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The Combined Effect of Chemical reaction, Radiation, MHD on Mixed Convection Heat and Mass Transfer Along a Vertical Moving Surface

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Abstract

This paper discusses the effect of Chemical reaction, Radiation and MHD on laminar mixed convection boundary layer flow and heat and mass transfer on continuously moving vertical surface. The fluid viscosity is assumed to vary as an inverse linear function of temperature and local similarity solutions are obtained for the boundary layer equations subject to isothermally moving vertical surface with uniform speed. The system of non-linear partial differential equations developed in the process is finally transformed into a set of ordinary differential equations with the help of similarity transformations involved in the problem. This set of equations is for different values of the various parameters. The results showing the effect of physical parameters on velocity, temperature and concentration have been computed and presented graphically to discuss their in details.

Keywords: MHD mixed convection; vertical moving surface; chemical reaction; radiation; Runge-Kutta method shooting technique

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1. Introduction

Continuously moving surface through an otherwise quiescent medium has many applications in manufacturing processes, such as wire drawing, metal extrusion, and paper production [Altan et al. (1979) and Tadmor et al. (1970)]. The pioneering work in this area was carried out by

Sakiadis (1961) who developed a numerical solution for the boundary layer flow field of a stretched surface. Many authors have reviewed this problem to study the hydrodynamic and thermal boundary layer due to moving surface [Magyari (1999), Keller (1999) and Tsou et al. (1967)]. Suction or injection of a uniform surface was introduced by Fox et al. (1968) and Erickson et al. (1966) for stretched surface velocity and temperature and by Gupta and Gupta (1977) for linearly moving surface. Magyari et al. (1999) have reported analytical and computational solution when the surface moves with rapidly decreasing velocities using the self-similar method.

Many researchers considered the effect of constant viscosity on boundary layers developed by continuously moving surface. The chemical equation is applicable to transformations of elementary particles as well as nuclear reaction. Numerous applications of chemical reaction are experiment in chemical engineering, in polymer production and manufacturing of ceramics etc. The importance of thermal radiation becomes intensified at high absolute-temperature levels due to basic difference between radiation and the convection and conduction energy-exchange mechanisms, some devices for space applications are designed to operate at high temperature levels in order to achieve high thermal efficiency. Hence, radiation must often be considered when calculating thermal effects in devices such as a rocket nozzle, a nuclear power plant, or a gaseous - core nuclear rocket.

It is known that the fluid viscosity changes with temperature [Herwig et al. (1986)]. Recently, in Ali et al. (2006), the effect of variable viscosity on a mixed convection heat transfer along a vertical moving surface was studied. Now in Present investigation we have seen the effects of MHD, chemical reaction and radiation and to get precious information about the flow, temperature and concentration.

2. Mathematical Analysis

Let us consider a steady two dimensional laminar flow due to vertically moving isothermal surface. Using boussinesq approximation for incompressible viscous fluid, the fluid viscosity is assumed to vary as an inverse linear function of temperature.

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma\left(\mathrm{T} - \mathrm{T}_\infty\right)\right], \text{ or } \frac{1}{\mu} = a\left(\mathrm{T} - \mathrm{T}_\infty\right)$$

where $a = \frac{\gamma}{\mu_\infty}$ and $\mathrm{T}_r = \mathrm{T}_\infty - \frac{1}{\gamma}$

The equations governing for convective variable viscosity fluid flow are

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

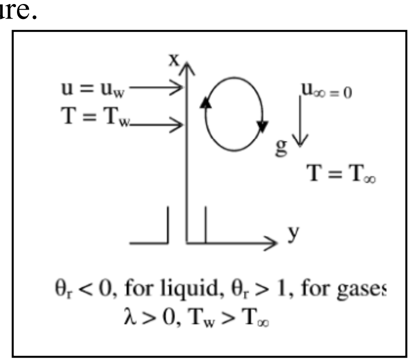

Figure 1. The surface moving upwards in the *X*-direction

Equation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=Sg\beta(T-T_\infty)+Sg\beta^*(C-C_\infty)+\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 u}{\rho_\infty} \tag{2}$$

Equation of energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho_\infty C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Equation of diffusion:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1\left(C-C_\infty\right), \tag{4}$$

subject to the following boundary conditions:

$$\begin{array}{llllll} u=U_w & v=0 & & \text{at} & y=0 \\ T=T_w & C=C_w & & \text{at} & y=0 \\ u\to 0 & T\to T_\infty & C\to C_\infty & \text{as} & y\to\infty. \end{array} \tag{5}$$

