Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 14, Issue 2 (December 2019), pp. 672 – 686

An $(S-1, S)$ Inventory System with Negative Arrivals and Multiple Vacations

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Received: September 15, 2018; Accepted: November 11, 2019

Abstract

In this paper, we consider a continuous review one-to-one ordering policy inventory system with multiple vacations and negative customers. The maximum storage capacity is S. The customers arrive according to a Poisson process with finite waiting hall. There are two types of customers: ordinary and negative. An ordinary customer, on arrival, joins the queue and the negative customer does not join the queue and takes away any one of the waiting customers. When the waiting hall is full, the arriving primary customer is considered to be lost. The service time and lead time are assumed to have independent exponential distribution. When the inventory becomes empty, the server takes a vacation and the vacation duration is exponentially distributed. The stationary distribution of the number of customers in the waiting hall, the inventory level and the server status for the steady state case. Some system performance measures and numerical illustrations are discussed.

Keywords: Base Stock Policy; Positive leadtime; Continuous review inventory system; Multiple vacations; Negative Customers

MSC 2010 No.: 90B05, 60J27

1. Introduction

The $(S-1,S)$ policy have been studied extensively by Schmidt and Nahmias (1985), Pal (1989), Kalpakam and Sapna (1995) and Kalpakam and Sapna (1996). In all these models, whenever the inventory level drops by one unit, either due to a demand or a failure, an order for one item is placed. Kalpakam and Arivaringnan (1998) discussed with a $(S-1,S)$ system with renewal demands for non-perishable items. Kalpakam and Shanthi (2000) have considered the modified base stock policy and random supply quantity. Recently, Gomathi et al. (2012) considered a two commodity inventory system for base-stock policy with service facility.

Berman and Kim (1999) considered a queueing - inventory system with Poisson arrivals, exponential service times and zero lead times. Berman and Sapna (2000) studied the concept of "queueing - inventory" system with service facility. Krishnamoorthy and Anbazhagan (2008) analyzed a perishable queueing inventory system with N policy, Poisson arrivals, exponential distributed lead times and service times. Yadavalli et al. (2018) considered a service facility inventory system with two stage services and repeated attempt for the customers. Jeganathan et al. (2013) and Jeganathan et al. (2016) studied a retrial inventory system with non-preemptive priority service and optional service provided for the customers.

In all the above models, the authors have considered that the arrival of customers to the service station should join the system until it is full. The customers who arrive at the service station are classified as ordinary (positive or regular) and negative customers. The arrival of ordinary customers to the service station increases the queue length by one and the arrival of negative customer to the service station causes to remove one ordinary customer, if any is present.This type of customer is called negative customer. Since the work is analysed by Gelenbe (1991), the research on queueing systems with negative arrivals has been greatly motivated by some practical applications in computers, neural networks and communication networks, etc. For comprehensive literature on queueing networks with negative arrivals, one may refer to Chao et al. (1999) and Gelenba (1998). A recent review can be found in Artalejo (2000).

In several situations, the server is unavailable to the customers due to server's failure, which may be engaged in other works such as maintenance or serving secondary customers, or may just go away and may not be waiting. The aim of studying the queueing model with vacation for utilizing the idle time of the server, by which the total average cost involved may be minimized. Applications arise naturally in call centres with multitask employees, customized manufacturing, telecommunication and computer networks, maintenance activities, production and quality control problems, etc. Danial and Ramanarayanan (1987) and Daniel and Ramanarayanan (1988) have first introduced the server vacation in inventory with two servers. Also they have studied an inventory system in which the server takes rest when the level of the inventory is zero. They assumed that the demands that occurred during stock-out period are assumed to be lost. Narayanan et al. (2008) studied on an (s,S) inventory policy with service time, vacation to server and correlated lead time. Sivakumar (2011) has considered a retrial inventory system with multiple server vacaion. He has assumed Poisson arrival and exponential service time. Padmavathi et al. (2015) developed a retrial inventory system with single and modified multiple vacation. They have assumed that two different vacation models when the inventory level zero. Rustamov and Adamov (2017) deal with a numerical method to perisable inventory systems with server vacations.

