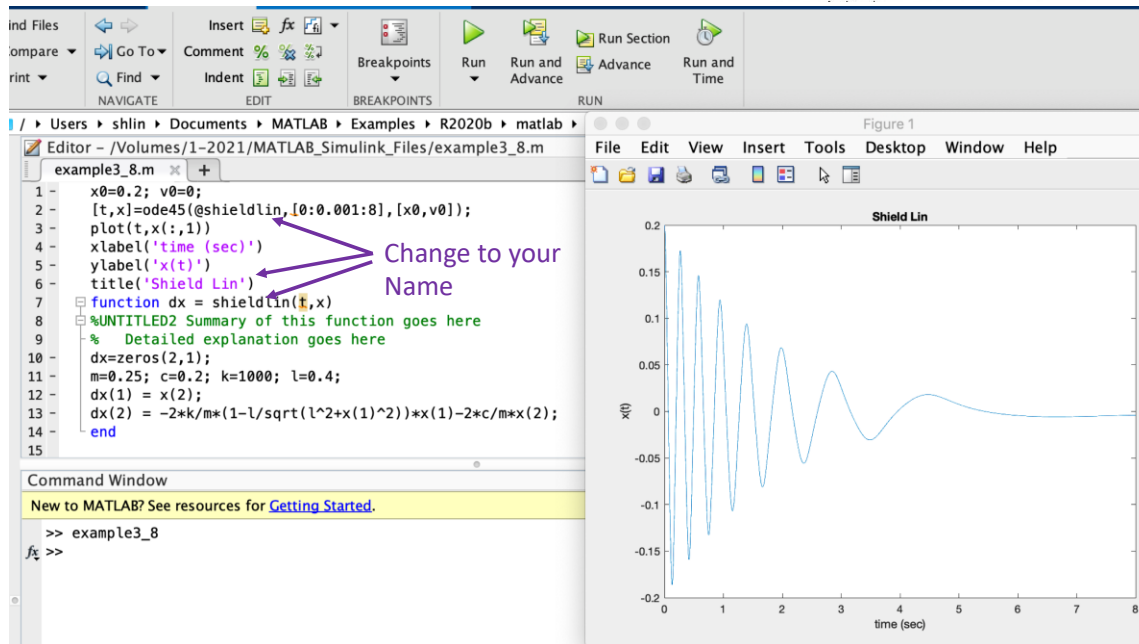
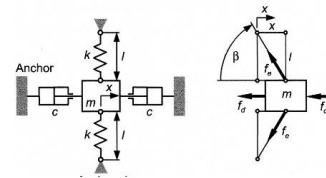
Title of Assignment: Homework 3

TOPICS COVERED IN HOMEWORK 3

1. MATLAB Exercise solving Differential Equation.
2. Derive Equations of Motion and Determine Natural Frequency.

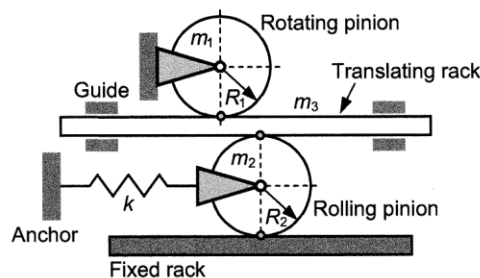
Homework

Matlab exercise for example 3.8



Homework

- 3.9** A fixed-center cylindrical pinion of mass m_1 and radius R_1 transmits its rotation to a translating rack of mass m_3 as illustrated in Figure 3.51. The mobile rack further interacts with another cylindrical pinion of mass m_2 and radius R_2 , which has its center connected to a spring of stiffness k , and rolls on a fixed rack. For no slippage between pinions and racks, demonstrate that this system has one DOF, formulate its mathematical model, and determine its natural frequency.



Homework

wheel can roll without slippage on a fixed wall under small angular motions of the rod. A heavy particle of mass m is also placed on the lever. At the rod's knowing that $N_1 = 24$, $N_2 = 48$, $N_3 = 32$, $N_4 = 40$, $J_1 = 0.001 \text{ kg m}^2$, $J_2 = 0.008 \text{ kg m}^2$, $J_3 = 0.002 \text{ kg m}^2$, $J_4 = 0.006 \text{ kg m}^2$. The two flexible shafts lengths are $l_1 = 0.1 \text{ m}$ and $l_2 = 0.08 \text{ m}$, the shafts constant circular cross-section diameters are $d_1 = 0.02 \text{ m}$ and $d_2 = 0.015 \text{ m}$, the flexible shafts shear modulus is $G = 150 \text{ GPa}$, and the mass density is $\rho = 7800 \text{ kg/m}^3$. Include the inertia contributions of the flexible shafts.

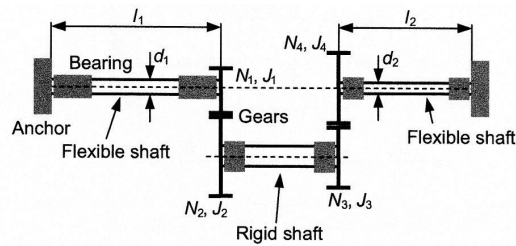
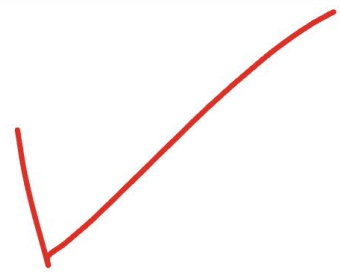
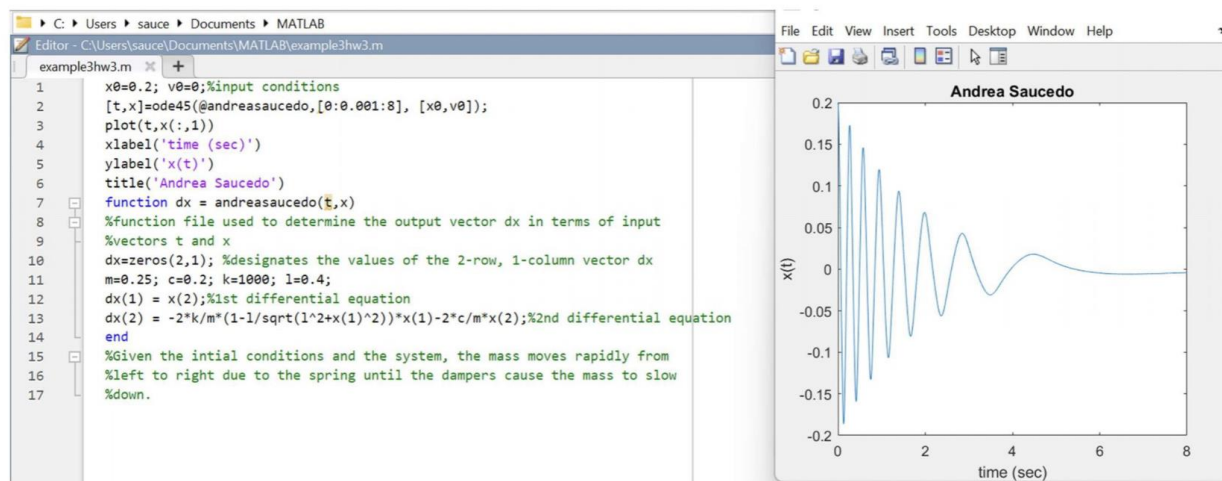


FIGURE 3.57

Three-Shaft Gear Train.

ABOVE AVERAGE



3.9 knowns: $m_1, m_2, R_1, R_2, m_3, k$
 (a) For no slippage between pinions and racks, demonstrate that the system has one single DOF.

of rotating parts - # of translating parts

$$2 - 1 = 1$$

B) Mathematical model and natural frequency

$$J_{eq,2} = J_2 + \left(\frac{N_2}{N_1}\right)^2 \cdot J_1 = J_2 + \left(\frac{\theta_1}{\theta_2}\right)^2 \cdot J_1$$

$$\theta_2 = \frac{R_1}{2R_2} \cdot \theta_1$$

$$J_{eq,2} = J_2 + \left(\frac{\theta_1}{\frac{R_1}{2R_2} \theta_1}\right)^2 \cdot J_1 \rightarrow J_{eq,2} = J_2 + \left(\frac{\theta_1^2}{\theta_1^2}\right) \left(\frac{2R_2}{R_1}\right)^2 J_1$$

$$J_{eq,2} = J_2 + \left(\frac{2R_2}{R_1}\right)^2 J_1$$

Analysis: the pinions are cylinders

$$J_c = \frac{1}{2} m R^2$$

$$J_1 = \frac{1}{2} m_1 R_1^2 \quad J_2 = \frac{1}{2} m_2 R_2^2$$

$$J_{eq,2} = \left[\frac{1}{2} m_2 R_2^2 + m_2 R_2^2\right] + \frac{1}{2} m_1 R_1^2 \left(\frac{2R_2}{R_1}\right)^2$$

$$= \left(\frac{1}{2} m_2 + m_2\right) R_2^2 + \frac{1}{2} m_1 R_1^2 \left(\frac{4R_2^2}{R_1^2}\right)$$

$$= \frac{1}{2} (3m_2) R_2^2 + \frac{1}{2} m_1 (4R_2^2)$$

$$J_{eq,2} = \frac{1}{2} (3m_2 + 4m_1) R_2^2$$

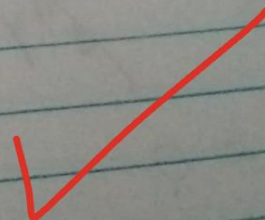
$$F_s = kx_2$$

$$M_s = F_s \cdot R_2 = (kx_2)R_2 = k(R_2\theta_2)R_2 = kR_2^2\theta_2$$

$$K_{eq2} = k R_2^2$$

$$\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} = \sqrt{\frac{\cancel{kR_2^2} \times 2}{\frac{1}{2} \times 2 (4m_1 + 3m_2) \cancel{R_2^2}}}$$

$$\omega_n = \sqrt{\frac{2k}{(4m_1 + 3m_2)}}$$



3.16

It is 1 degree of freedom because the only type of motion is rotation.

Find its natural frequency.

$$\omega_n = \sqrt{k_{eq}/J_{eq}}$$

$$K_1 = \frac{G I_{P1}}{L_1} = \frac{G \pi d_1^4}{32 L_1} = \frac{(150 \times 10^9 \text{ N/m}^2) \pi (0.02 \text{ m})^4}{32 (0.1 \text{ m})}$$

$$K_1 = 23,561.95 \text{ Nm}$$

$$K_2 = \frac{G \pi d_2^4}{32 L_2} = \frac{(150 \times 10^9 \text{ N/m}^2) \pi (0.015 \text{ m})^4}{32 (0.08 \text{ m})}$$

$$K_2 = 9318.933 \text{ N.m}$$

$$k_{eq} = K_1 \left(\frac{N_2}{N_1} \right)^2 + K_2 \left(\frac{N_3}{N_4} \right)^2$$

$$k_{eq} = (23,561.95 \text{ Nm}) \left(\frac{48}{24} \right)^2 + (9318.933 \text{ N.m}) \left(\frac{32}{40} \right)^2$$

$$k_{eq} = 100,211.9171 \text{ N.m}$$

$$J_{fsl} = \frac{1}{2} m R^2 = \frac{1}{2} \left(\rho L_1 \frac{\pi d_1^2}{4} \right) \left(\frac{d_1}{2} \right)^2$$

$$J_{fsl} = \frac{1}{2} \left(7800 \text{ kg/m}^3 \cdot 0.1 \text{ m} \cdot \frac{\pi (0.02 \text{ m})^2}{4} \right) \left(\frac{0.02 \text{ m}}{2} \right)^2$$

$$J_{fsl} = 1.225 \times 10^{-5} \text{ kg m}^2$$

$$J_{fsR} = \frac{1}{2} \left(\rho L_2 \frac{\pi d_2^2}{4} \right) \left(\frac{d_2}{2} \right)^2$$

$$J_{fsR} = \frac{1}{2} \left(7800 \text{ kg/m}^3 \right) (0.08 \text{ m}) \frac{\pi (0.015 \text{ m})^2}{4} \left(\frac{0.015 \text{ m}}{2} \right)^2$$

$$J_{fsR} = 3.1 \times 10^{-6} \text{ kg m}^2$$

$$J_{eq} = J_2 + J_3 + \left(J_R \right) \left(\frac{N_3}{N_4} \right)^2 + \left(J_L \right) \left(\frac{N_2}{N_1} \right)^2$$

Moment of inertia of a shaft with one end fixed = $1/3$ * moment of inertia of the shaft with free ends. (See Table 2.7)

$$J_R = J_{fsR} + J_A = (3.1 \times 10^{-6} \text{ kg m}^2) + (0.006 \text{ kg m}^2)$$

$$J_R = 0.0060031 \text{ kg m}^2$$

$$J_L = J_{fsL} + J_I = (1.225 \times 10^{-5} \text{ kg m}^2) + (0.0001 \text{ kg m}^2)$$

$$J_L = 0.00101225 \text{ kg m}^2$$

$$J_{eq} = J_2 + J_3 + J_R \left(\frac{N_3}{N_4} \right)^2 + J_L \left(\frac{N_2}{N_1} \right)^2$$

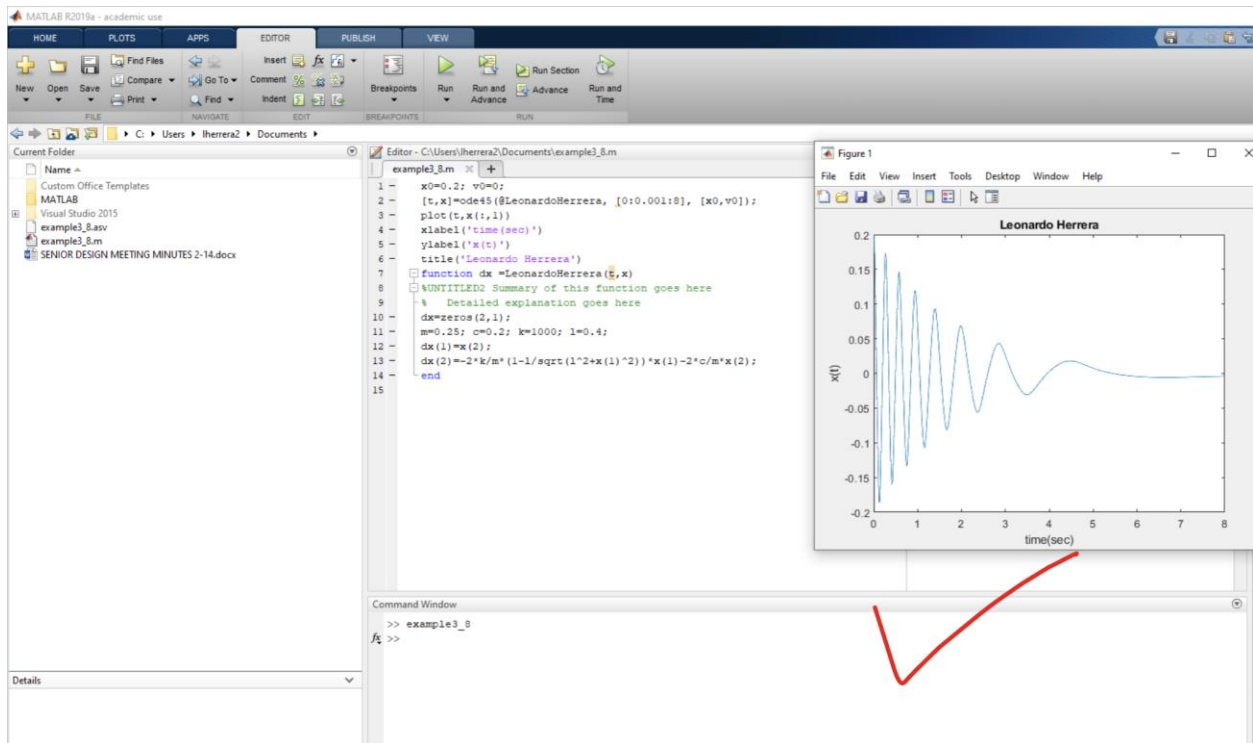
$$J_{eq} = (0.008 \text{ kg m}^2) + (0.002 \text{ kg m}^2) + (0.0060031 \text{ kg m}^2) \left(\frac{32}{40} \right)^2 + (0.00101225 \text{ kg m}^2) \left(\frac{48}{24} \right)^2$$

$$J_{eq} = 0.01789 \text{ kg m}^2$$

$$\omega_n = \sqrt{k_{eq} / J_{eq}}$$

$$\omega_n = \sqrt{\frac{100211}{0.01789}} = 2366.75 \text{ rad/s} = \omega_n$$

AVERAGE

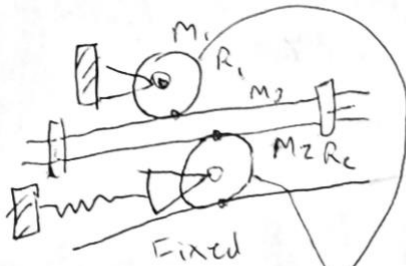


3.9

1 DoF

$$E = T + U$$

$$E = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} k x^2 + \frac{1}{2} m_3 \dot{x}^2$$



$$\theta_1 = \frac{x}{R_1} \quad \dot{\theta}_1 = \frac{\dot{x}}{R_1}$$

$$\theta_2 = \frac{x}{R_2} \quad \dot{\theta}_2 = \frac{\dot{x}}{R_2}$$

$$J_1 = \frac{1}{2} m_1 R_1^2$$

$$J_2 = \frac{1}{2} m_2 R_2^2$$

$$E = \frac{1}{2} \left(\frac{1}{2} m_1 R_1^2 \right) \dot{\theta}_1^2 + \frac{1}{2} \left(\frac{1}{2} m_2 R_2^2 \right) \dot{\theta}_2^2 + \frac{1}{2} m_3 \dot{x}^2 + \frac{1}{2} k x^2$$

$$E = \frac{1}{2} \left(\frac{1}{2} m_1 R_1^2 \right) \left(\frac{\dot{x}}{R_1} \right)^2 + \frac{1}{2} \left(\frac{1}{2} m_2 R_2^2 \right) \left(\frac{\dot{x}}{R_2} \right)^2 + \frac{1}{2} m_3 \dot{x}^2 + \frac{1}{2} k x^2$$

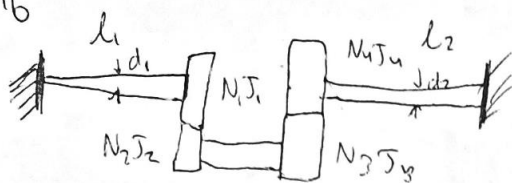
$$E = \frac{1}{2} \left(\frac{m_1 + m_2 + 2m_3}{2} \right) \dot{x}^2 + \frac{1}{2} k x^2$$

$$\frac{dE}{dt} = \frac{1}{2} \cdot 2 \left(\frac{m_1 + m_2 + 2m_3}{2} \right) \ddot{x} + \frac{1}{2} \cdot 2 \cdot k x = 0$$

$$\omega_n = \sqrt{\frac{k}{\frac{m_1 + m_2 + 2m_3}{2}}} = \sqrt{\frac{2k}{m_1 + m_2 + 2m_3}}$$

X

3.16



$$K_{eq} = K_{1eq} + K_{2eq} = K_1 \left(\frac{N_2}{N_1} \right)^2 + K_2 \left(\frac{N_3}{N_4} \right)^2$$

$$K_1 = \frac{G \pi d_1^4}{32 l_1}$$

$$K_2 = \frac{G \pi d_2^4}{32 l_2}$$

$$K_{eq} = \frac{G \pi}{32} \left(\frac{d_1^4}{l_1} \left(\frac{N_2}{N_1} \right)^2 + \frac{d_2^4}{l_2} \left(\frac{N_3}{N_4} \right)^2 \right)$$

$$K_{eq} = \frac{150 \times 10^9 \cdot \pi}{32} \left(\frac{0.02^4}{0.1} \left(\frac{48}{24} \right)^2 + \frac{0.015^4}{0.08} \left(\frac{32}{40} \right)^2 \right)$$

$$K_{eq} = 100,211.8969 \text{ N.m}$$

$$J_{eq} = J_2 + J_3 + J_{1eq} + J_{4eq} \quad \text{missing moment of inertia for shafts.}$$

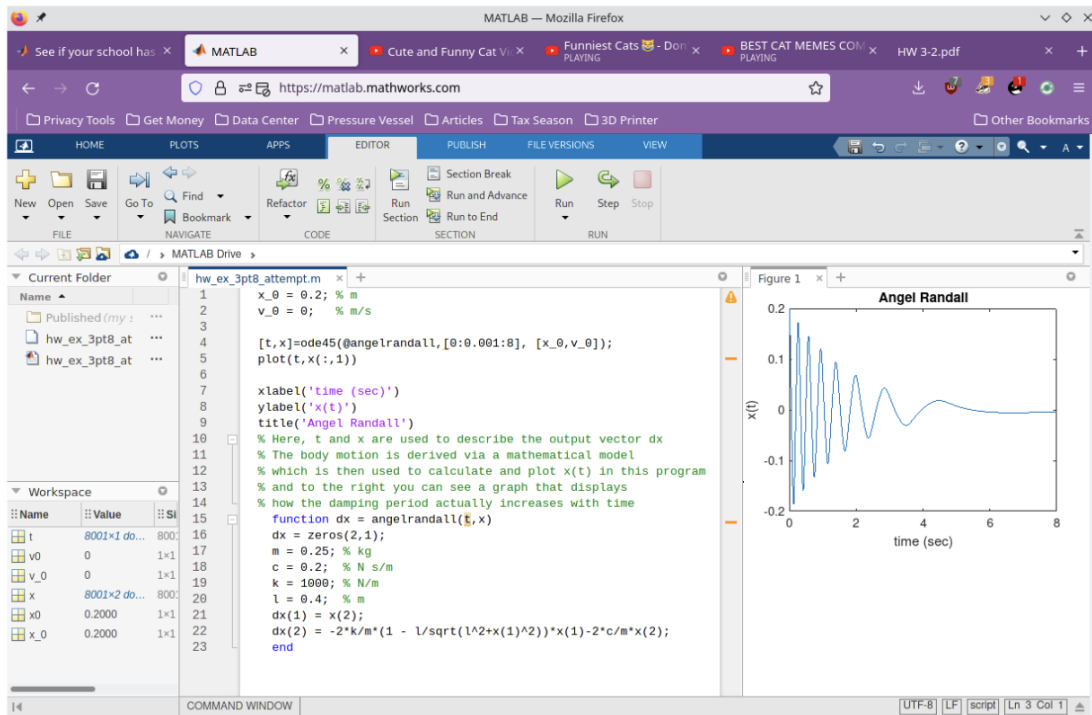
$$J_{eq} = J_2 + J_3 + J_1 \left(\frac{N_2}{N_1} \right)^2 + J_4 \left(\frac{N_3}{N_4} \right)^2$$

$$J_{eq} = 0.008 + 0.002 + 0.001 \left(\frac{48}{24} \right)^2 + 0.006 \left(\frac{32}{40} \right)^2$$

$$J_{eq} = 0.01784 \text{ kg.m}^2 \quad \text{kg.m}^2/\text{s}^2 \cdot \text{m} \quad \text{kg.m}^2/\text{s}^2$$

$$\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} = \sqrt{\frac{100,211.8969 \text{ N.m}}{0.01784 \text{ kg.m}^2}} = 2370.08$$

BELOW AVERAGE



I don't know why Gnu Octave displays an error whereas Matlab displays a plot. Nevertheless, I'm just happy to see that a graph was produced!



It's good that you obtained the output solution.

Given:

$$m_1, m_2, m_3$$

$$R_1, R_2$$

$$k$$

- Find: (a) Proof of 1 DOF
(b) Mathematical model
(c) Natural Frequency, ω_n

Assumes:

no slippage b/w pinion and rack

homogeneous equation

t is negligible

Solution:

(a) ~~3 - 2 = 1~~ $\Rightarrow 3 - 2 = 1$

of free coordinates - # of constraints = 1 DOF
(w/ two fixed axis pinions)

(b) ~~$\frac{dE}{dt} = 0$~~

~~$x = X \sin(\omega t + \phi)$~~

$$x = X \sin(\omega t + \phi)$$

$$\omega_n = \omega = \sqrt{k/m}$$

$$\text{where } x = \frac{\sqrt{\omega_n^2 \cdot x_0^2 + v_0^2}}{\omega_n} \sin\left(\sqrt{k/m} t + \tan^{-1}\left(\frac{\omega_n \cdot x_0}{v_0}\right)\right)$$

(c)

$$\omega_n = \omega = \sqrt{k/m} = \sqrt{\frac{k_{eq}}{(m_1 + m_2 + m_3)}}$$

Given:

$$n_1 = 24$$

$$n_2 = 48$$

$$n_3 = 32$$

$$n_4 = 40$$

$$l_1 = 0.1 \text{ m}$$

$$l_2 = 0.08 \text{ m}$$

$$d_1 = 0.02 \text{ m}$$

$$d_2 = 0.015 \text{ m}$$

$$G = 150 \text{ GPa}$$

$$\rho = 7800 \text{ kg/m}^3$$

$$J_1 = 0.001 \text{ kg m}^2$$

$$J_2 = 0.008 \text{ kg m}^2$$

$$J_3 = 0.002 \text{ kg m}^2$$

$$J_4 = 0.006 \text{ kg m}^2$$

Find: ω_n

Assumption: 1-DOF system, ~~fixed~~ $k=1$

Solution:

~~Equation~~

$$\omega_n = \sqrt{\frac{k_{eq}}{J_{eq}}}$$

Moment of inertia of shafts should be included.

~~Equation~~

$$\begin{aligned} k_{eq} &= \left[k \cdot \left(\frac{n_2}{n_1} \right)^2 + k \right] + \left[k \cdot \left(\frac{n_4}{n_3} \right)^2 + k \right] \\ &= \left[1 \cdot \left(\frac{48}{24} \right)^2 + 1 \right] + \left[1 \cdot \left(\frac{40}{32} \right)^2 + 1 \right] = 7.56 \text{ N.m} \end{aligned}$$

$$\begin{aligned} J_{eq} &= \left[J_1 + \left(\frac{n_1}{n_2} \right)^2 \cdot J_2 \right] + \left[J_3 + \left(\frac{n_3}{n_4} \right)^2 \cdot J_4 \right] \\ &= \left[0.001 + \left(\frac{24}{48} \right)^2 \cdot 0.008 \right] + \left[0.002 + \left(\frac{32}{40} \right)^2 \cdot 0.006 \right] \\ &= 0.00884 \text{ kg.m}^2 \end{aligned}$$

$$\omega_n = \sqrt{\frac{7.56 \text{ N.m}}{0.00884 \text{ kg.m}^2}} = 29.24 \text{ n/100.m}$$

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 5

TOPICS COVERED IN HOMEWORK 5

1. Derive Equations of Motion for Two Degrees of Freedom System and Use MATLAB to obtain Eigen Values and Eigen Vectors.

Homework

- 3.36** Derive the mathematical model of the mechanical system shown in Figure 3.71, which is formed of a pinion, two racks, a body, and several springs. The pinion comprises two concentric disks forming a single solid part that rotates around a fixed axis passing through its center C . The pinion meshes (without slippage) with two racks that consequently undergo translatory and parallel motions. Calculate the eigenvalues, unit-norm eigenvectors, and describe the modes of the mechanical system. Use the MATLAB specialized command to check the analytical eigenvalues and eigenvectors. Known are $m_1 = 1 \text{ kg}$, $m_2 = 2.1 \text{ kg}$, $J_C = 0.05 \text{ kg m}^2$, $m_3 = 0.6 \text{ kg}$, $k_1 = 120 \text{ N/m}$, $k_2 = 100 \text{ N/m}$, $k_3 = 110 \text{ N/m}$, $R_2 = 2R_1 = 0.03 \text{ m}$.

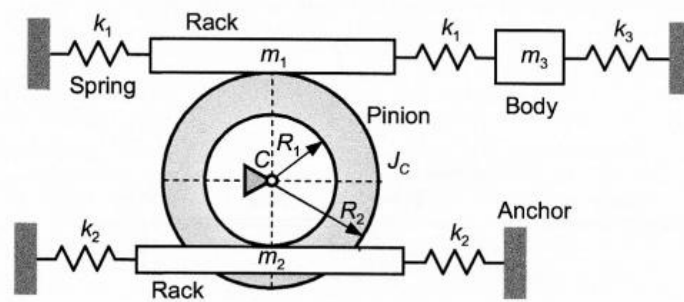


FIGURE 3.71

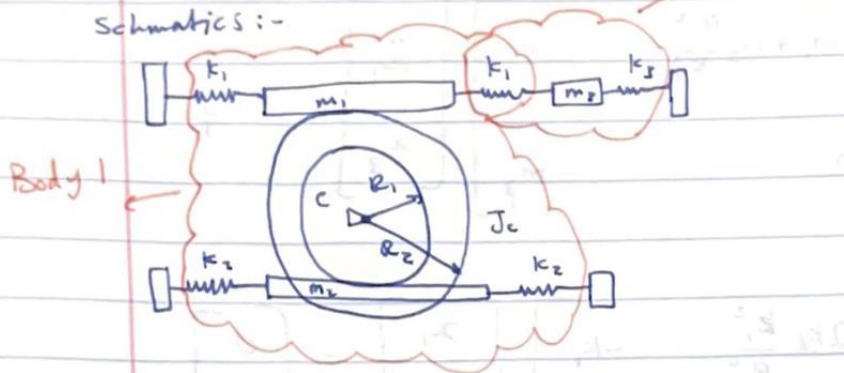
Conservative Mechanical System With Pinion, Racks, Translating Mass, and Linear-Motion Springs.

