Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 15, Issue 2 (December 2020), pp. 957 – 969

Impulse Effect on the Food-Limited Population Model with Piecewise Constant Argument

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Received: October 3, 2019; Accepted: June 9, 2020

Abstract

The qualitative study of mathematical models is an important area in applied mathematics. In this paper, a version of the food-limited population model with piecewise constant argument under impulse effect is investigated. Differential equations with piecewise constant arguments are related to difference equations. First, a representation for the solutions of the food-limited population model is stated in terms of the solutions of corresponding difference equation. Then using linearized oscillation theory for difference equations, a sufficient condition for the oscillation of the solutions about positive equilibrium point is obtained. Moreover, asymptotic behavior of the non-oscillatory solutions are investigated. Later, applying the same theory, non-impulsive model is also studied. Numerical examples are given to compare the results of impulsive model with the results of non-impulsive case. The results show that when the solutions of impulsive differential equation model are oscillatory about positive equilibrium under suitable conditions the solutions of the corresponding non-impulsive model is not oscillatory. This situation indicates to the importance of impulse effects on the asymptotic behavior of the solutions.

Keywords: Food-limited population model; Piecewise constant argument; Impulse; Difference equation; Linearized oscillation

MSC 2010 No.: 34K11, 39A60

1. Introduction

Mathematical modeling is an important tool to understand the dynamic behavior of biological systems. One of the food-limited population model was proposed by Smith (1963). It is considered food requirement of the populations in the growing stage as well as the permanence stage. Later, the delay differential equation model was investigated by Gopalsamy et al. (1988; 1990a). So-Yu (1995) studied stability properties of a general food-limited population model with delay. Then, Berezansky-Braverman (2003) studied oscillation of non-autonomous delay food limited population model. Moreover, oscillatory and stability properties of some generalized food-limited population models under impulse effect have been dealt with by Liu-Ge (2003), Tang-Chen (2004), and Wang-Yan (2007)). Recently, asymptotic behavior of a food-limited population model with Markovian switching and Levy jumps has been investigated by Liu et al. (2018).

Studies with mathematical models involving piecewise constant delays instead of continuous delays started with the work of Busenberg-Cooke (1982). They investigated dynamics of vertically transmitted diseases. Then, stability, oscillation and existence of periodic solutions of the linear differential equations were investigated by Cooke-Wiener (1984), Aftabizadeh-Wiener (1985), Aftabizadeh et al. (1987), Shen-Stavroulakis (2000), Bereketoglu et al. (2017) and the references cited therein. Moreover the book of Wiener (1994) is a well known source for whom would like to work on functional differential equations. One of the logistic model with piecewise constant arguments was investigated by Gopalsamy et al. (1990b) and a necessary and sufficient condition for the oscillation of the positive solutions was established. Recently, asymptotic behavior of the solutions of the population model with piecewise constant argument has been studied by Karakoç (2017). The existing results about some of the biological models can be found in the book of Agarwal et al. (2014).

Moreover, some physical and engineering systems with piecewise constant forces was studied by Dai-Singh (1994; 2003). They improved an analytical and numerical method for solving linear and nonlinear vibration problems and they showed that a function with piecewise constant argument is a good approximation to the given nonlinear function.

In real world problems, it is known that an exterior effect changes the asymptotic behavior of the solutions and the solutions of the mathematical models may be discontinuous. Because of this reality, studies on the impulsive differential equations with piecewise constant arguments have been started with the works of Karakoç et al. (2010; 2018) and Chiu (2015; 2017).

The aim of the present paper is to show how can an exterior effect change the asymptotic behavior of the food-limited population model. The study is organized as follows. Section 2 presents the study and fundamental concepts. In Section 3, we prove the main results. We also give some corollaries for the non-impulsive case. Finally, in Section 4, we consider some examples to compare the results of impulsive differential equation models with non-impulsive differential equation models.

