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# Transient thermal stresses due to axisymmetric heat supply in a semi-infinite thick circular plate

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## Abstract

The present paper deals with the determination of thermal stresses in a semi-infinite thick circular plate of a finite length and infinite extent subjected to an axisymmetric heat supply. A thick circular plate is considered having constant initial temperature and arbitrary heat flux is applied on the upper and lower face. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in terms of Bessel's function. The thermoelastic behavior has been computed numerically and illustrated graphically for a steel plate.

**Keywords:** Heat Conduction; Michell's function; Thermal Stresses; Thick Plate; Goodier's thermoelastic potential; Thermoelasticity

**MSC 2010 No.:** 30E25, 35K05, 44A10, 74F05, 35A22

## 1. Introduction

Nowacki (1957) has determined steady state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Roy choudhary (1972, 1973) and Wankhede (1982) determined Quasi-static thermal stresses in thin circular plate. Gogulwar et al. (2005) determined thermal stresses in thin circular plate with heat source. Also, Tikhe et al. (2005) studied transient thermoelastic deformation in a thin circular plate. Qian and Batra (2004) studied transient thermoelastic deformation of thick functionally graded plate. Moreover, Sharma et al. (2004) studied the behavior of themoelastic thick plate under lateral loads an obtained the results for radial and axial displacements and temperature change have been computed numerically and illustrated graphically for different theories of generalized thermoelasticity. Also EI-Maghraby (2004, 2005) solved two-dimensional problem of thick plate with heat sources in generalized thermoelasticity. Kulkarni et al. (2007) has determined quasi-static thermal stresses in a thick circular plate. The problem of determination of thermal stresses in a semi-infinite solid with a finite area of plane boundary subjected to different temperature distribution has been studied by number of authors. Kulkarni et al. (2008) has studied the quasi-static thermal stresses in thick circular plate under study temperature field. Kedar et al. (2011) has studied the estimation of temperature distribution and thermal stresses in a thick circular plate.

The present paper deals with a thick plate of thickness $2b$ occupying space $D$ defined by $0 \le r < \infty, -b \le z \le b$. The plate is subjected to a transient axisymmetric temperature field dependent on the radial and axial directions of the cylindrical co-ordinate system. The initial temperature $T_i(r,z)$ in the thick plate is given by a constant temperature $T_i$ and the heat flux $QF(r)$ is prescribed on the upper and lower boundary surfaces. Under these conditions, the thermoelasticity in a semi-infinite thick circular cylinder are required to determine.

The result presented here will be useful in engineering problems particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. The missile skill material is assumed to have physical properties independent of temperature, so that the temperature $T(r,z,t)$ is a function of radius, thickness and time only.

## 2. Formulation of a problem

The differential equation governing the displacement potential function $\phi(r,z,t)$ is given by Noda (2003)

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=K\tau \, , \tag{1}$$

where $K$ is the restraint coefficient and temperature change $\tau = T - T$, $T_i$ is initial temperature, displacement function $\phi$ is known as Goodier's thermoelastic potential.

The temperature of the cylinder at time $t$ satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial T}{\partial t} \, , \tag{2}$$

with the boundary conditions

$$\lambda \frac{\partial T}{\partial z} = \pm Q F(r), \qquad \text{on} \quad z = \pm b, \tag{3}$$

$$T = T_i, \qquad \text{at} \quad r \to \infty, \tag{4}$$

and initial condition

$$T = T_i, \qquad \text{at} \quad t = 0, \tag{5}$$

where $k$ is the thermal diffusivity of the material of the cylinder.