ABOVE AVERAGE

Homework 3-3

Bashir Jarad

Schematics:-



Given:-

$$m_1 = 1 \text{ kg}, \quad m_2 = 2.1 \text{ kg}, \quad m_3 = 0.6 \text{ kg} \quad J_c = 0.05 \text{ kg} \cdot \text{m}^2$$

$$k_1 = 120 \text{ N/m}, \quad k_2 = 100 \text{ N/m}, \quad k_3 = 110 \text{ N/m} \quad R_2 = 2R_1 = 0.03 \text{ m}$$

Solution:- $\therefore \text{DOF} = 2$

$$\begin{aligned} T_1 &= \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \\ T_2 &= \frac{1}{2} m_3 \dot{x}_3^2 \end{aligned} \quad \left. \begin{aligned} x_2 &= \theta \cdot R_1 \Rightarrow \\ \theta &= \frac{x_2}{R_1} = \frac{x_1}{R_2} \end{aligned} \right\}$$

general terms:

$$\begin{aligned} U_1 &= \frac{1}{2} k_1 (x_1 - x_3)^2 + \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 + \frac{1}{2} k_3 x_3^2 \\ U_2 &= \frac{1}{2} k_1 (x_1 - x_3)^2 + \frac{1}{2} k_3 x_3^2 \end{aligned} \quad \left. \begin{aligned} x_1 &= \frac{x_2 R_1}{R_2} \end{aligned} \right\}$$

$$\begin{aligned} \text{combined terms:} \quad T_1 &= \frac{1}{2} J_c \left(\frac{\dot{x}_1}{R_2} \right)^2 + \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \left(\frac{x_1 R_1}{R_2} \right)^2 \\ T_2 &= \frac{1}{2} m_3 \dot{x}_3^2 \end{aligned}$$

$$\begin{aligned} U_1 &= \frac{1}{2} k_1 (x_1 - x_3)^2 + \frac{1}{2} k_1 x_1^2 + k_2 \left(\frac{x_1 R_1}{R_2} \right)^2 \\ U_2 &= \frac{1}{2} k_1 (x_1 - x_3)^2 + \frac{1}{2} k_3 x_3^2 \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \frac{dT}{dx_1} + \frac{dU}{dx_1} &= 0 \quad \left\{ \begin{aligned} J_c \left(\frac{\dot{x}_1}{R_2} \right) \cdot \frac{1}{R_2} + m_1 \dot{x}_1 + m_2 \left(\frac{\dot{x}_1 R_1}{R_2} \right) \left(\frac{R_1}{R_2} \right) + k_1 (x_1 - x_3) \\ + k_1 x_1 + 2k_2 \left(\frac{x_1 R_1}{R_2} \right) \left(\frac{R_1}{R_2} \right) &= 0 \end{aligned} \right. \\ \frac{d}{dt} \frac{dT}{dx_3} + \frac{dU}{dx_3} &= 0 \quad \left\{ \begin{aligned} m_3 \dot{x}_3 - k_1 (x_1 - x_3) + k_3 x_3 &= 0 \end{aligned} \right. \end{aligned}$$

$$\begin{bmatrix} \frac{J_c}{R_z^2} + m_1 + m_2 \frac{R_1^2}{R_z^2} & 0 \\ 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{bmatrix} +$$

$$\begin{bmatrix} 2k_1 + 2k_2 & \frac{k_1^2}{R_e^2} \\ -k_1 & k_1 + k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$


Users ▸ bashirsmacbook ▸ Documents ▸ MATLAB

Editor - untitled *

```
1 m1=1 ; m2=2.1; m3=0.6 ; Jc=0.05;
2 k1=120; k2=100 ; k3=110;
3 R2=0.03 ; R1=0.015;
4 M_mat = [Jc/(R2^2)+m1+m2*((R1/R2)^2),0 ; 0 , m3];
5 K_mat = [2*k1+2*k2*((R1/R2)^2), -k1 ; -k1 , k1+k3];
6 [vector, Lambda] = eig(inv(M_mat)*K_mat)
7 Omegan = sqrt(Lambda)
```

Command Window

```
vector =
    -0.8846    0.0055
    -0.4664   -1.0000

Lambda =
    3.9722    0
    0   384.4417

Omegan =
    1.9930    0
    0   19.6072

fx >>
```



AVERAGE

3.36 $m_1 = 1 \text{ kg}$, $m_2 = 2.1 \text{ kg}$, $J_c = 0.05 \text{ kg} \cdot \text{m}^2$, $m_3 = 0.6 \text{ kg}$, $K_1 = 120 \text{ N/m}$,
 $K_2 = 100 \text{ N/m}$, $K_3 = 110 \text{ N/m}$, $R_2 = 2R_1 = 0.03 \text{ m}$

$$J_{eq} = J_c + m_1 \cdot R_2^2 + m_2 \cdot R_1^2 + m_3 \cdot R_2^2$$

$$K_{eq} = 2K_2 \cdot R_1^2 + 2K_1 \cdot R_2^2 + K_3 \cdot R_2^2$$

$$[J_c + m_2 \cdot R_1^2 + (m_1 + m_3) \cdot R_2^2] \ddot{\theta} + [2K_2 \cdot R_1^2 + (2K_1 + K_3) \cdot R_2^2] \theta = 0$$

$$[\frac{1}{2} J_c + (m_1 + m_3) \cdot R_2^2] \ddot{\theta}_2 + [(2K_1 + K_3) \cdot R_2^2] \theta_2 = 0$$

$$[\frac{1}{2} J_c + m_2 \cdot R_1^2] \ddot{\theta}_1 + [2K_2 \cdot R_1^2] \theta_1 = 0$$

$$\begin{bmatrix} \frac{1}{2} J_c + m_2 \cdot R_1^2 & 0 \\ 0 & \frac{1}{2} J_c + (m_1 + m_3) \cdot R_2^2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 2K_2 \cdot R_1^2 & 0 \\ 0 & (2K_1 + K_3) \cdot R_2^2 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

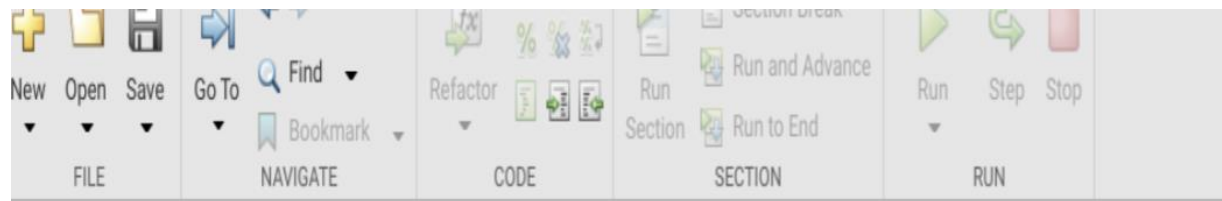
$$[M] = \begin{bmatrix} \frac{1}{2} J_c + m_2 \cdot R_1^2 & 0 \\ 0 & \frac{1}{2} J_c + (m_1 + m_3) \cdot R_2^2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 2K_2 \cdot R_1^2 & 0 \\ 0 & (2K_1 + K_3) \cdot R_2^2 \end{bmatrix}$$



Scanned with
CamScanner

There are two independent coordinates in the system.
 One can be angle of the pinion, the other one must be x_3 for the body mass m_3 .

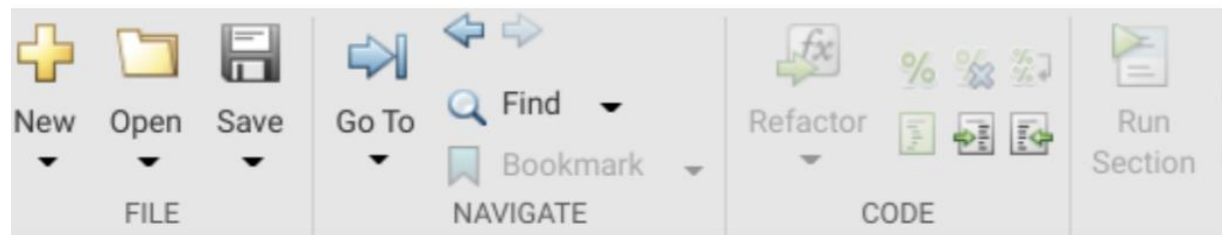


/ > MATLAB Drive > Published > Zoe FEA

```

untitled.asv x +
1  m1=1; m2=2.1; m3=0.6; J=0.05; k1=120; k2=100; k3=110; R2=0.03; R1=0.015;
2  M_mat=[(J/2)+m2*(R1)^2 0; 0 (J/2)+(m1+m3)*(R2)^2];
3  K_mat=[(2*k2)*(R1)^2 0; 0 (2*k1+k3)*(R2)^2];
4  eigenvalues= eig(M_mat\K_mat);
5  [V,D]=eig(M_mat\K_mat);
6

```



/ > MATLAB Drive > Published > Zoe FEA

```

untitled.asv
Command Window
>> M_mat

M_mat =

    0.0255    0
         0    0.0264

>> K_mat

K_mat =

    0.0450    0
         0    0.3150

```

← → 📁 📁 📁 / > MATLAB Drive > Published > Zoe FEA

untitled.asv

Command Window

eigenvalues =

1.7666
11.9138

>> [V,D]=eig(M_mat\K_mat)

V =

1 0
0 1

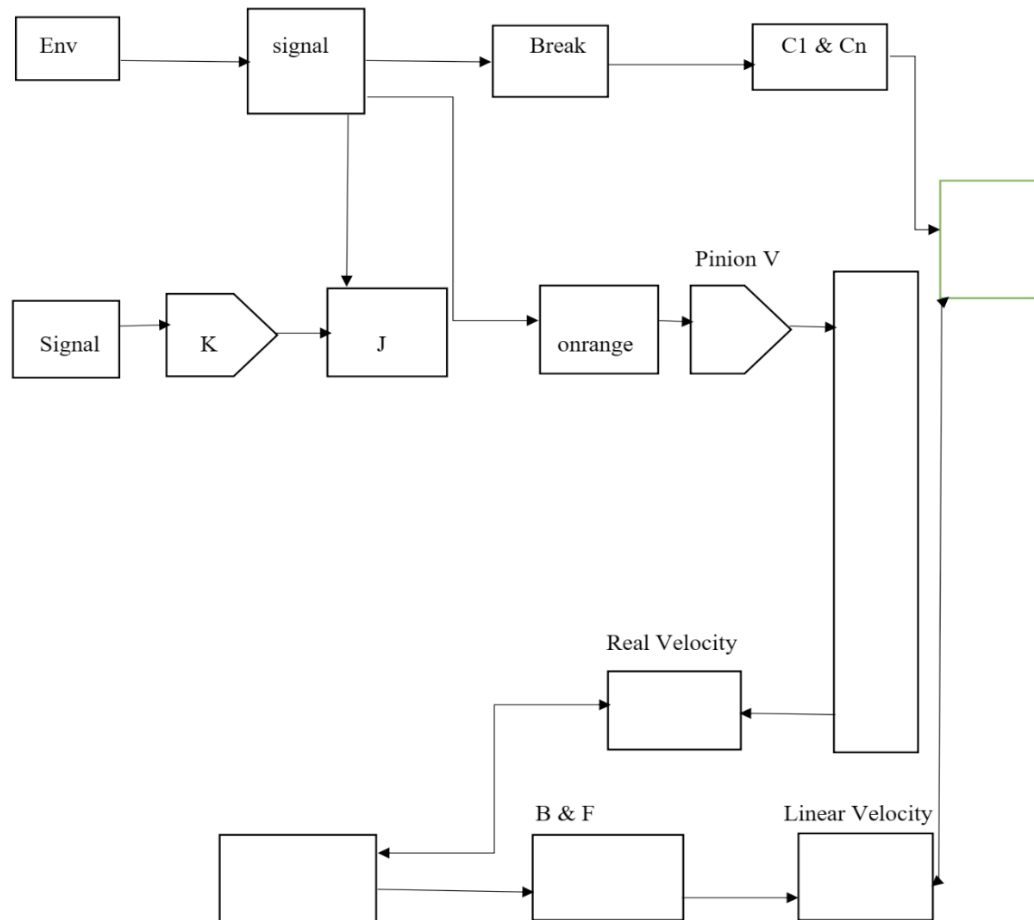
D =

1.7666 0
0 11.9138



BELOW AVERAGE

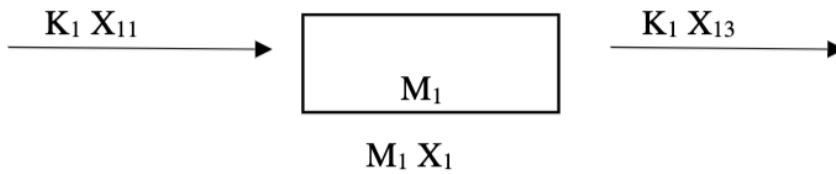
A mathematical model is developed for the design synthesis of rack and pinion steering linkage and general objective is to minimize the difference between the steering center over the full range of steering input angle while fitting into a reasonable space because there is a substantial amount of design are in the systems and the mathematical representation is not clear, the model constrains and objective actually “evolve” to the eventual desired from the problem has multiple optima and practical and heuristic consideration are used to choose suboptimal but more realistic solutions once satisfactory optimal solutions are identified these involve manipulation of the objective function constraints set and initial guess both leading and trailing link designs are considered the form being slightly the better limitations are also considered.



Not sure how the flowchart is used for derivating a mathematical model.

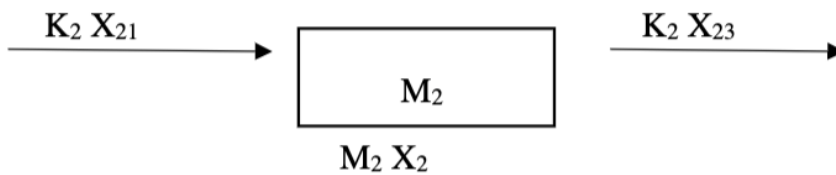
Now we will draw free body diagram of all the components.

FBD of M1:



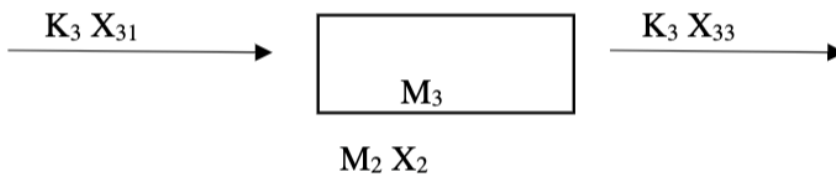
$$M_1 X_1 + K_1 X_{11} + K_1 X_{13} = 0$$

FBD of M2:



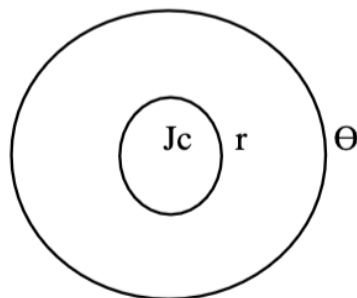
$$M_2 X_2 + K_2 X_{21} + K_2 X_{23} = 0$$

FBD of M3:



$$M_3 X_3 + K_3 X_{31} + K_3 X_{33} = 0$$

FBD of Pinion:



$$J_c \times R_1 \times \Theta + 2K_2 \times R_1 \times \Theta = 0$$

$$J_c \times R_2 \times \Theta + 2K_2 \times R_2 \times \Theta = 0$$

$$[m][X] + [K][X] = 0$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \\ \mathbf{M}_3 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{bmatrix}$$


$$\mathbf{k} = \begin{bmatrix} k_1 & 0 & k_1 + k_3 \\ k_2 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Good efforts! however, this problem is a 2 DOF system, only allow 2 independent variables.

Code Matlab:

```
K = [ 120    0    230
      0    100    100
      0     0    110 ];
M = [ 1 ; 2.1 ; 0.6] ;
X = [ 3    -2    -9
      -2     4     1
       3     1    -1 ];
% Eigenvalues and right and left eigenvectors
% of matrix K
[V,D,W] = eig(K);
disp("Diagonal matrix of Eigenvalues :");
disp(D);
disp("Right eigenvectors :")
disp(V);
disp("Left eigenvectors :")
disp(W);
% Eigenvalues and right and left eigenvectors
% of matrix A
[V,D,W] = eig(X);
disp("Diagonal matrix of Eigenvalues :");
disp(D);
disp("Right eigenvectors :")
disp(V);
disp("Left eigenvectors :")
disp(W);
```

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 6

TOPICS COVERED IN HOMEWORK 6

1. Repeat a Simulink example; change function to MATLAB function.
2. Derive Equations of Motion, Construct Block Diagram of a Dynamic System and Plot System Response by Simulink

Note that there is no damping, which means that $F = 0$ and the corresponding derivatives of F in Eq. (3.121) are also zero. Eq. (3.127) is combined with the definition Eq. (3.123) and they result in Eq. (3.125).

Eq. (3.125) is rewritten in the form:

$$\begin{cases} \ddot{x}_1 = -\frac{2k_1 + k}{m_1} \cdot x_1 + \frac{k}{m_1} \cdot x_2 + \frac{1}{m_1} \cdot f \\ \ddot{x}_2 = \frac{k}{m_2} \cdot x_1 - \frac{2k_2 + k}{m_2} \cdot x_2 \end{cases} \quad \text{or} \quad \begin{cases} \ddot{x}_1 = -a_{11} \cdot x_1 + a_{12} \cdot x_2 + \frac{1}{m_1} \cdot f \\ \ddot{x}_2 = a_{21} \cdot x_1 - a_{22} \cdot x_2 \end{cases} \quad (3.128)$$

The stiffnesses k_1 and k_2 are given in Table 2.2 for fixed-guided beams, whereas the stiffness of the serpentine spring k is expressed in Eq. (2.10); they are $k_1 = 12E \cdot l_z / l_1^3$; $k_2 = 12E \cdot l_z / l_2^3$; $k = 6E \cdot l_z / (5l^3)$ with l_z being the circular cross-sectional area moment of inertia, $l_z = \pi \cdot d^4 / 64$.

With the numerical values of this example, the parameters of Eq. (3.128) are $k_1 = 0.380$ N/m, $k_2 = 1.555$ N/m, $k = 0.304$ N/m, $a_{11} = 2.952913017 \cdot 10^9$ m²/s², $a_{12} = 8.436894333 \cdot 10^8$ m²/s², $a_{21} = 1.214912784 \cdot 10^9$ m²/s², $a_{22} = 1.365561969 \cdot 10^{10}$ m²/s², $1/m_1 = 2.777777777 \cdot 10^9$ kg⁻¹. The block diagram of Figure 3.38 integrates Eq. (3.128). The input force on the mass m_1 is produced by combining the Ramp block from the Source library and the Math Function from the Math Operations library. While the unit ramp (Slope parameter is 1) provides the time input (which is interpreted as a variable u), the following expression needs to be specified for the Math Function: $0.000002 \cdot \exp(-10 \cdot u(1))$, which constitutes f of the example. The remaining

(Values on this page, such as a_{11} , a_{12} , m_1 , etc. are needed for the homework)

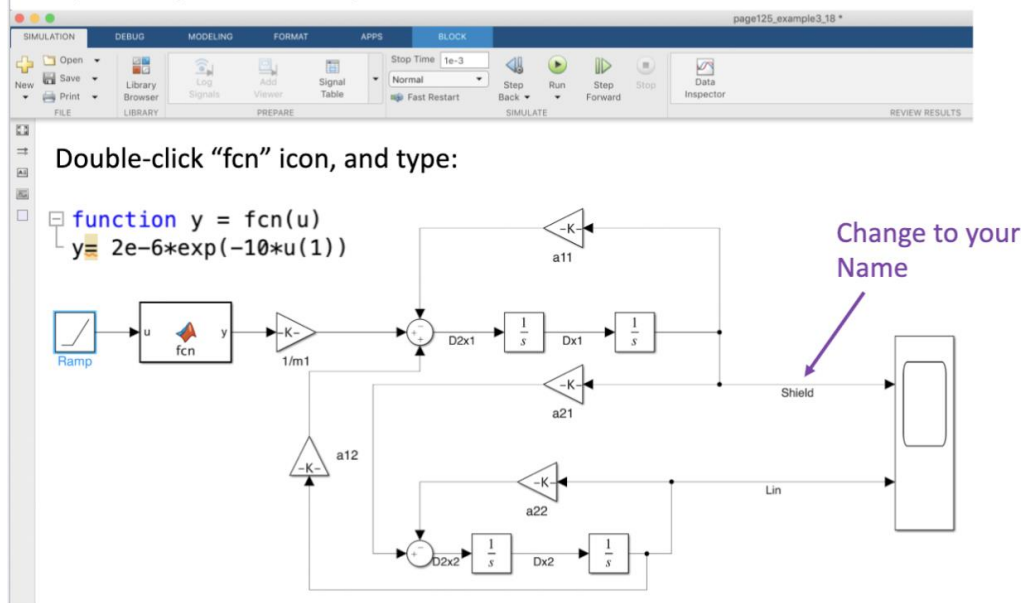
Homework

Simulink for Example 3.18, page 125

(set "Stop Time" = 1e-3)

$$\begin{cases} \ddot{x}_1 = -a_{11} \cdot x_1 + a_{12} \cdot x_2 + \frac{1}{m_1} \cdot f \\ \ddot{x}_2 = a_{21} \cdot x_1 - a_{22} \cdot x_2 \end{cases}$$

(note that coefficients a_{11} , a_{12} , m_1 , etc. are shown on prior page)



Homework

- 3.41** In the mechanical system of Problem 3.36 and shown in Figure 3.71, the spring k_3 is replaced by a damper of viscous damping coefficient $c = 32 \text{ N s/m}$, as indicated in Figure 3.76. A force $f = 180 + 20 \cdot e^{-4 \cdot t} \cdot \sin(10\pi \cdot t) \text{ N}$ acts now on the body of mass m_3 . All other parameters provided in Problem 3.36 being the same, plot the system's time response by means of Simulink.

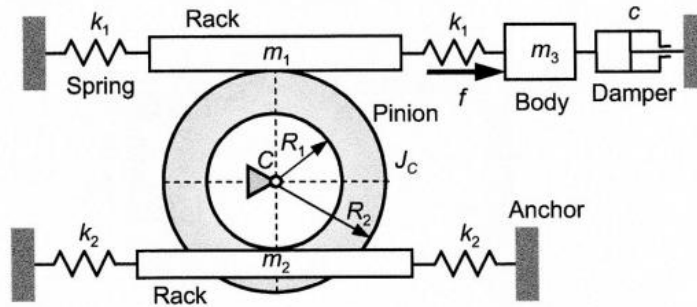


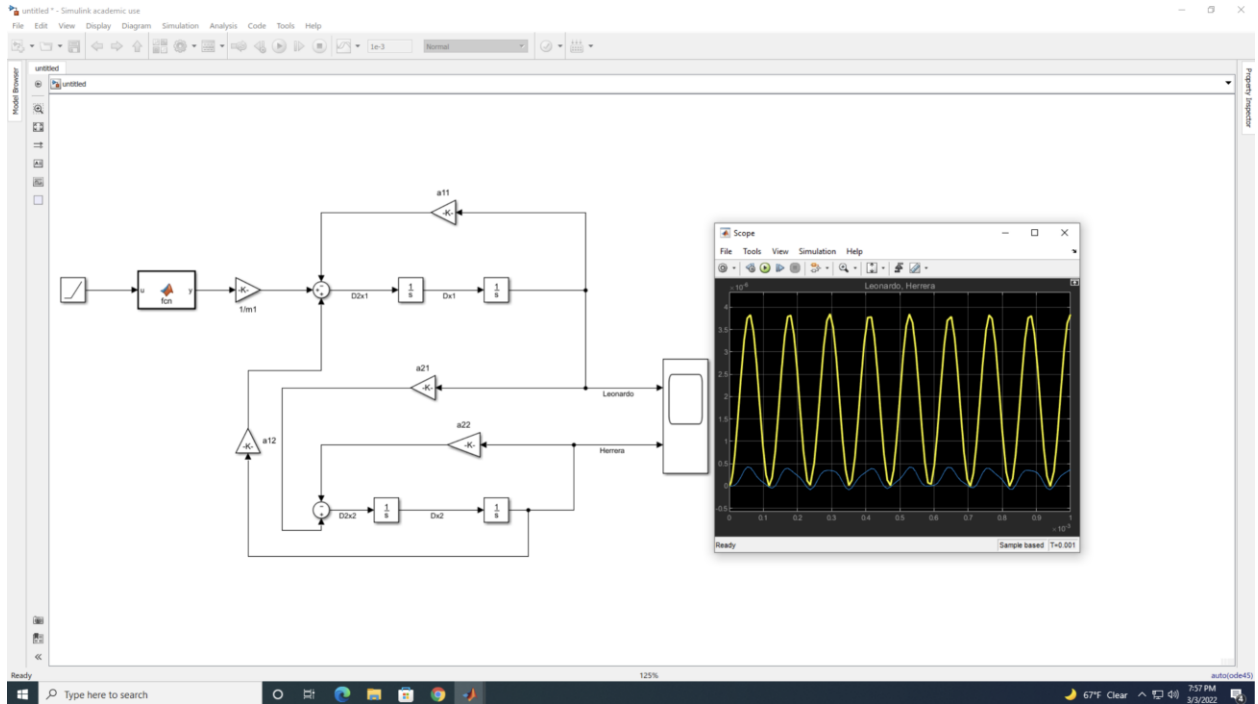
FIGURE 3.76

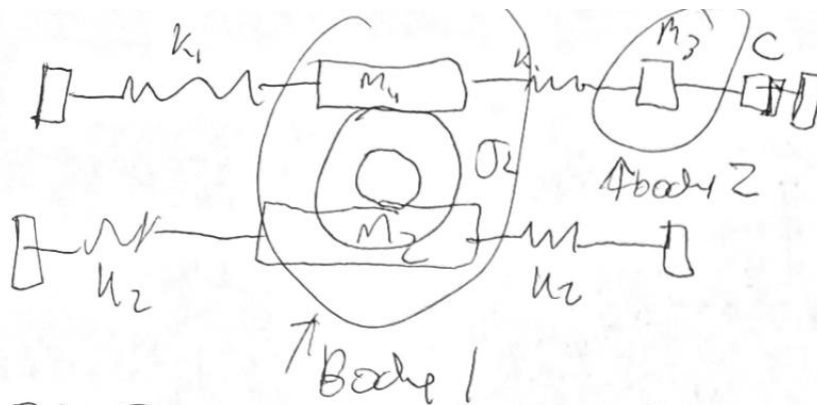
Mechanical System With Pinion, Racks, Translating Mass, Linear-Motion Springs, Damper, and External Force.

Parameters given in problem 3.36 are: $m_1 = 1 \text{ kg}$, $m_2 = 2.1 \text{ kg}$,
 $J_C = 0.05 \text{ kg m}^2$, $m_3 = 0.6 \text{ kg}$, $k_1 = 120 \text{ N/m}$, $k_2 = 100 \text{ N/m}$,
 $R_2 = 2R_1 = 0.03 \text{ m}$.

ABOVE AVERAGE

Example 3.18.