In real life situations, the sale agencies deal with two different items with high cost like email server and data server, refrigerator and washing machine, etc. Keeping them in stock for sales purpose is high risk but yields high profit, wherein the waiting customers may be wooed or taken away by new arriving customers from a large population. Many companies are looking for prospective customers at others' sales centers. Motivated by such situations, this paper focuses on $(S-1, S)$ policy stochastic inventory system under continuous review at a service facility with a finite waiting hall for customers. The customers arriving to the service station are classified as ordinary and negative customers. The server takes a vacation of exponential length each time when the inventory level becomes empty. When the vacation ends he finds the inventory level is still zero and the server takes another vacation; otherwise, he terminates his vacation is ready to serve any arriving demands. The joint probability distribution of the number of customers in the waiting hall, the inventory level and the server status is obtained for the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

The remainder of this paper is organized as follows. In Section 2, we present the mathematical model and the notations. Analysis of the model and the steady state solution are given in Section 3. In Section 4, we derive various measures of system performance in steady state. In Section 5, the total expected cost rate is calculated and numerical study is presented. The last section is meant for conclusion.

2. Model description

We consider a single server continuous review stochastic inventory system adopted by one-to-one ordering policy. According to this policy, orders are placed for one unit as and when the inventory level drops due to a demand. The maximum inventory level is denoted by S. Customers arrive at the system one by one in according to a Poisson stream with arrival rate $\lambda(>0)$. If the server is busy, the arriving customer first try to queue up in a finite waiting room of capacity M. Finding that at full, he considered to be lost. The probability that a customer is an ordinary is p and a negative is $q(=1-p)$. We have assumed that the negative customer removes any one of the ordinary waiting customers from the system including the one at the service point. Each customer requires a single item having random duration of service which follows an exponential distribution with parameter μ. The lead time is exponentially distributed with the rate β. The server takes a vacation of random duration once the inventory level becomes empty. On return from vacation, if the inventory level is positive, otherwise he takes another vacation. The vacation duration is exponentially distributed with rate θ. We assume that the inter-demand times between primary customers, the lead times, service times and the sever vacation times are mutually independent random variables.

Notations

$[A]_{ij}$: The element/submatrix at (i, j)th position of A.

$\mathbf{0}$	:	Zero matrix.
$\mathbf{I}$	:	Identity matrix.
$\mathbf{e}$	:	A column vector of $1'$s of appropriate dimension.
δ_{ij}	:	$\begin{cases} 1, \text{ if } i = j. \\ 0, \text{ otherwise.} \end{cases}$
$\bar{\delta}_{ij}$	:	$1 - \delta_{ij}$,
$Y(t)$	:	$\begin{cases} 0, & \text{if server is on vacation at time } t. \\ 1, & \text{if server is not on vacation at time } t. \end{cases}$

3. Analysis

Let $X(t)$, $L(t)$ and $Y(t)$ denote the number of customers in the waiting hall, the inventory level of the commodity and the server status at time t. From the assumptions made on the input and output processes, it can be shown that the triplet $\{(X(t), L(t), Y(t)),\ t \geq 0\}$ is a continuous time Markov chain with state space given by E,

$$E = \{(i, 0, 0) : i = 0, 1, 2, \ldots, M\} \cup \{(i, k, m) : i = 0, 1, 2, \ldots, M, \quad k = 1, 2, \ldots, S, \quad m = 1, 0\}.$$

To determine the infinitesimal generator,

$$P = (h((i, k, m), (j, l, n))), \qquad (i, k, m), (j, l, n) \in E,$$

of this process we use the following arguments:

∗ Transitions due to the arrival of an ordinary customers:

- $(i, k, m) \to (i + 1, k, m)$: the rate is $p\lambda$, for $0 \leq i \leq M - 1$, $1 \leq k \leq S$, $m = 1, 0$.
- $(i, 0, 0) \to (i + 1, 0, 0)$: the rate is $p\lambda$, for $0 \leq i \leq M - 1$.

∗ Transitions due to the arrival of a negative customers:

- $(i, k, m) \to (i - 1, k, m)$: the rate is $q\lambda$, for $1 \leq i \leq M$, $1 \leq k \leq S$, $m = 1, 0$.
- $(i, 0, 0) \to (i - 1, 0, 0)$: the rate is $q\lambda$, for $1 \leq i \leq M$.

∗ Transitions due to service completion in the system:

- $(i, k, 1) \to (i - 1, k - 1, 1)$: the rate is μ, for $1 \leq i \leq M$, $2 \leq k \leq S$.
- $(i, 1, 1) \to (i - 1, 0, 0)$: the rate is μ, for $1 \leq i \leq M$.

