Analysis of heat absorption viscoelastic exothermic chemical reactive fluid with temperature dependent viscosity under bimolecular kinetic

[1]*S. O. Salawu and [2] R.A. Kareem

[1] Department of Mathematics
Landmark University
Omu-Aran, Nigeria
kunlesalawu2@gmail.com;

[2] Department of Mathematics
Lagos State Polytechnic
Ikorodu, Nigeria

*Corresponding author

Abstract

This study examines the boundary layer flow of variable viscosity, incompressible exothermic chemical reactive fluid with thermal radiation and asymmetric convective cooling under Bimolecular kinetic. The viscoelastic fluid flow along a vertical channel in the presence of a thermal buoyancy force and pressure gradient. Rosseland approximation is defined for the thick radiation heat flux in the energy equation with gray radiating liquid, non-scattering but with heat absorbing depending on wavelength. The convective heat exchange with the sorrounding temperature at the channel surface satisfied Newton's law of cooling. The computational analysis of the dimensionless nonlinear governing equations is obtained using Weighted residual method (WRM). The solutions are employed to obtain the velocity field, energy profile, skin friction and Nusselt number of the flow. These are graphically presented and discussed to show the effect of some pertinent fluid parameters on the non-Newtonian liquid flow. It is observed that the heat source reaction terms need to be cautiously monitored and controlled to avoid reaction solution blow up.

1. Introduction

Flow of liquids through a vertical boundary layer, non-movable plate with heat absorption continue to gain substantial attention due to its numerous usefulness and applications for a wide spectrum of industrial system, such as combustion chamber, rocket engine, thermal insulation, geothermal reservoirs and so on, Hassan and Salawu (2019). Previous prime studies regarding convective flow over plates with thermal radiation immersed in an ambient fluid have been established in the research work of (Baker et al. (1976); Makinde (2009); Dada and Salawu (2017)).

The flow of non-Newtonian fluid through a channel is a vital area of study due to its industrial and engineering applications, for example, polymer extrusion, nuclear reactor design, geophysics, energy storage systems, crude oil, Salawu and Fatunmbi (2017). A wide collection of mathematical formulations has been established to prompt the nonlinear stress-strain features of such fluids that show viscous and elastic characteristics. A thorough study of such models that comprises of the upper convected Maxwell, second order Reiner-Rivlin and Walters-B models were given in Zahorski (1982). For greatly elastic fluids, including melts polymer, the upper convected Maxwell (UCM) classical formulation has shown to be dependable. This viscoelastic fluid flow model gives the generality of the Maxwell material classical model for the distortion of material by the applying upper converted derivative that was initiated by Oldroyd (1950). Several theoretical, experimental and computational analyses have been exploited for this model. In Horikawa (1987), the flow of UCM fluid by applying the perturbation technique via finite difference scheme for an inclined cylinder with fixed length was reported. It was obtained from the studied that the non-Newtonian fluid tends to flow axially in the neighborhood of the cylinder. The spectral computational results for three dimensional flows under motionless loading conditions for a UCM fluid was presented by Roberts and Walters (1992). The results show that viscoelasticity enhances the load bearing capacity. Many studies relating to heat transfer of viscoelastic third grade liquids have been investigated (Makinde (2009); Yurusoy and Pakdemirli (2002)), but the studies have a deficiency in rational and systematic investigation of the thermodynamics cases with reference to the responses of varying viscosity and reactive chemical kinetics of the flow system.

The study of reacting viscous liquids is very vital in hydrodynamics lubricant and engineering heat transfer systems, (Chinyoka (2008); Makinde and Sibanda (2000); Chinyoka and Makinde (2011)). Many reactive lubricants are used in industry and engineering practices, e.g. synthetic esters, hydrocarbon oils, polyphenylethers and many more; their effectiveness relies basically on time to time temperature difference, Okedoye and Salawu (2019). Hence, the influence of variable viscosity and thermal conductivity on dissipation heat transfer and inclined magnetic field was examined by Salawu and Dada (2016), while (Kareem and Salawu (2017); Salawu and Amoo (2016)) investigated the impacts of thermal conductivity and variable viscosity on soret and dufour in non-Darcy porous media with dissipation and magnetic field. The second law thermodynamic for a reactive variable viscosity couette flow under Arrhenius kinetics in a channel was analyzed by (Kobo and Makinde (2010); Makinde and Aziz (2010)). It is therefore essential to examine the thermal loading characteristics and heat transport conditions of reactive viscous fluids to scale their significance as lubricants.