The displacement function in the cylindrical co-ordinate system are represented by the Michell's function is given by Noda (2003)

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z}, \tag{6}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}. \tag{7}$$

The Michell's function $M$ must satisfy

$$\nabla^2 \nabla^2 M = 0, \tag{8}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{9}$$

The components of the stresses are represented by the thermoelastic displacement potential $\phi$ and Michell's function $M$ as

$$\sigma_{rr} = 2G\left[\frac{\partial^2 \phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right], \tag{10}$$

$$\sigma_{\theta\theta} = 2G\left[\frac{1}{r}\frac{\partial \phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left(\nu \nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right], \tag{11}$$

$$\sigma_{zz} = 2G\left[\frac{\partial^2 \phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left(\left[2-\nu\right]\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right], \tag{12}$$

$$\sigma_{rz} = 2G\left[\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left(\left[1-\nu\right]\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right], \tag{13}$$

where $G$ and $\nu$ are the shear modulus and Poisson's ratio respectively, and the stresses are traction free on upper and lower surfaces i.e.,

$$\sigma_{zz} = \sigma_{rz} = 0\,, \qquad \text{at} \qquad z = \pm\, b\,. \tag{14}$$

Equations (1) to (14) constitute mathematical formulation of the problem.

## 3. The solution of the problem

Introducing the temperature change $\tau = T - T_i$ , into equations (1) to (4) and applying the integral transform and their inversions defined in Ozisik (1968), one obtains the expression of temperature change as

$$\tau(r,z,t) = \frac{Q}{\lambda}\int_0^\infty \overline{F}(\beta) J_0(\beta r) \times \left[ \frac{\cosh\beta z}{\sinh\beta b} - \frac{1}{\beta b} e^{-k\beta^2 t} + 2\beta b \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos n\pi\left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)} \right] d\beta\,. \tag{15}$$

Now assume Michell's function $M$ which satisfies condition (8) as,

$$M = \left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r)\left[B(\beta)\sinh(\beta z) + C(\beta)\beta z\cosh(\beta z)\right] d\beta\,, \tag{16}$$

where $B(\beta)$ and $C(\beta)$ are the arbitrary functions, which can be determined by using condition (14).

To obtain displacement potential $\phi$ using equation (15) in equation (5), one has

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = \left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r) \times \left[ \frac{\cosh\beta z}{\sinh\beta b} - \frac{1}{\beta b} e^{-k\beta^2 t} + 2\beta b \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos n\pi\left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)} \right] d\beta\,. \tag{17}$$

Considering first term of equation (17) as

$$\frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_1}{\partial r} + \frac{\partial^2 \phi_1}{\partial z^2} = \left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r)\left[\frac{\cosh \beta z}{\sinh \beta b}\right] d\beta . \tag{18}$$

To solve equation (18), assume $\phi_1$ which satisfies equation (5), as

$$\phi_1 = \int_{\beta=0}^{\infty} D(\beta) J_0(\beta r)\left[z \sinh \beta z\right] d\beta . \tag{19}$$

Using equation (19) in equation (18), one obtains

$$D(\beta) = \left(\frac{QK}{\lambda}\right)\frac{\overline{F}(\beta)}{2\beta \sinh \beta b} . \tag{20}$$

Hence,

$$\phi_1 = \left(\frac{QK}{2\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r)\left[\frac{z \sinh \beta z}{\beta \sinh \beta b}\right] d\beta . \tag{21}$$

Now considering second and third term of equation (17) as

$$\frac{\partial^2 \phi_2}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_2}{\partial r} + \frac{\partial^2 \phi_2}{\partial z^2} = \left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r) \times \left[ -\frac{1}{\beta b} e^{-k\beta^2 t} + 2\beta b \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos n\pi \left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2 + \beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2 + \beta^2 b^2)} \right] d\beta . \tag{22}$$

To solve equation (22) using

$$\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \approx \frac{1}{k}\frac{\partial}{\partial t}, \tag{23}$$

in equation (22) and on integrating w.r.to $t$, one obtains

$$\phi_2 = \left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta)\ J_0(\beta r) \times \left[ \frac{1}{\beta^3 b} e^{-k\beta^2 t} + 2\beta b^3 \sum_{n=1}^{\infty} \frac{(-1)^{n} \cos n\pi \left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2 + \beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2 + \beta^2 b^2)^2} \right] d\beta . \tag{24}$$

