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Integrated Farm Model for Optimal Allocation of Resources- A Linear Programming Approach

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Abstract

The mathematical model for optimal allocation of farm resources, especially land and water are proposed to optimize the resources that contribute to increase farm revenues. A study is being carried out, to analyze the cropping practice adopted by growers, depending on availability and accessibility of resources. Different crop-combinations and cropping patterns are being analyzed in districts of Rajasthan. Rajasthan has arid topography with varying weather conditions. Thus, a diverse crop variety is being cultivated in a region. Being a state with inadequate water resources, the formulated model proposed different crop combinations alternatives. A crop-mix model is developed to reduce the input cost and maximize farm revenues. The Multi-Objective Linear Programming approach is applied to determine the feasibility of decision variables. The model solution lies within the feasible region; hence, it must be in the convex hull of previous year planting decisions. Consequently, the model finds an optimal crop combination to optimize the objective function under prevailing climatic conditions. To avoid the uncertainties involved in sector some of the farm parameters such as climatic and market fluctuations were kept constant. The model solution is achieved by imposing constraints, that limit the decision variables within the feasible range. LINGO 18.0 Software is used to determine the optimal values of the decision variables.

Keywords: Decision analysis; mathematical model; crop combination; crop-mix; convex hull, Multi-Objective Linear Programming; Rajasthan; farm model

MSC 2010: 90C05, 90C90

1. Introduction

Food security has become a major concern for both producers and consumers. Thus, a need for an integrated farm management system has emerged. Allocation of the crop, crop combinations, operational activities involved, for better productivity are some of the most significant decision variables that need to optimize. Thus, a study is carried out to analyze the cropping scenario in different districts of Rajasthan. By interviewing the farmers, it is observed that there are many crops that provide more economic benefits, but due to financial constraints and inadequate availability of water resources in a region, growers are not able to cultivate these crops.

Rajasthan being an arid and semi-arid region, often face water inadequacy situations. Most of the cultivation in a region is rain-fed. However, water inadequacy emerges as one of the major factors affecting the agriculture sector. Many regions across the globe were facing water crises due to population growth, urbanization and increased food production. Consequently, the need to allocate water resources efficiently emerged. Hence, for optimizing water resources, an optimal model is developed by Shangguan et al. (2002) for the allocation of irrigation resources under water-deficient conditions. The results indicate that farm benefits depend on market conditions and operational costs. Thus, it is necessary to consider the cropping pattern and allocation of resources according to availability, accessibility and prevailing market demand.

The availability of water resources is one of the most significant parameters that need to be focused to maximize the farm revenues, particularly in arid regions. In case, of limited availability of water supply, a proper irrigation schedule will lead to increased crop production. During the growing season, many crops were allocated simultaneously on the same piece of land. Rao et al. (1990) addressed the problem of allocating water resources for irrigating a field cultivated with multiple crops. Further, Li and Guo (2014) proposed a Multi-Objective optimal water resource allocation model. The model incorporates the associated uncertainties too. The results of the model support the existing irrigation pattern in a region and, evaluated the desired water allocation for multiple uncertainties.

Mathematical output or representation of farm constraints enables the policy-makers to determine the areas in which the changes will be carried out to reduce the losses and increase the profits. Thus, different mathematical techniques are used by Adekanmbi and Olugbara (2015) and Aparnathi (2014) for crop allocation depending on available resources.

Multi-Objective Linear Programming approach applied for analysis provides information about the fixed resources such as available arable land, crop combination. Further, to evaluate the feasibility of a region Sarker and Quaddus (2002) proposed a mathematical model by using the multi-objective approach. The formulated model provides an insight into how a multi-objective approach can be used for optimizing farm resources.

1.1. Cropping Scenario in Rajasthan

The regional agricultural model is based on the data collected from different districts of Rajasthan. The mathematical model will provide an ‘optimal’ land-use and crop combination allocation plan. Linear Programming approach is used to get an optimized result of crop production subject to the set of constraints. Hence, Linear Programming is considered as one of the appropriate tools to get an optimized result of a model for a farm scenario.

From the literature reviewed, crop allocation is another constraint that posed a challenge to get an optimize farm revenues. These decision parameters were further investigated by Dury et al. (2012) to determine the effect of short and long term decisions on crop productivity. Since ancient times different crop-pattern is being practiced by the growers to increase the farm yield sustainably. However, during farm survey in different districts of Rajasthan crop rotation is observed as one of the most commonly practiced crop patterns adopted by growers to improve the crop yield. The practice improves the crop yield to about 5%.