In the present study, the flow is assumed to be driven by thermal convection and axial pressure gradient. The aim of this current study is to carry out an analysis on the radiative heat transfer of variable viscosity third-grade exothermic chemical reactive flow with heat absorption and asymmetric convective cooling under bimolecular chemical kinetic. The mathematical model for the exothermic reactive flow is offered in section 2. Section 3 shows the review and implementation of Weighted residual method for the solution process while in section 4, the computational and graphical results are established and quantitatively discussed with reference to different existing fluid parameters in the system.

2. Mathematical Formulation

Consider laminar, incompressible, isotropic chemical reaction of variable viscosity third-grade liquid flow along non-moveable parallel plates with thermal radiation as portrayed in Figure 1. The non-Newtonian formulation is used to create the viscoelastic influences. The flow is propelled by Bimolecular chemical kinetics and assumed to be driven by the combined effect of the thermal buoyancy force and axial pressure gradient. The flow is along the direction of x with y-axis perpendicular to the flow. The channel surfaces are subjected to convective exchange of heat with the ambient temperature. Following Chinyoka and Makinde (2011), and ignoring time-dependent and the fluid reactive viscose consumption. The velocity and temperature balance equation governing the flow are as follows:
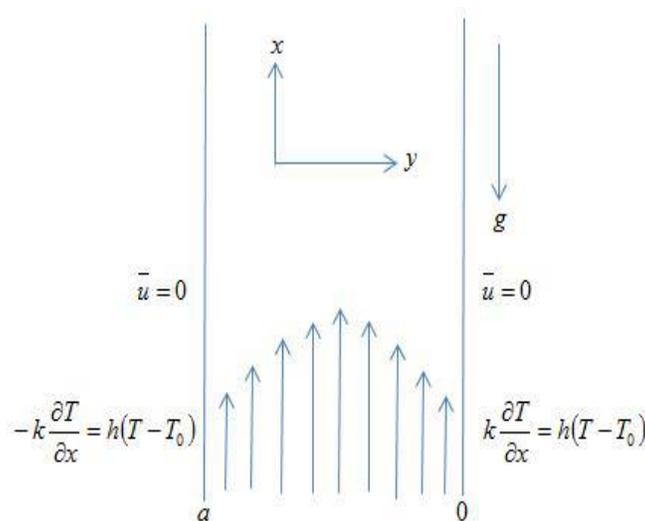

Figure 1. Geometry of the flow

$$-\frac{d\overline{P}}{d\overline{x}}+\frac{d}{d\overline{y}}\left[\overline{\mu}(T)\frac{d\overline{u}}{d\overline{y}}\right]+6\psi\frac{d^2\overline{u}}{d\overline{y}^2}\left(\frac{d\overline{u}}{d\overline{y}}\right)^2+\rho g\beta(T-T_0)=0, \tag{1}$$

$$\frac{d}{d\bar{y}}\left(k\frac{dT}{d\bar{y}}-q^r\right)+\left(\frac{d\bar{u}}{d\bar{y}}\right)^2\left[\bar{\mu}(T)+2\psi\left(\frac{d\bar{u}}{d\bar{y}}\right)^2\right]+QCA\left(\frac{KT}{\upsilon l}\right)^m\exp\left(-\frac{E}{RT}\right)-Q_0(T-T_0)=0,$$

(2)

With the boundary conditions as:

$$\bar{y}=a,\bar{u}=0,-k\frac{dT}{d\bar{y}}=h(T-T_0),$$

$$\bar{y}=a,\bar{u}=0,k\frac{dT}{d\bar{y}}=h(T-T_0).$$

(3)

Here, $\bar{u}$, T, T_0, a, $\bar{P}$, ρ, ψ and β are respectively the axial velocity of the fluid, fluid temperature, temperature of the ambient, channel width, fluid pressure, density, material coefficients and expansivity coefficient. The terms E, k, h, Q, Q_0, R, A, C, l, K, m and υ are the activation energy, thermal conductivity, heat transfer coefficient, heat of reaction, heat source, universal gas constant, reaction rate constant, initial species concentration, Plancks number, Boltzmanns constant, numerical exponent and vibration frequency respectively.

