

**PRAIRIE VIEW A&M UNIVERSITY
ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
MCEG 3307 Automatic Controls
Fall 2021**

I. COURSE DESCRIPTION IN ABET FORMAT

II. ASSESSMENT RESULTS FOR CURRENT AND PAST THREE PERIODS

Program Outcomes Assessed in Course

Assessment Summary of Class Semester Averages for Outcomes

End of Semester Course Outcomes Assessment Reports

III. TESTS

Summary of Topics covered by Test

Original Test Questions & Master Solution/grading scheme used

(Summary of topics can be a separate page or include on the assignment itself)

Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

IV. QUIZZES

Summary of Topics covered by Quiz

Original Quiz Questions & Master Solution/grading scheme used

Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

V. HOMEWORK ASSIGNMENTS

Summary of Topics covered by Homework

Original Homework Questions & Master Solution/grading scheme used

Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

VI. PROJECT

Summary of Topics Covered by Project

Project Problem Statement and Grading Scheme

Above Average (Graded original work)

Average (Graded original work)

Below Average (Graded original work)

Course Description in ABET Format

MECHANICAL ENGINEERING DEPARTMENT MCEG 3307 Automatic Controls

- | | |
|--|---|
| 1. Course number and name: | MCEG 3307 Automatic Controls |
| 2. Credits and contact hours: | 3 credit hours |
| 3. Instructor's or course coordinator's name: | Dr. Mingli Han |
| 4. Text book: Title: | "Control Systems Engineering", ISBN9781118170519, |
| Author: | Norman S. Nise |
| Publisher, Year & Edition: | Wiley, 2015 |
| a. Other Supplemental material | "Feedback Control of Dynamic Systems", 7 th edition, G. F. Franklin, J. D. Powell, and A. Emami-Naeini, Pearson, NJ, ISBN: 0133496597, 2014. |
| 5. Specific course information | |
| a. Brief description of the content of the course (catalog description) | Analysis and synthesis of continuous time control systems, transfer function, block diagrams, stability, root locus, state space representation, and design considerations for feedback control system. |
| b. Prerequisites or Co-requisites | |
| c. *Indicate whether a required, elective, or selected elective (as per definition provided) course in the program | Elective |

- | | |
|---|--|
| 6. a. Specific goals for the course specific outcomes of instruction (e.g. The student will be able to explain the significance of current research about a particular topic.) | b. Explicitly indicate which of the student outcomes listed in Criterion 3 or any other outcomes are addressed by the course. |
|---|--|

Students will be able to :

The specific goals for the course specific outcomes listed on the left column will address the following student learning outcomes:

1. Derive mathematical model for mechanical, electrical and electromechanical systems
2. Apply Laplace transform techniques to determine the time response of physical systems
3. Model a system of components by block diagram and determine the input/output behavior of the system based on such a model

4. Generate state-space models for linear systems
5. Use time domain techniques to analyze systems and to design controllers that achieve prescribed requirements
6. Use frequency domain techniques to analyze systems and to design controllers that achieve prescribed requirements

7 Brief list of topics to be covered

1. Introduction
2. Laplace transform
3. Transfer function
4. Modeling of dynamic systems
5. Time response analysis
6. Stability and performance
7. Root locus
8. Controller design

***Required** courses are required of all students in the program, **elective** courses (often referred to as open or free electives) are optional for students, and **selected elective** courses are those for which students must take one or more courses from a specified group

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II ASSESSMENT RESULTS FOR CURRENT AND PAST THREE PERIODS

Program Outcomes Assessed in this course

Technical elective course is not assessed for current and past periods.

End of Semester Course Assessment Reports Fall 2021 and Last Three Assessment Periods.

Each summary contains the following Information

1. Performance Statistics

- Total number of students in class
- The class average performance (**stated as %**) in each course outcome assessed in this class and the **two previous times course was assessed**.
- The acceptable class average for the class in each outcome area as set by your Department.
- Percentage of students who scored below the expected average in each outcome.

2. Implementation Summary

- Brief summary of plans implemented during the semester based on last assessment report

3. Perceived Problems

- Instructor's **critical evaluation** of perceived problems that affected students' performance. **Realistic problems should be identified throughout the semester and summarized in this section of the report.**

4. New Plans for Addressing Problems

- Instructor's plans for addressing the perceived problems the next time the class is taught. **These should be specific enough for another instructor to implement. Most of these should be things that, you, the instructor can implement to help students to improve. Be creative.**

5. Overall Trend over last three assessment periods

- Instructors summary of overall performance trend for each outcome measured over the last three periods.

5. Were Expectations Met?

- Answer **YES** if the **class average was above the expected average** and the **stated percentage of students** scored at or above the expected average, else answer NO.

**Automatic Controls
Fall 2021**

Title of Assignment: Midterm Exam

TOPICS COVERED

Midterm Exam covered the following topics:

1. Introduction
2. Laplace transform
3. Transfer function
4. Modeling of dynamic systems

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

Student Name (Print):

Midterm Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Closed book, closed homework, no USB drive, no cell phone during the exam. Only pen/pencil, eraser, ruler and calculator can be used in the exam
- Any one minute late submission will result in a 10% penalty of the grade

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1

(1) What is unit step function? (10 points)

(2) What is the Laplace Transform of the unit step function? (5 points)

(3) Please find the Laplace Transform of $f(t) = e^{-t} \sin \omega t u(t)$. (10 points)

(4) For electrical elements, impedance is the relationship between voltage and current, i.e. $Z(s) = \frac{V(s)}{I(s)}$. Assuming C, R and L are the coefficients for capacitor, resistor and inductor, respectively, what is the impedance of capacitor, resistor and inductor? (10 points)

Problem 2

- (1) Consider the block diagram shown in Figure 1 and Figure 2, what is the transfer function between $Y(s)$ and $R(s)$? (20 points)

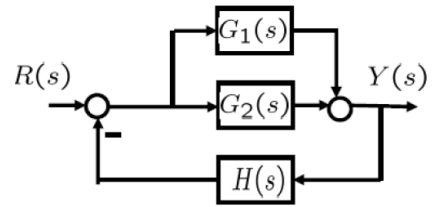


Figure 1 Block diagram.

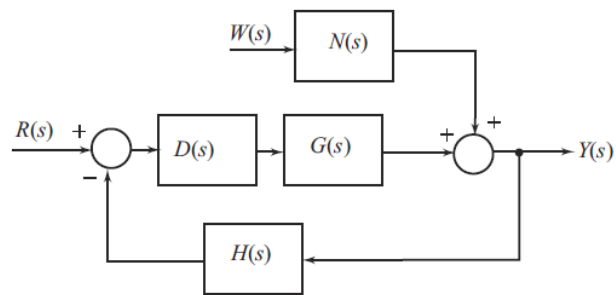
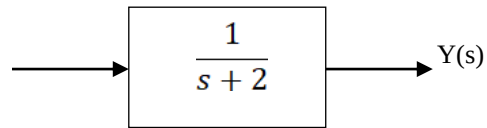


Figure 2 Block diagram.

Hint: assuming $W(s) = 0$

- (2) Consider the following system, if the unit impulse function is given as the input, what is the output of the system, $Y(s)$. (15 points)



Problem 3

- (1) A system is described by the following differential equation:

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

Assume all the initial conditions are zero, find the expression for the transfer function of the system, $\frac{Y(s)}{X(s)}$. (15 points)

(2) Solve for $y(t)$ in the below Ordinary Differential Equation using Laplace Transform and Partial-Fraction Expansion, assume all initial conditions are zero. (15 points)

$$\frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2\delta(t)$$

Hint:

$$s^3 + 5s^2 + 8s + 4 = (s^3 + 4s^2 + 4s) + (s^2 + 4s + 4) = s(s^2 + 4s + 4) + (s + 2)^2 = s(s + 2)^2 + (s + 2)^2 = (s + 1)(s + 2)^2$$

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at} u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$

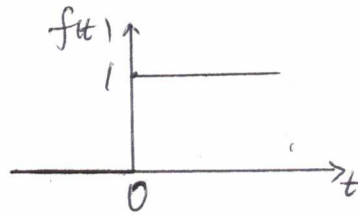
TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st} dt$	Definition
2.	$\mathcal{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathcal{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathcal{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathcal{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathcal{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{(k-1)}(0-)$	Differentiation theorem
10.	$\mathcal{L}\left[\int_{0-}^t f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$	Final value theorem ¹
12.	$f(0+) = \lim_{s \rightarrow \infty} sF(s)$	Initial value theorem ²

Problem 1

(1) What is unit step function? (10 points)

$$f(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



(2) What is the Laplace Transform of the unit step function? (5 points)

$$\frac{1}{s}$$

(3) Please find the Laplace Transform of $f(t) = e^{-t} \sin \omega t u(t)$. (10 points)

$$\mathcal{L}(\sin \omega t) = \frac{\omega}{s^2 + \omega^2}$$

according to shift property $\mathcal{L}(e^{-t} \sin \omega t) = \frac{\omega}{(s+1)^2 + \omega^2}$

(4) For electrical elements, impedance is the relationship between voltage and current, i.e. $Z(s) = \frac{V(s)}{I(s)}$. Assuming C, R and L are the coefficients for capacitor, resistor and inductor, respectively, what is the impedance of capacitor, resistor and inductor? (10 points)

capacitor: $\frac{V(s)}{I(s)} = \frac{1}{sC}$

inductor resistor: $\frac{V(s)}{I(s)} = sL$

resistor: $\frac{V(s)}{I(s)} = R$

Problem 2

- (1) Consider the block diagram shown in Figure 1 and Figure 2, what is the transfer function between $Y(s)$ and $R(s)$? (20 points)

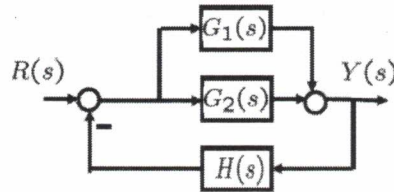


Figure 1 Block diagram.

$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g}, \quad F_g = G_1(s) + G_2(s)$$

$$L_g = -(G_1 + G_2(s)) H(s)$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{G_1(s) + G_2(s)}{1 + (G_1(s) + G_2(s)) H(s)} = \frac{G_1(s) + G_2(s)}{1 + (G_1(s) + G_2(s)) H(s)}$$

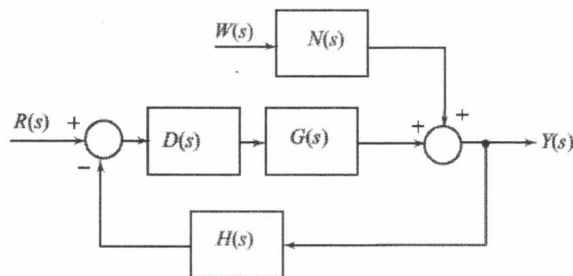


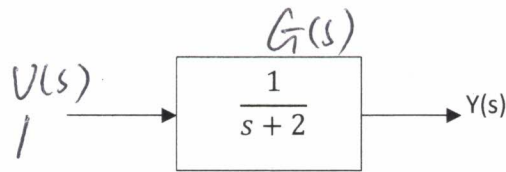
Figure 2 Block diagram.

$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g}, \quad F_g = D(s) G(s)$$

$$L_g = -D(s) G(s) H(s)$$

$$\frac{Y(s)}{R(s)} = \frac{D(s) G(s)}{1 + D(s) G(s) H(s)}$$

- (2) Consider the following system, if the unit impulse function is given as the input, what is the output of the system, $Y(s)$. (15 points)



$$Y(s) = U(s) G(s) = \frac{1}{s+2}$$

Problem 3

- (1) A system is described by the following differential equation:

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

Assume all the initial conditions are zero, find the expression for the transfer function of the system, $\frac{Y(s)}{X(s)}$. (15 points)

$$s^3 Y(s) + 3s^2 Y(s) + 5s Y(s) + Y(s) = s^3 X(s) + 4s^2 X(s) + 6s X(s) + 8 X(s)$$

$$(s^3 + 3s^2 + 5s + 1) Y(s) = (s^3 + 4s^2 + 6s + 8) X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

(2) Solve for $y(t)$ in the below Ordinary Differential Equation using Laplace Transform and Partial-Fraction Expansion, assume all initial conditions are zero. (15 points)

$$\frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2\delta(t)$$

Hint: $s^3 + 5s^2 + 8s + 4 = (s^3 + 4s^2 + 4s) + (s^2 + 4s + 4) = s(s^2 + 4s + 4) + (s + 2)^2 = s(s + 2)^2 + (s + 2)^2 = (s + 1)(s + 2)^2$

$$s^3 Y(s) + 5s^2 Y(s) + 8s Y(s) + 4 Y(s) = 2$$

$$Y(s) = \frac{2}{s^3 + 5s^2 + 8s + 4} = \frac{2}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{(s+2)^2} + \frac{C}{s+2}$$

multiply $s+1$: $\left. \frac{2}{(s+2)^2} \right|_{s \rightarrow -1} = A + (s+1) \left. \frac{B}{(s+2)^2} \right|_{s \rightarrow -1} + (s+1) \left. \frac{C}{s+2} \right|_{s \rightarrow -1}$
 $\therefore A = 2$

multiply $(s+2)^2$: $\left. \frac{2}{s+1} \right|_{s \rightarrow -2} = (s+2)^2 \left. \frac{A}{s+1} \right|_{s \rightarrow -2} + B + (s+2) \left. C \right|_{s \rightarrow -2}$
 $\therefore B = -2$

To get C : $\frac{2}{s+1} = (s+2)^2 \frac{A}{s+1} + B + (s+2) C$

take derivative : $\left. \frac{-2}{(s+1)^2} \right|_{s \rightarrow -2} = \frac{2(s+2)(s+1) - (s+2)^2}{s+1} A \Big|_{s \rightarrow -2} + C$

$$\therefore C = -2$$

$$\therefore Y(s) = \frac{2}{s+1} - \frac{2}{(s+2)^2} - \frac{2}{s+2}$$

$$\therefore y(t) = (2e^{-t} - 2te^{-2t} - 2e^{-2t}) u(t)$$

ABOVE AVERAGE

(Note: All Pages shown in Light Blue should be printed on Light Blue Paper)

Test 1

94

1. Unit step function

$$f(t) = x(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Values change abruptly, example on/off switch of voltage.

2. Laplace transform of unit step function

$$F(s) = \int_0^{\infty} 1 \cdot e^{-st} dt = -\frac{1}{s} [e^{-st}]_0^{\infty} = \boxed{\frac{1}{s}}$$

3. Laplace transform of $f(t) = e^{-t} \sin \omega t u(t)$

- From table

$$\propto (e^{-at} u(t)) \rightarrow x(s+a)$$

$$\propto (\sin \omega t u(t)) \rightarrow \frac{\omega}{s^2 + \omega^2}$$

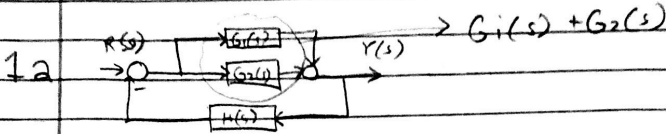
$$e^{-t} \sin \omega t u(t) \rightarrow \boxed{\frac{\omega}{(s+a)^2 + \omega^2}}$$

4. Resistance $v(t) = Ri(t)$ $\frac{V(s)}{I(s)} = \boxed{R}$

Inductance $v(t) = L \frac{di}{dt}$ $\frac{V(s)}{I(s)} = \boxed{sL}$

Capacitance $i(t) = \frac{d \cdot v(t)}{dt}$ $\frac{V(s)}{I(s)} = \boxed{\frac{1}{sC}}$

Problem 2



$$\frac{Y(s)}{R(s)} = \frac{G_1(s) + G_2(s)}{1 + (G_1(s) + G_2(s))H(s)}$$

→ looking at circuit

1b Figure 2 Block Diagram

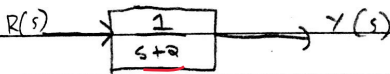
Forward path → $Y(s)$ to $R(s)$

= $D(s)$ to $G(s)$

Loops: $1 - (-D(s)G(s)H(s))$

$$\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 - (-D(s)G(s)H(s))} \quad \left[\frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)} \right]$$

2. Unit impulse function → $r(t) = \delta(t) = R(s) = 1$



$$Y(s) = R(s) \frac{1}{2s+1}$$

$$Y(s) = 1 \cdot \frac{1}{2s+1} \quad \times$$

$$Y(s) = \frac{1}{2s+1} \Rightarrow \frac{1}{2(s+\frac{1}{2})} \quad -2$$

$$y(t) = \frac{1}{2} e^{-1/2 t} u(t)$$

Output is $\frac{1}{2s+2}$

Problem 3

1 - Find transfer function

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

$$\mathcal{L}\left[\frac{d^3 y}{dt^3}\right] = s^3 Y(s) - s^2 y(0) - sy'(0) - y''(0)$$

If initial conditions are 0

$$y(0) = 0 \quad y'(0) = 0 \quad y''(0) = 0$$

$$\mathcal{L}\left[\frac{d^3 y}{dt^3}\right] = s^3 Y(s)$$

$$\mathcal{L}\left[\frac{d^2 y}{dt^2}\right] = s^2 Y(s)$$

$$\mathcal{L}\left[\frac{dy}{dt}\right] = s Y(s)$$

$$\mathcal{L}\left[\frac{d^3 x}{dt^3}\right] = s^3 X(s)$$

$$\mathcal{L}\left[\frac{d^2 x}{dt^2}\right] = s^2 X(s)$$

$$\mathcal{L}\left[\frac{dx}{dt}\right] = s X(s)$$

Putting replacements in given equation

$$s^3 Y(s) + 3s^2 Y(s) + 5s Y(s) + Y(s) = s^3 X(s) + 4s^2 X(s) + 6s X(s) + 8X(s)$$

$$Y(s) [s^3 + 3s^2 + 5s + 1] = X(s) [s^3 + 4s^2 + 6s + 8]$$

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

$$2. \quad \frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2 \delta(t)$$

- Applying Laplace to both sides

$$\mathcal{L}[d^3] + 5\mathcal{L}[d^2] + 8\mathcal{L}[d] + 4\mathcal{L}[y] = 2\mathcal{L}[\delta(t)]$$

$$s^3 + 5s^2 + 8s + 4 = 2e^{-t}$$

$$\mathcal{L} y(t) = \frac{e^{-t}}{(s+1)(s^2+4s+4)} \Rightarrow \frac{e^{-t}}{s+1(s+2)^2}$$

Inverse Laplace

$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\frac{1}{(s+1)(s+2)^2} = \frac{A(s+2)^2 + B(s+1)(s+2) + C(s+1)}{(s+1)(s+2)^2}$$

$$1 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

When $s = -2$ then $C = -1$

When $s = -1$ then $A = 1$ X

$0 = A + B \Rightarrow B = -A$ so $B = -1$

$$\frac{1}{s+1(s+2)^2} = \frac{1}{s+1} + \frac{-1}{s+2} + \frac{-1}{(s+2)^2}$$

so $\mathcal{L}^{-1} \left\{ e^{-t} \left(\frac{1}{s+1} + \frac{-1}{s+2} + \frac{-1}{(s+2)^2} \right) \right\}$ X

$$(e^{-(t-1)} - e^{-1(t-1)} - (t-1)e^{-1(t-1)}) u(t-1)$$

- 4

AVERAGE

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

76

Student Name (Print):

Zalayna
Jenkins

Midterm Exam

MCEG3307 Automatic Controls

Fall 2021

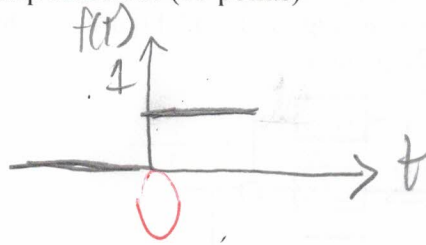
- Please stay on this page until you are asked to start the exam
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Note:

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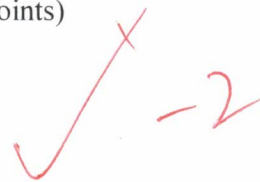
Problem 1

(1) What is unit step function? (10 points)



(2) What is the Laplace Transform of the unit step function? (5 points)

$$\mathcal{L} = \int_{-\infty}^{\infty} \delta(t) g(t) dt = g(0)$$



(3) Please find the Laplace Transform of $f(t) = e^{-t} \sin \omega t u(t)$. (10 points)

$$\mathcal{L} = \frac{1}{s-1} \cdot \frac{\omega}{s^2 + \omega^2}$$



(4) For electrical elements, impedance is the relationship between voltage and current, i.e. $Z(s) = \frac{V(s)}{I(s)}$. Assuming C, R and L are the coefficients for capacitor, resistor and inductor, respectively, what is the impedance of capacitor, resistor and inductor? (10 points)

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

(Capacitor)

$$\frac{V(s)}{I(s)} = R$$

(Resistor)

$$\frac{V(s)}{I(s)} = sL$$

(Inductor)



Problem 2

- (1) Consider the block diagram shown in Figure 1 and Figure 2, what is the transfer function between $Y(s)$ and $R(s)$? (20 points)

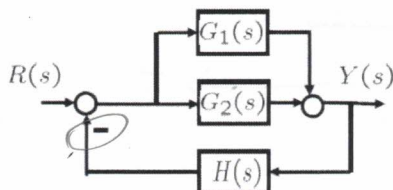


Figure 1 Block diagram.

$$\frac{Y(s)}{R(s)} = \frac{F}{1-Lg} = \frac{G_1(s) + G_2(s)}{1 - G_1(s)G_2(s)H(s)} = \frac{G_1(s) + G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

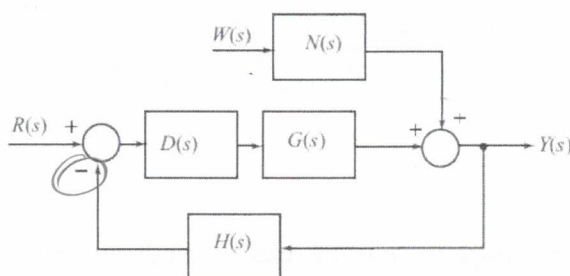
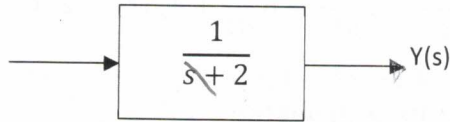


Figure 2 Block diagram.

