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Application of Differential Transform Method to the Generalized Burgers–Huxley Equation

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Abstract

In this paper, the differential transform method (DTM) will be applied to the generalized Burgers-Huxley equation, and some special cases of the equation, say, Huxley equation and Fitzhugh-Nagoma equation. The DTM produces an approximate solution for the equation, with few and easy computations. Numerical comparison between differential transform method, Adomian decomposition method and Variational iteration method for Burgers-Huxley, Huxley equation and Fitzhugh-Nagoma equation reveal that differential transform method is simple, accurate and efficient.

Keywords: Differential transform method; Adomian decomposition method; Variational iteration method; Burgers-Huxley equation; Huxley equation; Fitzhugh-Nagoma equation.

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1. Introduction

The differential transform method (DTM) is a semi-analytical-numerical method that uses Taylor series for the solution of differential equations. The DTM is an alternative procedure for

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obtaining the Taylor series solution of the given differential equation and is promising for various other types of differential equations. With this method, it is possible to obtain highly accurate results or exact solutions for differential equations. The concept of the differential transform method was first proposed by [Zhou (1986)], who solved linear and nonlinear initial value problems in electric circuit analysis.

Nonlinear partial differential equations (NPDEs) are encountered in various disciplines, such as physics, chemistry, biology, mathematics and engineering. In this paper, the differential transform method (DTM) is applied to the generalized nonlinear Burgers–Huxley equation, which models the interaction between reaction mechanisms, convection effects and diffusion transports [Satsuma (1987)], and some special cases of the equation, which usually appear in mathematical modeling of some real world phenomena, i.e., two known cases are: Huxley equation, which is an evolution equation that describes the nerve propagation in biology from which molecular CB properties can be calculated.

It also gives a phenomenological description of the behavior of the myosin heads II [Wazwaz (2005)] and Fitzhugh-Nagoma equation, an important nonlinear reaction-diffusion equation used in circuit theory, biology and population genetics [Hariharan and Kannan (2010)]. Many methods have been used to solve the Burgers-Huxley equation [Javidi (2006, 2006), Hashim et al. (2006), Wazwaz (2008), Batiha et al. (2008), Javidi and Golbabai (2009), Kyrychko et al. (2005), Guha (2006), Kaushik (2008) and Khattak (2009)].

2. The Model Problem

The analysis presented in this paper is based on the generalized nonlinear Burgers-Huxley equation,

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u\left(1 - u^{\delta}\right)\left(u^{\delta} - \gamma\right), 0 \leq x \leq 1\,, t \geq 0\,. \tag{1}$$

With initial condition

$$u(x,0) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh(\sigma\gamma x)]^{1/\delta}. \tag{2}$$

The exact solution of this equation is as follows,

$$u(x,t) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x - \left\{\frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(\rho-\alpha)}{2(1+\delta)}\right\} t\right)\right\}]^{1/\delta}, \tag{3}$$

where

$$\sigma = \frac{\delta(\rho-\alpha)}{4(1+\delta)}, \rho = \sqrt{\alpha^2 + 4\beta(1+\delta)}\,. \tag{4}$$

When $\alpha = 0$, and $\delta = 1$, Equation (1) is reduced to the Fitzhugh-Nagoma nerve conduction equation that it is the form, [Hodgkin and Huxley (1952)]

$$u_t - u_{xx} = \beta u(1-u)(u-\gamma), \ 0 \le x \le 1, \ \ t > 0. \tag{5}$$

With initial condition

$$u(x,0) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh(\sigma\gamma x)] . \tag{6}$$

The exact solution of this equation is as follows,

$$u(x,t) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x + \tfrac{(2-\gamma)\rho}{4}t\right)\right\}] , \tag{7}$$

where

$$\sigma = \tfrac{\rho}{8}, \ \rho = \sqrt{8\beta} .$$

When $\alpha = 0$, Equation (1) is reduced to the Huxley equation that it is the form, [Wazwaz (2005)]

$$u_t - u_{xx} = \beta u(1-u^{\delta})(u^{\delta}-\gamma), \ 0 \le x \le 1, \ \ t > 0. \tag{8}$$

