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Local Non-Similar Solution of Powell-Eyring Fluid flow over a Vertical Flat Plate

[1]Hemangini Shukla, [2]Hema C. Surati and [3]M.G. Timol

[1]Department of Mathematics
Government Engineering College
Gandhinagar-382028
Gujarat Technological University
Gujarat, India
Email-hsshukla94@gmail.com

[2]Department of Mathematics
Sarvajanik College of Engineering and Technology
Surat-395001, Gujarat, India
E-mail: hema.surati@scet.ac.in

[3]Department of Mathematics
Veer Narmad South Gujarat University
Magdalla Road
Surat-395007, Gujarat, India
E-mail: mgtimol@gmail.com

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Abstract

Our objective is to obtain the non-similarity solution of non-Newtonian fluid for Powell-Eyring model by a local non-similarity method. Here, free stream velocity is considered in power-law form ($U = x^m$). The governing equations are transformed using non-similar transformations and derived equations are treated as ordinary differential equations. Non-similar solutions are obtained for different values of power-law index m and stream-wise location ξ. Influence of various parameters on velocity and temperature field are presented graphically using MATLAB bvp4c solver.

Keywords: Non-Newtonian fluid; Powell-Eyring model; local non-similarity method; Non-similar solution; similarity solution; free stream velocity

MSC 2010 No.: 35Q35, 76M99, 34B15, 35G30, 65L10

Nomenclature:

A, B - Constants
u ,v - Velocity component in X and Y directions respectively
x, y - Cartesian co-ordinates
ρ - The fluid density
α - The thermal diffusivity
ϑ - kinematic viscosity
T - Temperature of fluid
Ψ - Stream function
ξ, η - Transformed variables
μ - Dynamic viscosity
β, C - Powell-Eyring fluid parameter
U - Free stream velocity
m - Power law index
T_w - Surface temperature
T_∞ - Ambient temperature
f, g, h, θ - Dependent functions
pr - Prandtl number
τ_{yx} - Stress component

1. Introduction

The concept of similarity solution depends on invariance postulate. If any of the governing equations or boundary conditions are not followed by this invariance postulate, in such circumstances similarity solution does not exist, hence one can depend on the non-similarity solution.

Hansen and Na (1968) found a similarity solution of non-Newtonian Powell-Eyring fluid model using a linear group of transformations and observed the possibility of similarity solution only for the flow over a 90-degree wedge. Na (1994) analyzed the two-dimensional Reiner-Philippoff, non-Newtonian fluid model. He found a similarity solution for a boundary layer flow over a 90-degree wedge and non-similarity solution on boundary layer flow over any body shape using the finite difference method. Patil et al. (2015) also conclude that the similarity solution exists only for the flow past a 90-degree wedge for non-Newtonian fluid model characterized by composite and implicit types of stress-strain relationship. He analyzed the three-dimensional Reiner-Philippoff model and established non-similarity on flow past any body shape other than a 90-degree wedge. The similarity solution is found recently by Shukla et al. (2017) for forced convection flow of Powell-Eyring and Prandtl-Eyring model and had taken free stream velocity U in power-law form with power $1/3$ for boundary layer flow over a 90-degree wedge. The similarity solution exists only for the value of $m = 1/3$ when free stream velocity is in power-law form $U = x^m$.

Stream-wise variations in the free-stream velocity, surface mass transfer, transverse curvature, stream-wise variations in surface temperature, surface heat flux, volume heat generation etc… are causes which give non-similarity in a boundary layer (Sparrow et al. 1970, Sparrow and Yu 1971, Massoudi 2001).

Sparrow et al. (1970) had introduced the method of local non-similarity and applied it on different non-similar velocity boundary value problems. Thermal boundary value problems are analyzed using the local non-similarity method by Sparrow and Yu. (1971). Many researchers like to apply the local non-similarity method because of simplicity in concept and computation. There are two attractive features of the method one is local solutions independent of upstream information and second is obtained equations can be treated as ordinary differential equations as in similarity solutions.