Hint: assuming $W(s) = 0$

$$\frac{Y(s)}{R(s)} = \frac{F}{1-Lg} = \frac{D(s) \times G(s) + N(s)}{1 - D(s)G(s)H(s)} = \frac{D(s) \times G(s) + N(s)}{1 + D(s)G(s)H(s)}$$

- (2) Consider the following system, if the unit impulse function is given as the input, what is the output of the system, $Y(s)$. (15 points)



$$Y(s) = \frac{1}{2}$$

-10

Problem 3

- (1) A system is described by the following differential equation:

$$\frac{d^3 y}{dt^3} + 3 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + y = \frac{d^3 x}{dt^3} + 4 \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 8x$$

Assume all the initial conditions are zero, find the expression for the transfer function of the system, $\frac{Y(s)}{X(s)}$. (15 points)

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 3s^2 + 5s + 1}{s^3 + 4s^2 + 6s + 8} = \frac{s^3 + 3s^2 + 5s + 1}{(s+2)(s+2)^2}$$

$$(s^3 + 4s^2 + 2s) + (s^2 + 4s + 2)$$

$$s(s^2 + 4s + 2)(s+2)^2$$

$$s(s+4)^2 + (s+4)^2$$

$$= (s+2)(s+2)$$

-2

- (2) Solve for $y(t)$ in the below Ordinary Differential Equation using Laplace Transform and Partial-Fraction Expansion, assume all initial conditions are zero. (15 points)

$$\frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2\delta(t) - 1$$

Hint: $s^3 + 5s^2 + 8s + 4 = (s^3 + 4s^2 + 4s) + (s^2 + 4s + 4) = s(s^2 + 4s + 4) + (s+2)^2 = s(s+2)^2 + (s+2)^2 = (s+1)(s+2)^2$

$$s^3 + 5s^2 +$$

$$\frac{2\delta(t)}{(s+1)(s+2)^2}$$

$$\frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

$$s+1=0$$

$$-1+1=0$$

$$A + (s+1) \frac{B}{s+2} + (s+1) \frac{C}{s+2}$$

$$\cancel{s+1} \frac{2}{(s+2)(s+2)} = 2$$

$$(-1+2)(-1+2)$$

$$B = \frac{A}{(s+2)} + B + \frac{C(s+2)}{s+2}$$

$$s+2=0$$

$$-2+2=0$$

$$\cancel{s+2} \frac{2}{(s+1)(s+2)^2} = -2$$

$$-2+1$$

$$C = \frac{A}{s+1} + \frac{B}{s+2} + C$$

$$\cancel{(s+2)} \frac{2}{(s+1)(s+2)} = -2$$

$$(2+1)$$

$$y(t) = (2e^{-5t} - 2e^{-8t} - 2e^{-t})$$

3

BELOW AVERAGE

PRAIRIE VIEW A&M UNIVERSITY

ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING

Total Grade:

48

Student Name (Print): Ke'Andre March

Midterm Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Closed book, closed homework, no USB drive, no cell phone during the exam. Only pen/pencil, eraser, ruler and calculator can be used in the exam
- Any one minute late submission will result in a 10% penalty of the grade

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1

(1) What is unit step function? (10 points)

A unit step function consist of an open system and a closed system. A closed system is able to correct itself and an open system cant.

X
-9

(2) What is the Laplace Transform of the unit step function? (5 points)

$$\mathcal{L}[f(t)] = F(s)$$

$$= \int_0^{\infty} f(t)e^{-st} dt, \mathcal{L}[kf(t)] = kF(s)$$

-3

X

(3) Please find the Laplace Transform of $f(t) = e^{-t}\sin\omega t u(t)$. (10 points)

$$\mathcal{L}[f(t)] = \mathcal{L}[e^{-t}\sin\omega t u(t)]$$

$$e^{-t} = \frac{1}{s+a} \quad a=1$$

$$\sin\omega t u(t) = \frac{\omega}{s^2 + \omega^2}$$

$$\frac{1}{s+1} \rightarrow \frac{1}{s} \rightarrow u(t)$$

$$\mathcal{L}[f(t)] \text{ of } e^{-t}\sin\omega t u = u(t) \left(\frac{\omega}{s^2 + \omega^2} \right)$$

-2

(4) For electrical elements, impedance is the relationship between voltage and current, i.e.

$Z(s) = \frac{V(s)}{I(s)}$. Assuming C, R and L are the coefficients for capacitor, resistor and inductor,

respectively, what is the impedance of capacitor, resistor and inductor? (10 points)

$$Z(s) = \frac{V(s)}{I(s)} \quad \text{voltage} \\ \text{current}$$

$$C, R, L \quad \text{inductor} \\ \text{capacitor} \quad \text{resistor}$$

Impedance

$$Z(s) = \frac{C(s)}{R}$$

X

-7

Problem 2

- (1) Consider the block diagram shown in Figure 1 and Figure 2, what is the transfer function between $Y(s)$ and $R(s)$? (20 points)

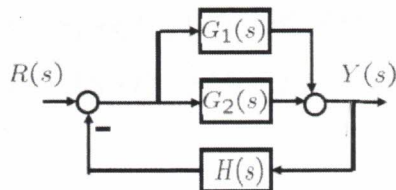


Figure 1 Block diagram.

$$u(t) = \frac{1}{s}$$

$$W(s) = 0$$

$$tu(t) = \frac{1}{s^2}$$

$$Y(s) \rightarrow H(s) \rightarrow R(s) \rightarrow G_1(s) \rightarrow G_2(s)$$

$\frac{1}{s_H} \quad \frac{1}{H} \quad \frac{1}{R} \quad \frac{1}{s_R} \quad \frac{1}{s^2} \quad \frac{1}{s^2}$

$\times \quad R(s) \leftarrow$

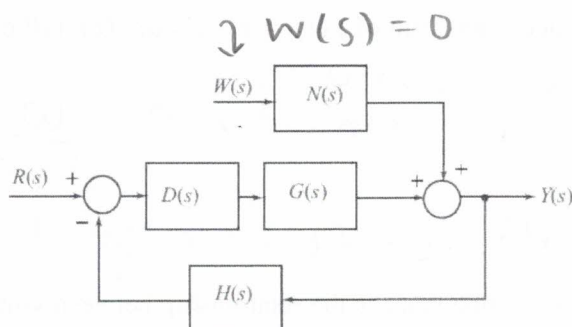


Figure 2 Block diagram.

Hint: assuming $W(s) = 0$

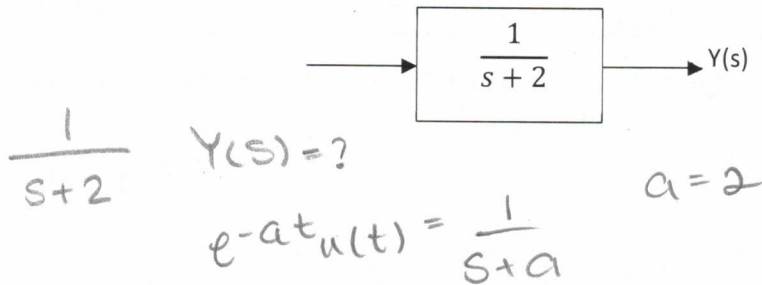
$$0 \rightarrow N(s) \rightarrow$$

$\times$

$$R(s) \rightarrow \frac{1}{D} \rightarrow \frac{1}{G} \rightarrow Y(s)$$

$\uparrow$ $H(s)$ $\downarrow$ $\frac{1}{H}$

- (2) Consider the following system, if the unit impulse function is given as the input, what is the output of the system, $Y(s)$. (15 points)



$$\frac{1}{s+2} = e^{-2t} = Y(s)$$

output function
of the system

Problem 3

- (1) A system is described by the following differential equation:

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$$

Assume all the initial conditions are zero, find the expression for the transfer function of the system, $\frac{Y(s)}{X(s)}$. (15 points)

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$$

$$\frac{Y(s)}{X(s)} = s^3 + 3s^2 + 5s + 1 = a^3 + 4a^2 + 6a + 8x$$

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 3s^2 + 5s + 1}{a^3 + 4a^2 + 6a + 8x} - \frac{1}{8x}$$

$$C_1 = 0$$

initial conditions

(2) Solve for $y(t)$ in the below Ordinary Differential Equation using Laplace Transform and Partial-Fraction Expansion, assume all initial conditions are zero. (15 points)

$$\frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2\delta(t)$$

Hint: $s^3 + 5s^2 + 8s + 4 = (s^3 + 4s^2 + 4s) + (s^2 + 4s + 4) = s(s^2 + 4s + 4) + (s+2)^2 = s(s+2)^2 + (s+2)^2 = (s+1)(s+2)^2$

$$\frac{d^3 y}{dt^3} + 5 \frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 4y(t) = 2\delta(t) \quad \mathcal{L}\delta(t) = 1$$

$$s^3 + 5s^2 + 8s + 4 = 2(1)$$

$$(s+2)^2 = s(s+2)^2 + (s+2)^2 = (s+1)(s+2)^2$$

$$Y(s) = \frac{(s+1)(s+2)^2}{2}$$

$$s^3 + 5s^2 + 8s + 4(Y(s)) = 2(1)$$

$$s^3 + 5s^2 + 8s - 2(1) = -4(Y(s))$$

$$s^3 + 5s^2 + 8s - 2 = -4(Y(s))$$

$$s^3 + 4s^2 + 4s + (s^2 + 4s - 2)$$

$$s(s^2 + 4s - 2) + (s+2)^2$$



10

**Automatic Controls
Fall 2021**

Title of Assignment: Final Exam

TOPICS COVERED

Final Exam covered the following topics:

1. Modeling of dynamic systems
2. Time response analysis
3. Stability and performance
4. Root locus
5. Controller design

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

Student Name (Print):

Final Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Any one minute late submission will result in a 10% penalty of the grade
- Any dishonest behavior will result in “F” grade immediately

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1 (9 points)

True or False

- (1) If the transfer function of the closed-loop system has at least one simple pole and no repeated poles on the $j\omega$ -axis, the system is stable.
- (2) The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.
- (3) Poles and zeros marked on the root locus are the poles and zeros of the closed-loop transfer function

Problem 2 (51 points)

- (1) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{5(s+2)}{(s-1)(s^2+s+1)}$$

- (2) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{1}{s^2}$$

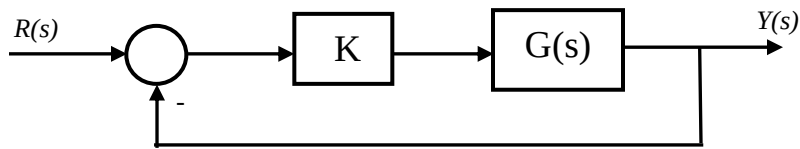
(3) The standard form of the second-order system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

If the poles location moves to the left side of the s-plane, the settling time will increase or decrease? (5 points) Why? (6 points)

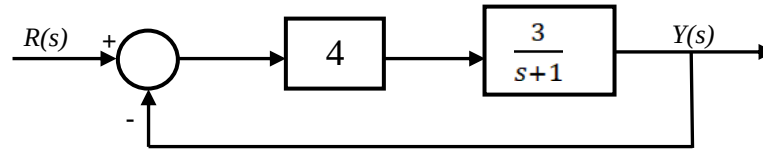
(4) Please find the range of K to keep the following system stable. (20 points)

$$G(s) = \frac{s + 1}{s(s + 2)}$$



Problem 3 (40 points)

Consider the following unity feedback system:



- (1) What is the characteristic equation? (10 points)
- (2) What is the closed-loop transfer function of this system? (10 points)
- (3) Assuming a unit step function is given to the system, please calculate the steady state value, y_{ss} . (10 points)
- (4) Assuming a unit step function is given to the system, please calculate the steady state error, e_{ss} . (10 points)

Problem 1 (9 points)

True or False

- (1) If the transfer function of the closed-loop system has at least one simple pole and no repeated poles on the $j\omega$ -axis, the system is stable.

False

- (2) The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.

True

- (3) Poles and zeros marked on the root locus are the poles and zeros of the closed-loop transfer function

False

Problem 2 (51 points)

- (1) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{5(s+2)}{(s-1)(s^2+s+1)}$$

poles: $s-1=0$, $s^2+s+1=0$

$$s_1 = 1 \quad s_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2}$$

s_1 is positive. $\therefore$ The system is unstable

- (2) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{1}{s^2}$$

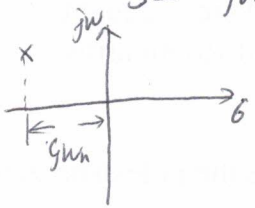
$$s^2=0 \Rightarrow s_1=0, s_2=0$$

There are two repeated poles on the $j\omega$ -axis
 $\therefore$ the system is unstable

(3) The standard form of the second-order system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

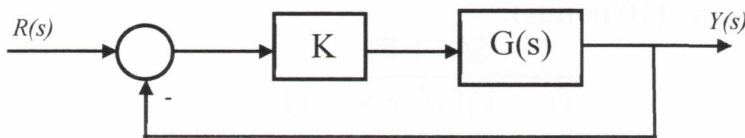
If the poles location moves to the left side of the s-plane, the settling time will increase or decrease? (5 points) Why? (6 points)

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$


If poles location moves to the left side of the s-plane, $\zeta\omega_n \uparrow$, $\zeta\omega_n$ will increase, settling time = $\frac{3}{\zeta\omega_n}$ will decrease.

(4) Please find the range of K to keep the following system stable. (20 points)

$$G(s) = \frac{s+1}{s(s+2)}$$



$$1 + K \cdot \frac{s+1}{s(s+2)} = 0$$

$$s(s+2) + k(s+1) = 0$$

$$s^2 + (k+2)s + k = 0$$

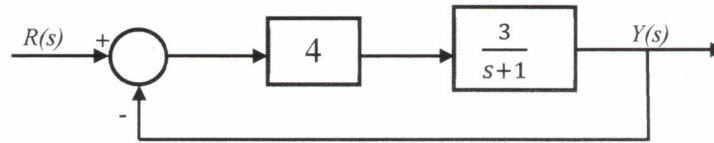
s^2		1	K
s		$k+2$	0
s^0		k	

$$k+2 > 0, \quad k > 0$$

$$\therefore k > 0$$

Problem 3 (40 points)

Consider the following unity feedback system:



- (1) What is the characteristic equation? (10 points)
- (2) What is the closed-loop transfer function of this system? (10 points)
- (3) Assuming a unit step function is given to the system, please calculate the steady state value, y_{ss} . (10 points)
- (4) Assuming a unit step function is given to the system, please calculate the steady state error, e_{ss} . (10 points)

$$(1) \quad 1 + 4 \cdot \frac{3}{s+1} = 0$$

$$s+13=0$$

$$(2) \quad G_c(s) = \frac{\frac{12}{s+1}}{1 + \frac{12}{s+1}} = \frac{12}{s+13}$$

$$(3) \quad \text{For step input:}$$
$$y_{ss} = \lim_{s \rightarrow 0} s \cdot G_c(s) \cdot \frac{1}{s} = G_c(0) = \frac{12}{13} = 0.923$$

$$(4) \quad \text{For step input:}$$

$$e_{ss} = \frac{R}{1+k_p} = \frac{1}{1+G(0)} = \frac{1}{1+12} = \frac{1}{13} = 0.077$$

ABOVE AVERAGE

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

89

Student Name (Print): Jade A. Coleman

Final Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Any one minute late submission will result in a 10% penalty of the grade
- Any dishonest behavior will result in "F" grade immediately

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1 (9 points)

True or False

- (1) If the transfer function of the closed-loop system has at least one simple pole and no repeated poles on the $j\omega$ -axis, the system is stable.

False ✓

- (2) The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.

False ✗

- (3) Poles and zeros marked on the root locus are the poles and zeros of the closed-loop transfer function

True ✗ -6

Problem 2 (51 points)

- (1) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{5(s+2)}{(s-1)(s^2+s+1)}$$

unstable
pole in RHP ✓

$$(s-1)=0$$

$$s=1$$

$$s^2+s+1=0$$

$$s=-1\pm\sqrt{3}$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{a^2}$$

$$s = \frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{(1)^2}$$

- (2) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{1}{s^2}$$

$$\sqrt{s^2}=0$$

$$s=0$$

$$\frac{-1\pm\sqrt{1-4}}{1}$$

$$= -1\pm\sqrt{-3}$$

Stable ✗

has one simple pole
no repeated poles
no pole in RHP

(3) The standard form of the second-order system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

If the poles location moves to the left side of the s-plane, the settling time will increase or decrease? (5 points) Why? (6 points)

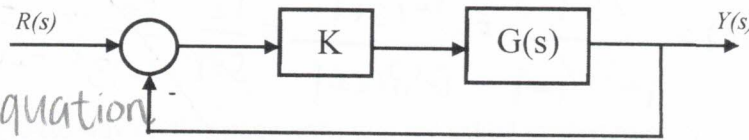
The settling time will decrease because moving to the left side will decrease the initial response time, which ultimately effects the range, causing the settling time to decrease as well.

(4) Please find the range of K to keep the following system stable. (20 points)

$$G_c(s) = \frac{F_g}{1-L_g}$$

$$F_g = \frac{s+1}{s(s+2)} \cdot K \quad L_g = \frac{s+1}{s(s+2)} (K)(-1)$$

$$G(s) = \frac{s+1}{s(s+2)}$$



Characteristic equation

$$1 + K \cdot \frac{(s+1)}{s(s+2)} = 0$$

$$\frac{s(s+2)}{s(s+2)} \cdot 1 + \frac{K(s+1)}{s(s+2)} \rightarrow \frac{s(s+2)}{s(s+2)} + \frac{K(s+1)}{s(s+2)} = 0$$

$$\frac{s(s+2) + K(s+1)}{s(s+2)} = 0$$

$$s(s+2) + K(s+1) = 0$$

$$s^2 + 2s + Ks + K = 0 \rightarrow s^2 + s(2+K) + K = 0$$

$$\begin{array}{c|cc} s^2 & 1 & K \\ s^1 & 2+K & 0 \\ s^0 & K & \end{array}$$

$$\frac{K(2+K) - (1)(0)}{2+K} = \frac{K(2+K)}{2+K} = K$$

= stable
no sign change

$$= \boxed{K > 0}$$

$$Y(s) = G(s)R(s)$$

$$G_c(s) = \frac{s+1}{s(s+2)}$$

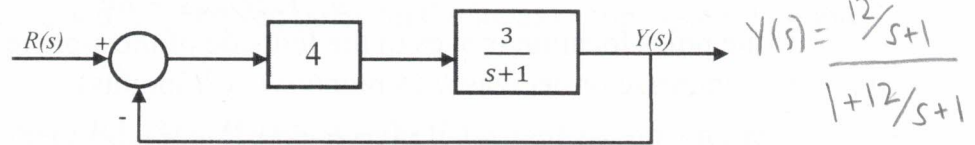
$$1 + K \left(\frac{s+1}{s(s+2)} \right)$$

$$\frac{s+1}{s(s+2)}$$

$$\frac{s^2 + s(2+K) + K}{s^2 + s(2+K)}$$

Problem 3 (40 points)

Consider the following unity feedback system:



- (1) What is the characteristic equation? (10 points)
- (2) What is the closed-loop transfer function of this system? (10 points)
- (3) Assuming a unit step function is given to the system, please calculate the steady state value, y_{ss} . (10 points) *Find y_{ss}*
- (4) Assuming a unit step function is given to the system, please calculate the steady state error, e_{ss} . (10 points) *Find e_{ss}*

$$\begin{aligned}
 1) & \quad 1 + \left[4 \cdot \frac{3}{s+1} \right] = 1 + \frac{12}{s+1} \Rightarrow \frac{s+1}{s+1} + \frac{12}{s+1} \Rightarrow \frac{s+1+12}{s+1} \Rightarrow \frac{s+13}{s+1} = 0 \Rightarrow \boxed{s+13=0} \\
 2) & \quad \frac{F_g}{1-L_g} \rightarrow F_g = \frac{12}{s+1} \quad L_g = -\frac{12}{s+1} \quad Y(s) = \frac{12/s+1}{1+12/s+1} = \frac{12/s+1}{s+13/s+1} = \frac{12}{s+1} \cdot \frac{s+1}{s+13} = \frac{12}{s+13} = G_c(s) \\
 3) & \quad y_{ss} = G_c(0) = \frac{12}{(0)+13} = \frac{12}{13} = \boxed{0.923} \\
 4) & \quad e_{ss} = 1 - y_{ss} = 1 - 0.923 = \boxed{0.0769}
 \end{aligned}$$

AVERAGE

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

71

Student Name (Print):

Zalayna
Jenkins

Final Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Any one minute late submission will result in a 10% penalty of the grade
- Any dishonest behavior will result in "F" grade immediately

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1 (9 points)

True or False

- (1) If the transfer function of the closed-loop system has at least one simple pole and no repeated poles on the $j\omega$ -axis, the system is stable.