The Rosseland approximation is adopted for the expression of thick radiation heat flux q^r in the heat equation that can be defined as $q^r=\frac{16\sigma T_\infty^3}{3\delta}\frac{d^2T}{dy^2}$ where σ is the Stefan-Boltzmann and δ is the mean absorption coefficient. The temperature dependent viscosity ($\bar{\mu}$) is defined to be $\bar{\mu}(T)=\mu_0e^{-\varepsilon(T-T_0)}$, where ε is the variation viscosity term and μ_0 is the initial viscosity dynamic of the fluid.

Introducing the dimensionless quantities (4) into equations (1) to (3) along with the temperature dependent viscosity and thermal radiation.

$$x=\frac{\bar{u}}{a},y=\frac{\bar{y}}{a},u=\frac{\rho a\bar{u}}{\mu_0},G=-\frac{dP}{dx},\theta=\frac{E(T-T_0)}{RT_0^2},P=\frac{\bar{P}\rho a^2}{\mu_0^2},N=1+\frac{4}{3}R,$$

$$r=\frac{\varepsilon RT_0^2}{E},\gamma=\frac{\psi\mu_0}{\rho^2a^4},n=\frac{RT_0}{E},\omega=\frac{\mu_0^3e^{\frac{E}{RT}}}{\rho^2QAa^4C}\left(\frac{\upsilon l}{RT_0}\right)^m,Gt=\frac{\rho^2a^3g\beta RT_0^2}{\mu_0^2E},\mu=\frac{\bar{\mu}}{\mu_0},$$

(4)

$$Q=\frac{Q_0T_0^2RKe^{\frac{E}{RT}}}{E^2QAC}\left(\frac{\upsilon l}{KT_0}\right)^m,\lambda=\frac{QEAa^2Ce^{-\frac{E}{RT}}}{RKT_0^2}\left(\frac{KT_0}{\upsilon l}\right)^m,Bi=\frac{ah}{K},R=\frac{4\sigma T_\infty^3}{\delta k}.$$

Therefore, the main equations becomes:

$$G + e^{-r\theta} \frac{d^2 u}{dy^2} - re^{-r\theta} \frac{d\theta}{dy} \frac{du}{dy} + 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy}\right)^2 + Gt\theta = 0 , \tag{5}$$

$$N\frac{d^2\theta}{dy^2} + \lambda\left\{(1+n\theta)^m e^{\frac{\theta}{1+n\theta}} + \omega\left(\frac{du}{dy}\right)^2 \left[e^{-r\theta} + 2\gamma\left(\frac{du}{dy}\right)^2\right] - Q\theta\right\} = 0, \tag{6}$$

The boundary conditions are obtained as follows:

$$u = 0, \; \frac{d\theta}{dy} = -Bi\theta \quad y = 1,$$

$$u = 0, \; \frac{d\theta}{dy} = Bi\theta \quad y = 0. \tag{7}$$

where $Gt, G, \lambda, \gamma, n, N, \omega, r, Bi$ and Q represent the Grashof number, pressure gradient parameter, Frank-kamenetskii parameter, non-Newtonian parameter, activation energy parameter, radiation parameter, viscous heating parameter, variable viscosity parameter, Biot number and heat absorption parameter respectively.