ZDOF

$$T_1 = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$U_1 = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_1 (x_1 - x_3)^2 + k_2 x_2^2$$

$$x_2 = R_1 \theta \quad \theta = \frac{x_2}{R_1} = \frac{x_1}{R_2}$$

$$x_1 = R_2 \theta$$

$$\left. \begin{aligned} T_1 &= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} J_c \frac{\dot{x}_1^2}{R_2^2} + \frac{1}{2} m_2 \left(x_1 \frac{R_1}{R_2} \right)^2 \\ U_1 &= \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_1 (x_1 - x_3)^2 + k_2 \left(x_1 \frac{R_1}{R_2} \right)^2 \end{aligned} \right\} \begin{aligned} F_1 &= 0 \\ Q_1 &= 0 \end{aligned}$$

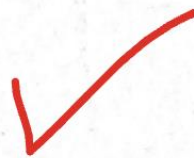
Body 2

$$T_2 = \frac{1}{2} m_3 \dot{x}_3^2$$

$$U_2 = \frac{1}{2} k_1 (x_1 - x_3)^2$$

$$F_2 = \frac{1}{2} c \dot{x}_3^2$$

$$Q_2 = f$$



$$\frac{d}{dt} \frac{\partial T_1}{\partial \dot{x}_1} + \frac{\partial V_1}{\partial x_1} + \frac{\partial F}{\partial \dot{x}_1} = 0$$

$$\frac{d}{dt} \frac{\partial T_1}{\partial \dot{x}_1} = m_1 \ddot{x}_1 + J_C \ddot{x}_1 / R_2^2 + m_2 \ddot{x}_1 \frac{R_1^2}{R_2^2}$$

$$\frac{\partial V_1}{\partial x_1} = k_1 x_1 + k_1 (x_1 - x_3) + 2k_2 x_1 \frac{R_1^2}{R_2^2}$$

$$\ddot{x}_1 \left(m_1 + \frac{J_C}{R_2^2} + \frac{m_2 R_1^2}{R_2^2} \right) + x_1 (2k_1 + 2k_2 \frac{R_1^2}{R_2^2}) - k_1 x_3 = 0$$

$$\ddot{x}_1 (57.08) + x_1 (290) - 120 x_3 = 0$$

$$\ddot{x}_1 = \frac{120 x_3 - 290 x_1}{57.08}$$

$$\ddot{x}_1 = \frac{120 x_3}{57.08} - \frac{290}{57.08} x_1 = 2.102 x_3 - 5.0805 x_1$$

$a_{11} \quad a_{12}$

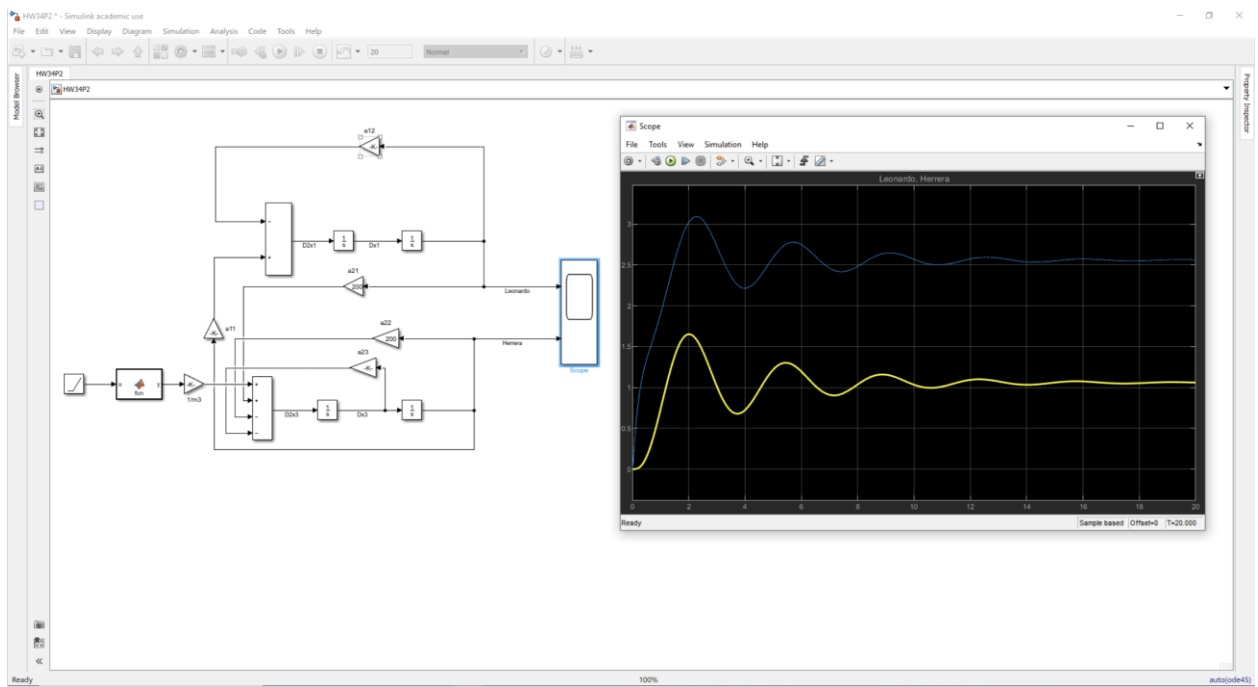
$$\frac{d}{dt} \frac{\partial T_2}{\partial \dot{x}_3} + \frac{\partial V_2}{\partial x_3} + \frac{\partial F}{\partial \dot{x}_3} = f$$

$$m_3 \ddot{x}_3 + k_1 x_3 - k_1 x_1 + c \dot{x}_3 = f$$

$$\ddot{x}_3 = \frac{f}{m_3} + \frac{k_1 x_1}{m_3} - \frac{k_1 x_3}{m_3} - \frac{c \dot{x}_3}{m_3}$$

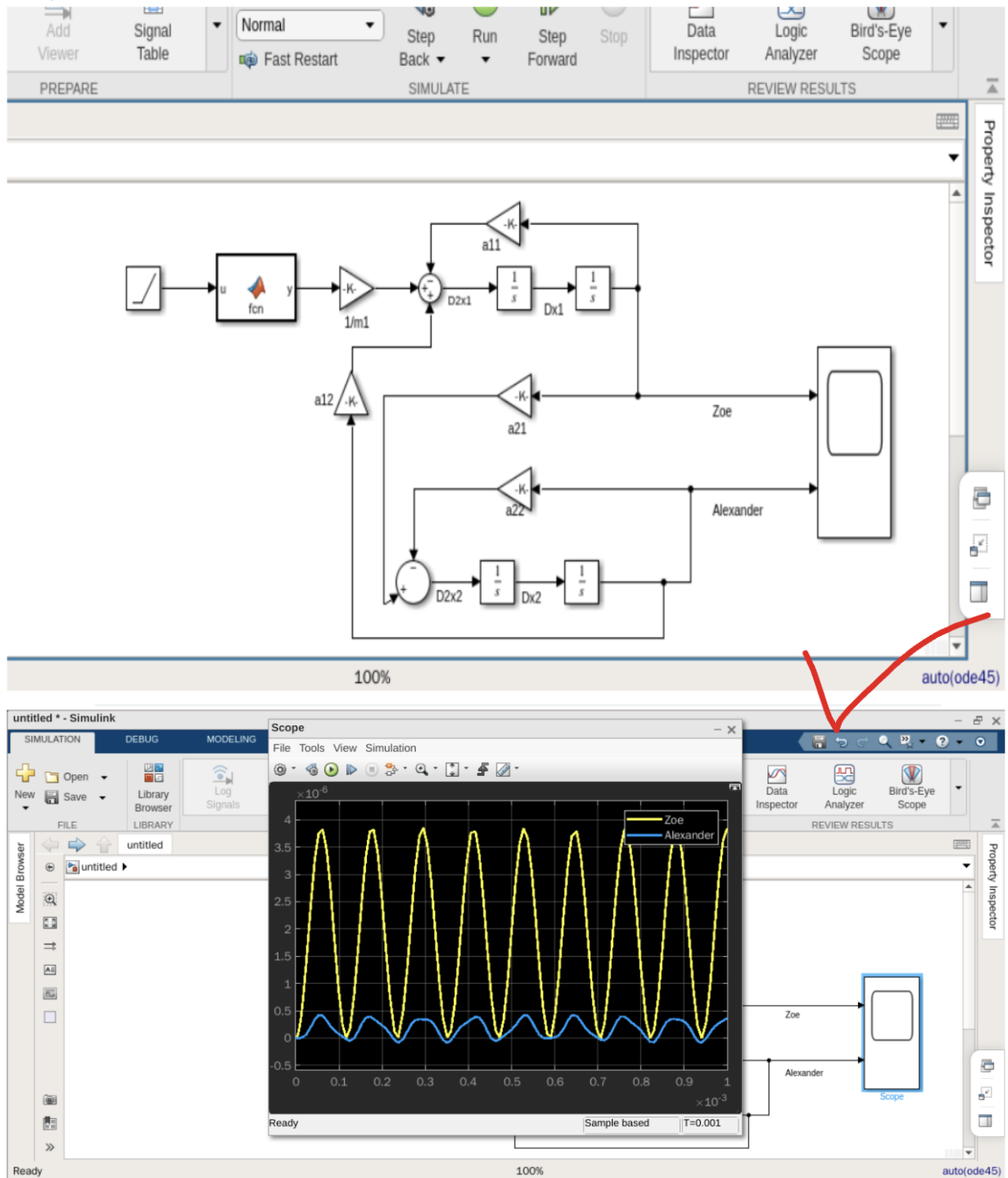
$$\ddot{x}_3 = 1.666 f + 200 x_1 - 200 x_3 - 53.33 \dot{x}_3$$

$a_{21} \quad a_{22} \quad a_{23}$



AVERAGE

3.18)



3.41)

Zoe Alexander

$$3.41) (\mathcal{J}_c + m_1 R_2^2 + m_2 R_1^2) \ddot{\theta} + (k_1 R_2^2 + 2k_2 R_1^2 - k_1 R_2) \theta = 0$$

$$(0.051375 \text{ kg} \cdot \text{m}^2) \ddot{\theta} - (3.447) \theta = 0$$

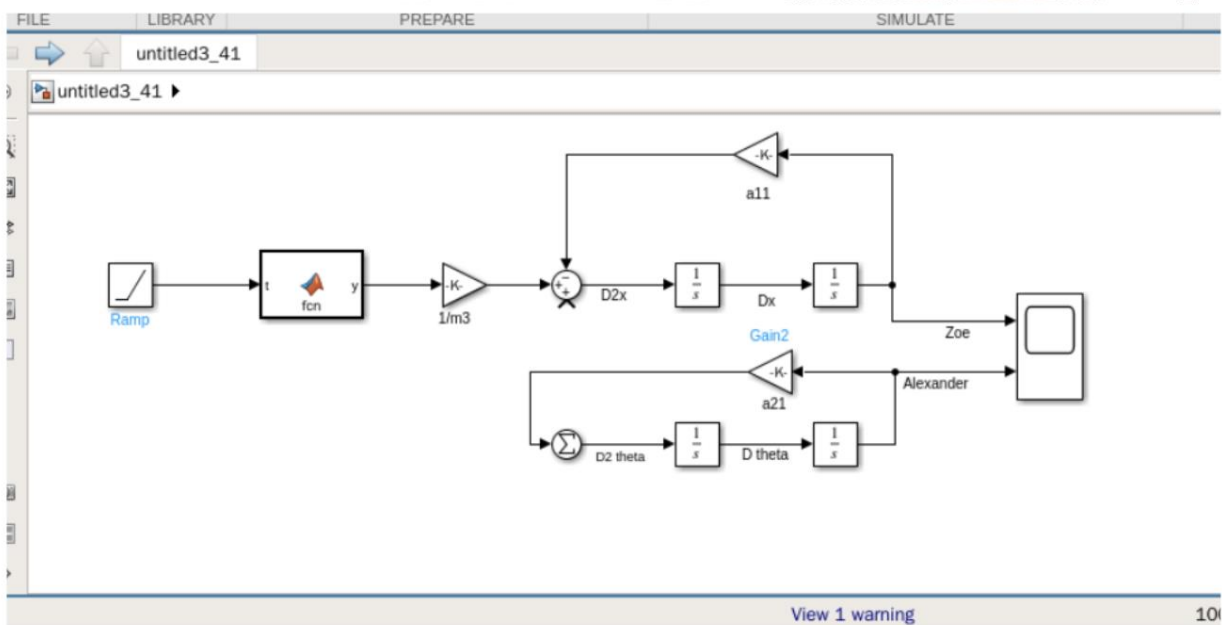
$$\ddot{\theta} = 67.098 \cdot \theta \rightarrow \ddot{\theta} = a_{21} \cdot \theta$$

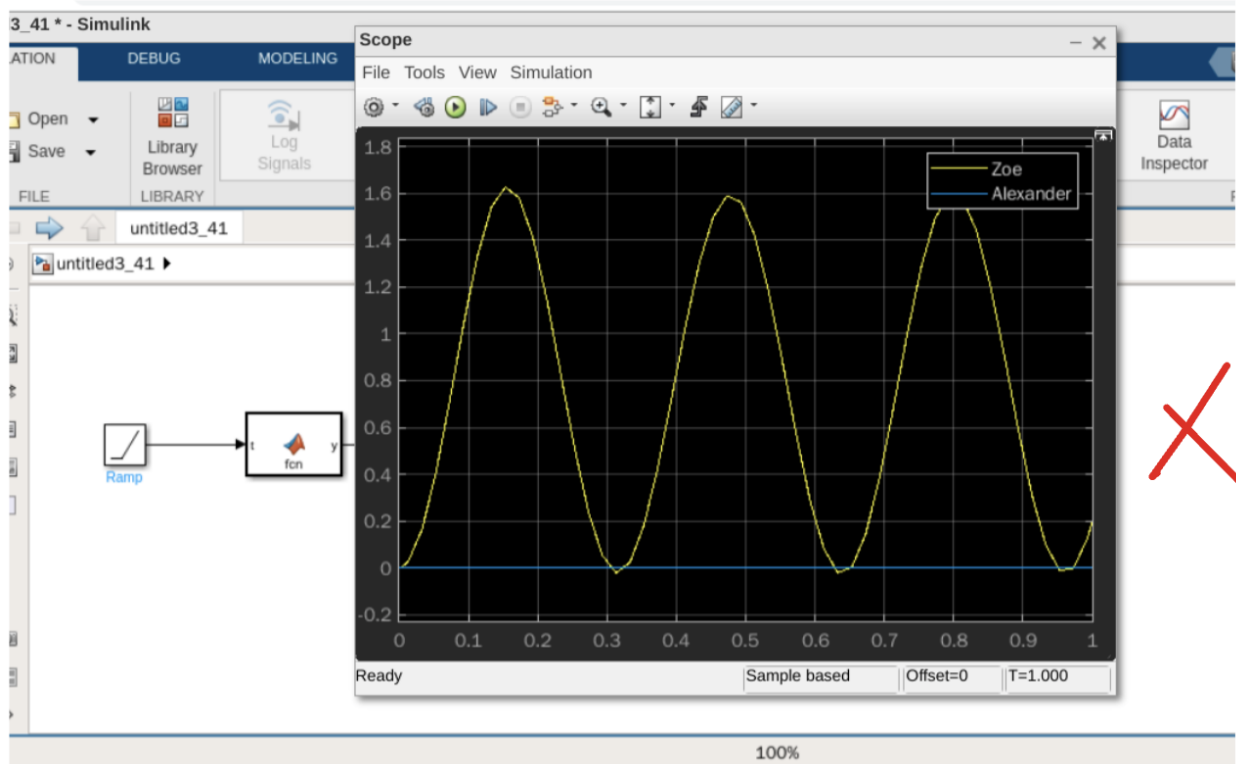
$$m_3 \ddot{x}_3 + (k_3 + k_1 - k_1 R_2) x_3 = 0$$

$$(0.6 \text{ kg}) \ddot{x}_3 + (228.2) x_3 = 0$$

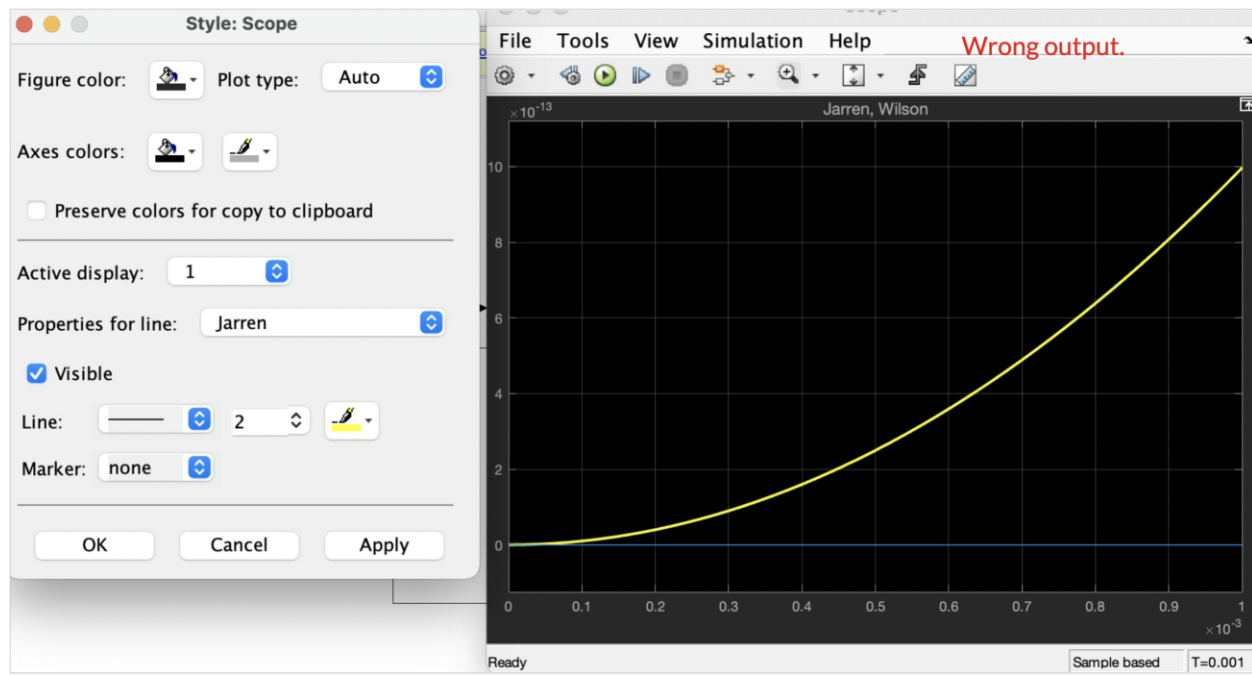
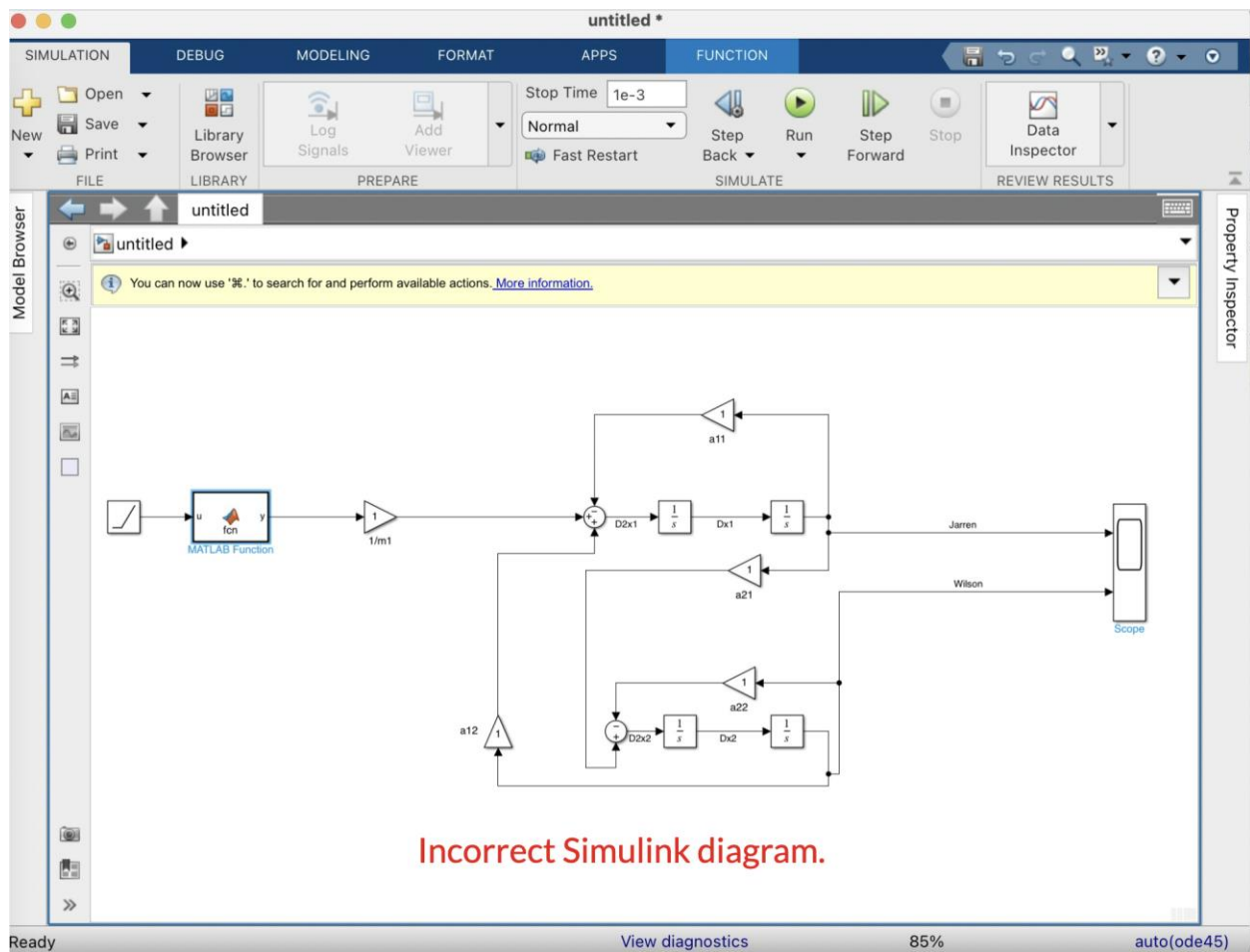
$$\ddot{x}_3 = -380.33 x_3 + \frac{1}{0.6} \cdot f \quad \text{where } f = 180 + 20 e^{-4t} \cdot \sin(10\pi t)$$

Equations are incorrect.





BELOW AVERAGE



3.41

Body 1: $T_1 = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} J_C \left(\frac{\dot{x}_1}{R_2} \right)^2 + \frac{1}{2} m_2 \left(\dot{x}_1 \frac{R_1}{R_2} \right)^2$

$U_1 = \frac{1}{2} k_1 x_1^2 + 2 \left(\frac{1}{2} k_2 \left(\frac{x_1}{R_2} \right)^2 \right) + \frac{1}{2} k_1 (x_1 - x_3)^2$

Body 2: $T_2 = \frac{1}{2} m_3 \dot{x}_3^2$ $U_2 = \frac{1}{2} k_1 (x_1 - x_3)^2$

$F_2 = \frac{1}{2} c x_3^2$ $Q_2 = 180 + 20 \cdot e^{-4t} \cdot \sin(10\pi t) \cdot U$

Body 1:

$\frac{d}{dt} \frac{\partial T_1}{\partial \dot{x}_1} + \frac{\partial U_1}{\partial x_1} + \frac{\partial F_1}{\partial x_1} = Q_1$ $\frac{\partial T_1}{\partial \dot{x}_1} = m_1 \dot{x}_1 + J_C \left(\frac{\dot{x}_1}{R_2} \right) \frac{1}{R_2} + m_2 \left(\dot{x}_1 \frac{R_1}{R_2} \right) \frac{R_1}{R_2}$

$\frac{\partial U_1}{\partial x_1} = k_1 x_1 + k_1 (x_1 - x_3) + \left(k_2 \left(\frac{x_1}{R_2} \right)^2 \right)$

Body 2:

$\frac{d}{dt} \frac{\partial T_2}{\partial \dot{x}_3} + \frac{\partial U_2}{\partial x_3} + \frac{\partial F_2}{\partial x_3} = Q_2$

$\frac{\partial T_2}{\partial \dot{x}_3} = m_3 \dot{x}_3$

$\frac{\partial U_2}{\partial x_3} = k_1 (x_1 - x_3)$

$\frac{\partial F_2}{\partial x_3} = c x_3$

did not complete equation derivation, and did not obtain Simulink results.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 7

TOPICS COVERED IN HOMEWORK 7

1. Use MATLAB to find Laplace transform of given time functions.
2. Use MATLAB to find time functions from given Laplace transform functions.

Homework

6.9 Use MATLAB to find the Laplace transform of the following time functions.

(i) $f(t) = e^{-t} \cdot \cos(\omega \cdot t + \phi)$.

(ii) $f(t) = e^{-5t} \cdot t^{1/2}$.

(iii) $f(t) = e^{-4t} \cdot \sin(2t) \cdot \cos(3t)$.

6.11 Use MATLAB to find the Laplace transform of the time functions in Figure 6.12.

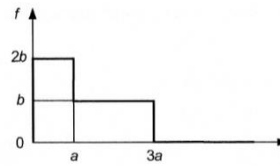


FIGURE 6.12

Multistep Function.

6.28 Use inverse Laplace transform function in MATLAB to find the time functions of the following Laplace transforms.

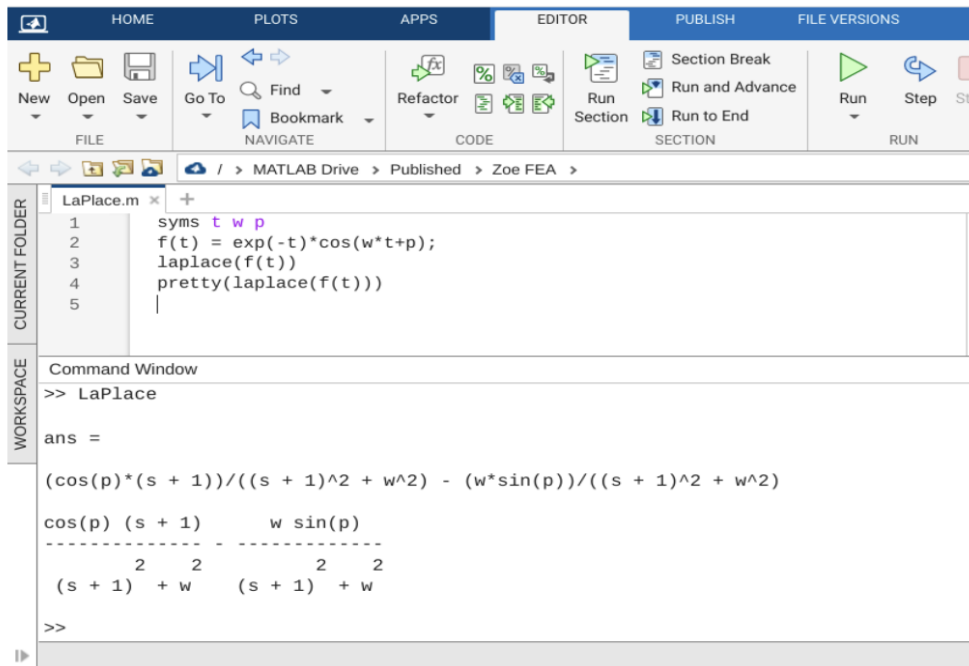
(i) $Y(s) = 2\omega \cdot s / (s^2 + \omega^2)^2$.

(ii) $Y(s) = 1 / (s^3 + a \cdot s^2)$.