False

marginally stable

- (2) The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.

True

- (3) Poles and zeros marked on the root locus are the poles and zeros of the closed-loop transfer function

False (open-loop)

Problem 2 (51 points)

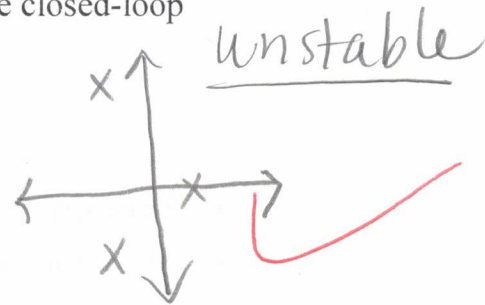
- (1) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{5(s+2)}{(s-1)(s^2+s+1)}$$

$$s^2 + s + 1 = 0$$

$$s = -1 \pm \sqrt{1-4}$$

$$s_{2,3} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$$



- (2) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{1}{s^2}$$

unstable

The duplicate roots makes it unstable.

(3) The standard form of the second-order system is

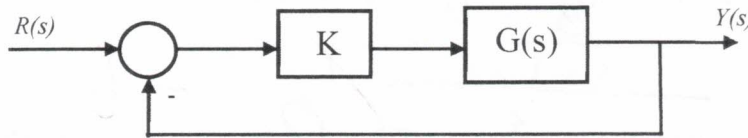
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

If the poles location moves to the left side of the s-plane, the settling time will increase or decrease? (5 points) Why? (6 points)

The location of the poles allows for t_s to decrease as ω_d increases.

(4) Please find the range of K to keep the following system stable. (20 points)

$$G(s) = \frac{s+1}{s(s+2)} \quad s^2 + 2s = 0$$

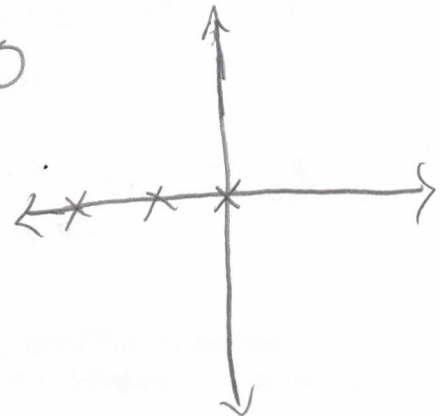


$$G_c(s) = \frac{Fg}{1-Lg} = \frac{\frac{s+1}{s(s+2)}}{1 + \frac{s+1}{s(s+2)}}$$

$$1 + K \frac{s+1}{s(s+2)} = 0$$

$$s^2 + 2s + K = 0$$

$$s_1 = \frac{-2 \pm \sqrt{2^2 - 4K}}{2}$$



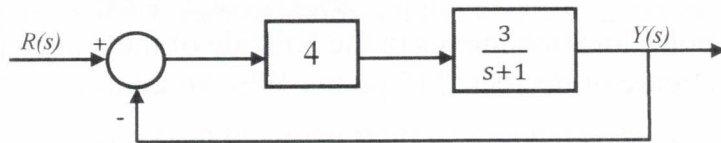
$$K > 1$$

$$s = -1 \pm \sqrt{1-K}$$

-6

Problem 3 (40 points)

Consider the following unity feedback system:



- (1) What is the characteristic equation? (10 points)
- (2) What is the closed-loop transfer function of this system? (10 points)
- (3) Assuming a unit step function is given to the system, please calculate the steady state value, y_{ss} . (10 points)
- (4) Assuming a unit step function is given to the system, please calculate the steady state error, e_{ss} . (10 points)

1) Characteristic equation

$$1 + 4 \cdot \frac{3}{s+1} = 0 \quad -6$$

$$2. G_c(s) = \frac{4}{s} \cdot \frac{3}{s+1} \quad X = 0 \quad -7$$

$$3. y_{ss} = \lim_{s \rightarrow 0} \frac{3}{s+1} \cdot \frac{4}{s} = s \cdot \frac{4}{s} \cdot \frac{3}{s+1} = 1.5 \quad -5$$

$y_{ss} = 1.5$

$$4. e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s + \frac{1}{1+G(s)} R(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{3}{s+1} = 1.5 \quad -5$$

BELOW AVERAGE

PRAIRIE VIEW A&M UNIVERSITY

**ROY G. PERRY COLLEGE OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING**

Total Grade:

33

Student Name (Print): Ke'Andre March

Final Exam

MCEG3307 Automatic Controls

Fall 2021

- Please stay on this page until you are asked to start the exam
- Any one minute late submission will result in a 10% penalty of the grade
- Any dishonest behavior will result in "F" grade immediately

Note:

- Academic dishonesty of any sort (plagiarism, cheating, fabrication or facilitation) is prohibited under any circumstance. Any detected actions of dishonesty will cause the failure of the class immediately.

Problem 1 (9 points)

True or False

- (1) If the transfer function of the closed-loop system has at least one simple pole and no repeated poles on the $j\omega$ -axis, the system is stable.

True

- (2) The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.

False

- (3) Poles and zeros marked on the root locus are the poles and zeros of the closed-loop transfer function

True

Problem 2 (51 points)

- (1) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{5(s+2)}{(s-1)(s^2+s+1)}$$

stable, only one sign change. approaches 0

$$\frac{5(1+2)}{(1-1)(1^2+1+1)} \quad s=1 \quad \text{pole @ } 1, -1$$

- (2) Consider the stability of the following system with the closed-loop transfer function: (10 points)

$$\frac{1}{s^2}$$

This function is stable, approaches zero, no sign change,

$$\frac{1}{s^2} \quad s=1, 2, 3, \dots \quad \text{approaches 0.}$$

(3) The standard form of the second-order system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

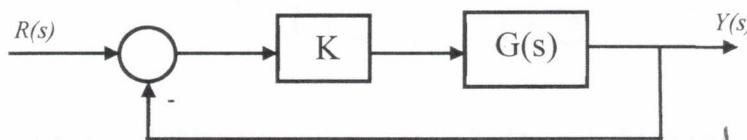
If the poles location moves to the left side of the s-plane, the settling time will increase or decrease? (5 points) Why? (6 points)

The settling time should increase, the system is stable, and it will take more time for the system to settle with negative values, poles etc.

-6

(4) Please find the range of K to keep the following system stable. (20 points)

$$G(s) = \frac{s+1}{s(s+2)}$$



original

$$G(s) = \frac{s+1}{s(s+2)} \rightarrow \frac{K(s+1)}{s(s+2)}$$



pole @ -2

$$\frac{1}{2} < K < 2$$

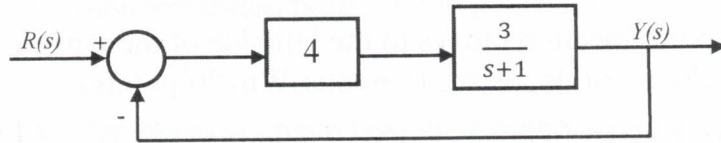
• without the graph changing and becoming unstable, the values from $\frac{1}{2}$ to 2, keeps it looking stable.

• Finding the range using the unit step function + looking for poles etc.

-12

Problem 3 (40 points)

Consider the following unity feedback system:



- (1) What is the characteristic equation? (10 points)
- (2) What is the closed-loop transfer function of this system? (10 points)
- (3) Assuming a unit step function is given to the system, please calculate the steady state value, y_{ss} . (10 points)
- (4) Assuming a unit step function is given to the system, please calculate the steady state error, e_{ss} . (10 points)

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} \quad \checkmark, \quad \frac{4(3)}{s+1} = \frac{12}{s+1} \quad \times$$

pole @ -1

(steady state error)

$$\frac{12}{-1+1} = \frac{12}{0}$$

$$K_p = G(0) = \frac{12}{1+1} = \frac{12}{2} = 6 \quad \times$$

-30

Final value (steady state)

$$\frac{3}{s(s+1)} \quad \frac{12}{s(s+1)}$$

$$s = -1$$

$$s = 0$$

$$s = -1$$

**Automatic Controls
Fall 2021**

Title of Assignment: Quiz 1

TOPICS COVERED IN QUIZ 1

1. Introduction
2. Laplace transform
3. Transfer function

MCEG 3307 Automatic Controls

Fall 2021

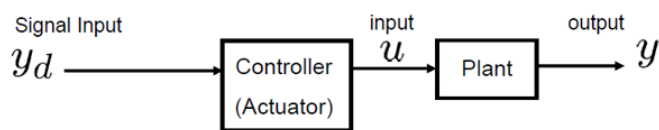
Quiz 1 (40 mins)

Student Name (Print):

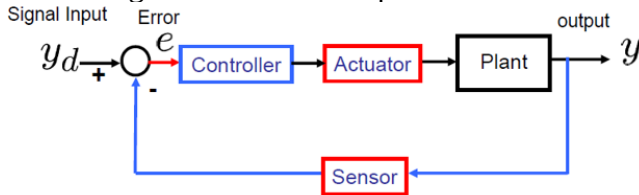
Grade:

1. What is the difference between open-loop control and close loop control? (20 points)

Block diagram of Open-loop control:



Block diagram of Closed-loop control:



2. What is the Laplace transforms for the time function: te^{-3t} (15 points)
3. What is the Laplace transform for the time function: $t^3u_s(t)$? (15 points)
4. Solve for $y(t)$ in the below equation, assume all initial conditions are zero. (50 points)

$$\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 32y = 32u_s(t).$$

MCEG 3307 Automatic Controls

Fall 2021

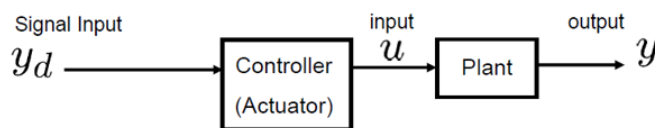
Quiz 1 (40 mins) Solution

Student Name (Print):

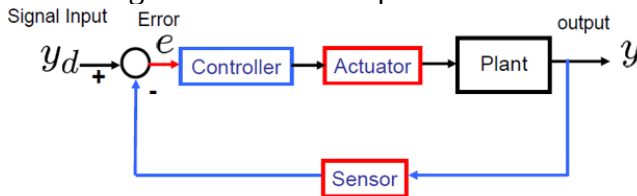
Grade:

1. What is the difference between open-loop control and close loop control? (20 points)

Block diagram of Open-loop control:



Block diagram of Closed-loop control:



For open-loop control, there is no sensor element, and output and desired input will not be compared. Calibration should be conducted to achieve good control outcome. When disturbance appears, re-calibration should be conducted.

For closed-loop control, there is a sensor element, and output and desired input will be compared. When disturbance appears, closed-loop system can adjust by itself. No need to do re-calibration or re-modeling.

2. What is the Laplace transforms for the time function: te^{-3t} (15 points)

$$\frac{1}{(s+3)^2}$$

3. What is the Laplace transform for the time function: $t^3u_s(t)$? (15 points)

$$\frac{6}{s^4}$$

4.

$$s^2 Y(s) + 12s Y(s) + 32 Y(s) = \frac{32}{s}$$

$$Y(s) (s^2 + 12s + 32) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s+4)(s+8)}$$

$$= \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

Multiply both sides by s , and let s goes to 0:

$$s Y(s) \Big|_{s \rightarrow 0} = A + s \cdot \frac{B}{s+4} \Big|_{s \rightarrow 0} + s \cdot \frac{C}{s+8} \Big|_{s \rightarrow 0}$$

$$\therefore A = s Y(s) = s \cdot \frac{32}{s(s+4)(s+8)} = \frac{32}{(s+4)(s+8)} \Big|_{s \rightarrow 0} = 1$$

Multiply both sides by $s+4$, and let s goes to -4 :

$$(s+4) Y(s) \Big|_{s \rightarrow -4} = \frac{s+4}{s} \cdot A \Big|_{s \rightarrow -4} + B + \frac{s+4}{s+8} \cdot C \Big|_{s \rightarrow -4}$$

$$\therefore B = (s+4) \cdot \frac{32}{s(s+4)(s+8)} = \frac{32}{s(s+8)} \Big|_{s \rightarrow -4} = -2$$

Multiply both sides by $s+8$, and let s goes to -8 :

$$(s+8) Y(s) \Big|_{s \rightarrow -8} = \frac{s+8}{s} \cdot A \Big|_{s \rightarrow -8} + \frac{s+8}{s+4} \cdot B \Big|_{s \rightarrow -8} + C$$

$$\therefore C = (s+8) \cdot \frac{32}{s(s+4)(s+8)} \Big|_{s \rightarrow -8} = \frac{32}{s(s+4)} \Big|_{s \rightarrow -8} = 1$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$\therefore y(t) = \mathcal{L}^{-1}(Y(s)) = (1 - 2e^{-4t} + e^{-8t}) u_s(t)$$

ABOVE AVERAGE

Asher Corson

95

- 1.) A closed loop system takes the actual behavior produced by the system, compares it to the desired behavior, and make corrections based on the differences or errors. An open system is a once-through system that has one input and one output, it does not adjust based on a desired behavior.

$$2.) t e^{-3t} = \mathcal{L}\{t e^{-3t}\} = \frac{n!}{(s-a)^{n+1}} = \frac{1!}{(s+3)^{1+1}} = \frac{1}{(s+3)^2}$$

$n=1, a=-3$

$$3.) t^3 u_s(t) = \mathcal{L}\{t^3 u_s(t)\} = \frac{n!}{s^{n+1}} \cdot \frac{1}{s} = \frac{3!}{s^{3+1}} \cdot \frac{1}{s} = \frac{6}{s^4} \cdot \frac{1}{s} = \frac{6}{s^5}$$

$$4.) \frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 32y = 32u_s(t)$$

$$y(0) = 0$$

$$y'(0) = 0$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\left\{\frac{dy}{dx}\right\} = sY(s) - y(0)$$

$$\mathcal{L}\left\{\frac{d^2 y}{dx^2}\right\} = s^2 Y(s) - s \cdot y(0) - y'(0)$$

$$\mathcal{L}\{u_s(t)\} = \frac{1}{s}$$

$$s^2 Y(s) - \cancel{s \cdot y(0)} - \cancel{y'(0)} + 12s Y(s) - \cancel{12y(0)} + 32Y(s) = \frac{32}{s}$$

$$s^2 Y(s) + 12s Y(s) + 32 \cdot Y(s) = \frac{32}{s}$$

$$Y(s)(s^2 + 12s + 32) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2 + 12s + 32)}$$

$$s(s^2 + 12s + 32)$$

$$Y(s) = \frac{32}{s(s+4)(s+8)}$$

$$s(s+4)(s+8)$$

$$(s+4)(s+8)$$

$$s^2 + 4s + 4s + 32$$

$$s^2 + 12s + 32$$

4
cont.) $Y(s) = \frac{32}{s(s+4)(s+8)}$

$$\frac{32}{s(s+4)(s+8)} = \left(\frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8} \right) \frac{s(s+4)(s+8)}{s(s+4)(s+8)}$$

$$32 = A(s+4)(s+8) + B(s)(s+8) + C(s)(s+4)$$

① $s=0$

$$32 = A(0+4)(0+8) + B(0)(0+8) + C(0)(0+4)$$

$$32 = A(4)(8)$$

$$32 = A(32)$$

$$\frac{32}{32} = A$$

$$1 = A$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$\mathcal{L}\{Y(s)\} = \mathcal{L}\left\{\frac{1}{s}\right\} - \mathcal{L}\left\{\frac{2}{s+4}\right\} + \mathcal{L}\left\{\frac{1}{s+8}\right\}$$

$$y(t) = 1 - \sin 2t + e^{-8t}$$

② $s=-4$

$$32 = A(-4+4)(-4+8) + B(-4)(-4+8) + C(-4)(-4+4)$$

$$32 = B(-4)(-4+8)$$

$$32 = B(-4)(4)$$

$$32 = B(-16)$$

$$\frac{32}{-16} = B$$

$$-2 = B$$

③ $s=-8$

$$32 = A(-8+4)(-8+8) + B(-8)(-8+8) + C(-8)(-8+4)$$

$$32 = C(-8)(-8+4)$$

$$32 = C(-32)$$

$$\frac{32}{-32} = C$$

$$-1 = C$$

-3

AVERAGE

78

Quiz 1

Jade A. Coleman

1. The difference between open-loop and close-loop control is that close-loop requires feedback, unlike open-loop. -2

2. $t e^{-3t}$ $n=1, a=-3$ $f(t) = t^n e^{at} \rightarrow F(s) = \frac{n!}{(s-a)^{n+1}}$
 $\mathcal{L}\{t e^{-3t}\} = \frac{1!}{[s-(-3)]^{(1)+1}} = \frac{1}{(s+3)^2}$ ✓

3. $t^3 u_5(t)$ $\mathcal{L}\{t^3 u_5(t)\} = \boxed{t^3/s}$ ✗ -10

4. $\frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 32y = 32 u_5(t)$

$y(0)=0$ Find $y(t)$
 $y'(0)=0$

$$s^2 Y(s) - s y(0) - y'(0) + 12 \{s Y(s) - y(0)\} + 32 Y(s) = 32/s$$

$$s^2 Y(s) - s(0) - 0 + 12 \{s Y(s) - 0\} + 32 Y(s) = 32/s$$

$$s^2 Y(s) + 12 s Y(s) + 32 Y(s) = 32/s$$

$$Y(s) [s^2 + 12s + 32] = 32/s$$

$$Y(s) [(s+8)(s+4)] = 32/s$$

$$Y(s) = \frac{32}{s(s+8)(s+4)} = \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+4}$$

$$\mathcal{L}^{-1} \left\{ Y(s) = \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+4} \right\}$$

$$A(s+8)(s+4) + B(s)(s+4) + C(s)(s+8) = 32$$

$$A[s^2 + 12s + 32] + B[s^2 + 4s] + C[s^2 + 8s] = 32$$

$$s^2 [A+B+C] + s [12A+4B+8C] + 32A = 32$$

$$32A = 32$$

$$A+B+C=0$$

$$12A+4B+8C=0$$

$$B+C=-1$$

$$4B+8C=-12$$

$$C=-1-B$$

$$4(B+2C)=-12$$

$$B+2C=-3$$

$$B=-3+2C$$

$$3B=-5$$

$$B=-3+2(-1-B)$$

$$B=-5/3$$

$$A-B=-3-2-2B$$

$$B+2/3=-1$$

$$-2/3-2/3-3+2C+C=-1$$

$$B=-7/3-2/3-3+3C=-1$$

$$B=-5/3 \quad C=2/3 \quad 3C=2$$

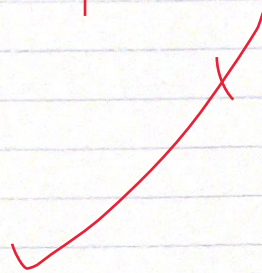
$$\mathcal{L}^{-1}\{1/s\} = 1$$

$$- \mathcal{L}^{-1}\left\{\frac{5}{3(s+2)}\right\} = \frac{5}{3} e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{3(s+4)}\right\} = \frac{2}{3} e^{-4t}$$

$$y(t) = 1 - \frac{5}{3} e^{-2t} + \frac{2}{3} e^{-4t}$$

-10



BELOW AVERAGE

Joshua Guyton

40

1) A open-loop control does not depend on the output.

