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New Lie group of transformation for the non-Newtonian fluid flow narrating differential equations

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Abstract

In this endeavour, a new Lie point of transformation for the fluid flow narrating differential equations are proposed. For this purpose a non-Newtonian fluid named tangent hyperbolic fluid is considered towards the flat surface in a magnetized flow field. In addition, equation of concentration admits the role of chemically reactive species. A mathematical model in terms of the coupled PDE's is constructed. Lie group of analysis is implemented to yield the new Lie point of transformation for tangent hyperbolic fluid flow narrating differential equations when the heat and mass transfer individualities are considered. The resultant system of PDE's is reduced into system of ODE's via obtained set of transformation. The self-coded computational scheme is accomplished and the outcomes are reported by way of graphs. It is noticed that tangent hyperbolic fluid velocity, temperature and concentration is decreasing function of magnetic field parameter, Prandtl number and chemical reaction parameter respectively.

Keywords: Lie point of transformation; Non-Newtonian liquid; Numerical method

MSC 2010 No.: 76N20, 76N17

1. Introduction

There is no doubt that the fluid science remains an exploratory and interesting topic for the human beings. Firstly the efforts were made towards flow field properties of viscous fluid. This was made possible by Sir Isaac Newton after industrial revolution at the end of 19th Century. Newton initiated the idea of physical interpretation of fluid flow regime in terms of mathematical models. Such concept with constant viscosity was introduced in his article under titled “Principia”. Later, the mathematical flow equations for inviscid fluid was subsequently contributed by Daniel Bernoulli and Leonhard Euler. Such mathematical equations are also termed as Euler’s inviscid equations. Even, Adhémar St.Venant, Siméon Denis Poisson, Augustin-Louis Cauchy and Claude-Louis Navier contributed a lot for the developments of mathematical modelling towards fluids flow. They added their thoughts in context of frictional force as well. The ultimate outclass mathematical treatment was proposed by Sir George Stokes in 1845. These mathematical equations were for the motion of viscous fluid. The Newtonian terms were considered by Stokes and these equations were known by Navier-Stokes equations. Researchers and scientists are still busy to formulate flow field of various fluids by utilizing Navier-Stokes equations. The exact solution in this direction seems a though job therefore researchers having affiliation with fluid science seek an approximate solution. An impossibility of an exact solution is due to the existence of non-linear character of flow narrating differential equations because for complete description of flow model, all equations (equation of continuity for mass, Navier-Stokes equations for momentum, energy equation for the first law of thermodynamics and concentration equation) are considered simultaneously. In short, mathematical modelling under fundamental laws yields system of partial differential equations (PDE’s) and these PDE’s admits non-linearity due to which we need to find acceptable numerical solutions. Prandtl (1938) introduced a revolutionary concept of the “boundary layer” subject to fluid flows over a surfaces. Since than investigators namely Ahmad and Mubeen (1995), Rees et al. (1996), Kumari et al. (1997), Lesnic et al. (199), Temam and Xiaoming (2001), Lok et al. (2003), Khan and Emmanuel (2005), Hayat and Sajid (2007), Yam et al. (2009), Hameed and Ellahi (2011), Ellahi et al. (2013), and Khan et al. (2018) used the concept of boundary layer and interpret acceptable traits in the field of fluid science.

In actual, the active part of flow narrating PDE’s is retained under usual boundary layer approximations. The remained system is than solved by appropriate computational algorithm. One of the step in this direction to reduce the number of independent variables representing domains towards said flow problem. To be more specific, the obtained system of PDE’s seems impossible to solve analytically. Therefore, investigators firstly reduce the system of PDE’s into system of coupled ODE’s and then computational scheme is used to report the acceptable solution. The reduction in an independent variables is attained via similarity variables. In this direction, order reduction of differential equations is one of the application of theory of Lie symmetry. This idea was proposed by Sophus Lie, see Helgason (1990).