Non-Newtonian power law model is investigated by Massoudi (2001). He discussed similarity, local-similarity, and local non-similarity method by considering the free stream velocity, the injection velocity, and the surface temperature as varying functions of the streamwise coordinate x for the flow over a porous wedge. Isomen et al. (2015) done similarity and non-similarity analysis to study the effect of buoyancy force on velocity and temperature for the steady incompressible flow of fluid over an impermeable wedge and obtained ordinary equations are solved using Runge-Kutta Gill with Shooting method.

Governing equations of Casson non-Newtonian fluid flow are converted in non-similar form by Subba Rao et al. (2016) and employed Keller-Box implicit difference method to solve the non-similar equations. The MHD Powell-Eyring fluid flow over a vertical plate in a porous medium is examined numerically by applying the implicit finite difference Keller-Box method by Readdy et al. (2018). Mureithi and Mason (2010) obtained non-similarity solution for a forced–free convection boundary layer flow over a horizontal plate with power-law variations in the freestream velocity and wall temperature using local non-similarity method. Chiam (1993) demonstrated the efficiency of local non-similarity method using the two-equation model and three-equation model by analysing the MHD boundary layer flow over continuously moving flat plate.

Yian & Amin (2002) had applied the local non-similarity method to study laminar free convection boundary layer flow over a vertical flat plate with an exponential variation in surface temperature. Effect of different physical parameters over forced convective Hiemenz flow in porous media is studied by Isomen et al. (2015). By applying the local non-similarity method. Akgul and Pakdemirli (2012) had obtained the local non-similarity solution for the flow of an electrically conducting fluid over a Microcantilever-Based Sensor by considering mass transfer and chemical reaction at the sensor surface.

Abdullah et al. (2018) studied unsteady mixed convection in the stagnation flow on a heated vertical surface embedded in a Nano fluid-saturated porous medium. The governing system of nonlinear partial differential equations is transformed using the Sparrow-Quack-Boerner local non-similarity method and the obtained system is considered as a system of ordinary differential equations.

Since more work is still needed to understand the effect of various parameters involving different non-Newtonian models and the formulation of accurate method of analysis for any body shape of engineering significance. So, from the literature review, we got inspiration to find the non-similar solution and examined effects of the different physical parameter on velocity and temperature profile for two dimensional steady incompressible, laminar flow over a flat plate for Non-Newtonian fluid model namely Powell-Eyring fluid model by applying local non-similarity method. The aim of this paper is to study the boundary layer flow whose non-similarity is caused by variations in the freestream velocity by considering shapes other than a 90-degree wedge by entering the expression of the free stream velocity into a power law form x^m.

2. Governing Equation

The basic equations of continuity, momentum, and energy of two-dimensional, steady, incompressible, laminar flow over a vertical flat plate with a Cartesian co-ordinate system in usual notations are (Shukla et al. (March-2017), MJIS):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\frac{1}{\rho}\frac{\partial}{\partial y}(\tau_{yx}). \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Subject to the boundary conditions:

$$y=0{:}\, u=0, v=0\,, T=T_w. \tag{4}$$

$$y=\infty{:}\, u=U(x), T=T_\infty. \tag{5}$$

Here, u, v are velocity components in the x, y directions respectively. T - The fluid temperature, ρ - The fluid density, α - the thermal diffusivity, T_w- Surface temperature, T_∞ - ambient temperature, U - free stream velocity.

We define stream function $\psi(x, y)$, to reduce one dependent variable which satisfies equation (1).

$$u=\frac{\partial\psi}{\partial y},\; v=-\frac{\partial\psi}{\partial x}. \tag{6}$$

Equations (1) to (5) transform as follows:

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U\frac{dU}{dx}+\frac{1}{\rho}\frac{\partial}{\partial y}(\tau_{yx}). \tag{7}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{8}$$

With boundary conditions:

$$y=0{:}\ \frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial x}=0, T=T_w. \tag{9}$$

$$y = \infty: \frac{\partial \psi}{\partial y} = U(x),\ T = T_\infty. \tag{10}$$

Mathematically, Powell-Eyring model is written as

$$\tau_{yx} = \mu \frac{\partial u}{\partial y} + \frac{1}{\beta} sinh^{-1}(\frac{1}{C}\frac{\partial u}{\partial y}), \tag{11}$$

where μ is dynamic viscosity, β and C are material constants of Powell-Eyring fluid.