A closed-loop control is ~~is~~ where the input depends on the output. -10

2) $t e^{-3t}$ $t^n e^{at}$, $n=1, 2, 3$ $F(s) = \frac{n!}{(s-a)^{n+1}}$

$$F(s) = \frac{1!}{(s+3)^{1+1}} = \frac{1}{(s+3)^2}$$

$$F(s) = \frac{1}{(s+3)^2}$$

3) $t^3 u_s(t)$

e^{-s3}
s

~~is~~ -10

$$4) \frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 32y = 32 u_s(t)$$

-40

**Automatic Controls
Fall 2021**

Title of Assignment: Quiz 2

TOPICS COVERED IN QUIZ 2

1. Stability and performance

MCEG 3307 Automatic Controls

Quiz 2

Student Name (Print):

Grade:

1. Consider the stability of the following system with the closed-loop transfer function. (40 points)

$$\frac{5(s+2)}{(s+1)(s^2+s+1)}$$

2. Using the Routh-Hurwitz criterion and the unity feedback system of Figure 1 with

$$G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

Tell whether or not the closed-loop system is stable.

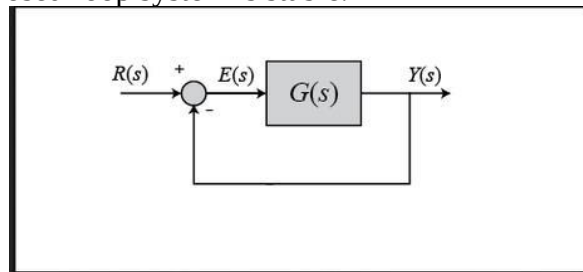


Figure 1 A unity feedback system.

- (1) Find the close-loop transfer function. (30 points)
(2) Identify the stability of the system using the Routh-Hurwitz criterion. (30 points)

MCEG 3307 Automatic Controls

Quiz 2

Student Name (Print):

Grade:

1. Consider the stability of the following system with the closed-loop transfer function. (40 points)

$$\frac{5(s+2)}{(s+1)(s^2+s+1)}$$

pole: $(s+1)(s^2+s+1)=0$
 $s_1 = -1, \quad s_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$

all the poles are on the open left half plane, $\therefore$ the system is stable

2. Using the Routh-Hurwitz criterion and the unity feedback system of Figure 1 with

$$G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

Tell whether or not the closed-loop system is stable.

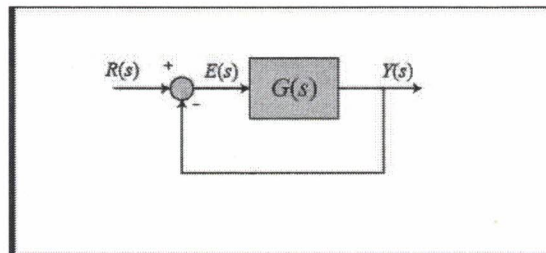


Figure 1 A unity feedback system.

- (1) Find the close-loop transfer function. (30 points)
 (2) Identify the stability of the system using the Routh-Hurwitz criterion. (30 points)

(1) $G_c = \frac{F_g}{1-L_g} = \frac{\frac{1}{2s^4+5s^3+s^2+2s}}{1 + \frac{1}{2s^4+5s^3+s^2+2s}} = \frac{1}{2s^4+5s^3+s^2+2s+1}$

(2) Routh array:

s^4	2	1	1
s^3	5	2	
s^2	$\frac{1}{5}$	1	
s^1	-23		
s^0	1		

$$b_1 = \frac{5 \times 1 - 2 \times 2}{5} = \frac{1}{5}$$

$$b_2 = \frac{5 \times 1 - 2 \times 0}{5} = 1$$

$$c_1 = \frac{2 \times \frac{1}{5} - 5 \times 1}{\frac{1}{5}} = -23$$

from $\frac{1}{5} \rightarrow -23 \rightarrow 1$, there are two sign changes.

$\therefore$ the system is unstable

ABOVE AVERAGE

100

Asher Carson

11/1/2021

Quiz #2

$$1.) G_c = \frac{5(s+2)}{(s+1)(s^2+s+1)}$$

$$D(s) = (s+1)(s^2+s+1)$$

$$(s+1)(s^2+s+1) = 0$$

$$s+1=0$$

$$s^2+s+1=0$$

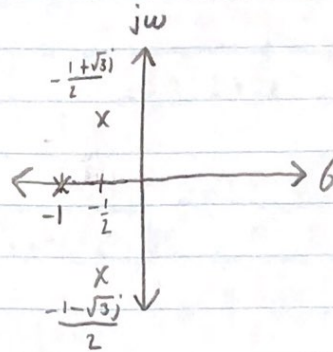
$$s_1 = -1$$

$$s_{2,3} = \frac{-1 \pm \sqrt{1^2 - 4}}{2}$$

$$s_{2,3} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$s_2 = \frac{-1 + \sqrt{3}j}{2}, s_3 = \frac{-1 - \sqrt{3}j}{2}$$

$$s_{2,3} \text{ real} = -\frac{1}{2}$$



Stable, all roots are in
Open LHP

$$2.) G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

$$G_c = \frac{F_g}{1 - L_g}$$

$$F_g = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$$

$$L_g = (-1) \left(\frac{1}{2s^4 + 5s^3 + s^2 + 2s} \right)$$

$$G_c = \frac{1}{2s^4 + 5s^3 + s^2 + 2s} = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1} \cdot \frac{(2s^4 + 5s^3 + s^2 + 2s)}{(2s^4 + 5s^3 + s^2 + 2s + 1)}$$

$$G_c = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1}$$

$$Q(s) = 2s^4 + 5s^3 + s^2 + 2s + 1$$

s^4	2	1	1
s^3	5	2	
s^2	$\frac{1}{5}$	1	
s^1	-23		
s^0	1		

$$b_1 = \frac{(5 \cdot 1) - (2 \cdot 2)}{5}$$

$$b_1 = \frac{1}{5}$$

$$c_1 = \left(\frac{1}{5} \cdot 2 \right) - (5 \cdot 1)$$

$$d_1 = \frac{(-23 \cdot 1) - (\frac{1}{5} \cdot 0)}{-23}$$

$$b_2 = \frac{(5 \cdot 1) - (2 \cdot 0)}{5}$$

$$c_1 = -23$$

$$d_1 = 1$$

$$b_2 = 1$$

sign changes: 2 ($\frac{1}{5} \rightarrow -23 \rightarrow 1$) } $\rightarrow$ unstable
2 roots in open RHP

AVERAGE

85

Automatic Controls

Quiz #2

Andrew Taylor

$$\#1) \quad \frac{5(s+2)}{(s+1)(s^2+s+1)}$$

Consider stability w/ closed loop transfer function

$$(s+1)(s^2+s+1) = 0$$

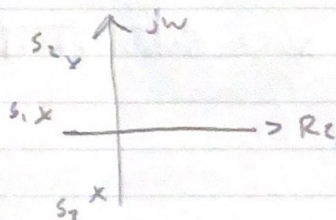
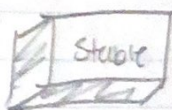
$$s_1 = -1, \quad s_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

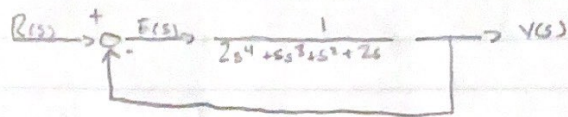
$$Re(s) < 0$$

$$ax^2 + bx + c = 0$$

$$s^2 + s + 1 = 0$$



#2.) $G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$



~~$2s^4 + 5s^3 + s^2 + 2s = 0$~~ $1 + 2s^4 + 5s^3 + s^2 + 2s = 0$

s^4	2	1	1
s^3	5		
s^2	$\frac{5-4}{5} = \frac{1}{5}$	2	
s^1	x		
s^0	1		

$$x = \frac{\frac{1}{5}(2) - 5}{\frac{1}{5}}$$

$$= \frac{2 - 25}{1}$$

$$\underline{\underline{x = -23}}$$

sign change in 1st column

not stable

1.) Closed loop transfer function?

2.) stability?

Marginally stable

-25

BELOW AVERAGE

K. March
Automatic Controls
Dr. Han
Nov 1, 2021

55

Q412

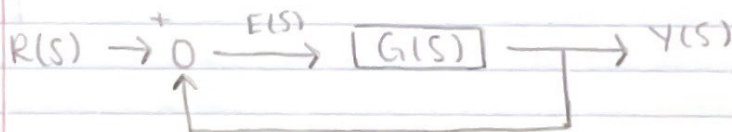
1) $\frac{5(s+2)}{(s+1)(s^2+s+1)}$ zeros: -2 ✓
poles: -1 ✗

$\frac{5(s+2)}{(s+1)(s^2+s+1)} = 0$ $x = -2$

-5

This should be stable ✓ since the root is -2 ✗

2) $G(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s}$



$\frac{1}{2s^4 + 5s^3 + s^2 + 2s} = 0$ roots = no solution

The closed loop isn't stable since s cannot be solved.

$\frac{Y(s)}{R(s)} = F(s) = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + K}$?

-40

$s^4 = 2$	s	$10s + 2s$
$s^3 = 5$	1	2
$s^2 = 1$	$s^2 + 2s$	
$s^1 = 1$	$2s + K$ ✗	
$s^0 = 0$		

**Automatic Controls
Fall 2021**

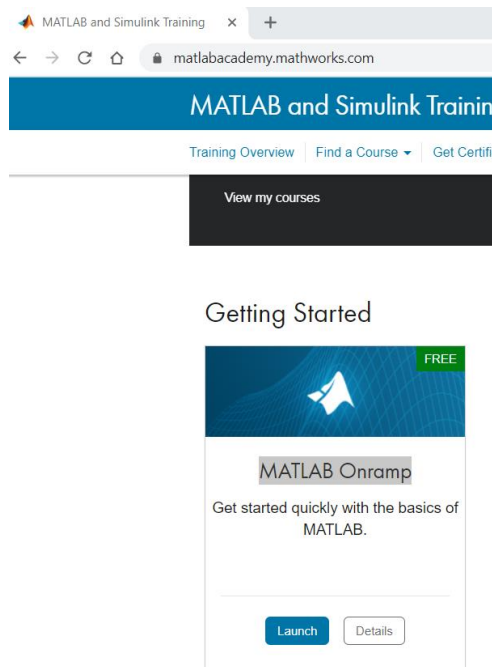
HOMEWORK 1

TOPICS COVERED IN HOMEWORK 1

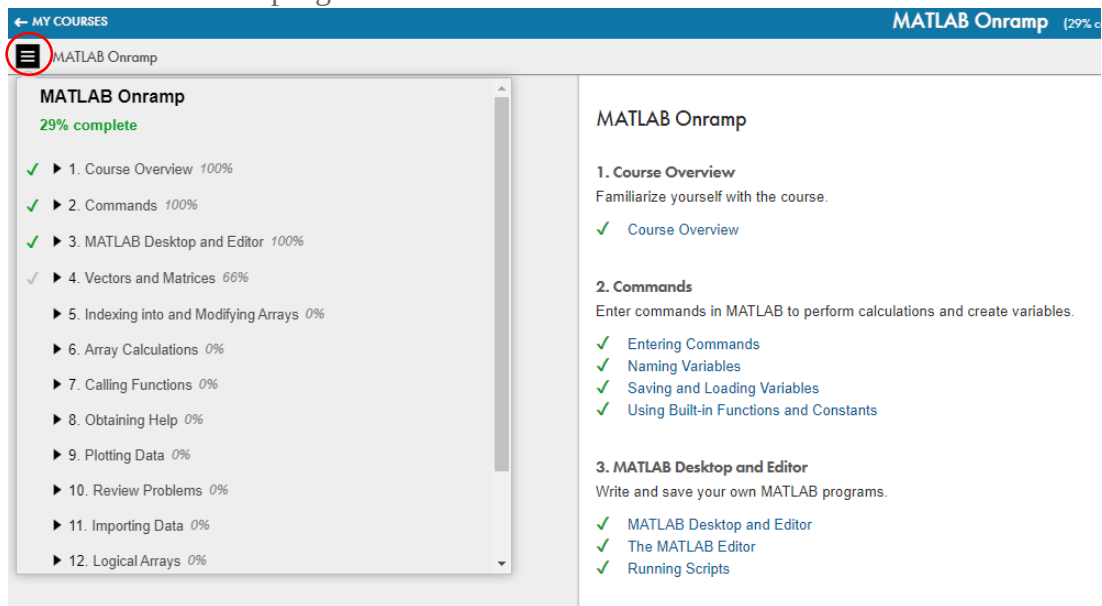
1. Matlab

Homework 1

1. Go to <https://matlabacademy.mathworks.com/>, to complete **MATLAB Onramp** course. The course takes around 2 hours to complete. You can take breaks and resume it.

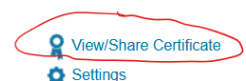
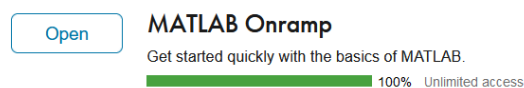


2. Click the icon at the top right corner to resume the courses:

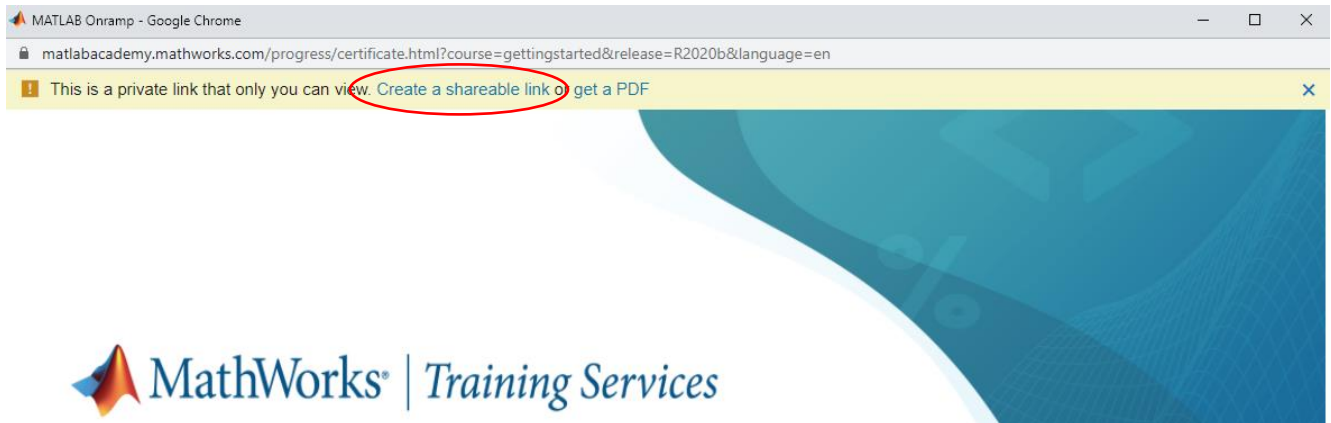


3. After completion, go to <https://matlabacademy.mathworks.com/> . Click “**View/Share Certificate**” as shown below:

Getting Started



4. Choose “Create a shareable link”:



5. Copy and paste the sharable link on ecourses. For example:

<https://matlabacademy.mathworks.com/progress/share/certificate.html?id=0af14726-c4ee-4a3d-8e44-8dce9f626874>

ABOVE AVERAGE

Course Completion Certificate

Asher Carson

has successfully completed **100%** of the self-paced training course

MATLAB Onramp



DIRECTOR, TRAINING SERVICES

28 August 2021

AVERAGE

Course Completion Certificate

Priscille djomo

has successfully completed **100%** of the self-paced training course

MATLAB Onramp



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BELOW AVERAGE

Course Completion Certificate

Joshua Guyton

has successfully completed **100%** of the self-paced training course

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Automatic Controls
Fall 2021

HOMEWORK 2

TOPICS COVERED IN HOMEWORK 2

- 1. Laplace transform**
- 2. Transfer function**

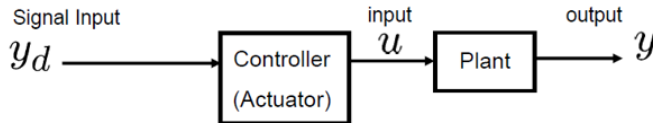
Homework 2

1. How do closed-loop systems differ from open-loop systems?
2. Please write the equation and plot it for the following function:
 - (a) Unit step function
 - (b) Unit impulse function
 - (c) Unit ramp function.
3. Using the Laplace transform pairs of Table 2.1 and the properties of Laplace transform in Table 2.2, derive the Laplace transforms for the following time functions:
 - a) $e^{-at} \cos \omega t u(t)$
 - b) $t^3 u(t)$
4. Find the inverse Laplace transform of $F(s) = \frac{10}{s(s+2)(s+3)^2}$. Please show the calculation process.

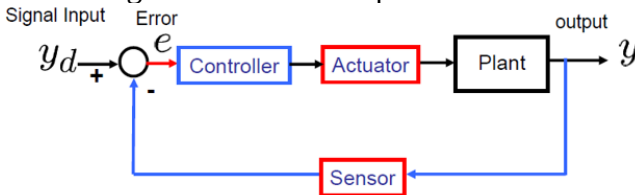
Homework 2

1. How do closed-loop systems differ from open-loop systems?

Block diagram of Open-loop control:



Block diagram of Closed-loop control:



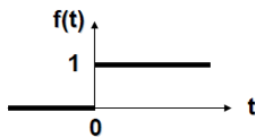
For open-loop control, there is no sensor element, and output and desired input will not be compared. Calibration should be conducted to achieve good control outcome. When disturbance appears, re-calibration should be conducted.

For closed-loop control, there is a sensor element, and output and desired input will be compared. When disturbance appears, closed-loop system can adjust by itself. No need to do re-calibration or re-modeling.

2. Please write the equation and plot it for the following function:

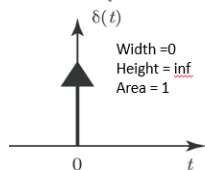
(a) Unit step function

$$f(t) = u_s(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



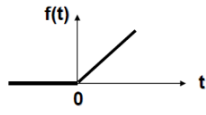
(b) Unit impulse function

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



(c) Unit ramp function.

$$f(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



3. Using the Laplace transform pairs of Table 2.1 and the properties of Laplace transform in Table 2.2, derive the Laplace transforms for the following time functions:

c) $e^{-at} \cos \omega t u(t)$

d) $t^3 u(t)$

4. Find the inverse Laplace transform of $F(s) = \frac{10}{s(s+2)(s+3)^2}$. Please show the calculation process.

3307 HW2

3.

$$(a) \mathcal{L}[\cos wt] = \frac{s}{s^2 + w^2}$$

$\therefore e^{at} g(t) \rightarrow G(s-a)$ shift property

$$\mathcal{L}[e^{-at} \cos wt] = \frac{(s+a)}{(s+a)^2 + w^2}$$

$$(b) \therefore \mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\therefore \mathcal{L}[t^3 u(t)] = \frac{3!}{s^4} = \frac{6}{s^4}$$

$$4. F(s) = \frac{10}{s(s+2)(s+3)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)^2} + \frac{D}{s+3}$$

To obtain A:

$$s F(s) \Big|_{s \rightarrow 0} = A + s \cdot \frac{B}{s+2} \Big|_{s \rightarrow 0} + s \cdot \frac{C}{(s+3)^2} \Big|_{s \rightarrow 0} + s \cdot \frac{D}{s+3} \Big|_{s \rightarrow 0}$$

$$\therefore A = \frac{10}{2 \times 9} = \frac{5}{9}$$

To obtain B:

$$(s+2) F(s) \Big|_{s \rightarrow -2} = (s+2) \cdot \frac{A}{s} \Big|_{s \rightarrow -2} + B + (s+2) \cdot \frac{C}{(s+3)^2} \Big|_{s \rightarrow -2} + (s+2) \frac{D}{s+3} \Big|_{s \rightarrow -2}$$

$$\therefore B = \frac{10}{(-2)(-2+3)^2} = -5$$

To obtain C:

$$(s+3)^2 F(s) \Big|_{s \rightarrow -3} = (s+3)^2 \frac{A}{s} \Big|_{s \rightarrow -3} + (s+3)^2 \cdot \frac{B}{s+2} \Big|_{s \rightarrow -3} + C + (s+3) \cdot D \Big|_{s \rightarrow -3}$$

$$\therefore C = \frac{10}{-3(-3+2)} = \frac{10}{3}$$

To obtain D:

3307 HW2

$$\frac{10}{s(s+2)} = (s+3)^2 \frac{A}{s} + (s+3)^2 \frac{B}{s+2} + C + (s+3)D$$

$$\frac{0-10(2s+2)}{s^2(s+2)^2} = \frac{2(s+3)s-(s+3)^2}{s^2} A + \frac{2(s+3)(s+2)-(s+3)^2}{(s+2)^2} B + D$$

$$\text{Let } s \rightarrow -3 \quad \therefore D = \frac{40}{9}$$

$$\therefore F(s) = \frac{5}{9s} - \frac{5}{s+2} + \frac{10}{3(s+3)^2} + \frac{40}{9(s+3)}$$

$$\therefore f(t) = \left(\frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t} \right) u(t)$$

ABOVE AVERAGE

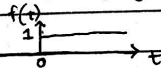
HW #2

1. Open loop: calibration is key, can be sensitive to disturbances.

Closed loop: compare actual behavior with desired, make corrections based on error. Closed loop can self correct, open loops cannot.