The x-coordinate is measured along the moving surface from the point where the surface originates and y-coordinate is measured normal to it (Figure1), where u and v are the velocity components in x-and y-directions respectively. S is a dummy parameter stands for 0, +1, -1. The stream function and following transformation have been used.

$$\psi=\sqrt{2}\nu_\infty \mathrm{Re}_x^{\frac{1}{2}} f(\eta) \qquad \eta=\frac{y}{x\sqrt{2}}\mathrm{Re}_x^{\frac{1}{2}}$$

$$\theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)} \qquad \phi=\frac{(C-C_\infty)}{(C_w-C_\infty)}$$

The radioactive heat flux q_r under Rosseland approximation by Brewster (1992) has the form:

$$q_r=-\frac{4\sigma\partial T^4}{3\chi\partial y},$$

where σ is Stefan-Boltzmann constant and χ is the mean absorption coefficient. We assume that the temperature differences within the flow are so small that T^4 can be expressed as a linear function of T_∞. This is obtained by expending T^4 in Taylor series about T_∞ and neglecting the higher order terms. Thus we get:

$$T^4 = 4T_\infty^3 T' - 3T_\infty^4 .$$

Here, u and v is the velocity components along in the x-and y-direction.

$$u = \frac{\nu_\infty}{x}\operatorname{Re}_x f'(\eta), \qquad \operatorname{Re}_x = \frac{U_w x}{\nu_\infty}, \qquad v = \frac{\nu_\infty \operatorname{Re}_x^{\frac{1}{2}}}{x\sqrt{2}}\left(f'\eta - f\right), \tag{6}$$

where f' and θ are the dimensionless velocity and temperature respectively, η is similarity variable. Putting all the values in equations (2)- (4), we get

$$f''' - \frac{(\theta-\theta_r)}{\theta_r} ff'' - \frac{f''\theta'}{(\theta-\theta_r)} - 2\lambda\frac{(\theta-\theta_r)}{\theta_r}\theta - 2\delta\frac{(\theta-\theta_r)}{\theta_r} - 2Mf' = 0 \tag{7}$$

$$\theta''(1+R) + \Pr f\theta' = 0 \ , \tag{8}$$

$$\phi'' + Sc\left(f\phi' + 2L\phi\right) = 0 \ . \tag{9}$$

The transformed boundary conditions are given by:

$$f'(0) = 1 \ , \qquad f(0) = 0 \ , \qquad \theta(0) = 1 \ , \qquad \varphi(0) = 1,$$

$$f'(\infty) \to 0, \qquad \theta(\infty) \to 0, \qquad \varphi(\infty) \to 0, \tag{10}$$

where θ_r is constant viscosity/temperature parameter defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\gamma\left(T_w - T_\infty\right)}.$$

Introducing the following non-dimensional parameters:

$$\lambda = \frac{SG_r}{\operatorname{Re}_x^{\ 2}} \ , \quad \delta = \frac{SG_c}{\operatorname{Re}_x^2} \ , \quad G_r = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2} \ , \quad G_c = \frac{g\beta\left(C_w - C_\infty\right)x^3}{\nu^2},$$

$$M = \frac{\sigma B_0^2 x}{\rho_\infty U_w} \ , \quad R = \frac{16\sigma T_\infty^3}{3k\chi} \ , \quad \Pr = \frac{\nu_\infty}{\alpha} \ , \quad Sc = \frac{\nu_\infty}{D} \ , \quad L = \frac{xk_1}{U_w},$$

where λ is buoyancy parameter, M is the magnetic parameter, Sc is the Schmidt number, Pr is the prandtle number, L is the chemical reaction parameter, R is the radiation parameter, G_r and G_c are the Grashof number for heat and mass transfer.

Keeping in view of engineering aspects, the most important characteristics of the flow are rate of skin-friction and heat transfer coefficient, which can be written as

$$C_f\sqrt{\mathrm{Re}_x} = \frac{\sqrt{2}\theta_r}{(\theta_r - 1)} f''(0,\theta_r), \qquad \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\frac{1}{\sqrt{2}}\theta'(0,\theta_r).$$

3. Results and Discussion

The couple nonlinear ordinary differential equation (7) to (9) are solved numerically by using the fourth order Runge-Kutta method. Local similarity solutions of the differential equation (7) to (9) subject to the boundary condition (10) were obtained for increasing values of δ at each constant θ_r. At each new θ_r we start from the known solution of the equations with $\lambda = 1$, where $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are known.