ABOVE AVERAGE

6.9

i)



The MATLAB Editor interface shows the following code in the Editor window:

```

1 syms t w p
2 f(t) = exp(-t)*cos(w*t+p);
3 laplace(f(t))
4 pretty(laplace(f(t)))
5

```

The Command Window displays the result:

```

>> LaPlace

ans =

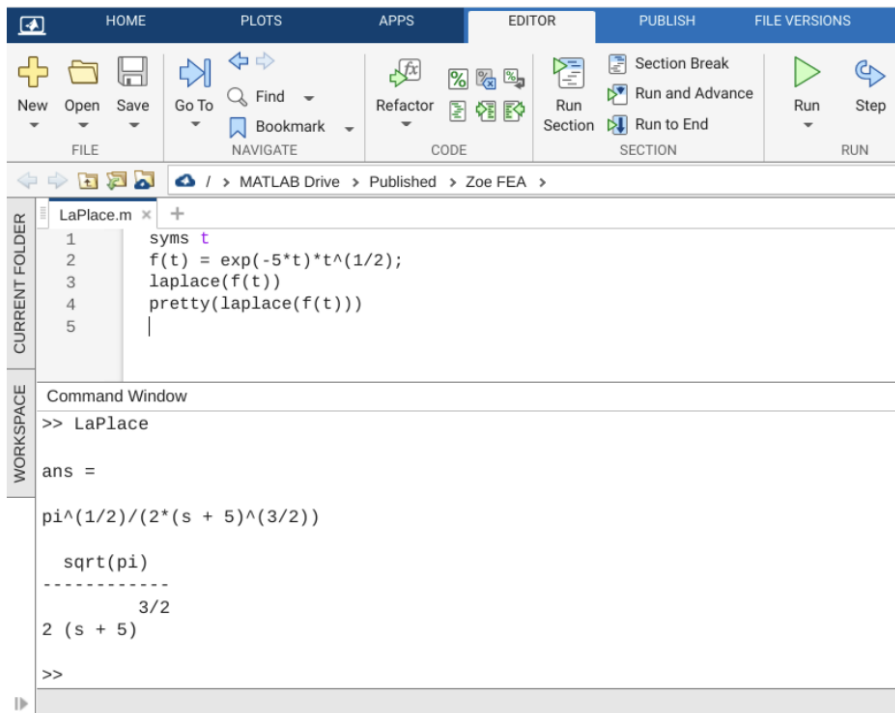
(cos(p)*(s + 1))/((s + 1)^2 + w^2) - (w*sin(p))/((s + 1)^2 + w^2)

cos(p) (s + 1)      w sin(p)
-----
      2      2      2      2
(s + 1) + w      (s + 1) + w

```



ii)



The MATLAB Editor interface shows the following code in the Editor window:

```

1 syms t
2 f(t) = exp(-5*t)*t^(1/2);
3 laplace(f(t))
4 pretty(laplace(f(t)))
5

```

The Command Window displays the result:

```

>> LaPlace

ans =

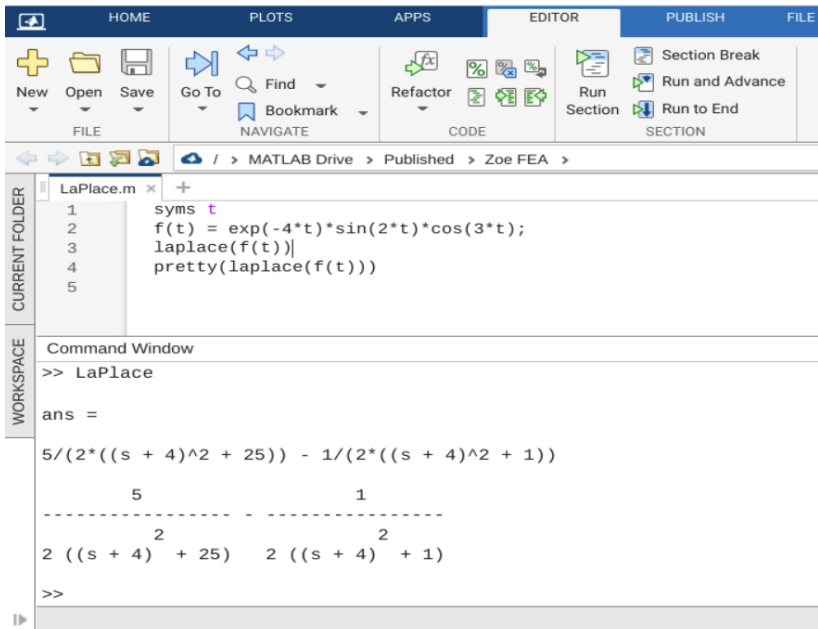
pi^(1/2)/(2*(s + 5)^(3/2))

sqrt(pi)
-----
      3/2
2 (s + 5)

```



iii)



The image shows the MATLAB Editor interface. The top menu bar includes HOME, PLOTS, APPS, EDITOR, PUBLISH, and FILE. The EDITOR tab is active, showing a script named 'LaPlace.m'. The script contains the following code:

```
1 syms t
2 f(t) = exp(-4*t)*sin(2*t)*cos(3*t);
3 laplace(f(t))
4 pretty(laplace(f(t)))
5
```

The Command Window shows the output of the script:

```
>> LaPlace

ans =

5/(2*((s + 4)^2 + 25)) - 1/(2*((s + 4)^2 + 1))

-----
2 ((s + 4)2 + 25) 2 ((s + 4)2 + 1)

>>
```



6.11

$$f(t) = \begin{cases} 2b, & 0 \leq t \leq a \\ b, & a \leq t \leq 3a \end{cases} \quad \begin{matrix} f_1 = 2b \\ f_2 = b \end{matrix}$$

$$\mathcal{L}\{f(t)\} = \int_0^a 2b \cdot e^{-s \cdot t} dt + \int_a^{3a} b \cdot e^{-s \cdot t} dt$$

Scanned with CamScanner

HOME PLOTS APPS EDITOR PUBLISH FILE VERSIONS

FILE NAVIGATE CODE SECTION RUN

Matlab Drive > Published > Zoe FEA >

LaPlace.m x +

```

1 syms a b s t
2 f1 = 2*b; f2 = b;
3 Fs = simplify(int(f1*exp(-s*t),t,0,a)+int(f2*exp(-s*t),t,a,3*a));
4 pretty(Fs)

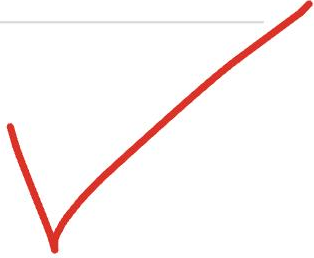
```

Command Window

```

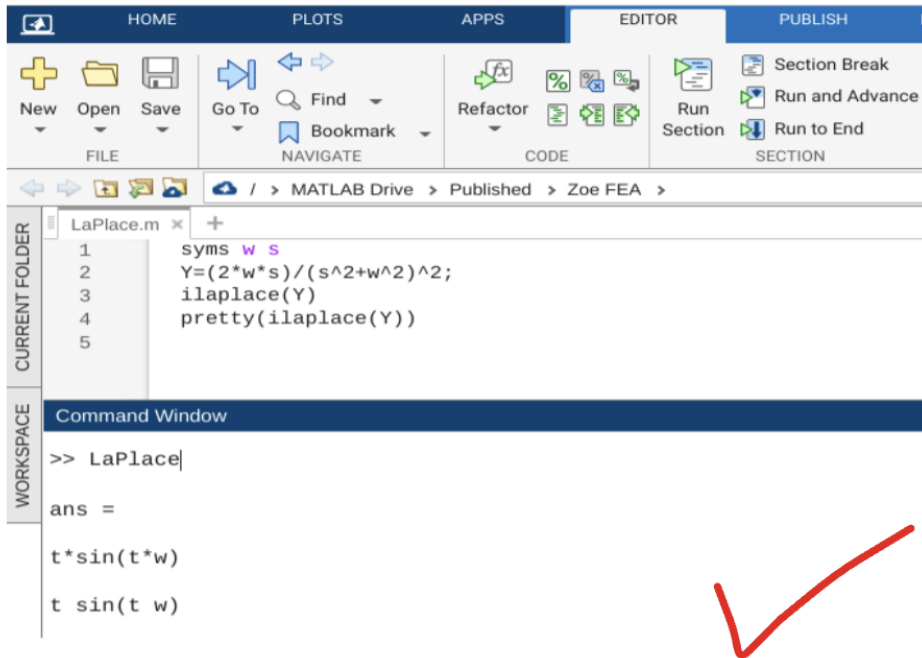
>> LaPlace
    b exp(-a s) - 2 b + b exp(-3 a s)
-----
                      s
>>

```



6.28

i)



The MATLAB Editor interface shows the following components:

- Toolbars:** HOME, PLOTS, APPS, EDITOR, PUBLISH.
- Current Folder:** LaPlace.m
- Code Editor:**

```

1 syms w s
2 Y=(2*w*s)/(s^2+w^2)^2;
3 ilaplace(Y)
4 pretty(ilaplace(Y))
5

```
- Command Window:**

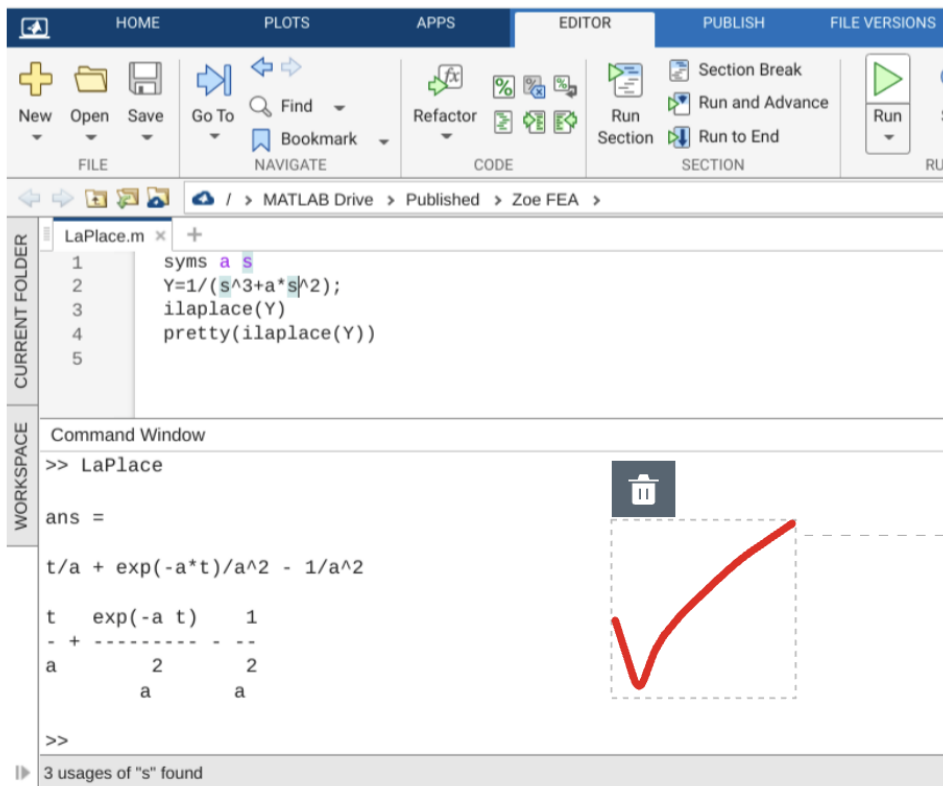
```

>> LaPlace
ans =
t*sin(t*w)
t sin(t w)

```

A red checkmark is drawn next to the Command Window output.

ii)



The MATLAB Editor interface shows the following components:

- Toolbars:** HOME, PLOTS, APPS, EDITOR, PUBLISH, FILE VERSIONS.
- Current Folder:** LaPlace.m
- Code Editor:**

```

1 syms a s
2 Y=1/(s^3+a*s^2);
3 ilaplace(Y)
4 pretty(ilaplace(Y))
5

```
- Command Window:**

```

>> LaPlace
ans =
t/a + exp(-a*t)/a^2 - 1/a^2
t exp(-a t) 1
- + ----- -
a      2      2
a      a

```

A red checkmark is drawn next to the Command Window output. A dashed box highlights the output, and a trash icon is visible above it.

AVERAGE

6.9 Use MATLAB to find the Laplace transform of the following time functions.

(i) $f(t) = e^{-t} \cdot \cos(\omega \cdot t + \phi)$.

(ii) $f(t) = e^{-5t} \cdot t^{1/2}$.

(iii) $f(t) = e^{-4t} \cdot \sin(2t) \cdot \cos(3t)$.

```

NAVIGATE EDIT BREAKPO
Users > Zion > Documents >
Editor - C:\Users\Zion\Documents\laplacefunctions.m
laplacefunctions.m x +
1 %Homework Problem (i,ii,iii)
2 syms w t phi
3 % (i)
4 f1 = exp(-t)*cos(w*t+phi);
5 f2 = exp(-5*t)*t^(1/2);
6 f3 = exp(-4*t)*sin(2*t)*cos(3*t);
7
8 %Display Laplace of function 1
9 disp 'Function 1:'
10 L1 = laplace(f1);
11 pretty(L1)
12
13 %Display Laplace of function 2
14 disp 'Function 2:'
15 L2 = laplace(f2);
16 pretty(L2)
17
18 %Display Laplace of function 3
19 disp 'Function 3:'
20 L3 = laplace(f3);
21 pretty(L3)

```

```

Users > Zion > Documents >
Editor - C:\Users\Zion\Documents\laplacefunctions.m
Command Window
>> laplacefunctions
Function 1:
cos(phi) (s + 1) w sin(phi)
-----
      2      2      2      2
(s + 1) + w (s + 1) + w

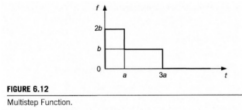
Function 2:
sqrt(pi)
-----
      3/2
2 (s + 5)

Function 3:
      5      1
-----
      2      2
2 ((s + 4) + 25) 2 ((s + 4) + 1)

fx>>

```

6.11 Use MATLAB to find the Laplace transform of the time functions in Figure 6.12.



$$f(t) = \begin{cases} 2b, & 0 < t < a \\ b, & 0 < t < 3a \end{cases}$$

$$L[f(t)] = \int_0^a f_1(t) \cdot e^{-s \cdot t} dt + \int_a^{3a} f_2(t) \cdot e^{-s \cdot t} dt + \int_{3a}^{\infty} f_3(t) \cdot e^{-s \cdot t} dt$$

$$\hookrightarrow L[f(t)] = \int_0^a 2b \cdot e^{-s \cdot t} dt + \int_a^{3a} b \cdot e^{-s \cdot t} dt$$

$$\hookrightarrow = 2b \left[\frac{e^{-s \cdot t}}{-s} \right]_0^a + b \left[\frac{e^{-s \cdot t}}{-s} \right]_a^{3a}$$

$$= \frac{2b}{-s} (e^{-as} - 1) + \frac{b}{-s} (e^{-3as} - e^{-as})$$

$$= \frac{2b}{s} - \frac{2be^{-as}}{s} - \frac{be^{-3as}}{s} + \frac{be^{-as}}{s}$$

$$f(s) = \frac{b}{s} (2 - e^{-as} - e^{-3as})$$

```
C:\Users\Zion\Documents
Editor - C:\Users\Zion\Documents\laplacefunctions.m
laplacefunctions.m x +
1 syms b t
2
3 f = 2*b*heaviside(t)-b*heaviside(t-1)-b*heaviside(t-3)
4
5 disp ' laplace multistep function'
6
7 L = laplace(f);
8
9 pretty(L)
```

Command Window

```
>> laplacefunctions
laplace multistep function
2 b b exp(-s) b exp(-3 s)
-----
s s s
```

The time intervals are from 0 to a, (not 0 to 1) & a to 3a (not 1 to 3).

6.28 Use inverse Laplace transform function in MATLAB to find the time functions of the following Laplace transforms.

(i) $Y(s) = 2\omega \cdot s / (s^2 + \omega^2)^2$.

(ii) $Y(s) = 1 / (s^3 + a \cdot s^2)$.

```
C:\Users\Zion\Documents
Editor - C:\Users\Zion\Documents\laplacefunctions.m
laplacefunctions.m x +
1 syms w s a
2
3 Y1 = 2*w*s/(s^2+w^2)^2;
4 Y2 = 1/(s^3+a*s^2);
5
6 disp ' Inverse laplace of Y1'
7
8 inv1 = ilaplace(Y1);
9 pretty(inv1)
10
11 disp ' Inverse laplace of Y2'
12
13 inv2 = ilaplace(Y2);
14 pretty(inv2)
```

Command Window

```
Inverse laplace of Y1
t sin(t w)

Inverse laplace of Y2
t exp(-a t) 1
- + ----- -
a 2 2
a a
```


BELOW AVERAGE

PUBLISH VIEW

Breakpoints Run Run and Advance Run Section Advance Run and Time

Editor - H:\homework.m

homework.m

```

1 - clc;clear;close;
2 - %Prob(6.9)
3 - %define variables
4 - syms t w phi
5 - %write f(t)
6 - f1 = exp(-t)*cos(w*t+phi);
7 - f2 = exp(-5*t)*t^(1/2);
8 - f3 = exp(-4*t)*sin(2*t)*cos(3*t);
9 - %calculate laplace
10 - fprintf('Laplace of f1\n\n')
11 - L1 = laplace(f1);
12 - pretty(L1)
13 - fprintf('Laplace of f2\n\n')
14 - L2 = laplace(f2);
15 - pretty(L2)
16 - fprintf('Laplace of f3\n\n')
17 - L3 = laplace(f3);
18 - pretty(L3)

```

Command Window

New to MATLAB? See resources for [Getting Started](#).

Laplace of f1

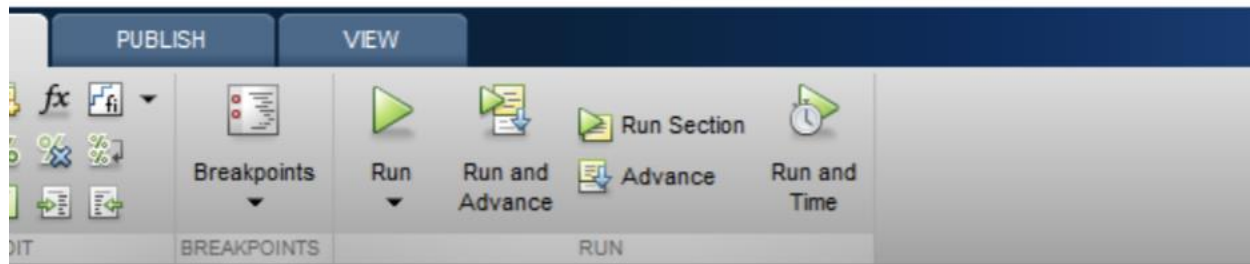
$$\frac{\cos(\phi) (s + 1)}{(s + 1)^2 + w^2} - \frac{w \sin(\phi)}{(s + 1)^2 + w^2}$$

Laplace of f2

$$\frac{\sqrt{\pi}}{2 (s + 5)^{3/2}}$$

Laplace of f3

$$\frac{5}{2 ((s + 4)^2 + 25)} - \frac{1}{2 ((s + 4)^2 + 1)}$$

```
Editor - H:\6.11.m
homework.m x 6.11.m x +
1 %Prob(6.11)
2 %define variables
3 - syms b t
4 %here take a = 1
5 %write f
6 - f = 2*b*heaviside(t)-b*heaviside(t-1)-b*heaviside(t-3);
7 %determine laplace
8 - fprintf('laplace of multistep function\n\n')
9 L = laplace(f);
10 - pretty(L)
11
```

cannot assume a=1

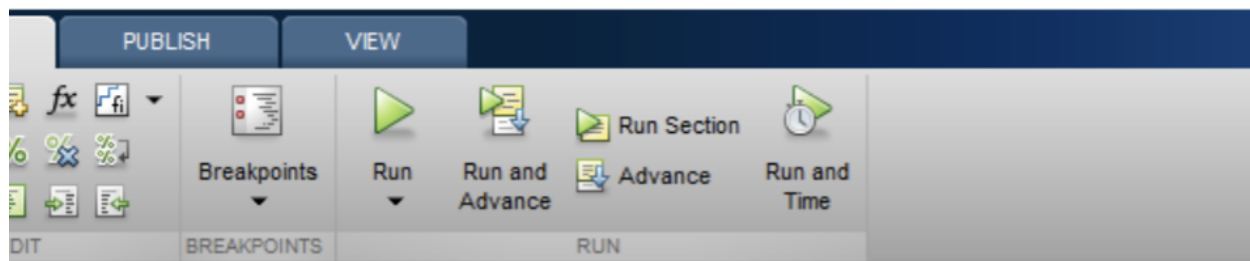
```
Command Window
New to MATLAB? See resources for Getting Started.

>> 6.11

ans =

    6.1100
```

?



```
Editor - H:\6.28.m
homework.m 6.11.m 6.28.m +
1 %Prob(6.28)
2 %define variables
3 syms w s a
4 %write Y(s)
5 Y1 = 2*w*s/(s^2)+w^2)^2;
6 Y2 = 1/(s^3+a*s^2);
7 %calculate inverse laplace
8 fprintf('Inverse laplace of Y1\n\n')
9 I1 = ilaplace(Y1);
10 pretty(I1)
11 fprintf('Inverse laplace of Y2\n\n')
12 I2 = ilaplace(Y2);
13 pretty(I2)
```

Command Window

New to MATLAB? See resources for [Getting Started](#).

```
>> 6.28

ans =

    6.2800
```

(A large red question mark is drawn next to the output 6.2800)

(A red dot is drawn below the question mark)

fx >>

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 8

TOPICS COVERED IN HOMEWORK 8

1. Formulate transfer function of a dynamic system, and plot the output using impulse as the input function in MATLAB.

Homework

- 7.35** The MEMS of Figure 7.46 is formed of two pairs of beams and a shuttle mass and is subjected to capacitive actuation. Use a lumped-parameter model to formulate the system's mathematical model for the motion about the y direction by also taking into account the damping between the fixed and mobile capacitor plates. All beams have a circular cross section of diameter $d = 1.8 \mu\text{m}$. Known also are the shuttle mass $m = 8 \cdot 10^{-11} \text{ kg}$, $l_1 = 180 \mu\text{m}$, $l_2 = 90 \mu\text{m}$, Young's modulus $E = 160 \text{ GPa}$, and the damping coefficient $c = 0.4 \text{ N s/m}$. Plot the system's response using its transfer function and MATLAB's `impz` function when an impulsive electrostatic force $f = 1.2 \cdot 10^{-6} \delta(t) \text{ N}$ is applied by the capacitive unit superimposed to an initial displacement $y(0) = 1.4 \mu\text{m}$. The transfer function is $G(s) = Y(s)/F(s)$. Ignore the mass contribution from the springs and verify whether the initial capacitive gap $g_0 = 10 \mu\text{m}$ is sufficient for the mobile plate motion.

(Add your name
to the title of the plots)

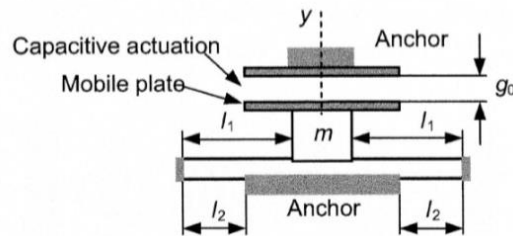


FIGURE 7.46

Physical Model of Translatory Mechanical Microsystem With Capacitive Actuation.

ABOVE AVERAGE

Homework

$$d = 1.8 \mu\text{m}$$

$$m = 8 \cdot 10^{-11} \text{ kg}$$

$$l_1 = 180 \mu\text{m}$$

$$l_2 = 90 \mu\text{m}$$

$$E = 160 \text{ GPa}$$

$$c = 0.4 \text{ N s/m}$$

$$f = 1.2 \cdot 10^{-6} \delta(t) \text{ N}$$

$$y(0) = 1.4 \mu\text{m}$$

$$G(s) = \frac{Y(s)}{F(s)}$$

$$g_0 = 10 \mu\text{m}$$

- 7.35** The MEMS of Figure 7.46 is formed of two pairs of beams and a shuttle mass and is subjected to capacitive actuation. Use a lumped-parameter model to formulate the system's mathematical model for the motion about the y direction by also taking into account the damping between the fixed and mobile capacitor plates. All beams have a circular cross section of diameter $d = 1.8 \mu\text{m}$. Known also are the shuttle mass $m = 8 \cdot 10^{-11} \text{ kg}$, $l_1 = 180 \mu\text{m}$, $l_2 = 90 \mu\text{m}$, Young's modulus $E = 160 \text{ GPa}$, and the damping coefficient $c = 0.4 \text{ N s/m}$. Plot the system's response using its transfer function and MATLAB's `impz` function when an impulsive electrostatic force $f = 1.2 \cdot 10^{-6} \delta(t) \text{ N}$ is applied by the capacitive unit superimposed to an initial displacement $y(0) = 1.4 \mu\text{m}$. The transfer function is $G(s) = Y(s)/F(s)$. Ignore the mass contribution from the springs and verify whether the initial capacitive gap $g_0 = 10 \mu\text{m}$ is sufficient for the mobile plate motion.

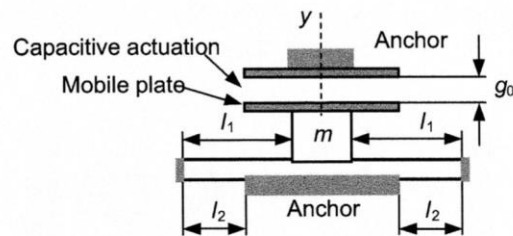


FIGURE 7.46

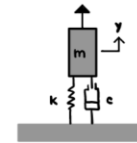
Physical Model of Translatory Mechanical Microsystem With Capacitive Actuation.

$$U(s) = 1/s \text{ for } u(t) = h(t)$$

$$U(s) = 1 \text{ for } \delta(t) \text{ and } h(t)$$

$$U(s) = 1 \rightarrow G_{eq}(s) = G(s) \cdot [a_2 \cdot y(0) \cdot s + a_2 \cdot \dot{y}(0) + a_1 \cdot y(0) + 1]$$

$$= \frac{a_2 \cdot y(0) \cdot s + a_2 \cdot \dot{y}(0) + a_1 \cdot y(0) + 1}{a_2 \cdot s^2 + a_1 \cdot s + a_0}$$



$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = f(t)$$

AFTER L(s) w/ ZERO INITIAL CONDITIONS

$$m(s^2 Y(s) - y(0)s) + c(sY(s) - \dot{y}(0)) + kY(s) = F(s)$$

$$Y(s) = \frac{m\dot{y}(0) + cy(0) + F(s)}{ms^2 + cs + k} \rightarrow \frac{m\dot{y}(0)s + cy(0) + f}{ms^2 + cs + k} \cdot 1 = G(s)U(s)$$

$$\frac{m\dot{y}(0)s + cy(0) + f}{ms^2 + cs + k} \cdot 1 = G(s)U(s)$$

$$k = 2 \frac{k_1 k_2}{k_1 + k_2} = 2 \frac{\frac{12EI}{l_1^3} \cdot \frac{12EI}{l_2^3}}{\frac{12EI}{l_1^3} + \frac{12EI}{l_2^3}} = \frac{24E}{l_1^3 + l_2^3} \cdot \frac{\pi d^4}{64} = \frac{3\pi d^4 E}{8(l_1^3 + l_2^3)}$$

$$m = 8e^{-11}$$

$$l_1 = 180e^{-6}$$

$$l_2 = 90e^{-6}$$

$$E = 160e^9$$

$$c = 0.4$$

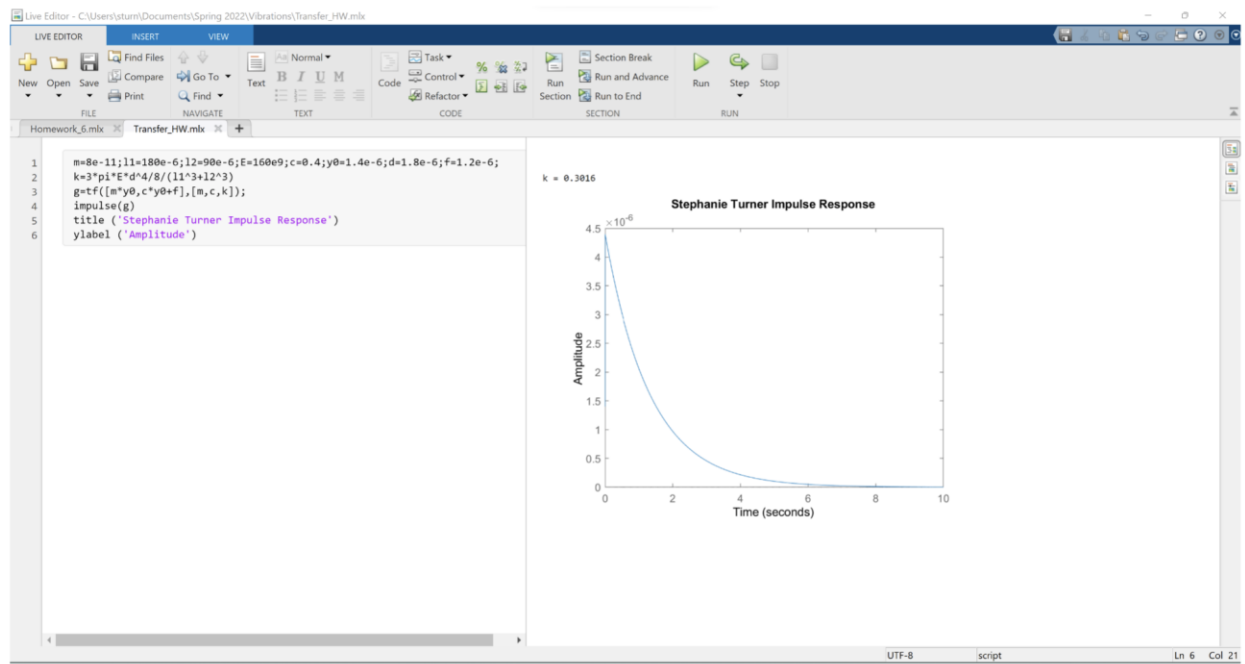
$$y_0 = 1.4e^{-6}$$

$$d = 1.8e^{-6}$$

$$f = 1.2e^{-6}$$

$$k = 3\pi E d^4 / 8 / (l_1^3 + l_2^3)$$

INPUTS



Is the gap sufficient?

AVERAGE

$$1/K_{\text{half}} = 1/k_1 + 1/k_2$$

$$K_e = 2 * K_{\text{half}}$$

m For calculating K_e we have.

$$\frac{1}{K_e} = \frac{2}{K_1} + \frac{2}{K_2} \quad \text{--- (i)}$$

where K_1 and K_2 are defined as

$$K_1 = \frac{12 E \cdot l_1}{l_1^3} \quad ; \quad K_2 = \frac{12 E \cdot l_2}{l_2^3} \quad \text{--- (ii)}$$

$$\text{and } l_2 = \frac{\pi d^4}{64} = \frac{\pi (1.8 \times 10^{-6})^4}{64}$$

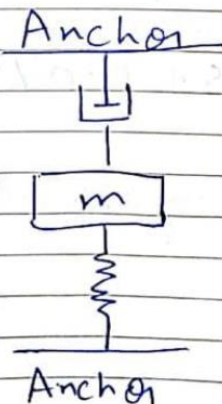
$$l_2 = 5152.99 \times 10^{-28}$$

By putting l_1 , l_2 and E in eq (ii) we get

$$K_1 = 0.1696 \quad ; \quad K_2 = 10357$$

$$\text{Hence } \frac{1}{K_e} = 13.2662 \text{ and } K_e = 0.07538$$

Now to simplify our system



we can write equation as

$$m s^2 + C s + K = u(t)$$

Now by using following eq

$$\frac{a_2 \cdot y(0) \cdot s + a_2 \cdot \dot{y}(0) + a_1 \cdot y(0) + 1}{a_2 \cdot s^2 + a_1 \cdot s + a_0} \quad \text{--- (iii)}$$

Here

$$a_2 = m = 8 \times 10^{-11} \text{ OR } 80 \times 10^{-12}$$

$$a_1 = C = 0.4$$

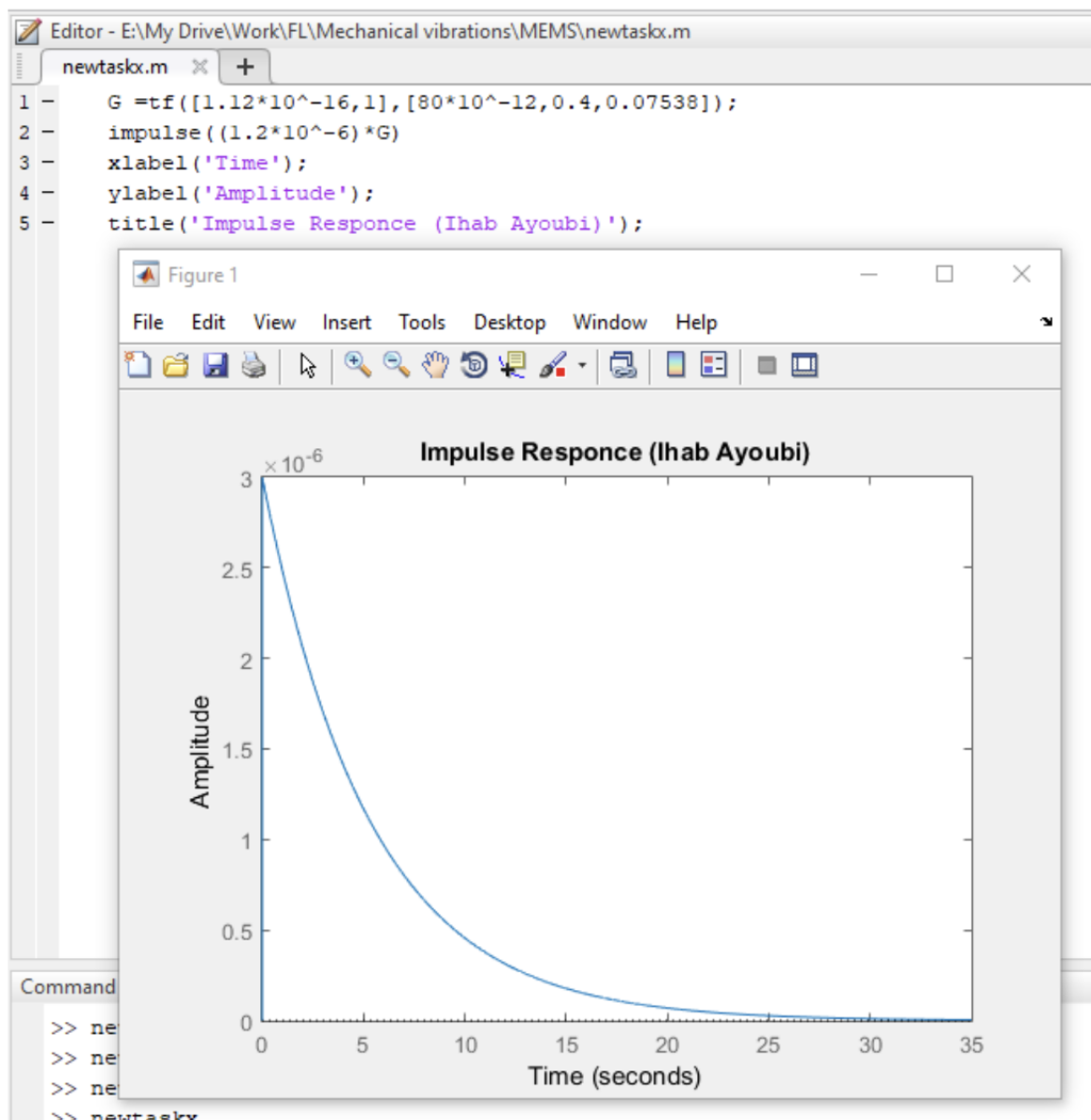
$$a_0 = K = 0.07538$$

By putting above values and $y(0) = 1.4 \times 10^{-6}$,
 $\dot{y}(0) = 0$ in eq (iii) we get

$$\frac{(1.12 \times 10^{-16}) s + \text{missing } a_1 \text{ and } y(0) \text{ in transfer function.}}{(80 \times 10^{-12}) s^2 + (0.4) s + 0.07538}$$

$$(80 \times 10^{-12}) s^2 + (0.4) s + 0.07538$$

Plotting above eq in MATLAB we get.