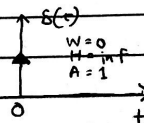
- 2a Unit step function $f(t) = u_0(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$F(s) = \int_0^{\infty} 1 \cdot e^{-st} dt = -\frac{1}{s} [e^{-st}]_0^{\infty} = \frac{1}{s}$$



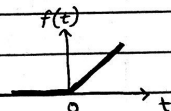
- 2b Unit impulse function

$$\int_{-\infty}^{\infty} \delta(t) g(t) dt = g(0)$$



- 2c Unit ramp function

$$\int_a^b uv' dt = [uv]_a^b - \int_a^b u'v dt$$



Laplace Transform from 2.1 & 2.2

$$3. \quad \alpha(f(t))(\alpha) = \int_0^{\infty} f(t) e^{-st} dt$$

$$\text{From table} \Rightarrow \alpha(\cos \omega t)(s) = \frac{s}{s^2 + \omega^2}$$

$$\alpha(e^{-at} u(t))(s) = \frac{s}{s^2 + \omega^2}$$

$$\alpha(e^{-at} f(t)) = F(s+a) \Rightarrow \alpha(f(t) X(s+a)) = F(s+a)$$

$$\alpha(e^{-at} \cos \omega t u(t)) = \alpha(\cos \omega t u(t))(s+a)$$

$$\alpha(e^{-at} \cos \omega t u(t)) = \frac{s+a}{(s+a)^2 + \omega^2}$$

3b $t^3 u(t)$

From table $\Rightarrow \mathcal{L}\{t^n u(t)\} = \frac{n!}{s^{n+1}}$

$\mathcal{L}\{t^3 u(t)\} = \frac{3!}{s^{3+1}} = \frac{6}{s^4}$

$\mathcal{L}\{t^3 u(t)\} = \frac{6}{s^4}$

4. $F(s) = \frac{10}{s(s+2)(s+3)^2}$

$\frac{10}{s(s+2)(s+3)^2} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+3)} + \frac{D}{(s+3)^2}$

$10 = A(s+2)(s+3)^2 + B(s)(s+3)^2 + C(s)(s+2)(s+3) + Ds(s+2)$

When $s=0$

$10 = A(2)(3^2) + 0 + 0 + 0 \quad A = \frac{5}{9}$

When $s=-2$

$10 = 0 + B(-2)(-2+3)^2 + 0 + 0 \quad B = -5$

When $s=-3$

$10 = 0 + 0 + 0 + D(-3)(-3+2) \quad D = \frac{10}{3}$

$10 = D \cdot 3$

$-3D(-3+2)$

$+9D+6D$

$10 = \frac{30}{3}$

$C = \frac{40}{9}$

$f(s) = \frac{5}{9s} - \frac{5}{(s+2)} + \frac{40/9}{s+3} + \frac{10/3}{(s+3)^2}$

Inverse Laplace

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1, \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = e^{-2t}, \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = e^{-3t}, \mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2}\right\} = te^{-3t}$

$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{40}{9}e^{-3t} + \frac{10}{3}te^{-3t}$

AVERAGE

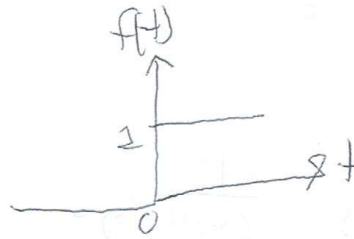
Joshua Guyton

1). A open-loop control does not depend on the output.

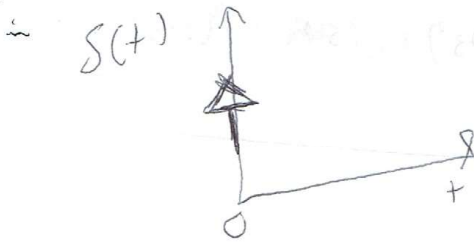
A closed-loop control is where the input depends on the output.

2.1

$$a) f(t) = u_s(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

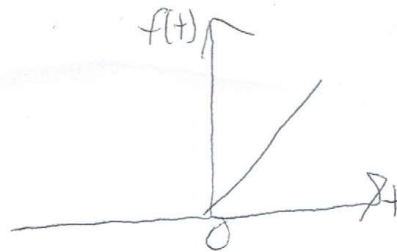


$$b) \delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases} \quad \int_{-\infty}^{\infty} \delta(t) g(t) dt = g(0)$$



$$c) f(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(s) = \int_0^{\infty} t e^{-st} dt = -\frac{1}{s} [t e^{-st}]_0^{\infty} + \frac{1}{s} \int_0^{\infty} e^{-st} dt = \frac{1}{s^2}$$



3) a) $e^{-at} \cos \omega t$ $\rightarrow \frac{\omega}{(s+a)^2 + \omega^2}$

b) $t^3 u(t) \rightarrow \frac{n!}{s^{n+1}} \quad \frac{3!}{s^{3+1}} = \frac{6}{s^4}$ ✓

4) $[-] \quad F(s) = \frac{10}{s(s+2)(s+3)^2}$

$= \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{s+2} + \frac{D}{(s+3)^2}$ X multiply all and cancel out

$A(s+2)^2(s+3)^2 + B(s+1)(s+2)(s+3)^2 + C(s+1)(s+2)^2(s+3) + D(s+1)(s+2)^2(s+2)$
 $\frac{A(s+2)^2(s+3)^2 + B(s+1)(s+2)(s+3)^2 + C(s+1)(s+2)^2(s+3) + D(s+1)(s+2)^2(s+2)}{(s+1)(s+2)^2(s+3)^2}$

$10 = \frac{A(s+2)(s+3)^2 + Bs(s+3)^2 + Cs(s+3) + Ds(s+2)}{s(s+2)(s+3)^2}$

BELOW AVERAGE

Asher Corson

missing one problem

- 1.) A closed loop system takes the actual behavior produced by the system, compares it to the desired behavior, and make corrections based on the differences or errors. An open system is a once-through system that has one input and one output, it does not adjust based on a desired behavior.

$$2.) t e^{-3t} = \mathcal{L}\{t e^{-3t}\} = \frac{n!}{(s-a)^{n+1}} = \frac{1!}{(s+3)^{1+1}} = \frac{1}{(s+3)^2}$$

$n=1, a=-3$

$$3.) t^3 u_s(t) = \mathcal{L}\{t^3 u_s(t)\} = \frac{n!}{s^{n+1}} \cdot \frac{1}{s} = \frac{3!}{s^{3+1}} \cdot \frac{1}{s} = \frac{6}{s^4} \cdot \frac{1}{s} = \frac{6}{s^5} \quad \times$$

$$4.) \frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 32y = 32u_s(t)$$

$$y(0) = 0$$

$$y'(0) = 0$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\left\{\frac{dy}{dx}\right\} = sY(s) - y(0)$$

$$\mathcal{L}\left\{\frac{d^2 y}{dx^2}\right\} = s^2 Y(s) - s \cdot y(0) - y'(0)$$

$$\mathcal{L}\{u_s(t)\} = \frac{1}{s}$$

$$s^2 Y(s) - \cancel{s \cdot y(0)} - \cancel{y'(0)} + 12s Y(s) - \cancel{12y(0)} + 32Y(s) = \frac{32}{s}$$

$$s^2 Y(s) + 12s Y(s) + 32 \cdot Y(s) = \frac{32}{s}$$

$$Y(s)(s^2 + 12s + 32) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2 + 12s + 32)}$$

$$s(s^2 + 12s + 32)$$

$$Y(s) = \frac{32}{s(s+4)(s+8)}$$

$$s(s+4)(s+8)$$

$$(s+4)(s+8)$$

$$s^2 + 4s + 4s + 32$$

$$s^2 + 12s + 32$$

4
cont.) $Y(s) = \frac{32}{s(s+4)(s+8)}$

$$\frac{32}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

$$32 = A(s+4)(s+8) + B(s)(s+8) + C(s)(s+4)$$

① $s=0$

$$32 = A(0+4)(0+8) + B(0)(0+8) + C(0)(0+4)$$

$$32 = A(4)(8)$$

$$32 = A(32)$$

$$\frac{32}{32} = \frac{A(32)}{32}$$

$$1 = A$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$$

$$\mathcal{L}\{Y(s)\} = \mathcal{L}\left\{\frac{1}{s}\right\} - \mathcal{L}\left\{\frac{2}{s+4}\right\} + \mathcal{L}\left\{\frac{1}{s+8}\right\}$$

$$y(t) = 1 - \sin 2t + e^{-8t}$$

② $s=-4$

$$32 = A(-4+4)(-4+8) + B(-4)(-4+8) + C(-4)(-4+4)$$

$$32 = B(-4)(-4+8)$$

$$32 = B(-4)(4)$$

$$32 = B(-16)$$

$$\frac{32}{-16} = \frac{B(-16)}{-16}$$

$$-2 = B$$

③ $s=-8$

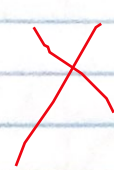
$$32 = A(-8+4)(-8+8) + B(-8)(-8+8) + C(-8)(-8+4)$$

$$32 = C(-8)(-8+4)$$

$$32 = C(32)$$

$$\frac{32}{32} = \frac{C(32)}{32}$$

$$1 = C$$



**Automatic Controls
Fall 2021**

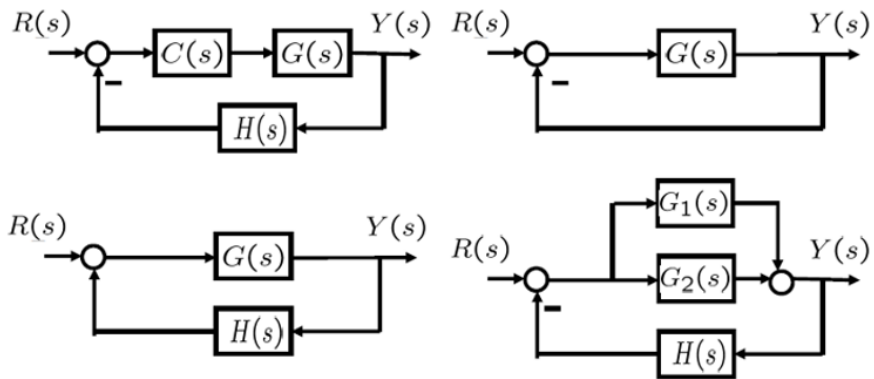
HOMEWORK 3

TOPICS COVERED IN HOMEWORK 3

- 1. Modeling of dynamic systems**
- 2. Time response analysis**

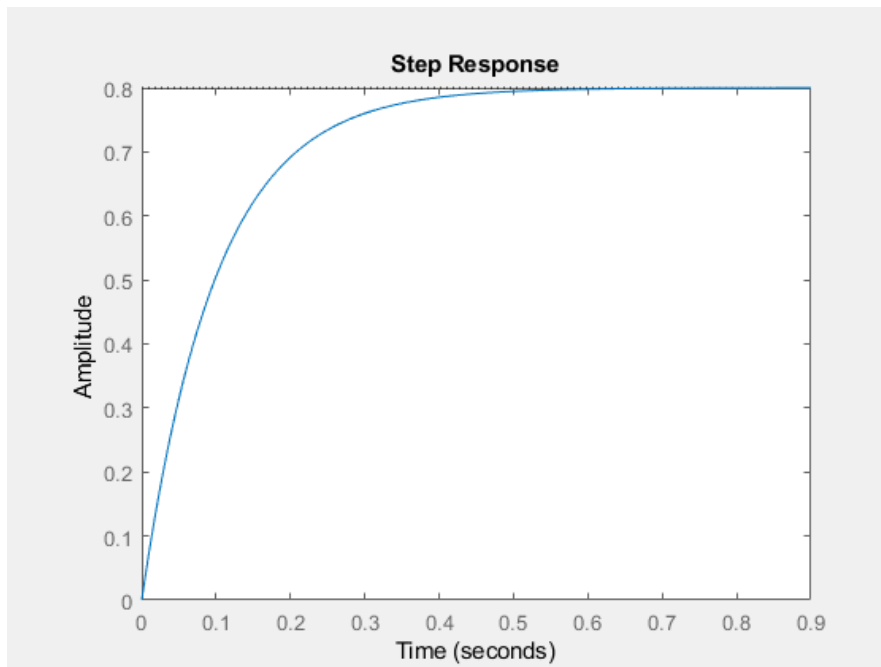
Homework 3

1. Please compute the transfer function from $R(s)$ to $Y(s)$



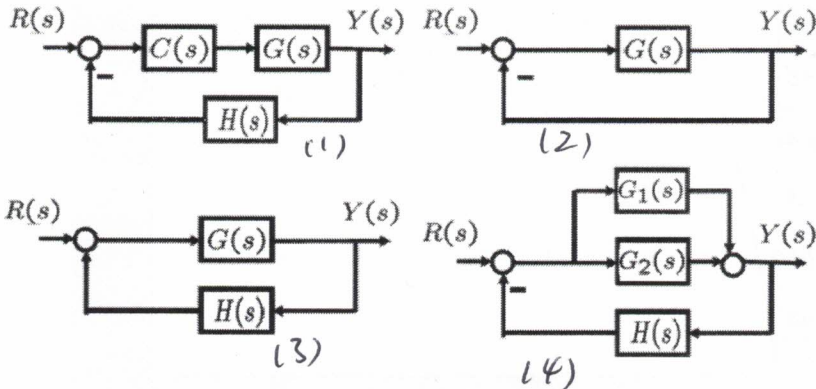
2. Problem 24 in Chapter 2 of the textbook.

3. Based on the unit step response given in the figure below, (1) Please find the transfer function of the system. (2) Please plot the unit ramp response for the system.



Homework 3

1. Please compute the transfer function from $R(s)$ to $Y(s)$



$$(1) \quad \frac{Y(s)}{R(s)} = \frac{F_g}{1-L_g} = \frac{C(s)G(s)}{1+C(s)G(s)H(s)}$$

$$(2) \quad \frac{Y(s)}{R(s)} = \frac{F_g}{1-L_g} = \frac{G(s)}{1+G(s)}$$

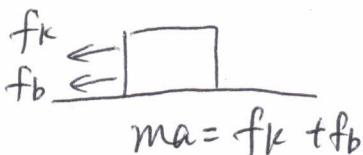
$$(3) \quad \frac{Y(s)}{R(s)} = \frac{F_g}{1-L_g} = \frac{G(s)}{1-G(s)H(s)}$$

$$(4) \quad \frac{Y(s)}{R(s)} = \frac{F_g}{1-L_g} = \frac{G_1(s)+G_2(s)}{1+(G_1(s)+G_2(s))H(s)}$$

2. Problem 24 in Chapter 2 of the textbook.



Assume the displacement of the spring is $x_2(t)$
 $5(x_2 - x_1) = f(t)$ ①



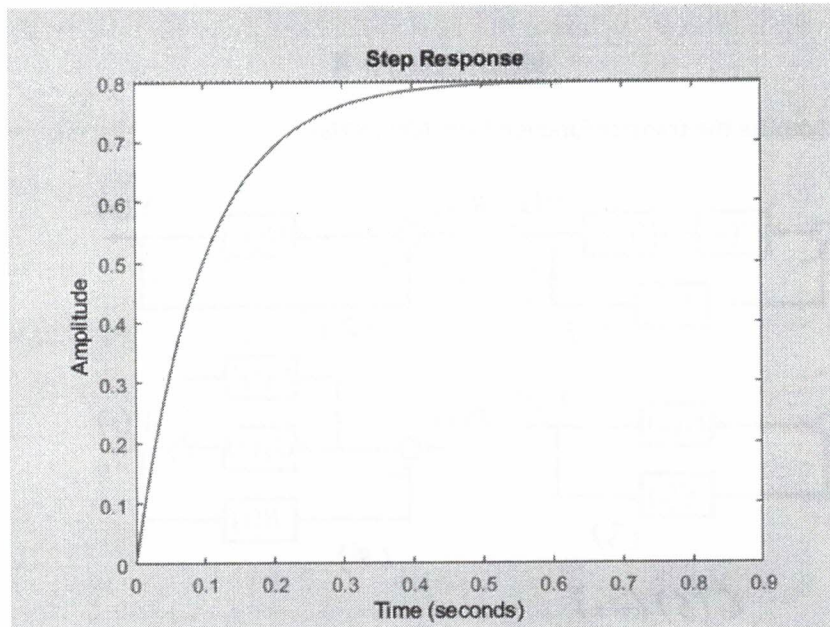
$$m\ddot{x}_1(t) = -5(x_1 - x_2) - 4\dot{x}_1 \quad (2)$$

from ①
 $-5x_1(s) + 5x_2(s) = F(s)$

from ②
 $(5s^2 + 4s + 5)x_1(s) - 5x_2(s) = F(s)$
 $(5s^2 + 4s)x_1(s) = F(s)$

$$\therefore \frac{x_1(s)}{F(s)} = \frac{1}{s(5s+4)}$$

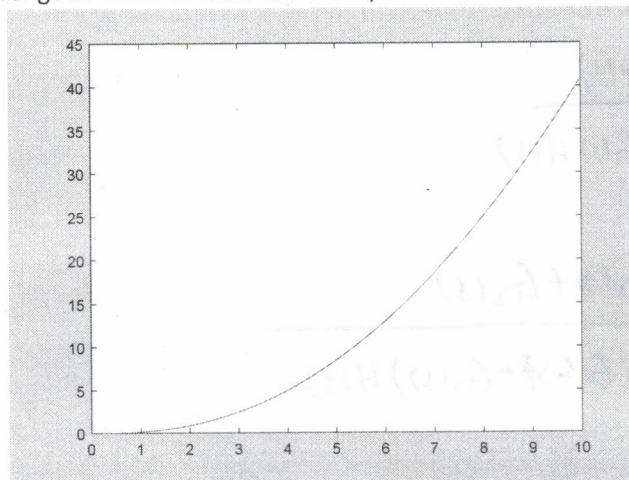
3. Based on the unit step response given in the figure below, (1) Please find the transfer function of the system. (2) Please plot the unit ramp response for the system.



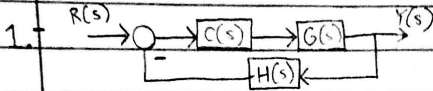
(1)

$$tf = \frac{8}{s + 10}$$

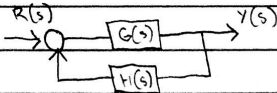
(2) use step command to tf/s, the ramp response is shown below (may need to adjust the range of x-axis to show the curve):



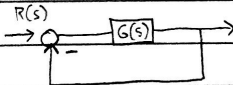
ABOVE AVERAGE

HW #3

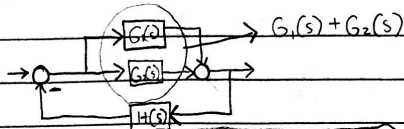
$$\frac{Y(s)}{R(s)} = \frac{C(s) G(s)}{1 + G(s) C(s) H(s)}$$



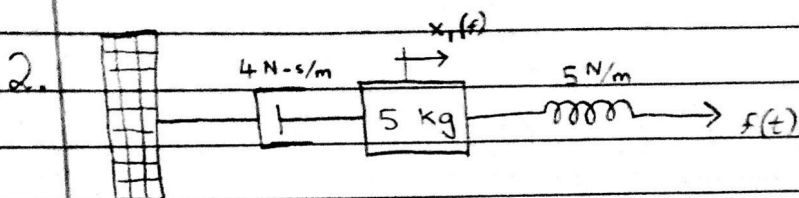
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$$



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s) \times 1}$$



$$\frac{Y(s)}{R(s)} = \frac{G_1(s) + G_2(s)}{1 + (G_1(s) + G_2(s)) H(s)}$$



$x_2(t)$ displacement

$$(5s^2 + 4s + 5)x_1(s) - 5x_2(s) = 0$$

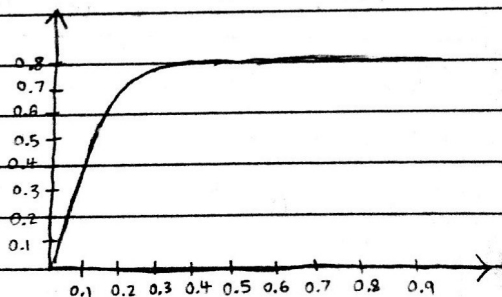
$$-5x_1(s) + 5x_2(s) = F(s)$$

Adding equations

$$(5s^2 + 4s)x_1(s) = F(s)$$

$$G(s) = \frac{x_1(s)}{F(s)} = \frac{1}{s(5s + 4)}$$

3.