With initial condition

$$u(x,0) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh(\sigma\gamma x)]^{1/\delta}. \tag{9}$$

The exact solution of this equation is as follows,

$$u(x,t) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x + \tfrac{(1+\delta-\gamma)\rho}{2(1+\delta)}t\right)\right\}]^{1/\delta} , \tag{10}$$

where

$$\sigma = \frac{\delta\rho}{4(1+\delta)}, \ \rho = \sqrt{4\beta(1+\delta)} .$$

3. Differential Transform Method

The basic definitions and fundamental operations of one and two dimensional differential transform are defined as follows,[Erturk and Momani (2007), Ayaz (2004), Kangalgil and Ayaz (2009) and Yang et al. (2006)]. In what follows, we assume that for each function involved in our study, all its derivatives existent and are continuous in the region of interest.

Definition 3.1.

a) The differential transform of a function $w(x)$ is defined as

$$W(k) = \frac{1}{k!}\frac{d^k w(x)}{dx^k}\bigg|_{x=x_0}, k \geq 0, \tag{11}$$

b) The inverse differential transform of a sequence, $\{W(k)\}_{k=0}^{\infty}$ is defined as

$$w(x) = \sum_{k=0}^{\infty} W(k)(x - x_0)^k, \tag{12}$$

c) The two-dimensional differential transform of a function $w(x,y)$, is defined as

$$W(k,h) = \frac{1}{k!h!}\frac{\partial^{k+h} w(x,y)}{\partial x^k \partial y^h}\bigg|_{\substack{x=x_0 \\ y=y_0}}, \ k \geq 0, h \geq 0, \tag{13}$$

d) The inverse two-dimensional differential transform of a sequence, $\{W(k,h)\}_{k,h=0}^{\infty}$ is defined as

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h)(x - x_0)^k (y - y_0)^h. \tag{14}$$

When (x_0, y_0) are taken as $(0,0)$, and in real applications, the function $w(x,y)$ can be shown as truncated series of Equation (8) can be written as the following,

$$w(x,y) = \sum_{k=0}^{n}\sum_{h=0}^{m} W(k,h) x^k y^h. \tag{15}$$

Which will be equal to (8), as $m, n \to \infty$.

The fundamental operations of two-dimensional differential transform method are listed in Table 1 below [Ayaz (2004), Kangalgil and Ayaz (2009), Yang et al. (2006) and Chen and Ho (1999)].

4. Application

4.1. Burger-Huxley Equation

The Burgers-Huxley equation has been solved by the differential transform method, for two different values of δ.

$\delta = 1$

The two-dimensional transform of Equation (1) for $\delta = 1$, is written as follows:

$$u_t + \alpha u u_x - u_{xxx} - \beta u(1-u)(u-\gamma), 0 \le x \le 1, t \ge 0. \tag{16}$$

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(\sigma\gamma x)\right]. \tag{17}$$

Table 1. Operations for two-dimensional differential transform method.

Original function	**Transformed function**
$w(x,y) = u(x,y) \mp v(x,y)$	$W(k,h) = U(k,h) \mp V(k,h)$
$w(x,y) = \alpha u(x,y)$	$W(k,h) = \alpha U(k,h)$
$w(x,y) = \frac{\partial u(x,y)}{\partial x}$	$W(k,h) = (k+1)U(k+1,h)$
$w(x,y) = \frac{\partial u(x,y)}{\partial y}$	$W(k,h) = (h+1)U(k,h+1)$
$w(x,y) = \frac{\partial^{r+s} u(x,y)}{\partial x^r \partial y^s}$	$W(k,h) = (k+1)\ldots(k+r)$ $(h+1)\ldots(h+s)U(k+r,h+s)$
$w(x,y) = (x-x_0)^m (y-y_0)^n$	$W(k,h) = \delta(k-m,h-n) = \delta(k-m)\delta(h-n)$ $\delta(k-m,h-n) = \begin{cases} 1 & k = m \text{ and } h = n \\ 0 & \text{else} \end{cases}$
$w(x,y) = u(x,y)v(x,y)$	$W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)V(k-r,s)$
$w(x,y) = u(x,y)v(x,y)q(x,y)$	$W(k,h) = \sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)V(t,s)Q(k-r-t,p)$