For a given value of δ the values of $f''(0), \theta'(0)$ and $\phi'(0)$ were estimated and the differential equations (7) to (9) were integrated using Runge-Kutta method until the boundary condition at infinity f', θ and ϕ decay exponentially to zero ($\leq 10^{-4}$ where the solution to be accepted and solution with f', θ and ϕ > 10^{-4} will not be considered). If the boundary condition are not satisfied then the numerical routine uses a half interval method to calculate correction to the estimated values of $f''(0), \theta'(0)$ and $\phi'(0)$. The process is repeated iteratively until exponentially decaying solution in f', θ and ϕ is obtained.

The local solutions were obtained for different values of $-10 \leq \theta_r \leq 10$ by Ali et al. (2006). The values of η_∞ was chosen as large as possible between 3.5 and 25 depending upon the Prandtl number and the viscosity / temperature parameter without causing numerical oscillations in the values f', θ and ϕ. The effects of these parameters on the velocity, temperature and concentration profiles have been analyzed with the help of graphical representation through Figures 1-16. Figures (1)-(4) shows that the velocity increases with an increase in the value of viscosity/temperature parameter, chemical reaction parameter, radiation parameter, and buoyancy parameter.

While velocity decreases with an increase in the values of magnetic parameter, Prandtl number, Schmidt number in Figures (5)-(7). It is clear from figures (8) and (9) temperature increases with an increase in the value of radiation and magnetic parameter while figure (10) shows that temperature decreases with increase in the value of buoyancy parameter. And concentration profile increases with increase in the value of chemical reaction parameter and magnetic parameter while concentration profile decreases with increase in the value of Schmidt number and buoyancy parameter. It is worth mentioning that small values of Pr ($\ll 1$) physically correspond to liquid metals, which have high thermal conductivity but low viscosity, while Pr ~

1 corresponds to di-atomic gases including air. On the other hand, large values of Pr (>> 1) correspond to high-viscosity oils. Table 1 describes the effects of numerical values of various parameters in skin friction coefficient and local Nusselt number.

Table 1: Skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$

Physical parameters	Values	$f''(0)$	$-\theta'(0)$
θ_r	2	-0.2199235	0.47501189
	4	0.0879965	0.4826875
	10	0.2855465	0.4864125
L	0	-0.2578636	0.47181189
	0.2	-0.2199235	0.47501189
Pr	0.71	-0.2199235	0.47501189
	7	-0.2299201	1.41100222
δ	0.5	-0.4869535	0.46185222
	1	-0.2199235	0.47501189
M	1	-0.2199235	0.47501189
	2	-0.9926025	0.44391922

4. Conclusion

The authors have theoretically studied how the governing parameters viscosity/temperature parameter, chemical reaction parameter, radiation parameter, buoyancy parameter and Prandtl number influence the boundary layer flow and heat transfer characteristics on the moving surface. The investigation of the effects of viscosity/temperature parameter and Prandtl number on the skin friction coefficient and on local Nusselt number reveals that both the skin friction coefficient and the Nusselt number increase as the viscosity/temperature parameter and the Prandtl number increase. The magnetic field also increases the skin friction, but reduces the heat transfer

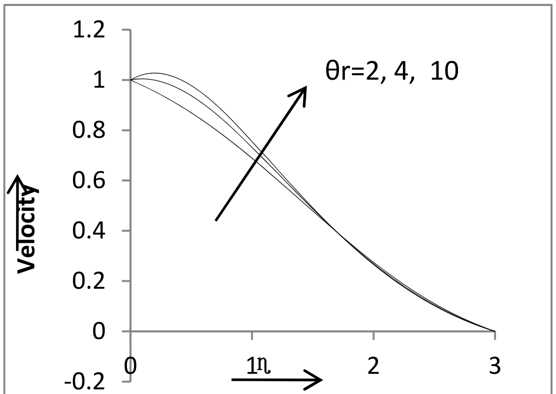

Figure 1. **Velocity profile for various value of θr at M=1, Pr=0.71, R=1,λ=1,L=0.2,Sc=0.5 δ=1**

