Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 14, Issue 2 (December 2019), pp. 820 – 835

Regular Semiopen Sets on Intuitionistic Fuzzy Topological Spaces in Ŝostak's Sense

[1]G. Saravanakumar, [2]S. Tamilselvan and [3]A. Vadivel

[1]Department of Mathematics
Annamalai University
Annamalainagar, Tamil Nadu-608 002
saravananguru2612@gmail.com

[2]Mathematics Section (FEAT)
Annamalai University
Annamalainagar, Tamil Nadu-608 002
tamil_au@yahoo.com

[3]Department of Mathematics
Government Arts College (Autonomous)
Karur, Tamil Nadu-639 005
avmaths@gmail.com

Received: September 29, 2018; Accepted: February 8, 2019

Abstract

We introduce the concepts of fuzzy (r, s)-regular semi (resp. (r, s)-α, (r, s)-pre, (r, s)-β) open sets, their respective interior and closure operators on intuitionistic fuzzy topological spaces in Ŝostak's sense and then we investigate some of their characteristic properties.

Keywords: Intuitionistic fuzzy topology in Ŝostak's sense; Fuzzy (r, s)-regular semi open set; Fuzzy (r, s)-regular interior and fuzzy (r, s)-regular closure

MSC 2010 No.: 54A40

1. Introduction

The concept of fuzzy sets was introduced by Zadeh (1965). Chang (1968) defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Ŝostak (1985), used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay et al. (1992) and by Ramadan (1992). As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov (1986). Recently, Coker and Haydar Es (1995), Coker (1997), and Gurcay et al. (1997) introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci (1996) defined intuitionistic fuzzy topological spaces in Ŝostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy (r, s)-regular semi (resp. (r, s)-α, (r, s)-pre, (r, s)-β) open sets, their respective interior and closure operators on intuitionistic fuzzy topological spaces in Ŝostak's sense and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement of $\tilde{1} - \mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair

$$A = (\mu_A, \gamma_A),$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and degree of non-membership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. (Atanassov (1986))

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then,

(i) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$,
(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = (\gamma_A, \mu_A)$,
(iv) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$,
(v) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$,
(vi) $\underset{\sim}{0} = (\tilde{0}, \tilde{1})$ and $\underset{\sim}{1} = (\tilde{1}, \tilde{0})$.

Definition 2.2. (Atanassov (1986))

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be a intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then,

(i) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(ii) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

Definition 2.3. (Ramadan (1992))

A smooth fuzzy topology on X is a map $T : I^X \to I$ which satisfies the following properties:

(i) $T(\tilde{0}) = T(\tilde{1}) = \underset{\sim}{1}$,
(ii) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$,
(iii) $T(\vee \mu_i) \geq \wedge T(\mu_i)$.

The pair (X, T) is called a smooth fuzzy topological space.

Definition 2.4. (Coker and Demirci (1996))

An intuitionistic fuzzy topology on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(i) $\underset{\sim}{0}, \underset{\sim}{1} \in T$,
(ii) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$,
(iii) If $A_i \in T$ for all i, then $\cup A_i \in T$.

The pair (X, T) is called an intuitionistic fuzzy topological space.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.5. (Coker (1997))

Let X be a nonempty set. An intuitionistic fuzzy topology in $\hat{S}$ostak's sense (SoIFT for shor) $T = (T_1, T_2)$ on X is a map $T : I(X) \to I \otimes I$ which satisfies the following properties:

(i) $T_1(\underset{\sim}{0}) = T_1(\underset{\sim}{1}) = 1$ and $T_2(\underset{\sim}{0}) = T_2(\underset{\sim}{1}) = 1$,
(ii) $T_1(A \cap B) \geq T_1(A) \wedge T_1(B)$ and $T_2(A \cap B) \leq T_2(A) \vee T_2(B)$,
(iii) $T_1(\cup A_i) \geq \wedge T_1(A_i)$ and $T_2(\cup A_i) \leq \vee T_2(A_i)$.

The $(X,T) = (X,T_1,T_2)$ is said to be an intuitionistic fuzzy topological space in $\hat{S}$ostak's sense (SoIFTS, for short). Also, we call $T_1(A)$ a gradation of openness of A and $T_2(A)$ a gradation of nonopenness of A.