Yes, the initial gap $g_0 = 10\mu\text{m}$ is enough for the movement of plate as max amplitude is $3\mu\text{m}$.

The correct max amplitude is $4.4 \text{ e-}6\text{m}$.

BELOW AVERAGE

7.35)

Kinematic $T = \frac{1}{2} m \dot{y}^2$ $\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) = m \ddot{y}$

Potential $V = \frac{1}{2} k_1 y^2 + \frac{1}{2} k_2 y^2$ $\frac{\partial V}{\partial y} = k_1 y + k_2 y$ K for fixed guided beam

Dissipation $F = \frac{1}{2} c \dot{y}^2 - f \dot{y}$ $\frac{\partial F}{\partial \dot{y}} = c \dot{y} + f$

$k_1 = \frac{12 E I}{L_1^3} = \frac{12 E \pi d_1^4}{64 L_1^3} = \frac{3 E \pi d_1^4}{16 L_1^3}$

$k_2 = \frac{3 E \pi d_2^4}{16 L_2^3}$

$\mathcal{L} \{ m \ddot{y} \} + \mathcal{L} \{ k_1 y \} + \mathcal{L} \{ k_2 y \} + \mathcal{L} \{ c \dot{y} \} - \mathcal{L} \{ f \} = \mathcal{L} \{ 0 \}$

$m s^2 Y(s) + k_1 Y(s) + k_2 Y(s) + c s Y(s) - F(s) = 0$

$(m s^2 + c s + k_1 + k_2) Y(s) = F(s)$

$G(s) = \frac{Y(s)}{F(s)} = \frac{1}{m s^2 + c s + k_1 + k_2}$

Missing initial condition $y(0)$ in Laplace transform.

Scanned with CamScanner

HOME PLOTS APPS EDITOR PUBLISH FILE VERSIONS VIEW

New Open Save Go To Find Refactor Run Section Run Run to End Run Step Stop

FILE NAVIGATE CODE SECTION RUN

MATLAB Drive > Published > Zoe FEA >

CH7HW.m

```

1 m = 8e-11; l1 = 180e-6; l2 = 90e-6;
2 d = 1.8e-6; E = 160e9; c = 0.4;
3 f = 1.2e-6; y0 = 1e-6; g0 = 10e-6;
4 k1 = (3*E*d^4)/(16*l1^3);
5 k2 = (3*E*d^4)/(16*l2^3);
6
7 G = tf(1, [m, c, k1, k2]);
8 impulse(G);
9 title('Zoe Alexander Impulse Response');

```

Missing pi in the calculation

$K_{eq} = 2 * k_1 * k_2 / (k_1 + k_2)$

Figure 1: Zoe Alexander Impulse Response

COMMAND WINDOW UTF-8 LF script Ln 5 Col 26

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

Title of Assignment: Homework 9

- 1. Create bode plot in frequency domain for the transfer function between transmitted force and applied force.**

Homework for Chapter 9

- 9.28** A sinusoidal force acts on the mechanical system illustrated in Figure 9.49. Considering that $m = 0.6$ kg and $k = 220$ N/m, analyze the transmissibility of the input force to the force on the wall for $c_1 = 10$ N s/m and then for $c_2 = 15$ N s/m during the steady-state regime. (when a dynamic system in the steady-state, initial conditions have no effect to the output.)

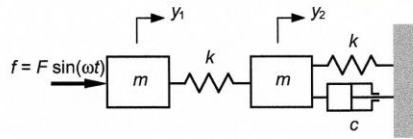
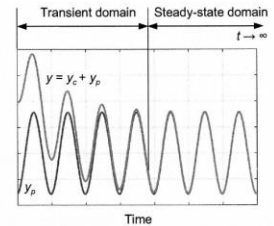


FIGURE 9.49

Mechanical System With Sinusoidal Input.



- (1) Make Bode Plot for Magnitude (absolute value) in MATLAB by adding:

`opts = bodeoptions('cstprefs');`

`opts.MagUnits = 'abs';`

- (2) Include your name in the title by adding:

`opts.Title.String = 'your name Bode Plot';`

ABOVE AVERAGE

$$G = \frac{F_b(s)}{F(s)}$$



$$m(s^2 Y_1) = -k(Y_1 - Y_2) + F$$

$$(m \cdot s^2 + k) Y_1 = k Y_2 + F$$

$$m(s^2 Y_2) = -k(Y_2 - Y_1) - c Y_2 - c s Y_2$$

$$(m s^2 + 2k) Y_2 = (k - c s) Y_1$$

$$f_b = f_d + f_c$$

$$f_b = c \dot{y}_2 + k y_2 \quad F_b = c(s \cdot Y_2 \rightarrow \dot{y}_2) + k Y_2$$

HW ch. 9

$$m = 0.6 \text{ kg}, k = 220 \text{ N/m}, c_1 = 10 \text{ N/sm}, c_2 = 15 \text{ N}$$

$$m \ddot{y}_1 + f + f_s$$

$$m \ddot{y}_2 + f_s + f_d$$

$$\begin{cases} m \ddot{y}_1 = -k(y_1 - y_2) + f \\ m \ddot{y}_2 = -k(y_2 - y_1) - c \dot{y}_2 \end{cases}$$

$$f_b = c \dot{y}_2 + k y_2$$

$$\textcircled{1} (m \cdot s^2 + k) Y_1 = k Y_2 + F$$

$$(m \cdot s^2 + c \cdot s + 2k) Y_2 = k Y_1 \quad \rightarrow Y_2 = \frac{k}{m s^2 + c s + 2k} Y_1$$

$$F_b = (c \cdot s + k) Y_2 \quad \rightarrow F_b = (c s + k) \left(\frac{k}{m s^2 + c s + 2k} \right) Y_1$$

$$F_b(s) = \frac{(c \cdot s + k)(k)}{m \cdot s^2 + c \cdot s + 2k} Y_1$$

$$(m \cdot s^2 + k) Y_1 = k \cdot \frac{k}{m s^2 + c s + 2k} Y_1 + F(s)$$

$$\left(m \cdot s^2 + k - \frac{k^2}{m s^2 + c s + 2k} \right) Y_1 = F_s$$

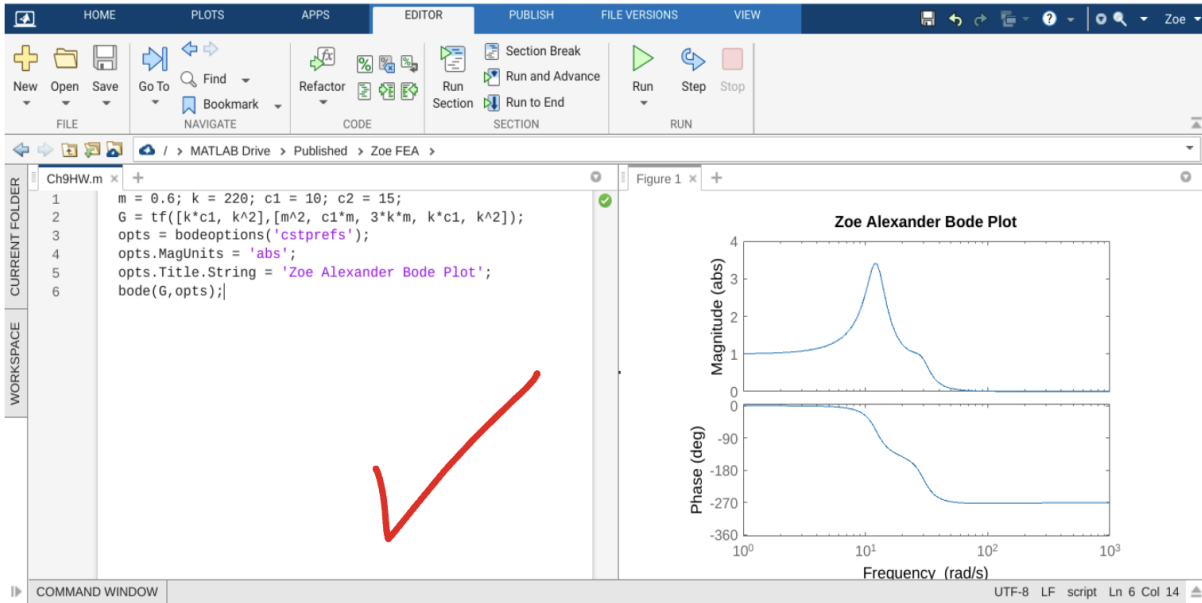
$$\frac{(m \cdot s^2 + k)(m s^2 + c s + 2k) - k^2}{m \cdot s^2 + c s + 2k} Y_1 = F_s$$

C

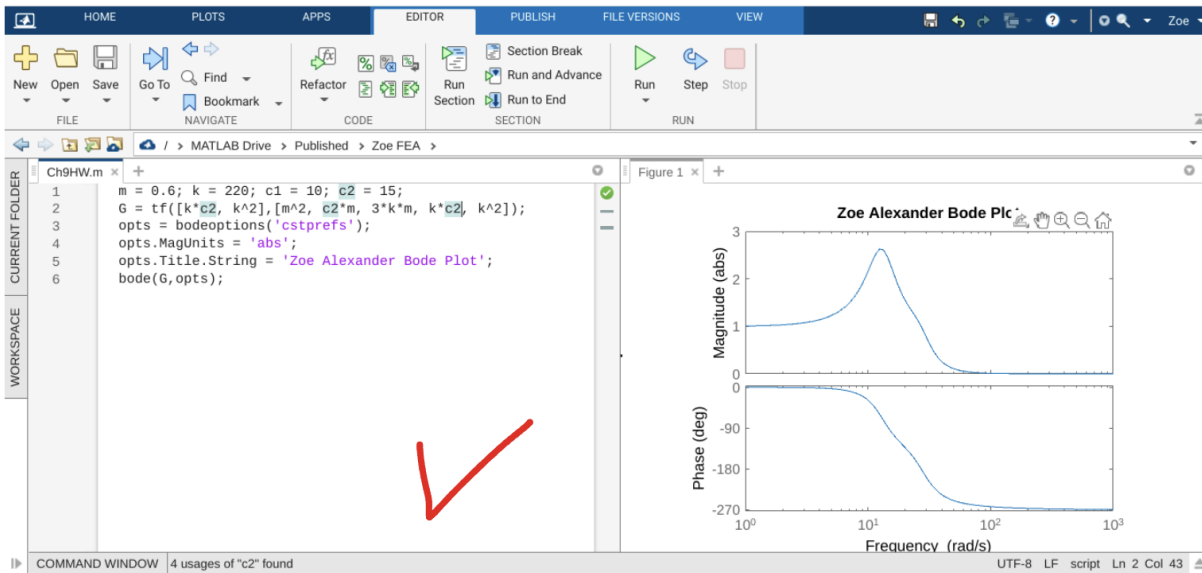
$$G = \frac{F_b}{F(s)} = \frac{(c \cdot s + k)(k)}{m \cdot s^2 + c s + 2k} \cdot \frac{m \cdot s^2 + c s + 2k}{(m \cdot s^2 + k)(m s^2 + c s + 2k) - k^2}$$

$$G = \frac{(c \cdot s + k)(k)}{(m \cdot s^2 + k)(m s^2 + c s + 2k) - k^2} = \frac{k c \cdot s + k^2}{m^2 \cdot s^4 + c m s^3 + 3 k \cdot m \cdot s^2 + k \cdot c \cdot s + k^2}$$

For $c_1=10$ N s/m:



For $c_2=15$ N s/m:



AVERAGE

the 1st equation is in time domain.

$$m \ddot{y}_1 + K(y_1 - y_2) = \cancel{F(s)} \quad f(t) \text{ ————— (i)}$$

$$(ms^2 + K)Y_1 - KY_2 = F(s) \text{ ————— (ii)}$$

Now for 2nd mass

$$m \ddot{y}_2 + K(y_2 - y_1) + K(y_2) + Cy_2 = 0 \text{ ————— (iii)}$$

$$(ms^2 + 2K + Cs)Y_2 - KY_1 = 0 \text{ ————— (iv)}$$

Also;

$$F_b(s) = (Cs + K)Y_2 \text{ ————— (v)}$$

Substituting value of Y_2 in eq (ii) & (v) we get

You have not shown $Y_2 = ?$

$$F_b(s) = \frac{(Cs + K) - K}{(ms^2 + Cs + 2K)}$$

$$F(s) = \frac{(ms^2 + K)(ms^2 + Cs + 2K) - K^2}{(ms^2 + Cs + 2K)}$$

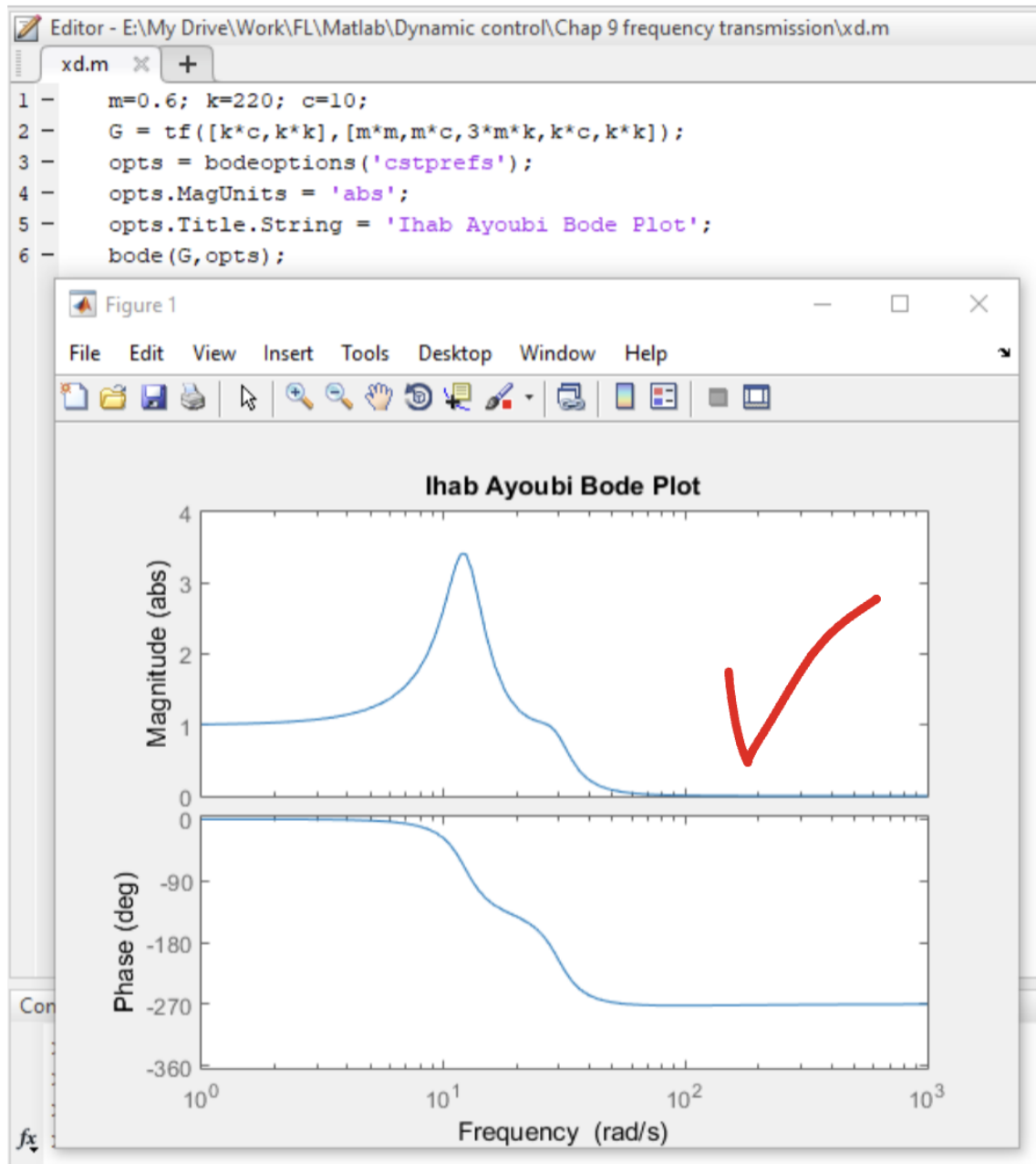
Simplifying we get

$$\frac{F_b(s)}{F(s)} = \frac{KCs + K^2}{m^2s^4 + mCs^3 + 3mKs^2 + KCs + K^2}$$

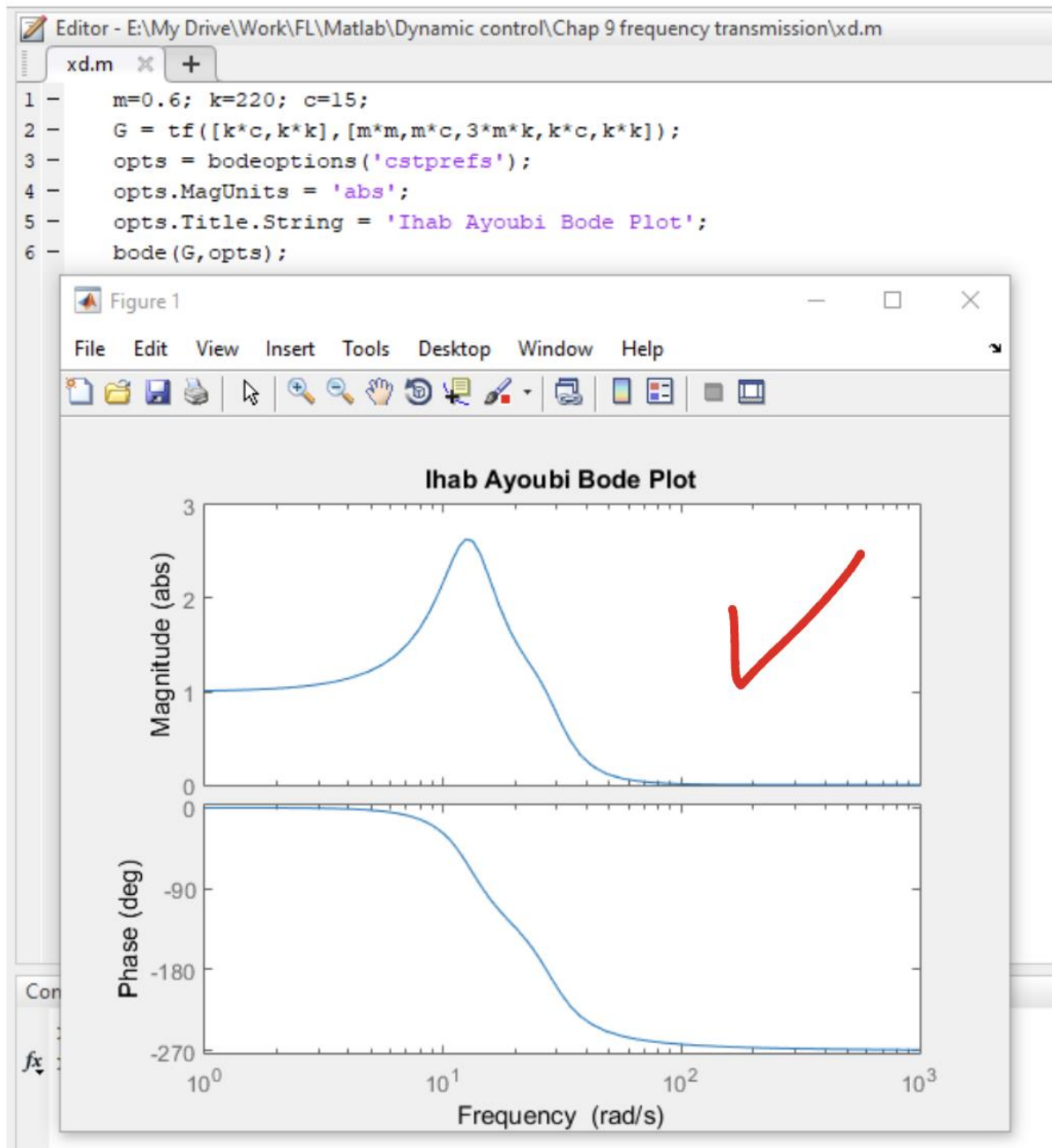
Now by using MATLAB we get

Now we plot the above equation in MATLAB

First, we use $c = 10$;

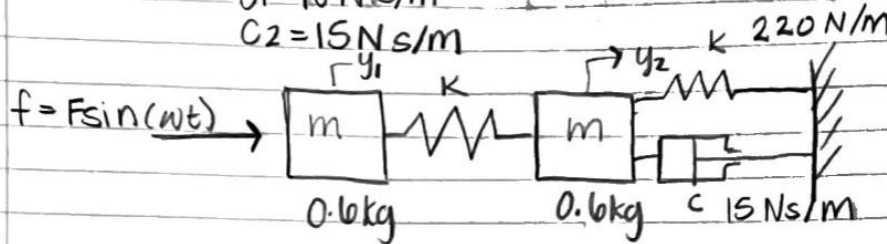


Now for $c = 15$



BELOW AVERAGE

9.28 Given: $m = 0.6 \text{ kg}$
 $k = 220 \text{ N/m}$
 $C_1 = 10 \text{ N.s/m}$
 $C_2 = 15 \text{ N.s/m}$



$$0.6 \frac{d^2 y_2}{dt^2} + 220(y_2 - y_1) = 0$$

Apply Laplace Transform

$$0.6 s^2 y_2(s) + 220 y_2(s) - 220 y_1(s) = 0$$

$$y_2(s) [0.6 s^2 + 220] = 220 y_1(s)$$

$$y_2(s) = \frac{220 y_1(s)}{0.6 s^2 + 220} \quad \text{--- (1)}$$

@ y_1

$$0.6 \frac{d^2 y_1}{dt^2} + 220(y_1 - y_2) + 220 y_1 + 10 \frac{dy_1}{dt} + 15 \frac{dy_1}{dt} = F$$

Laplace Transform

$$0.6 s^2 y_1(s) + 220 [y_1(s) - y_2(s)] + 220 y_1(s) + 10 y_1(s) + 15 y_1(s) = F(s)$$

$$\Rightarrow y_1(s) [0.6 s^2 + 10s + 15s + 440] - 220 y_2(s) = F(s) \quad \text{--- (2)}$$

$$= y_1(s) [0.6 s^2 + 25s + 440] - \frac{220^2 y_1(s)}{0.6 s^2 + 220} = F(s)$$

$$= y_1(s) \left[\frac{0.6 s^2 + 25s + 440}{0.6 s^2 + 220} - \frac{220^2}{0.6 s^2 + 220} \right] = F(s)$$

$$= y_1(s) \left[\frac{(0.6 s^2 + 25s + 440) - 220^2}{0.6 s^2 + 220} \right] = F(s)$$

$$= y_1(s) \left[\frac{0.36 s^4 + 15 s^3 + 396 s^2 + 5500 s + 48400}{0.6 s^2 + 220} \right] = F(s)$$

Transmitted force F_t = spring force + damping force at wall, not Y_1 which is the displacement of mass 1.

Transfer Function

$$\frac{U_1(s)}{F(s)} = \frac{0.1s^2 + 220}{0.36s^4 + 15s^3 + 391s^2 + 5500s + 48400}$$

It is better to keep the parameters (m, c, & k) in the equation derivation. Use MATLAB to do the calculation for you.

taylorodavis ▸ Documents ▸ MATLAB ▸

```
Editor - /Users/taylorodavis/Documents/MATLAB/HwEx9_28.m
HwEx9_28.m
1 m=0.6; k=220; c=10; c2=15;
2 G= tf([c*m, k*m, c*k],[m*c, m*k+m*k+m*k, k*c]);
3 opts = bodeoptions('cstprefs');
4 opts.MagUnits='abs';
5 opts.Title.String='Taylore Davis Bode Plot';
6 bode(G,opts);
```

No bode plot.

**DYNAMIC SYSTEMS AND CONTROLS
SPRING 2022**

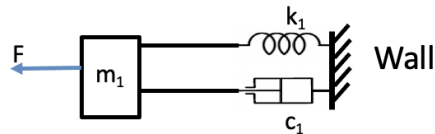
Title of Assignment: Homework 10

- 1. Use Simulink to plot the output of a mechanical dynamic system**
- 2. Add a PID controller and a linear actuator to the dynamic system in the purpose of reducing the output (transmitted force).**

Application of a PID controller

A dynamic system can be modeled as a mass, spring, damper system as shown in the schematic diagram. The system oscillates under a Sine wave force and transmits force to the fixed wall.

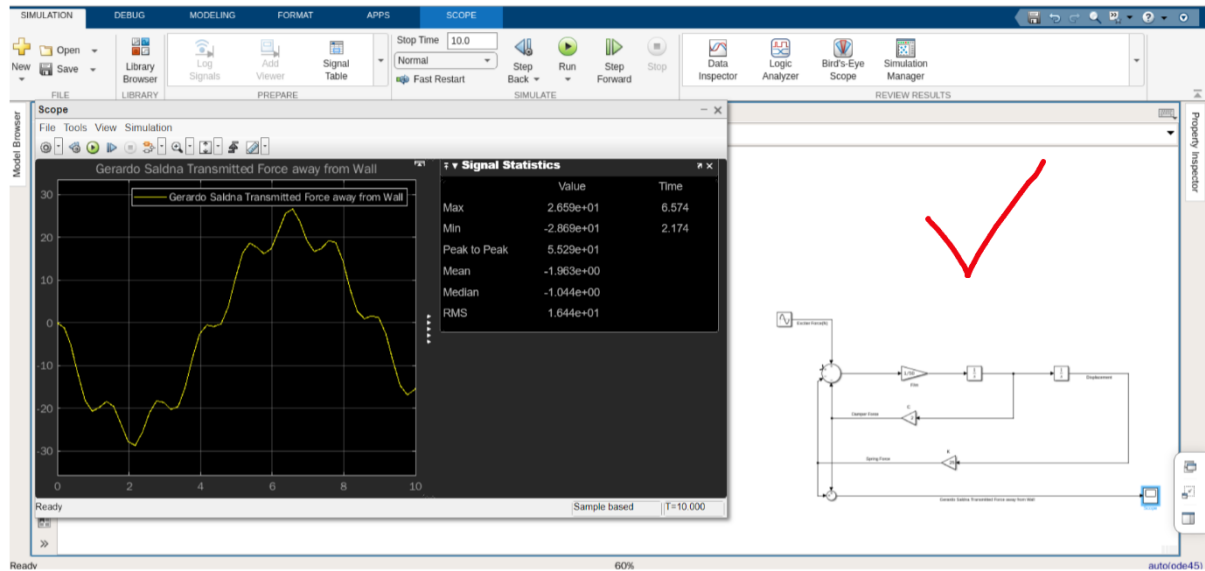
Where $F(t) = 180 \sin(5t)$ N, $M_1 = 50$ kg, $C_1 = 2$ Ns/m, $K_1 = 25$ N/m



- (1) Simulate the dynamic motion of the system using Simulink, Plot the transmitted force to the Wall.
- (2) Add a PID controller to command a linear actuator to provide actuation force to the system in the purpose of reducing transmitted force to the wall.