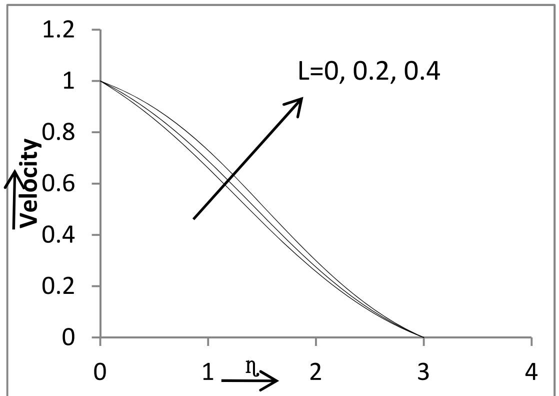

Figure 2. Velocity profile for various value of L at M=1, Pr=0.71, R=1,λ=1,θr=22,Sc=0.5 δ=1

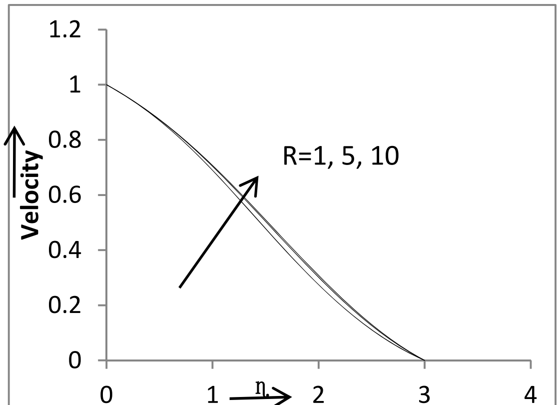

Figure 3. Velocity profile for various value of R at M=1, Pr=0.71, L=0.2,λ=1,θr=2,Sc=0.5 δ=1

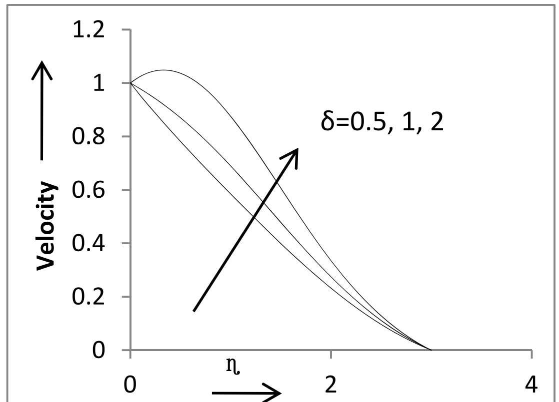

Figure 4. . Velocity profile for various value of R at M=1, Pr=0.71, L=0.2,λ=1,θr=2,Sc=0.5 δ=1

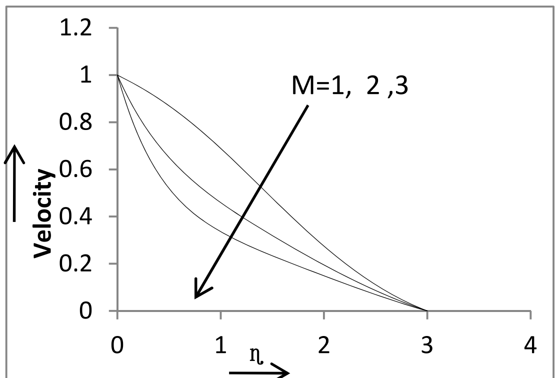

Figure 5. Velocity profile for various value of R at M=1, Pr=0.71, L=0.2,λ=1,θr=2,Sc=0.5 δ=1

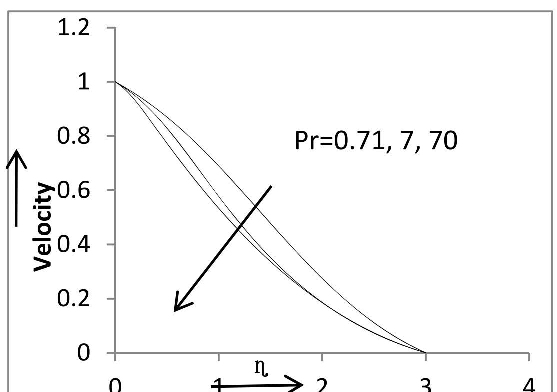

Figure 6. Velocity profile for various value of Pr at M=1, R=1, L=0.2,λ=1,θr=2,Sc=0.5 δ=1

