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Remarkable Applications of Measure of Non-compactness For Infinite System of Differential Equations

[1]Merve İlkhan and [2]Emrah Evren Kara

Department of Mathematics
Düzce University
Düzce, Turkey
[1]merveilkhan@gmail.com, [2]karaeevren@gmail.com

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Abstract

The essential goal of our study is to search for a solution of an infinite system of differential equations in two different Banach spaces under certain assumptions by the aid of measure of non-compactness. Also, we establish some interesting examples related to our results.

Keywords: Measure of non-compactness; Infinite system of differential equations; Banach spaces

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1. Introduction

The theory of infinite systems of differential equations is an outstanding field of study for functional analysis and applied mathematics since it has many applications in the theory of artificial neural networks, branching processes, the theory of dissociation of polymers and so on (cf. Hille (1961), Zautykov (1965), Oğuztöreli (1972), Deimling (1977)). Also, Persidskii (1959), Persidskii (1961) and Zautykov and Valeev (1974) have studied various problems in mechanics which lead to infinite systems of differential equations.

Recently, Banaś and Lecko (2001) and Mursaleen and Mohiuddine (2012) have studied the solvability of infinite systems of differential equations in classical Banach spaces by using the measure of non-compactness. For further results and discussions related to this topic, we refer to the papers of Banaś (2012), Mursaleen and Alotaibi (2012), Banaś and Sadarangani (2013), Mursaleen (2013), Alotaibi et al. (2015), Mohiuddine et al. (2016), Demiriz (2017), Banaś and Krajewska (2017), Srivastava et al. (2018) and various references given therein. A large number of these results have been formulated from the point of measures of non-compactness. Also, Olszowy (2009) and Olszowy (2010) have studied the infinite systems of integral equations in Frechet spaces with measure of non-compactness.

For the foundation of the Hausdorff measure of non-compactness, we refer to Kara et al. (2015). The theory of measure of non-compactness has been handled to establish the classes of compact operators on some sequence spaces (see Mursaleen and Noman (2010a), Mursaleen and Noman (2010b), Mursaleen et al. (2011), Kara and Başarır (2011), Başarır and Kara (2012), Alotaibi et al. (2015)).

For some papers related to sequence spaces, one can consult the papers of Kara (2013), Candan (2015), Başarır et al. (2016), Kara and İlkhan (2016) and Ellidokuzoğlu et al. (2018).

In this paper, by applying the technique of measure of non-compactness, we search for the solvability of infinite systems of differential equations in some Banach spaces. Also, we give an example of infinite system of differential equations which has a solution in these spaces but has no solution in the classical sequence spaces c_0 and ℓ_p.

2. Background and notation

In this section, we give some basic definitions, notations and results about sequence spaces and the Hausdorff measure of noncompactness (special case of measure of noncompactness). These can be found in the papers of Koshy (2001) and Banaś et al. (2017).

Let $\mathbb{N} = \{0, 1, 2, ...\}$ and $\mathbb{R}$ be the set of all real numbers. By ω and $\mathbb{R}^\infty$, we denote the vector space of all real sequences $u = (u_k)_{k\in\mathbb{N}}$. Let c_0 be the set of all null sequences. We write $\ell_p = \{u \in \omega : \sum_k |u_k|^p < \infty\}$, for $1 \leq p < \infty$. The spaces c_0 and ℓ_p are Banach spaces with the norms $\|u\|_\infty = \sup_k |u_k|$ and $\|u\|_p = \left(\sum_k |u_k|^p\right)^{1/p}$, respectively.

Let (X, ρ) $((X, \|.\|))$ be a metric space (a normed space). By $\mathcal{B}(u_0, r)$ and $\overline{\mathcal{B}}(u_0, r)$, we denote the open ball and closed ball in X centered u_0 and with radius $r > 0$. Moreover, let M_X be the collection of all non-empty and bounded subsets of X. If $Y \in M_X$, then the *Hausdorff measure of noncompactness* of the set Y, denoted by $\chi(Y)$, is defined by

$$\begin{aligned}\chi(Y) &:= \inf\left\{\varepsilon > 0 : Y \subset \bigcup_{i=1}^{n} \mathcal{B}(u_i, r_i),\ u_i \in X,\ r_i < \varepsilon\ (i = 1, 2, ..., n),\ n \in \mathbb{N} - \{0\}\right\} \\ &:= \inf\{\varepsilon > 0 : Y \text{ has a finite } \varepsilon\text{-net in } X\}.\end{aligned}$$

The function $\chi : M_X \to [0,\infty)$ is called the Hausdorff measure of non-compactness.