Finally $\phi = \phi_1 + \phi_2$,

$$\phi = \left(\frac{QK}{\lambda}\right) \int_{\beta=0}^{\infty} \overline{F}(\beta) J_0(\beta r)$$

$$\times \left[ \frac{z \sinh \beta z}{2\beta \sinh \beta b} + \frac{1}{\beta^3 b} e^{-k\beta^2 t} + 2\beta b^3 \sum_{n=1}^{\infty} \frac{(-1)^n \cos n\pi \left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2 + \beta^2 b^2)^2} \right] d\beta \,. \qquad (25)$$

Now using equations (15), (16) and (25) in equations (6), (7) and (10) to (13), one obtains the expressions for displacements and stresses respectively as

$$u_r = \left(\frac{QK}{\lambda}\right) \int_{\beta=0}^{\infty} \overline{F}(\beta) J_1(\beta r) \left\{ B(\beta)\beta \cosh \beta z + C(\beta)\beta \left[\cosh \beta z + \beta z \sinh \beta z\right] \right.$$

$$\left. - \left[ \frac{z \sinh \beta z}{2\beta \sinh \beta b} + \frac{1}{\beta^3 b} e^{-k\beta^2 t} + 2\beta b^3 \sum_{n=1}^{\infty} \frac{(-1)^n \cos n\pi \left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2 + \beta^2 b^2)^2} \right] \right\} d\beta \,, \qquad (26)$$

$$u_z = -\left(\frac{QK}{\lambda}\right) \int_{\beta=0}^{\infty} \overline{F}(\beta) J_1(\beta r) \left\{ B(\beta)\beta^2 \sinh \beta z - C(\beta)\beta^2 \left[2(1-2v)\sinh \beta z - \beta z \cosh \beta z\right] \right.$$

$$\left. - \frac{\left[\sinh \beta z + \beta z \cosh \beta z\right]}{2\beta \sinh \beta b} + 2\beta b^2 \pi \sum_{n=1}^{\infty} \frac{n(-1)^n \sin n\pi \left(\frac{z}{b}\right) e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2 + \beta^2 b^2)^2} \right\} d\beta \,, \qquad (27)$$

$$\sigma_{rr} = \left(\frac{2QKG}{\lambda}\right) \int_{\beta=0}^{\infty} \overline{F}(\beta) \left( B(\beta)\beta^3 \left[ J_0(\beta r) - \frac{J_1(\beta r)}{\beta r} \right] \cosh \beta z \right.$$

$$+ C(\beta)\beta^3 \left\{ 2vJ_0(\beta r) \cosh \beta z + \left[ J_0(\beta r) - \frac{J_1(\beta r)}{\beta r} \right] (\cosh \beta z + \beta z \sinh \beta z) \right\}$$

$$- \left[ J_0(\beta r) - \frac{J_1(\beta r)}{\beta r} \right] \frac{\beta z \sinh \beta z}{2 \sinh \beta b} - J_0(\beta r) \frac{\cosh \beta z}{\sinh \beta b} + \frac{J_1(\beta r)}{\beta r \beta b} e^{-k\beta^2 t}$$

$$\left. + 2\beta b \sum_{n=1}^{\infty} \frac{(-1)^n \cos n\pi \left(\frac{z}{b}\right)}{(n^2\pi^2 + \beta^2 b^2)^2} \left[ \frac{\beta^2 b^2}{\beta r} J_1(\beta r) + n^2\pi^2 J_0(\beta r) \right] e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]} \right) d\beta \,, \qquad (28)$$

$$\sigma_{\theta\theta} = \left(\frac{2QKG}{\lambda}\right)\int_{\beta=0}^{\infty} \overline{F}(\beta)\Bigg( B(\beta)\beta^3 \frac{J_1(\beta r)}{\beta r}\cosh\beta z$$

$$+ C(\beta)\beta^3\left\{2\nu J_0(\beta r)\cosh\beta z + \frac{J_1(\beta r)}{\beta r}(\cosh\beta z + \beta z\sinh\beta z)\right\}$$

$$- J_0(\beta r)\frac{\cosh\beta z}{\sinh\beta b} - \frac{J_1(\beta r)}{\beta r}\frac{\beta z\sinh\beta z}{2\sinh\beta b} + \frac{1}{\beta b}\left[J_0(\beta r) - \frac{J_1(\beta r)}{\beta r}\right]e^{-k\beta^2 t}$$