Final Value : 0.8

Time constant T : $63\% \times 0.8 = .50$

At 0.5, $T = 0.1s$

$K \Rightarrow 0.8$

$Ts + 1 \quad 0.1s + 1$

Unit ramp response From chart

$$\frac{C(s)}{R(s)} = \frac{0.8}{1+Ts}$$

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{1}{s^2} \left(\frac{0.8}{1+Ts} \right) = \frac{0.8/T}{s^2(s+1/T)}$$

$$C(s) = 0.8 \left(\frac{1/T}{s^2(s+1/T)} \right) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1/T}$$

$$1/T = A \left(s(s+1/T) \right) + B \left(s+1/T \right) + C(s^2)$$

$$s=0 \Rightarrow B \left(\frac{1}{T} \right) = \frac{1}{T} \quad B=1$$

$$s=1 \Rightarrow \frac{1}{T} = A \left(1+\frac{1}{T} \right) + 1 \left(1+\frac{1}{T} \right) + T \cdot 1$$

$$\Rightarrow \frac{A(T+1)}{T} = -(1+T) \Rightarrow A = -T$$

$$s = -1/T \Rightarrow \frac{1}{T} = C \left(\frac{1}{T^2} \right) \Rightarrow C = T$$

$$C(s) = -T/s + 1/s^2 + 1/s + 1/T$$

$$C(t) = (t - T + 1e^{-t/T}) u(t)$$

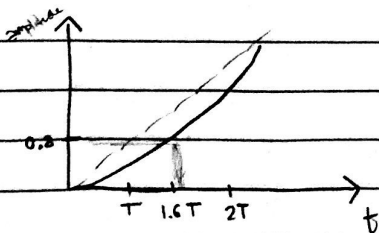
$$c(t) = t - T(1 - e^{-t/T})$$

T_s from transfer function is $1.6T$

$$C(t) = 1.6T - T [1 - e^{-1.6T/T}]$$

$$C(t) = 1.6T - T(0.8)$$

$$C(t) = 0.8T$$



AVERAGE

Priscille Djomo Djomo

MCEG 3307

Homework #3

3)

We assume the given system is 1st order syst.

Let input = $R(s) \Rightarrow r(t)$

output = $C(s) \Rightarrow c(t)$

Transfer function of the syst is

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

$$\Rightarrow C(s) = R(s) \frac{1}{1+Ts}$$

Given the plot, we observe that:

Desired output = 0.8

$$\frac{C(s)}{R(s)} = \frac{0.8}{1+Ts}$$

$$C(s) = R(s) \left[\frac{0.8}{1+Ts} \right] = \frac{1}{s} \left[\frac{0.8}{1+Ts} \right]$$

$$C(s) = 0.8 \left[\frac{1/T}{s \left[\frac{1}{T} + s \right]} \right]$$

T=?

$$C(s) = 0.8 \left[\frac{1}{s} - \frac{1}{s + \frac{1}{T}} \right]$$

$$\frac{1}{T} = A \left[s \left(s + \frac{1}{T} \right) \right] + B \left(s + \frac{1}{T} \right) + C \left[s' \right]$$

$$s = 0 \Rightarrow B \left(\frac{1}{T} \right) = \frac{1}{T} \Rightarrow B = 1$$

$$s = -\frac{1}{T} \Rightarrow \frac{1}{T} = -C \left(\frac{1}{T} \right) \Rightarrow C = T$$

$$\Rightarrow \frac{1}{T} = A \left(1 + \frac{1}{T} \right) + 1 + \frac{1}{T} + T$$

$$\Rightarrow \frac{A(T+1)}{T} = - (1+T) \Rightarrow A = -T$$

$$C(s) = -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s + \frac{1}{T}}$$

$$C(t) = \left[t - T + T e^{\frac{t}{T}} \right] u(t)$$

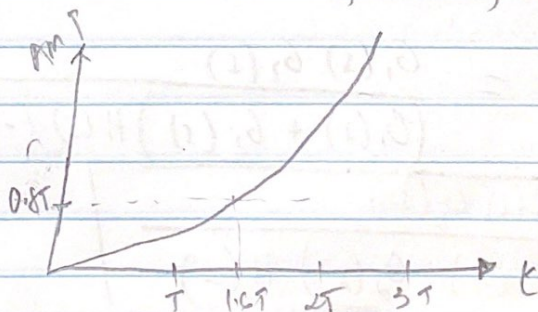
We know settling time $t \approx 1.6 T$

$$C(t) = 1.6 T - T \left(1 - e^{-\frac{1.6 T}{T}} \right)$$

$$C(t) = 1.6 T - T (0.8)$$

$$C(t) \approx 0.8 T$$

• Unit Ramp Response



1)

$$\frac{Y(s)}{Z(s)} = \frac{F_g}{1 - L_g} = \frac{C(s) G(s)}{1 - C(s) G(s) H(s) (-1)}$$

$$\boxed{\frac{Y(s)}{Z(s)} = \frac{C(s) G(s)}{1 + C(s) G(s) H(s)}}$$

2)

$$\frac{Y(s)}{Z(s)} = \frac{F_g}{1 - L_g} \Rightarrow F_g = G(s)$$

$$L_g = G(s) (-1) \Rightarrow L_g = -G(s)$$

$$\boxed{\frac{Y(s)}{Z(s)} = \frac{G(s)}{G(s) + 1}}$$

3)

$$\frac{Y(s)}{Z(s)} = \frac{F_g}{1 - L_g} \Rightarrow F_g = G(s)$$

$$L_g = G(s) H(s)$$

$$\boxed{\frac{Y(s)}{Z(s)} = \frac{G(s)}{1 - G(s) H(s)}}$$

$$\textcircled{4} \quad \frac{Y(s)}{Z(s)} = \frac{F_g}{1 - L_g} = \frac{G_1(s) G_2(s)}{(G_1(s) + G_2(s)) H(s) (-1)}$$

$$\boxed{\frac{Y(s)}{Z(s)} = \frac{G_1(s) G_2(s)}{G_1(s) + G_2(s) H(s)}} \quad \times$$

24

According to Newton's 2nd law.

$$F = ma$$

Considering the total force acting on sky

$$\frac{d^2 x_1}{dt^2} = 5 \frac{dx_1}{dt} - 2x_1 + f(t)$$

$$f(t) = \frac{d^2 x_1}{dt^2} + 5 \frac{dx_1}{dt} + 2x_1$$

Converting to s-domain from time domain

$$f(s) = s^2 x_1(s) + 5s x_1(s) + 2x_1(s)$$

$$\frac{f(s)}{x_1(s)} = s^2 + 5s + 2$$

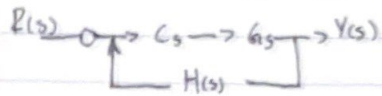
$$\boxed{\frac{x_1(s)}{f(s)} = \frac{1}{s^2 + 5s + 2}}$$

BELOW AVERAGE

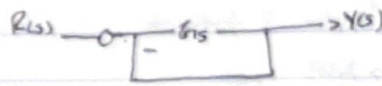
Automatic Controls

Homework # 3

Andrew Taylor



$$\frac{-Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}$$



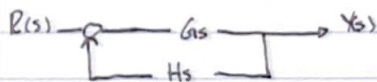
~~$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g} = \frac{E(s)G(s)}{1 - G(s)}$$~~

$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g} = \frac{E(s)G(s)}{1 - G(s)}$$

$$F_g = G(s)$$

$$L_g = G(s) - 1$$

$$\frac{G(s)}{1 + G(s)}$$

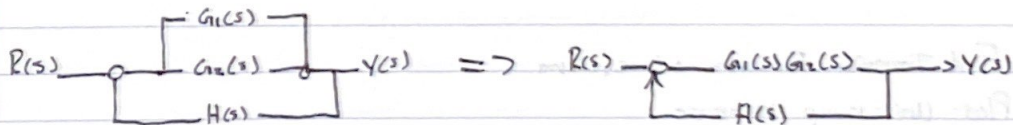


$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g}$$

$$F_g = G(s)$$

$$L_g = G(s)H(s)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$



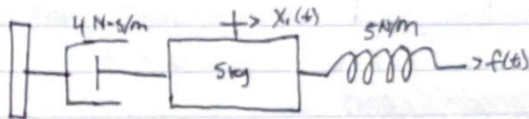
$$\frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g}$$

$$F_g = G_1(s) + G_2(s)$$

$$L_g = (G_1(s) + G_2(s))H(s) - 1$$

$$\frac{Y(s)}{R(s)} = \frac{G_1(s) + G_2(s)}{1 + (G_1(s) + G_2(s))H(s)}$$

#2.) Problem 24: Chapter 2 textbook



$$(s^2 + 4s + 5) X_1(s) - 5 X_2(s) = 0$$

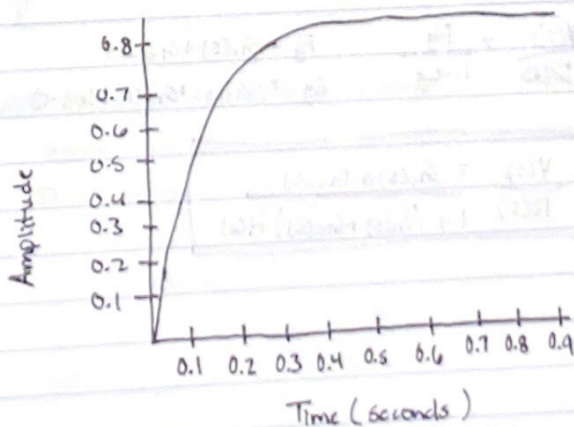
$$-5 X_1(s) + 5 X_2(s) = F(s)$$

$$(s^2 + 4s) X_1(s) = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{s(s+4)}$$

$$G(s) = \frac{X_1(s)}{F(s)} = \frac{1}{s(s+4)}$$

#3.) Find Transfer function of system
Plot Unit ramp response



**Automatic Controls
Fall 2021**

HOMEWORK 4

TOPICS COVERED IN HOMEWORK 4

- 1. Stability and performance**

Homework 4

- Using Routh-Hurwitz criterion, determine whether the unity feedback system of Figure 1 is stable if

$$G(s) = \frac{240}{(s+1)(s+2)(s+3)(s+4)}$$



Figure 1 A unity feedback system.

- Determine the stability of the system shown in Figure 2 using Routh-Hurwitz criterion. Notice that there is positive feedback.

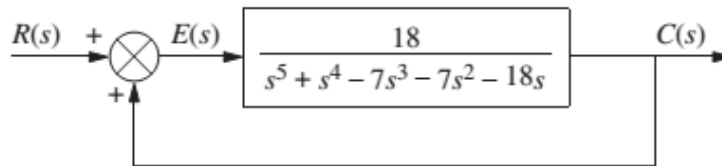


Figure 2 A system with positive unity feedback.

- Find the range of K to keep the system shown in Figure 3 stable.

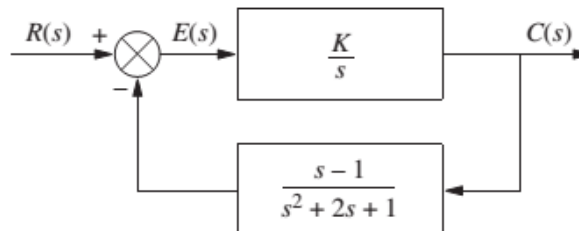


Figure 3 A feedback control system.

HW 4:

240

$$1. \quad \frac{G(s)}{1+G(s)} = \frac{240}{(s+1)(s+2)(s+3)(s+4)+240}$$

$$Q(s) = (s^2+3s+2)(s+3)(s+4) + 240$$

$$= (s^3+3s^2+3s^2+9s+2s+6)(s+4) + 240$$

$$= (s^3+6s^2+11s+6)(s+4) + 240$$

$$= s^4 + 4s^3 + 6s^3 + 24s^2 + 11s^2 + 44s + 6s + 24 + 240$$

$$= s^4 + 10s^3 + 35s^2 + 50s + 264$$

s^4		1	35	264
s^3		10	50	
s^2		30	264	
s^1		-38		
s^0		264		

2 sign change, 2 rhp poles
unstable

2.

$$1 - \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s} = 0$$

$$s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18 = 0$$

s^5		1	-7	-18
s^4		1	-7	-18
s^3		4	-14	
s^2		-3.5	-18	
s^1		-34.6		
s^0		-18		

$$(s^4 - 7s^2 - 18)' = 4s^3 - 14s$$

1 sign change, unstable

$$3. \quad 1 + \frac{k}{s} \cdot \frac{s-1}{s^2+2s+1} = 0$$

$$s^3 + 2s^2 + s + ks - k = 0$$

$$s^3 + 2s^2 + (k+1)s - k = 0$$

$$\begin{array}{l|ll} s^3 & 1 & k+1 \\ s^2 & 2 & -k \\ s^1 & \frac{3k+2}{2} & \\ s^0 & -k & \end{array}$$

$$\frac{2(k+2)+k}{2} = \frac{3k+2}{2}$$

$$\frac{3k+2}{2} > 0 \Rightarrow k > -\frac{2}{3}$$

$$-k > 0 \Rightarrow k < 0$$

$$-\frac{2}{3} < k < 0$$

ABOVE AVERAGE

$$1) G(s) = \frac{240}{(s+1)(s+2)(s+3)(s+4)}$$

$$= \frac{G}{1+G} = \frac{\frac{240}{(s+1)(s+2)(s+3)(s+4)}}{1 + \frac{240}{(s+1)(s+2)(s+3)(s+4)}}$$

$$= \frac{240}{(s+1)(s+2)(s+3)(s+4) + 240}$$

$$= \frac{240}{(s^2+2s+5+2)(s^2+4s+3s+12)+240} = \frac{240}{(s^2+2s+5+2)(s^2+7s+12)+240}$$

$$= \frac{240}{s^4 + 7s^3 + 12s^2 + 3s^3 + 21s^2 + 36s + 2s^2 + 14s + 24 + 240}$$

$$= \frac{240}{s^4 + 10s^3 + 35s^2 + 50s + 264}$$

s^4	1	35	264
s^3	10	50	
s^2	30	264	
s^1	-38		
s^0	264		

$$\frac{(50 \times 10) - 50}{10} = 30$$

$$\frac{(50 \times 30) - (10 \times 264)}{30} = -38$$

$$\frac{(-38 \times 264) - 0}{-38} = 264$$

$$\frac{10 \times 264}{10} = 264$$

Two sign changes mean system is unstable

$$2) G(s) = \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}$$

$$= \frac{\frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}}{1 - \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}} = \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18}$$

$$s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18$$

s^5	1	-7	-18
s^4	1	-7	-18
s^3	0=4	0=-14	zero
s^2	-3.5	-18	
s^1	=34.57		
s^0	18		

two sign changes
system is unstable

row use derivative

$$\frac{(4x-7) - (-14x)}{4} = 3.5$$

$$\frac{(-14x-18) - (0)}{-14} = -1.8$$

$$\frac{(-3.5x-14) - (4(-1.8))}{-3.5} = -34.57$$



$$3) \frac{k}{s}$$

$$\frac{s-1}{s^2+2s+1}$$

$$1 + \frac{k}{s} \frac{s-1}{s^2+2s+1} = 1 + \frac{k(s-1)}{s^3+2s^2+s}$$

$$= s^3 + 2s^2 + ks - k =$$

$$= s^3 + 2s^2 + s(k+1) - k$$

s^3	1	$k+1$
s^2	2	$-k$
s^1	$\frac{2(k+1)+k}{2}$	
s^0	$-k$	

$$\frac{2(k+1) - (-k)}{2(k+1)} = \frac{2(k+1)+k}{2}$$

$$\frac{2(k+1)+k}{2} = \frac{2k+2+k}{2} = \frac{3k+2}{2} > 0$$

$$\frac{3k+2}{2} > 0$$

$$\frac{3k}{3} > \frac{-2}{3}$$

$$k > -\frac{2}{3}$$

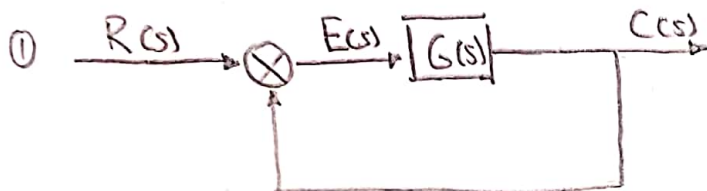
$$-\frac{2}{3} < k < 0$$



AVERAGE

Marcelo Epsm One.

94



$$G(s) = \frac{240}{(s+1)(s+2)(s+3)(s+4)}$$

$$\frac{F_g}{1 - L_g} = \frac{\frac{240}{(s+1)(s+2)(s+3)(s+4)}}{1 + \frac{240}{(s+1)(s+2)(s+3)(s+4)}}$$

$$\frac{240}{(s+1)(s+2)(s+3)(s+4)} \cdot \frac{(s+1)(s+2)(s+3)(s+4)}{[(s+1)(s+2)(s+3)(s+4) + 240]}$$

$$= \frac{240}{s^4 + 10s^3 + 35s^2 + 50s + 264}$$

$$D(s) = s^4 + 10s^3 + 35s^2 + 50s + 264$$

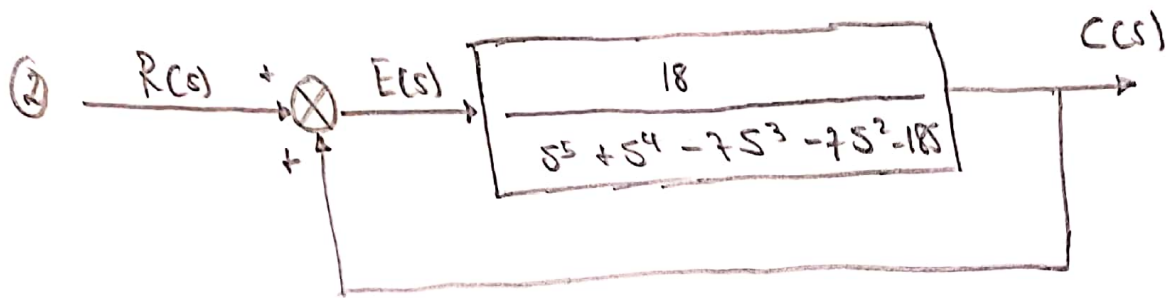
s^4	1	35	264
s^3	10	50	0
s^2	30	264	0
s^1	-38	0	
s^0	264		

$$\frac{(10 \times 35) - 50}{10} = \frac{308}{10} = 30$$

$$\frac{10 \times 264 - 30}{10} = 261$$

$$\frac{(30 \times 264) - (10 \times 264)}{30}$$

since they are two sign changes,
They are two points on the R.H.S. This means that the system is unstable.