∗ Transitions due to replenishments:

- $(i,k,m) \rightarrow (i,k+1,m)$: the rate is $(S-k)\beta$, for $0 \le i \le M$, $1 \le k \le S$, $m = 1, 0$.
- $(i,0,0) \rightarrow (i,1,0)$: the rate is $S\beta$, for $0 \le i \le M$.

$*$ Transitions due to vacation completion:

- $(i,k,0) \rightarrow (i,k,1)$: the rate is θ, for $0 \le i \le M$, $1 \le k \le S$.

$*$ We observe that no transition other than the above is possible.

Hence, we have $h((i,k,m),(j,l,n))$=

$$
\begin{cases}
p\lambda, & \begin{array}{lll} j=i+1, & l=k, & n=m, \\ i=0,1,\dots,M-1, & k=S,S-1,\dots,1, & m=1,0, \end{array} \\
 & \qquad\qquad \textit{or} \\
 & \begin{array}{lll} j=i+1, & l=k, & n=m, \\ i=0,1,\dots,M-1, & k=0, & m=0, \end{array} \\
q\lambda, & \begin{array}{lll} j=i-1, & l=k, & n=m, \\ i=1,\dots,M, & k=S,S-1,\dots,1, & m=1,0, \end{array} \\
 & \qquad\qquad \textit{or} \\
 & \begin{array}{lll} j=i-1, & l=k, & n=m, \\ i=1,\dots,M, & k=0, & m=0, \end{array} \\
\theta, & \begin{array}{lll} j=i, & l=0, & n=1, \\ i=0,1,2,\dots,M, & k=S,S-1,\dots,1, & m=0, \end{array} \\
\mu, & \begin{array}{lll} j=i-1, & l=k-1, & n=m, \\ i=1,2,\dots,M, & k=S,S-1,\dots,2, & m=1, \end{array} \\
 & \qquad\qquad \textit{or} \\
 & \begin{array}{lll} j=i-1, & l=k, & n=0, \\ i=1,\dots,M, & k=1, & m=1, \end{array} \\
(S-k)\beta, & \begin{array}{lll} j=i, & l=k+1, & n=m, \\ i=0,1,\dots,M, & k=S-1,S-2,\dots,1 & m=1,0, \end{array} \\
 & \qquad\qquad \textit{or} \\
 & \begin{array}{lll} j=i, & l=1, & n=m, \\ i=0,1,\dots,M, & k=0, & m=0, \end{array} \\
-((S-k)\beta+p\lambda+\delta_{0m}\theta), & \begin{array}{lll} j=i, & l=k, & n=m, \\ i=0, & k=S,S-1,\dots,1, & m=1,0, \end{array} \\
-(S\beta+p\lambda), & \begin{array}{lll} j=i, & l=k, & n=m, \\ i=0, & k=0, & m=0, \end{array} \\
-((S-k)\beta+\lambda+\delta_{1m}\mu+\delta_{0m}\theta), & \begin{array}{lll} j=i, & l=k, & n=m, \\ i=1,2,\dots,M-1, & k=S,S-1,\dots,1, & m=1,0, \end{array}
\end{cases}
$$

$$\begin{cases} -(S\beta+\lambda), & j=i, \quad & l=k, \quad & n=m, \\ & i=1,2,\ldots,M-1, & k=0, & m=0, \\[1em] -((S-k)\beta+q\lambda+\delta_{1m}\mu+\delta_{0m}\theta), & j=i, & l=k, & n=m, \\ & i=M, & k=S,S-1,\ldots,1, & m=1,0, \\[1em] -(S\beta+p\lambda), & j=i, & l=k, & n=m, \\ & i=0, & k=0, & m=0, \\[1em] 0, & \text{otherwise.} & & \end{cases}$$

Denote $\mathbf{q} = ((q,0,0),(q,1,0),(q,1,1),(q,2,0),(q,2,1),\ldots,(q,S,0),(q,S,1))$ for $q = 0,1,\ldots,M$. By ordering states lexicographically, the infinitesimal generator A can be conveniently expressed in a block partitioned matrix with entries

$$[A]_{ij} = \begin{cases} A_2,\ j=i, & i=M, \\ A_1,\ j=i, & i=1,2,\ldots,M-1, \\ A_0,\ j=i, & i=0, \\ B,\ \ j=i+1, & i=0,1,2,\ldots,M-1, \\ C,\ \ j=i-1, & i=1,2,\ldots,M, \\ \mathbf{0},\ \ \text{otherwise}, & \end{cases}$$