The current pagination contains analysis on new scaling group of transformation for coupled differential system appeared in fluid science. To be specific, a non-Newtonian liquid is considered towards flat surface in a magnetized flow field with both heat and mass properties. Moreover, the fluid concentration admits the role of chemically reactive species. A mathematical model is construct against said problem. Lie group analysis is executed to proposed set of Lie point of transformations. The yielded Lie transformations are used for the order reduction. The reduced system is solved computationally. The obtained variations are offered by means of graphical trends. The layout of article is designed in such way that the limited literature survey is presented in Section-1. The mathematical modelling along with the group theoretic scheme is explained in Section 2. The computational scheme is presented in the Section 3. The obtained outcomes are discussed in Section 4. The summary of analysis is offered in Section 5.

2. Mathematical Treatment

The tangent hyperbolic fluid (THF) is equipped in the region $\breve{y} > 0$. The flow field is interacted with applied magnetic field. Moreover, heat source/sink, chemically reactive species, velocity and temperature slip effects are taken into account. The fluid flow is induced due to stretching of flat surface. The tensor of tangent fluid model (see Akbar et al. (2013)) is termed as

$$\overleftrightarrow{\tau} = \left[\mu_\infty + (\mu_0 + \mu_\infty) \tanh(\Gamma \dot{\gamma}_1)^n \right] \dot{\gamma}_1, \tag{2.1}$$

Here $\dot{\gamma}_1$ is defined by

$$\dot{\gamma}_1 = \sqrt{\frac{\sum_i \sum_j (\dot{\gamma}_1)_{ij} (\dot{\gamma}_1)_{ji}}{2}} = \sqrt{\frac{\pi}{2}}, \text{ with } \pi = \frac{\text{trace}[\text{grad}\vec{V} + (\text{grad}\vec{V})^t]^2}{2}. \tag{2.2}$$

The flow materials with shearing characteristics can be studied by THF model. For this purpose, we consider $\Gamma\dot{\gamma}_1 < 1$ and $\mu_\infty = 0$. From Equation (2.1) one can obtain

$$\vec{\tau} = \mu_0 \dot{\gamma}_1[(\Gamma\dot{\gamma}_1)^n] = \mu_0\dot{\gamma}_1[1+\Gamma\dot{\gamma}_1 - 1]^n = \mu_0\dot{\gamma}_1[1+n(\Gamma\dot{\gamma}_1 - 1)]. \tag{2.3}$$

The utilizing of extra tensor of THF via fundamentals laws one can obtain the ultimate equations

$$\breve{u}_{\breve{x}} + \breve{v}_{\breve{y}} = 0, \tag{2.4}$$

$$\breve{u}\breve{u}_{\breve{x}} + \breve{v}\breve{u}_{\breve{y}} = \nu_1(1-n)\breve{u}_{\breve{y}\breve{y}} + \sqrt{2}\,\nu_1 n\Gamma\breve{u}_{\breve{y}}\breve{u}_{\breve{y}\breve{y}} - \frac{\sigma B^2}{\rho}\breve{u}, \tag{2.5}$$

$$\breve{u}\hat{T}_{\breve{x}} + \breve{v}\hat{T}_{\breve{y}} = (\frac{\kappa}{c_p\rho})\hat{T}_{\breve{y}\breve{y}} + \frac{Q_1}{c_p\rho}(\hat{T} - \hat{T}_\infty), \tag{2.6}$$

$$\breve{u}\hat{C}_{\breve{x}} + \breve{v}\hat{C}_{\breve{y}} = D_c\hat{C}_{\breve{y}\breve{y}} - k_1(\hat{C}_w - \hat{C}_\infty), \tag{2.7}$$

with

$$\hat{C} = \hat{C}_w,\ \hat{T} = \hat{T}_w + D_1\hat{T}_{\breve{y}},\ \breve{v} = 0,\ \ \breve{u} = b\breve{x} + L_1\breve{u}_{\breve{y}},\ \text{ for } \breve{y} = 0, \tag{2.8}$$
$$\hat{C} \to \hat{C}_\infty,\ \hat{T} \to \hat{T}_\infty,\ \breve{u} \to 0,\ \text{ when } \breve{y} \to \infty.$$