$$+ 2\beta b\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\pi\left(\frac{z}{b}\right)}{(n^2\pi^2+\beta^2 b^2)^2}$$

$$\times\left[\left(n^2\pi^2+\beta^2b^2\right)J_0(\beta r) - \frac{\beta^2 b^2}{\beta r}J_1(\beta r)\right]e^{-k\left[(n^2\pi^2+\beta^2b^2)\frac{t}{b^2}\right]}\Bigg)d\beta, \tag{29}$$

$$\sigma_{zz} = \left(\frac{2QKG}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)J_0(\beta r)\Bigg\{ -B(\beta)\beta^3\cosh\beta z + C(\beta)\beta^3\left[(1-2\nu)\cosh\beta z - \beta z\sinh\beta z\right]$$

$$+\frac{\beta z\cosh\beta z}{2\sinh\beta b} + \frac{1}{\beta b}e^{-k\beta^2 t} + 2\beta^3 b^3\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\pi\left(\frac{z}{b}\right)}{(n^2\pi^2+\beta^2b^2)^2}e^{-k\left[(n^2\pi^2+\beta^2b^2)\frac{t}{b^2}\right]}\Bigg\}d\beta, \tag{30}$$

$$\sigma_{rz} = \left(\frac{2QKG}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)J_1(\beta r)\Bigg\{ B(\beta)\beta^3\sinh\beta z + C(\beta)\beta^3\left[2\nu\sinh\beta z + \beta z\cosh\beta z\right]$$

$$-\frac{\left[\sinh\beta z + \beta z\cosh\beta z\right]}{2\sinh\beta b} + 2\beta^2 b^2\pi\sum_{n=1}^{\infty}\frac{(-1)^n n\sin n\pi\left(\frac{z}{b}\right)}{(n^2\pi^2+\beta^2b^2)^2}e^{-k\left[(n^2\pi^2+\beta^2b^2)\frac{t}{b^2}\right]}\Bigg\}d\beta. \tag{31}$$

In order to satisfy condition (14), solving equations (30) and (31) for $B(\beta)$ and $C(\beta)$, one obtains

$$B(\beta) = \frac{1-2\nu}{2\beta^3\sinh\beta b} + \frac{\left[2\nu\sinh\beta b + \beta b\cosh\beta b\right]}{\beta^3(\sinh\beta b\cosh\beta b + \beta b)}$$

$$\times\left[\frac{1}{\beta b}e^{-k\beta^2 t}+2\beta^3 b^3\sum_{n=1}^{\infty}\frac{e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right], \tag{32}$$

$$C(\beta)=\frac{1}{2\beta^3\sinh\beta b}-\frac{\sinh\beta b}{\beta^3(\sinh\beta b\cosh\beta b+\beta b)}$$

$$\times\left[\frac{1}{\beta b}e^{-k\beta^2 t}+2\beta^3 b^3\sum_{n=1}^{\infty}\frac{e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right]. \tag{33}$$

Using these values of $B(\beta)$ and $C(\beta)$ in equations (26) to (31), one obtain the expression for displacements and stresses respectively as

$$u_r=\left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)J_1(\beta r)\frac{1}{\beta}$$

$$\times\left\{\frac{(1-v)\cosh\beta b}{\sinh\beta b}-\frac{\beta b}{2}-\frac{\sin^2\text{h}\,\beta b}{\sinh\beta b\cosh\beta b+\beta b}\frac{1}{2}\left[2(1+2v)+\beta b\right]\right.$$

$$\left.\times\left[\frac{1}{\beta^3 b}e^{-k\beta^2 t}+2\beta b^3\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\pi\left(\frac{z}{b}\right)e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right]\right\}d\beta\,, \tag{34}$$

$$u_z=-\left(\frac{QK}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)J_0(\beta r)\left\{\frac{(1-2v)(\beta-2)}{2\beta}+\frac{2(1-v)\sin^2\text{h}\,\beta b}{\beta\left(\sinh\beta b\cosh\beta b+\beta b\right)}\right.$$