2. Preliminaries

Let us consider following delay food-limited population model under impulse conditions:

$$N'(t) = rN(t)\frac{K - N^p([t-k])}{K + crN^p([t-k])},\ t \geq 0,\ t \neq n,\ n = 1, 2, ... \tag{1}$$

$$N(n^+) = N(n^-)\left(\frac{K^{1/p}}{N(n-l)}\right)^q,\ n = 1, 2, ... \tag{2}$$

where $r, K \in (0, \infty),\ c \in [0, \infty)$ are constants, $k \in \mathbb{Z}^+ = \{1, 2, 3, ...\}$ and $l \in \{2, 3, ...\}$ are fixed numbers, $p > 0$, $q \geq 0$ are constants, $[.]$ denotes the greatest integer function, $N(n^+) = \lim_{t\to n^+} N(t)$ and $N(n^-) = \lim_{t\to n^-} N(t)$. Define $k_0 = \max\{k, l\}$.

By a solution of Equation (1)-(2) we mean a function $N(t)$ defined on the set $\{-k_0, 1-k_0, ..., -1\} \cup [0, \infty)$ such that N is continuous on $\mathbb{R}^+$ with the possible exception of the points $[t] \in [0, \infty)$, right continuous and has left-hand limit at the points $[t] \in [0, \infty)$, differentiable and satisfies Equation (1) for any $t \in \mathbb{R}^+$, with the possible exception of the points $[t] \in [0, \infty)$ where one-sided derivatives exist, and N satisfies impulse conditions (2) for $n \in \mathbb{Z}^+$.

As given in Karakoç et al. (2018), we say that a function $x(t)$ defined on $[0, \infty)$ is called oscillatory about zero if there exist two real valued sequences $\{t_n\}_{n\geq 0},\ \{t'_n\}_{n\geq 0} \subset [0, \infty)$ such that $t_n \to +\infty$, $t'_n \to +\infty$ as $n \to +\infty$ and $x(t_n) \leq 0 \leq x(t'_n)$ for $n \geq N_1$ where N_1 is sufficiently large. Otherwise, the function $x(t)$ is called non-oscillatory. Thus, it is clear that a piecewise continuous function $x : [0, \infty) \to \mathbb{R}$ can be oscillatory even if $x(t) \neq 0$ for all $t \in [0, \infty)$. On the other hand, we deal with oscillatory behavior of the equation (1)-(2) about equilibrium point. So, as given in Gyori-Ladas (1991), we say that a function $x(t)$ is called oscillatory about K^* if the function $(x(t) - K^*)$ is oscillatory about zero.

Difference equations are main tool for the investigation of differential equations with piecewise constant arguments. First, we consider corresponding difference equation to obtain a sufficient condition for the oscillation of the solutions. Note that as given in Gyori-Ladas (1991), a solution $\{y_n\}$ of a difference equation is said oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is called non-oscillatory. Linearized oscillation for difference equations has been stated by Gyori-Ladas (1991). They proved that if $\lim_{u\to 0}\frac{f_i(u)}{u} = 1$ for $i = 1, 2, ..., m$ and there exists a positive constant δ such that

$$\begin{cases} \text{either } f_i(u) \leq u \text{ for } 0 \leq u \leq \delta \text{ and } i = 1, 2, ..., m, \\ \text{or } f_i(u) \geq u \text{ for } -\delta \leq u \leq 0 \text{ and } i = 1, 2, ..., m, \end{cases}$$

then every solution of equation

$$a_{n+1} - a_n + \sum_{i=1}^{m} p_i f_i(a_{n-k_i}) = 0,\ n = 0, 1, 2, ... \tag{3}$$

oscillates if and only if every solution of its linearized equation

$$b_{n+1} - b_n + \sum_{i=1}^{m} p_i b_{n-k_i} = 0,\ n = 0, 1, 2, ... \tag{4}$$

oscillates, where $p_i \in (0,\infty)$ and $k_i \in \{0,1,2,...\}$ for $i=1,2,...,m$ with $\sum_{i=1}^{m}(p_i+k_i) \neq 1,\ f_i \in C(\mathbb{R},\mathbb{R})$ and $uf_i(u) > 0$ for $u \neq 0$.