where

$$[A_0]_{ij} = \begin{cases} F_{S-i},\ j=i, & i=S,S-1\ldots,1, \\ F_{01},\ \ j=1, & i=0, \\ F_{00},\ \ j=0, & i=0, \\ \mathbf{0},\ \ \ \text{otherwise}, & \end{cases}$$

$$[F_{S-i}]_{mn} = \begin{cases} \theta, & n=1, \quad & m=0, \\ (S-i)\beta, & n=i, & m=1,0, \\ -((S-i)\beta+p\lambda), & n=1, & m=1, \\ -((S-i)\beta+p\lambda+\theta), & n=0, & m=0, \\ \mathbf{0}, & \text{otherwise.} & \end{cases}$$

For $i=S,S-1,\ldots,1,$

$$F_{01} = \begin{array}{c} \\ 0 \end{array} \begin{array}{cc} 1 & 0 \\ \big(0 & S\beta\big) \end{array}, \quad F_{00} = \begin{array}{c} \\ 0 \end{array} \begin{array}{c} 0 \\ \big(-(S\beta+p\lambda)\big) \end{array},$$

$$[A_1]_{ij} = \begin{cases} Q_{S-i}, \, j = i, & i = S, S-1 \ldots, 1, \\ F_{01}, \quad j = 1, & i = 0, \\ Q_{00}, \quad j = 0, & i = 0, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[Q_{S-i}]_{mn} = \begin{cases} \theta, & n = 1, \quad m = 0, \\ (S-i)\beta, & n = i, \quad m = 1, 0, \\ -((S-i)\beta + \lambda + \mu), & n = 1, \quad m = 1, \\ -((S-i)\beta + \lambda + \theta), & n = 0, \quad m = 0, \\ \mathbf{0}, & \text{otherwise}. \end{cases}$$

For $i = S, S-1, \ldots, 1,$

$$Q_{00} = \begin{matrix} & 0 \\ 0 & \big(-(S\beta + \lambda)\big) \end{matrix},$$

$$[A_2]_{ij} = \begin{cases} P_{S-i}, \, j = i, & i = S, S-1 \ldots, 1, \\ F_{01}, \quad j = 1, & i = 0, \\ P_{00}, \quad j = 0, & i = 0, \\ \mathbf{0}, \quad \text{otherwise}, \end{cases}$$

$$[P_{S-i}]_{mn} = \begin{cases} \theta, & n = 1, \quad m = 0, \\ (S-i)\beta, & n = i, \quad m = 1, 0, \\ -((S-i)\beta + q\lambda + \mu), & n = 1, \quad m = 1, \\ -((S-i)\beta + q\lambda + \theta), & n = 0, \quad m = 0, \\ \mathbf{0}, & \text{otherwise}. \end{cases}$$

For $i = S, S-1, \ldots, 1,$

$$P_{00} = \begin{matrix} & 0 \\ 0 & \big(-(S\beta + q\lambda)\big) \end{matrix},$$

$$B = p\lambda I_{(2S+1)\times(2S+1)}, \quad C = q\lambda I_{(2S+1)\times(2S+1)}.$$

It may be noted that the matrices A_0, A_1, A_2, B and C are square matrices of order $(2S+1)$.

3.1. Steady State Analysis

The structure of A that the homogeneous Markov process $\{(X(t), L(t), Y(t)),\ t \geq 0\}$ on the finite state space E is irreducible. Hence, the limiting distribution

$$\pi^{(i,j,k)} = \lim_{t\to\infty} pr\{X(t) = i, L(t) = j, Y(t) = k|X(0), L(0), Y(0)\},$$

exists. Let

$$\mathbf{\Pi}= (\mathbf{\Pi^{(0)}}, \mathbf{\Pi^{(1)}}, \mathbf{\Pi^{(2)}}, \ldots, \mathbf{\Pi^{(M)}}).$$