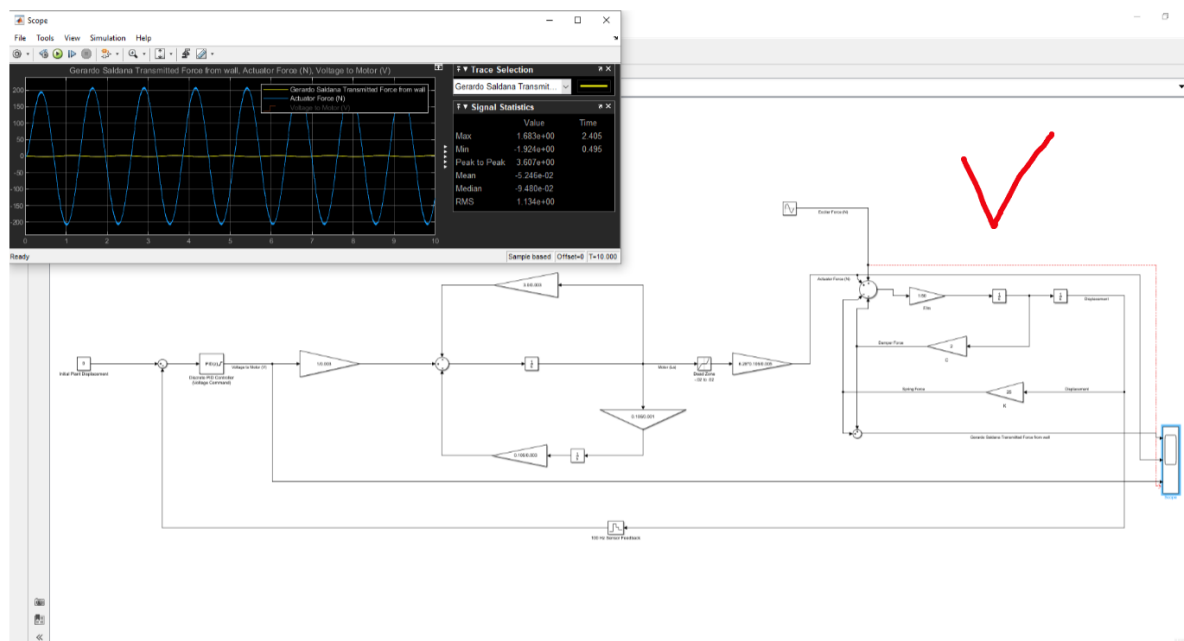
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(1)



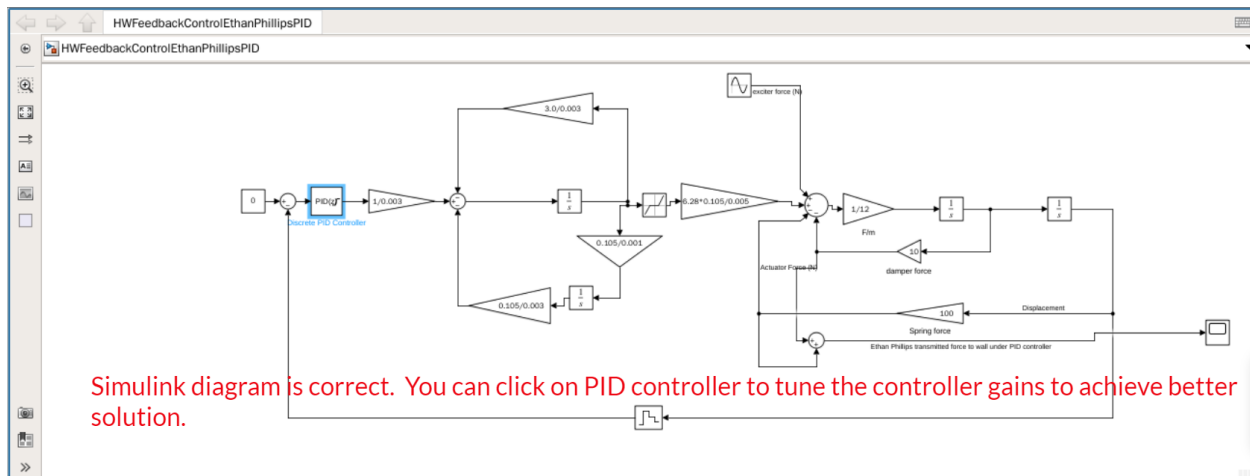
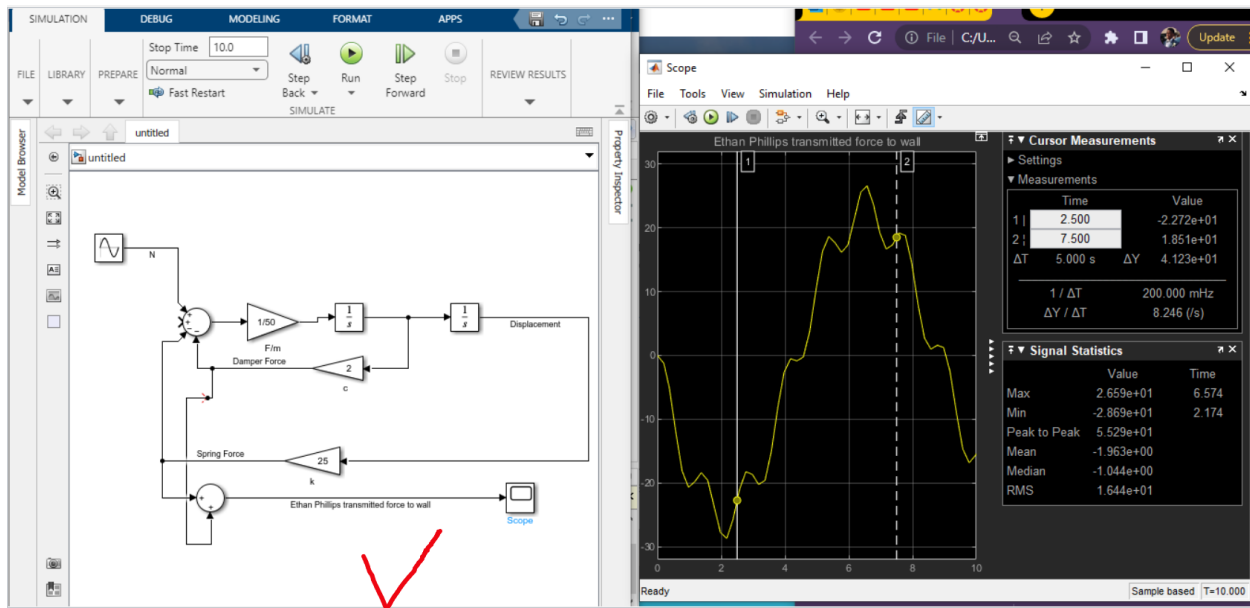
- In the original dynamic system, the vibration is simulated as it runs through the machine, if for example this were the suspension of a car the car would bounce around and shake erratically as it is shown on the graph. Note that largest displacement occurs over time at 6.574s with a max value of 2.659e+01 where the force is at its highest value resulting in a more violent reaction.

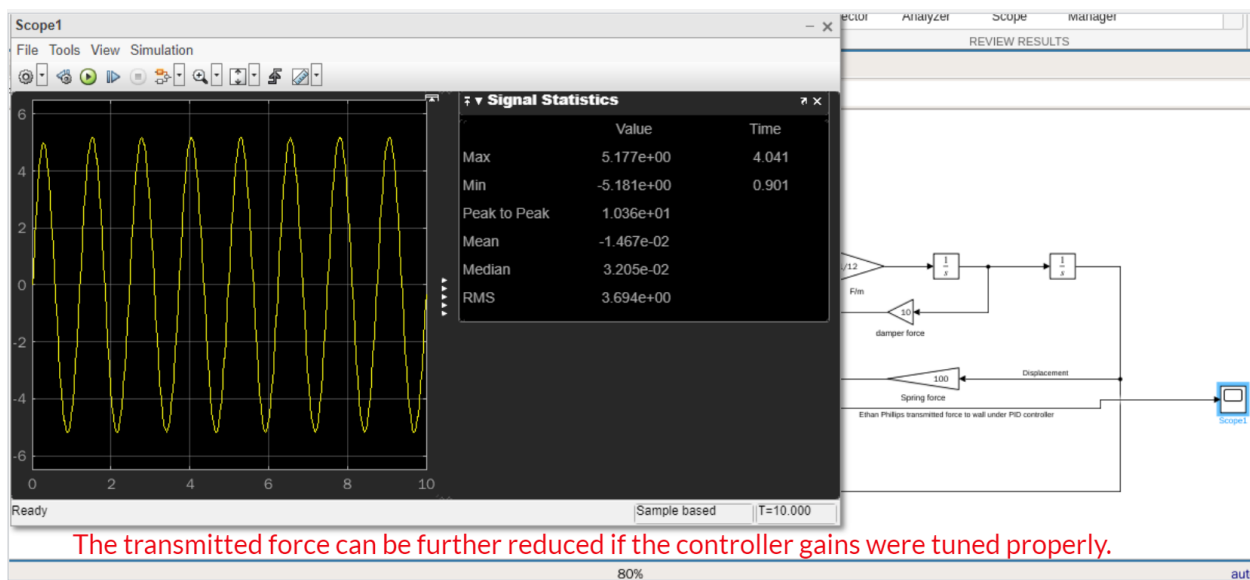
(2)



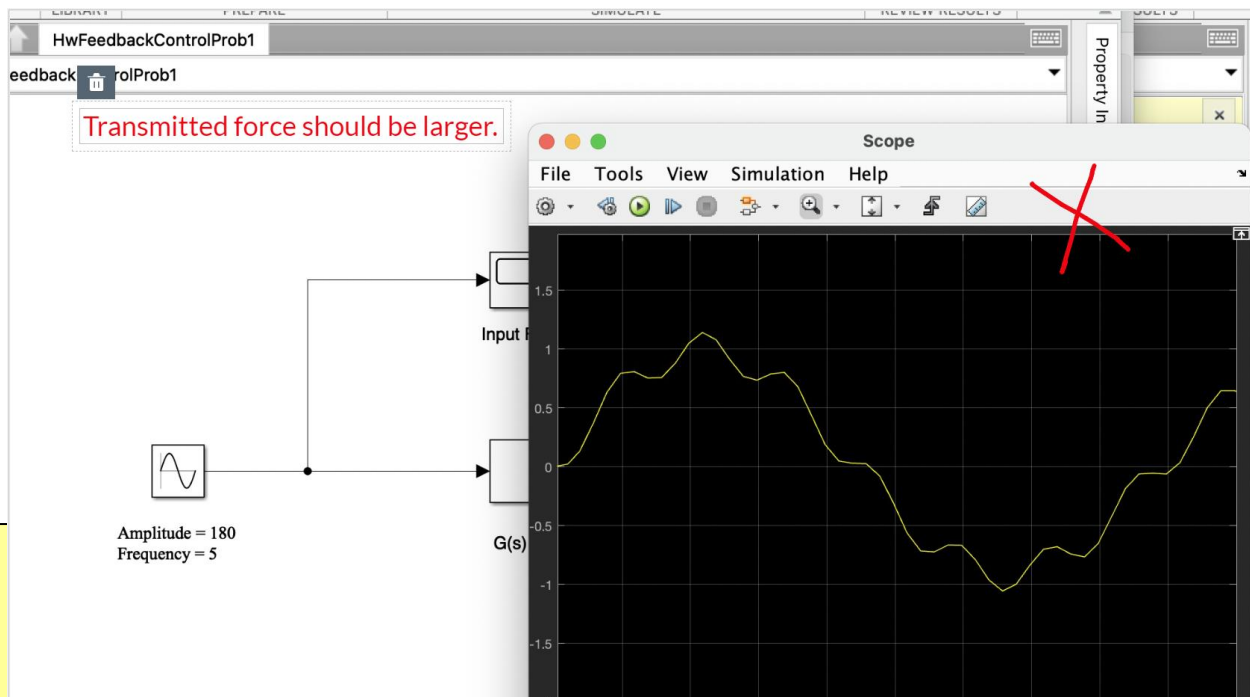
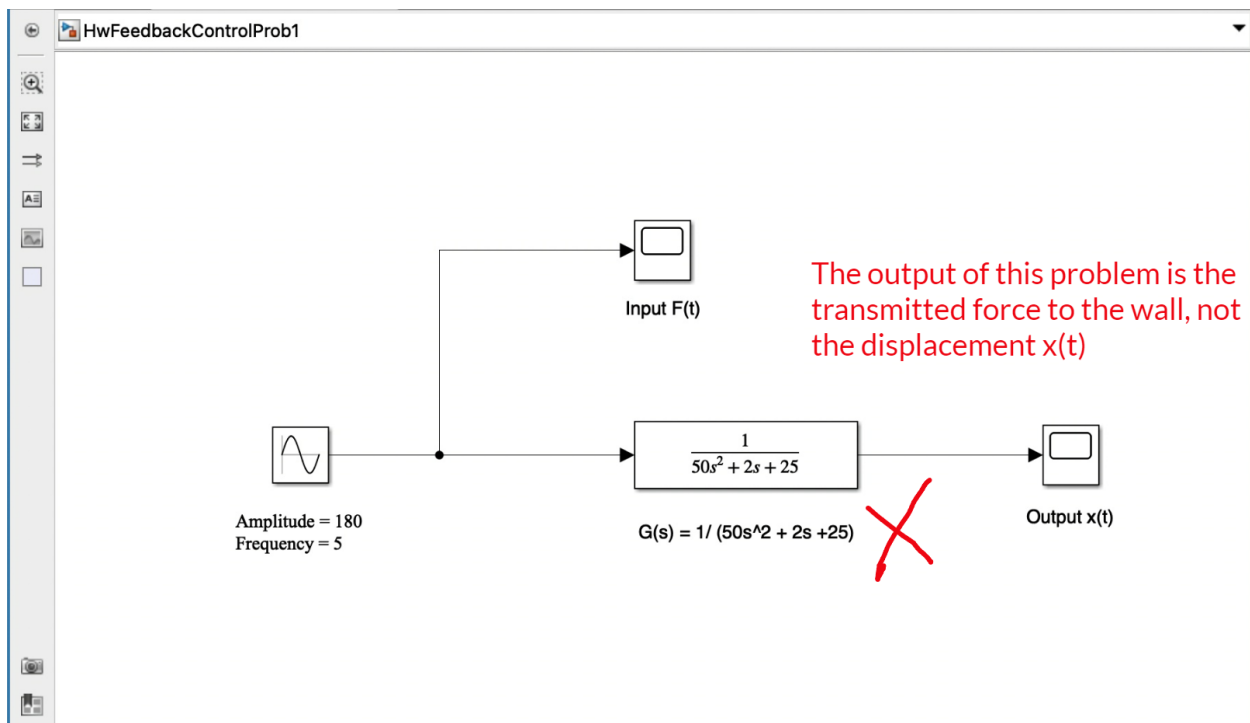
- Comparing the two Simulink diagrams you can see the results vary significantly. In the first image the Simulink diagram represents the Vibration system as it is without the PID controller. With the PID controller you see the force lessens as it dampens the force via the actuator. The actuator reacts along with the exciter force lessening the impact as the force leaves the wall. As it illustrates the size of the vibrations are more stable and less erratic over time, giving for a more smoother experience

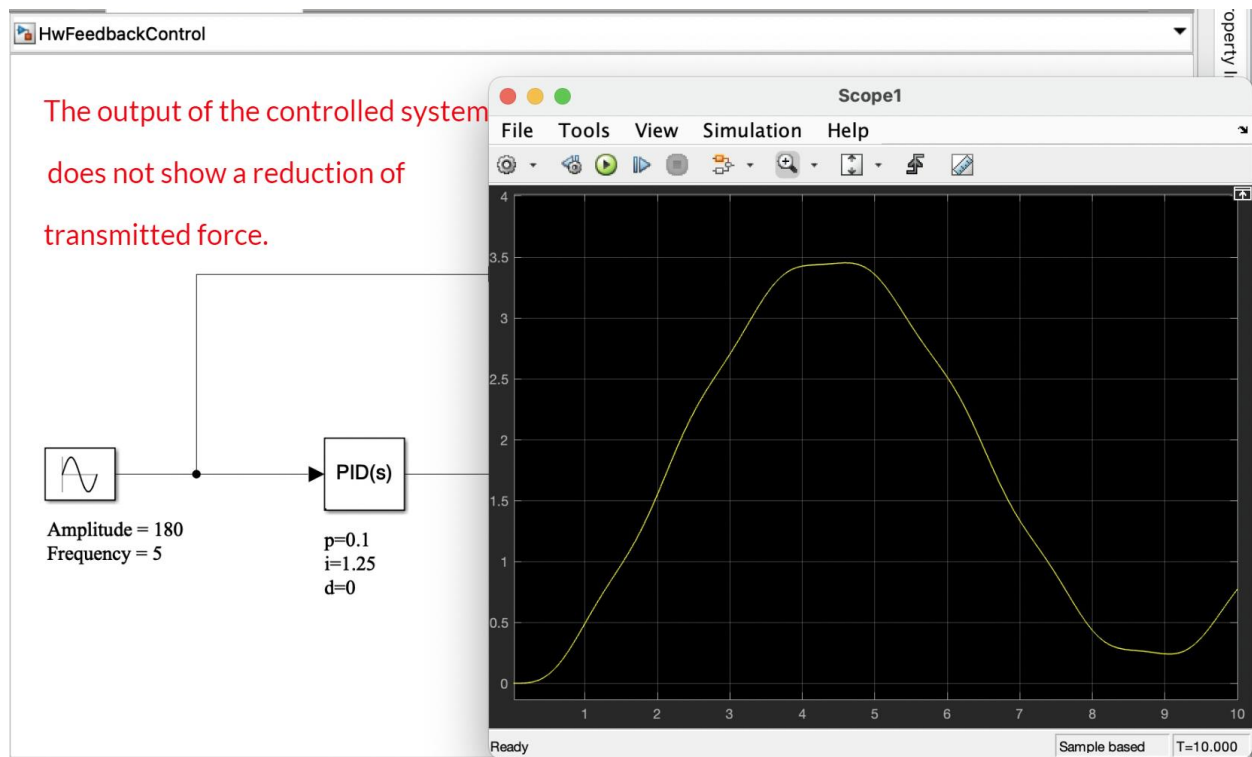
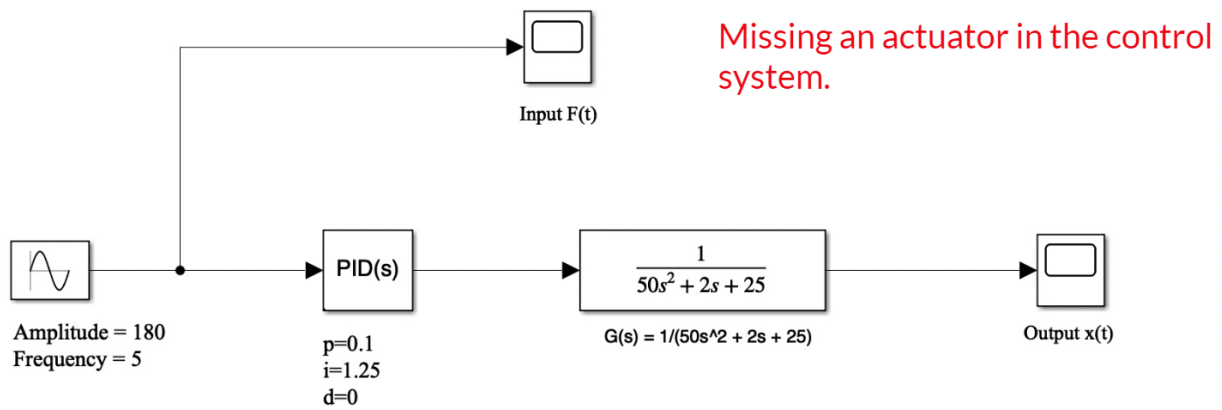
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DYNAMIC SYSTEMS AND CONTROLS SPRING 2022

Title of Assignment: Design Project

TOPICS COVERED IN DESIGN PROJECT

This project is used to evaluate students' ability to apply the knowledge learned in the class for understanding the dynamics and controls of an active suspension system of a vehicle. The students are required to pick a topic of a particular active suspension system. Describe the functionality of the active suspension system, draw a schematic diagram of the system, define the main variables in the system, derive equations of motion of the suspension system, collect or calculate values for the parameters in the dynamic system, assume an external load, use MATLAB/Simulink to simulate the dynamic equations, and plot the system response of your defined variables. Explain the system response and how the response illustrates the functionality of the active suspension system.

Project Title: **Active Suspension System for Vehicle**

Suspension system is an important sub-system for vehicle design. We have probably all experienced the uncomfortable feeling when we drive a car without slowing down over a pothole on the road. Active suspension is a method to go beyond a traditional passive suspension system which has only a fixed spring and a fixed damper. Active suspension technologies allow automobile manufacturer to achieve a greater degree of ride comfortability and car handling.

The students are required to research active or semi-active suspension systems for vehicle. You may pick a topic of particular active suspension system. Describe the functionality of the active suspension system, draw a schematic diagram of the system, define the main variable (or variables) in the system, derive equations of motion of the suspension system, collect or calculate values for the parameters in the dynamic system, assume an external load, use MATLAB/Simulink to simulate the dynamic equations, and plot the system response of your defined variables. Explain the system response and how the response illustrates the functionality of the active suspension system.

video demonstration of Active Suspension Systems

Bose:

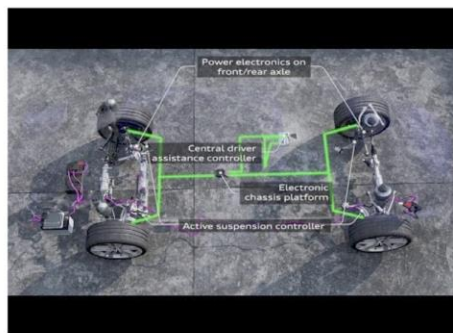
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Audi

<https://youtu.be/jE8s1qZy61Q>

Audi 2

<https://youtu.be/P7QQXthHyQ>



Project Grading Rubric

For MCEG 4306 – Dynamic Systems and Controls, Spring 2022

Title: Active Suspension System for Vehicle

Grading Rubric

Item	Content Description	Assigned Points	Points Received	Relevant to assessment
1. Description of an Active Suspension System for Vehicle including the Schematic Drawings.	Research active suspension used in vehicle design; pick an active suspension system that you are interested. Describe how the system works and include its schematic drawings.	20		<i>An ability to acquire and apply new knowledge as needed, using appropriate learning strategies.</i>
2. Information Research & Equation Derivation.	Derive dynamic equation(s) of motion of the active suspension system. Collect or calculate parameter values used in the equations.	20		<i>An ability to identify, and formulate complex engineering problems.</i>
3. Use of MATLAB/Simulink	Based on the equations of motion, simulate the motion using MATLAB script file or Simulink.	20		<i>An ability to solve complex engineering problems by applying engineering skills.</i>
4. Conclusion and Discussions.	From the MATLAB/Simulink output of the active suspension system, discuss the results and how the results illustrate the functionality of the active suspension system.	20		<i>An ability to solve complex engineering problems by applying engineering skills</i>
5. Report Writing	Use of technical writing format which generally includes abstract, introduction, equations, figures, tables, conclusion, references, etc. All parameters used in the analysis must be clearly justified by referring to the origin of technical data or calculated using appropriate methods.	20		<i>an ability to communicate effectively</i>
	Total	100		

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Abstract

For the last twenty years, active suspension technologies have dominated vehicle dynamics research. Passenger comfort and safety are both influenced by the suspension system design, which is a critical component of automobile design. Today's automobiles are equipped with active suspension control systems because of their ability to balance ride comfort with vehicle performance. A typical example of a distributed control system may be found here. Due to their energy regeneration capabilities, high bandwidth, simplified structure, precise force control, improved handling, and drive characteristics, active suspensions were found to be the best choice for vehicles. Real-time simulations of the controlled system are carried out using the Matlab/Simulink. Making a comparison of passive and active systems is the primary function of this report.

1. Introduction

From the horse-drawn carriage, which had flexible leaf springs affixed at the four corners, to current high performance vehicles with complicated control algorithms, suspension systems are an intricate part. Every vehicle traversing the extreme variety of roads in this country is subjected to the vibrations produced. This can have a detrimental impact to both the comfort of the passengers and the longevity of the vehicle itself. Whether a vehicle tilts during cornering (otherwise known as rolling) or dips to the front while braking, different disturbances occur. Additionally, when traveling over rough terrain, the vehicle's body may experience unwelcome vertical vibrations (bouncing). Because the tires may lose their grip on the road, these jerky movements are often uncomfortable, but they are also potentially hazardous to the occupants of the vehicle. A car's suspension is therefore primarily responsible for stability as well as passenger comfort across a wide range of road conditions. This, in turn, would have a direct effect on the user's safety.

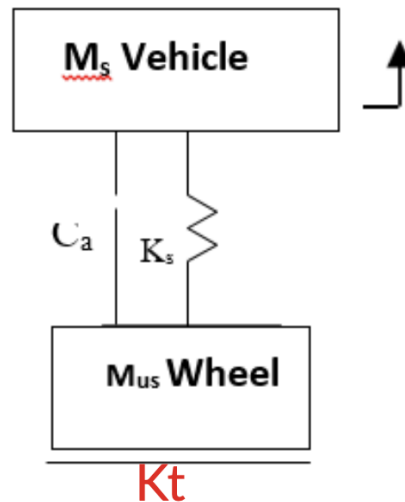


Figure 1.1 – Passive system

Figure 1 depicts a typical automotive suspension system in schematic form. Suspension systems should be able to give a smooth ride and good control over a suitable range of deflection. In addition, these standards are influenced by the vehicle's intended use. Luxury sedans typically have softer suspensions, but poor road handling, while sports vehicles typically have stiff, harsh suspensions. Both road and load perturbations are important factors to consider while developing a vehicle's system of stability and performance. Smaller-amplitude high-frequency road disturbances have features similar to those of hills that are large at low frequencies but modest at high frequencies (such as road roughness). Acceleration, braking, and cornering all cause loads to fluctuate, which is what we call a load disturbance. Therefore, with a suitable suspension design, it is important to ensure that the outputs are not adversely affected by the suspension (e.g. vehicle height etc). Suspension systems need to be "soft" for road disturbances and "stiff" for load disturbances in order to have sufficient cushioning.

When it comes to vehicle handling, a heavily damped suspension will transfer a lot of the vehicle's road input to the vehicle body, whereas a lightly damped suspension will provide a more comfortable ride but significantly reduce the vehicle's ability to handle turns, lane changes, or negotiating an exit ramp, for example. A suspension design, then, is a balancing these two objectives is the art of compromise. Passive suspensions that are well-designed can improve riding comfort and road holding ability to some extent, but they cannot eliminate this trade-off completely.

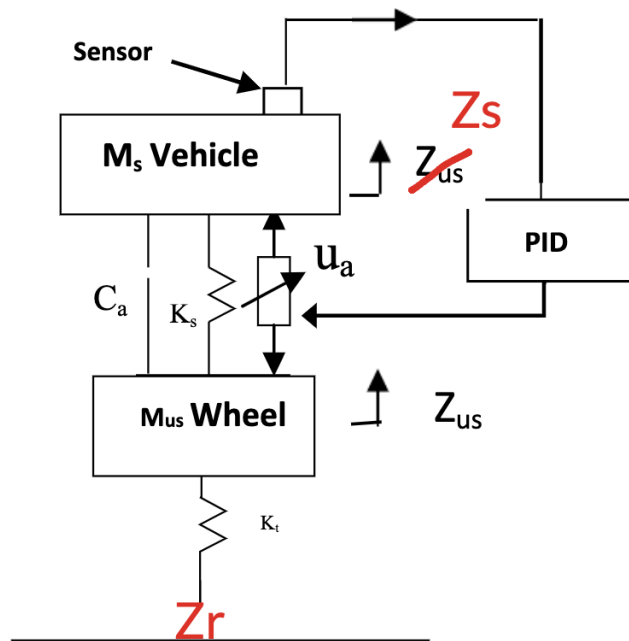


Figure 1.2 – Active system

It's important to note that an active suspension system (Figure 1.2 above) has a built-in actuator that may provide control forces to dampen the motions listed above. In addition, the tire-road contact forces' dynamic nature has increased road holding. There has been a lot of interest in the usage of active suspension systems, which can overcome some of the limits of passive suspension systems in recent years. Automakers are considering the use of active suspensions to improve ride comfort and controllability on the road. Electronically controlled active suspension systems have the potential to enhance both the vehicle's ride quality and its road handling at the same time. Varying road conditions need different behavior from an active suspension system, and it should be able to do so without exceeding its travel restrictions.

There are several advantages to using a system of active suspension rather than passive suspension, but these systems are difficult to execute since they require a lot of sophisticated electronic sensors and actuators. Adaptable dampers, springs, sensors, and actuators have made these systems more widely applicable in the last few years. Because of this, the vehicle provides a superior balance of ride and handling qualities in a variety of situations, compared to conventional suspension systems. ECS systems

are often found in luxury cars and high-priced sport utility vehicles since they are more expensive than traditional suspension systems.