$$sinh^{-1}(\frac{1}{C}\frac{\partial u}{\partial y}) \cong \frac{1}{C}\frac{\partial u}{\partial y} - \frac{1}{6}(\frac{1}{C}\frac{\partial u}{\partial y})^3 \text{ for } \left|\frac{1}{C}\frac{\partial u}{\partial y}\right| \leq 1. \tag{12}$$

Substituting values from equations (11) and (12) in equation (7) we get,

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U\frac{dU}{dx} + \frac{1}{\rho}\left(\mu + \frac{1}{\beta C}\right)\frac{\partial^3 \psi}{\partial y^3} - \frac{1}{2\rho\beta C^3}(\frac{\partial^2 \psi}{\partial y^2})^2\frac{\partial^3 \psi}{\partial y^3}. \tag{13}$$

3. Transformation of the Governing equation

Now, defining new variables ξ and η (Akgul and Pakdemirli (2012)).

$$\xi = x^{\frac{1-m}{2}}, \eta = \frac{y\sqrt{U}}{\sqrt{\vartheta x}}, f(\xi,\eta) = \frac{\psi}{\sqrt{x\vartheta U}}, U = x^m, \theta(\xi,\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \tag{14}$$

where variable η is chosen as a similarity variable if the boundary layer is similar which depends on variable x and y. Variable ξ depends only on a variable x.

Applying above transformations we had converted equations (7) to (13) in the following form.

$$(1+A)f''' - AB\xi^{\frac{2(3m-1)}{1-m}}(f'')^2 f''' + m(1-(f')^2) + \frac{m+1}{2}ff''$$
$$= \frac{1-m}{2}\xi[f'\frac{\partial^2 f}{\partial \xi \partial \eta} - f''\frac{\partial f}{\partial \xi}]\ . \tag{15}$$

$$\frac{1}{pr}\theta'' + \frac{m+1}{2}f\theta' = \frac{m-1}{2}\ \xi[\theta'\frac{\partial f}{\partial \xi} - f'\frac{\partial \theta}{\partial \xi}]. \tag{16}$$

With boundary conditions:

$$f(\xi,0) = 0, f'(\xi,0) = 0, f'(\xi,\infty) = 1, \theta(\xi,0) = 1, \theta(\xi,0) = 0, \tag{17}$$

where 'des' on the functions denotes the differentiation with respect to η and

$$A = \frac{1}{\mu\beta c}, B = \frac{1}{2\vartheta c^2}, pr = \frac{\vartheta}{\alpha}.$$

4. Local Non-similarity method

Sparrow et al. (1970, 1971) introduced the so-called method of local non-similarity. Differentiating the original governing equations with respect to a variable ξ and considering the obtained equations as auxiliary equations combined with original equation. Then considering the variable ξ in this partial differential equation to be a constant so, we can reduce the system as a system of ordinary differential equation.

Take $g(\xi,\eta) = \frac{\partial f(\xi,\eta)}{\partial \xi}$, $h(\xi,\eta) = \frac{\partial \theta(\xi,\eta)}{\partial \xi}$ in equation (15) - (17) with boundary conditions. Equations are transformed as follows.

$$(1+A)f''' - AB\xi^{\frac{2(3m-1)}{1-m}}(f'')^2 f''' + m(1-(f')^2) + \frac{m+1}{2} f f'' = \frac{1-m}{2}\xi[f'g' - f''g]. \quad (18)$$

$$\frac{1}{pr}\theta'' + \frac{m+1}{2} f\theta' = \frac{m-1}{2}\xi[\theta' g - f' h]. \quad (19)$$

With boundary conditions:

$$f(\xi,0) = 0, f'(\xi,0) = 0, f'(\xi,\infty) = 1, \theta(\xi,0) = 1, \theta(\xi,0) = 0. \quad (20)$$