We partition the vector $\mathbf{\Pi^{(i)}}$ into as follows,

$$\mathbf{\Pi^{(i)}}= (\mathbf{\Pi^{(i,0)}}, \mathbf{\Pi^{(i,1)}}, \mathbf{\Pi^{(i,2)}}, \ldots, \mathbf{\Pi^{(i,S)}}), \quad i = 0, 1, 2, \ldots, M,$$

which is partitioned as follows,

$$\mathbf{\Pi^{(i,j)}} = (\pi^{\mathbf{(i,0,0)}}),$$
$$\mathbf{\Pi^{(i,j)}} = (\pi^{\mathbf{(i,j,0)}}, \pi^{\mathbf{(i,j,1)}}).$$

for $i = 0, 1, 2, \ldots, M, \quad j = 1, 2, \ldots, S.$

Then, the limiting probability $\mathbf{\Pi}$ satisfies

$$\mathbf{\Pi} A = 0, \qquad \mathbf{\Pi} e = 1. \tag{1}$$

From the structure of A, it is a finite QBD matrix, therefore, its steady state vector $\mathbf{\Pi}$ can be computed by using the following algorithm described by Gaver et al. (1984).

Algorithm :

1. Determine recursively the matrices

$$F_0 = A_0,$$
$$F_i = A_1 + B(-F_{i-1}^{-1})C, \qquad i = 1, 2, \ldots, M-1,$$
$$F_M = A_2 + B(-F_{M-1}^{-1})C.$$

2. Compute recursively the vectors $\mathbf{\Pi}^{(i)}$ using

$$\mathbf{\Pi}^{(i)} = \mathbf{\Pi}^{(i+1)} B(-F_i^{-1}), \qquad i = 0, 1, 2, \ldots, M-1.$$

3. Solve the system of equations

$$\mathbf{\Pi}^{(M)} F_M = 0 \text{ and } \sum_{i=0}^{M} \mathbf{\Pi}^{(i)} e = 1.$$

From the system of equations $\mathbf{\Pi}^{(M)}F_M = 0$, vector $\mathbf{\Pi}^{(M)}$ could be determined uniquely, up to a multiplicative constant. This constant is decided by

$$\mathbf{\Pi}^{(i)} = \mathbf{\Pi}^{(i+1)}B(-F_i^{-1}), \quad i = 0, 1, 2, \ldots, M-1 \text{ and } \sum_{i=0}^{M} \mathbf{\Pi}^{(i)}e = 1.$$

4. System Performance Measures

In this section some performance measures of the system under consideration in the steady state are derived.

4.1. Expected inventory level

Let ρ_I denote the mean inventory level in the steady state. Then,

$$\rho_I = \sum_{i=0}^{M}\sum_{j=1}^{S} j\left[\pi^{(i,j,1)} + \pi^{(i,j,0)}\right].$$

4.2. Expected reorder rate

Let ρ_R denote the expected reorder rate in the steady state. Then,

$$\rho_R = \sum_{i=1}^{M}\sum_{j=1}^{S} \mu\left[\pi^{(i,j,1)}\right].$$

4.3. Expected number of demands in the waiting hall

Let ρ_W denote the expected number of demands in the waiting hall in the steady state. Then,

$$\rho_W = \sum_{i=1}^{M}\sum_{j=1}^{S} i\left[\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}\right].$$

4.4. Fraction of time server is on vacation

Let ρ_{FV} denote the server is on vacation in the steady state. Then,

$$\rho_{FV} = \sum_{i=0}^{M}\sum_{j=0}^{S} \left[\pi^{(i,j,0)}\right].$$

4.5. Expected blocking rate

Let ρ_B denote the expected blocking rate in the steady state. Then,

$$\rho_B = \sum_{j=1}^{S} p\lambda \left[\pi^{(M,j,1)} + \pi^{(M,j,0)} + \pi^{(M,0,0)}\right].$$

4.6. Mean rate of arrivals of negative customers

Let ρ_{Ng} denote mean rate of arrivals of negative customers in the steady state. Then,

$$\rho_{Ng} = \sum_{i=1}^{M}\sum_{j=1}^{S} q\lambda \left[\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}\right].$$

5. Cost Analysis

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be
c_h : the inventory holding cost per unit item per unit time,
c_s : the inventory setup cost per unit item per unit time,
c_b : cost per blocking customer,
c_w : waiting cost of a customer in the waiting hall per unit time,
c_n : cost of loss per unit time due to arrival of a negative customer.