3. Main Results

In this section, first we obtain a representation for the solutions of Equation (1)-(2). Then we deal with oscillation about positive equilibrium point as well as asymptotic behavior of non-oscillatory solutions. By the biological meaning of the problem we consider differential equation (1)-(2) with the initial conditions

$$N(-k_0) = N_{-k_0} > 0,\ N(1-k_0) = N_{1-k_0} > 0, ..., N(-1) = N_{-1} > 0,\ N(0) = N_0 > 0. \quad (5)$$

By applying method of steps it can be seen that all solutions of the equation (1)-(2) with the positive initial conditions (5) are positive. Moreover, it is clear that $N^* = K^{1/p}$ is the positive equilibrium point of the equation (1)-(2). Now, using the substitution $N(t) = N^* e^{x(t)}$, the following equation is obtained:

$$x'(t) = r\left(\frac{1-e^{px([t-k])}}{1+cre^{px([t-k])}}\right),\ t \neq n,\ n=1,2,..., \quad (6)$$

$$x(n^+) - x(n^-) = -qx(n-l),\ n=1,2,... \quad (7)$$

It is clear that the solution $N(t)$ of the equation (1)-(2) oscillates about positive equilibrium point N^* if and only if the solution $x(t)$ of the equation (6)-(7) is oscillatory. So, we investigate the properties of the equation (6)-(7). The differential equation (6)-(7) is considered with the initial conditions

$$x(-k_0) = \ln\frac{N_{-k_0}}{N^*} = x_{-k_0}, ...,\ x(-1) = \ln\frac{N_{-1}}{N^*} = x_{-1},\ x(0) = \ln\frac{N_0}{N*} = x_0. \quad (8)$$

The following result gives the solution of the initial value problem (6)-(8) in terms of the solution of corresponding difference equation.

Theorem 3.1.

The unique solution $x(t)$ defined on $\{-k_0, 1-k_0, ..., -1\} \cup [0,\infty)$ of the initial value problem (6)-(8) has the following representation

$$x(t) = y(n) + r\left(\frac{1-e^{py(n-k)}}{1+cre^{py(n-k)}}\right)(t-n),\ n \leq t < n+1,\ n \in \mathbb{N}, \quad (9)$$

where the sequence $y(n)$ is the unique solution of the difference equation

$$y(n+1) - y(n) - r\left(\frac{1-e^{py(n-k)}}{1+cre^{py(n-k)}}\right) + qy(n-l+1) = 0, \quad (10)$$

with the initial conditions

$$y(-k_0) = x_{-k_0}, ..., y(-1) = x_{-1},\ y(0) = x_0. \quad (11)$$

Proof:

Let $x_n(t) \equiv x(t)$ be a solution of (6)-(7) on $n \leq t < n+1$. Equation (6) is rewritten as

$$x'(t) = r\left(\frac{1-e^{px(n-k)}}{1+cre^{px(n-k)}}\right),\ n < t < n+1. \tag{12}$$

Integrating both sides of Equation (12) from n to t we obtain that

$$x_n(t) = x\left(n^+\right) + r\left(\frac{1-e^{px(n-k)}}{1+cre^{px(n-k)}}\right)(t-n),\ n < t < n+1. \tag{13}$$

Similarly, if $x_{n-1}(t)$ is a solution of Equation (6)-(7) on $n-1 \leq t < n$, then we get

$$x_{n-1}(t) = x\left((n-1)^+\right) + r\left(\frac{1-e^{px(n-1-k)}}{1+cre^{px(n-1-k)}}\right)(t-n+1),\ n-1 < t < n. \tag{14}$$

Using the impulse conditions (7), from (13) and (14) we obtain that

$$x\left(n^+\right) - x\left((n-1)^+\right) - r\left(\frac{1-e^{px(n-1-k)}}{1+cre^{px(n-1-k)}}\right) = -qx(n-l).$$

Since x is right continuous at the points $t = n,\ n = 1, 2, ...$, the above equation gives the difference equation (10). Applying the initial conditions (11), the unique solution of equation (10) is obtained. Moreover, it is seen from (9) that $x(t)$ satisfies (6) for $t \neq n$ and (7) for $t = n$. ■

Now, we shall obtain a sufficient condition for the oscillation of the difference equation (10).

Theorem 3.2.

Suppose that $q > 0$ and

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} + q\frac{l^l}{(l-1)^{l-1}} > 1. \tag{15}$$

Then every solution of equation (10) is oscillatory.