One can use the set of variables:

$$\phi = \frac{\hat{C} - \hat{C}_\infty}{\hat{C}_w - \hat{C}_\infty},\ \theta = \frac{\hat{T} - \hat{T}_\infty}{\hat{T}_w - \hat{T}_\infty},\ v = \frac{\breve{v}}{\sqrt{b\nu_1}},\ u = \frac{\breve{u}}{\sqrt{b\nu_1}},\ y = \sqrt{\frac{b}{\nu_1}}\,\breve{y},\ x = \sqrt{\frac{b}{\nu_1}}\,\breve{x}, \tag{2.9}$$

incorporating Equation (2.9) into Equations (2.4)-(2.8) one can conclude

$$u_x + v_y = 0, \tag{2.10}$$

$$uu_x + vu_y = (1-n)u_{yy} + \sqrt{2}\,b\,n\Gamma u_y u_{yy} - \frac{\sigma B^2}{\rho b}u, \tag{2.11}$$

$$u\theta_x + v\theta_y = (\frac{\kappa}{c_p\mu})\theta_{yy} + \frac{Q_1}{c_p\rho b}\theta, \tag{2.12}$$

$$u\phi_x + v\phi_y = \frac{D_c}{\nu_1}\phi_{yy} - \frac{k_1}{b}\phi, \tag{2.13}$$

with

$$u = x + \sqrt{\frac{b}{\nu_1}}L_1 u_y,\ v = 0,\ \theta = 1 + \sqrt{\frac{b}{\nu_1}}D_1\theta_y,\ \ \phi = 1\ \text{ for } y = 0, \tag{2.14}$$

$$u \to 0,\ \theta \to 0,\ \phi \to 0,\ \text{ when } y \to \infty.$$

Further, we have stream function

$$u=\Psi_y, v=-\Psi_x, \tag{2.15}$$

by utilizing Equation (2.15) into Equations (2.10)-(2.14), we have

$$\Psi_{xy}-\Psi_{yx}=0, \tag{2.16}$$

$$\Psi_y\Psi_{xy}-\Psi_x\Psi_{yy}=(1-n)\Psi_{yyy}+\sqrt{2}\,bn\Gamma\Psi_{yy}\Psi_{yyy}-\frac{\sigma B^2}{\rho b}\Psi_y, \tag{2.17}$$

$$\Psi_y\theta_x-\Psi_x\theta_y=(\frac{\kappa}{c_p\mu})\theta_{yy}+\frac{Q_1}{c_p\rho b}\theta, \tag{2.18}$$

$$\Psi_y\,\phi_x-\Psi_x\phi_y=\frac{D_c}{\nu_1}\phi_{yy}-\frac{k_1}{b}\phi, \tag{2.19}$$

while the reduced endpoint conditions are

$$\Psi_y=x+\sqrt{\frac{b}{\nu_1}}L_1\Psi_{yy},\;\Psi_x=0,\;\theta=1+\sqrt{\frac{b}{\nu_1}}D_1\theta_y,\;\;\phi=1\;\text{ for }\;y=0, \tag{2.20}$$

$$\Psi_y\to 0,\;\theta\to 0,\;\phi\to 0,\;\text{when}\;\;y\to\infty.$$

For order reduction of the Equations (2.10)-(2.13) we need set of scaling group of transformation (see Rehman et al. (2018)) via Lie group analysis. In this context, one can consider set of point transformation

$$X_1:\;\;\phi^*=\phi e^{\varepsilon\lambda_6},\;\Gamma^*=\Gamma e^{\varepsilon\lambda_5},\;\theta^*=\theta e^{\varepsilon\lambda_4},\Psi^*=\Psi e^{\varepsilon\lambda_3},\;y_1=y\,e^{\varepsilon\lambda_2},x_1=x\,e^{\varepsilon\lambda_1}. \tag{2.21}$$