3. Method of solution

The Weighted residual method see (Kareem et al. (2020); Salawu et al. (2019)) is about looking for an approximate solution, in the polynomial form to the differential equation in the form

$$D[u(y)] = f \; in \; the \; domain \; S, \quad B_\mu[u] = \gamma_\mu \; on \; \partial S, \tag{8}$$

where $D[u]$ denotes a differential operator for spatial derivatives of the dependent variables, u, f are the function for a known position, $B_\mu[u]$ represents the approximate number of boundary conditions with S been the domain and ∂S the boundary. By assuming a solution to $u(y)$, an expression of the form

$$u(y) \approx w(y, c_1, c_2, c_3...c_n), \tag{9}$$

which depends on a number of unknowns $c_1, c_2, c_3...c_n$ and is such that for arbitrary value c_i's the boundary conditions are satisfied and the residual equation become

$$E(y, c_i) = L(w(y, c_i)) - f(y), \tag{10}$$

The aim is to reduce the residual $E(y, c)$ to zero in some average sense over the domain. That is

$$\int_Y E(y, c) W_i dy = 0 \quad i = 1, 2, 3 \, ,n, \tag{11}$$

where the number of weight functions W_i is the same as the number of unknown constants c_i in w. therefore, the weighted functions are taken to be Dirac delta functions. That is, $W_i(y) = \delta(y - y_i)$, such that the error is zero at the chosen nodes y_i. That is, integration of equation (11) with $W_i(y) = \delta(y - y_i)$ results in $E(y, c_i) = 0$.

Applying WRM to equations (5) to (7), supposing a polynomial with unknown coefficients or parameters to be determined later, this polynomial is called the basis function which are taken as follow:

$$u(y) = \sum_{i=0}^{n} a_i y^i, \qquad \theta(y) = \sum_{i=0}^{n} b_i y^i. \tag{12}$$

By imposing the boundary conditions (7) on the basis functions (12) and substitute the basis functions on equations (5) and (6) to obtain the residual as

$$u_r = G + e^{-r\left(b_{10}y^{10} + b_9 y^9 + b_8 y^8 + b_7 y^7 + b_6 y^6 + b_5 y^5 + b_4 y^4 + b_3 y^3 + b_2 y^2 + b_1 y + b_0\right)} \left(90 y^8 a_{10} + 72 y^7 a_9\right.$$
$$\left. + 56 y^6 a_8 + 42 y^5 a_7 + 30 y^4 a_6 + 20 y^3 a_5 + 12 y^2 a_4 + 6 y a_3 + 2 a_2\right) - ... \tag{13}$$

$$\theta_r = N\left(90 y^8 b_{10} + 72 y^7 b_9 + 56 y^6 b_8 + 42 y^5 b_7 + 30 y^4 b_6 + 20 y^3 b_5 + 12 y^2 b_4 + 6 y b_3 + 2 b_2\right)$$
$$+ \lambda\left(\left(n\left(b_{10} y^{10} + b_9 y^9 + b_8 y^8 + b_7 y^7 + b_6 y^6 + b_5 y^5 + b_4 y^4 + b_3 y^3 + b_2 y^2 + ...\right.\right.\right. \tag{14}$$

Reducing the error to zero by collocating the residual equations at regular interval within the boundary domain when $Gt = 2, m = 0.5, \gamma = 1, n = 1, r = 0.1, Bi = 1, \lambda = 0.5, Q = 1, N = 1, G = 0.5$ and $\beta = 1$. That is, $y_r = \dfrac{(b-a)r}{R}$, where $r = 1, 2, ..., R-1$ and $a = 0, b = 1, R = 10$. The unknown constant coefficients are obtained using MAPLE software.

Therefore, the dimensionless momentum and temperature equations are gotten as follows:

$$u = u = -24.2254 y^{10} + 121.1279 y^9 - 258.3851 y^8 + 306.7782 y^7 - 222.6386 y^6$$
$$+ 102.9260 y^5 - 30.7468 y^4 + 5.6042 y^3 - 0.8884 y^2 + 0.4490 y \tag{15}$$

$$Q = \theta = 1.5982 \times 10^{-37} y^{10} - 8.0192 \times 10^{-37} y^9 + 1.7374 \times 10^{-36} y^8 - 2.1280 \times 10^{-36} y^7$$
$$+ 1.6202 \times 10^{-36} y^6 - 7.9379 \times 10^{-37} y^5 + 2.5035 \times 10^{-37} y^4 - 4.9373 \times 10^{-38} y^3 + ... \tag{16}$$

The weighted residual method scheme is repeated for variation in values of the parameters $Gt, m, \gamma, n, r, Bi, \lambda, Q, N, G$ and β.