Proof:

We use linearized oscillation for difference equations to prove the result. Equation (10) can be written as

$$y(n+1) - y(n) + p_1 f_1(y(n-k)) + p_2 f_2(y(n-l+1)) = 0,$$

where $p_1 = \dfrac{rp}{1+cr} > 0,\ f_1(u) = \dfrac{(1+cr)}{p}\dfrac{(e^{pu}-1)}{1+cre^{pu}} \in C(\mathbb{R},\mathbb{R}),\ p_2 = q > 0,\ f_2(u) = u \in C(\mathbb{R},\mathbb{R})$. It is clear that $\sum_{i=1}^{2}(p_i + k_i) = \dfrac{rp}{1+cr} + k + q + l - 1 \neq 1$. Moreover, it is satisfied that $uf_i(u) > 0$ for $u \neq 0$ and $\lim_{u\to 0}\dfrac{f_i(u)}{u} = 1,\ i = 1, 2$. On the other hand, it is shown that if $cr \geq 1$, then

$$\frac{df_1}{du} = \frac{e^{pu}(1+cr)^2}{(1+cre^{pu})^2} \leq 1 \text{ for } u \geq 0.$$

So, we have

$$\frac{d}{du}(f_1(u) - u) \leq 0 \text{ for } u \geq 0,$$

and

$$f_1(u) \leq u \text{ for } u \geq 0.$$

If $cr \leq 1$, then we obtain that

$$\frac{df_1}{du} = \frac{e^{pu}\left(1+cr\right)^2}{\left(1+cre^{pu}\right)^2} \leq 1 \text{ for } u \leq 0.$$

Hence, for $u \leq 0$ it is observed that $f_1(u) \geq u$. Since Equation (10) is in the form (3), by the result of Gyori-Ladas (1991), every solution of Equation (10) is oscillatory if and only if every solution of linearized equation

$$y(n+1) - y(n) + \frac{rp}{1+cr}y(n-k) + qy(n-l+1) = 0, \tag{16}$$

is oscillatory. On the other hand, Equation (16) is in the form (4) and it is known that if

$$\sum_{i=1}^{m} p_i \frac{(k_i+1)^{k_i+1}}{k_i^{k_i}} > 1,$$

then every solution of Equation (4) is oscillatory (Gyori-Ladas (1991), Theorem 7.3.1). So, under the condition (15) every solution of Equation (16) is oscillatory which completes the proof. ■

Corollary 3.1.

Let assume that $q > 0$. Every solution of Equation (1)-(2) oscillates about $N^* = K^{1/p}$ if the condition (15) is satisfied.

Proof:

It is observed that if $y(n)$ is the oscillatory solution of difference equation (10) with the conditions (11), then $x(t)$ is the oscillatory solution of the initial value problem (6)-(8). Considering the substitution $N(t) = N^* e^{x(t)}$, the result is obtained. ■

The following result is related to asymptotic behavior of the non-oscillatory solutions.

Theorem 3.3.

Assume that $q > 0$. If a solution $N(t)$ of Equation (1)-(2) is non-oscillatory about $N^* = K^{1/p}$, then $\lim_{t\to\infty} N(t) = K^{1/p}$.

Proof:

Recalling the substitution $N(t) = N^* e^{x(t)}$, it is sufficient to show that for every non-oscillatory solution $x(t)$ of the Equation (6)-(7) $\lim_{t\to\infty} x(t) = 0$. We prove the statement for an eventually

positive solution $x(t)$ of Equation (6)-(7). In the case of the solution $x(t)$ is eventually negative, the same result is also obtained. From Equation (6) for $n < t < n+1$, we get

$$x'(t) = -p_1 f_1\left(x([t-k])\right) < 0,$$

where $p_1 = \dfrac{rp}{1+cr} > 0,\ f_1(u) = \dfrac{(1+cr)}{p}\dfrac{(e^{pu}-1)}{1+cre^{pu}} > 0$ for $u > 0$. On the other hand, from the impulse conditions (7), we have