$$\left.\times\left[\frac{1}{\beta^3 b}e^{-k\beta^2 t}+2\beta b^3\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\pi\left(\frac{z}{b}\right)e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right]\right\}d\beta\,, \tag{35}$$

$$\sigma_{rr}=\left(\frac{2QKG}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)\left(2(v-1)\frac{J_1(\beta r)}{\beta r}\frac{\cosh\beta b}{\sinh\beta b}-vJ_0(\beta r)\cosh\beta b\right.$$

$$+\left[\frac{\beta b J_0(\beta r)+(1-2v)\sinh\beta b\cosh\beta b\,\dfrac{J_1(\beta r)}{\beta r}}{\sinh\beta b\cosh\beta b+\beta b}\right]\times\left[\frac{1}{\beta b}e^{-k\beta^2 t}+2\beta^3 b^3\sum_{n=1}^{\infty}\frac{e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right]$$

$$+\frac{J_1(\beta r)}{\beta r\beta b}e^{-k\beta^2 t}$$

$$+2\beta b\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\pi}{(n^2\pi^2+\beta^2 b^2)^2}\left[\frac{\beta^2 b^2}{\beta r}J_1(\beta r)+n^2\pi^2 J_0(\beta r)\right]e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}\Bigg)d\beta\,, \tag{36}$$

$$\sigma_{\theta\theta}=\left(\frac{2QKG}{\lambda}\right)\int_{\beta=0}^{\infty}\overline{F}(\beta)\left((v-1)\left[J_0(\beta r)-\frac{J_1(\beta r)}{\beta r}\right]\frac{\cosh\beta b}{\sinh\beta b}\right.$$

$$+\left[\frac{\beta b}{\sinh\beta b\cosh\beta b+\beta b}\frac{J_1(\beta r)}{\beta r}+\frac{(2v-1)\sinh\beta b\cosh\beta b}{\sinh\beta b\cosh\beta b+\beta b}\frac{J_1(\beta r)}{\beta r}\right.$$

$$\left.-\frac{2v\sinh\beta b\cosh\beta b J_0(\beta r)}{\sinh\beta b\cosh\beta b+\beta b}\right]\times\left[\frac{1}{\beta b}e^{-k\beta^2 t}+2\beta^3 b^3\sum_{n=1}^{\infty}\frac{e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}}{(n^2\pi^2+\beta^2 b^2)^2}\right]$$

$$+\frac{1}{\beta b}\left[J_0(\beta r)-\frac{J_1(\beta r)}{\beta r}\right]e^{-k\beta^2 t}+\ 2\beta b\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{(n^2\pi^2+\beta^2 b^2)^2}\times(n^2\pi^2+\beta^2 b^2)$$

$$\times\left[J_0(\beta r)-\frac{\beta^2 b^2}{\beta r}J_1(\beta r)\right]e^{-k\left[(n^2\pi^2+\beta^2 b^2)\frac{t}{b^2}\right]}\Bigg)d\beta\,, \tag{37}$$

$$\sigma_{zz}=0\,, \tag{38}$$

$$\sigma_{rz}=0\,. \tag{39}$$

## 4. Numerical Calculations and discussion

For sake of simplicity we consider $f(r)=e^{-\omega r},\ \omega>0$ , here we consider the function $f(r)$ which falls of exponentially as one moves away from the center of the circular plate in the radial direction.

Mathematical model is prepared with steel plate for purposes of numerical computations. The dimensions and material properties are given below

Radius $r = 0, 10, 20, 30, 40, 50, \cdots \to \infty$ in meter, Thickness $-5 \le z \le 5$, i.e., $b = 5\ m$,

Thermal diffusivity $k = 15.9 \times 10^{-6}\ m^2 s^{-1}$, $t = 5$ sec, $\beta = 100$, $n = 100$, $\omega = 5$.