The coordinates $(\phi,\,\theta,\,\Gamma,\Psi,\;y,x)$ can be transformed into $(\phi^*,\,\theta^*,\,\Gamma^*,\Psi^*,\;y_1,x_1)$ under the set of scaling group of transformation given by Equation (2.21). The effort in this direction is given as:

$$e^{\varepsilon(\lambda_1+2\lambda_2-2\lambda_3)}\left(\Psi^*_{y_1}\,\Psi^*_{x_1y_1}-\Psi^*_{x_1}\,\Psi^*_{y_1y_1}\right)=e^{\varepsilon(3\lambda_2-\lambda_3)}(1-n)\Psi^*_{y_1y_1y_1} \tag{2.22}$$
$$+e^{\varepsilon(5\lambda_2-2\lambda_3-\lambda_5)}\sqrt{2}\,bn\Gamma^*\Psi^*_{y_1y_1}\,\Psi^*_{y_1y_1y_1}-e^{\varepsilon(\lambda_2-\lambda_3)}\,\frac{\sigma B^2}{\rho b}\Psi^*_{y_1},$$

$$e^{\varepsilon(\lambda_1+\lambda_2-\lambda_3-\lambda_4)}\left(\Psi^*_{y_1}\theta^*_{x_1}-\Psi^*_{x_1}\theta^*_{y_1}\right)=e^{\varepsilon(2\lambda_2-\lambda_4)}(\frac{\kappa}{c_p\mu})\theta^*_{y_1y_1}+e^{-\varepsilon\lambda_4}\,\frac{Q_1}{c_p\rho b}\theta^*, \tag{2.23}$$

$$e^{\varepsilon(\lambda_1+\lambda_2-\lambda_3-\lambda_6)}\left(\Psi^*_{y_1}\;\phi^*_{x_1}-\Psi^*_{x_1}\phi^*_{y_1}\right)=e^{\varepsilon(2\lambda_2-\lambda_6)}\,\frac{D_c}{\nu_1}\phi^*_{y_1y_1}-e^{-\varepsilon\lambda_6}\,\frac{k_1}{b}\phi^*, \tag{2.24}$$

here, structure given by Equations (2.22)-(2.24) will be preserved under the scaling group of transformation X_1 via relation given below

$$\lambda_1 + 2\lambda_2 - 2\lambda_3 = 3\lambda_2 - \lambda_3 = 5\lambda_2 - 2\lambda_3 - \lambda_5 = \lambda_2 - \lambda_3, \tag{2.25}$$

$$\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4 = 2\lambda_2 - \lambda_4 = -\lambda_4,$$

$$\lambda_1 + \lambda_2 - \lambda_3 - \lambda_6 = 2\lambda_2 - \lambda_6 = -\lambda_6,$$

from endpoint conditions one can easily conclude $\lambda_4 = 0,$ and $\lambda_6 = 0.$ Along with these values the common practice towards Equation (2.25) yields

$$\lambda_1 = \lambda_1,\ \lambda_2 = 0,\ \lambda_3 = \lambda_1,\ \lambda_4 = 0,\ \lambda_5 = -\lambda_1 \text{ and } \lambda_6 = 0. \tag{2.26}$$

In result of Equation (2.26), our one parameter point transformation can be written as

$$\mathrm{X}_1:\ x_1 = x e^{\varepsilon\lambda_1},\ y_1 = y,\ \Psi^* = \Psi e^{\varepsilon\lambda_1},\ \theta^* = \theta,\ \Gamma^* = \Gamma e^{-\varepsilon\lambda_1},\ \phi^* = \phi. \tag{2.27}$$

Further, the Taylor's expansion for X_1 around $\varepsilon = 0$ with $O(\varepsilon)$ restriction yields

$$\mathrm{X}_1:\ x_1 - x = x\varepsilon\lambda_1 + O(\varepsilon),\ y_1 - y = 0 + O(\varepsilon),\ \Psi^* - \Psi = \Psi\varepsilon\lambda_1 + O(\varepsilon), \tag{2.28}$$

$$\theta^* - \theta = 0 + O(\varepsilon), \Gamma^* - \Gamma = -\Gamma\varepsilon\lambda_1 + O(\varepsilon),\ \phi^* - \phi = 0 + O(\varepsilon).$$

The characteristic equation subject to Equation (2.28) is

$$\frac{dx}{x} = \frac{dy}{0} = \frac{d\Psi}{\Psi} = \frac{d\theta}{0} = \frac{d\Gamma}{-\Gamma} = \frac{d\phi}{0}. \tag{2.29}$$