The other quantities of engineering interest are the skin friction (τ) and the wall heat transfer rate (Nu) defined as follows:

$$\tau = \frac{\partial u}{\partial y}, \quad Nu = -\frac{\partial \theta}{\partial y}. \tag{17}$$

4. Discussion of Results

In order to get clear insight into the work, a semi analytical solution to the dimensionless momentum, energy, skin friction and wall temperature are computed using Weighted residual method coupled with collocation technique. All graphs agreed with the default parameter values except otherwise specified on the respective graph.

The effect of variational rise in the values of the thermal Grashof number (Gt) on the momentum field is portrayed in Figure 2. It is noticed that a rise in the values of the thermal buoyancy force relative to the hydrodynamic viscosity forces in the boundary layer result in an enhancement in the velocity distributions as shown in the figure. Figure 3 shows the impact of the third-grade parameter (γ) on the flow rate profile. From the figure, the flow velocity decreases as the non-Newtonian material parameter increases. The behaviour is due to a rise in the fluid particle bonding force that causes the liquid to be more viscoelastic thereby reduces the flow within the system. The sliding movement is as a result of the disproportion between the asymmetric convective cooling and temperature of the plates as the viscoelastic parameter rises.
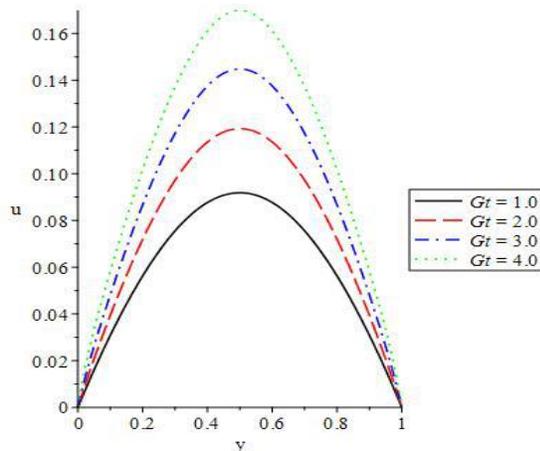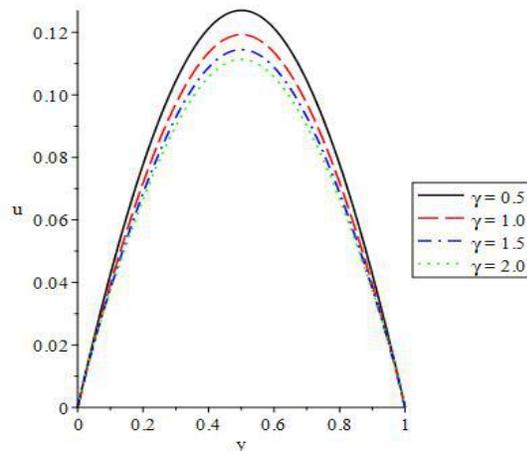