With exact solution

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x - \left\{\frac{\gamma\alpha}{2} - \frac{(2-\gamma)(\rho-\alpha)}{4}\right\}t\right)\right\}\right], \tag{18}$$

where

$$\sigma = \frac{(\rho - \alpha)}{8}, \; \rho = \sqrt{\alpha^2 + 8\beta} \; . \tag{19}$$

Using the related definitions in Table 1, we have

$$(h+1)U(k,h+1) + \alpha \sum_{r=0}^{k} \sum_{s=0}^{h} U(r,h-s)(k+1-r)U(k-r+1,s) - (k+1)(k+2)U(k+2,h) = (\beta + \beta\gamma) \sum_{r=0}^{k} \sum_{s=0}^{h} U(r,h-s)U(k-r,s) - \beta \sum_{r=0}^{k} \sum_{t=0}^{k-r} \sum_{s=0}^{h} \sum_{p=0}^{h-s} U(r,h-s-p)U(t,s)U(k \;\; r \;\; t,p) \;\; \beta\gamma U(k,h) \tag{20}$$

Using Equation (14) and when (x_0, y_0) are taken as $(0,0)$, $u(x,0)$ will be obtained as

$$u(x,0) = \sum_{k=0}^{\infty} U(k,0)x^k. \tag{21}$$

By equating the coefficients of the terms with identical powers of x, in Equations (17) and (21) the following results can be obtained.

$$U(k,0) = 0, \; k = 2,4,6, \ldots \; , U(0,0) = \frac{\gamma}{2}, U(1,0) = \frac{\gamma^2\sigma}{2}, U(3,0) = -\frac{\gamma^4\sigma^3}{6}, \; \ldots \; . \tag{22}$$

By substituting Equations (22) into Equation (20), and by recursive method, following some values of $U(k,h)$ will be obtained.

$$U(0,1) = -\frac{\beta\gamma^2}{4} + \frac{\beta\gamma^3}{8} - \frac{\alpha\gamma^3\sigma}{4},$$

$$U(2,1) = \frac{\gamma^4\sigma^2\beta}{4} - \frac{\beta\gamma^5\sigma^2}{8} + \frac{\alpha\gamma^5\sigma^3}{4},$$

$$U(7,1) = -\frac{31\gamma^{10}\sigma^8\beta}{630} + \frac{31\beta\gamma^{11}\sigma^8}{1260} - \frac{31\alpha\gamma^{11}\sigma^9}{630},$$

$$U(0,2) = -\frac{\beta^2\gamma^4}{32} + \frac{\beta\gamma^4\alpha\sigma}{16} + \frac{\beta^2\gamma^5}{64} - \frac{3\beta\gamma^5\alpha\sigma}{32} + \frac{\gamma^4\sigma^2\beta}{4} - \frac{\beta\gamma^5\sigma^2}{8} + \frac{\alpha\gamma^5\sigma^2}{2} + \frac{\gamma^5\sigma^2\alpha^2}{8}, \qquad \vdots$$

By substituting all values of $U(k,h)$ into Equation (15), and some manipulations, the series form solutions of Equation (16) with initial condition (17) will be obtained as follows,

$$u(x,t) = \left\{\frac{\gamma}{2} + \frac{\gamma^2\sigma}{2}x + \cdots\right\} + \left\{\left(-\frac{\beta\gamma^2}{4} + \frac{\beta\gamma^3}{8} - \frac{\alpha\gamma^3\sigma}{4}\right) + \left(\frac{\beta\gamma^4\sigma}{8} - \gamma^4\sigma^2 - \frac{\alpha\gamma^4\sigma^2}{4}\right)x + \cdots\right\}t + \cdots \; .$$

Tables 2, 3 and 4, show the absolute errors for Equation (16), by the methods; differential transform, Adomian decomposition (AD) [Hashim et al. (2006)], and Variational iteration (VI) [Batiha et al. (2008)] methods. The solutions by (AD), and (VI), methods, in the references, are obtained for less values of parameters, but in this paper these methods have been used for more values of parameters. Numerical results in Tables 2, 3, and 4, reveal that DTM, with less and easier computations, has the same results, as ADM and VIM, for Burgers-Huxley equation, with $\delta = 1$.