The long run total expected cost rate is given by

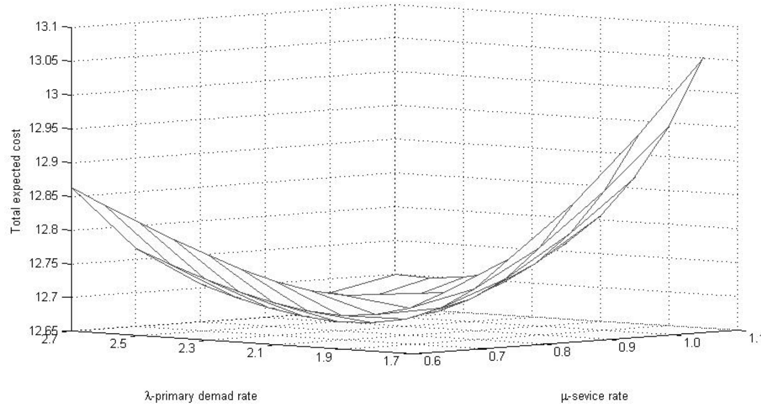
$$TC(S,M) = c_h\rho_I + c_s\rho_R + c_b\rho_B + c_w\rho_W + c_n\rho_{Ng}.$$

Substituting ρ's into the above equation, we obtain

$$\begin{aligned} TC(S,M) =& c_h \sum_{i=0}^{M}\sum_{j=1}^{S} j\left[\pi^{(i,j,1)} + \pi^{(i,j,0)}\right] + c_s \sum_{i=1}^{M}\sum_{j=1}^{S} \mu\left[\pi^{(i,j,1)}\right] \\ &+ c_b \sum_{j=1}^{S} p\lambda\left[\pi^{(M,j,1)} + \pi^{(M,j,0)} + \pi^{(M,0,0)}\right] + c_w \sum_{i=1}^{M}\sum_{j=1}^{S} i\left[\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}\right] \\ &+ c_n \sum_{i=1}^{M}\sum_{j=1}^{S} q\lambda\left[\pi^{(i,j,1)} + \pi^{(i,j,0)} + \pi^{(i,0,0)}\right] \end{aligned}$$

6. Numerical Examples

In this section we illustrated the sensitivity investigation is given by considering the following parameters and cost values. Table 1 presents the optimal value of the total expected cost rate for various combinations of the primary demand rate λ and the service rate μ. We have assumed constant values for other parameters and costs. Namely, $S = 5$, $M = 5$ $\theta = 0.05$, $\beta = 0.7$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$. The optimal value of the total expected cost rate is $TC^*(2.3, 0.9)$ = 12.659332 for the values of $\lambda = 2.3$ and $\mu = 0.9$. The value that is shown bold is the least among the values in that column and the value that is shown underlined is the least in that row. Convexity of the total cost for various combinations of λ and μ is given in Figure 1.



$S = 5$, $M = 5$ $\theta = 0.05$, $\beta = 0.7$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$

Figure 1. Convexity of the total cost for various combinations of λ and μ

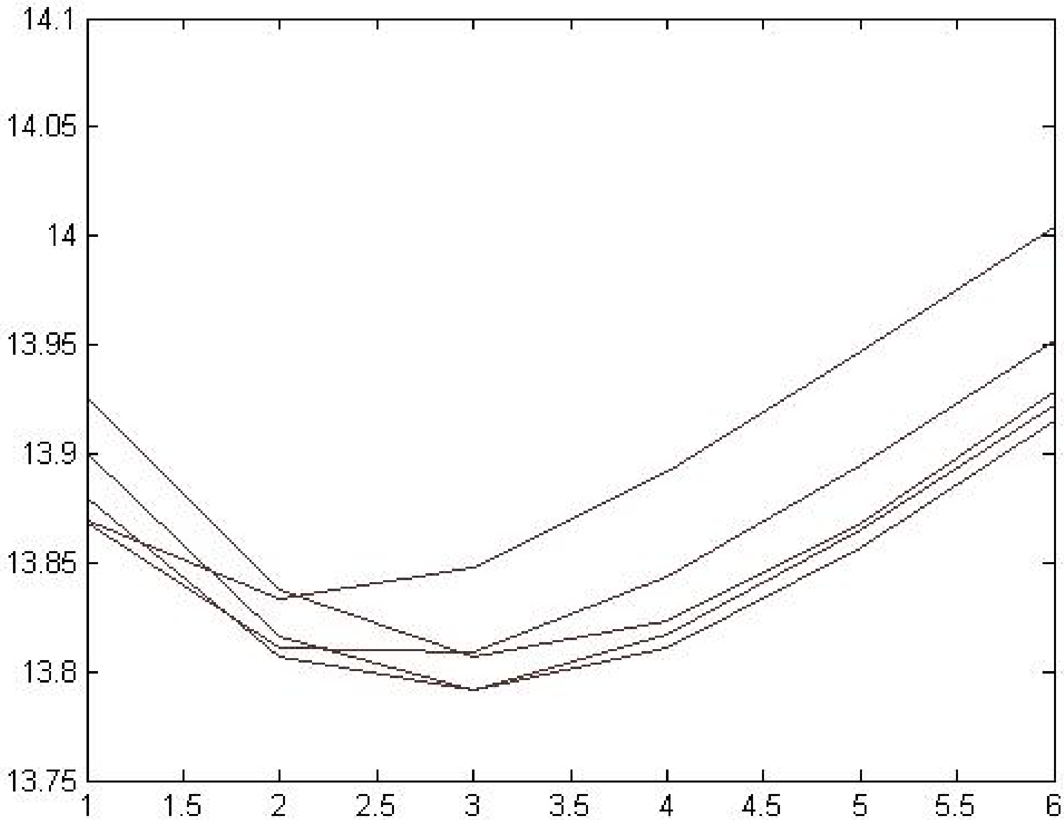
Table 1. Total expected cost rate as a function of λ and μ