The possible combination yields

$$\xi = y,\ \Psi = xF(\xi),\ \theta = \theta(\xi),\ \Gamma = x^{-1}\Gamma_0,\ \phi = \phi(\xi), \tag{2.30}$$

incorporating Equation (2.30) into Equations (2.17)-(2.20), we obtain system of non-linear ODE's

$$\frac{d^3F(\xi)}{d\xi^3}(1-n) - \left(\frac{dF(\xi)}{d\xi}\right)^2 + \frac{d^2F(\xi)}{d\xi^2}F(\xi) + nW_b\frac{d^3F(\xi)}{d\xi^3}\frac{d^2F(\xi)}{d\xi^2} - \gamma^2\frac{dF(\xi)}{d\xi} = 0, \tag{2.31}$$

$$\frac{d^2\theta(\xi)}{d\xi^2} + \Pr\left(F(\xi)\frac{d\theta(\xi)}{d\xi} + Q^{\pm}\theta(\xi)\right) = 0, \tag{2.32}$$

$$\frac{d^2\phi(\xi)}{d\xi^2} + Sc\left(F(\xi)\frac{d\phi(\xi)}{d\xi}\right) - ScR_c\phi(\xi) = 0, \tag{2.33}$$

the reduced endpoint conditions are

$$\phi(\xi)=1,\ \theta(\xi)=1+\alpha_2\frac{d\theta(\xi)}{d\xi},\ F(\xi)=0, \frac{dF(\xi)}{d\xi}=1+\alpha_1\frac{d^2F(\xi)}{d\xi^2},\ \text{at}\ \xi=0, \tag{2.34}$$
$$\phi(\xi)\to 0,\ \theta(\xi)\to 0,\ \frac{dF(\xi)}{d\xi}\to 0,\ \ \text{when}\ \ \xi\to\infty.$$

The physical quantities namely skin friction (SF), local Nusselt number (LNN) and local Sherwood number (LSN) are acknowledged as

$$C_F=\frac{\vec{\tau}_w}{\rho(bx)^2},\ Nu_x=\frac{x\,q_w}{\kappa(\hat{T}_w-\hat{T}_\infty)},\ Shu_x=\frac{xq_m}{D_c(\hat{C}_w-\hat{C}_\infty)}, \tag{2.35}$$
$$\vec{\tau}_w=(1-n)u_y+\frac{n\Gamma}{\sqrt{2}}(u_y)^2,\ q_w=-\kappa\hat{T}_{\bar{y}},\ q_m=-D_c\hat{C}_{\bar{y}},$$

the corresponding dimensionless forms of these quantities are

$$\sqrt{\mathrm{Re}}C_F=\left((1-n)\frac{d^2F(0)}{d\xi^2}+\frac{n}{2}W_b\left[\frac{d^2F(0)}{d\xi^2}\right]^2\right),\ \frac{Nu_x}{\sqrt{\mathrm{Re}}}=-\frac{d\theta(0)}{d\xi},\ \frac{Shu_x}{\sqrt{\mathrm{Re}}}=-\frac{d\phi(0)}{d\xi}. \tag{2.36}$$

The involved parameters namely power law index (n), Hartmann number (γ), heat generation/absorption parameter $(Q^{\pm})$, Prandtl number $(\Pr)$, Weissenberg number (W_b), Schmidt number (Sc), velocity slip parameter (α_1), thermal slip parameter (α_2) and chemical reaction parameter (R_c) are defined as:

$$W_b=\sqrt{2}\Gamma_0 b,\ \ \gamma=\sqrt{\frac{\sigma B^2}{\rho b}},\ \Pr=\frac{\mu c_p}{\kappa},\ Q^{\pm}=\frac{Q_1}{\rho c_p b},\ \ Sc=\frac{\nu_1}{D_c}, \tag{2.37}$$
$$\alpha_1=\sqrt{\frac{b}{\nu_1}}L_1, \alpha_2=\sqrt{\frac{b}{\nu_1}}D_1, R_c=\frac{k_1}{b}.$$