Table 2. Absolute errors for $\gamma = 0.001$, $\alpha = \beta = 1$, by using 5-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.05	1.87406E-08	1.87405E-08	1.87406E-08
	0.1	3.74812E-08	3.74813E-08	3.74813E-08
	1	3.74812E-07	3.74812E-07	3.748125E-07
0.5	0.05	1.87406E-08	1.87405E-08	1.87406E-08
	0.1	3.74812E-08	3.74813E-08	3.74813E-08
	1	3.74812E-07	3.74813E-07	3.748125E-07
0.9	0.05	1.87406E-08	1.87405E-08	1.87406E-08
	0.1	3.74812E-08	3.74813E-08	3.74813E-08
	1	3.74812E-07	3.74813E-07	3.748125E-07

Table 3. Absolute errors for $\gamma = \alpha = 0.1$, $\beta = 0.001$, by using 5-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.05	1.3634E-07	1.3608E-07	1.3608E-07
	0.1	2.7243E-07	2.7216E-07	2.7216E-07
	1	2.72200E-06	2.72151E-06	2.72151E-06
0.5	0.05	1.3736E-07	1.3608E-07	1.3608E-07
	0.1	2.7345E-07	2.7216E-07	2.7216E-07
	1	2.72302E-06	2.72151E-06	2.72151E-06
0.9	0.05	1.3838E-07	1.3608E-07	1.3608E-07
	0.1	2.7447E-07	2.7216E-07	2.7216E-07
	1	2.72404E-06	2.72151E-06	2.72151E-06

Table 4. Absolute errors for $\alpha = 0.01$, $\beta = \gamma = 0.0001$, by using 5-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.05	2E-14	2E-14	2E-14
	0.1	4E-14	3E-14	3E-14
	1	3.7E-13	3.7E-13	3.7E-13
0.5	0.05	2E-14	2E-14	2E-14
	0.1	4E-14	3E-14	3E-14
	1	3.7E-13	3.7E-13	3.7E-13
0.9	0.05	2E-14	2E-14	2E-14
	0.1	4E-14	3E-14	3E-14
	1	3.7E-13	3.7E-13	3.7E-13

$\delta = 2;$

The two-dimensional transform of Equation (1) for $\delta = 2$, is as follows.

$$u_t + \alpha u^2 u_x - u_{xx} = (\beta + \beta\gamma)u^3 - \beta\gamma u - \beta u^2 u^3,\ 0 \le x \le 1, t \ge 0\,. \tag{23}$$

$$u(x,0) = \{\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh(\sigma\gamma x)\}^{1/2}\,. \tag{24}$$

with the exact solution

$$u(x,t) = [\tfrac{\gamma}{2} + \tfrac{\gamma}{2}\tanh\left\{\sigma\gamma\left(x - \left\{\frac{\gamma\alpha}{3} - \frac{(\lambda-\gamma)(\rho-\alpha)}{6}\right\}t\right)\right\}]^{1/2}\,, \tag{25}$$

where

$$\sigma = \frac{\rho-\alpha}{6},\ \rho = \sqrt{\alpha^2 + 12\beta}\,. \tag{26}$$

Using related definitions in Table 1, the following results will obtain

$$\begin{aligned}&(h+1)U(k,h+1) + \alpha\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}U(r,h-s-p)U(t,s)(k-r-t+\\&1)U(k-r-t+1,p) - (k+1)(k+2)U(k+2,h) =\\&(\beta+\beta\gamma)\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}U(r,h-s-p)U(t,s)U(k-r-t,p) - \beta\gamma U(k,h) -\\&\beta\sum_{r=0}^{k}\sum_{s=0}^{h}\{(\sum_{t=0}^{r}\sum_{p=0}^{h-s}U(t,h-s-p)U(r-t,p))\times(\sum_{i=0}^{k-r}\sum_{j=0}^{k-r-i}\sum_{e=0}^{s}\sum_{f=0}^{s-e}U(i,s-\\&e-f)U(j,e)U(k-r-i-j,f))\}\,.\end{aligned} \tag{27}$$