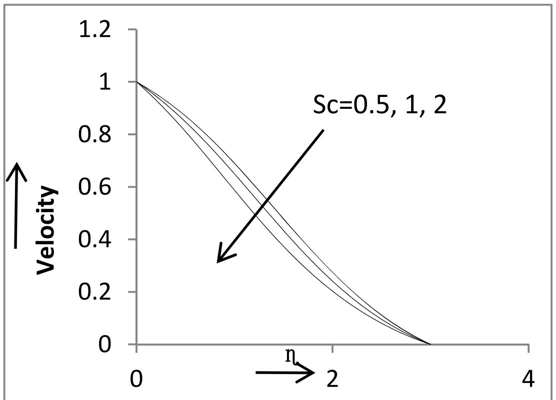

Figure 7. Velocity profile for various value of Sc at M=1, Pr=0.71, L=0.2, λ=1, θr=2,R=1 δ=1, Sc=0.5

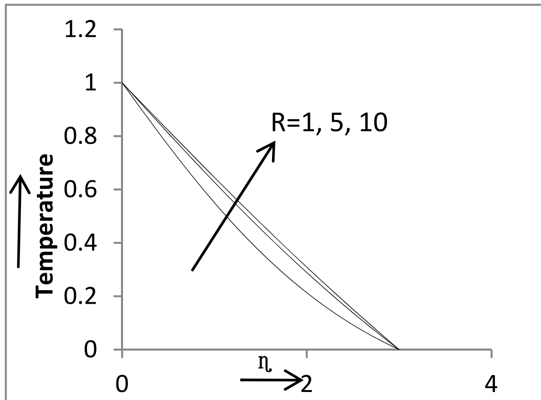

Figure 8.Temperature profile for various value of R at M=1, Pr=0.71, L=0.2, λ=1, θr=2, δ=1, Sc=0.5

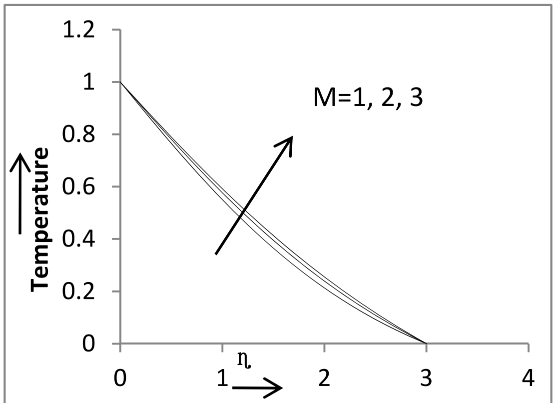

Figure 9. Temperature profile for various value of M at R=1, Pr=0.71, L=0.2, λ=1, θr=2, δ=1, Sc=0.5

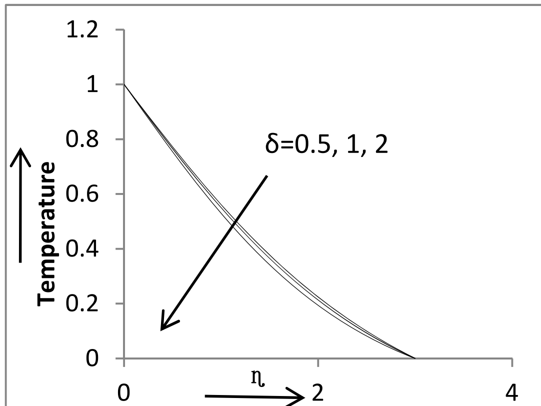

Figure 10. Temperature profile for various value of δ at R=1, Pr=0.71, L=0.2, λ=1, θr=2, M=1, Sc=0.5

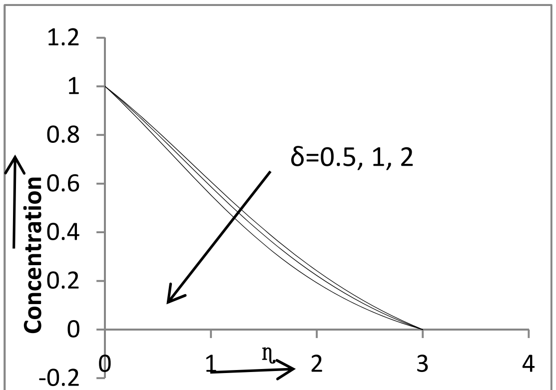

Figure 11. Concentration profile for various value of δ at R=1, Pr=0.71, L=0.2, λ=1, θr=2, δ=1, Sc=0.5

