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On General Matrix Application Of Quasi Power Increasing Sequences

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Abstract

In this paper, we give a general theorem dealing with absolute matrix summability by using quasi σ-power increasing sequences. This theorem includes some results concerning absolute summability methods.

Keywords: Summability factors; Absolute matrix summability; Almost increasing sequence; Quasi power increasing sequence; Infinite series; Hölder inequality; Minkowski inequality

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1. Introduction

Summability theory plays an important role in Analysis, Applied Mathematics and Engineering Sciences. The aim in this theory is to bring a value to the indefinite divergent series. Various summability methods have been introduced by researchers to find this value. Some of these methods are Cesàro, Abel, Nörlund, Riesz, matrix summability, etc.

In the 1700s, Bernoulli intuitively assigned values to the divergent series. He assigned the value 1/2 for the series $\sum(-1)^n$. The significant rise of summability began in the latter part of the 19th century. In 1890, Cesàro made a study dealing with the multiplication of series and Cesàro became the first mathematician who founded summability (see Hardy (1949)). Das (1969) gave the definition of absolute summability. Then Kishore and Hotta (1970) defined the summability factor. By using lower triangular matrices, the definition of $|A|_k$ summability was given by Tanović-Miller (1979). Bor (1985) defined $\left|\bar{N}, p_n\right|_k$ summability, and later he (1993) defined $\left|\bar{N}, p_n; \delta\right|_k$ summability of an infinite series. The definition of $|A, p_n|_k$ summability of an infinite series was given by Sulaiman (2003). Then, the more general $|A, p_n; \delta|_k$ summability method was defined by Özarslan and Öğdük (2004).

There is an important application area of these methods. They have some different applications on sequences such as positive non-decreasing, almost increasing and quasi power increasing sequences. The purpose of this paper is to obtain a general theorem on absolute matrix summability of an infinite series.

2. Notation and Preliminaries

A positive sequence (b_n) is said to be almost increasing if there exist a positive increasing sequence (c_n) and two positive constants L and M such that $Lc_n \leq b_n \leq Mc_n$ (see Bari and Stečkin (1956)). A positive sequence (μ_n) is said to be a quasi σ-power increasing sequence if there exists a constant $K = K(\sigma, \mu) \geq 1$ such that $Kn^\sigma \mu_n \geq m^\sigma \mu_m$ holds, for all $n \geq m \geq 1$ (see Leindler (2001)). It should be noted that every almost increasing sequence is a quasi σ-power increasing sequence for any non-negative σ, but the converse need not be true as can be seen by taking the example $\mu_n = n^{-\sigma}$ for $\sigma > 0$. A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| = \sum_{n=1}^{\infty} |\lambda_n - \lambda_{n+1}| < \infty$.

Let $\sum a_n$ be a given infinite series with partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad \text{as} \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1).$$

The sequence-to-sequence transformation

$$\omega_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v,$$

defines the sequence (ω_n) of the $(\bar{N}, p_n)$ means of the sequence (s_n), generated by the sequence of coefficients (p_n) (see Hardy (1949)). The series $\sum a_n$ is said to be summable $\mid \bar{N}, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$, and β is a real number, if (see Gürkan (1998))

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k + k - 1)} \mid \omega_n - \omega_{n-1} \mid^k < \infty.$$

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then, A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ...$$

We say that the series $\sum a_n$ is summable $\mid A, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$, and β is a real number, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid A_n(s) - A_{n-1}(s) \mid^k < \infty.$$

For $\beta = 1$, $\mid A, p_n, \beta; \delta \mid_k$ summability reduces to $\mid A, p_n; \delta \mid_k$ summability (see Özarslan and Öğdük (2004)). Additionally, if we take $\beta = 1$ and $\delta = 0$, then $\mid \bar{N}, p_n, \beta; \delta \mid_k$ summability reduces to $\mid \bar{N}, p_n \mid_k$ summability (see Bor (1985)).