By equating the coefficients of the terms with identical powers of x, in Equations (21, 24) the following results can be obtained.

$$U(0,0) = \frac{\sqrt{2\gamma}}{2},\ U(1,0) = \frac{\sqrt{2}\gamma^{3/2}\sigma}{4},\ U(2,0) = -\frac{\sqrt{2}\gamma^{5/2}\sigma^2}{16}, \dots\,. \tag{28}$$

Substituting Equations (28) into Equation (27) and using recursive method, the values of $U(k,h)$ will be obtained. Some results have been derived as follows:

$$U(0,1) = -\frac{\alpha\sqrt{2}\gamma^{\frac{5}{2}}\sigma}{8} - \frac{\beta\gamma^{\frac{3}{2}}\sqrt{2}}{4} + \frac{\beta\gamma^{\frac{5}{2}}\sqrt{2}}{8},$$

$$U(2,1) = \frac{9\alpha\sqrt{2}\gamma^{\frac{7}{2}}\sigma^3}{64} + \frac{3\beta\gamma^{\frac{5}{2}}\sqrt{2}\sigma^2}{32} - \frac{9\beta\gamma^{\frac{7}{2}}\sqrt{2}\sigma^2}{64},$$

$$U(1,2) = -\frac{5\gamma^{11/2}\alpha^2\sqrt{2}\sigma^3}{128} - \frac{\alpha\gamma^{9/2}\beta\sqrt{2}\sigma^2}{4} + \frac{3\alpha\gamma^{11/2}\beta\sqrt{2}\sigma^2}{32} - \frac{5\gamma^{7/2}\sigma\beta^2\sqrt{2}}{32} + \frac{\gamma^{9/2}\sigma\beta^2\sqrt{2}}{4} - \frac{7\gamma^{11/2}\beta^2\sigma\sqrt{2}}{128},$$

$\vdots$

Substitution all values of $U(k,h)$, into Equation (15), and some manipulations, doing the series form of the solutions of Equation (23), will be obtained as follows,

$$u(x,t)=\left\{\frac{\sqrt{2\gamma}}{2}+\frac{\sqrt{2}\gamma^{\frac{3}{2}}\sigma}{4}x+\cdots\right\}+\left\{\left(\left(-\frac{\alpha\sqrt{2}\gamma^{\frac{3}{2}}\sigma}{8}-\frac{\beta\gamma^{\frac{3}{2}}\sqrt{2}}{4}+\frac{\beta\gamma^{\frac{3}{2}}\sqrt{2}}{8}\right)+\left(-\frac{\alpha\sqrt{2}\gamma^{\frac{7}{2}}\sigma^{2}}{16}+\frac{\beta\gamma^{\frac{5}{2}}\sqrt{2}\sigma}{8}+\frac{\beta\gamma^{\frac{7}{2}}\sqrt{2}\sigma}{16}\right)x+\cdots\right\}t+\cdots$$

Tables 5, 6 and 7, shows the absolute errors for Equation (23), by using differential transform (DT), Adomian decomposition (AD) [Hashim, Noorani and Al-Hadidi (2006)], and Variational iteration (VI) [Batiha, Noorani and Hashim (2008)], methods. Numerical results in Tables 5, 6 and 7, show that the DTM, with less and easer computations, is effective and convenient for Burgers-Huxley equation for $\delta = 2$.