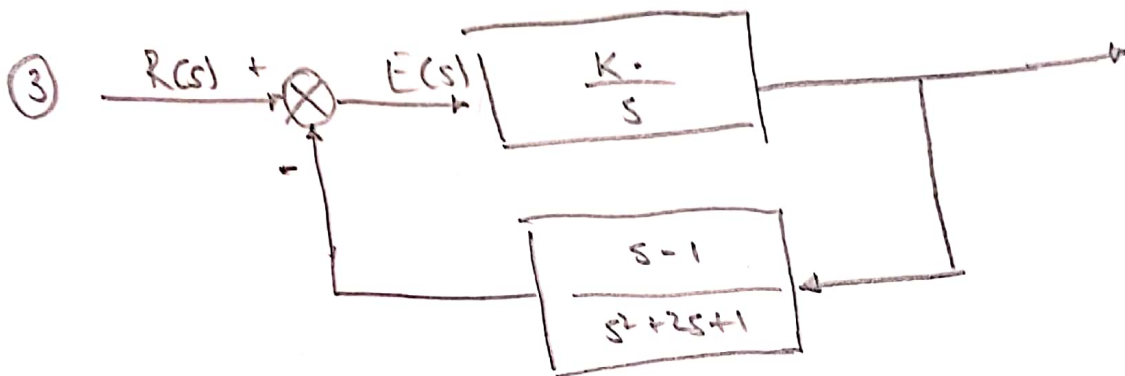
$$\frac{\bar{f}_g}{1 - f_g} = \frac{\frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}}{1 - \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}} = \frac{\frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}}{\frac{s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s}}$$

$$= \frac{18}{s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18}$$

$$C(s) = s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18$$

s^5	1	-7	-18	$\frac{1 \cdot (-7) - 1 \cdot (-18)}{1} = \frac{0}{1}$
s^4	1	-7	-18	
s^3	$\textcircled{0} \rightarrow 4$	$\textcircled{0} \rightarrow 14$	0	
s^2	-3.5	-18	0	
s^1	-34.571	0		
s^0	18			

Three sign changes which means
They are three points on the R.H.S.
The system is unstable.



$$\begin{aligned}
 \frac{T_g}{1 - L_g} &= \frac{\frac{K}{s}}{1 + \frac{K}{s} \cdot \frac{s-1}{s^2+2s+1}} = \frac{\frac{K}{s}}{1 + \frac{K(s-1)}{s(s^2+2s+1)}} \\
 &= \frac{\frac{K}{s}}{\frac{s(s^2+2s+1) + K(s-1)}{s(s^2+2s+1)}} = \frac{K \cdot \cancel{s} (s^2+2s+1)}{s [s(s^2+2s+1) + K(s-1)]} \\
 &= \frac{K \cdot (s^2+2s+1)}{s^3+2s^2+s + Ks - K} = \frac{K(s^2+2s+1)}{s^3+2s^2+(1+K)s - K}
 \end{aligned}$$

$$D(s) = s^3 + 2s^2 + (K+1)s - K$$

s^3	1	$K+1$ ✓	$\frac{K+1}{2} > 0$
s^2	2	$-K$ ✓	$K+1 > 0$
s^1	$\frac{K+1}{2}$ ✗	0	$K > -1$
s^0	$-K$		

Range of K ✗ That when is stable

- 6

BELOW AVERAGE

HW #4

Asher Carson

$$1.) G(s) = \frac{240}{(s+1)(s+2)(s+3)(s+4)}$$

$$\frac{(s+1)(s+2)}{s^2+2s+2} \quad \frac{(s+3)(s+4)}{s^2+4s+3s+12}$$

$$Q(s) = (s+1)(s+2)(s+3)(s+4)$$

$$s^2+3s+2 \quad s^2+7s+12$$

$$Q(s) = s^4 + 10s^3 + 35s^2 + 50s + 24$$

$$\begin{array}{c|ccc} & s^2 & 3s & 2 \\ \hline s^2 & s^4 & 3s^3 & 2s^2 \\ 7s & 7s^3 & 21s^2 & 14s \\ 12 & 12s^2 & 36s & 24 \end{array}$$

$$s^4 + 10s^3 + 35s^2 + 50s + 24$$

$$s^4 \mid 1 \quad 35 \quad 24$$

$$s^3 \mid 10 \quad 50 \quad 0$$

$$s^2 \mid 30 \quad 24$$

$$s^1 \mid 42 \quad 0$$

$$s^0 \mid 24$$

$$b_1 = \frac{(10 \cdot 35) - (1 \cdot 50)}{10}$$

$$b_2 = \frac{(24 \cdot 10) - (1 \cdot 6)}{10}$$

$$b_1 = 30$$

$$b_2 = 24$$

sign changes: 0
stable

$$c_1 = \frac{(30 \cdot 50) - (10 \cdot 24)}{30}$$

$$c_2 = \frac{(30 \cdot 0) - (10 \cdot 0)}{30}$$

$$c_1 = 42$$

$$c_2 = 0$$

$$d_1 = \frac{(42 \cdot 24) - (30 \cdot 0)}{42}$$

$$d_1 = 24$$

$$2.) G(s) = \frac{14}{s^5 + s^4 - 7s^3 - 7s^2 - 14s}$$

$$Q(s) = s^5 + s^4 - 7s^3 - 7s^2 - 14s$$

$$Q(s) = s(s^4 + s^3 - 7s^2 - 7s - 14)$$

$$\bar{Q}(s)$$

$$s^4 \mid 1 \quad -7 \quad -14$$

$$s^3 \mid 1 \quad -7 \quad 0$$

$$s^2 \mid \cancel{0} \quad -14$$

$$s \mid \frac{-7\epsilon + 14}{\epsilon} \quad 0$$

$$s^0 \mid -14$$

$$b_1 = \frac{(-7 \cdot 1) - (-7 \cdot 1)}{1}$$

$$b_2 = \frac{(-14 \cdot 1) - (1 \cdot 0)}{1}$$

$$b_1 = 0$$

$$b_2 = -14$$

$$c_1 = \frac{(-7\epsilon) - (-14 \cdot 1)}{\epsilon}$$

$$c_2 = \frac{(\epsilon \cdot 0) - (1 \cdot 0)}{\epsilon}$$

$$c_1 = \frac{-7\epsilon + 14}{\epsilon}$$

$$c_2 = 0$$

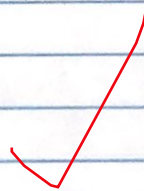
$$d_1 = \frac{(-14 \cdot \frac{-7\epsilon + 14}{\epsilon}) - (0 \cdot \epsilon)}{\frac{-7\epsilon + 14}{\epsilon}}$$

$$d_1 = -14$$

sign changes: 1 unstable

$$\frac{-7\epsilon + 14}{\epsilon} \rightarrow -14$$

one root in RHP



$$G(s) = \frac{F_g}{1-L_g}$$

$$F_g = \frac{K}{s}$$

$$L_g = \frac{K}{s} \left(\frac{s-1}{s^2+2s+1} \right) (-1)$$

$$= \frac{-K(s-1)}{s(s^2+2s+1)} = \frac{-Ks+K}{s^3+2s^2+s} \quad \times$$

$$G(s) = \frac{\frac{K}{s}}{1 - \left(\frac{-Ks+K}{s^3+2s^2+s} \right)} = \frac{\frac{K}{s}}{\frac{s^3+2s^2+s}{s^3+2s^2+s} + \frac{Ks-K}{s^3+2s^2+s}} = \frac{\frac{K}{s}}{\frac{s^3+2s^2+(1+K)s-K}{s^3+2s^2+s}}$$

$$G(s) = \frac{K}{s} \cdot \frac{s^3+2s^2+s}{s^3+2s^2+(1+K)s-K}$$

$$\begin{array}{c|ccc} s^4 & 1 & 1+K & -K \\ s^3 & 2 & 0 & 0 \\ s^2 & 1+K & -K & \\ s^1 & \frac{2K}{1+K} & 0 & \\ s^0 & -2K & & \end{array} \quad \times$$

$$G(s) = \frac{K(s^3+2s^2+s)}{s^4+2s^3+(1+K)s^2+0s-K}$$

$$b_1 = \frac{[(1+K) \cdot 2] - [1 \cdot (0)]}{2}$$

$$b_1 = \frac{2+2K}{2} = \frac{2(1+K)}{2} \Rightarrow b_1 = 1+K$$

$$b_2 = \frac{(-K \cdot 2) - (1 \cdot 0)}{2} = \frac{-2K}{2} = -K$$

$$d_1 = \frac{-2K \cdot \frac{1+K}{1+K}}{2K} = -2K$$

$$c_1 = \frac{(1+K)(0) - (-K \cdot 2)}{(1+K)} \quad c_2 = 0$$

$$c_1 = \frac{2K}{1+K}$$

$$d_1 = \frac{\left[\left(\frac{2K}{1+K} \right) (-K) \right] - \left[(1+K)(0) \right]}{\frac{2K}{1+K} - 1+K}$$

$$1+k > 0$$

$$k > -1$$

$$\frac{2k}{1+k} > 0$$

$$\frac{-2k > 0}{-2 \quad -2}$$

$$k < 0$$

$$1+k \left(\frac{2k}{1+k} \right) > (0) 1+k$$

$$\frac{2k > 0}{2 \quad 2}$$

$$k > 0$$

For it to remain stable:

$$K = \begin{cases} 1+k, & k > -1 \\ \frac{2k}{1+k}, & k > 0 \\ -2k, & k < 0 \end{cases}$$

Automatic Controls
Fall 2021

PROJECT

- 1. Simulink**
- 2. Modeling of dynamic system**
- 3. Stability and performance**

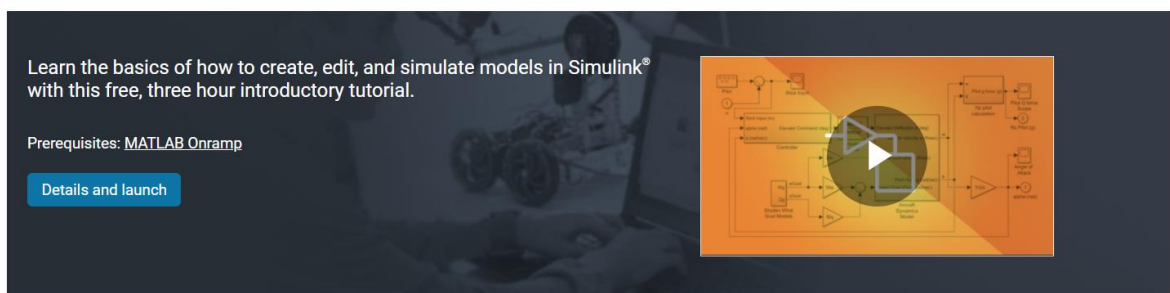
MCEG 3307 Automatic Controls Project

This is an individual project

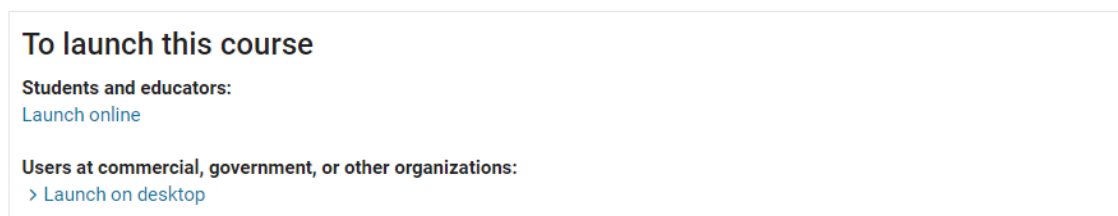
Part I:

1. Please go to <https://www.mathworks.com/learn/tutorials/simulink-onramp.html> , to complete MATLAB Simulink Onramp course. The course takes around 3 hours to complete. You can take breaks and resume it.
2. Click **Details and launch**:

Simulink Onramp



3. Click “launch online”:



4. After completion, submit the certificate and the sharable link of the certificate.

Part II:

1. Refer to the Antenna Control Case study on Page 92 of the textbook. Complete the challenge question on Page 93, which is to find the transfer function of each subsystem using configuration 2 data.
2. Refer to the Antenna Control Case study on Page 272, find the close-loop transfer function using Configuration 2 data. You can refer to the solution of Problem a on this page to complete this task.
3. Based on the derived closed-loop transfer function in the previous task, find the range of preamplifier gain K to keep the closed-loop system stable.

Submission requirements:

A final report in electronic format should be documented in word format and submitted to Canvas. There is no specific format and page limitation, but the report should include:

- Certificate of the MATLAB Simulink Onramp course
- Deriving process of the closed-loop transfer function of the antenna position control system based on the requirements in Part II (block diagram, equations, etc.)
- Calculation process to find the range of K (block diagram, tables, etc.)
- Conclusion

**ABOVE AVERAGE
PROJECT REPORT**

Prairie View A&M University



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Automatic Controls

MCEG 3307 P01

Project

11/23/2021

Asher Carson

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Antenna Control Case Study

This section of the project was focusing on the Antenna Azimuth Position Control System in the textbook. In this, we have three main questions we were asked to answer. For the first part, we were instructed to first the transfer function for each of the separate subfunctions shown in the block diagram, shown in Figure 1. To help find the subfunctions, I referenced the schematic to determine the transfer functions, shown in Figure 2. With these two diagrams, we can start determining each transfer function. We are also using the data for configuration 2, which will be shown in Table 1 below.

Block Diagram

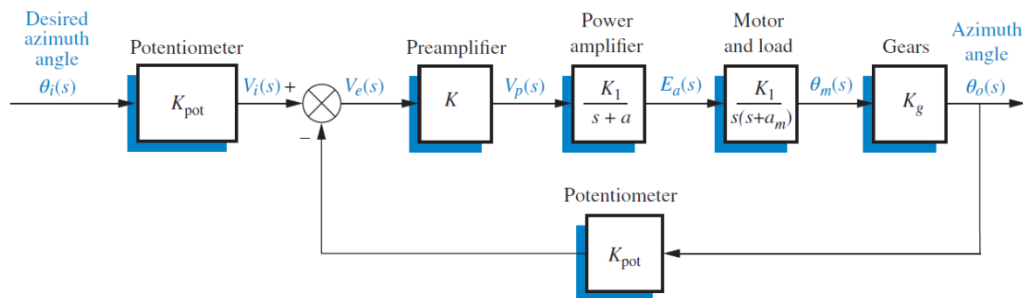


Figure 1 Block Diagram

Schematic

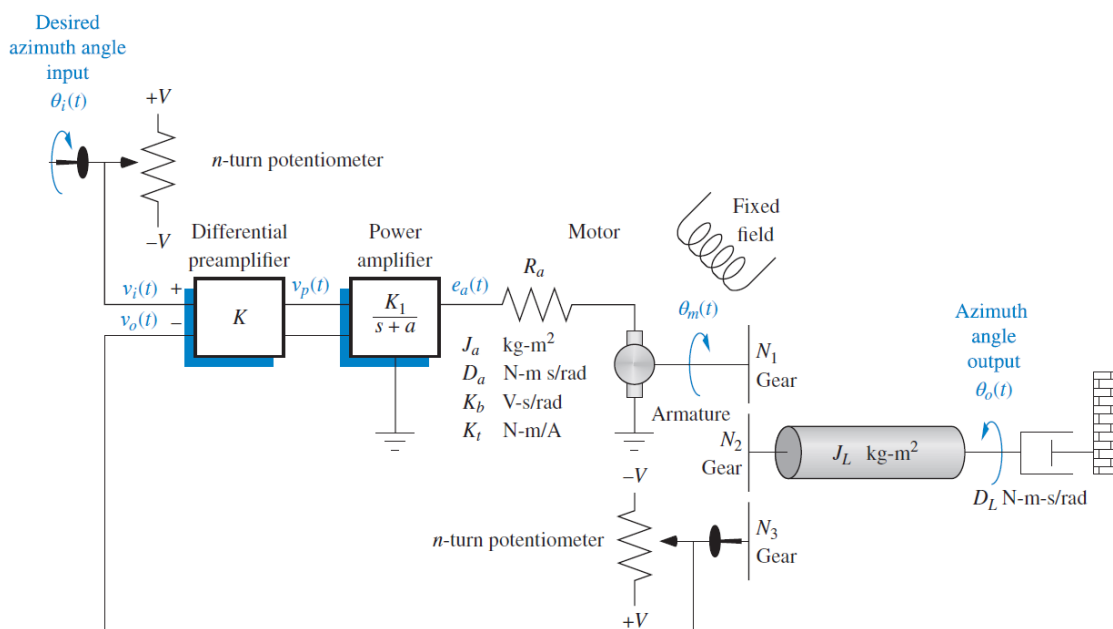


Figure 2 Schematic Diagram

Schematic Parameters

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	—	—	—
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_t	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250
J_L	1	5	5
D_L	1	3	3

1.) Subfunction Transfer Functions

Table 1 Configuration Data

Transfer Functions describes as the relationship between the input and output of a system.

$$\text{Transfer Function} = \frac{\text{Output}}{\text{Input}}$$

Potentiometer

Starting with the Potentiometer, the transfer function is found by dividing the voltage change, $V_i(s)$, and the angular displacement, $\theta_i(s)$, and is shown below.

$$\frac{V_i(s)}{\theta_i(s)} = \frac{V}{n\pi}$$

When you insert the data from Table 1, the first transfer function is then,

$$\frac{V_i(s)}{\theta_i(s)} = \frac{V}{n\pi} = \frac{10}{10\pi} = 3.142$$

Therefore, the transfer function, K_{pot} , is equal to 3.142.

Preamplifier

The preamplifiers transfer function, K , has no value according to Table 1 so it remains K .

Power Amplifier

The Power Amplifier's transfer function is shown as,

$$\frac{E_a(s)}{V_p(s)} = \frac{K_1}{s + a}$$

Once I input the values from Table 1, the transfer function is,

$$\frac{E_a(s)}{V_p(s)} = \frac{K_1}{s + a} = \frac{150}{s + 150}$$

Motor and Load

The book gives us a specific equation for the motor and load transfer function. Equation 2.153 shows the motor and load transfer function as,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\left(\frac{K_t}{R_a * J_m} \right)}{s \left[s + \frac{1}{J_m \left(D_m + \frac{K_t * K_b}{R_a} \right)} \right]}$$

We were also given the formula for J_m and D_m as shown below.

$$J_m = J_a + J_L \left(\frac{N_1}{N_2} \right)^2$$

$$D_m = D_a + D_L \left(\frac{N_1}{N_2} \right)^2$$

First, we must solve for all the variables used in the transfer function, so taking the values from Table 1 will help us solve for J_m and D_m .

$$J_m = J_a + J_L \left(\frac{N_1}{N_2} \right)^2 = 0.05 + 5 \left(\frac{50}{250} \right)^2 = 0.25$$

$$D_m = D_a + D_L \left(\frac{N_1}{N_2} \right)^2 = 0.01 + 3 \left(\frac{50}{250} \right)^2 = 0.13$$

From here, we can take the known values and plug it into the Motor and Load transfer function.

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\left(\frac{K_t}{R_a * J_m}\right)}{s \left[s + \frac{1}{J_m \left(D_m + \frac{K_t * K_b}{R_a} \right)} \right]} = \frac{\left(\frac{1}{5 * 0.25}\right)}{s \left[s + \frac{1}{0.25 \left(0.13 + \frac{1 * 1}{5} \right)} \right]} = \frac{0.28}{s[s + 1.32]}$$

In the block diagram, the transfer function is shown as,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m}{s(s + a_m)}$$

So, when we set the two equations equal to each other we can find the value of K_m and a_m .

$$\frac{K_m}{s(s + a_m)} = \frac{0.28}{s[s + 1.32]}$$

So, we can infer that K_m is equal to 0.28 and a_m is 1.32.

Gears

Lastly, we have the gears subsystem, which can be found by simply finding the gear ratio.

$$\frac{\theta_o(s)}{E_a(s)} = \frac{N_1 \theta_m(s)}{N_2 E_a(s)} = 0.2$$

Therefore, K_g is equal to 0.2.

2.) Transfer Function

Now that we have all the subsystems accounted for, our next step is to find the transfer function of the system. We will also be basing this information on 2nd configuration. Based on the block diagram, we can use the following formula to calculate the transfer function.

$$G_c = \frac{F_g}{1 - L_g}$$

Where F_g is the forward gain and the L_g is the Loop gain. Both values will be shown below.

$$F_g: K_{pot} * K * \left(\frac{K_1}{s + a}\right) * \frac{K_m}{s(s + a_m)} * K_g = \frac{(K_{pot} * K * K_1 * K_m * K_g)}{(s + a)s(s + a_m)}$$

$$L_g: -K_{pot} * K * \left(\frac{K_1}{s + a}\right) * \frac{K_m}{s(s + a_m)} * K_g * K_{pot} = -\frac{(K_{pot} * K * K_1 * K_m * K_g * K_{pot})}{(s + a)s(s + a_m)}$$

So, the Transfer Function would be as follows,

$$G_c = \frac{F_g}{1 - L_g} = \frac{\frac{(K_{pot} * K * K_1 * K_m * K_g)}{(s + a)s(s + a_m)}}{\left(1 + \frac{(K_{pot} * K * K_1 * K_m * K_g * K_{pot})}{(s + a)s(s + a_m)}\right)}$$

After simplification, the transfer function becomes,

$$G_c = \frac{(K_{pot} * K * K_1 * K_m * K_g)}{(s + a)s(s + a_m) + (K_{pot} * K * K_1 * K_m * K_g * K_{pot})}$$

$$= \frac{3.14 * K * 150 * 0.8 * 0.2}{(s + 150)(s)(s + 1.32) + (3.14 * K * 150 * 0.8 * 0.2 * 3.14)}$$

Making our transfer function,

$$= \frac{75.36K}{s^3 + 151.32s^2 + 198s + 236.6K}$$

3.) Stability

Finally, I will use the Routh Array to find the value of K that will keep the closed-loop stable. Using the denominator of the transfer function as Q(s).

$$Q(s) = s^3 + 151.32s^2 + 198s + 236.6K$$

The Routh Array is shown in the figure below.

$$s^3 + 151.32s^2 + 198s + 236.6k$$

s^3	1	198
s^2	151.32	236.6k
s^1	$198 - 1.5638k$	
s^0	236.6k	

$$b_1 = \frac{(151.32 \cdot 198) - (1 \cdot 236.6k)}{151.32}$$

$$b_1 = \frac{29961.36 - 236.6k}{151.32}$$

$$b_1 = 198 - 1.5638k$$

$$236.6k > 0$$

$$k > 0$$

$$198 - 1.5638k > 0$$

$$198 > 1.5638k$$

$$\frac{198}{1.5638} > k$$

$$126.62 > k$$

$$C_1 = \frac{[(198 - 1.6039k)(242.7k)] - (151.32 \cdot 0)}{198 - 1.6039k}$$

$$C_1 = \frac{(198 - 1.6039k) \cdot 236.6k}{(198 - 1.6039k)}$$

$$C_1 = 242.7k$$

$$0 < k < 126.62$$

The process is correct

With this information we can see that K is stable between the values of 0 and 126.62.