Given a normal matrix $A = (a_{nv})$, two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ are defined as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, \ldots, \tag{1}$$

and

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{2}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we write

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v, \tag{3}$$

and

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{4}$$

Bor (2008) obtained the following theorem.

Theorem 2.1.

Let (X_n) be an almost increasing sequence and let there be sequences (γ_n) and (λ_n) such that

$$\mid \Delta\lambda_n \mid \leq \gamma_n, \tag{5}$$

$$\gamma_n \to 0 \quad \text{as} \quad n \to \infty, \tag{6}$$

$$\sum_{n=1}^{\infty} n \mid \Delta\gamma_n \mid X_n < \infty, \tag{7}$$

$$\mid \lambda_n \mid X_n = O(1), \tag{8}$$

where $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$. If

$$\sum_{v=1}^{n} \frac{\mid s_v \mid^k}{v} = O(X_n) \quad \text{as} \quad n \to \infty, \tag{9}$$

and (p_n) is a sequence such that

$$P_n = O(np_n), \tag{10}$$

$$P_n \Delta p_n = O(p_n p_{n+1}), \tag{11}$$

then the series $\sum_{n=1}^{\infty} a_n \frac{P_n \lambda_n}{np_n}$ is summable $\mid \bar{N}, p_n \mid_k$, $k \geq 1$.

Lemma 2.2.

Let (X_n) be a quasi σ-power increasing sequence for some $0 < \sigma < 1$. If conditions (6) and (7) are satisfied, then

$$nX_n \gamma_n = O(1), \tag{12}$$

$$\sum_{n=1}^{\infty} \gamma_n X_n < \infty, \tag{13}$$

(see Leindler (2001)).

3. Main Result

Many works concerning absolute matrix summability methods have been done (see Özarslan (2013, 2014, 2015, 2019a, 2019b, 2019c), Özarslan and Yavuz (2013, 2014)). The aim of this paper is to generalize Theorem 2.1 to $|A, p_n, \beta; \delta|_k$ summability by using a quasi σ-power increasing sequence instead of an almost increasing sequence.

Now, we shall prove the following theorem.

Theorem 3.1.

Let $A = (a_{nv})$ be a positive normal matrix such that

$$\overline{a}_{n0} = 1, \ n = 0, 1, ..., \tag{14}$$

$$a_{n-1,v} \geq a_{nv}, \quad \text{for} \quad n \geq v + 1, \tag{15}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{16}$$

$$\mid \hat{a}_{n,v+1} \mid = O(v \mid \Delta_v(\hat{a}_{nv}) \mid), \tag{17}$$

where $\Delta_v(\hat{a}_{nv}) = \hat{a}_{nv} - \hat{a}_{n,v+1}$. Let (X_n) be a quasi σ-power increasing sequence, for some $0 < \sigma < 1$. If $(\lambda_n) \in \mathcal{BV}$, and the conditions (5)-(8), (10)-(11) of Theorem 2.1 and

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} \frac{|s_n|^k}{n} = O(X_m) \quad \text{as} \quad m \to \infty, \tag{18}$$

$$\sum_{n=v+1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\Delta_v(\hat{a}_{nv})| = O\left(\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k}\right), \tag{19}$$

$$\sum_{n=v+1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1} |\hat{a}_{n,v+1}| = O\left(\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}\right), \tag{20}$$

are satisfied, then the series $\sum_{n=1}^{\infty} a_n \frac{P_n \lambda_n}{np_n}$ is summable $\mid A, p_n, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$, and $-\beta(\delta k + k - 1) + k > 0$.

If we take (X_n) as an almost increasing sequence, we get another result for $\mid A, p_n, \beta; \delta \mid_k$ summability (see Özarslan and Karakaş (2018)). If we take (X_n) as an almost increasing sequence, $\beta = 1$, $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then we get Theorem 2.1.