Table 5. Absolute errors for $\gamma = 0.001$, $\alpha = \beta = 1$, using 3-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.1	1.74857E-06	1.74992E-06	1.74982E-06
	0.2	3.49911E-06	3.49983E-06	3.49962E-06
	0.3	5.24970E-06	5.24971E-06	5.24940E-06
	0.4	7.00033E-06	6.99958E-06	6.99916E-06
	0.5	8.75099E-06	8.74942E-06	8.74890E-06
0.3	0.1	1.74459E-06	1.74983E-06	1.74971E-06
	0.2	3.49506E-06	3.49967E-06	3.49943E-06
	0.3	5.24557E-06	5.24948E-06	5.24915E-06
	0.4	6.99612E-06	6.99927E-06	6.99881E-06
	0.5	8.74670E-06	8.74903E-06	8.74848E-06
0.5	0.1	1.74062E-06	1.74976E-06	1.74954E-06
	0.2	3.49101E-06	3.49953E-06	3.49918E-06
	0.3	5.24145E-06	5.24925E-06	5.24881E-06
	0.4	6.99193E-06	6.99897E-06	6.99843E-06
	0.5	8.74243E-06	8.74866E-06	8.74801E-06

Table 6: Absolute errors for $\gamma = \alpha = 0.1$, $\beta = 0.001$, using 3-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.1	1.2954E-06	1.2911E-06	1.2874E-06
	0.2	2.5864E-06	2.5821E-06	2.5748E-06
	0.3	3.8773E-06	3.8732E-06	3.8621E-06
	0.4	5.1683E-06	5.1641E-06	5.1496E-06
	0.5	6.4593E-06	6.4552E-06	6.4370E-06
0.3	0.1	1.3043E-06	1.2909E-06	1.2858E-06
	0.2	2.5951E-06	2.5820E-06	2.5731E-06
	0.3	3.8860E-06	3.8728E-06	3.8603E-06
	0.4	5.1769E-06	5.1638E-06	5.1477E-06
	0.5	6.4677E-06	6.4546E-06	6.4349E-06
0.5	0.1	1.3131E-06	1.2908E-06	1.2827E-06
	0.2	2.6037E-06	2.5817E-06	2.5700E-06
	0.3	3.8945E-06	3.8724E-06	3.8570E-06
	0.4	5.1853E-06	5.1633E-06	5.1444E-06
	0.5	6.4760E-06	6.4541E-06	6.4315E-06

Table 7. Absolute errors for $\alpha = 0.01$, $\beta = \gamma = 0.0001$ using 3-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.1	6E-12	6E-12	6E-12
	0.2	1.2E-11	1.2E-11	1.3E-11
	0.3	1.8E-11	1.8E-11	1.8E-11
	0.4	2.2E-11	2.2E-11	2.3E-11
	0.5	2.8E-11	2.8E-11	2.8E-11
0.3	0.1	6E-12	6E-12	6E-12
	0.2	1.1E-11	1.1E-11	1.2E-11
	0.3	1.8E-11	1.8E-11	1.8E-11
	0.4	2.3E-11	2.3E-11	2.4E-11
	0.5	2.9E-11	2.9E-11	2.9E-11
0.5	0.1	6E-12	5E-12	6E-12
	0.2	1.1E-11	1E-11	1.2E-11
	0.3	1.7E-11	1.6E-11	1.7E-11
	0.4	2.3E-11	2.2E-11	2.4E-11
	0.5	2.9E-11	2.8E-11	2.9E-11

4.2. Fitzhugh-Nagoma Equation

The Fitzhugh-Nagoma equation has been solved by the differential transform method.
By using the related definitions in Table 1, two-dimensional differential transform Equation (5), will be obtained as follows,

$$(h+1)U(k,h+1) - (k+1)(k+2)U(k+2,h) = (\beta+\beta\gamma)\sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)U(k-r,s) - \beta\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)U(t,s)U(k-r-t,p) - \beta\gamma U(k,h) \quad (29)$$

By equating the coefficients of the terms with identical powers of x, in Equations (6, 21) the following results is obtained.

$$U(k,0) = 0,\ k = 2,4,6,\dots\quad, U(0,0) = \frac{\gamma}{2}, U(1,0) = \frac{\gamma^2\sigma}{2}, U(3,0) = -\frac{\gamma^4\sigma^3}{6},\quad \dots \quad (30)$$

By substituting Equations (30) into Equation (29), and by recursive method, all values of $U(k,h)$ will be obtained.