Figure 2. Effects of (Gt) on Velocity   **Figure 3.** Effects of (γ) on Velocity

Figure 4 depicts the response of the fluid velocity to variation in the variable viscosity parameter (r). It is observed from the plot that an increase in the parameter (r) reduces the viscosity of the liquid due to a rise in the heat transfer within the system and thereby consistently decreases the bonding force opposing the flow. This fundamentally leads to strengthening in the fluid velocity. Figure 5 demonstrates the effect of variations in the pressure gradient (G) on the fluid velocity. An increase in the pressure gradient causes an increase in the fluid momentum, i.e. the maximum velocity occur as the pressure gradient increases, which implies that the more the pressure applied in the channel, the faster the fluid flow.
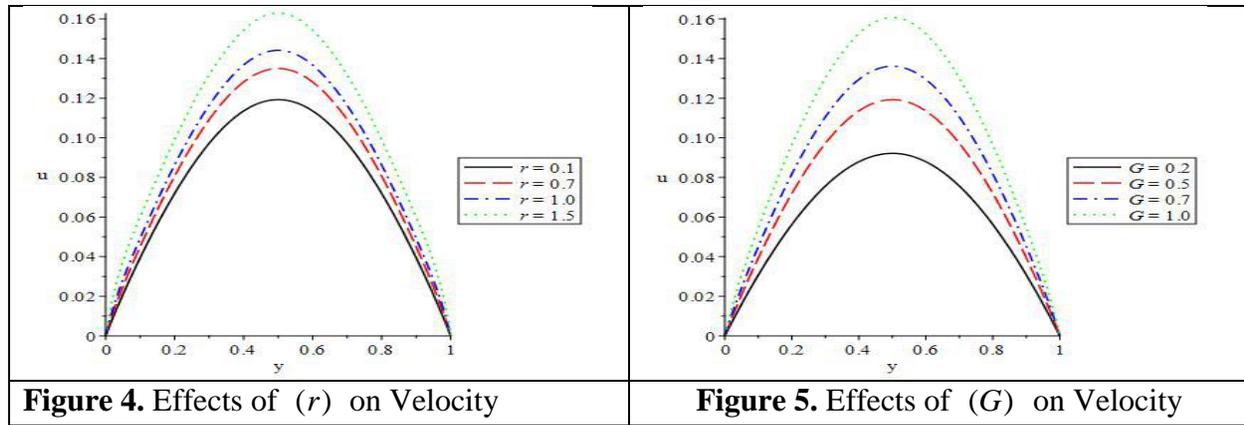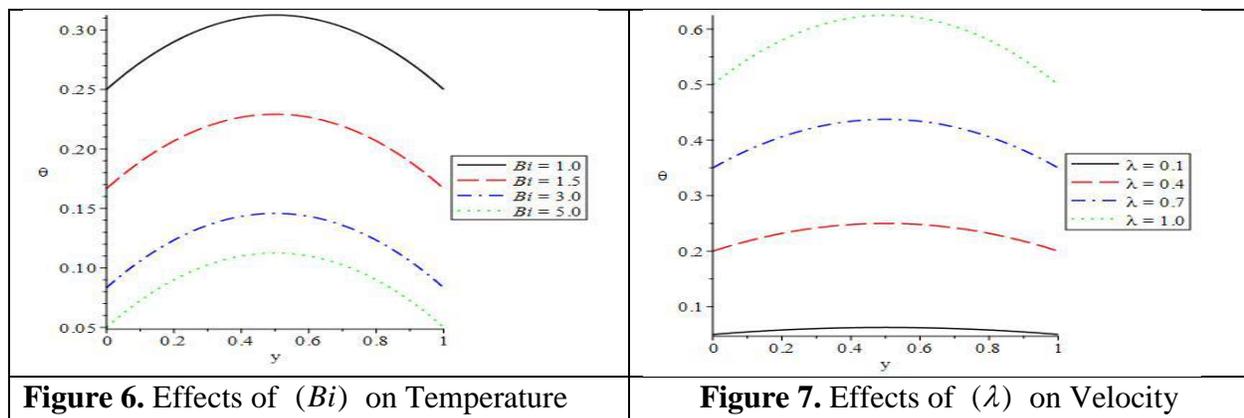
| Figure 4. Effects of (r) on Velocity | Figure 5. Effects of (G) on Velocity |