The following result gives the Hausdorff measure of non-compactness of a bounded set in the spaces c_0 and ℓ_p for $1 \le p < \infty$.

Lemma 2.1. (Rakočević (1998))

Let $X = \ell_p$ or $X = c_0$ and $Y \in M_X$. If $P_m : X \to X$ $(m \in \mathbb{N})$ is the operator defined by $P_m(u) = (u_0, u_1, ..., u_m, 0, 0, ...)$ for all $u = (u_k) \in X$, then we have

$$\chi(Y) = \lim_{m\to\infty}\left(\sup_{u\in Y}\|(I-P_m)(u)\|\right),$$

where I is the identity operator on X.

The set

$$\lambda_A = \{u = (u_n) \in \omega : Au \in \lambda\},$$

is called as the matrix domain of an infinite matrix A in the sequence space λ.

If $A = (a_{nk})$ is triangle, that is, $a_{nk} = 0$ for $k > n$ and $a_{nn} \neq 0$ for all $n \in \mathbb{N}$, then the spaces λ_A and λ are linearly isomorphic.

For $r, s \in \mathbb{R}\backslash\{0\}$, the generalized difference matrix $B(r,s) = \{b_{nk}(r,s)\}$ is given by

$$b_{nk}(r,s) = \begin{cases} r, & k = n, \\ s, & k = n-1, \\ 0, & 0 \le k < n-1 \text{ or } k > n, \end{cases}$$

for all $n, k \in \mathbb{N}$.

Kirişçi and Başar (2010) defined the following spaces

$$\widehat{\ell_p} = \left\{u = (u_n) \in \omega : \sum_{n=0}^{\infty}|ru_n + su_{n-1}|^p < \infty\right\},$$

for $1 \le p < \infty$, and

$$\widehat{c_0} = \left\{u = (u_n) \in \omega : \lim_{n\to\infty}|ru_n + su_{n-1}| = 0\right\}.$$

Here $u_{-1} = 0$. The spaces $\widehat{\ell_p}$ and $\widehat{c_0}$ are Banach spaces with the norms given by

$$\|u\|_{\widehat{\ell_p}} = \left(\sum_{n=0}^{\infty}|ru_n + su_{n-1}|^p\right)^{1/p} \quad \text{and} \quad \|u\|_{\widehat{c_0}} = \sup_{n\in\mathbb{N}}|ru_n + su_{n-1}|,$$

respectively.

One can redefine the spaces $\widehat{\ell_p}$ and $\widehat{c_0}$ as

$$\widehat{\ell_p} = \{\ell_p\}_{B(r,s)} \text{ and } \widehat{c_0} = \{c_0\}_{B(r,s)}.$$

Theorem 2.2. (Kirişçi and Başar (2010))

Let $\lambda \in \{\ell_p, c_0\}$ and $B = B(r,s)$. Then,

(i) $\lambda = \lambda_B$ if $|s/r| < 1$.

(ii) $\lambda \subset \lambda_B$ is strict if $|s/r| \geq 1$.

Throughout the study, we assume that $|s| \geq |r|$. Otherwise, our solutions coincide with the solutions given for the spaces ℓ_p and c_0.

3. Solutions for infinite systems of differential equations in $\widehat{\ell_p}$ and $\widehat{c_0}$

In this section, by applying the technique of measure of non-compactness, we search for the solvability of infinite systems of differential equations in Banach spaces $\widehat{\ell_p}$ and $\widehat{c_0}$. Also, we give an example of infinite system of differential equations which has a solution in these spaces but has no solution in the spaces c_0 and ℓ_p.

Consider the ordinary differential equation

$$u^{'} = g(a,u), \tag{1}$$

with the initial condition

$$u(0) = u_0. \tag{2}$$

Now, let $(X, \|.\|)$ be a real Banach space and take the interval $I = [0, A]$, $A > 0$. Then, we give the following result proved by Banaś and Goebel (1980), and modified by Banaś and Lecko (2001).