$S = 5$, $M = 5$ $\theta = 0.05$, $\beta = 0.7$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$

μ / λ	0.6	0.7	0.8	0.9	1.0	1.1
1.7	12.733593	12.729977	12.772855	12.848242	12.943846	13.050069
1.8	**12.728964**	12.703866	12.724866	12.779530	12.856327	12.945879
1.9	12.732824	12.690244	12.693029	12.730161	12.790871	12.866053
2.0	12.742915	**12.686194**	12.673904	12.696304	12.743353	12.806243
2.1	12.757527	12.689439	12.664740	12.674827	12.710343	12.762782
2.2	12.775365	12.698203	**12.663342**	12.663189	12.689012	12.732606
2.3	12.795448	12.711102	12.667966	**12.659332**	12.677039	12.713172
2.4	12.817035	12.727060	12.677224	12.661596	**12.672532**	12.702388
2.5	12.839564	12.745239	12.690016	12.668646	12.673950	**12.698534**
2.6	12.862613	12.764987	12.705470	12.679410	12.680042	12.700207

We assign the following values to the parameters: $S = 5$, $M = 5$ $\theta = 0.05$, $\lambda = 2$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$. For different values of β and μ, the total expected cost rate presented in Table 2. The value that is shown bold is the least among the values in that column and the value that is shown underlined is the least in that row. The two dimensional convexity of the total cost for various combinations of β and μ is given in Figure 2.

From Table 3, we observe that an increase in the arrival rate and server vacation rate makes a decrease in measures like expected inventory level, expected number of demands in the waiting hall, expected blocking rate and the total expected cost rate. However, the expected reorder rate and mean rate of arrivals of negative customers are increased considerably.



$S = 5$, $M = 5$ $\theta = 0.05$, $\lambda = 2$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$

Figure 2. Effect for various combinations of β and μ

Table 2. Total expected cost rate as a function of β and μ

$S = 5$, $M = 5$ $\theta = 0.05$, $\lambda = 2$, $p = 0.7$, $q = 0.3$, $c_h = 0.98$, $c_s = 1.2$, $c_w = 2.09$, $c_n = 0.03$

μ / β	1.3	1.4	1.5	1.6	1.7
0.9	13.869944	13.869042	13.880297	13.900157	13.925797
1.1	**13.833343**	13.810877	13.806440	13.816461	13.837569
1.3	13.848195	**13.809033**	**13.790986**	**13.791285**	**13.806810**
1.5	13.891897	13.843180	13.816835	13.810995	13.823088
1.7	13.947261	13.894733	13.865039	13.857013	13.868605
1.9	14.004568	13.951894	13.922257	13.914978	13.928403

Table 4 indicates that increase in arrival rate and lead time makes expected inventory level, expected reorder rate and total expected cost rate. However, expected number of demands in the waiting hall, expected blocking rate and mean rate of arrivals of negative customers decrease to the considerable extent.

Table 5 shows that the expected inventory level, expected reorder rate and total expected cost rate increase when increase in lead time and service rate. However, expected number of demands in the waiting hall, expected blocking rate and mean rate of arrivals of negative customers decrease.