Proof:

Let (I_n) denotes A-transform of the series $\sum_{n=1}^{\infty} a_n \frac{P_n \lambda_n}{np_n}$. Then, by (3) and (4), we obtain

$$\bar{\Delta} I_n = \sum_{v=1}^{n} \hat{a}_{nv} \frac{a_v P_v \lambda_v}{v p_v}.$$

Applying Abel's transformation to this sum, we get that

$$\begin{aligned}
\bar{\Delta} I_n &= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv} P_v \lambda_v}{v p_v}\right) \sum_{r=1}^{v} a_r + \frac{\hat{a}_{nn} P_n \lambda_n}{np_n} \sum_{r=1}^{n} a_r \\
&= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv} P_v \lambda_v}{v p_v}\right) s_v + \frac{a_{nn} P_n \lambda_n}{np_n} s_n \\
&= \sum_{v=1}^{n-1} \frac{\hat{a}_{n,v+1} P_{v+1} \Delta \lambda_v}{(v+1) p_{v+1}} s_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \Delta(\frac{P_v}{v p_v}) \lambda_v s_v + \sum_{v=1}^{n-1} \frac{\Delta_v(\hat{a}_{nv}) P_v \lambda_v}{v p_v} s_v + \frac{a_{nn} P_n \lambda_n}{np_n} s_n \\
&= I_{n,1} + I_{n,2} + I_{n,3} + I_{n,4}.
\end{aligned}$$

For the proof of Theorem 3.1, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,r} \mid^k < \infty, \quad \text{for} \quad r = 1, 2, 3, 4.$$

Using (10) and applying Hölder's inequality, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\,|\Delta\lambda_v||\,|s_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\,|\Delta\lambda_v|\,|s_v|^k\right)\\ &\quad\times\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\,|\Delta\lambda_v|\right)^{k-1}.\end{aligned}$$

Here, by virtue of (1), (2), (14) and (15), we find

$$\begin{aligned}\hat{a}_{n,v+1}=\bar{a}_{n,v+1}-\bar{a}_{n-1,v+1} &= \sum_{i=v+1}^{n}a_{ni}-\sum_{i=v+1}^{n-1}a_{n-1,i}\\ &= \sum_{i=0}^{n}a_{ni}-\sum_{i=0}^{v}a_{ni}-\sum_{i=0}^{n-1}a_{n-1,i}+\sum_{i=0}^{v}a_{n-1,i}\\ &= 1-\sum_{i=0}^{v}a_{ni}-1+\sum_{i=0}^{v}a_{n-1,i}\\ &= \sum_{i=0}^{v}(a_{n-1,i}-a_{ni})\\ &\geq 0,\end{aligned}$$

and so, by (1), (2), (15), we get

$$\begin{aligned}|\hat{a}_{n,v+1}| = \hat{a}_{n,v+1} &= \bar{a}_{n,v+1}-\bar{a}_{n-1,v+1}\\ &= a_{nn}+\sum_{i=v+1}^{n-1}(a_{ni}-a_{n-1,i})\\ &\leq a_{nn}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\,\gamma_v\,|s_v|^k\right)\\ &\quad\times\left(\sum_{v=1}^{n-1}|\Delta\lambda_v|\right)^{k-1}.\end{aligned}$$

Now, using the fact that $a_{nn}=O(\frac{p_n}{P_n})$ by (16) and $(\lambda_n)\in\mathcal{BV}$, we obtain

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\,\gamma_v\,|s_v|^k\right)\\
&= O(1)\sum_{v=1}^{m}\gamma_v\,|s_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\hat{a}_{n,v+1}|\\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1} v\gamma_v\frac{|s_v|^k}{v}\\
&= O(1)\sum_{v=1}^{m-1}\Delta(v\gamma_v)\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\beta(\delta k+k-1)-k+1}\frac{|s_r|^k}{r}\\
&\quad +O(1)m\gamma_m\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}\frac{|s_v|^k}{v}\\
&= O(1)\sum_{v=1}^{m-1}v|\Delta\gamma_v|X_v+O(1)\sum_{v=1}^{m-1}\gamma_vX_v+O(1)m\gamma_mX_m\\
&= O(1)\quad \text{as}\quad m\to\infty,\end{aligned}$$

by (7), (12), (13), (18) and (20).