To determine the effect of adopting crop rotation on farm revenues a mathematical model is formulated by Castellazzi et al. (2008) . A systematic and concise mathematical representation of crop rotations was proposed. Crop sequence considered in each season is quantified as a transition matrix, expresses that the crop allocated in a season depends on previous year allocation. Such matrices facilitate a mathematical model uncertainty and provide an insight into the long-term profit to be produced by cyclic rotations. Such cropping pattern makes the use of available resources effectively and thus contributes to cost reduction.

In farm planning, most crucial decision is the selection of crops in each season. These are the major decisions that are needed to be optimized in each growing season. Rădulescu (2012) develop a crop allocation model by using a decision making tool under risk. The model considers both climatic and market risks.

Small farmers in Rajasthan often practice a mix-farming pattern with the aim to utilize the available resources optimally. By interviewing the farmers in a region, it is analyzed that such a farm scenario provides a model of integrated farm scenarios. Filippi et al. (2017) proposed a mathematical model for crop-mix to optimize land resources. The complexity of a sector is due to several varying decision variables such as pest attack, climatic variations, soil moisture content and so on. Thus, two-stage mathematical model is proposed. To validate the proposed model with the decisions made by farmers' historical farm data is compared with the optimal returns of the farm. Along with the crop-mix, the concept of mixed-cropping patterns became significant in the sector. (Mohamed et al., 2016; Ruhul A. Sarker et al., 1997) developed a mathematical model to determine how the spatial crop distribution affects the crop productivity in crop-mix scenario

As a result of scarce availability and uneven distribution of resources, the concept of a mixed cropping pattern is being practiced in some regions of Rajasthan. Small farmers (owning 1-7 bighas) of land adopted mix-cropping pattern. Such an arrangement of crops optimizes land and other resources to their full extent and provides better outputs. Mixed cropping results in more crop yield, the stability of production, reduces the pest attack, improves soil content and reduces the risk of crop failure. Thus, to evaluate the benefits of mixed cropping system Jolayemi and Olaomi, (1995) develop a mathematical formulation for efficient integration and management of all the farm components.

In mixed cropping patterns, one of the most crucial decisions is to consider the crops that would compete with each other in terms of resources. During the survey, it is observed that the farmers generally support their mixed cropping farm scenario with livestock. Thus, this results in an integrated farm scenario. The farm considered is cultivated with grams, wheat, lemon, chilies, gourd, okra, tomatoes and coriander. Along with crop allocation farmer allocates some part of his farm for livestock (goat, cows, buffaloes and poultry) too. Due to livestock keeping the cost of fertilization and manuring almost slashes down to $\frac{3}{4}^{th}$ of the cost that is invested otherwise, in fertilization. Moreover, because of the use of natural fertilizer, the farm output also rises to about 2%. Based on an integrated farm system, a mathematical model has been formulated. In a mixed cropping pattern, it is not possible to allocate a defined area to each crop that is considered for cultivation. Large farmers generally, did not adopt this cropping pattern as they have sufficient land and other resources to allocate the crops.

In this paper, the seasonal allocation for a crop is developed to maximize the farm revenues from crop-water production by allocating land and water resources optimally to the crops cultivated using Multi-Objective Linear Programming. This is followed by the intra-seasonal crop allocation model to maximize the relative yield. Some of the stochastic parameters such as climate, soil properties, crop distribution and allocation of water resources were not taken under consideration.

Further, the section of the paper includes a formulation of a mathematical model for optimal crop-mix considering, eight crops combination simultaneously. The model depicts the farm practicing a mix-crop combinations pattern in the district of Rajasthan. The following section includes result analysis of the model using LINGO18.0, discussion and conclusion.

2. Mathematical Model

A mathematical model developed for mixed cropping pattern is as follows:

2.1. Notations

r_{ij} : expected profit per hectare from j^{th} crop in i^{th} season

e_{ij} : effect of j^{th} crop when planted as a mixed crop in i^{th} season

e_{kj} : effect of k^{th} crop when planted as a mixed crop with j^{th} crop

s_{ij} : the seeds procured for j^{th} crop in i^{th} season

x_{ij} : j^{th} crop cultivated in i^{th} season

P: the amount for the procurement of seedlings

p_{ij} : amount of fertilizers and pesticides required in the growth of j^{th} crop in i^{th} season