3. Numerical Scheme

Our interested is to solve Equations (2.31)-(2.34) by numerical method named as shooting method. For this purpose firstly the system will be transformed into an initial value problem. To achieve this, the dummy variables are introduced that are

$$y_1=F(\xi),\ \ y_2=\frac{dF(\xi)}{d\xi}, y_3=\frac{d^2F(\xi)}{d\xi^2},\ y_4=\theta(\xi),\ y_5=\frac{d\theta(\xi)}{d\xi},\ y_6=\phi(\xi),\ y_7=\frac{d\phi(\xi)}{d\xi}, \tag{3.1}$$

the system given in Equations (2.31)-(2.34) can be equivalently written as

$$\frac{dy_1(\xi)}{d\xi}=y_2(\xi),$$
$$\frac{dy_2(\xi)}{d\xi}=y_3(\xi), \tag{3.2}$$
$$\frac{dy_3(\xi)}{d\xi}=\frac{\left(y_2(\xi)\right)^2-y_3(\xi)y_1(\xi)-\gamma^2y_2(\xi)}{(1-n)+nW_by_3(\xi)},$$

$$\frac{dy_4(\xi)}{d\xi} = y_5(\xi),$$
$$\frac{dy_5(\xi)}{d\xi} = -\Pr\left(y_1(\xi)y_5(\xi) + Q^{\pm}y_4(\xi)\right),$$
$$\frac{dy_6(\xi)}{d\xi} = y_7(\xi),$$
$$\frac{dy_7(\xi)}{d\xi} = -Sc\left(y_1(\xi)y_7(\xi)\right) + R_c y_6(\xi),$$

along with far endpoint conditions

$$y_2(\xi) \to 0, y_4(\xi) \to 0, y_6(\xi) \to 0, \text{ when } \xi \to \infty. \tag{3.3}$$

Such scheme is utilized by means of MATLAB code and the obtained observations are offered by graphical trends.

4. Analysis

The non-Newtonian fluid model is considered. The said problem is translated in terms of mathematical model see Equations (2.4)-(2.8) in this regard. Since we are looking for solution of this system and due to coupled non-linearity we face problem to solve exactly. For numerical solution we need to transform PDE's into ODE's and such step can be attain with the help of scaling group of transformation. Mostly researchers move-on with so-called available transformation from literature. The obtain solution by this does not depicts complete physical outcomes. Therefore, we prefer to construct particular scaling group of transformation for our problem. To obtain such transformation a Lie analysis is implemented. The resultant system of ODE's is solved by employing self-coded computational algorithm. Some particular trends are validated with existing results namely, Figure 1 depicts the effect of magnetic field on tangent hyperbolic fluid velocity (THFV). It is detected that the THFV decline towards higher values of γ. The positive values enhance the Lorentz force and due to this the THF particles faced significant resistance as a results THFV curves shows decline nature. This observation is similar with Li et al. (2016) and Soid et al. (2018). Figure 2 is schemed to report the influence of *Pr* and *Sc* on tangent hyperbolic fluid temperature (THFT) and tangent hyperbolic fluid concentration (THFC) respectively. Both THFT and THFC reflects inverse trends towards positive values of *Pr* and *Sc*. This is due to inverse relation of thermal diffusivity with Pr. Similarly *Sc* variations confirms opposite nature towards mass diffusivity. One can validate these trends with Rehman et al. (2018). The impact of R_c on THFC is examined and provided via Figure 3. One can see that the graphical results are similar with Nayak et al. (2017).