1.1 Constraints

Active suspension systems add hydraulic actuators to the passive components of suspension system as shown in Figure 1.2. The advantage of such a system is that even if the active hydraulic actuator or the control system fails, the passive components come into action.

Primary variables used are derived from the Free body diagram seen in Figure 1.2 above.

M_s = Mass of Vehicle

M_{us} = Mass of wheel

C_a = Damper coefficient

K_s = Spring coefficient

U_a = Actuator coefficient – control force

Z_s = z movement of car body

Z_{us} = z movement of wheel

The constraining equations are as follows,

$$M_s \ddot{Z}_s + K_s(Z_s - Z_{us}) + C_a(\dot{Z}_s - \dot{Z}_{us}) - u_a = 0 \quad (1)$$

$$M_{us} \ddot{Z}_{us} + K_s(Z_{us} - Z_s) + C_a(\dot{Z}_{us} - \dot{Z}_s) + K_t(Z_{us} - Z_r) + u_a = 0 \quad (2)$$

Where u_a is the controlling force from the actuator. It can be noted that if the control force $u_a = 0$, then Eqn (1) becomes the equation of passive suspension system. Considering u_a as the control input, Eqn (1) becomes

$$(\dot{Z}_1) = Z_2$$

$$\dot{Z}_2 = -\frac{1}{M_s}[K_s(Z_1 - Z_3) + C_a(Z_2 - Z_4)]$$

$$(\dot{Z}_3) = Z_4$$

$$\dot{Z}_4 = -\frac{1}{M_{us}}[K_s(Z_1 - Z_3) + C_a(Z_2 - Z_4) + K_t(Z_3 - Z_r)]$$

Where, $Z_1 = Z_s$, $Z_2 = \dot{Z}_s$, $Z_3 = Z_{us}$, & $Z_4 = \dot{Z}_{us}$

1.2 Known values for quarter suspension

$$M_b = 300 \text{ Kg}$$

$$K_t = 190000 \text{ N/m}$$

$$M_{us} = 30 \text{ Kg}$$

$$\beta = 1 \text{ sec}^{-1}$$

$$K_a = 32000 \text{ N/m}$$

$$P_s = 10.55 \text{ MPa}$$

$$C_a = 1000 \text{ N/(m/sec)}$$

Did not use in this report.

2. Simulations

This sections shows the simulations used based on the equations above. First the system was run as a passive system to establish a base line of response. Then a PID controller was added to the Simulink diagram. Once the PID was installed, the system becomes active and we can see the quicker response and less dramatic impact when the vehicle comes in contact with the bump.

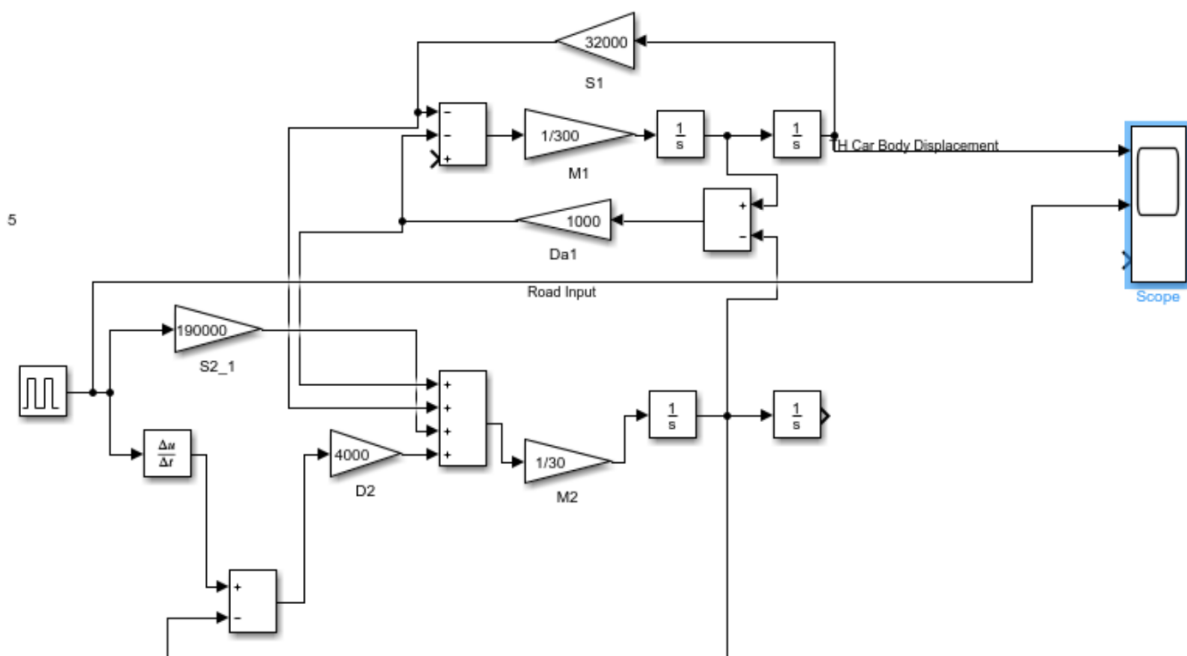


Figure 2.1 – Simulink of a Passive system

As the vehicle approaches and overcomes a bump in the road we can see from the graph the response of the passive system. The displacement of the vehicle upward $\frac{1}{4}$ the way through the bump is a constant response. Then we see the system recover over the next few seconds but the initial reaction is pretty significant.

Need to add a description here for bump (pulse generator).

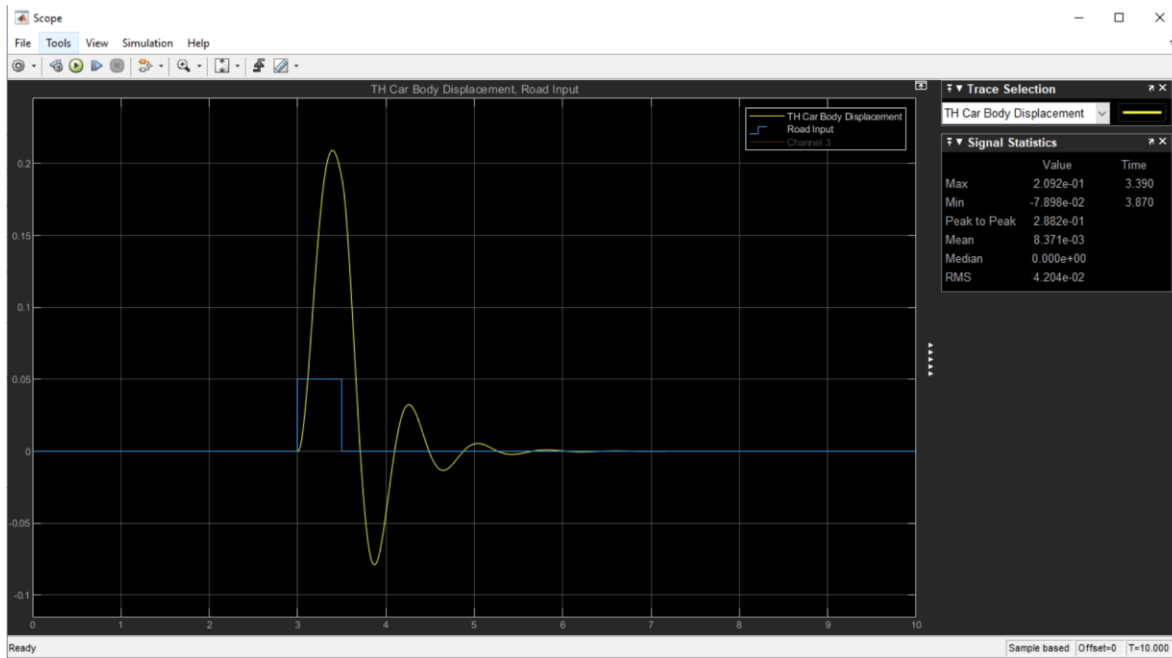


Figure 2.1 – Passive system response

The Simulink diagram shown here shows the system laid out with the 2 bodies from the schematic as well as the PID controller/actuator setup which completes the active nature of the suspension. The road conditions are simulated by a pulse generator. The specifics of the generator are an amplitude of .05, period of 100 secs, pulse width of .5 and a phase delay of 3 secs. The values placed for the springs, mass and damper are listed above.

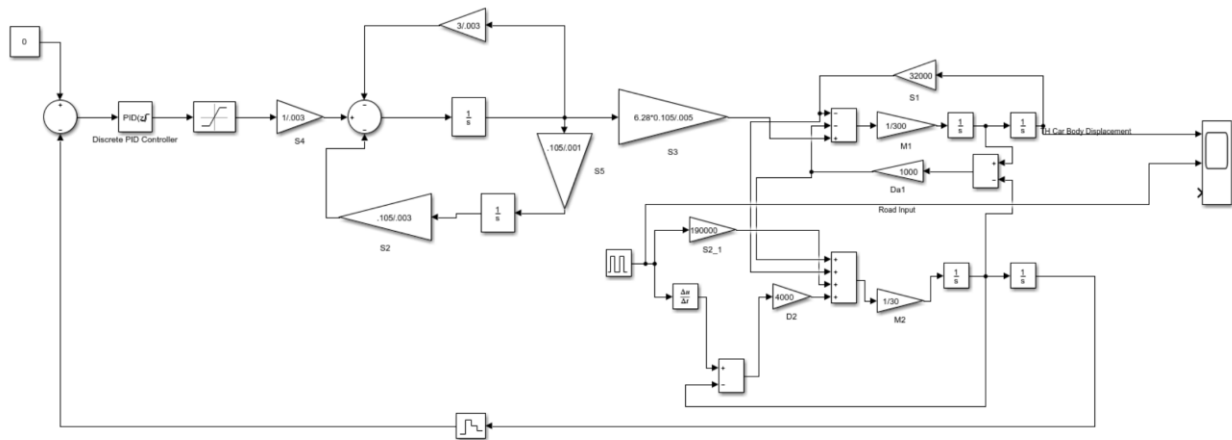


Figure 2.2 – Simulink diagram

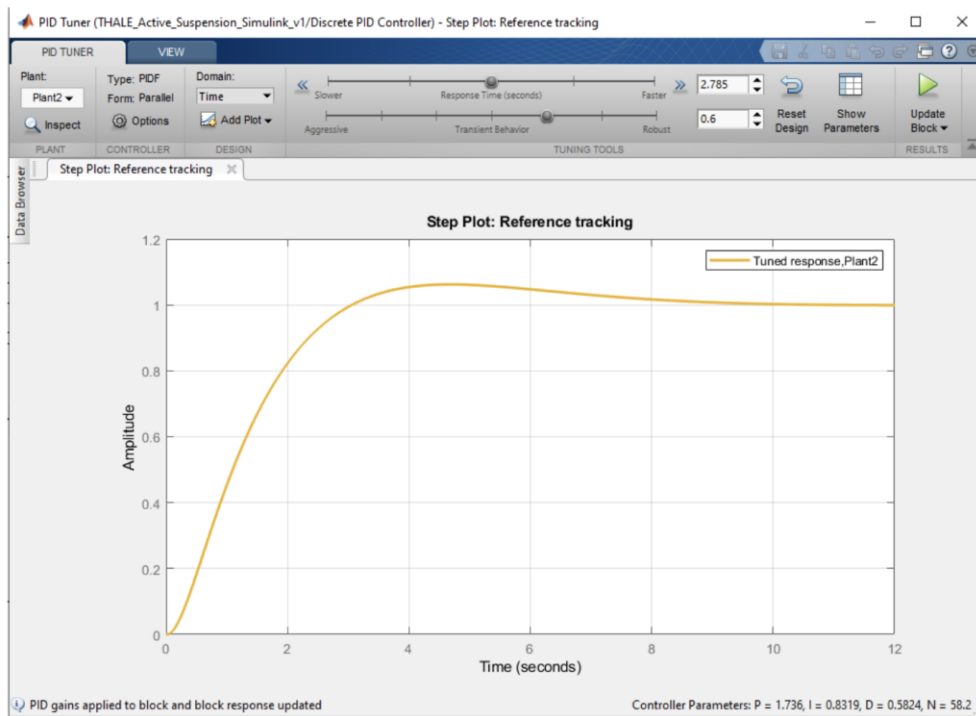


Figure 2.1 - Tuned PID controller

It would be better to include the real output values in the summary statement and make a comparison between the passive system and the active controlled system.

Below you can find the graphical representation of the active suspension responding to a bump in the roadway and the quickly responding. As you can see the addition of the PID controller aids the system significantly.

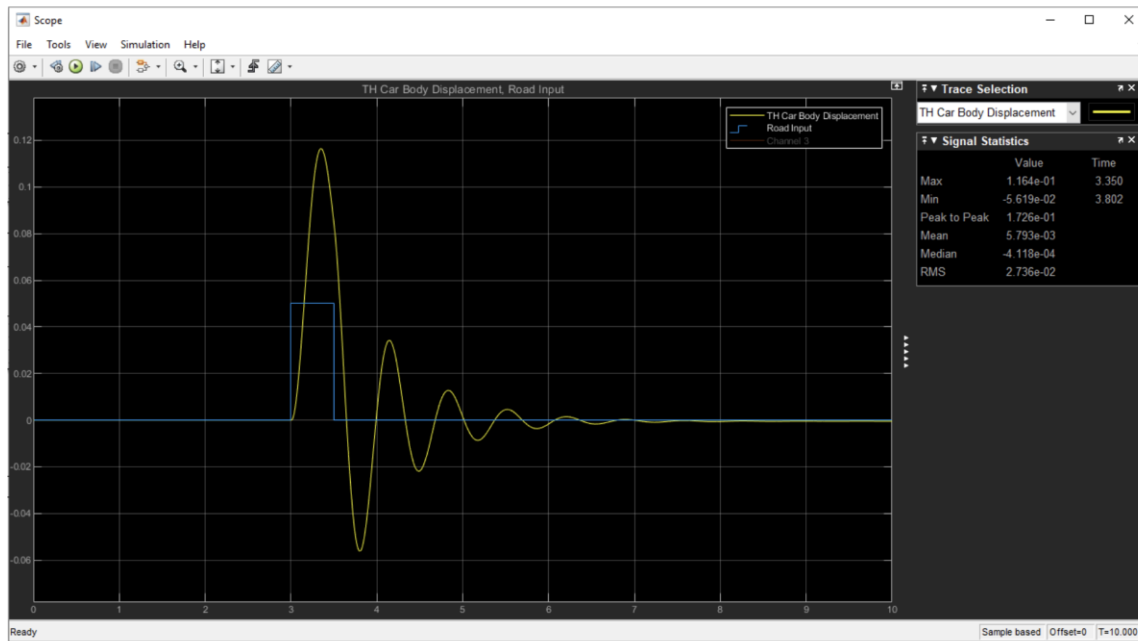


Figure 2.4 – Graphical representation of the Active suspension

Conclusion

From the information found during research for this project, we can see that an obvious choice for economic to middle class vehicles is the passive suspension system. Being a cheaper system, it fits the needs of the average person. While it will handle normal road conditions at basic comfort levels, it certainly has room for improvement.

The major improvement to a passive system is to add a pid controller, which can and does respond to changes in road conditions very quickly which increases ride comfort and handling. Designed for rough roads, potholes, and other irregular driving conditions. PID-controlled active suspension enhances ride comfort, according to the simulation results. It has also been demonstrated that the PID controller may be a successful tool in building active suspension systems, despite its simplicity in design and the availability of well-known standard hardware.

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Goszczak, Jarosław, et al. "The Study of Damping Control in Semi-Active Car Suspension | JVE Journals." *The Study of Damping Control in Semi-Active Car Suspension | JVE Journals*, www.extrica.com, 30 June 2020, <https://www.extrica.com/article/20578>.

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ABSTRACT

The purpose of this project is to learn about active and passive suspension systems and reinforces our understanding on feedback controls and dynamic equations. We did this so through research, deriving dynamic equations, and creating Simulink diagrams of active and passive suspension systems. In this report, I will prove how an active suspension system provides passengers' a higher quality riding experience in comparison to a passive suspension system.

CHAPTER 1

ABILITY TO CONDUCT EXPERIMENTS

1.1 Objectives of the Experiment

1. Research active or semi-active suspension systems for vehicle.
2. Describe the functionality of the active suspension system
3. Define main variables in the system
4. Assume an external load
5. Use MATLAB/Simulink to simulate the dynamic equations
6. Plot system response of defined variables
7. Explain the system response and how the response illustrates the functionality of the active suspension.
8. Compare how an active suspension system is better than a passive suspension system by simulating the system response of both systems going over a 0.12 m-deep pothole.

1.2 Theory Associated with Experiment

A suspension system helps drivers keep control while being on the road by maximizing the friction between the tires and the road surface. However, the downfall of a suspension system is that when a vehicle passes over road disturbances like a pothole, the body of the car oscillates and may provide discomfort for the passengers of the vehicle. However, active suspension systems have been made to prevent and reduce that discomfort. An active suspension system is a computer-controlled system, which actively controls the vertical movement of the vehicle in response to road disturbance. It consists of a feedback controller, springs, dampers, and masses concerning the body and suspension.

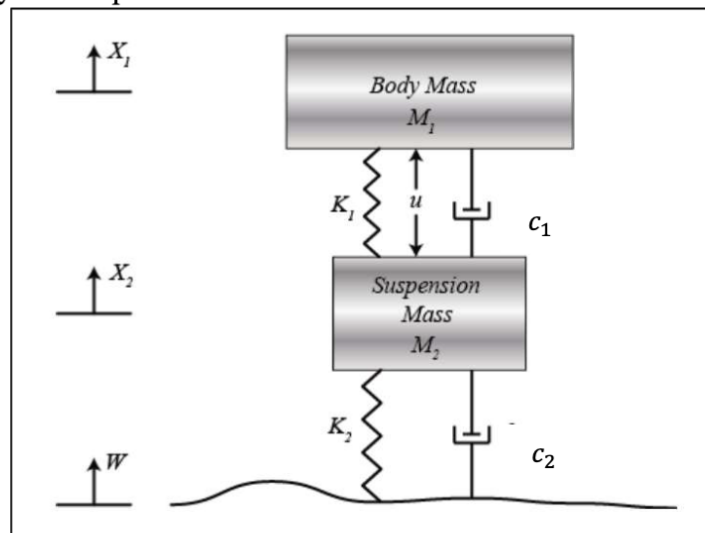


Figure 1.1 Model of Bus Suspension System (1/4 bus)

A suspension system, also known as a passive suspension system, consists of dampers, suspension mass, body mass, and suspension mass.

1.3 Literature Review

Spring: An item that becomes deformed when force is placed upon it but can also return to its original shape after the force has been removed.

k : The symbol that represents the strength of a spring in calculations.

Damper: A mechanism that restrains and reduces vibrations over time.

c : The symbol that represents the strength of dampers in calculations.

Body Mass: mass of the system being supported by the suspension. (Example: mass of the body of vehicle, passengers, etc.)

m_1 : The symbol that represents body mass in the calculations.

Suspension Mass: mass of wheels and its moving parts.

m_2 : The symbol that represents the suspension mass in calculations.

Feedback Controller: A device that measures the output of a process and then manipulates the input as needed to drive the process variable toward the desired setpoint.

It's good to apply Lagrange's method for deriving the dynamic equations. However, pothole W should be included in the equations.

CHAPTER 2

ABILITY TO ANALYZE EXPERIMENTAL DATA

2.1 Deriving Equations of the System based on Figure 1.1

$$T = \frac{1}{2} m_1 \dot{X}_1^2 + \frac{1}{2} m_2 \dot{X}_2^2$$

$$U = \frac{1}{2} k_1 (X_2 - X_1)^2 + \frac{1}{2} k_2 X_2^2$$

$$F = \frac{1}{2} c_1 (\dot{X}_2 - \dot{X}_1)^2 + \frac{1}{2} c_2 \dot{X}_2^2$$

$$\left\{ \begin{array}{l} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{X}_1} \right) = m_1 \ddot{X}_1; \quad \frac{\partial F}{\partial \dot{X}_1} = -c_1 (\dot{X}_2 - \dot{X}_1); \quad \left(\frac{\partial U}{\partial X_1} \right) = -k_1 (\dot{X}_2 - X_1); \\ \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{X}_2} \right) = m_2 \ddot{X}_2; \quad \frac{\partial F}{\partial \dot{X}_2} = c_2 \dot{X}_2; \quad \left(\frac{\partial U}{\partial X_2} \right) = k_2 X_2; \end{array} \right\}$$

$$\left\{ \begin{array}{l} m_1 \ddot{X}_1 + c_1 \dot{X}_1 - c_1 \dot{X}_2 + k_1 X_1 - k_1 X_2 = 0 \\ m_2 \ddot{X}_2 + c_2 \dot{X}_2 + k_2 X_2 = 0 \end{array} \right\}$$

$$\left\{ \begin{array}{l} \ddot{X}_1 = -\frac{1}{m_1} (c_1 \dot{X}_1 - c_1 \dot{X}_2 + k_1 X_1 - k_1 X_2) \\ \ddot{X}_2 = -\frac{1}{m_2} (c_2 \dot{X}_2 + k_2 X_2) \end{array} \right\}$$

2.2 Defining the Quantities of the System

m1 = 2700 kg
m2 = 310 kg
k1 = 85000 N/m
k2 = 510000 N/m
c1 = 450 N·s/m
c2 = 15000 N·s/m

These equations are for passive system. The active system should include a control force.

2.3 Presentation of Passive and Active Suspension System Simulink Diagrams

To create these Simulink diagrams many blocks of integrators, gains, step functions, out1, ln1, scopes, and a subsystem were used. The purpose of these Simulink diagrams is to simulate how each system responds to passing over a 0.12 m-deep pothole. This is primarily done so by the

step functions. The step function of W helps us to represent the smoothness of the road at a certain moment by inputting a final value for W . If the road was perfectly smooth, W 's final-value would be zero. However, to simulate the system's response to the pothole, W 's final value is -0.12.

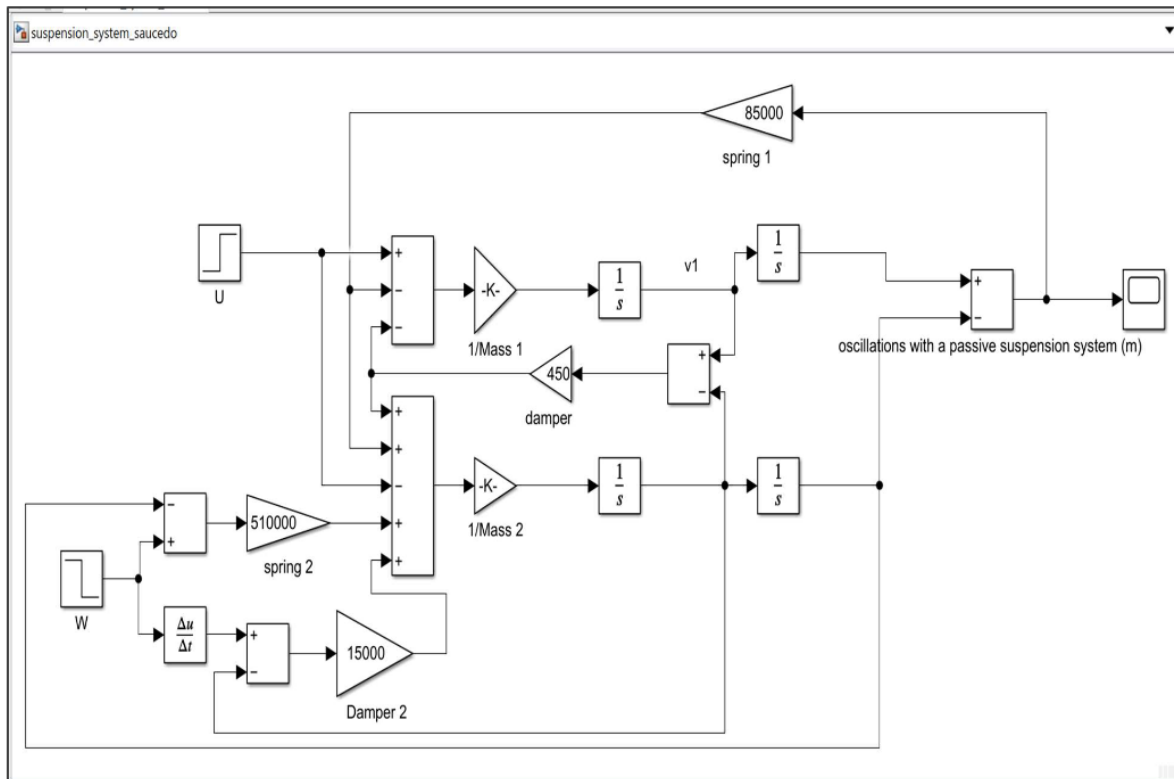


Figure 2.1 Simulink of a Passive Suspension System

In the passive suspension system, the U step function is the force that acts between the masses.

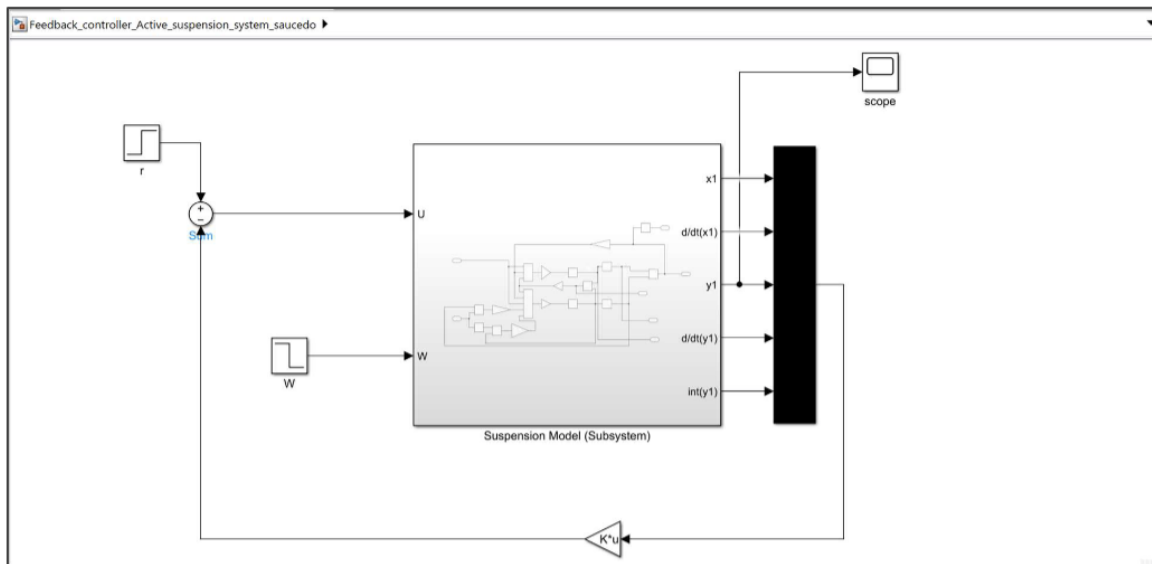


Figure 2.2 The Feedback Controller system of the Active Suspension System

In the feedback controller system, the “r” step function’s values for step time and final value are zero, which means that system is commanding the vehicle to stay at a level.

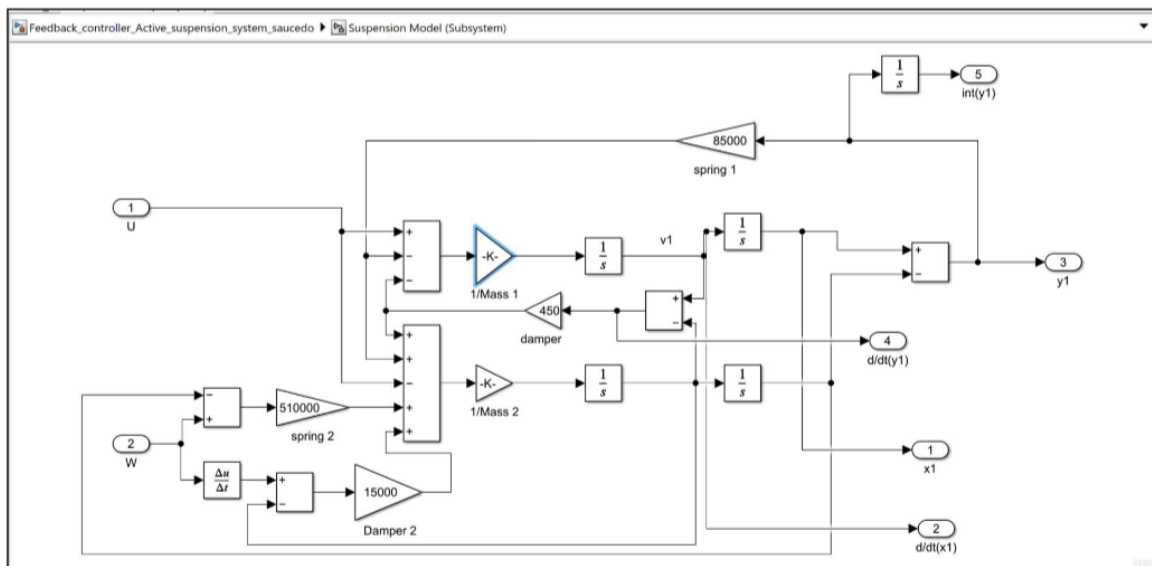


Figure 2.3 Subsystem of Active Suspension System

The subsystem of the active suspension system is a suspension system, but with inputs from the feedback controller and outputs informing the feedback controller system of how the suspension system is currently doing. Therefore, if there were road disturbances, the input begins to manipulate the suspension system to reduce the oscillations of the bus until the bus is properly leveled.