Differentiating equations (18)-(19) with boundary conditions with respect to ξ we get

$$(1+A)g''' - AB\xi^{\frac{2(3m-1)}{1-m}}(f'')^2 g''' - AB\frac{2(3m-1)}{1-m}\xi^{\frac{2(3m-1)}{1-m}-1}(f'')^2 f''' - 2AB\xi^{\frac{2(3m-1)}{1-m}} f'' f''' g'' - 2mf'g' + \frac{m+1}{2} g f'' + \frac{m+1}{2} f g'' + \frac{m-1}{2}[f'g' - f''g] = \frac{m-1}{2}\xi\frac{\partial}{\partial \xi}[-f'g' + f''g]. \quad (21)$$

$$\frac{1}{pr}h'' + \frac{m+1}{2} f h' + \frac{m+1}{2} g\theta' - \frac{m-1}{2}[\theta' g - f' h] = \frac{m-1}{2}\xi\frac{\partial}{\partial \xi}[\theta' g - f' h], \quad (22)$$

with

$$f(\xi,0) = 0, f'(\xi,0) = 0, f'(\xi,\infty) = 1, \theta(\xi,0) = 1, \theta(\xi,0) = 0,$$

$$g(\xi,0) = 0, g'(\xi,0) = 0, g'(\xi,\infty) = 0, h(\xi,0) = 0, h(\xi,\infty) = 0. \quad (23)$$

Equations (21) - (23) are auxiliary equations to the governing equations (18) - (20) with their boundary conditions in equation (23). Now, deleting the terms from the auxiliary equations (21) - (22) containing the differentiation with respect to stream-wise co-ordinate from the right- hand side of equations as discussed by Sparrow and Yu (1971). With the above assumption, the momentum and energy boundary-layer equations (18)-(19) and its auxiliary equations (21) - (22) could be brought together with their boundary conditions as

$$(1+A)f''' - AB\xi^{\frac{2(3m-1)}{1-m}}(f'')^2 f''' + m(1-(f')^2) + \frac{m+1}{2}ff''$$
$$-\frac{1-m}{2}\xi[f'g' - f''g] = 0. \quad (24)$$

$$(1+A)g''' - AB\xi^{\frac{2(3m-1)}{1-m}}(f'')^2 g''' - AB\,\frac{2(3m-1)}{1-m}\,\xi^{\frac{2(3m-1)}{1-m}-1}(f'')^2 f''' - 2mf'g'$$
$$-2AB\xi^{\frac{2(3m-1)}{1-m}} f''f'''g'' + \frac{m+1}{2}gf'' + \frac{m+1}{2}fg'' + \frac{m-1}{2}[f'g' - f''g] = 0. \quad (25)$$

$$\frac{1}{pr}\theta'' + \frac{m+1}{2}f\theta' - \frac{m-1}{2}\,\xi[\theta'g - f'h]=0.$$
(26)

$$\frac{1}{pr}h'' + \frac{m+1}{2}fh' + \frac{m+1}{2}g\theta' - \frac{m-1}{2}\,[\theta'g - f'h] = 0,$$
(27)

with

$$f(\xi,0) = 0, f'(\xi,0) = 0, f'(\xi,\infty) = 1, \theta(\xi,0) = 1, \theta(\xi,0) = 0,$$

$$g(\xi,0) = 0, g'(\xi,0) = 0, g'(\xi,\infty) = 0, h(\xi,0) = 0, h(\xi,\infty) = 0. \quad (28)$$

By considering ξ as a constant parameter, equations (24) to (27) may be treated as a system of ordinary differential equations.

5. Result and Discussion

The system of ordinary differential equations (24) to (27) with boundary conditions (28) are solved numerically using MATLAB bvp4c solver. Results are presented graphically for different physical parameters of the flow model. Figure 1 shows the effect of different Prandtl numbers on the temperature profile. Figure 1 indicates that the temperature profile move towards the boundary when Prandtl number is increased. As Physical aspect of Prandtl number this is attributed to the fact that a larger Prandtl number has a relatively lower thermal diffusivity causing a reduction in the thermal boundary layer thickness.

Figure 2 indicates the effect of Powell-Eyring fluid parameter A on temperature profile for $m = 0$ means on a flat plate. From observation of Figure 2 temperature θ enhances when Powell-Eyring fluid parameter A is increased. The thermal boundary layer thickness is negligibly affected by varying fluid parameter B at a wedge of 90 degree depicted from Figure 3. From the zoom area of Figure 3, we observed that as B increases the thermal boundary layer thickness decreases.