Definition 2.6. (Lee and Im (2001))

Let A be an intuitionistic fuzzy set in a SoIFTS (X,T_1,T_2) and $(r,s) \in I \otimes I$. Then, A is said to be

(i) fuzzy (r,s)-open if $T_1(A) \geq r$ and $T_2(A) \leq s$,
(ii) fuzzy (r,s)-closed if $T_1(A^c) \geq r$ and $T_2(A^c) \leq s$.

Definition 2.7. (Lee and Im (2001))

Let (X,T_1,T_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy (r,s)-interior is defined by

$$int(A,r,s) = \cup\{B \in I(X) | A \supseteq B, B \text{ is fuzzy}(r,s)\text{-open}\}.$$

and the fuzzy (r,s)-closure is defined by

$$cl(A,r,s) = \cap\{B \in I(X) | A \subseteq B, B \text{ is fuzzy}(r,s)\text{-closed}\}.$$

The operators $int : I(X) \times I \otimes I \to I(X)$ and $cl : I(X) \times I \otimes I \to I(X)$ are called the fuzzy interior operator and fuzzy closure operator in (X,T_1,T_2), respectively.

Lemma 2.8. (Lee and Im (2001))

For an intuitionistic fuzzy set A in a SoIFTS (X,T_1,T_2) and $(r,s) \in I \otimes I$

(i) $int(A,r,s)^c = cl(A^c,r,s)$,
(ii) $cl(A,r,s)^c = int(A^c,r,s)$.

Definition 2.9. (Lee and Im (2001))

Let (X,T) be an intuitionistic fuzzy topological space in $\hat{S}$ostak's sense. Then, it is easy to see that for each $(r,s) \in I \otimes I$, the family $T_{(r,s)}$ defined by

$$T_{(r,s)} = \{A \in I(X) | T_1(A) \geq r \text{ and } T_2(A) \leq s\}.$$

is an intuitionistic fuzzy topology on X.

Definition 2.10. (Lee and Im (2001))

Let (X,T) be an intuitionistic fuzzy topological space $(r,s) \in I \otimes I$. Then, the map $T^{(r,s)}$:

$I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1,0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ (r,s), & \text{if } A \in T - \{\underset{\sim}{0}, \underset{\sim}{1}\}, \\ (0,1), & \text{otherwise,} \end{cases}$$

becomes an intuitionistic fuzzy topology in $\hat{S}$ostak's sense on X.

Definition 2.11. (Lee (2004))

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, A is said to be

(i) fuzzy (r, s)-semi open if there is a fuzzy (r, s)-open set B in X such that $B \subseteq A \subseteq cl(B, r, s)$,
(ii) fuzzy (r, s)-semi closed if there is a fuzzy (r, s)-closed B in X such that $int(B, r, s) \subseteq A \subseteq B$.

3. Fuzzy (r, s)-regular semi (resp. (r, s)-α, (r, s)-pre, (r, s)-β) open sets

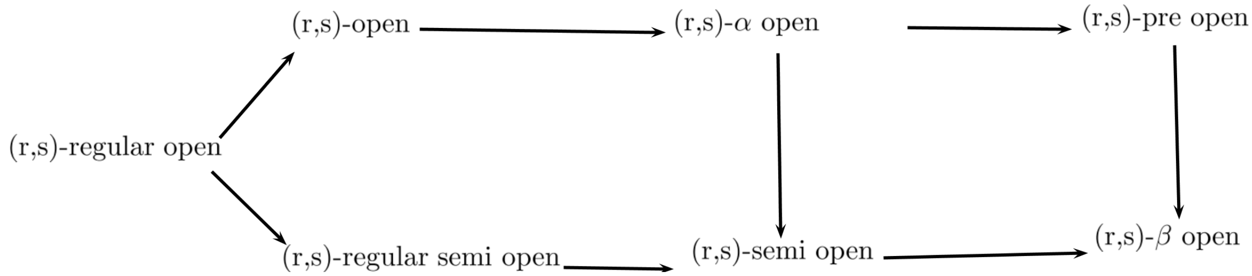
Definition 3.1.