$$x(n^+) < x(n^-).$$

So, $\lim\limits_{t\to\infty} x(t) = A \geq 0$ exists. Since $x(t) = y(n)$ for $t = n$, $\lim\limits_{n\to\infty} y(n) = A$. We claim that $A = 0$. Otherwise, taking the limit of both sides of equation (10) as $n \to \infty$, we obtain that

$$0 = A - A = r\left(\frac{1-e^{pA}}{1+cre^{pA}}\right) - qA < 0,$$

which is a contradiction. So, $A = 0$ and the proof is completed. ■

If $q = 0$, then we have non-impulsive differential equation with piecewise constant argument

$$N'(t) = rN(t)\frac{K - N^p([t-k])}{K + crN^p([t-k])},\ t \geq 0. \tag{17}$$

Note that by a solution of Equation (17) we mean a function $N(t)$ defined on the set $\{-k, 1-k, ..., -1\} \cup [0,\infty)$ such that N is continuous on $\mathbb{R}^+$, differentiable and satisfies (17) for any $t \in \mathbb{R}^+$ with the possible exception of the points $[t] \in [0,\infty)$ where one-sided derivatives exist. The following results for the solutions of the equation (17) are obtained easily.

Corollary 3.2.

If

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} > 1,$$

then every solution of Equation (17) oscillates about $K^{1/p}$.

Proof:

It can be seen that the corresponding difference equation to Equation (17) is

$$y(n+1) - y(n) - r\left(\frac{1-e^{py(n-k)}}{1+cre^{py(n-k)}}\right) = 0,$$

which is a special case of difference equation (10) with $q = 0$. Thus, the proof can be done easily by using the similar arguments of the proof Theorem 3.2 and Corollary 3.1. ■

Corollary 3.3.

If a solution $N(t)$ of equation (17) is non-oscillatory about $N^* = K^{1/p}$, then $\lim\limits_{t\to\infty} N(t) = K^{1/p}$.

Proof:

Applying the substitution $N(t) = N^* e^{x(t)}$ to the Equation (17), the following non-impulsive differential equation

$$x'(t) = r\left(\frac{1 - e^{px([t-k])}}{1 + cre^{px([t-k])}}\right),\ t \geq 0$$

is obtained. The rest of the proof is similar to proof of Theorem 3.3. ■

4. Numerical Examples

In this section we consider some examples to illustrate our results. Especially, we compare the solutions of impulsive differential equation with corresponding non-impulsive differential equation.

Example 4.1.

(i) Let us consider the following non-impulsive food-limited population model

$$N'(t) = \frac{1}{8}N(t)\left(2 - N([t-1])\right),\ t \geq 0. \tag{18}$$

It is clear that Equation (18) is a special case of (17) with $r = \frac{1}{4},\ c = 0,\ p = 1,\ k = 1$ and $K = 2$. It is easy to see that

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} = 1.$$

So, we can not apply Corollary 3.2. But, from Corollary 3.3, if a solution $N(t)$ of Equation (18) is non-oscillatory about the positive equilibrium point 2, then $\lim_{t\to\infty} N(t) = 2$. The solution $N(t)$ of the Equation (18) with the initial conditions $N(-2) = N(-1) = 1/4,\ N(0) = 1$ is demonstrated in Figure 1.

(ii) Now let us consider the same food-limited population model under impulse effect with $l = 2$, and $q = 1/2$

$$N'(t) = \frac{1}{8}N(t)\left(2 - N([t-1])\right),\ t \geq 0,\ t \neq n, \tag{19}$$

$$N(n^+) = N(n^-)\left(\frac{2}{N(n-2)}\right)^{1/2},\ n = 1, 2, ... \tag{20}$$

It is clear that

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} + q\frac{l^l}{(l-1)^{l-1}} > 1.$$

So, from Corollary 3.1, every solution of Equation (19)-(20) oscillates about the positive equilibrium point 2. The solution $N(t)$ of the Equation (19)-(20) with the initial conditions $N(-2) = N(-1) = 1/4,\ N(0) = 1$ is demonstrated in Figure 2.