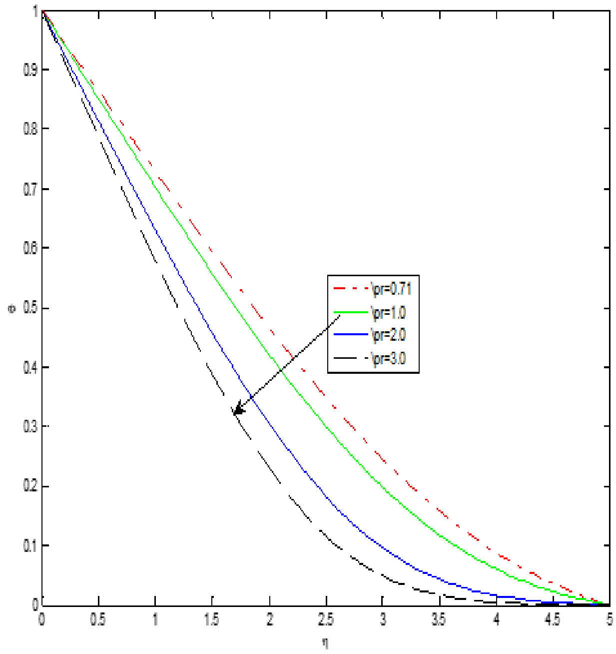

Figure 1. Effect of fluid parameter A on temperature profile for B=0.1, pr =1, m=0, ξ =0.1

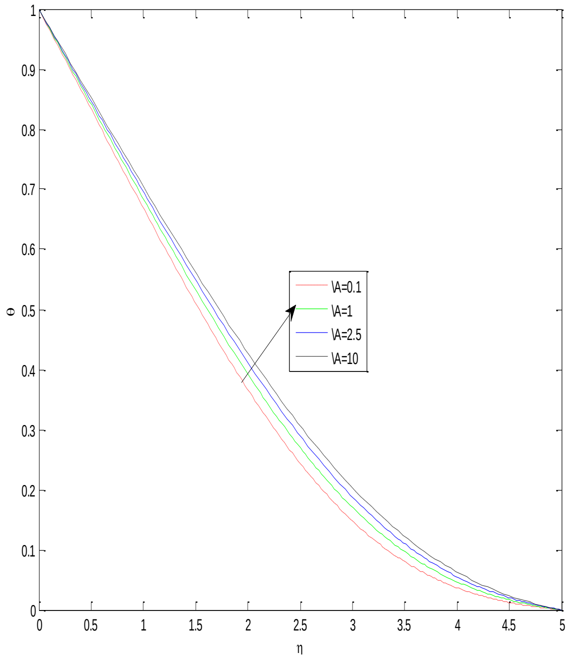

Figure 2. Effect of Prandtl number on temperature profile for A=5, B=0.1, m=0, ξ =0.1