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, A is said to be

(i) fuzzy (r, s)-regular open if $A = int(cl(A, r, s), r, s)$,
(ii) fuzzy (r, s)-regular closed if $A = cl(int(A, r, s), r, s)$,
(iii) fuzzy (r, s)-α open if $A \subseteq int(cl(int(A, r, s), r, s)r, s)$,
(iv) fuzzy (r, s)-α closed if $A \supseteq cl(int(cl(A, r, s), r, s)r, s)$,
(v) fuzzy (r, s)-pre open if $A \subseteq int(cl(A, r, s), r, s)$,
(vi) fuzzy (r, s)-pre closed if $A \supseteq cl(int(A, r, s), r, s)$,
(vii) fuzzy (r, s)-β open if $A \subseteq cl(int(cl(A, r, s), r, s)r, s)$,
(viii) fuzzy (r, s)-β closed if $A \supseteq int(cl(int(A, r, s), r, s)r, s)$,
(ix) fuzzy (r, s)-regular semi open if there is a fuzzy (r, s)-regular open set B in X such that $B \subseteq A \subseteq cl(B, r, s)$,
(x) fuzzy (r, s)-regular semi closed if there is a fuzzy (r, s)-regular closed set B in X such that $int(B, r, s) \subseteq A \subseteq B$.

Remark 3.2.

From the above definitions it is clear that the following implications are true for $(r, s) \in I \otimes I$.



The converses of the above implications are not true as the following examples show.

Example 3.3.

Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.6, 0.3),\ A_1(y) = (0.5, 0.3);$$
$$A_2(x) = (0.8, 0.1),\ A_2(y) = (0.5, 0.1).$$

Define $T : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right), & \text{if } A = A_1, A_2, \\ (0,1), & \text{otherwise.} \end{cases}$$

Then, clearly (T_1, T_2) is a SoIFT on X. The intuitionistic fuzzy set A_1 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-regular open. Also, A_2 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-semi open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-regular semi open.

Example 3.4.

Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.3, 0.6),\ A_1(y) = (0.3, 0.5);$$
$$A_2(x) = (0.4, 0.3),\ A_2(y) = (0.4, 0.3).$$

Define $T : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right), & \text{if } A = A_1, \\ (0,1), & \text{otherwise.} \end{cases}$$

Then, clearly (T_1, T_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-regular semi open and semiopen which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-regular open and α-open.

Example 3.5.

Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.4, 0.3),\ A_1(y) = (0.4, 0.3);$$
$$A_2(x) = (0.3, 0.1),\ A_2(y) = (0.3, 0.1).$$

Define $T : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right), & \text{if } A = A_2, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Then, clearly (T_1, T_2) is a SoIFT on X. The intuitionistic fuzzy set A_1 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-$\alpha$ open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-open.

Example 3.6.

Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.3, 0.6),\ A_1(y) = (0.3, 0.5);$$
$$A_2(x) = (0.1, 0.2),\ A_2(y) = (0.1, 0.2).$$

Define $T : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1, 0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right), & \text{if } A = A_1, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Then, clearly (T_1, T_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-$\beta$-open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-semi open.

Example 3.7.

Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.3, 0.6),\ A_1(y) = (0.3, 0.5);\ A_2(x) = (0.1, 0.3),\ A_2(y) = (0.1, 0.3).$$

Define $T : I(X) \to I \otimes I$ by

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0), & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ \left(\frac{1}{2}, \frac{1}{2}\right), & \text{if } A = A_1, \\ (0,1), & \text{otherwise.} \end{cases}$$

Then, clearly (T_1, T_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-$\beta$-open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-pre open. Also, A_1 is $\left(\frac{1}{2}, \frac{1}{2}\right)$-pre open which is not $\left(\frac{1}{2}, \frac{1}{2}\right)$-$\alpha$-open.

Theorem 3.8.

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, the following statements are equivalent:

(i) A is fuzzy (r, s)-regular semi open,
(ii) $int(A, r, s) = int(cl(A, r, s), r, s)$,
(iii) $\underset{\sim}{1} - A$ is fuzzy (r, s)-regular semi closed,
(iv) $cl(\underset{\sim}{1} - A, r, s) = cl(int(\underset{\sim}{1} - A), r, s), r, s)$.