CY_{ij} : crop yield due to mixed farm practice

MP_{ij} : market price of j^{th} crop in i^{th} season

CP_{ij} : cost of planting (in Rs.) j^{th} crop in i^{th} season per hectare

CC_i : capital available for cultivating in i^{th} season per hectare

C_i : total capital available in i^{th} season

HC_{ij} : harvesting cost in i^{th} season

H: capital allotted for harvesting

LA : total available land for cultivation

TCA_{ij} : capital available for cultivation of j^{th} crop in i^{th} season

CR_{ij} : cultivated area under j^{th} crop in i^{th} season

THC_i : total harvesting cost in i^{th} season

TOC_i : total farm operating cost in i^{th} season

WR_i : water required in i^{th} season for irrigation

TWA_i : total quantity (liters) of water available for irrigation in i^{th} season

2.2. Objective Functions

$$\text{Max. } X = \sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij}\, x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{m-1} e_{kj}\, x_{ki}\, r_{ij}, \qquad \text{(Total Farm Revenues)}$$

$$\text{Min. } Y = \sum_{i=1}^{n}(THC_I + CC_i + TOC_i), \qquad \text{(Total Farm Cost)}$$

and

$$\text{Min. } Z = \sum_{i=1}^{n} WR_i . \qquad \text{(Water Requirement)}$$

e_{kj} = (0, if j^{th} crop has no effect on the crops with which it is cultivated in i^{th} season; positive, if crop production (j^{th} crop) indicates a growth in i^{th} season; negative, if crop production (j^{th} crop) indicates a reduction in i^{th} season)

$$e_{kj} \begin{cases} = 0; \text{ no effect} \\ > 0; \text{ positive effect} \\ < 0; \text{ negative effect} \end{cases} \qquad (1)$$

Constraints

$$\sum_{i=1}^{n}\sum_{j=1}^{m} s_{ij}\, x_{ij} \leq P, \qquad (2)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}\, x_{ij} \geq 0, \qquad (3)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} CP_{ij}\, x_{ij} \leq \sum_{i=1}^{n} CC_i, \qquad (4)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} HC_{ij}\, x_{ij} \leq H, \qquad (5)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} CR_{ij} \leq LA, \qquad (6)$$

$$P + H + \sum_{i=1}^{n} CC_i \leq \sum_{i=1}^{n}\sum_{j=1}^{m} TCA_{ij}, \qquad (7)$$

and

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{m-1}(x_{ij} * x_{ki})\, WR_i \leq \sum_{i=1}^{n} TWA_i . \qquad (8)$$

The best possible way to get an optimal crop mix combination for a mixed cropping pattern is to select the crops of the same group.

Table 1. Effect on Crop Productivity of one Crop over other (Mixed Cropping)

Crop k	**Crop j**				
	1	**2**	**3**	**4**	**5**
1	0	1	-1	1	1
2	1	0	-1	-1	-1
3	0	1	0	0	0
4	1	1	0	0	1
5	1	1	1	1	1
6	1	-1	-1	1	-1
7	1	-1	-1	1	1
8	1	-1	-1	1	-1

Note that 1, 0 and -1 indicate, respectively, as the following:

the j^{th} crop has increased the production by the crop with which it is cultivated,
the j^{th} crop does not impact the production by the crop with which it is cultivated, and
the j^{th} crop has decrease the production by the crop with which it is cultivated.

Table 2. Cost of Resources used for Cultivation

Resources	**Cost of Resources (in 000' Rs.)**
A	20
B	7
C	15
D	8
E	20
F	32
G	20
H	22

Table 3. Cost of Cultivation of each Crop per Hectare (000'Rs.)

Cost (in Rs.)	**Crop i**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
x_1	8	9	7	6	4	8	9	8
x_2	4	3	6	5	2	1	4	4
x_3	4	3	3	2	5	6	1	5
x_4	8	4	8	9	6	5	6	4
x_5	3	2	4	1	7	8	4	4
x_6	9	2	4	5	8	7	8	3
x_7	8	6	5	7	2	4	5	7
x_8	7	6	5	7	2	4	5	2

Table 1 shows the effect of one crop over the other when cultivated in a mix pattern. The values indicate the effect on production due to other crops when cultivated in mix-crop pattern. Table 2 and 3 indicates the per hectare cost involved for cultivation. Eight crops were considered for mix-crop patterns. The crop combinations vary in each season. Table 4 illustrates the proposed mathematical model. The constraints that limit the model include land availability cost of preparing field, machinery cost, labour, water, seeds, manures & fertilizers, cultivation cost and lastly harvesting cost.