Theorem 3.1.

Assume that g is a function defined on $I \times X$ with values in X such that

$$\|g(a,u)\| \leq Q + R\|u\|,$$

for any $u \in X$, where Q and R are non-negative constants. Also, let g be uniformly continuous on $J_1 \times \overline{\mathcal{B}}(u_0, r)$, where $r = (QA_1 + RA_1\|u_0\|)/(1 - RA_1)$, and $I_1 = [0, A_1] \subset I$, $RA_1 < 1$. Further, assume that for any subset Y of $\overline{\mathcal{B}}(u_0, r)$ and for almost all $a \in I$ the following inequality holds:

$$\mu(g(a,Y)) \leq q(a)\mu(Y), \tag{3}$$

with a sublinear measure of non-compactness μ such that $\{u_0\} \in \ker \mu$. Then, the problem (1)-(2) has a solution u such that $\{u(a)\} \in \ker \mu$ for $a \in I_1$, where q is an integrable function on I and $\ker \mu = \{E \in M_X : \mu(E) = 0\}$ is the kernel of the measure μ.

Remark 3.2.

If we take the Hausdorff measure of non-compactness χ instead of μ in the above theorem, then the assumption of the uniform continuity of g can be replaced by the continuity of g.

We need the following lemma to prove our main results, which can be obtained from Lemma 2.1.

Lemma 3.3.

(i) If $Y \in M_{\widehat{\ell_p}}$, then

$$\chi(Y) = \lim_{n\to\infty}\left(\sup_{(u_k)\in Y}\sum_{k\geq n}|ru_k + su_{k-1}|^p\right),$$

holds.

(ii) If $Y \in M_{\widehat{c_0}}$, then

$$\chi(Y) = \lim_{n\to\infty}\left(\sup_{(u_k)\in Y}\left(\sup_{k\geq n}|ru_k + su_{k-1}|\right)\right),$$

holds.

We have the following theorem.

Theorem 3.4.

Let

$$u_i^{'} = h_i(a, u_1, u_2, ...), \quad (a \in I = [0, A]), \tag{4}$$

be the infinite system of ordinary differential equations with the initial condition

$$u_i(0) = u_i^0, \tag{5}$$

where $h_i : I \times \mathbb{R}^\infty \to \mathbb{R}$ for all $i = 1, 2, 3, ...$ Suppose that

i) $u_0 = (u_i^0) \in \widehat{c_0}$,

ii) the mapping $h = (h_1, h_2, ...) : I \times \widehat{c_0} \to \widehat{c_0}$ is continuous,

iii) there exists an increasing sequence (k_n) of natural numbers ($k_n \to \infty$ as $n \to \infty$) such that for any $a \in I$, $(u_i) \in \widehat{c_0}$ and for $n = 1, 2, ...$ the inequality

$$|h_n(a, u_1, u_2, ...)| \leq v_n(a) + w_n(a)\sup\{|ru_i + su_{i-1}| : i \geq k_n\},$$

holds, where v_n and w_n are real valued continuous functions on I such that $(v_n(a))$ converges uniformly on I to the function vanishing identically and $(w_n(a))$ is equibounded on I. Set

$$w(a) = \sup\{w_n(a) : n = 1, 2, ...\}; \ a \in I,$$

$$W_1 = \sup\{w(a) : a \in I\},$$

$$V_1 = \sup\{v_n(a) : a \in I, \ n = 1, 2, ...\}.$$

Then, the problem (4)-(5) has at least one solution $u = u(a) = (u_n(a))$ on $I_1 = [0, A_1] \subset I$, where $WA_1 < 1$ and $W = 2|s|W_1$. Also, $u(a) \in \widehat{c_0}$ for $a \in I_1$.