Proof:

(i)⇔(ii) Let A be an fuzzy (r, s)-regular semi open set. Then, there exists an fuzzy (r, s)-regular open set B such that $B \subseteq A \subseteq cl(B, r, s)$. Hence, $cl(A, r, s) = cl(B, r, s)$. Since

$$\begin{aligned} int(cl(A, r, s), r, s) &= B, \\ int(cl(A, r, s), r, s) &= B \subseteq int(A, r, s) \\ &\subseteq int(cl(B, r, s), r, s) \\ &= int(cl(A, r, s), r, s) \end{aligned}$$

Thus, $int(A, r, s) = int(cl(A, r, s), r, s)$.

(i)⇔(iii) Let A be an fuzzy (r, s)-regular semi open set. Then, there exists an fuzzy (r, s)-regular open set B such that $B \subseteq A \subseteq cl(B, r, s)$. Since $\underset{\sim}{1} - B$ is fuzzy (r, s)-regular closed set, $int(\underset{\sim}{1} - B, r, s)$ is an fuzzy (r, s)-regular open set such that $int(\underset{\sim}{1} - B, r, s) \subseteq \underset{\sim}{1} - A \subseteq \underset{\sim}{1} - B = cl(int(\underset{\sim}{1} - B, r, s), r, s)$. Thus, $\underset{\sim}{1} - A$ is fuzzy (r, s)-regular semi closed. The implication (i)⇔(iv) follow immediately by taking the complement of the two sides. ∎

Theorem 3.9.

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, the following statements are equivalent:

(i) A is fuzzy (r, s)-α open,
(ii) $A \subseteq int(cl(int(A, r, s), r, s)r, s)$,
(iii) $\underset{\sim}{1} - A$ is fuzzy (r, s)-α closed,

(iv) $\underset{\sim}{1} - A \supseteq cl(int(cl(\underset{\sim}{1} - A, r, s), r, s)r, s)$.

Proof:

It follows from Theorem 3.8. ■

Theorem 3.10.

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, the following statements are equivalent:

(i) A is fuzzy (r, s)-pre open,
(ii) $A \subseteq int(cl(A, r, s), r, s)$,
(iii) $\underset{\sim}{1} - A$ is fuzzy (r, s)-pre closed,
(iv) $\underset{\sim}{1} - A \supseteq cl(int(\underset{\sim}{1} - A, r, s), r, s)$.

Proof:

It follows from Theorem 3.8. ■

Theorem 3.11.

Let A be an intuitionistic fuzzy set in a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$. Then, the following statements are equivalent:

(i) A is fuzzy (r, s)-β open,
(ii) $A \subseteq cl(int(cl(A, r, s), r, s)r, s)$,
(iii) $\underset{\sim}{1} - A$ is fuzzy (r, s)-β closed,
(iv) $\underset{\sim}{1} - A \supseteq int(cl(int(\underset{\sim}{1} - A, r, s), r, s)r, s)$,

Proof:

It follows from Theorem 3.8. ■

Theorem 3.12.

Let (X, T_1, T_2) be a SoIFTS and $(r, s) \in I \otimes I$

(i) If A is fuzzy (r, s)-regular open set, then A is fuzzy (r, s)-regular semi open,
(ii) If A is fuzzy (r, s)-regular semi open and $int(A, r, s) \subseteq B \subseteq cl(A, r, s)$, then B is fuzzy (r, s)-regular semi open,
(iii) If A is fuzzy (r, s)-regular semi closed and $int(A, r, s) \subseteq B \subseteq cl(A, r, s)$, then B is fuzzy (r, s)-regular semi closed.

Proof:

(i) Let A be a fuzzy (r, s)-regular open, since $A \subseteq A \subseteq cl(A, r, s)$. Then, A is fuzzy (r, s)-regular

semi open.
(ii) Let A be a fuzzy (r,s)-regular semi open set and $int(A,r,s) \subseteq B \subseteq cl(A,r,s)$. Then, there is a fuzzy (r,s)-regular open set C such that $C \subseteq A \subseteq cl(C,r,s)$. It follows that

$$C = int(C,r,s) \subseteq int(A,r,s) \subseteq A \subseteq cl(A,r,s) \subseteq cl(cl(C,r,s),r,s) = cl(C,r,s),$$

and hence,

$$C \subseteq int(A,r,s) \subseteq B \subseteq cl(A,r,s) \subseteq cl(C,r,s).$$

Thus, B is a fuzzy (r,s)-regular semi open set.
(iii) Similar to (ii). ■

Theorem 3.13.