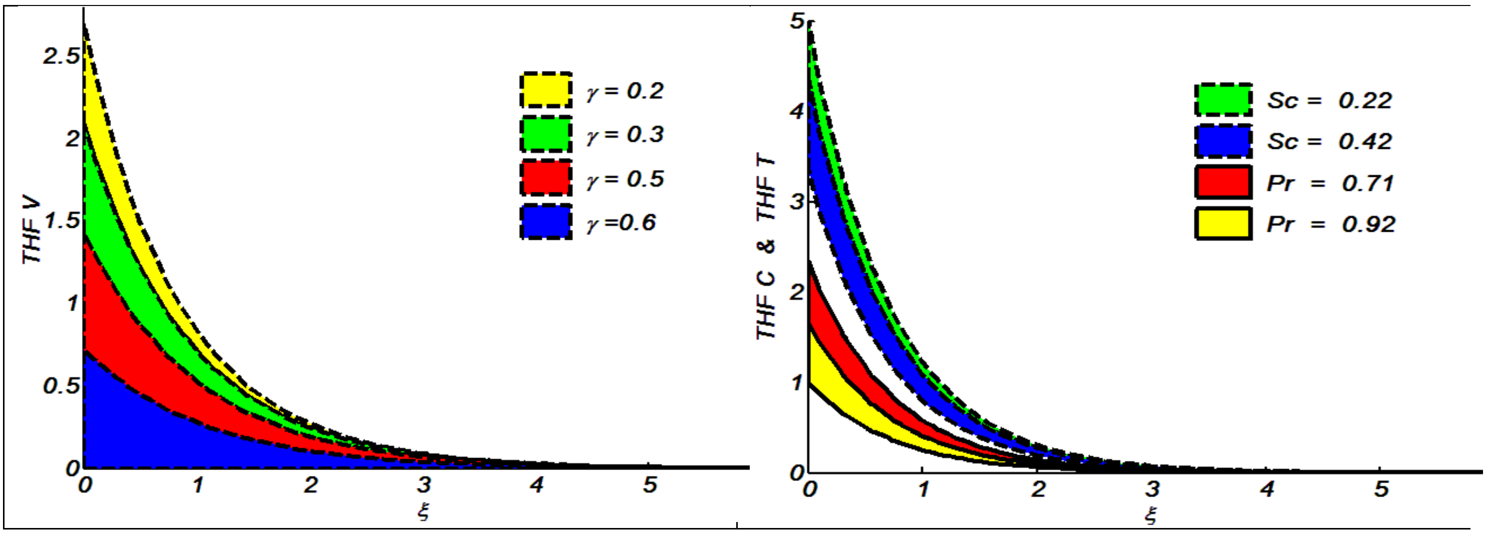

Figure 1. Impact of γ on THFV	**Figure 2.** Impact of Sc and $\Pr$ on THFC and THFT

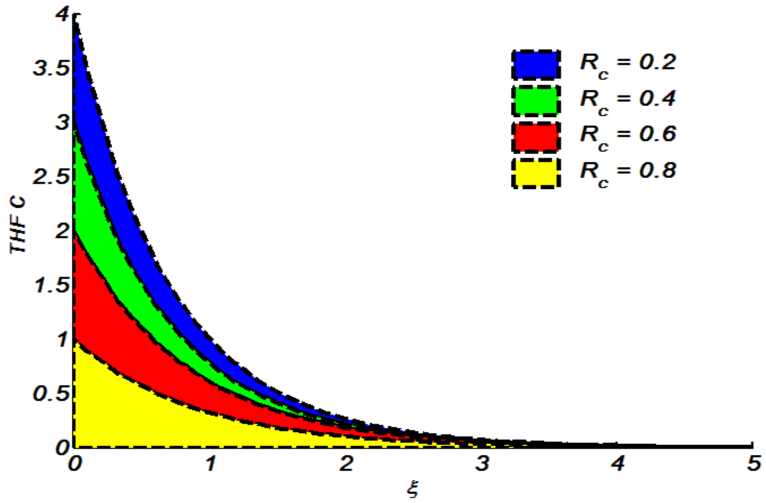

Figure 3. Impact of R_c on THFC.

5. Conclusion

The exact solution for non-linear coupled differential system subject to tangent hyperbolic fluid is not possible. For implementation of computational algorithm one should need to drop number of independent variables via suitable transformation. The strength of present pagination is offering a new scaling group of transformation for the non-Newtonian fluid manifested with magnetic field, heat generation/absorption, chemical reaction, velocity and temperature slips effects. One can extend this idea to various unsolved complex structured problem involved in the field of fluid science.

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