By substituting all values of $U(k,h)$, into Equation (15), and some manipulations, the series form solutions of Equation (5), with initial condition (6), will be obtained as follows,

$$u(x,t) = \left\{\frac{\gamma}{2} + \frac{\gamma^2\sqrt{2\beta}}{8}x + \cdots\right\} + \left\{\left(-\frac{\beta\gamma^2}{4} + \frac{\beta\gamma^3}{8}\right) + \cdots\right\}t + \cdots .$$

Tables 8, and 9, show the absolute errors for Equation (5), by the methods; differential transform (DT), Adomian decomposition (AD) [Hashim, Noorani and Al-Hadidi (2006)], and Variational iteration (VI) [Batiha, Noorani and Hashim (2008)], methods. Numerical results in Tables 8, and 9, reveal that DTM, with less and easier computations, has the same results, as ADM and VIM, for Fitzhugh-Nagoma equation.

Table 8. Absolute errors for $\gamma = 0.0001$, $\beta = 0.001$, by using 5-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.05	2.5E-13	2.5E-13	2.5E-13
	0.1	5E-13	5E-13	5E-13
	1	4.99E-12	5E-12	5E-12
0.5	0.05	2E-13	2.5E-13	2.5E-13
	0.1	4.5E-13	5E-13	5E-13
	1	4.95E-12	5E-12	5E-12
0.9	0.05	2E-13	2.5E-13	2.5E-13
	0.1	4.1E-13	5E-13	5E-13
	1	4.91E-12	5E-12	5E-12

Table 9. Absolute errors for $\gamma = 0.01$, $\beta = 1$, by using 5-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.05	2.485267E-06	2.487491E-06	2.487500E-06
	0.1	4.972766E-06	4.974998E-06	4.974999E-06
	1	4.974350E-05	4.974975E-05	4.974975E-05
0.5	0.05	2.476327E-06	2.487500E-06	2.487486E-06
	0.1	4.963818E-06	4.974981E-06	4.974980E-06
	1	4.973827E-05	4.974942E-05	4.974949E-05
0.9	0.05	2.467377E-06	2.487474E-06	2.487432E-06
	0.1	4.954852E-06	4.974946E-06	4.974917E-06
	1	4.972899E-05	4.978897E-05	4.974910E-05

4.3. Huxley equation

The Differential transform method is used to solve the Huxley equation for $\delta = 2$.
Considering By definitions in Table 1, two-dimensional differential transform Equation (8), for $\delta = 2$, will be obtained as follows,

$$(h+1)U(k,h+1) - (k+1)(k+2)U(k+2,h) = (\beta + \beta\gamma)\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)U(t,s)U(k-r-t,p) - \beta\gamma U(k,h) - \beta\sum_{r=0}^{k}\sum_{s=0}^{h}\{(\sum_{t=0}^{r}\sum_{p=0}^{h-s} U(t,h-s-p)U(r-t,p)) \times (\sum_{l=0}^{k-r}\sum_{j=0}^{k-r-l}\sum_{e=0}^{s}\sum_{f=0}^{s-e} U(l,s-e-f)U(j,e)U(k-r-l-j,f))\}. \tag{31}$$

By equating the coefficients of the terms with identical powers of x, in Equations (9) and (21) ,the following results are obtained.

$$U(0,0)=\frac{\sqrt{2\gamma}}{2},\ U(1,0)=\frac{\sqrt{2}\gamma^{3/2}\sigma}{4},\ U(2,0)=-\frac{\sqrt{2}\gamma^{5/2}\sigma^{2}}{16},\ \ldots \tag{32}$$

Substituting Equations (32) into Equation (31) and using recursive method, all the values of $U(k,h)$ will be obtained. Some results have been derived as follows:

$$U(0,1)=-\frac{\beta\gamma^{\frac{3}{2}}\sqrt{2}}{4}+\frac{\beta\gamma^{\frac{5}{2}}\sqrt{2}}{8},$$

$$U(2,1)=\frac{5\beta\gamma^{\frac{7}{2}}\sqrt{2}\sigma^{2}}{32}-\frac{9\beta\gamma^{\frac{9}{2}}\sqrt{2}\sigma^{2}}{64},$$

