DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
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PRAIRIE VIEW A&M UNIVERSITY
PRELIMINARY EXAMINATION – ENGINEERING MATHEMATICS
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CALCULATORS ARE NOT NECESSARY AND ARE NOT ALLOWED.

READ THIS CAREFULLY AND SIGN BELOW

I declare truthfully that the work I am presenting here is my own, and that I have not conducted myself in any manner unethical. I have not copied from anyone nor have I let anyone copy from me. I have not sought assistance from any external source. I am aware of the ethical requirements of my profession and I firmly believe in following and implementing them.

Name: ____

Signature: ____

PROBLEM 1

Find two matrices A and B such that AB is a null matrix, without any of the elements in A or B being zero.

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

PROBLEM 2

A solid body of total volume V and total surface area S is formed by joining together two cubes of different sizes so that every point on one side of the smaller cube is in contact with the larger cube. If S = 10 m^2, find the maximum and/or minimum values of V for which both cubes have non-zero volumes?

PROBLEM 3

This problem occurs in the analysis of a radioactive material. Consider a rod of homogeneous radioactive material lying along the x-axis, $0 \leq x \leq L$. The neutron density n(x,t) at position x and time t is affected by two processes – fission and diffusion. Conservation of neutrons leads to the following partial differential equation,

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2} + kn, \quad 0 < x < L, \qquad t > 0$$

where *D > 0* is a constant diffusion coefficient and *k > 0* is a fission constant. Suppose n(x,t) = 0 at the ends of the rod.

(i) Make the transformation $v(x,t) = n(x,t)e^{-kt}$, and show that

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2}, \quad 0 < x < L, \qquad t > 0$$

$v(0,t) = v(L,t) = 0, \qquad v(x,0) = f(x).$

(ii) Solve the above equation for $v(x,t)$ by employing the separation of variables technique.

(iii) Hence write down the complete solution for n(x,t).

(iv) Show that the rod will explode (n(x,t) $\to \infty$ as t $\to \infty$) only if

$$k > \frac{\pi^2 D}{L^2}.$$

PROBLEM 4

Ann will arrive at random between 8:15 and 8:45 and Bill will arrive between 8:00 and 9:00 (independently of Ann). Each agrees to wait up to five minutes for the other before leaving.

(1) Find the probability that Bill arrives first.

(2) Find the probability that they meet.

PROBLEM 5

Pick a point (X, Y) at random inside the circle $x^2 + y^2 = 1$. Find the conditional probability density function $f(y|x)$.

(a) By inspection:

(b) The long way:

PROBLEM 6

Evaluate the following integral.

$$\int \frac{\sqrt{x+4}}{x}\, dx.$$