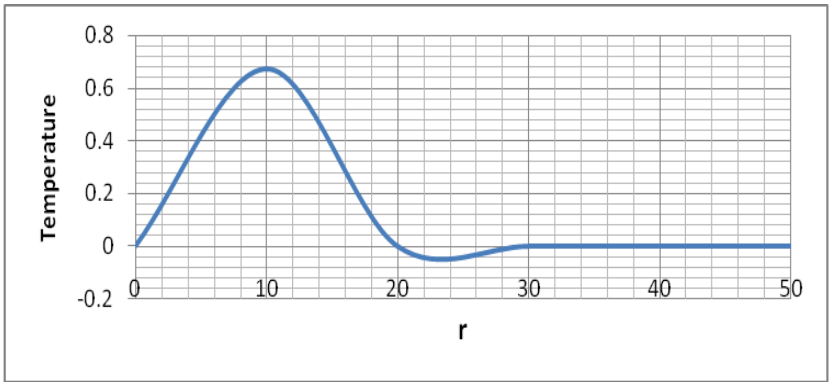


**Figure 1.** Temperature in radial direction

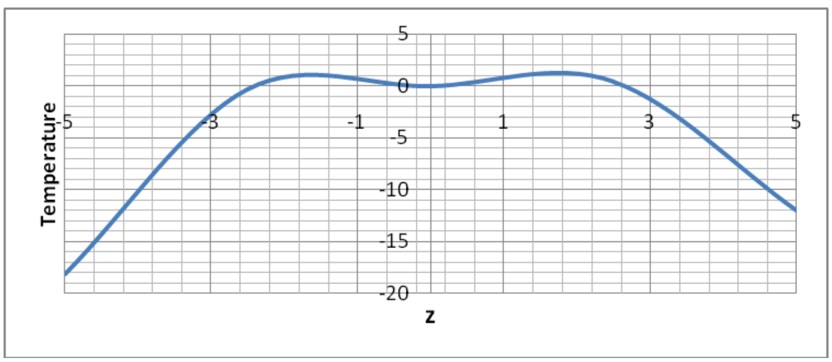


**Figure 2.** Temperature in axial direction

From Figure 1**,** it is observed that, temperature variation taken place within $0 \le r \le 20$ in the radial direction and then, it becomes stationary, whereas in Figure 2 temperature variation shown in the axial direction.

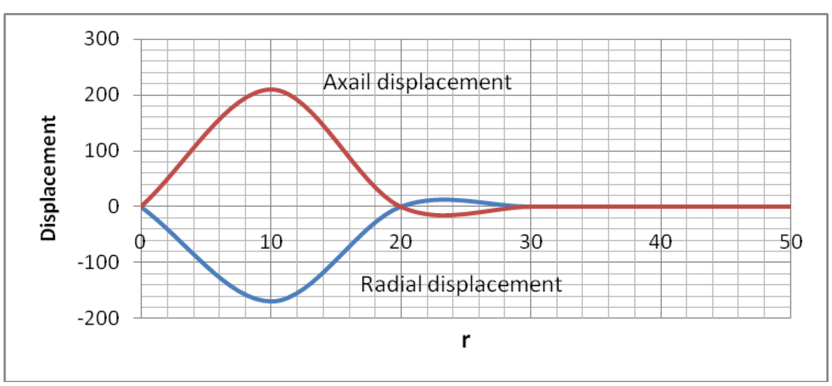


**Figure 3.** Displacement in radial direction

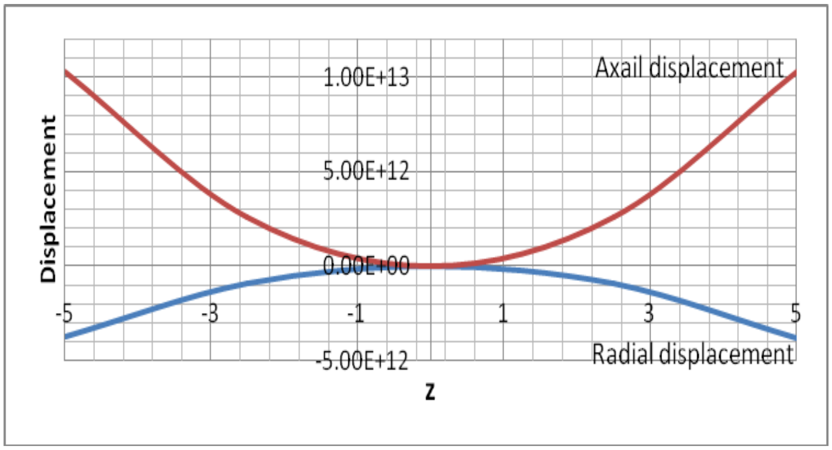


**Figure 4.** Displacement in axial direction

From Figure 3 and 4, it is seen that, displacement is proportional to temperature variation within $0 \le r \le 20$. Also, displacement takes place on upper and lower surface of thick circular plate, axisymmetrically in axial direction.