Again, applying Hölder's inequality, using the fact that $\Delta\left(\frac{P_v}{vp_v}\right)=O(\frac{1}{v})$ (see Mishra and Srivastava (1984)) and (17), we obtain

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,2}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}|\lambda_v||s_v|\right)^k\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}|\lambda_v|^k|s_v|^k\right)\\
&\quad\times\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}.\end{aligned}$$

Here, by (1) and (2), we have

$$\Delta_v(\hat{a}_{nv})=\hat{a}_{nv}-\hat{a}_{n,v+1}=\bar{a}_{nv}-\bar{a}_{n-1,v}-\bar{a}_{n,v+1}+\bar{a}_{n-1,v+1}=a_{nv}-a_{n-1,v}.$$

Hence, using (1), (14), and (15)

$$\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|=\sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv})\leq a_{nn}. \tag{21}$$

Then,

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,2}\mid^k = O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}|\lambda_v|^k|s_v|^k\right).$$

By using (16), we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,2}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}|\lambda_v|^k|s_v|^k\right)\\ &= O(1)\sum_{v=1}^{m}\frac{1}{v}|\lambda_v||s_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\hat{a}_{n,v+1}|\\ &= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}|\lambda_v|\frac{|s_v|^k}{v}\\ &= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{n=1}^{v}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\frac{|s_n|^k}{n}\\ &\quad+O(1)|\lambda_m|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}\frac{|s_v|^k}{v}\\ &= O(1)\sum_{v=1}^{m-1}\gamma_v X_v+O(1)|\lambda_m|X_m\\ &= O(1)\quad \text{as}\quad m\to\infty,\end{aligned}$$

by (5), (8), (13), (18) and (20).

Now, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}|I_{n,3}|^k &\le \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\frac{P_v}{vp_v}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v||s_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k|s_v|^k\right)\\ &\quad\times\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}.\end{aligned}$$

By virtue of (16) and (21),

$$\begin{aligned}\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}|I_{n,3}|^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\sum_{v=1}^{n-1}\left(\frac{P_v}{vp_v}\right)^k|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k|s_v|^k\right)\\ &= O(1)\sum_{v=1}^{m}\frac{P_v}{vp_v}|\lambda_v||s_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\Delta_v(\hat{a}_{nv})|\\ &= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\beta(\delta k+k-1)-k+1}|\lambda_v|\frac{|s_v|^k}{v}\\ &= O(1)\quad \text{as}\quad m\to\infty,\end{aligned}$$

as in $I_{n,2}$, in regard to the hypotheses of Theorem 3.1 and Lemma 2.2.

Finally, from Abel's transformation, as in $I_{n,2}$, we have

$$\begin{aligned}\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)} \mid I_{n,4}\mid^k &= \sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)}\left(\frac{P_n}{np_n}\right)^k a_{nn}^k|\lambda_n|^k|s_n|^k \\ &= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{n^k}|\lambda_n|^{k-1}|\lambda_n||s_n|^k \\ &= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\beta(\delta k+k-1)-k+1}|\lambda_n|\frac{|s_n|^k}{n} \\ &= O(1) \quad \text{as} \quad m\to\infty,\end{aligned}$$

in regard to the hypotheses of Theorem 3.1 and Lemma 2.2.

Therefore, Theorem 3.1 is proved. ■

4. Conclusion

In this paper, absolute matrix summability of an infinite series has been studied. A general theorem on $\mid A, p_n, \beta; \delta \mid_k$ summability method has been proved by using a quasi σ-power increasing sequence instead of an almost increasing sequence under weaker conditions. This general theorem brings a different perspective and studying field, and so it creates a basis for future research of interested researchers; also, the $\mid A, p_n, \beta; \delta \mid_k$ summability method can be used to generalize some different theorems on absolute summability.

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