Figure 6 represents the effects of the Biot number (Bi) on the energy fields. As it is observed in the temperature boundary conditions, the more the Biot numbers the higher the convective cooling at the channel surfaces. This is therefore correspondingly diminishes the temperature at the surfaces and the bulk fluid. The whole temperature profile reduces with an enhancement of the Biot number as the fluid persistently regulates the surface temperature. The decreased temperature congruently increases the fluid viscosity that in turn reduces the fluid momentum. Figure 7 illustrates the response of the heat transfer to an increase in the reaction parameter (λ). A rise in the parameter values (λ) causes a substantial rise in the viscous heating source which in turn increases the reaction rate and accordingly augments the temperature distribution. This is due to strengthening in the thermal viscous dissipation coupling that result in a sensible increase in the flow temperature field. The reaction of the fluid temperature to change in the thermal radiation is presented in the Figure 8. It is seen from the plot that as the values of parameter (N) rises, a decrease in the temperature field is experienced due to a reduction in the thermal boundary layer thickness that increases the amount of emission or transmission of energy in the form of waves or particles through the space or a material medium out of the system. Hence, heat transfer within the system reduces. Figure 9 shows the energy fields for various values of the heat sink parameter (Q). The figure shows that an increase in the values of (Q) causes a decrease in the energy distributions as expected. This is because the energy boundary layer diminishes as the values rises which allow more heat to diffuse out of the third-grade exothermic chemical reactive fluid flow system.
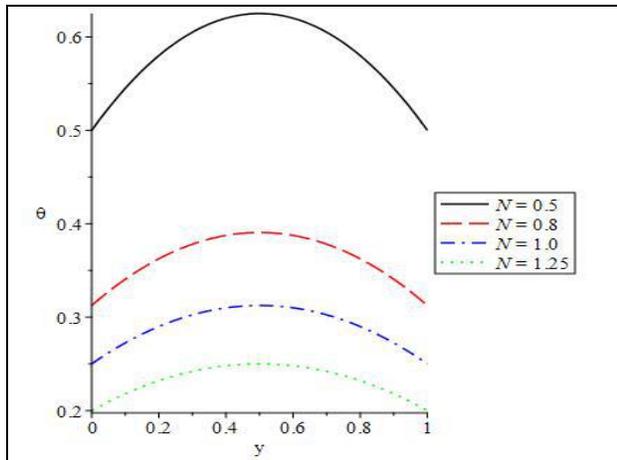

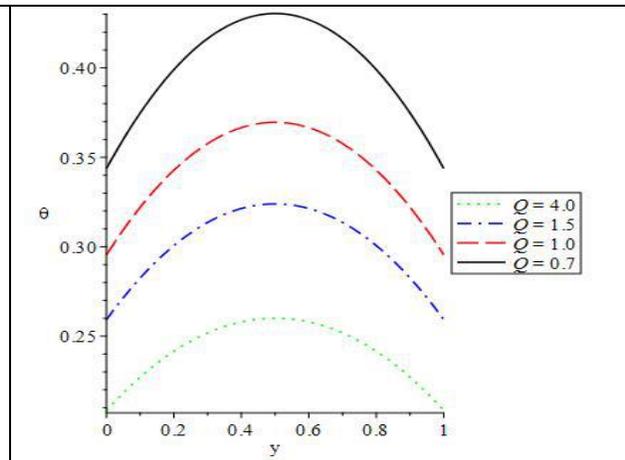| Figure 6. Effects of (Bi) on Temperature | Figure 7. Effects of (λ) on Velocity |

Figure 8. Effects of (N) on Temperature	**Figure 9.** Effects of (Q) on Velocity

Figures 10 and 11 depict the effects of (Bi) and (G) on the skin friction. From the figures, it can be observed that the skin friction decreases as the values of the parameters (Bi) increases, but rises as the parameter values (G) enhances within the range $0 \le y \le 0.5$. This is because the boundary walls get thicker and thinner respectively as the values of the parameters rises. But a reverse in the behaviour is noticed as it moves far away from the surfaces, i.e $y \ge 0.5$, due to a total turn around in the thermal and momentum boundary layers of the fluid flow far away from the surfaces.