Let (X, T_1, T_2) be a SoIFTS and $(r,s) \in I \otimes I$.

(i) If $\{A_i\}$ is a family of fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - open sets of X, then $\bigcup A_i$ is fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - open,
(ii) If $\{A_i\}$ is a family of fuzzy (r,s)-regular semi(resp. (r,s)-α, (r,s)-pre and (r,s)-β) - closed sets of X, then $\bigcap A_i$ is fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - closed.

Proof:

(i) Let $\{A_i\}$ be a collection of fuzzy (r,s)-regular semi open sets. Then, for each i, there is a fuzzy (r,s)-regular open set B_i such that $B_i \subseteq A_i \subseteq cl(B_i,r,s)$. Since $T_1(\bigcup B_i) \geq \bigwedge T_1(B_i) \geq r$ and $T_2(\bigcup B_i) \subseteq \bigvee T_2(B_i) \leq s$, $\bigcup B_i$ is a fuzzy (r,s)-regular open set. Moreover,

$$\bigcup B_i \subseteq \bigcup A_i \subseteq \bigcup cl(B_i,r,s) \subseteq cl(\bigcup B_i,r,s).$$

Hence, $\bigcup A_i$, is a fuzzy (r,s)-regular semi open set.
(ii) Similar (i)

The other cases are similar as in (i) and (ii). ■

Definition 3.14.

Let (X, T_1, T_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - interior is defined by

$$rsint(A,r,s) \text{ (resp. } \alpha int(A,r,s),\ pint(A,r,s) \text{ and } \beta int(A,r,s)) = \bigcup\{B \in I(X) | A \supseteq B,\ B \text{ is fuzzy } (r,s)\text{- regular semi (resp. } \alpha,\ \text{pre and } \beta) \text{ -open}\},$$

and the fuzzy (r,s)-regular semi (resp. (r,s)-α, (r,s)-pre and (r,s)-β) - closure is defined by

$$rscl(A,r,s) \text{ (resp. } \alpha cl(A,r,s),\ pcl(A,r,s) \text{ and } \beta cl(A,r,s)) = \bigcap\{B \in I(X) | A \subseteq B,\ B \text{ is fuzzy } (r,s)\text{-regular semi(resp. } \alpha,\ \text{pre and } \beta) \text{ -closed}\}.$$

Obviously $rscl(A,r,s)$ (respectively, $\alpha cl(A,r,s)$, $pcl(A,r,s)$ and $\beta cl(A,r,s)$) is the smallest fuzzy (r,s)-regular semi (respectively, α, pre and β) - closed set which contains A and $rsint(A,r,s)$ (resp. $\alpha int(A,r,s)$, $pint(A,r,s)$ and $\beta int(A,r,s)$) is the greatest fuzzy (r,s)-regular semi (respectively, α, pre and β) - open set which is contained in A. Also, $rscl(A,r,s)$ (respectively, $\alpha cl(A,r,s)$, $pcl(A,r,s)$ and $\beta cl(A,r,s)$) $=$ A for any fuzzy (r,s)-regular semi (respectively, α, pre and β) - closed set A and $rsint(A,r,s)$ (respectively, $\alpha int(A,r,s)$, $pint(A,r,s)$ and $\beta int(A,r,s)$) $= A$ for any fuzzy (r,s)-regular semi (respectively, α, pre and β) - open set A. Moreover, we have

$$int(A,r,s) \subseteq rsint(A,r,s) \subseteq A \subseteq rscl(A,r,s) \subseteq cl(A,r,s),$$

$$int(A,r,s) \subseteq \alpha int(A,r,s) \subseteq A \subseteq \alpha cl(A,r,s) \subseteq cl(A,r,s),$$

$$int(A,r,s) \subseteq pint(A,r,s) \subseteq A \subseteq pcl(A,r,s) \subseteq cl(A,r,s),$$

$$int(A,r,s) \subseteq \beta int(A,r,s) \subseteq A \subseteq \beta cl(A,r,s) \subseteq cl(A,r,s).$$

Also, we have the following results:

(i) $rscl(\underline{0},r,s) = \underline{0},\ rscl(\underline{1},r,s) = \underline{1}$,
(ii) $rscl(A,r,s) \supseteq A$,
(iii) $rscl(A \cup B,r,s) \supseteq rscl(A,r,s) \cup rscl(B,r,s)$,
(iv) $rscl(rscl(A,r,s),r,s) = rscl(A,r,s)$,
(v) $rsint(\underline{0},r,s) = \underline{0},\ rsint(\underline{1},r,s) = \underline{1}$,
(vi) $rsint(A,r,s) \subseteq A$,
(vii) $rsint(A \cap B,r,s) \subseteq rsint(A,r,s) \cap rsint(B,r,s)$,
(viii) $rsint(rsint(A,r,s),r,s) = rsint(A,r,s)$.

Theorem 3.15.

Let (X,T_1,T_2) be SoIFTS. For $A,\ B \in I^X$ and $(r,s) \in I \otimes I$. It satisfies the following statements:

(i) A is fuzzy (r,s)-regular semi open $\Leftrightarrow A = rsint(A,r,s)$,
(ii) A is fuzzy (r,s)-regular semi closed $\Leftrightarrow A = rscl(A,r,s)$,
(iii) $rscl(\underline{0},r,s) = \underline{0}$,
(iv) $rint(A,r,s) \subseteq rsint(A,r,s) \subseteq A \subseteq rscl(A,r,s) \subseteq rcl(A,r,s)$,
(v) $rscl(A,r,s) \cup rscl(B,r,s) \subseteq rscl(A \cup B,r,s)$,
(vi) $rcl(rscl(A,r,s),r,s) = rscl(rcl(A,r,s),r,s) = rcl(A,r,s)$.

Proof:

(i) Let A be fuzzy (r,s)-regular semi open. Then,

$$\begin{aligned} rsint(A,r,s) &= \bigcup\{G \in I^X : G \subseteq A,\ G \text{ is fuzzy } (r,s) \text{ -regular semi open}\} \\ &= A. \end{aligned}$$

Conversely, let $A = rsint(A,r,s)$. Since $rsint(A,r,s)$ is the arbitrary union of fuzzy (r,s)-regular semi open, then A is fuzzy (r,s)-regular semi open.

(ii) It is similar to part (i).

(iii) It is easily obtained from Definition 3.1.

(iv) Since

$$\begin{aligned} rint(A,r,s) &= \bigcup\{G \in I^X : G \subseteq A,\ G \text{ is fuzzy } (r,s) \text{ -regular open}\} \\ &\subseteq \bigcup\{G \in I^X : G \subseteq A, G \text{ is fuzzy } (r,s) \text{ -regular semi open}\} \\ &= rsint(A,r,s), \end{aligned}$$

it follows that, $rint(A,r,s) \subseteq rsint(A,r,s)$. Also,

$$\begin{aligned} rscl(A,r,s) &= \bigcap\{G \in I^X : G \supseteq A,\ G \text{ is fuzzy } r \text{ -regular semi closed}\} \\ &\subseteq rcl(A,r,s). \end{aligned}$$

Finally, we have

$$rint(A,r,s) \subseteq rsint(A,r,s) \subseteq A \subseteq rscl(A,r,s) \subseteq rcl(A,r,s).$$

(v) Since, $B \subseteq B \vee G,\ G \subseteq B \vee G$, then,

$$rscl(B,r,s) \subseteq rscl(B \cup G,r,s) \text{ and } rscl(G,r,s) \subseteq rscll(B \cup G,r,s).$$

Hence, $rscl(B,r,s) \cup rscl(G,r,s) \subseteq rscl(B \cup G,r,s)$.

(vi) Since $rcl(A,r,s)$ is fuzzy (r,s)-regular semi closed set, then

$$rscl(rcl(A,r,s),r,s) = rcl(A,r,s) \tag{1}$$

Now it remains to prove only the relation:

$$rcl(rscl(A,r,s),r,s) = rcl(A,r,s).$$

Since, $A \subseteq rscl(A,r,s)$, then $rcl(A,r,s) \subseteq rcl(rscl(A,r,s)$. It remains to prove: $rcl(rscl(A,r,s),r,s) \subseteq rcl(A,r,s)$. Let the contrary, that is, $rcl(rscl(A,r,s),r,s) \nsubseteq rcl(A,r,s)$.