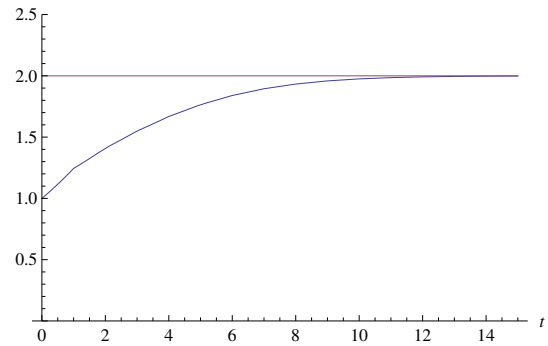

Figure 1. The solution $N(t)$ of equation (18) with $N(-2) = N(-1) = 1/4,\ N(0) = 1$

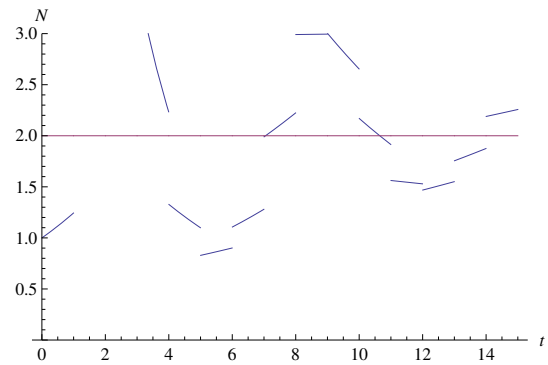

Figure 2. The solution $N(t)$ of equation (19)-(20) with $N(-2) = N(-1) = 1/4,\ N(0) = 1$

Example 4.2.

Let us consider the following food-limited population model

$$N'(t) = \frac{1}{3}N(t)\frac{3 - N([t-1])}{3 + N([t-1])},\ t \geq 0,\ t \neq n,\ n = 1, 2, ..., \tag{21}$$

with the impulse conditions

$$N(n^+) = N(n^-)\left(\frac{3}{N(n-2)}\right)^{1/12},\ n = 1, 2, ..., \tag{22}$$

where $K = 3$ is the positive equilibrium point of the Equation (21)-(22). It is easy to see that

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} + q\frac{l^l}{(l-1)^{l-1}} = 1.$$

So, the condition (15) is not satisfied. But, from Theorem 3.3, if a solution $N(t)$ of Equation (21)-(22) is non-oscillatory about the positive equilibrium point 3, then $\lim_{t\to\infty} N(t) = 3$. The solutions $N(t)$ of the Equation (21)-(22) with the initial conditions $N(-2) = N(-1) = N(0) = 1$ and $N(-2) = N(-1) = 2,\ N(0) = 5$ are demonstrated in Figure 3.

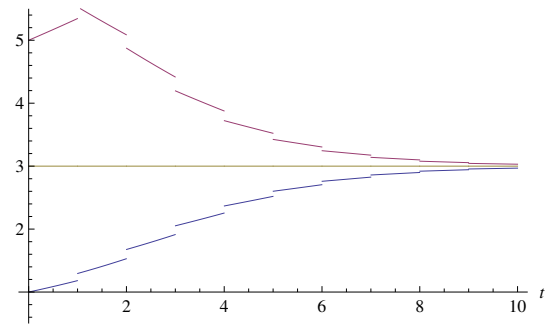

Figure 3. The solution $N(t)$ of equation (21)-(22) with $N(-2) = N(-1) = N(0) = 1$ and $N(-2) = N(-1) = 2,\ N(0) = 5$

Example 4.3.

(i) Let us consider the following non-impulsive food-limited population model

$$N'(t) = N(t)\frac{1 - N^{1/3}([t-2])}{1 + 2N^{1/3}([t-2])},\ t \geq 0,\ t \neq n,\ n = 1, 2, ... \tag{23}$$

Here, it is clear that $r = 1,\ c = 2,\ K = 1,\ p = 1/3$ and $k = 2$. It is easy to see that

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} < 1.$$

So, we can not apply Corollary 3.2. But, from Corollary 3.3, if a solution $N(t)$ of Equation (23) is non-oscillatory about the positive equilibrium point 1, then $\lim_{t\to\infty} N(t) = 1$. The solutions $N(t)$ of the Equation (23) with the initial conditions $N(-2) = N(-1) = N(0) = 1/2$ and with $N(-2) = N(-1) = N(0) = 2$ are demonstrated in Figure 4.