Proof:

Let us take an arbitrary $u = (u_n(a)) \in \widehat{c_0}$. By using the above assumptions, for any $a \in I$ and for a fixed n, we obtain

$$\begin{aligned}
|rh_n(a,u) + sh_{n-1}(a,u)| &= |rh_n(a,u_1,u_2,...) + sh_{n-1}(a,u_1,u_2,...)| \\
&\le |r| \left(v_n(a) + w_n(a) \sup\left\{|ru_i + su_{i-1}| : i \ge k_n\right\}\right) \\
&+ |s| \left(v_{n-1}(a) + w_{n-1}(a) \sup\left\{|ru_i + su_{i-1}| : i \ge k_{n-1}\right\}\right) \\
&\le |s| \left(V_1 + W_1 \sup\left\{|ru_i + su_{i-1}| : i \ge k_n\right\}\right) \\
&+ |s| \left(V_1 + W_1 \sup\left\{|ru_i + su_{i-1}| : i \ge k_{n-1}\right\}\right) \\
&\le V + W \left\|u\right\|_{\widehat{c_0}},
\end{aligned}$$

where $V = 2|s|V_1$ and $W = 2|s|W_1$.

Thus, we have

$$\left\|h(a,u)\right\|_{\widehat{c_0}} \le V + W \left\|u\right\|_{\widehat{c_0}}. \tag{6}$$

Now, let us take the closed ball $\overline{\mathcal{B}}(u_0, r)$ in $\widehat{c_0}$, where $r = \frac{VA_1 + WA_1\|u\|_{\widehat{c_0}}}{1 - WA_1}$. Let X be a subset of $\overline{\mathcal{B}}(u_0, r)$ and $a \in I_1$. Then, by using Lemma 3.3 (ii), we get

$$\begin{aligned}
\chi(h(a,X)) &= \lim_{n\to\infty} \left\{ \sup_{u\in X} \left\{ \sup\left\{ |rh_i(a,u) + sh_{i-1}(a,u)| : i \ge n \right\}\right\}\right\} \\
&= \lim_{n\to\infty} \left\{ \sup_{(u_i)\in X} \left\{ \sup\left\{ |rh_i(a,u_1,u_2,...) + sh_{i-1}(a,u_1,u_2,...)| : i \ge n \right\}\right\}\right\} \\
&\le \lim_{n\to\infty} \left\{ \sup_{(u_i)\in X} \left\{ |r| \sup_{i\ge n} \left\{ v_i(a) + w_i(a) \sup\left[|ru_p + su_{p-1}| : p \ge k_i \right]\right\}\right\}\right\} \\
&+ \lim_{n\to\infty} \left\{ \sup_{(u_i)\in X} \left\{ |s| \sup_{i\ge n} \left\{ v_{i-1}(a) + w_{i-1}(a) \sup\left[|ru_p + su_{p-1}| : p \ge k_{i-1} \right]\right\}\right\}\right\} \\
&\le |r| \lim_{n\to\infty} \left(\sup_{i\ge n} \left\{ v_i(a) \right\}\right) + |s| \lim_{n\to\infty} \left(\sup_{i\ge n} \left\{ v_{i-1}(a) \right\}\right) \\
&+ |r| w(a) \lim_{n\to\infty} \left(\sup_{(u_i)\in X} \left\{ \sup_{i\ge n} \left\{ \sup\left[|ru_p + su_{p-1}| : p \ge k_i \right]\right\}\right\}\right) \\
&+ |s| w(a) \lim_{n\to\infty} \left(\sup_{(u_i)\in X} \left\{ \sup_{i\ge n} \left\{ \sup\left[|ru_p + su_{p-1}| : p \ge k_{i-1} \right]\right\}\right\}\right) \\
&\le w_1(a)\chi(X),
\end{aligned} \tag{7}$$

where $w_1(a) = 2|s|w(a)$.

It follows from inequalities (6) and (7), Theorem 2.2 and Remark that there exists a solution $u = u(a)$ of the problem (4)-(5) such that $u(a) \in \widehat{c_0}$ for any $a \in I_1$. ■

Now, we consider the solvability of the problem (4)-(5) in the Banach space $\widehat{\ell_p}$.

Theorem 3.5.