Then, $rcl(rscl(A,r,s),r,s) \supset rcl(A,r,s)$. So, there exists fuzzy (r,s)-regular closed set $G \in I^X$, $G \supseteq A$ such that

$$rcl(A,r,s)(x) \subset G(x) \subset rcl(rscl(A,r,s),r,s)(x). \quad (2)$$

Since $A \subseteq G \Rightarrow rscl(A,r,s) \subseteq rscl(G,r,s) = rscl(rcl(A,r,s),r,s) = rcl(G,r,s)$, then, $rscl(A,r,s) \subseteq rcl(G,r,s)$ and this implies $rcl(rscl(A,r,s),r,s) \subseteq rcl(A,r,s)$, which contradicts to the relation (2). Hence, the result. ■

Theorem 3.16.

Let (X,T_1,T_2) be SoIFTS. For $A,\ B \in I^X$ and $(r,s) \in I \otimes I$. It satisfies the following statements:

(i) A is fuzzy (r,s)-αopen $\Leftrightarrow A = \alpha int(A,r,s)$,
(ii) A is fuzzy (r,s)-α closed $\Leftrightarrow A = \alpha cl(A,r,s)$,
(iii) $\alpha cl(\mathbb{0},r,s) = \mathbb{0}$,
(iv) $int(A,r,s) \subseteq \alpha int(A,r,s) \subseteq A \subseteq \alpha cl(A,r,s) \subseteq cl(A,r,s)$,
(v) $\alpha cl(A,r,s) \cup \alpha cl(B,r,s) \subseteq \alpha cl(A \cup B,r,s)$,
(vi) $\alpha cl(\alpha cl(A,r,s),r,s) = \alpha cl(cl(A,r,s),r,s) = cl(A,r,s)$.

Proof:

It follows from Theorem 3.15. ■

Theorem 3.17.

Let (X,T_1,T_2) be SoIFTS. For $A,\ B \in I^X$ and $(r,s) \in I \otimes I$. It satisfies the following statements:

(i) A is fuzzy (r,s)-pre open $\Leftrightarrow A = pint(A,r,s)$,
(ii) A is fuzzy (r,s)-pre closed $\Leftrightarrow A = pcl(A,r,s)$,
(iii) $pcl(\mathbb{0},r,s) = \mathbb{0}$,
(iv) $int(A,r,s) \subseteq pint(A,r,s) \subseteq A \subseteq pcl(A,r,s) \subseteq cl(A,r,s)$,
(v) $pcl(A,r,s) \cup pcl(B,r,s) \subseteq pcl(A \cup B,r,s)$,
(vi) $pcl(pcl(A,r,s),r,s) = pcl(cl(A,r,s),r,s) = cl(A,r,s)$.

Proof:

It follows from Theorem 3.15. ■

Theorem 3.18.

Let (X,T_1,T_2) be SoIFTS. For $A,\ B \in I^X$ and $(r,s) \in I \otimes I$. It satisfies the following statements:

(i) A is fuzzy (r,s)-βopen $\Leftrightarrow A = \beta int(A,r,s)$,
(ii) A is fuzzy (r,s)-β closed $\Leftrightarrow A = \beta cl(A,r,s)$,
(iii) $\beta cl(\mathbb{0},r,s) = \mathbb{0}$,
(iv) $int(A,r,s) \subseteq \beta int(A,r,s) \subseteq A \subseteq \beta cl(A,r,s) \subseteq cl(A,r,s)$,

(v) $\beta cl(A, r, s) \cup \beta cl(B, r, s) \subseteq \beta cl(A \cup B, r, s)$,
(vi) $\beta cl(\beta cl(A, r, s), r, s) = \beta cl(cl(A, r, s), r, s) = cl(A, r, s)$.

Proof:

It follows from Theorem 3.15. ∎

Theorem 3.19.

For an intuitionistic fuzzy set A of a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$, we have:

(i) $rsint(A, r, s)^c = rscl(A^c, r, s)$,
(ii) $rscl(A, r, s)^c = rsint(A^c, r, s)$.