$$U(1,2)=-\frac{5\gamma^{7/2}\sigma\beta^{2}\sqrt{2}}{32}+\frac{\gamma^{9/2}\sigma\beta^{2}\sqrt{2}}{4}-\frac{7\gamma^{11/2}\beta^{2}\sigma\sqrt{2}}{128},$$

$$\vdots$$

By substituting all values of $U(k,h)$, into Equation (15), and some manipulations, the series form solutions of Equation (8) with initial condition (9) will be obtained as follows,

$$u(x,t)=\left\{\frac{\sqrt{2\gamma}}{2}+\frac{\sqrt{2}\gamma^{\frac{3}{2}}\sigma}{4}x+\cdots\right\}+\left\{\left(-\frac{\beta\gamma^{\frac{3}{2}}\sqrt{2}}{4}+\frac{\beta\gamma^{\frac{5}{2}}\sqrt{2}}{8}\right)+\left(\frac{\beta\gamma^{\frac{5}{2}}\sqrt{2}\sigma}{8}+\frac{\beta\gamma^{\frac{7}{2}}\sqrt{2}\sigma}{16}\right)x+\cdots\right\}t+\cdots.$$

Tables 10, and 11, show the absolute errors for Equation (8), by the methods; differential transform (DT), Adomian decomposition (AD) [Hashim et al. (2006)], and Variational iteration (VI) [Batiha et al. (2008)], methods.

Table 10. Absolute errors for $\gamma=0.0001,\ \beta=0.001$, using 3-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
0.1	0.1	7E-11	7.1E-11	7.3E-11
	0.2	1.4E-10	1.42E-10	1.44E-10
	0.3	2.11E-10	2.13E-10	2.15E-10
	0.4	2.81E-10	2.83E-10	2.86E-10
	0.5	3.52E-10	3.54E-10	3.56E-10
0.3	0.1	6.9E-11	7.0E-11	7.3E-11
	0.2	1.37E-10	1.41E-10	1.44E-10
	0.3	2.08E-10	2.12E-10	2.15E-10
	0.4	2.78E-10	2.82E-10	2.85E-10
	0.5	3.49E-10	3.53E-10	3.56E-10
0.5	0.1	6.2E-11	7.0E-11	7.2E-11
	0.2	1.34E-10	1.42E-10	1.43E-10
	0.3	2.04E-10	2.12E-10	2.14E-10
	0.4	2.75E-10	2.83E-10	2.85E-10
	0.5	3.46E-10	3.54E-10	3.56E-10

Table 11. Absolute errors for $\gamma = 0.01$, $\beta = 1$, using 3-terms ADM, one iteration of VIM and one iteration of DTM.

x	t	ADM	VIM	DTM
	0.1	7.04318E-05	1.0670E-06	1.0671E-06
	0.2	1.40886E-04	2.1342E-06	2.1341E-06
0.1	0.3	2.11341E-04	3.2011E-06	3.2010E-06
	0.4	2.81795E-04	4.2682E-06	4.2680E-06
	0.5	3.52249E-04	5.3352E-06	5.3350E-06
	0.1	7.03450E-05	1.0670E-06	1.0672E-06
	0.2	1.40759E-04	2.1341E-06	2.1341E-06
0.3	0.3	2.11172E-04	3.2010E-06	3.2010E-06
	0.4	2.81586E-04	4.2682E-06	4.2681E-06
	0.5	3.51999E-04	5.3352E-06	5.3349E-06
	0.1	7.02583E-05	1.0670E-06	1.0671E-06
	0.2	1.40631E-04	2.1341E-06	2.1340E-06
0.5	0.3	2.11004E-04	3.2010E-06	3.2008E-06
	0.4	2.81376E-04	4.2681E-06	4.2678E-06
	0.5	3.51748E-04	5.3351E-06	5.3348E-06

5. Conclusion

The DTM has been successfully applied to find an approximate solution of the generalized Burgers–Huxley equation and some special cases of the equation. The results reveal that the DTM is more effective and convenient compared to the ADM and VIM. Another advantage of the DTM is that it does not require that many computations that as the other two methods. Computations of this paper have been done by Maple 10.

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