Consider the infinite system of ordinary differential equations (4) with (5). Suppose that

(i) $u_0 = (u_n^0) \in \widehat{\ell_p}$,

(ii) the mapping $h = (h_1, h_2, ...) : I \times \widehat{\ell_p} \to \widehat{\ell_p}$ is continuous,

(iii) there exist functions $w_i, t_i : I \to [0, \infty)$ for every $i \in \mathbb{N}$ such that

$$|h_i(a, u_1, u_2, ...)|^p \leq w_i(a) + t_i(a) \left| r u_i + s u_{i-1} \right|^p,$$

for $a \in I$, $u = (u_n) \in \widehat{\ell_p}$,

(iv) w_i is continuous on I and the the series $\sum\limits_{i=0}^{\infty} w_i(a)$ converges uniformly on I,

(v) the sequence $(w_i(a))$ is equibounded on I and the function $t(a) = \limsup_{i\to\infty} t_i(a)$ is integrable on I.

Then, the problem (4)-(5) has a solution $u(a) = (u_i(a))$ defined on the interval $I = [0, A]$ whenever $T_1 A < 2^{-(p+1)}$, where

$$T_1 = \sup \left\{ t_i(a) : a \in I,\ i = 0, 1, 2, ... \right\}.$$

Also, $u(a) \in \widehat{\ell_p}$ for any $a \in I$.

Proof:

For any $u(a) \in \widehat{\ell_p}$ and $a \in I$, we have

$$\begin{aligned}
\|h(a,u)\|_{\widehat{\ell_p}}^p &= \sum_{i=0}^{\infty} |r h_i(a,u) + s h_{i-1}(a,u)|^p \\
&\leq \sum_{i=0}^{\infty} 2^p \left(|r h_i(a,u)|^p + |s h_{i-1}(a,u)|^p \right) \\
&\leq |2s|^p \sum_{i=0}^{\infty} \left(|h_i(a,u)|^p + |h_{i-1}(a,u)|^p \right) \\
&\leq |2s|^p \sum_{i=0}^{\infty} \left(w_i(a) + t_i(a) \left| r u_i + s u_{i-1} \right|^p \right) + |2s|^p \sum_{i=0}^{\infty} \left(w_{i-1}(a) + t_{i-1}(a) \left| r u_{i-1} + s u_{i-2} \right|^p \right)
\end{aligned}$$

$$\begin{aligned}
&\leq |2s|^p \left(\sum_{i=0}^{\infty} w_i(a) + \sup_{i\geq 0}(t_i(a)) \sum_{i=0}^{\infty} |ru_i + su_{i-1}|^p \right) \\
&+ |2s|^p \left(\sum_{i=0}^{\infty} w_{i-1}(a) + \sup_{i\geq 0}(t_{i-1}(a)) \sum_{i=0}^{\infty} |ru_{i-1} + su_{i-2}|^p \right) \\
&= 2|2s|^p \left(\sum_{i=0}^{\infty} w_i(a) + \sup_{i\geq 0}(t_i(a)) \sum_{i=0}^{\infty} |ru_i + su_{i-1}|^p \right) \\
&\leq W + T \left\| u \right\|_{\widehat{\ell_p}}^p ,
\end{aligned}$$

where $W = 2|2s|^p \sup_{a\in I} \sum_{i=0}^{\infty} w_i(a)$ and $T = 2|2s|^p T_1$.

Now, let us take the closed ball $\overline{\mathcal{B}}(u_0, r)$ in $\widehat{\ell_p}$, where $r = (WA + TA \left\| u_0 \right\|_{\widehat{\ell_p}}^p)/(1 - TA)$.

Consider the operator $h = (h_i)$ on the set $I \times \overline{\mathcal{B}}(u_0, r)$ and let Y be subset of $\overline{B}(u_0, r)$. Then, we get

$$\begin{aligned}
\chi(h(a, Y)) &= \lim_{n\to\infty} \sup_{u\in Y} \left(\sum_{i\geq n} |rh_i(a, u) + sh_{i-1}(a, u)|^p \right) \\
&\leq |2s|^p \lim_{n\to\infty} \sup_{u\in Y} \left(\sum_{i\geq n} (|h_i(a, u)|^p + |h_{i-1}(a, u)|^p) \right) \\
&\leq |2s|^p \lim_{n\to\infty} \left(\sum_{i\geq n} w_i(a) + \sum_{i\geq n} w_{i-1}(a) \right) \\
&+ |2s|^p \lim_{n\to\infty} \left(\sup_{i\geq n} t_i(a). \sup_{(u_i)\in Y} \left[\sum_{i\geq n} |ru_i + su_{i-1}|^p \right] \right) \\
&+ |2s|^p \lim_{n\to\infty} \left(\sup_{i\geq n} t_{i-1}(a). \sup_{(u_i)\in Y} \left[\sum_{i\geq n} |ru_{i-1} + su_{i-2}|^p \right] \right) .
\end{aligned}$$