Table 4. Optimized Mathematical Formulation

Objective Function	$\text{Max } Z = 52x_1 + 80x_2 + 55x_3 + 60x_4 + 75x_5 + 75x_6 + 50x_7 + 70x_8$			
Constraints	Decision Variables	Sign Constraint	Requirement Vector	Constraints
	x_1	$\leq$	6	Land Availability
	x_2	$\leq$	6	Land Availability
	x_3	$\leq$	6	Land Availability
	x_4	$\leq$	6	Land Availability
	x_5	$\leq$	6	Land Availability
	x_6	$\leq$	6	Land Availability
	x_7	$\leq$	6	Land Availability
	x_8	$\leq$	6	Land Availability
	$8x_1 + 9x_2 + 7x_3 + 6x_4 + 4x_5 + 8x_6 + 9x_7 + 8x_8$	$\leq$	20000	Machine Cost
	$4x_1 + 3x_2 + 6x_3 + 5x_4 + 2x_5 + x_6 + 4x_7 + 4x_8$	$\leq$	7000	Labour
	$4x_1 + 3x_2 + 3x_3 + 2x_4 + 5x_5 + 6x_6 + x_7 + 5x_8$	$\leq$	15000	Fertilizers
	$8x_1 + 4x_2 + 8x_3 + 9x_4 + 6x_5 + 5x_6 + 6x_7 + 4x_8$	$\leq$	8000	Field Preparation
	$3x_1 + 2x_2 + 4x_3 + 1x_4 + 7x_5 + 8x_6 + 4x_7 + 4x_8$	$\leq$	20000	Water Cost
	$9x_1 + 2x_2 + 4x_3 + 5x_4 + 8x_5 + 7x_6 + 8x_7 + 3x_8$	$\leq$	32000	Seeds
	$8x_1 + 6x_2 + 5x_3 + 7x_4 + 2x_5 + 4x_6 + 5x_7 + 7x_8$	$\leq$	20000	Harvesting Cost
	$7x_1 + 6x_2 + 5x_3 + 7x_4 + 2x_5 + 4x_6 + 5x_7 + 2x_8$	$\leq$	22000	Cultivation Cost

3. Result Analysis

Table 5. Expected Profit per Hectare

Crops	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Profit (in Rs.)	0	480	0	0	99024.99	450	0	420
Optimized Solution	100375.00							

Table 6. Optimized Value of Decision Variables

Variable	**Value**
x_1	0
x_2	6
x_3	0
x_4	0
x_5	1320.33
x_6	6
x_7	0
x_8	6

The mix-crop pattern depends on historical observations of farmers. The model results (Table 5) show that the solution of the proposed model lies within the feasible region; hence, it must be in the convex hull of previous year planting decisions. Consequently, the model finds an optimal crop combination of those decision variables (Table 6) that optimizes the objective function under prevailing market conditions. This is achieved by imposing the constraints that limit the model solutions within the feasible range. The optimum results of the model are obtained at extreme points (corner points).

In Linear Programming, the convex combination of two feasible solutions is also feasible. Thus, crop-mix combinations are considered as a corner solution in decision space and optimum results would be a convex set in the decision space. Due to the recursive nature of the algorithm, decision parameters can be altered depending on the feasibility of decision variables. An optimal feasible solution of decision variables can be further approximated by performing Sensitivity Analysis. Such an analysis makes it easier for the decision-makers to have an in-depth analysis of the parameters that are needed to be altered. Variations in the cost vectors provide an upper and lower bound of the parameters taken under consideration.

Table 7. Sensitivity Analysis

Variable Cells							
			Final	Reduced	Objective	Allowable	Allowable
	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
	B2	variables x1	0	-48	52	48	1E+30
	C2	variables x2	6	0	80	1E+30	30
	D2	variables x3	0	-45	55	45	1E+30
	E2	variables x4	0	-52.5	60	52.5	1E+30
	F2	variables x5	1320.333333	0	75	15	25
	G2	variables x6	6	0	75	1E+30	12.5
	H2	variables x7	0	-25	50	25	1E+30
	I2	variables x8	6	0	70	1E+30	20
Constraints							
			Final	Shadow	Constraint	Allowable	Allowable
	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
	L10	<=	6	0	6	1E+30	0
	L11	<=	6	12.5	6	0	6
	L12	<=	0	0	6	1E+30	6
	L13	<=	6	20	6	1616.75	6
	L14	<=	5431.333333	0	20000	1E+30	14568.66667
	L15	<=	2688.666667	0	7000	1E+30	4311.333333
	L16	<=	6685.666667	0	15000	1E+30	8314.333333
	L17	<=	8000	12.5	8000	9148.857143	7922
	L18	<=	9326.333333	0	20000	1E+30	10673.66667
	L19	<=	10634.66667	0	32000	1E+30	21365.33333
	L20	<=	2742.666667	0	20000	1E+30	17257.33333
	L21	<=	2712.666667	0	22000	1E+30	19287.33333
	L7	<=	0	0	6	1E+30	6
	L8	<=	6	30	6	1980.5	6
	L9	<=	0	0	6	1E+30	6