The controller & actuator are missing in the diagram.

CHAPTER 3

ABILITY TO INTERPT EXPIERIMENTAL DATA

3.1 Presentation of Figures and Interpretation of Results Presented by Each Figure

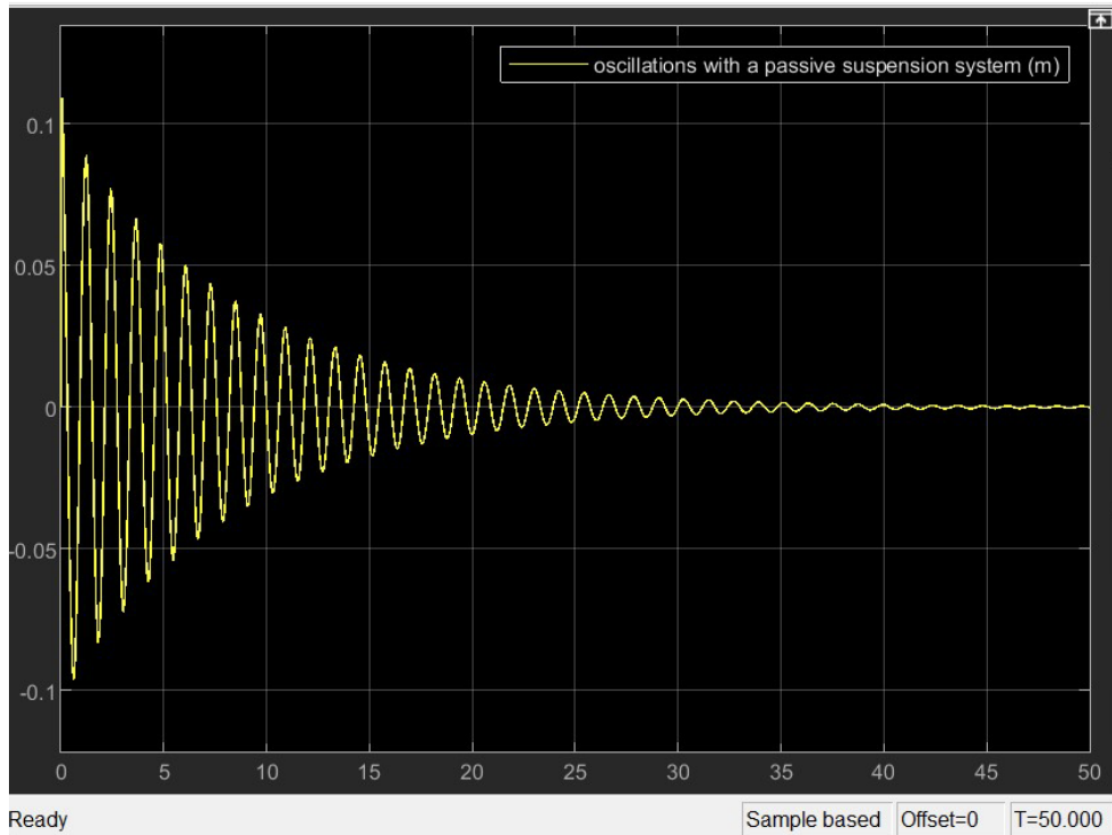


Figure 3.1 Oscillations with a Passive suspension system

These oscillations occurred after the bus passed over a 0.12 m deep pothole. These oscillations occur for about 50 seconds. The oscillations begin to range from ± 0.11 m and decrease over time.

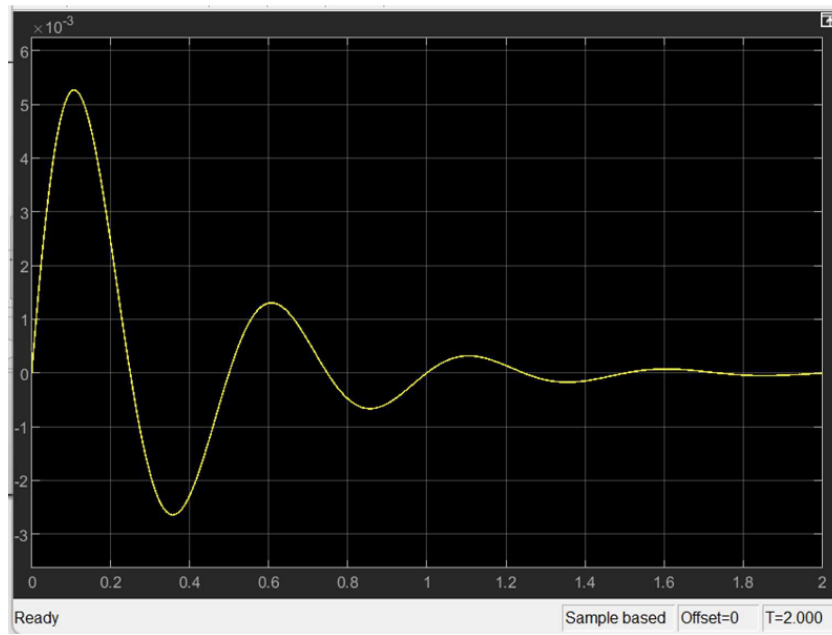


Figure 3.2 Oscillations with an Active Suspension System

The oscillations occurred after the bus passed a 0.12 m pothole. The largest oscillation to occur was 0.0052 m, but the oscillations of the bus decrease back to zero within two seconds.

3.2 Conclusions based on each Objective and Results

To summarize, an active suspension system is a computer-controlled system that actively controls the vertical movement of the vehicle in response to road disturbance. Therefore more comfort is provided to the passengers. An active suspension system consists of a feedback controller, dampers, springs, body mass, and suspension mass. In this project, we assumed a situation where a passive and active system would pass over a 0.12 m-deep pothole to demonstrate their differences in their performance when it comes to comfortability for the passengers. Furthermore, one can tell from the plotted responses that the active suspensions system is superior to the passive suspension system. By how long it takes for the active suspension system to return to a steady-state of zero oscillations and by how the oscillations of the active suspension system are less than half of the oscillations of the passive suspension system.

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3. CarBikeTech "What is Active Suspension or Adaptive Suspension?"
<https://carbiketech.com/active-suspension-adaptive-suspension/>

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Abstract

The purpose of this report is to explain how to improve the current ABC system to improve the vehicle's overall comfort and stability. Research on the system has been carried out through the last two weeks of class over the system although the knowledge for Simulink and MATLAB has been brushed upon since month 2 of this 2022 Spring Semester.

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1.2 Overview of ABC.....	6
Chapter 2. Components, FBD, and Equations	10
2.1 Suspension Components	10

Chapter 1. Overview and Background

In this chapter, there will be a discussion on where the DAS system originated from and what cars are using this system, and other systems with similar properties. From my understanding, Former 1 Racing cars use different types of suspension for being able to have more control while turning at high speeds while also riding low to decrease airflow between the ground and the bottom of the car. Active Suspension Systems has changed the way of driving for Former 1 Racing, and it changed the way we drive overall in everyday cars.

1.1 Overview of DAS

In 1983, an electronically adjustable damper would affect the curve of the shock absorption. The “electronically adjustable” is known as adaptive or semi-active. This type of system was first used by Toyota and then later a similar model system was conducted in 1989 which was used on the Cadillac Allante.

There are various types of suspension systems that are known to be active or semi-active. An **active suspension system** is a type of automotive suspension on a vehicle. Using an onboard system to control the vertical movement of the vehicle’s body rather than passive suspension provided by large springs where the movement is determined entirely by the road surface. A **semi-active suspension system** automotive system that controls the damping force of the shock absorber in response to input from the continuously varying road surfaces. The system itself also adjusts the actuators in the dampers to reduce vibration and improve riding safety and comfort. Semi-active/adaptive suspensions only vary shock absorber firmness to match changing road or dynamic conditions, active suspensions use some actuators to raise and lower the chassis independently at each wheel.

A **DAS (Delft Active Suspension) System** is a type of automotive suspension system that has been developed being a low-energy roll and pitch attitude control (low frequency). With the least amount of work, the DAS system is able to keep the car horizontal in circumstances, with roll and pitch canceled out. The suspension system denoted by Delft Active Suspension (DAS) is able to suppress the low frequent sprung mass motions (such as rolling during cornering or pitch during braking/accelerating) using only a small amount of energy. Besides being able to control the vehicle's attitude, DAS can also be applied for improving the ride when the vehicle is operated on an uneven road (vibration control).

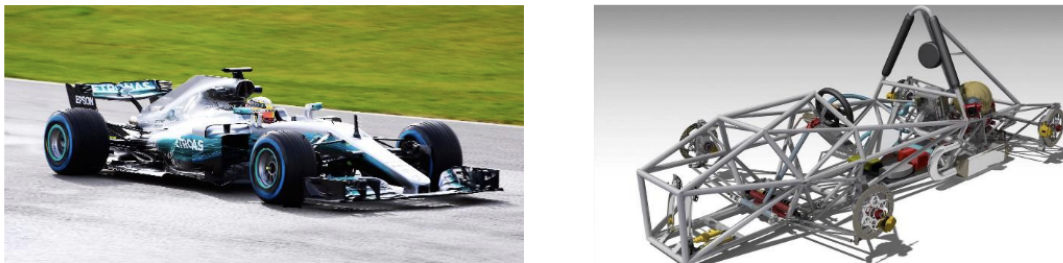


Figure 1.1 F1 Race Car and Suspension System

1.2 Overview of ABC

The **Active Body Control** system was originally introduced in 1999 on the Mercedes Benz CL-Class C215. In 2006, the C216 reintroduced the ABC system but as a second-generation Active Body

Suspension which was referred to as ABC Plus or ABC II in proper technical documentation. The Active Body Suspension is the only suspension system that can control individual spring and damping forces on each wheel. The suspension also controls the suppressing and rolling movements. The system itself has a clear focus on ultimately improving the suspension on each wheel to provide above satisfactory ride comfort, stability, and vehicle dynamics. Mercedes has been in the development of E-Active Body Control since 1999 and has been developed in-house by Mercedes-Benz individual engineering-based minds. The system uses hydraulic cylinders on top to control the low-frequency movements of the suspension. Below that hydraulic part is a spring and a dampener that can adjust to rebound and compression. The in-house developed suspension requires a 48V voltage operating power supply along with an AIRMATIC air suspension which is the base of the E-Active system.

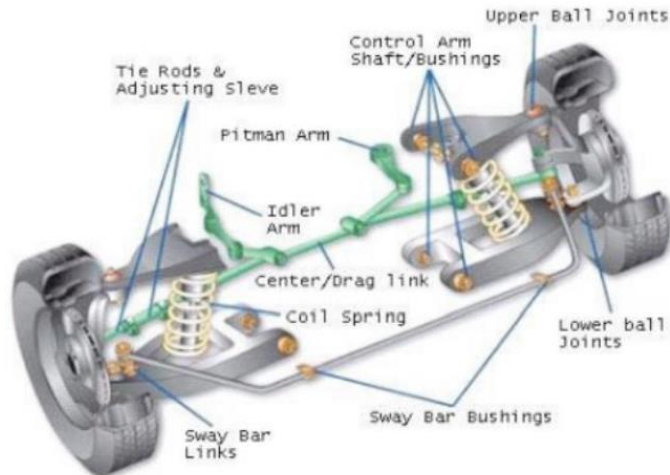


Figure 2 ABC Suspension System

The AIRMATIC system adjusts the amount of air for greater comfort and body roll as each air spring moves vertically. The self-leveling suspension system is a great suspension system for rough/rigid roads allowing a greater ground clearance. With the two combined, it provides an all-around level of control and stability keeping the vehicle level constant, no matter the driving conditions. The ABC suspension uses multiple accelerometers to measure the movement of the body during the cornering, braking, and acceleration process. Other accelerometers are able to measure if the body is moving up or down due to the wheel assembly.



Figure 3 AIRMATIC Suspension on Mercedes-Benz CL-Class

The E-Active Body Control actively supports and dampens the vehicle body as it's needed. This portion in particular is helpful to drivers while driving under uneven road conditions. During braking and accelerating, the vehicle tends to stay level above ground while various forces are acting on each wheel. The suspension system works by coordinating the air suspension with semi-supporting hydro-pneumatics. The hydro-pneumatic system provides water too automatically as defined by pressure & quantities in many locations from a single pump. These systems aren't involved in many of the newer cars only due to the fact of high development efforts and the additional cost of the suspension components. To provide excellent comfort and satisfaction to customers, it is necessary to have a suspension system that isolates the vehicle's chassis or drive from the unevenness of the ground. There are three different components applications in the suspension system:

- The Wheel or Axle Suspension
- Operator's Cab Suspension
- Boom Suspension
- Payload Suspension

A solid axle suspension houses the differential inside the axle itself, connected to the wheels by the rigid half-shafts. The entire axle moves as one, therefore if the left wheel were to drop into a pothole, the right wheel will move upward in response. Since the differential does not move with the axle, each wheel can have an independent suspension link to the vehicle. Since the suspension piece is shorter, it has a smaller turn radius to travel, therefore the wheel travel is reduced.

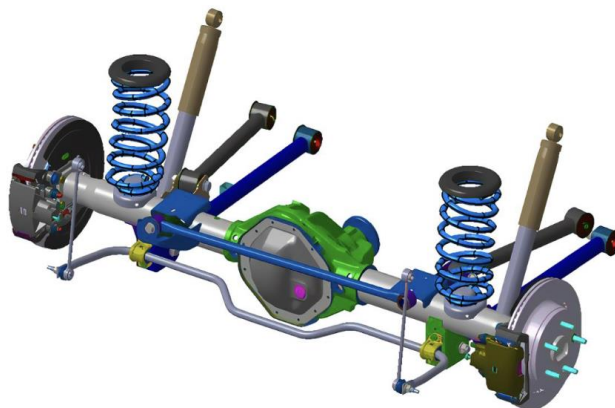


Figure 4 Solid Axle Suspension

Chapter 2. Components, FBD, and Equations

This chapter will discuss and show the different FBDs that are possible with the main focus on one. Important variables such as springs, mass, and dampers will also be discussed to show the significance of each component. This chapter is an important chapter that will correlate with the fourth chapter involving MATLAB and Simulink.

2.1 Suspension Components

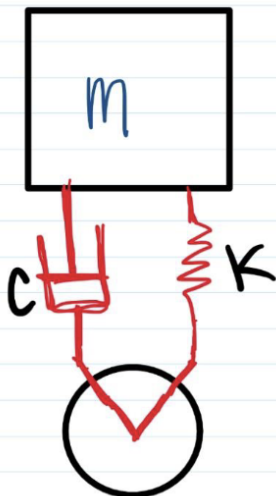
The main components that are a part of the Active Body Control system consist of a specialized pump, an active hydraulic cylinder, advanced struts, shock absorbers, springs, sensors, and 2 microcomputers. The type of pump that is used as the specialized pump in the active body control system is a high-pressure radial piston hydraulic pump that operates at 3,000 psi. The accumulators regulate the hydraulic pressure by means of an enclosed nitrogen bubble that is separated from the special hydraulic fluid by a membrane. The hydraulic pump is utilized by converting the mechanical energy of the body of the car into hydraulic energy. The active hydraulic cylinder could move up and down rapidly, a sensor in each cylinder to monitor its position, the microcomputers are able to detect the motion of the car being able to send a signal to the pump that will then move the hydraulic cylinders within the fraction of a millisecond. The Active system FBD will be a quarter-car model. Focusing on one suspension spring in the active system.

2.2 FBD of Quarter Vehicle

In the FBD below, the FBD is displaying the partial amount of the suspension system in a quarter-car. As in the Active Body Control system, the main components involve the spring and dampener. Utilizing the equation also in the figure is how the value can be determined if not already available.

Quarter Car
Model

X and Y were missing in the schematic diagram.



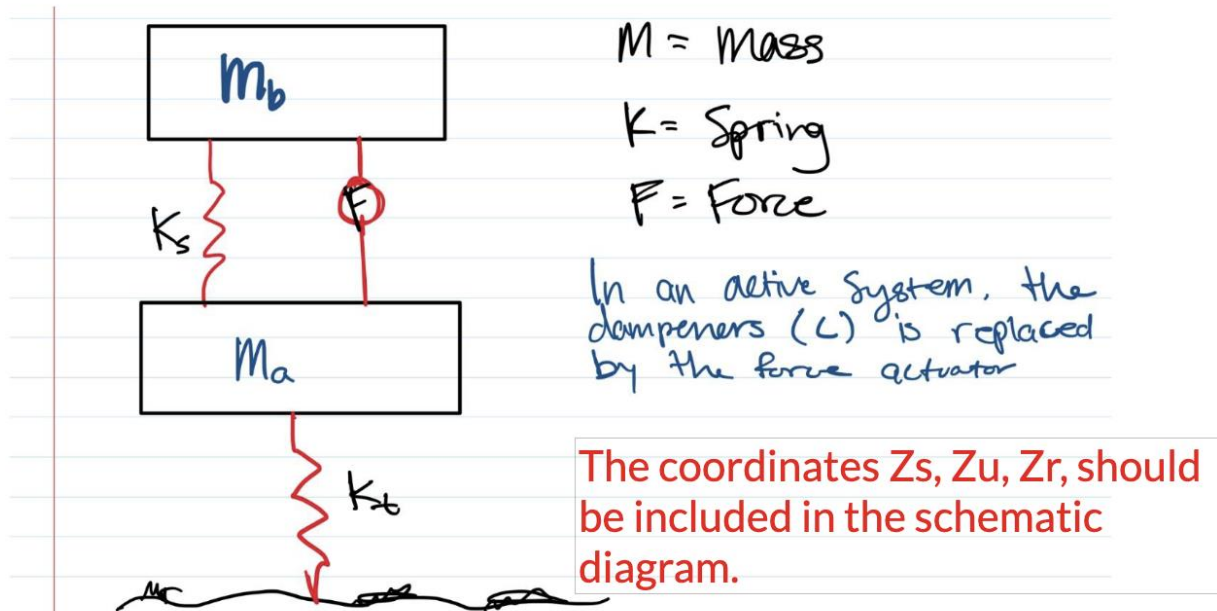
Equation:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$\ddot{x} = \frac{1}{m} \left((c\dot{y} + ky) - c\dot{x} - kx \right)$$

For a two degree of freedom system, there are two equations. You only derived one equation.

In this FBD, it is expressing a passive active system turning into an active system using the counterweights, springs, and pulleys to counterbalance the mass of the object. Changing the dampeners from a passive system into force actuators in a active utilizing the hydraulic pumps.



2.3 Equations

The mathematical model of electromagnetic active suspension is shown in the figure below. The friction force of the electromagnetic actuator is neglected. $M_s, M_u, K_s, K_t, Z_s, Z_u$, and Z_r all represent the same parameters and variables as described for a passive system. The electromagnetic force actuator is represented by F_a which is visualized as a control input.

$$M_s \ddot{Z}_s + K_s (Z_s - Z_u) = F_a$$

$$M_u \ddot{Z}_u - K_s (Z_s - Z_u) + K_t (Z_u - Z_r) = -F_a$$

In the figure below, the mathematical expression symbolizes the hydraulic active system in which an Active Body System is. An Active Body Control system utilizes the hydraulic pumps, springs, struts, sensors, and microcomputers.

Why the coordinates being changed to Z_u, Z_s and Z_r from the earlier equation with x and y ?

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = -F_f + F_A$$

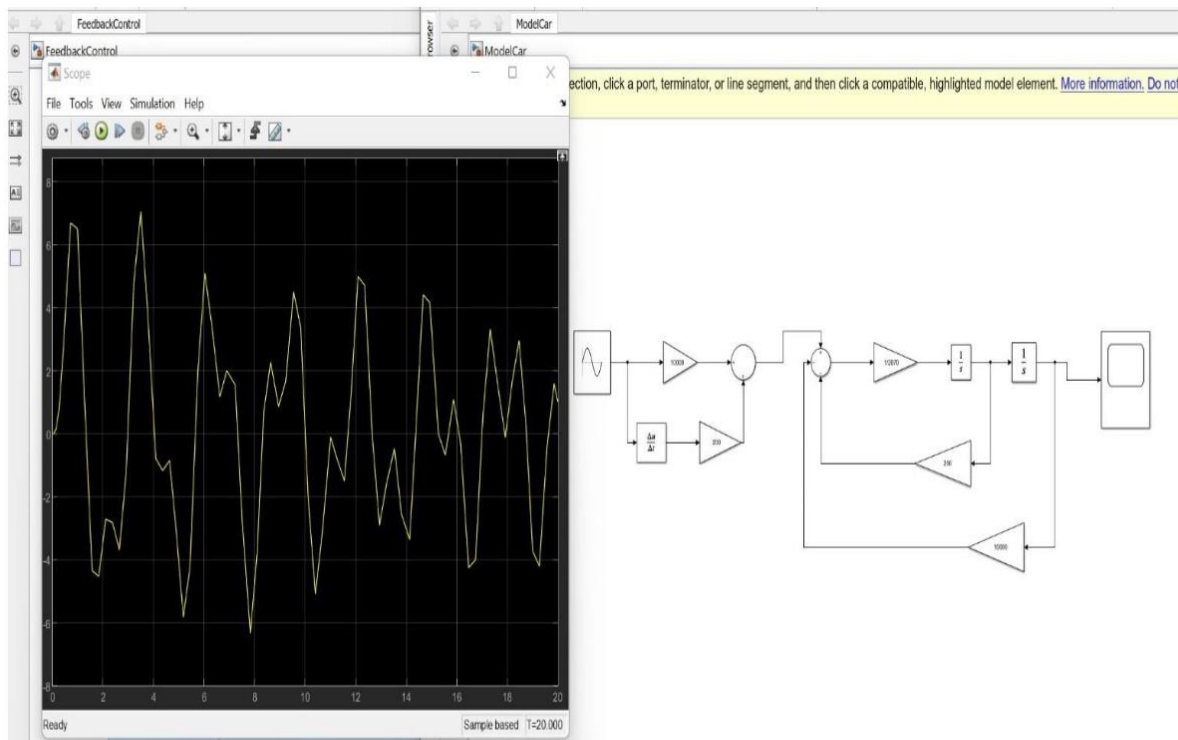
$$m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + k_e (z_u - z_r) = F_f - F_A$$

Chapter 3. Simulink

In this chapter, the Simulink for the suspension system will express the different vibrations and frequencies when the driver is on the open road. When driving harsh road conditions, some systems aren't able to respond quick enough.

3.1 Suspension System Simulink

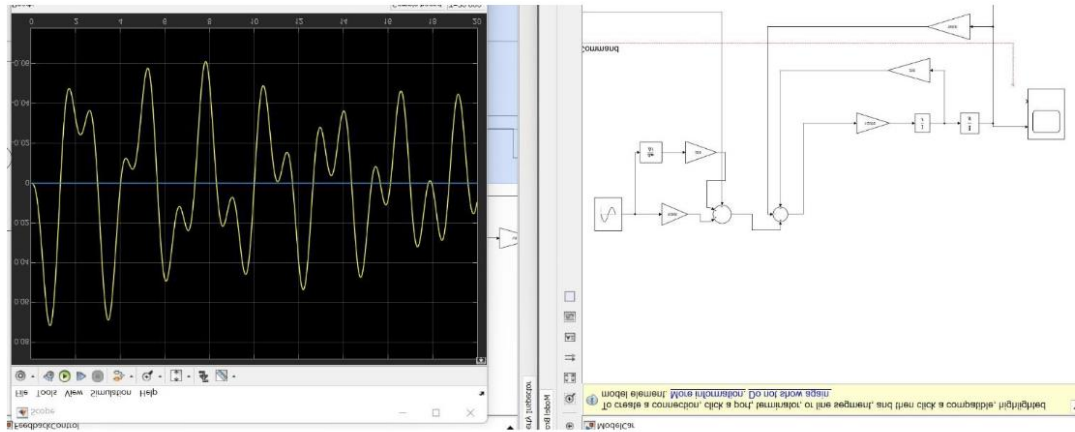
In the diagram below, the Simulink is being expressed through the MATLAB software showing the change and vibration frequencies that the user experiences while driving. This is the quarter-car model of a passive system which is in title with a different amplitude.



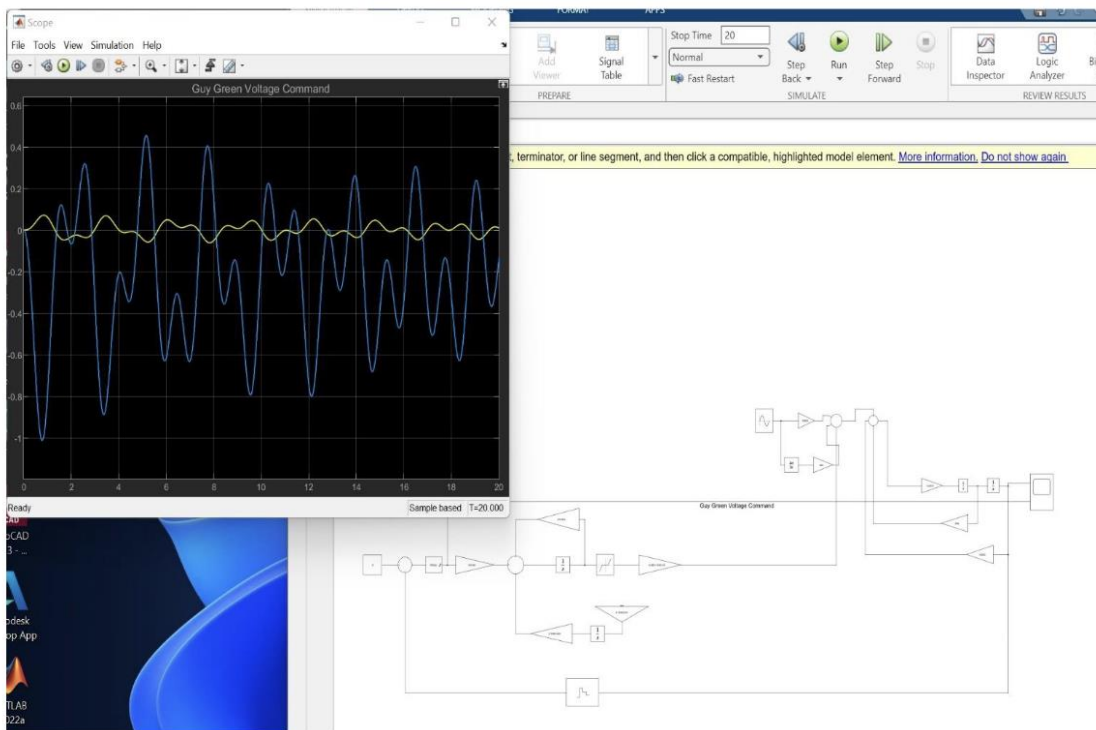
The input and output values of the Simulink diagram should be described and justified.

the specific values of amplitude and frequency must be included in the report.

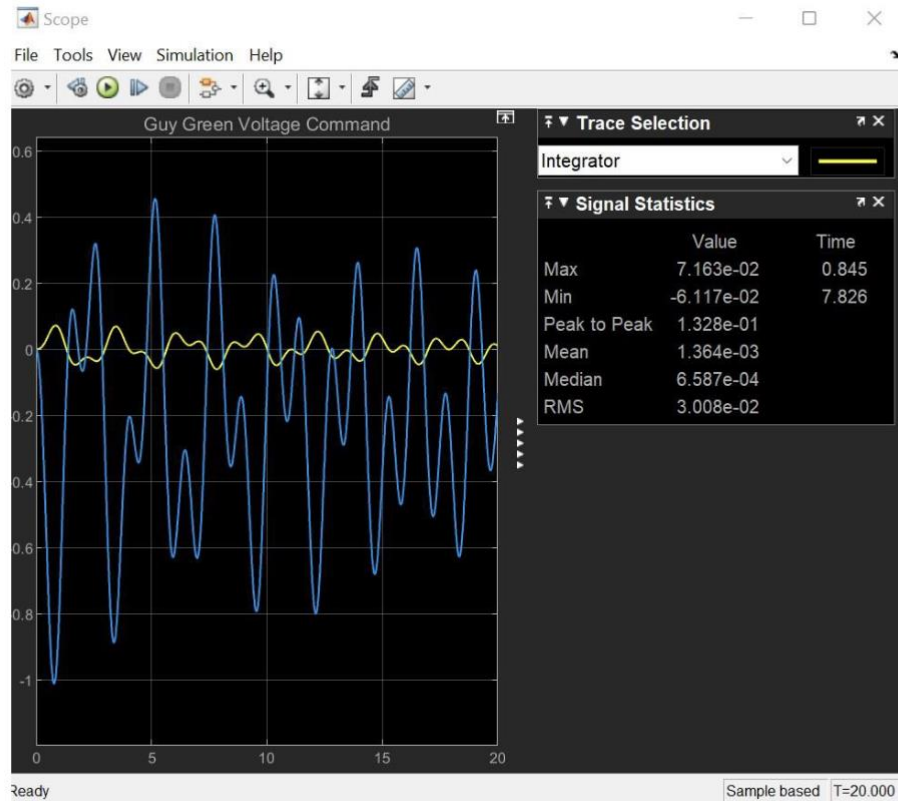
In the diagram below, the system is still passive but a change in amplitude and frequency has been made. The force is still in the same position but in a much smoother look.



In the figure below, the passive system has been changed to a active system with the PID tool. It took less than a second to reach it's max value but close to 10 seconds to reach it's lowest value.



The figure further expands on the maximum and minimum values that were met in the program.



Did not explain the meaning of the results from the Simulink output.

No conclusion.

No reference.