(ii) Now let us consider the same food-limited population model under impulse effect with $l = 3$, and $q = 1/2$

$$N'(t) = N(t)\frac{1 - N^{1/3}([t-2])}{1 + 2N^{1/3}([t-2])},\ t \geq 0,\ t \neq n, \tag{24}$$

$$N(n^+) = N(n^-)\left(\frac{1}{N(n-3)}\right)^{1/2},\ n = 1, 2, ... \tag{25}$$

It is clear that

$$\frac{rp}{1+cr}\frac{(k+1)^{k+1}}{k^k} + q\frac{l^l}{(l-1)^{l-1}} > 1.$$

So, from Corollary 3.1, every solution of Equation (24)-(25) oscillates about the positive equilibrium point 1. The solutions $N(t)$ of the Equation (24)-(25) with the initial conditions $N(-2) = N(-1) = N(0) = 1/2$ and $N(-2) = N(-1) = N(0) = 2$ are demonstrated in Figure 5 and Figure 6, respectively.

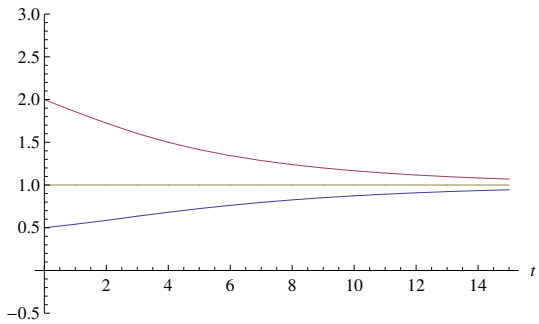

Figure 4. The solution $N(t)$ of equation (23) with $N(-2) = N(-1) = N(0) = 1/2$ and $N(-2) = N(-1) = 2,\ N(0) = 2$

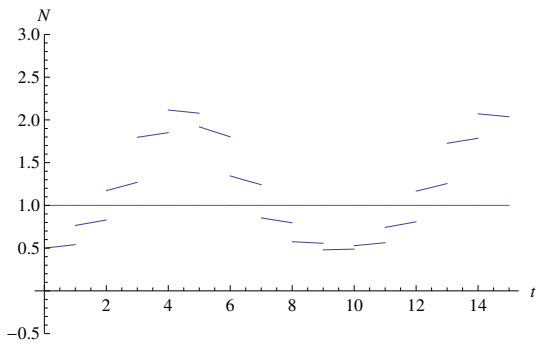

Figure 5. The solution $N(t)$ of equation (24)-(25) with $N(-2) = N(-1) = N(0) = 1/2$

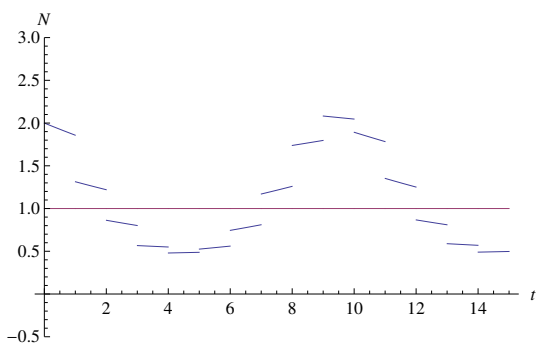

Figure 6. The solution $N(t)$ of equation (24)-(25) with $N(-2) = N(-1) = N(0) = 2$

5. Conclusion

In this work, we deal with impulse effect on the food-limited population model with piecewise constant arguments. Since the differential equation which is studied here can be written in terms of the solution of corresponding difference equation, linearized oscillation for difference equations is the main tool for our investigation. The technique we applied to obtain the main results can be used to investigate some of the other models. In the present paper it is seen that when the solutions of non-impulsive differential equation are non-oscillatory about positive equilibrium point, the solu-

tions of impulsive differential equation become oscillatory under suitable impulse conditions. So, we ask the following question: Under which type impulse conditions would the solutions of non-impulsive differential equation be oscillatory while the solutions of impulsive differential equation would be non-oscillatory?

Acknowledgment:

The author would like to thank the editor for his valuable comments and suggestions.

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