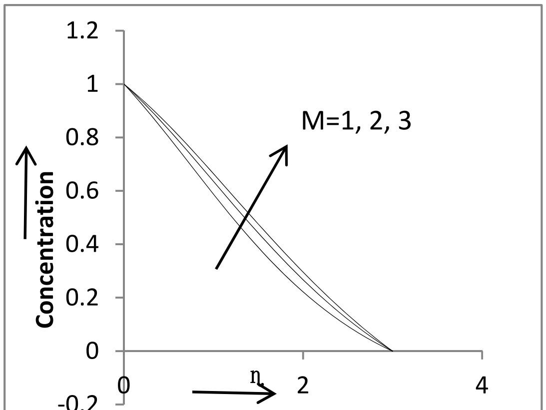

Figure 12. Concentration profile for various value of M at R=1, Pr=0.71, L=0.2, λ=1, θr=2, δ=1, Sc=0.5

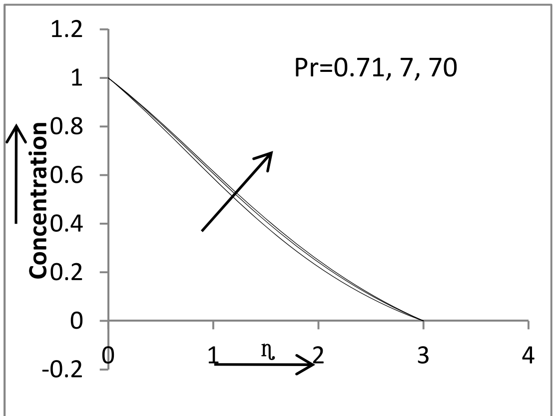

Figure 13. Concentration profile for various value of Pr at R=1, M=1, L=0.2, λ=1, θr=2, δ=1, Sc=0.5

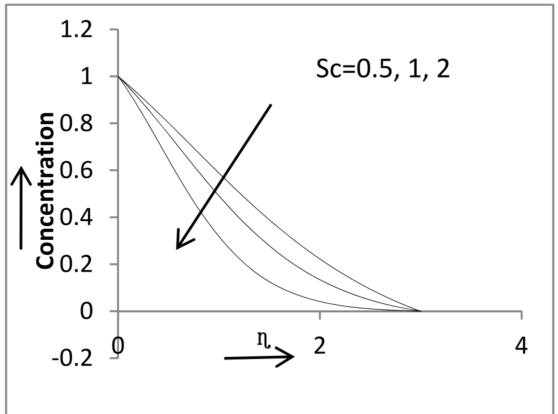

Figure 14. Concentration profile for various value of Pr at R=1, M=1, L=0.2, λ=1, θr=2, δ=1

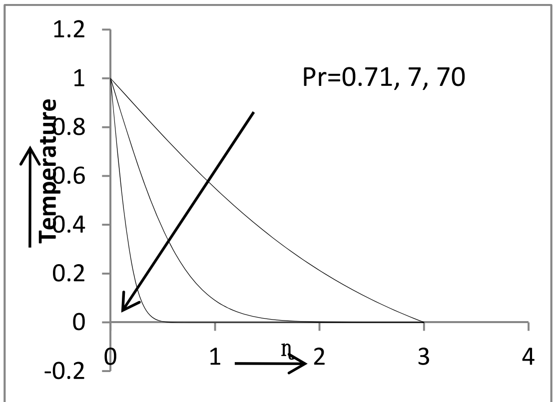

Figure 15. Temperature profile for various value of Pr at R=1, M=1, L=0.2, λ=1, θr=2, δ=1 Sc=0.5

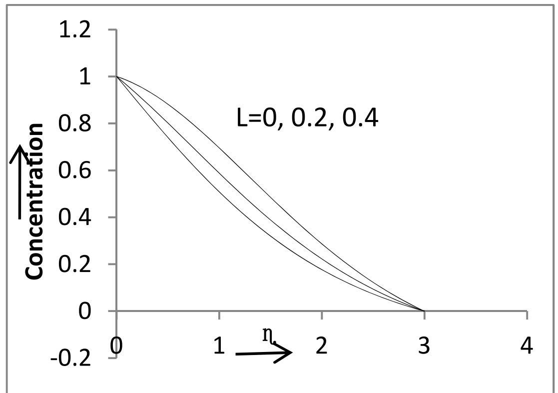

Figure 16. Concentration profile for various value of L at R=1, M=1, Sc=0.5, λ=1, θr=2, δ=1

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