Figures 12 and 13 show the influence of the reaction parameter (λ) and radiation parameter (N) on the temperature gradient. It is found that the heat effect on the wall increases and decreases, respectively as the parameter values rises due to the deposition of chemical reaction particles at the channel surfaces that increase the thermal boundary layer. This leads to loss of heat to the surroundings due to radiation within the range $0 \le y \le 0.5$, but the heat gradient later decreases and rises respectively as it moves farther from the surfaces, i.e. $y \ge 0.5$ toward the free stream flow in the system.
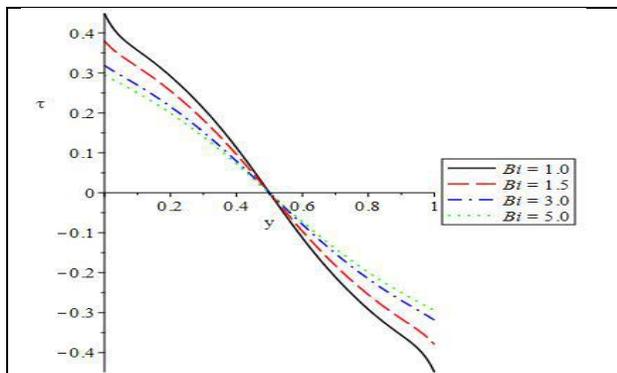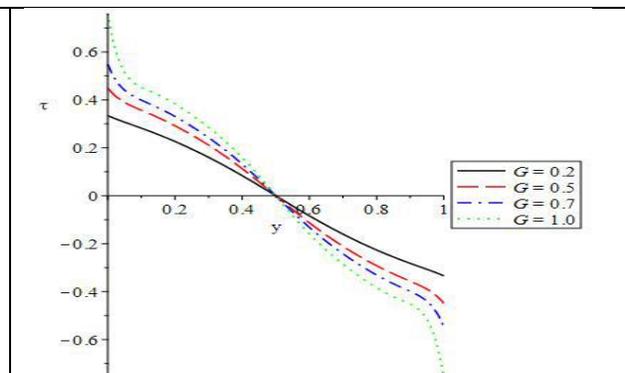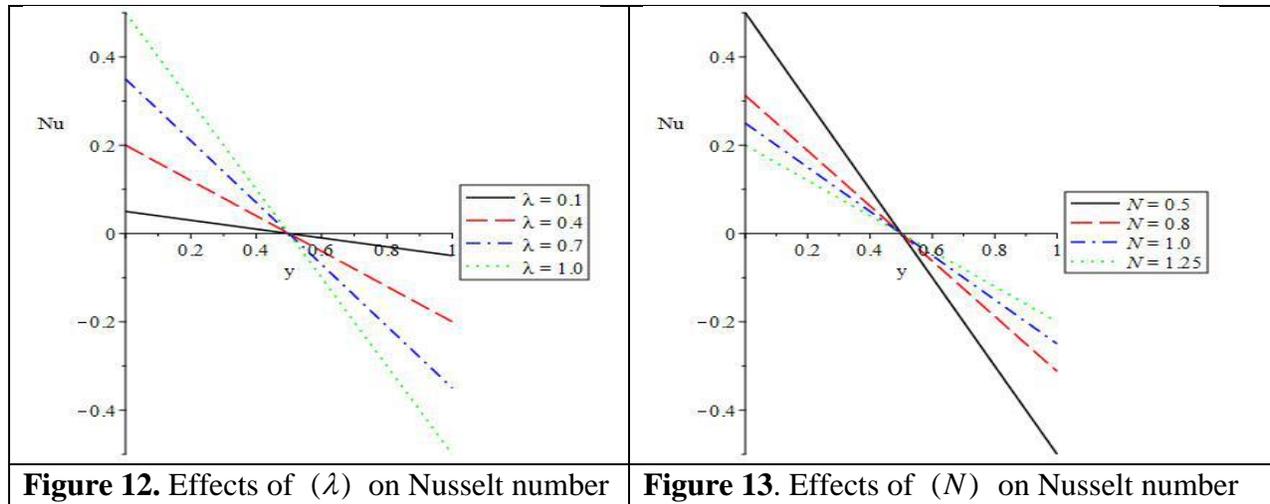

Figure 10. Effects of (Bi) on Skin friction	**Figure 11**. Effects of (G) on Skin friction

| **Figure 12.** Effects of (λ) on Nusselt number | **Figure 13**. Effects of (N) on Nusselt number |

5. Conclusion

A convergent and unconditionally stable Weighted residual method was developed and used to examine the radiative heat transfer of variable viscosity third-grade exothermic chemical reactive flow with heat absorption and asymmetric convective cooling under Bimolecular kinetic. It was noticed that there is an increase in the fluid momentum or energy field with an increase in the (i) thermal Grashof number (ii) fluid variable and (ii) pressure gradient. A decrease in the fluid momentum is noticed with a rise in the values of the non-Newtonian term, whereas high values of heat source parameter can cause a blow up of a reactive system. Hence, blow up in the system solution means that the reaction process needs to be cautiously monitored and controlled. More also, it is observed that the heat transfer rate within the system reduces as the values of Biot number, radiation and heat absorption increases due to a reduction in the thermal boundary layer thickness that causes heat to diffuse out of the system. The results from this study will assist in managing industrial and engineering processes where heat transfer is important.

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