4.) Conclusion

In this project, I was able to apply all the concepts we learned in this class and use it in a real-life scenario. Through this, I was able to find the stability range for the Antenna Control System. This information gives us the ranges for operating the antenna.



Course Completion Certificate

Asher Carson

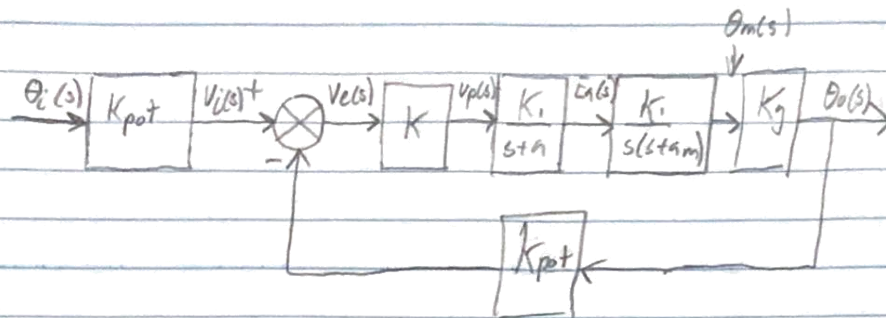
has successfully completed **100%** of the self-paced training course

Simulink Onramp


DIRECTOR, TRAINING SERVICES

17 November 2021





$$\frac{V_i(s)}{\Theta_i(s)} = \frac{V}{nr} = \frac{10}{nr} = \frac{V_p(s)}{V_e(s)} = \frac{K}{V_e(s)}$$

$$= 3.142$$

$$K_{pot} = 3.142 = \frac{10}{nr}$$

$$\frac{E_a(s)}{V_p(s)} = \frac{K_i}{s+a} = \frac{150}{s+150}$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_t / (R_a J_m)}{s \left[s + \frac{1}{J_m} \left(D_m + \frac{K_t K_b}{R_a} \right) \right]}$$

$$J_m = J_a + J_L \left(\frac{N_1}{N_2} \right)^2 ; D_m = D_a + D_L \left(\frac{N_1}{N_2} \right)^2$$

$$J_m = 0.05 + 5 \left(\frac{50}{250} \right)^2 ; D_m = 0.01 + 3 \left(\frac{50}{250} \right)^2$$

$$J_m = 0.25 ; D_m = 0.13$$

$$K_t = 1, R_a = 5, K_b = 1, R_a = 5$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{1/(5 \cdot 0.25)}{s \left[s + \frac{1}{0.25} \left(0.13 + \frac{1 \cdot 1}{5} \right) \right]}$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{0.4}{s[s+1.32]}$$

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{0.4}{s^2 + 1.32s}$$

$$\frac{\Theta_o(s)}{E_a(s)} = \frac{N_1}{N_2} \frac{\Theta_m(s)}{E_a(s)}$$

$$= \frac{50}{250} \frac{\Theta_m(s)}{E_a(s)}$$

$$= 0.2 \frac{\Theta_m(s)}{E_a(s)}$$

$$\begin{aligned}K_{p04} &= 3.14 \\K_i &= 150 \\a &= 130 \\K_m &= 0.8 \\a_m &= 1.32 \\K_g &= 0.2\end{aligned}$$

$$F_g: \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g}{s+a \cdot s(s+a_m)} \quad G_c = \frac{F_g}{1-L_g}$$

$$F_g: \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g}{(s+a)(s)(s+a_m)} \quad \frac{(s+a)(s)(s+a_m)}{(s+a)(s)(s+a_m)} + \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g \cdot K_{pot}}{(s+a)(s)(s+a_m)}$$

$$L_g: \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g \cdot K_{pot}}{(s+a)(s)(s+a_m)}$$

↓
✓

$$G_c = \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g}{(s+a)(s)(s+a_m)} = \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g}{(s+a)(s)(s+a_m) + K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g \cdot K_{pot}}$$

$$G_c = \frac{K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g}{(s+a)(s)(s+a_m) + K_{pot} \cdot K \cdot K_1 \cdot K_m \cdot K_g \cdot K_{pot}}$$

$$G_c = \frac{3.14 \cdot K \cdot 150 \cdot 0.4 \cdot 0.2}{(s+150)(s)(s+1.32) + (3.14 \cdot K \cdot 150 \cdot 0.4 \cdot 0.2 \cdot 3.14)}$$

$$G_c = \frac{75.36 K}{(s+150)(s)(s+1.32) + 236.6 K}$$

$$(s+150)(s)(s+1.32)$$

$$(s^2+150s)(s+1.32)$$

$$s^3+1.32s^2+150s^2+198s$$

$$G_c = \frac{76.32 K}{s^3+151.32s^2+198s+236.6 K}$$

$$s^3 + 151.32s^2 + 198s + 236.6K$$

s^3	1	198	$b_1 = (151.32 \cdot 198) - (1 \cdot 236.6K)$
s^2	151.32	236.6K	151.32
s^1	198 - 1.5638K		
s^0	236.6K	$b_1 = \frac{29961.36 - 236.6K}{151.32}$	

$$236.6K > 0$$

$$K > 0$$

$$b_1 = 198 - 1.5638K$$

$$198 - 1.5638K > 0$$

$$198 > 1.5638K$$

$$1.5638 > 1.5638$$

$$126.62 > K$$

$$C_1 = \frac{[(198 - 1.6039K)(242.7K)] - (151.32 \cdot 0)}{198 - 1.6039K}$$

$$C_1 = \frac{(198 - 1.6039K) 236.6K}{(198 - 1.6039K)}$$

$$C_1 = 242.7K$$

$$0 < K < 126.62$$

**AVERAGE
PROJECT REPORT**

AUTOMATIC CONTROLS PROJECT

85

MCEG 3307-P01

Jade A. Coleman

Part 1

<https://matlabacademy.mathworks.com/details/simulink-onramp/simulink>



Course Completion Certificate

Jade Coleman

has successfully completed **100%** of the self-paced training course

Simulink Onramp

A handwritten signature in black ink, appearing to read 'David H. Santos'.

DIRECTOR, TRAINING SERVICES



30 June 2021

Part 2

Schematic Parameters

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	—	—	—
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_f	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250
J_L	1	5	5
D_L	1	3	3

1. Refer to the Antenna Control Case study on Page 92 of the textbook. Complete the challenge question on Page 93, which is to find the transfer function of each subsystem using configuration 2 data.

1) $\frac{V(s)}{\Theta_v(s)} = \frac{10}{\pi}$ $\frac{V_p(s)}{V_e(s)} = K$ $\frac{E_a(s)}{V_p(s)} = \frac{150}{s+150}$

Motor & Load

$J_m = J_a + J_L (N_1/N_2)^2$, $J_a = 0.05$, $J_L = 5$, $N_1/N_2 = 50/250$
 $= (0.05) + (5)(50/250)^2$
 $= 0.25$

$D_m = D_a + D_L (N_1/N_2)^2$, $D_a = 0.01$, $D_L = 3$, $N_1/N_2 = 50/250$
 $= (0.01) + (3)(50/250)^2$
 $= 0.13$

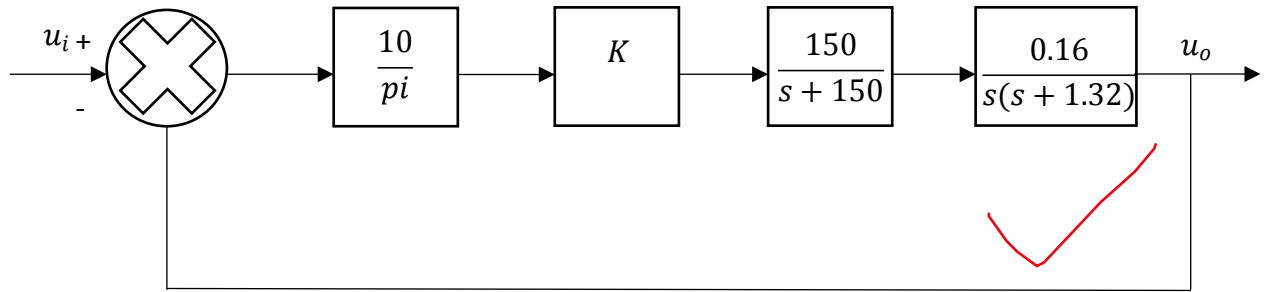
$\frac{\Theta_m(s)}{E_a(s)} = \frac{k_t / R_a J_m}{s \left[s + \frac{1}{J_m} (D_m + \frac{k_t k_b}{R_a}) \right]}$ $k_e = 1$, $R_a = 5$, $J_m = 0.25$,
 $D_m = 0.13$, $k_b = 1$

$= \frac{(1)/(5)(0.25)}{s \left[s + \frac{1}{(0.25)} \left(\frac{(0.13) + (1)(1)}{(5)} \right) \right]}$

$= \frac{0.2}{s[s+1.32]}$

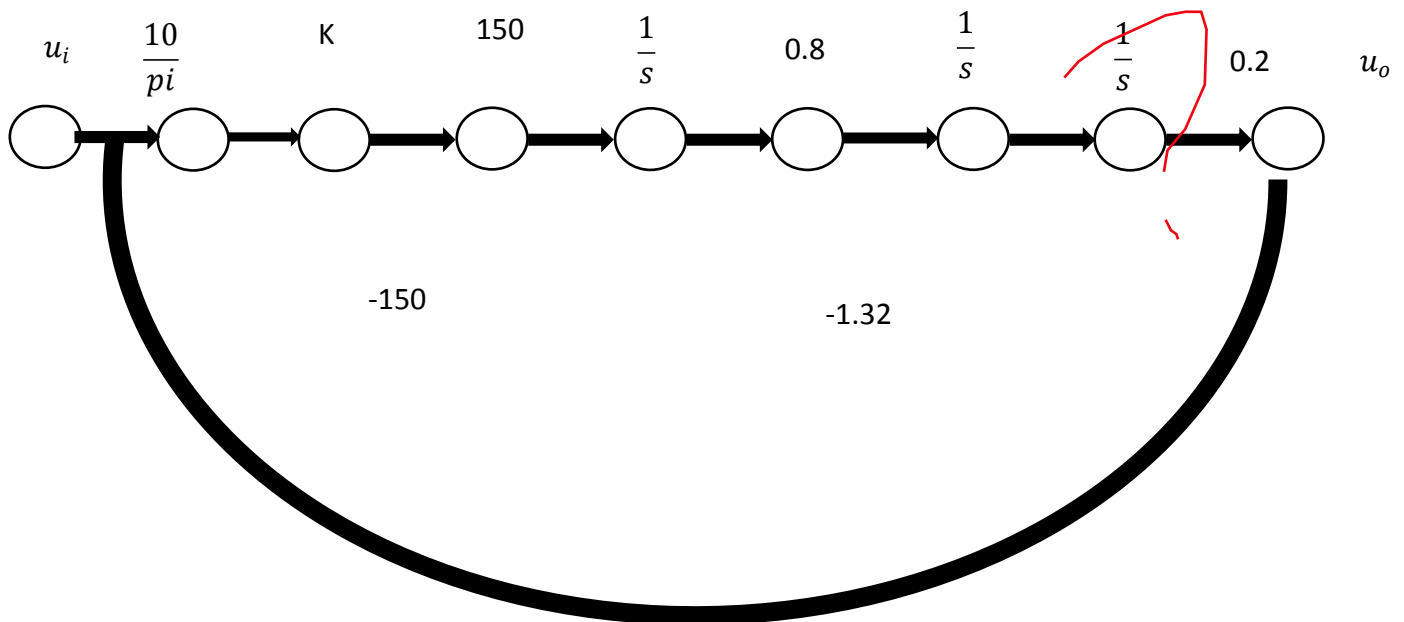
2. Refer to the Antenna Control Case study on Page 272, find the close-loop transfer function using Configuration 2 data. You can refer to the solution of Problem a on this page to complete this task.

a)



$$T(s) = \frac{76.39K}{s^3 + 151.32s^2 + 198s + 76.39K}$$

b)



$$2. c) T_i = \left(\frac{10}{\pi}\right)(K)(150) \left(\frac{1}{s}\right)(0.8) \left(\frac{1}{s}\right) \left(\frac{1}{s}\right)(0.2)$$

$$T_i = \frac{76.39}{s^3}$$

$$G_{u1} = -\frac{150}{s}$$

$$G_{u2} = (K)(150) \left(\frac{1}{s}\right)(0.8) \left(\frac{1}{s}\right) \left(\frac{1}{s}\right)(0.2) \left(-\frac{10}{\pi}\right)$$

$$G_{u2} = -\frac{1.32}{s}$$

$$G_{u2} = -\frac{76.39K}{s^2}$$

$$(G_{u1})(G_{u2}) = \left(-\frac{150}{s}\right) \left(-\frac{1.32}{s}\right) = \frac{198}{s^2}$$

$$\Delta = 1 - [G_{u1} + G_{u2} + G_{u3}] + [(G_{u1})(G_{u2})]$$

$$\Delta = 1 - \left[\left(-\frac{150}{s}\right) + \left(-\frac{1.32}{s}\right) + \left(-\frac{76.39K}{s^2}\right) \right] + \frac{198}{s^2}$$

$$= 1 + \frac{150}{s} + \frac{1.32}{s} + \frac{76.39K}{s^2} + \frac{198}{s^2}$$

$$\Delta_1 = 1$$

$$T(r) = \frac{T_i \Delta_1}{\Delta} = \frac{\left(\frac{76.39}{s^3}\right)(1)}{1 + \frac{150}{s} + \frac{1.32}{s} + \frac{76.39K}{s^2} + \frac{198}{s^2}}$$

$$T(r) = \frac{76.39K}{s^3 + 151.32s^2 + 198s + 7639K}$$

$$2. d) G(r) = \frac{10}{\pi} \frac{(0.16K)}{s(s+1.32)} \quad 2.55$$

$$T(r) = \frac{2.55}{s^2 + 1.32s + 2.55}$$

$$\omega_n = \sqrt{2.55} \quad 2.5\omega_n = 1.32 \quad \zeta = 0.213$$

$$e) C(s) = \frac{2.55}{s(s^2 + 1.32s + 2.55)}$$

$$C(s) = \frac{1}{s} - \frac{1}{25} \left(\frac{25s + 3.2}{s^2 + 1.32s + 2.55} \right)$$

$$c(t) = 1 - e^{-0.66t} [\cos(1.16t) + 0.7\sin(1.16t)]$$

$$f) G(s) = \frac{0.57K}{s(s+1.32)}, \quad T(s) = \frac{0.57}{s^2 + 1.32s + 0.57K} \quad \omega_n = \sqrt{0.57K} = 0.71K$$

$$2.5\omega_n = 2(0.57K)(0.71) = 0.798$$

3. Based on the derived closed-loop transfer function in the previous task, find the range of preamplifier gain K to keep the closed-loop system stable.

$$\text{Range} = K < 3.7 > 3.7$$

?

Conclusion

The overall objective of the project was to use the knowledge acquired from the course to evaluate the given loop systems. Using the calculations configured from the values given in the Configuration 1 column as a guide, the values derived from the Configuration 2 column were used to calculate the specified values. This ultimately allowed students to find the transfer functions and the range of the preamplifier required to keep the closed-loop system stable.

BELOW AVERAGE

Devonte Mitchell
Automatics Controls Fall 2021 Project
MCEG 3307

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Part 1: MathWorks Certificate



Course Completion Certificate

Devonté Mitchell

has successfully completed **59%** of the self-paced training course

Simulink Onramp

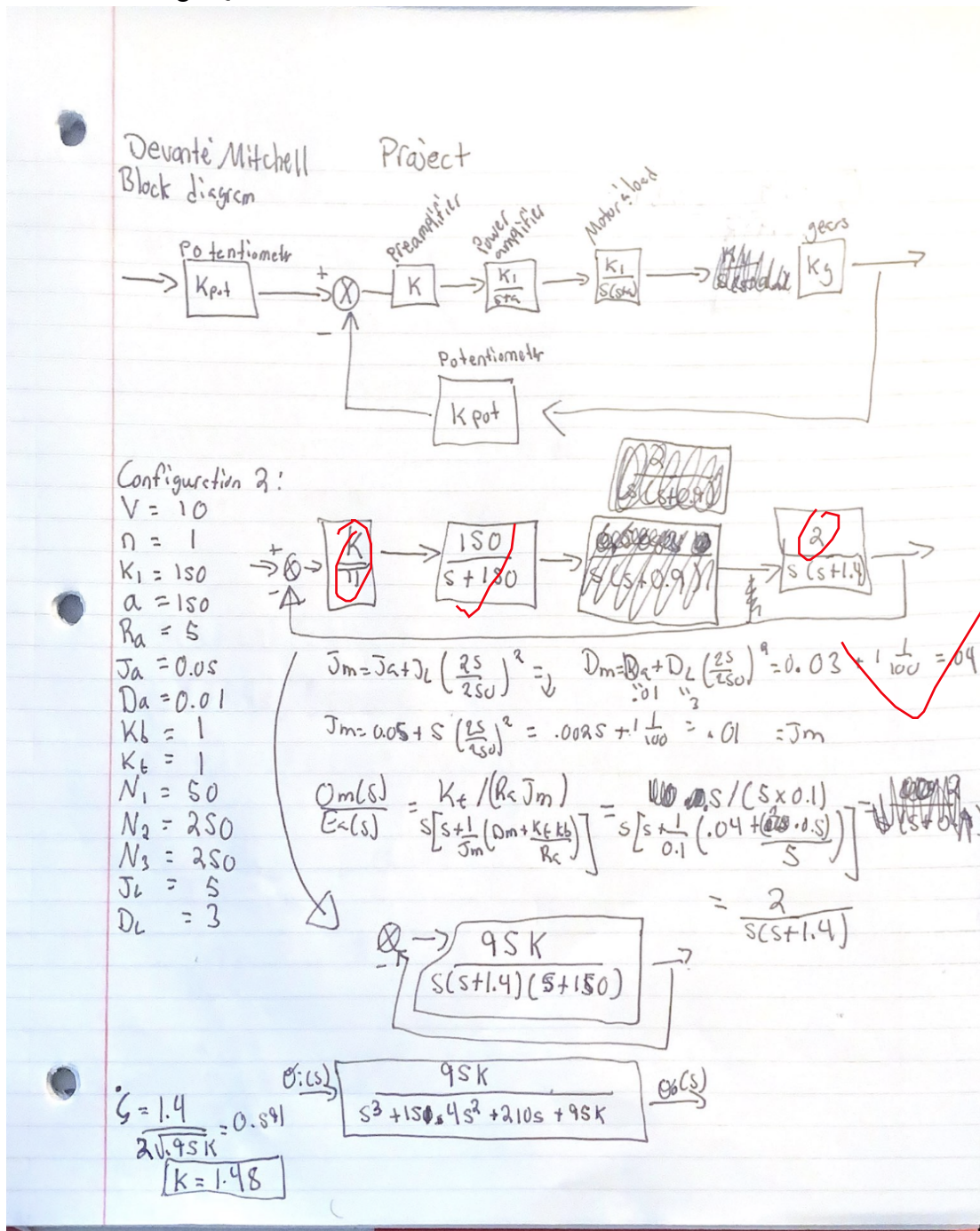


DIRECTOR, TRAINING SERVICES

23 November 2021

<https://matlabacademy.mathworks.com/progress/share/certificate.html?id=20a8c721-5983-417f-8a85-475b71a5e9ad&>

Part 2: Challenge Question



Conclusion:

Closed Block Function:

$$\frac{95k}{s^3 + 151.4s^2 + 210s + 95k}$$

$K=1.48$

should be a range