4. Discussion

With the increase in population and varying cropping pattern demand for optimal and systematic utilization of resources has been emerged. Thus, management and planning of farm allocation is a crucial decision for crop production. Both operational and managerial activities are significant in crop management. Nevo et. al, (1994) focusses that the production of crops in a region is mainly concerned with crop selection and area allocated under each crop. Due to the complex inter-relationships between the variables (climatic fluctuations, market conditions, agronomic conditions, resource availability) an optimal crop plan is difficult to achieve. To overcome this convolution, the mathematical model is proposed with the aim to observe the effect of a set of constraints on crop productivity. These include crop selection, production constraints and revenue estimation.

Jolayemi & Olaomi (1995) explain mixed-cropping as a system in which two or more crops were cultivated simultaneously on the same piece of land in a cropping period irrespective of the spatial arrangements or defined land allocated under each cultivated crop. Such cropping pattern results in higher productivity, better yield, reduces the risk of pests and, crop failure.

Thus, to obtain an optimized crop combination of a mixed-crop Linear Programming tool is used. To simulate the decision taken at the farm level mathematical model is widely used by at the regional level.

Thus, for the farmers owning a small piece of land mix-crop is one of the most appropriate crop patterns to optimize the farm output. By interviewing the farmers, it has been observed that in a region availability of water for irrigating field is one of the major constraints.

The farm model formulated cultivated eight crops in a mixed-crop pattern. By using Linear Programming feasibility of the decision variable is determined. All the decision variable lies within the feasibility region. The optimized value of some of the decision value is zero indicating that these variables do not contribute significantly in solutions optimality.
By performing a sensitivity analysis for a formulated model, lower and upper bounds (Table 7) of decision variables are identified. This range supports the decision-makers to alter the decisions to optimize the revenues.

Reduced cost associated with the non-zero optimal value (Table 7) of the decision variable explains the change in optimized value by changing the associated cost vector coefficient by one unit. Sensitivity Analysis of the tight constraint (non-binding) constraints with a zero-shadow price indicates that these crops were limited within the allocated region and cannot be elaborated further as these would not affect the solution optimality (Table 7). Allowable increase and decrease provide a range within which cost and activity vector can vary. The zero value of the decision variable is due to an undefined lower bound.

For predicting future crop production, based on previous season crop productivity simulation tools are gaining importance especially in the agriculture sector. These tools analyze the decision-making policy of the individual grower based on the availability of resources that is typically based on mathematical programming models. Berger (2001) presents a spatial multi-agent programming model. The Linear Programming tool is applied to determine the available alternatives to reduce input cost and increase farm revenue. The spatial framework considered, both economic and hydrologic aspects simultaneously. Results indicate that simulation is a powerful tool to provide a better understanding of agriculture scenarios.

5. Conclusion

A mathematical model for optimal allocation of land and water resources is proposed. Formulated Linear Programming model discusses the spatial crop distribution planning on a seasonal basis, assuming, some of the parameters to be static during the cultivation period. By interviewing the farmers, it had been analysed that there are some crops such as (tomatoes, cucumber) that are not cultivated by farmers due to high input cost and low shelf life. Due to dry topography, most of the field remains fallow in kharif season.

The model incorporates only a deterministic approach to evaluate solution feasibility. However, it will be further modified by including climatic, market and soil as some of the parameters to make the scenario stochastic. The recursive nature of the Linear Programming approach makes the decision-maker to alter the feasibility region by altering the coefficient of decision variables within a range of allowable increase and decrease. The algorithm provides a range within which the activity coefficients and requirement vector of the model can be altered to get an optimized result.

Moreover, the model approach can be further elaborated by including more constraints. The result obtains indicates that due to uneven distribution of crops in mixed crop scenario, some of the crops do not contribute significantly in optimal solution and thus, results out to be zero. Thus, crop distribution is one of the most crucial decisions that decides the optimality of the decision variables. Hence, there is a need to define a distribution function that will determine the crop yield.

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