It follows from assumptions (i)-(v) and Lemma 3.3 (i) that

$$\chi(h(a, Y)) \leq t_1(a)\chi(Y),$$

where $t_1(a) = 2|2s|^p t(a)$. This says that the operator h satisfies (3) of Theorem 3.1 and Remark. Thus, we obtain that there exists a solution $u = u(a)$ of problem (4)-(5) such that $u(a) \in \widehat{\ell_p}$ for any $a \in I$. ∎

Example 3.6.

Let us consider the infinite system of differential equations

$$u_i^{'} = u_i; \quad u_i(0) = \frac{1 + (-1)^i}{2}, \tag{8}$$

for $i = 0, 1, 2, \ldots$.

It is easily seen that the solution of (8) has the form

$$(u(a)) = \left(\frac{1+(-1)^i}{2}e^a\right),$$

on the interval $I = [0, A]$.

Now, for every $a \in I$, we will show that $u(a) \notin \widehat{c_0}$. Let $a \in I$. Then, we have

$$|ru_i(a) + su_{i-1}(a)| = |r|e^a,$$

for $i = 0, 2, 4, ...$ Also, for $i = 1, 3, 5, ...,$ we have

$$|ru_i(a) + su_{i-1}(a)| = |s|e^a.$$

Thus, we get $\lim_{i\to\infty} |ru_i(a) + su_{i-1}(a)| \neq 0$ for every $a \in I$. This shows that $u(a) \notin \widehat{c_0}$ for every $a \in I$. Therefore, the sequence space $\widehat{c_0}$ is not suitable to consider solvability of problem (8) in this space. Indeed, this situation appears quite naturally since the initial point $(u_i^0) = \left(\frac{1+(-1)^i}{2}\right)$ is not in the space $\widehat{c_0}$.

Corollary 3.7.

The problem in the above example has no solution in the space $\widehat{\ell_p}$.

Proof:

The proof is clear since the inclusion $\widehat{\ell_p} \subset \widehat{c_0}$ holds. ■

Example 3.8.

Let us consider the infinite system of differential equations

$$u_i^{'} = u_i; \quad u_i(0) = (-s/r)^i/r, \tag{9}$$

for $i = 0, 1, 2, ...$ and take the interval $I = [0, A]$. Then, the solution of (9) has the form

$$u(a) = (u(a)) = \left(((-s/r)^i/r)e^a\right) = ((1/r)e^a, (-s/r^2)e^a, (s^2/r^3)e^a, ...),$$

for every $a \in I$. Also, for every $a \in I$, we have that

$$|ru_i(a) + su_{i-1}(a)| = e^a \left|r(-s/r)^i/r + s(-s/r)^{i-1}/r\right| = 0,$$

for $i \geq 1$ and $|ru_i(a) + su_{i-1}(a)| = e^a$ for $i = 0$. Thus, the problem (9) has a solution in the space $\widehat{\ell_p}$ (or $\widehat{c_0}$). On the other hand, the initial condition $(u_i^0) = ((-s/r)^i/r)$ is not in c_0 (and so ℓ_p). Thus, the problem (9) has no solution in the spaces c_0 and ℓ_p according to Theorem 3 by Banaś and Lecko (2001) and Theorem 3.1 by Mursaleen and Mohiuddine (2012).

4. Conclusion

This study deals with the solution for an infinite system of ordinary differential equations with initial condition in some Banach sequence spaces. For this purpose, it is adopted the technique of measures of noncompactness. Also, it is observed with an example that this problem has a solution in the generalized Banach spaces $\widehat{\ell_p}$ or $\widehat{c_0}$ but it has no solution in the classical Banach spaces ℓ_p or c_0.

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