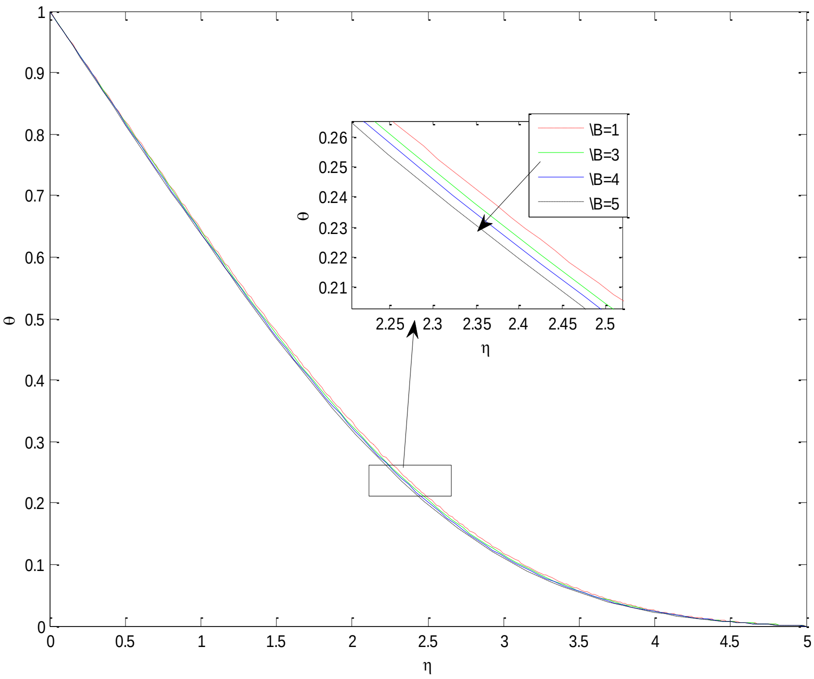

Figure 3. Effect of fluid parameter B on temperature profile for A=5, pr =1, m=1/3, ξ =0.1

The effects of the stream-wise location on the temperature profile are also shown in Figure 4.

Temperature profile enhances with increase of ξ. Effect of the free stream velocity power-law index m on temperature profile is shown in Figure 5. As an increasing value of m thermal boundary layer thickness reduces.

Figures 6 and 7 show the effects of the Powell-Eyring fluid parameters A and B, respectively, on the velocity profile. The velocity profiles decrease as A increases and increases as B increases.

Figure 8 shows the velocity profile for different wedge angle. It is found that velocity increase with the increase of free stream velocity power law index m.

The effects of the stream-wise location on the velocity profile are also shown in Figure 9.

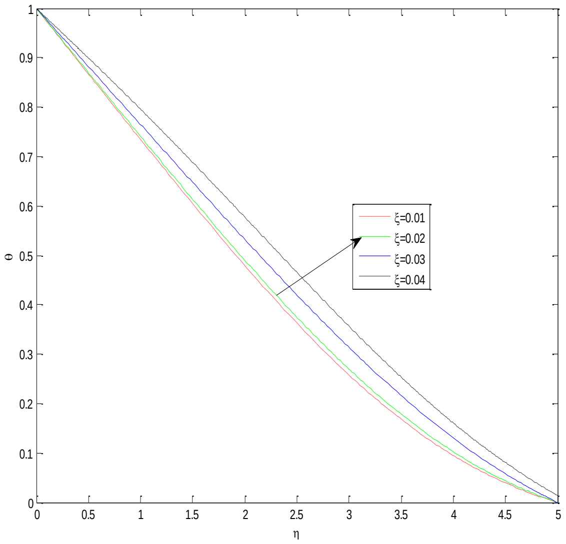

Figure 4. Effect of xi on temperature profile for A=1, B=0.1, pr =0.7, m=0

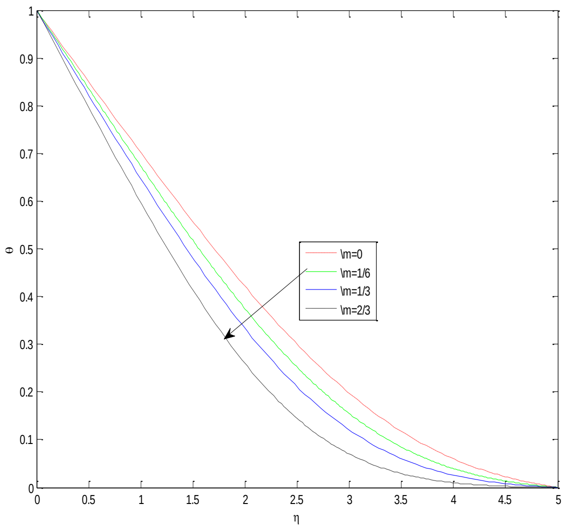

Figure 5. Effect of parameter m on temperature profile A=5, B=1, pr =1, ξ =0.1

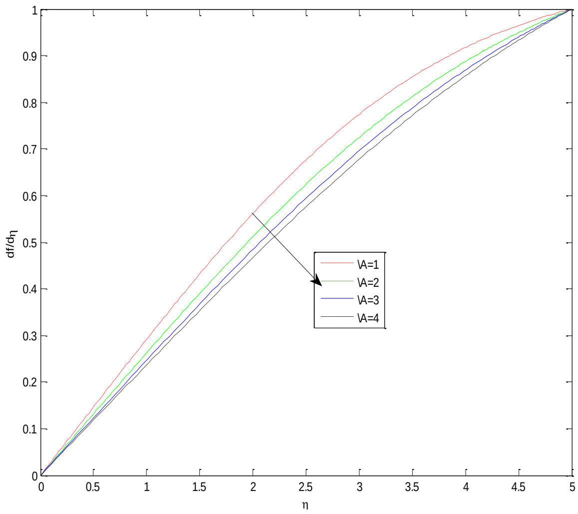

Figure 6. Effect of fluid parameter A on velocity profile for B=0.1, pr =1, m=0, ξ =0.1