Proof:

(i) Since $rsint(A, r, s) \subseteq A$ and $rsint(A, r, s)$ is fuzzy (r, s)-regular semi open in X, $A^c \subseteq rsint(A, r, s)^c$ and $rsint(A, r, s)^c$ is fuzzy (r, s)-regular semi closed in X. Thus,

$$\begin{aligned} rscl(A^c, r, s) &\subseteq rscl(rsint(A, r, s)^c, r, s) \\ &= rsint(A, r, s)^c \end{aligned}$$

Conversely, since $A^c \subseteq rscl(A^c, r, s)$ and $rscl(A^c, r, s)$ is fuzzy (r, s)-regular semi closed in X, $rscl(A^c, r, s)^c \subseteq A$ and $rscl(A^c, r, s)^c$ is fuzzy (r, s)-regular semi open in X. Thus,

$$\begin{aligned} rscl(A^c, r, s)^c &= rsint(rscl(A^c, r, s)^c, r, s) \\ &\subseteq rsint(A, r, s) \end{aligned}$$

and hence, $rsint(A, r, s)^c \subseteq rscl(A^c, r, s)$
(ii) Similar to (i). ∎

Theorem 3.20.

For an intuitionistic fuzzy set A of a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$, we have:

(i) $\alpha int(A, r, s)^c = \alpha cl(A^c, r, s)$,
(ii) $\alpha cl(A, r, s)^c = \alpha int(A^c, r, s)$.

Proof:

It follow from Theorem 3.19. ∎

Theorem 3.21.

For an intuitionistic fuzzy set A of a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$, we have:

(i) $pint(A, r, s)^c = pcl(A^c, r, s)$,
(ii) $pcl(A, r, s)^c = pint(A^c, r, s)$.

Proof:

It follow from Theorem 3.19. ■

Theorem 3.22.

For an intuitionistic fuzzy set A of a SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$, we have:

(i) $\beta int(A, r, s)^c = \beta cl(A^c, r, s)$,
(ii) $\beta cl(A, r, s)^c = \beta int(A^c, r, s)$.

Proof:

It follow from Theorem 3.19. ■

4. Conclusion

In the present paper we introduced fuzzy (r, s)-regular semi (respectively, (r, s)-α, (r, s)-pre, (r, s)-β) open sets, their respective interior and closure operators on intuitionistic fuzzy topological spaces in Ŝostak's sense. We investigate some of their characteristic properties and establish the relations between them with some counter examples.

Acknowledgment:

The authors wish to thank the reviewers for their careful reviews which improve the presentation of this paper considerably.

REFERENCES

Atanassov, K. (1986). Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol. 20, pp. 87–96.

Chang, C.L. (1968). Fuzzy topological spaces, J. Math. Anal. Appl, Vol. 24, pp. 182–190.

Chattopadhyay, K. C., Harza, R. N. and Samanta, S. K. (1992). Gradation of openness: fuzzy topology, Fuzzy Sets and Systems, Vol. 49, pp. 237–242.

Coker, D. (1997). An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, Vol. 88, pp. 81–89.

Coker, D. and Demirci, M. (1996). An introduction to intuitionistic fuzzy topologica spaces in Ŝostak's sense, BUSEFAL, Vol. 67, pp. 67–76.

Coker, D. and Haydar Es, A. (1995). On fuzzy compactness in intuitionistic fuzzy topological spaces, J. Fuzzy Math, Vol. 3, pp. 899–909.

Gurcay, H., Coker, D. and Haydar Es, A. (1997). On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math, Vol. 5, pp. 365–378.

Lee, E.P. (2004). Semiopen sets on intuitionistic fuzzy topological spaces in Ŝostak's sense, Journal of Korean institute of Intelligent Systems, Vol. 11, No. 2, pp. 234–238.

Lee, E.P. and Im, Y.B. (2001). Mated fuzzy topological spaces, J. of Fuzzy Logic and Intelligent Systems, Vol. 11, pp. 161–165.

Ramadan, A.A. (1992). Smooth topological spaces, Fuzzy Sets and Systems, Vol. 48, pp. 371–375.

Ŝostak, A. (1985). On a fuzzy topological structure, Supp. Rend. Circ. Math. Palermo (Ser. II), Vol. 11, pp. 89–103.

Zadeh, L.A. (1965). Fuzzy sets, Information and Control, Vol. 8, pp. 338–353.