Table 3. Effects of λ and θ on some performance measures and total cost

λ	θ	ρ_I	ρ_R	ρ_W	ρ_B	ρ_N	TC
2	1	5.308861	0.719987	3.113831	0.563783	0.834559	12.599611
	2	5.311536	0.722475	3.108817	0.561933	0.834901	12.594750
	3	5.312544	0.723274	3.107098	0.561407	0.835208	12.593113
3	1	4.799978	0.742670	3.431449	0.935586	1.154436	12.801543
	2	4.799326	0.745854	3.427138	0.933283	1.155487	12.795747
	3	4.799195	0.746897	3.425628	0.932667	1.156202	12.793736
4	1	4.554990	0.753809	3.619659	1.325271	1.470018	12.977649
	2	4.552364	0.757416	3.616049	1.322910	1.471833	12.971914
	3	4.551529	0.758613	3.614775	1.322369	1.472993	12.969904
$\mu = 0.8, \beta = 0.7, p = 0.7, q = 1 - p, c_h = 0.98, c_s = 1.2, c_w = 2.09, c_n = 0.03, S = 5, M = 5$							

Table 4. Effects of λ and β on some performance measures and total cost

λ	β	ρ_I	ρ_R	ρ_W	ρ_B	ρ_N	TC
3	5	6.624869	0.726192	3.100115	0.556737	0.817071	13.867555
	6	6.661288	0.726235	3.100041	0.556700	0.817065	13.903142
	7	6.687283	0.726254	3.100009	0.556684	0.817063	13.928574
4	5	6.020099	0.750882	3.419034	0.922619	1.126501	13.980331
	6	6.054287	0.750939	3.418965	0.922568	1.126494	14.013759
	7	6.078704	0.750963	3.418936	0.922546	1.126491	14.037657
5	5	5.716825	0.763314	3.608952	1.305218	1.431470	14.104119
	6	5.749639	0.763381	3.608890	1.305157	1.431463	14.136228
	7	5.773089	0.763410	3.608864	1.305131	1.431460	14.159189
$\mu = 0.8, \theta = 0.05, p = 0.7, q = 1 - p, c_h = 0.98, c_s = 1.2, c_w = 2.09, c_n = 0.03, S = 5, M = 5$							

Table 5. Effects of β and μ on some performance measures and total cost

β	μ	ρ_I	ρ_R	ρ_W	ρ_B	ρ_N	TC
2	1	8.730789	0.769228	2.126565	0.166029	0.464821	13.937712
	2	12.697318	1.027743	1.100815	0.048213	0.324373	15.987098
	3	14.780886	1.111886	0.773299	0.039019	0.249588	17.443215
3	1	9.008422	0.772386	2.117229	0.163509	0.464114	14.194049
	2	13.377446	1.047881	1.036016	0.034151	0.318011	16.542169
	3	15.801879	1.148591	0.653699	0.015223	0.236255	18.237470
4	1	9.142190	0.772955	2.115548	0.163056	0.463991	14.322307
	2	13.670081	1.051866	1.023190	0.031371	0.316772	16.806890
	3	16.216018	1.156294	0.628600	0.010233	0.233489	18.600030
$\lambda = 2, \theta = 0.05, p = 0.7, q = 1 - p, c_h = 0.98, c_s = 1.2, c_w = 2.09, c_n = 0.03, S = 5, M = 5$							

7. Conclusion

In this paper, we discussed continuous review inventory system with base stock policy and customers are of two type: ordinary and negative. Various system performance measures are derived in the steady state. The results are illustrated with numerically. This model addresses the intraction between inventory and the quality of service. The current situation faced by manufacturing companies and server, which are under instance pressure to reduce inventory and idle (vacation) time for utilize capacity, slow moving items and provide high holding cost service levels. And also the total expected cost rate is minimized.

Acknowledgment:

N. Anbazhagan's research has been financialy supported jointly by RUSA-Phase 2.0 grant sanctioned vide letter No. F 24-51/2014-U, Policy (TN Multi-Gen), Dept. of Edn. Govt. of India, Dt. 09.10.2018, UGC-SAP (DRS-I) vide letter No. F.510/8/DRS-I/2016(SAP-I) Dt. 23.08.2016, DST-PURSE 2nd Phase programme vide letter No. SR/PURSE Phase 2/38 (G) Dt. 21.02.2017 and DST (FST - level I) 657876570 vide letter No. SR/FIST/MS-I/2018/17 Dt. 20.12.2018.

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