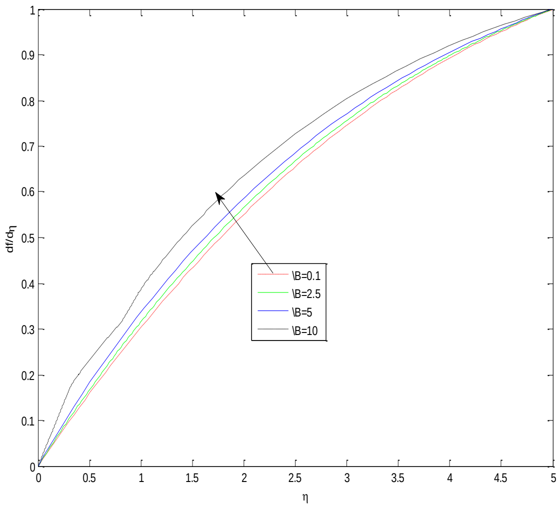

Figure 7. Effect of fluid parameter B on velocity profile for A=5, pr =1, m=1/3, ξ =0.1

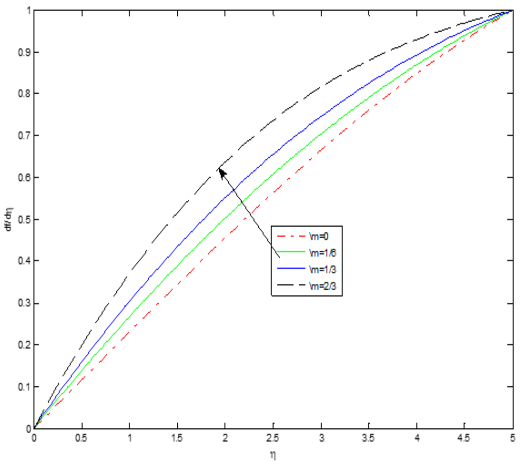

Figure 8. Effect of fluid parameter m on velocity profile for A=5, B=0.1, pr =1, ξ =0.1

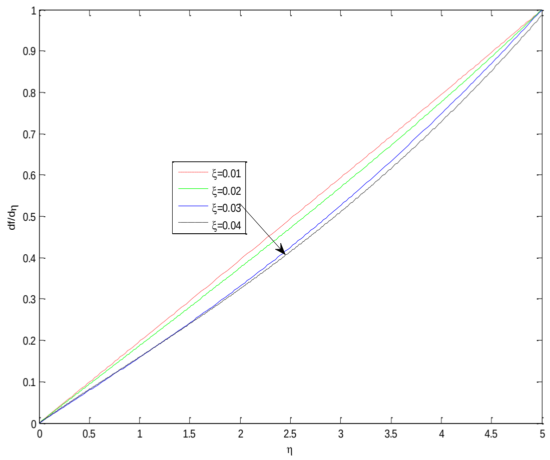

Figure 9. Effect of fluid parameter xi on velocity profile for A=1, B=0.1, pr =0.7, m=0

6. Conclusions

In this paper, we examined the influence of different physical parameters on Powell-Eyring fluid flow. We investigated the Powell-Eyring model by considering free-stream velocity in power-law form and found the non-similarity solution using the local non-similarity method at other than the 90-degree wedge. Transformed governing equations are considered as ordinary differential equations and solved graphically using MATLAB bvp4c solver. We had compared temperature and velocity profile for different values of power-law index *m* that means at different body shape. We also observed the effect of stream-wise co-ordinate on velocity and temperature profile.