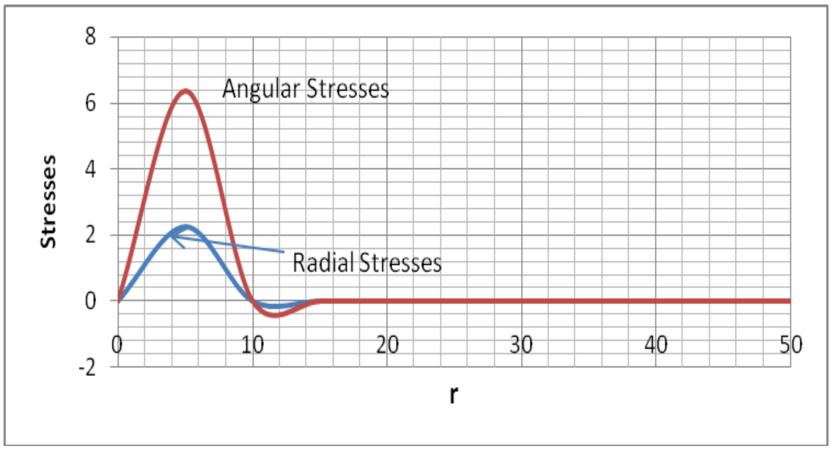


**Figure 5.** Thermal stresses in radial direction

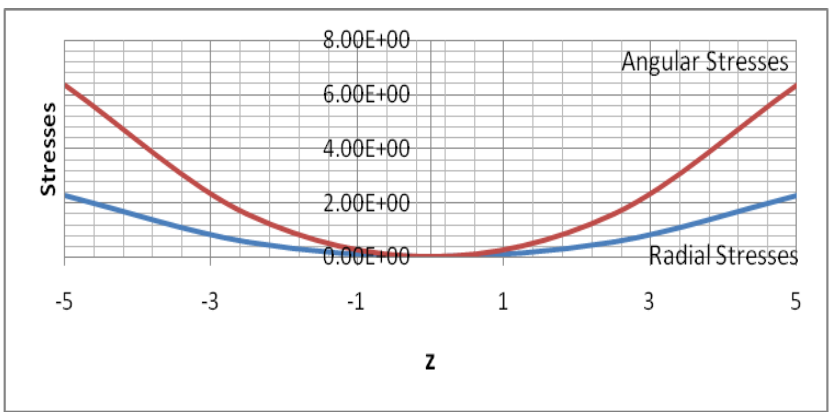


**Figure 6.** Thermal stresses in axial direction

From Figure 5 and 6**,** it is observed that, radial and angular stresses develops tensile stresses within $0 \le r \le 10$ and the stresses develops tensile stresses on upper and lower surface of thick circular plate, axisymmetrically.

## 5. Conclusion

In this paper a semi infinite thick circular plate is considered subjected to a transient axisymmetric temperature field on the radial and axial directions of the cylindrical body and determined the expressions for temperature, displacements and stress functions, due to axisymmetric heat supply. As a special case a mathematical model is constructed for $f(r) = e^{-\omega r}$ and performed numerical calculations. The thermoelastic behavior is examined with the help of axisymmetric heat supply on the upper and lower surface of plate are shown.

It is observed that, the effect of temperature, displacement, stresses are negligible for an infinite region within circular region, whereas effects of temperature, displacement, stresses are zero for an infinite extent. Also, it is observed that temperature, displacement, stresses are shown zero at middle surface and away due to axisymmetric heat supply.

From the numerical illustration it is concluded that temperature, displacement and stresses are proportional to each other. Stresses develop tensile stresses.

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