REFERENCES

Abdullah Abdullah, Fouad Ibrahim and Ali Chamkha (2018). Non-similar Solution of Unsteady Mixed Convection Flow Near the Stagnation Point of a Heated Vertical Plate in A Porous Medium Saturated with A Nanofluid. Journal of Porous Media, Vol. 21, No. 4, pp. 363–388 .

Akgul, M. B. and Pakdemirli, M. (2012). Magnetohydrodynamic and Slip Effects on the Flow and Mass Transfer over a Microcantilever-Based Sensor. Journal of Applied Mathematics, Vol.2012, Hindawi Publishing Corporation, doi:10.1155/2012(Article ID 289459), 11pages.

Chiam, T. C. (1993). Magnetohydrodynamic Boundary Layer Flow Due to a Continuously Moving Flat Plate . Computer and Mathematics with Application,Vol. 26, No. 4, pp. 1-7.

Hansen, A.G. and Na, T.Y. (1968). Similarity Solutions of Laminar, Incompressible Boundary Layer Equations of Non-Newtonian Fluids Journal of Basic Engineering , ASME, Vol. 90, No. 1, pp. 71-74.

Ismoen, M. , Karim, M. F. , Mohamad, R. and Kandasamy, R. (2015). Similarity and Non-similarity Solutions on Flow and Heat Transfer Over A Wedge with Power Law Stream Condition. International Journal of Innovation in Mechanical Engineering & Advanced Materials (IJIMEAM), Vol.1, No.1, pp.5-12.

Ismoen, M. , Majid, S. M. H. , Mohamad, R. and Kandasamy, R. (2015). Non-similar Boundary Layers over a Wedge due to Thermophoresis and Viscosity Effects. Indian Journal of Science and Technology, 8(32).

Lok Yian Yian and Norsarahaida Amin. (2002). Local Non-similarity Solution for Vertical Free Convection Boundary Layers. Matematika,Vol. 18, pp. 21–31.

Mureithi, E. W. and Mason, D.P. (2010). Local Non-Similarity Solutions for A Forced–Free Boundary Layer Flow with Viscous Dissipation. Mathematical and Computational Applications, Vol. 15, No. 4, pp. 558-573.

Na, T. Y. (1994). Boundary Layer Flow of Reiner-Philippoff Fluids. lnt. Journal of Non-Linear Mechanics, Vol. 29, No. 6, pp. 871-877.

Patil, V. S., Patil, N. S. and Timol, M.G. (2015). A remark on similarity analysis of boundary layer equations of a class of non-Newtonian fluids . International Journal of Non-Linear Mechanics, Vol.71, pp.127–131.

Reddy, M. S. and Nagaraja, L. (2018). Non-similar solution of steady flow of an Eyring-Powell fluid with MHD effect: a vertical porous plate . International Journal of Pure and Applied Mathematics , Vol. 119, No. 17, pp.1653-1667 .

Seshadri, R., and Na, T. Y. (1985). Group Invariance in Engineering Boundary Value Problems. New York: Springer.

Shukla, H. S. , Surati , H. C. and Timol, M. G. (2017). On the solution of forced heat convection Powell Eyring fluid flow by group-theoretic method. Surat, India: PRIME, ISBN:978-81-933591-5-0.

Shukla, H. S., Patel, J. , Surati, H. C., Patel, M. and Timol, M. G. (2017). Similarity Solution of Forced Convection Flow of Powell-Eyring & Prandtl-Eyring Fluids by Group-Theoretic Method. Mathematical Journal of Interdisciplinary Sciences , Vol.5, No. 2, pp.151–165.

Sparrow, E.M. , Quack, H. , and Boerner, C. J. (1970). Local non-similarity boundary-layer solutions. AIAA J ,Vol. 8, No. 11, pp. 1936-1942.

Sparrow, E. M. and Yu, H.S. (1971). Local non-similarity thermal boundary-layer solutions. J. Heat Transfer Trans.ASME , pp.328-334.

Subba Rao, Prasad, R., Nagendra, Reddy, B., and Anwar Beg, O. (2016). Non-Similar Computational Solution for Boundary Layer Flows of Non-Newtonian Fluid from an Inclined Plate with Thermal Slip, Journal of Applied Fluid Mechanics, Vol. 9, No. 2, pp. 795-807.